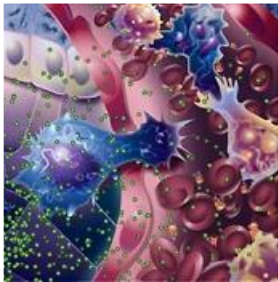
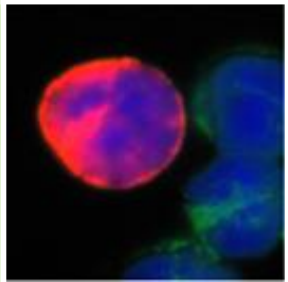
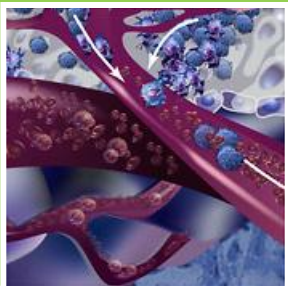


Cancer metastasis modeling at the Scripps Physical Science Oncology Center

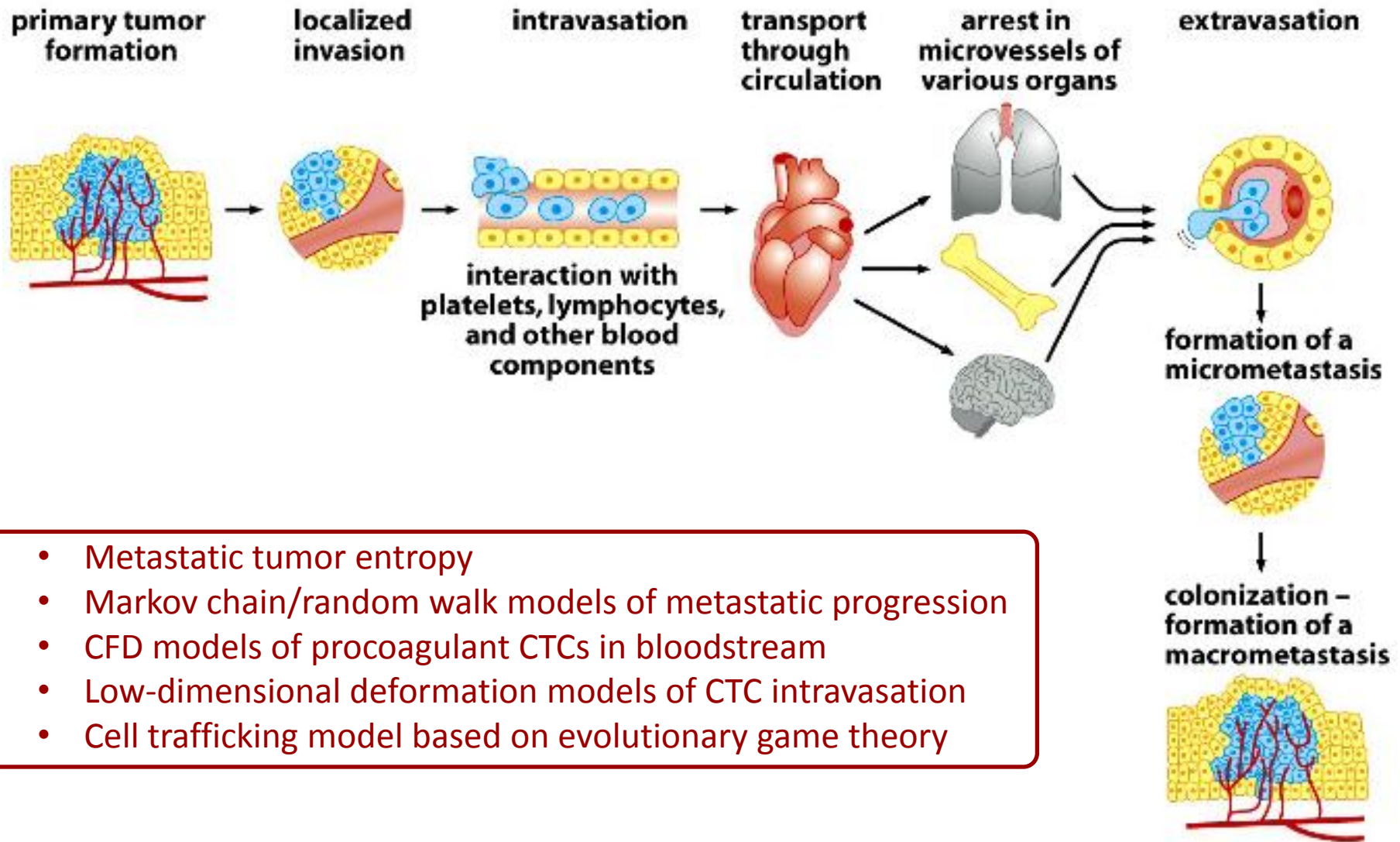
-Overview of several projects-



Paul K. Newton Ph.D.

Viterbi School of Engineering and Department of Mathematics
University of Southern California

Overview of models

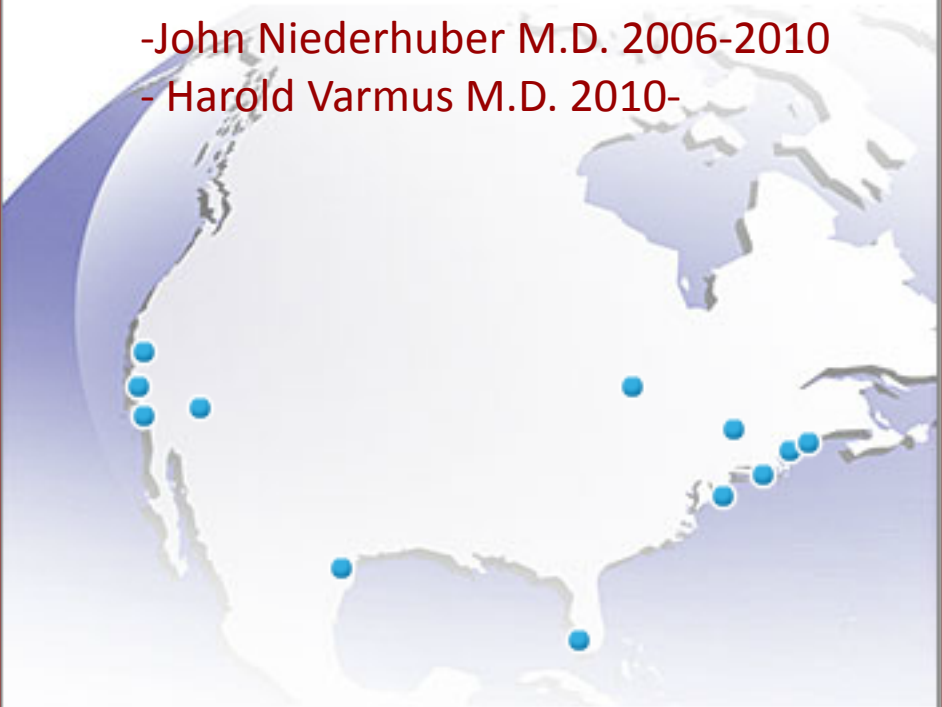


- Metastatic tumor entropy
- Markov chain/random walk models of metastatic progression
- CFD models of procoagulant CTCs in bloodstream
- Low-dimensional deformation models of CTC intravasation
- Cell trafficking model based on evolutionary game theory

Figure 14-4 The Biology of Cancer (© Garland Science 2007)

CENTERS

-John Niederhuber M.D. 2006-2010
- Harold Varmus M.D. 2010-



Physical Science-Oncology Centers

The National Cancer Institute (NCI) has awarded cooperative agreements to 12 leading institutions to build a collaborative network of Physical Science-Oncology Centers (PS-OCs).

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ology Center
a, NY

Michael L. Shuler, Ph.D.
Barbara L. Hempstead,

Center & Research

ology Center
Center & Research

bert A. Gatenby, M.D.
bert J. Gillies, Ph.D.

of Technology

ology Center
of Technology,

ott Manalis, Ph.D.
er Jacks, Ph.D.

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y, Chicago, IL

omas V. O'Halloran,
nathan D. Licht, M.D.

stitute

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stitute, La Jolla, CA

ter Kuhn, Ph.D.
lly J. Bethel, M.D.

California

ology Center
California, Los

Daniel Hillis, Ph.D.
vid B. Agus, M.D.



Scripps PS-OC: 'Mathematics and Physics of Cancer Metastasis'

Physical Sciences Oncology Center 2010-2015 National Cancer Institute

Mathematics & Physics



Peter Kuhn



Paul Newton



Owen McCarty

Oncology and Pathology



Kelly Bethel



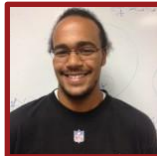
Jorge Nieva



Lyudmila Bazhenova

Collaborators

Math Modeling - USC



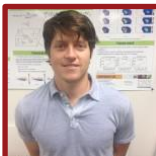
Jeremy Mason



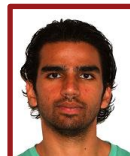
Angie Lee



Jeffrey West



Brian Hurt



Zaki Hasnain



Juliana Porter

MSKCC



Larry Norton

The 'particles' in the bloodstream

WBCs: 4,500-10,000 / μ l

neutrophils: 2,500-8,000 / μ l

lymphocytes: 1,000-4,000 / μ l

monocytes: 100-700 / μ l

eosinophils: 50-500 / μ l

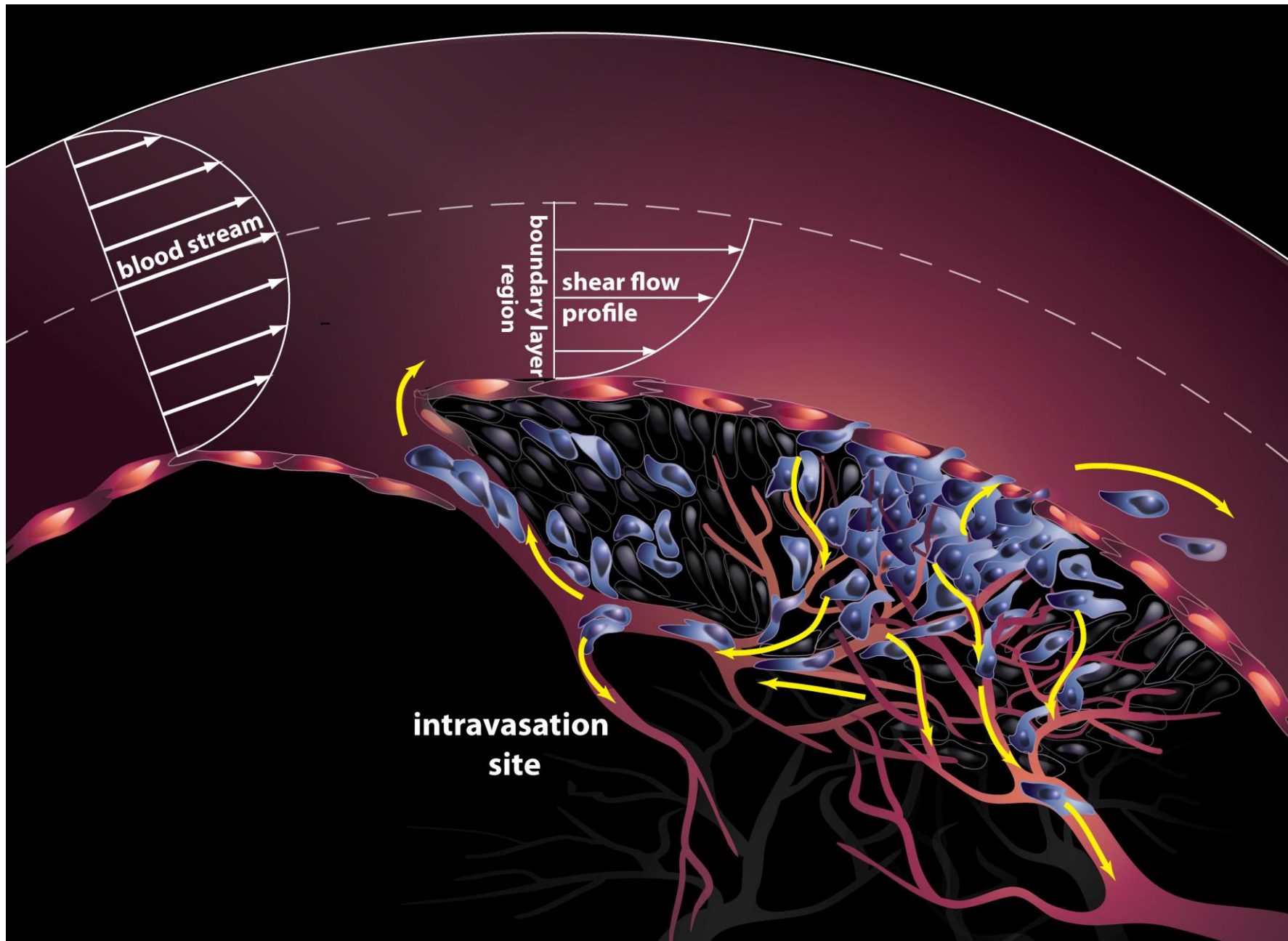
basophils: 25-100 / μ l

RBCs: 4-5 million / μ l

CTCs: 2-200 / ml

Platelets: 150-400 million / μ l

Phoenix Philms



T.R. Ashworth 1869

146

Hospital Reports.

[May,

HOSPITAL REPORTS.

MELBOURNE HOSPITAL.

A case of Cancer in which cells similar to those in the Tumours were seen in the blood after death. Reported by THOMAS RAMSDEN ASHWORTH, Resident Physician.

Richard J—, æt. 38, was admitted on Oct. 9th, 1868, suffering from what was understood to be "Rheumatism and Debility." He died of Marasmus on the 10th of the following March.

He had a number of subcutaneous tumours (about thirty) situated over the anterior wall of the thorax and abdomen, varying in size from that of a bean to that of a small orange. There was one between the scapule, and another on the inner side of the left thigh about four inches above the knee joint. In none of them had any softening as yet taken place.

These tumours on section, thirty hours after death, were found to consist of a thick opaque jelly-like substance, of an amber colour, having a thin fibrous covering, and in places a delicate fibrous stroma running through the substance of the tumours themselves. Examined by the microscope, under a power of four hundred diameters, they presented an unusual structural character. The jelly-like substance appeared perfectly transparent dotted over at nearly equal distances with what presented the appearance of cells containing one or more nuclei (Fig I.) Not being able to account for the appearance of these cells being equidistant from each other, some solution of magenta dye was added, and it was then seen that the cells were in reality only the nuclei containing nucleoli of large and beautifully pellucid cells, highly refractive, and not containing a trace of granular matter outside the nucleus. They were mostly circular in shape, a few being slightly ovoid. (Fig II.)

These tumours being so numerous, and evidently malignant, it was determined to examine the blood also, in the belief that it might possibly throw some light upon their multiplication in different parts of the body. A portion was accordingly obtained from the internal saphena vein of the right leg which was quite free from any tumour.

This blood was dark and fluid, when viewed by the microscope a little of the magenta solution being added, it was seen to contain comparatively few red corpuscles, these being mostly shrivelled and stellate, *besides* a few white corpuscles, some aggregated; together with masses, besides these some cells like white corpuscles, some larger, together with patches of granular matter which took the dye; but the most singular circumstance was that occasionally cells were seen exactly in shape, size, and appearance like to those of the tumours. On one examination three were seen in the field of view at the same time.—(Fig III.)

1869.]

District Dispensaries.

147

These cancer cells were seen not only by the narrator, but by Dr. Robertson, Dr. Moloney, and Dr. Lawrence.

One of the tumours was forwarded to Professor Halford, who expressed himself to the effect that he had never seen one of a similar character, but that it was undoubtedly a rare species of cancer, and he kindly pointed out in the English Journal of Anatomy and Physiology for May 1868, page 147, a description of a tumour of the chorda dorsalis, by Professor Turner, which if not identical in character, which was presented by Dr. Thomas Robertson to the Museum of the University of Edinburgh.

The fact of cells identical with those of the cancer itself being seen in the blood may tend to throw some light upon the mode of origin of multiple tumours existing in the same person. Whether these cells came from an existing cancer structure, or were formed in the blood itself during life, or after death, from the materies puræ which, although it has never been seen, is known to exist there, future investigation will perhaps point out. One thing is certain, that if they came from an existing cancer structure, they must have passed through the greater part of the circulatory system to be removed at the internal saphena vein of the sound leg.

Australian Medical Journal,

MAY, 1869.

DISTRICT DISPENSARIES.

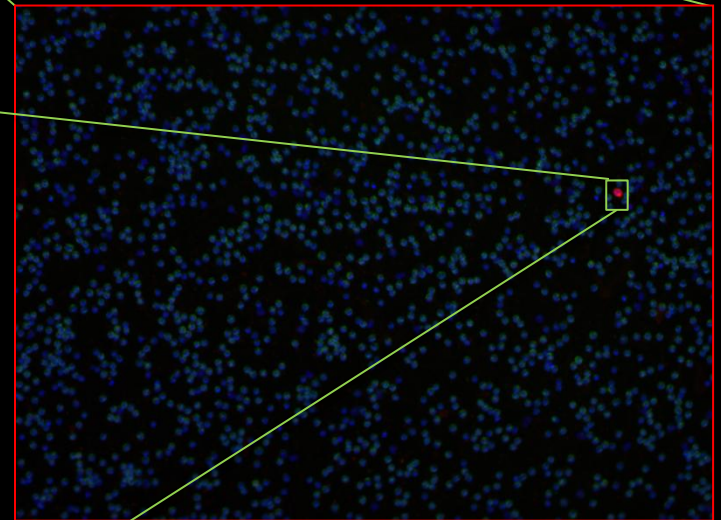
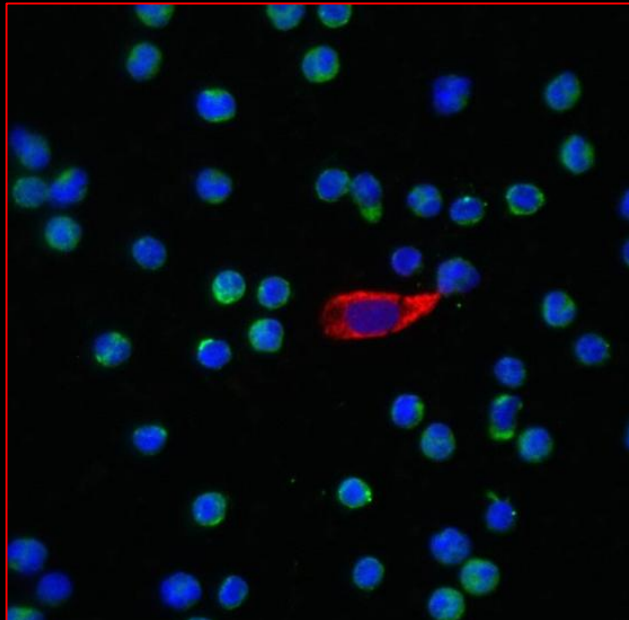
In the Report of the Sub-Committee of the Melbourne Hospital appointed to inquire into the present system of dispensing out-door relief, it is recommended, among other changes, that branch establishments should be formed in populous suburban municipalities, as a means of relieving the pressure of duties now performed at the Hospital itself, and which the report indirectly acknowledges are inefficiently performed. This extension of the operations of the leading hospital of Melbourne has long been needed. It was urged years ago by Mr. Gilbee, and if his suggestion had been adopted a good many of the abuses of this institution would have been avoided. It is simply a physical impossibility, as matters seem at present to be managed, to attend to the out-patients, as they should be attended, and not to neglect the in-patients. This would not be easy if even the honorary staff attended regularly, but as the report itself admits that there is "great irregularity" in the attendance of the honorary staff, the duty of prescribing for the out-patients

HD-CTC on a slide (Kuhn Lab TSRI)

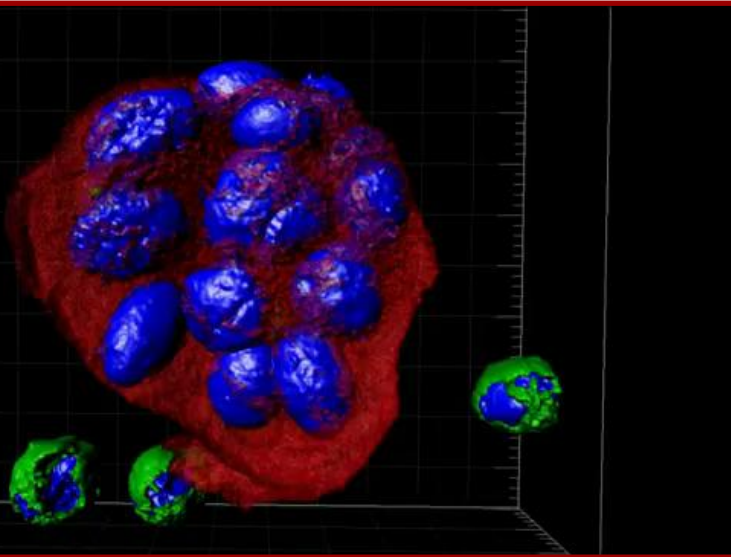


~3 Million Cells per slide

3-30 CTCs per slide

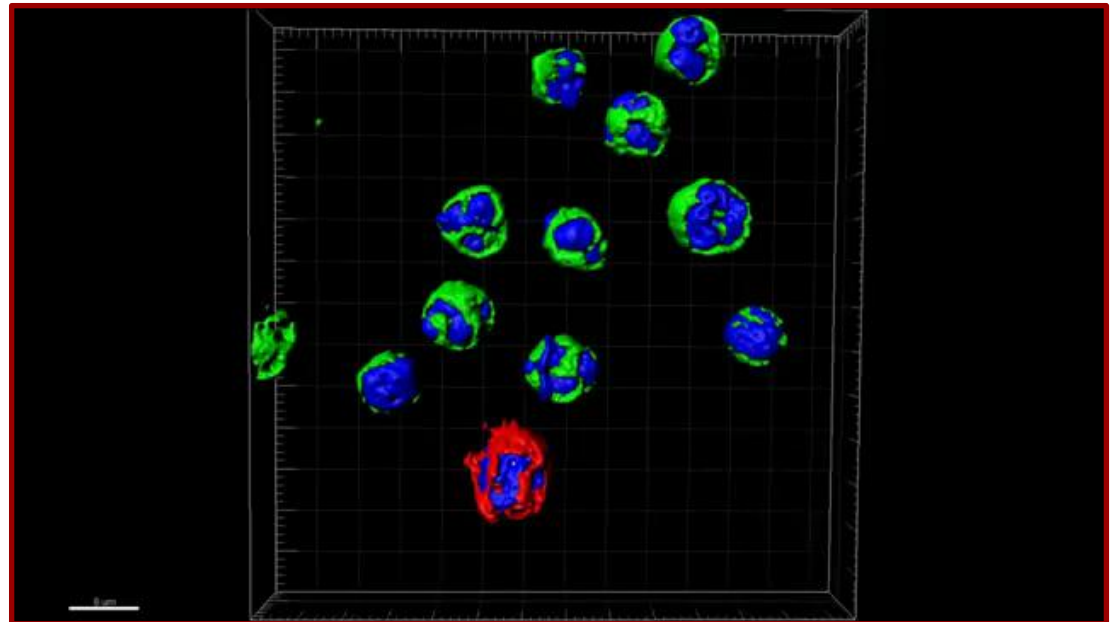


Circulating tumor cells (CTCs)



Lung CTC cluster

Prostate CTCs



Peter Kuhn Lab
The Scripps Research Institute

1. Metastatic entropy

Question: What is the best metric to use to compare the complexity of different cancers?

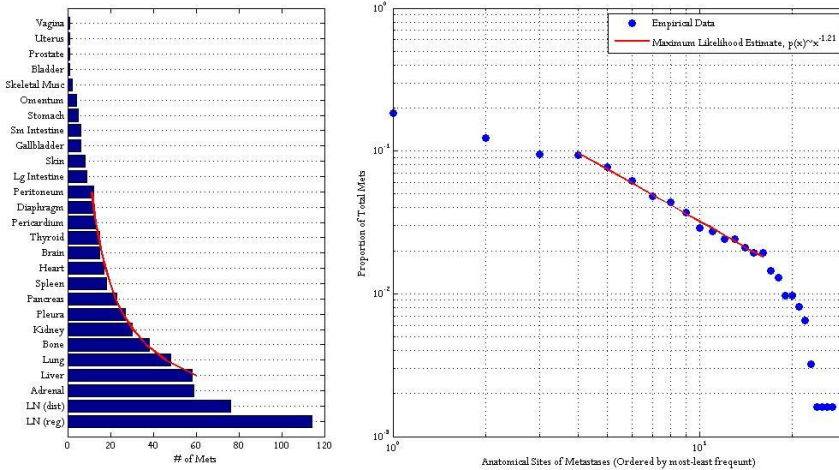
Metastatic signature of 4 cancer types



Lung

Metastatic Distribution for Primary Lung Cancer

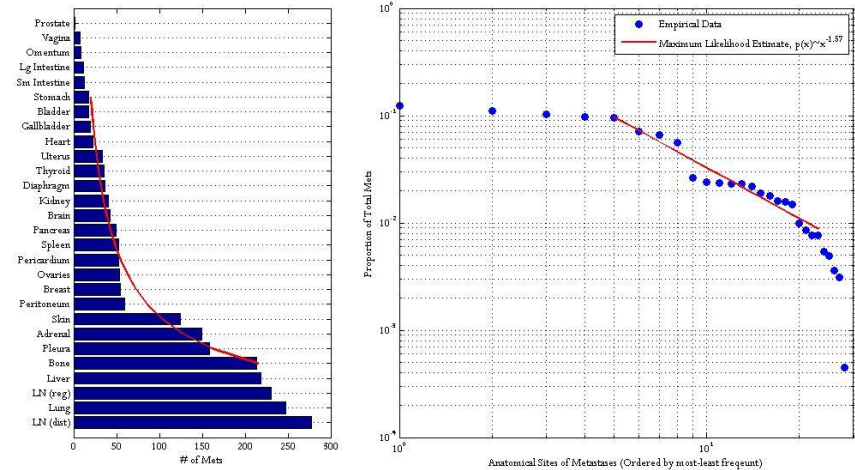
169 Patients with 619 Mets spanning 27 Anatomical Sites



Breast

Metastatic Distribution for Primary Breast Cancer

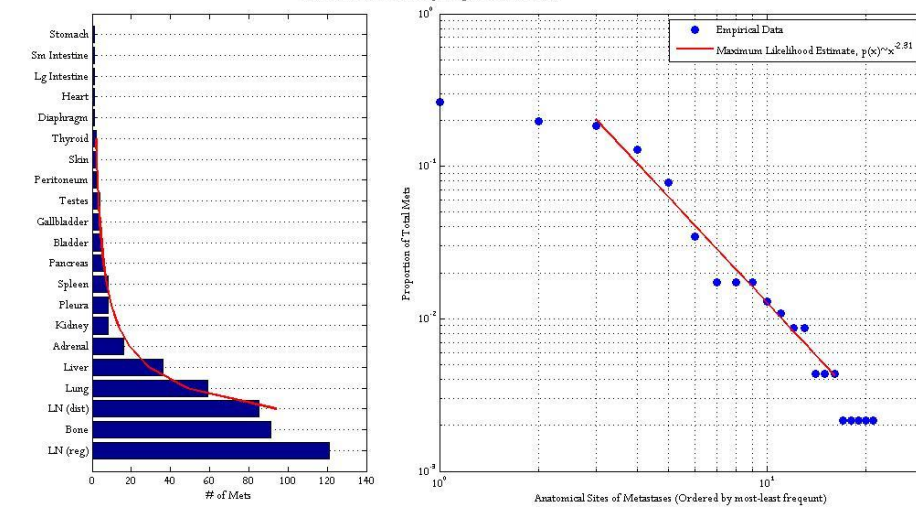
452 Patients with 2285 Mets spanning 28 Anatomical Sites



Prostate

Metastatic Distribution for Primary Prostate Cancer

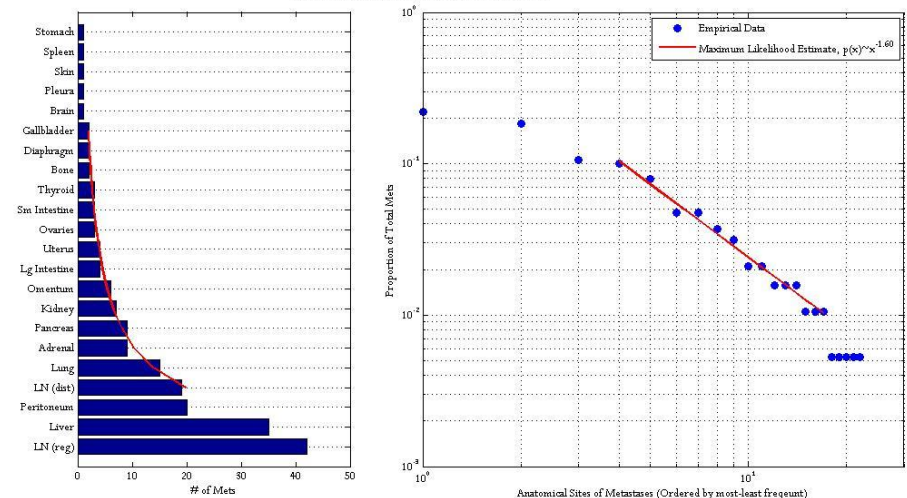
195 Patients with 462 Mets spanning 21 Anatomical Sites



Colon

Metastatic Distribution for Primary Colon Cancer

125 Patients with 190 Mets spanning 22 Anatomical Sites





Two main 'drivers' of metastatic complexity

1. Number of metastatic sites 'N'
 - Larger N increases complexity
2. Probabilistic distribution to those sites
 - More even distribution (flatter) increases complexity

Metastatic entropy



$$E_N = - \sum_{i=1}^N p_i \log p_i$$

p_i % metastatic tumors to site 'i'

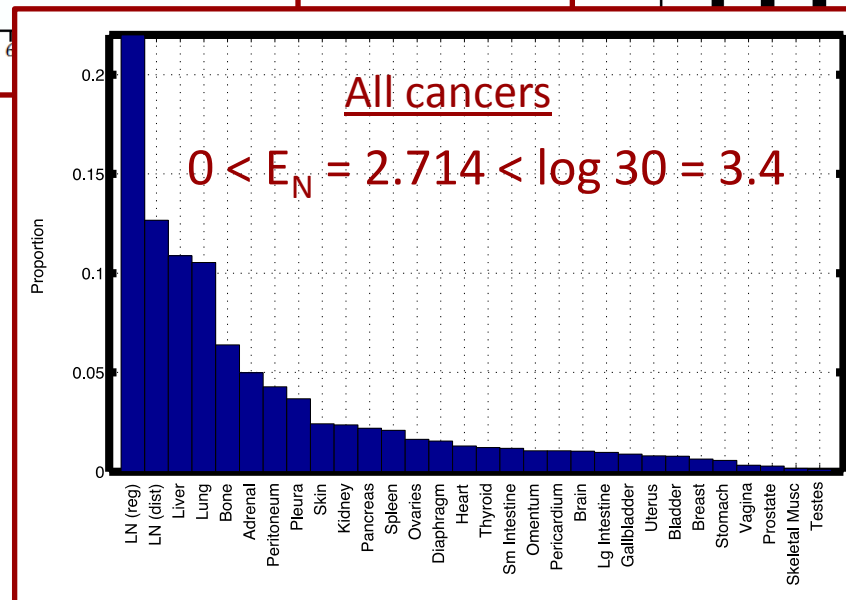
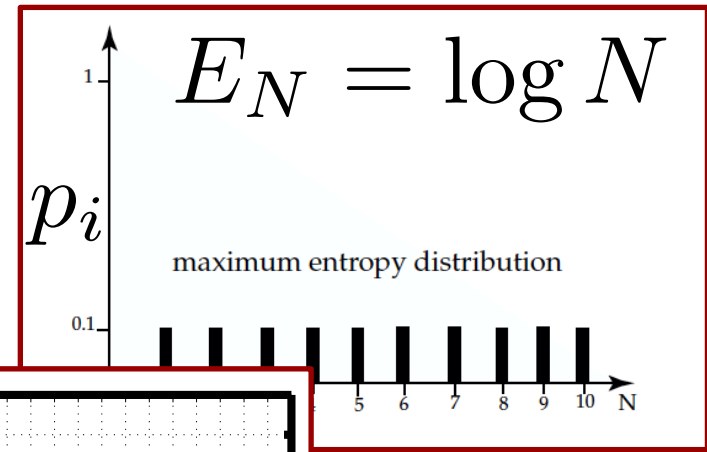
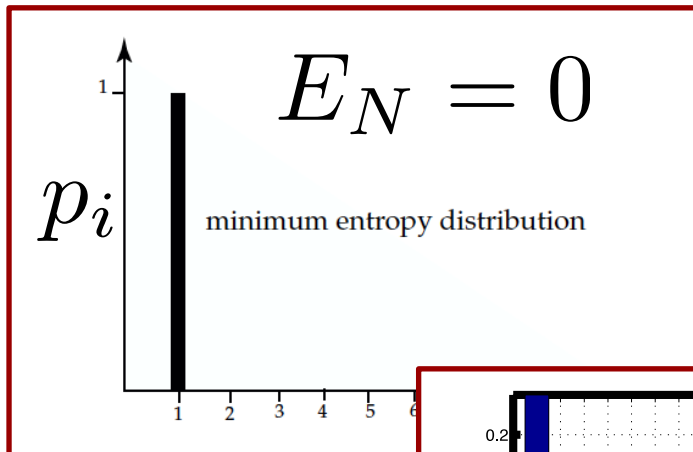
Metastatic entropy



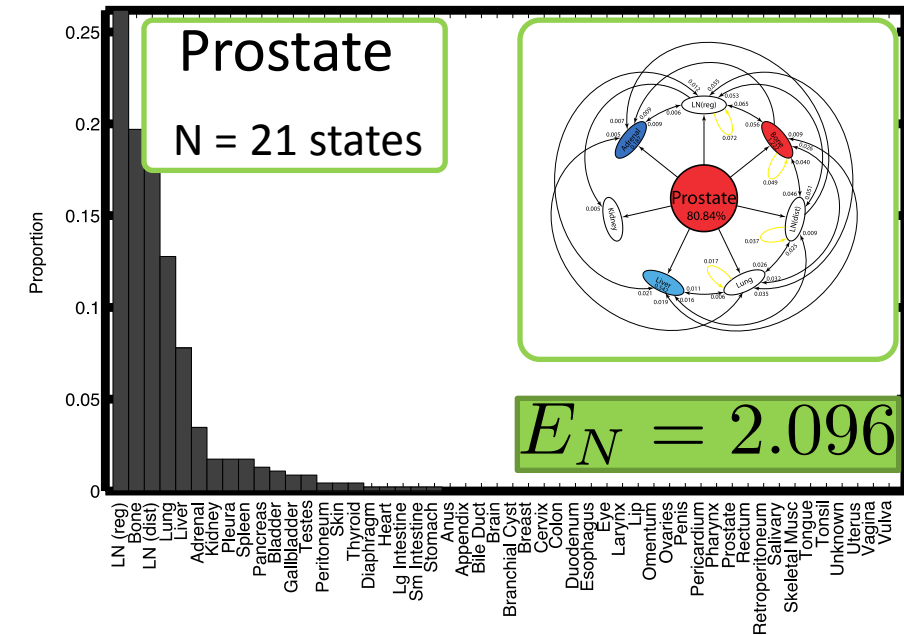
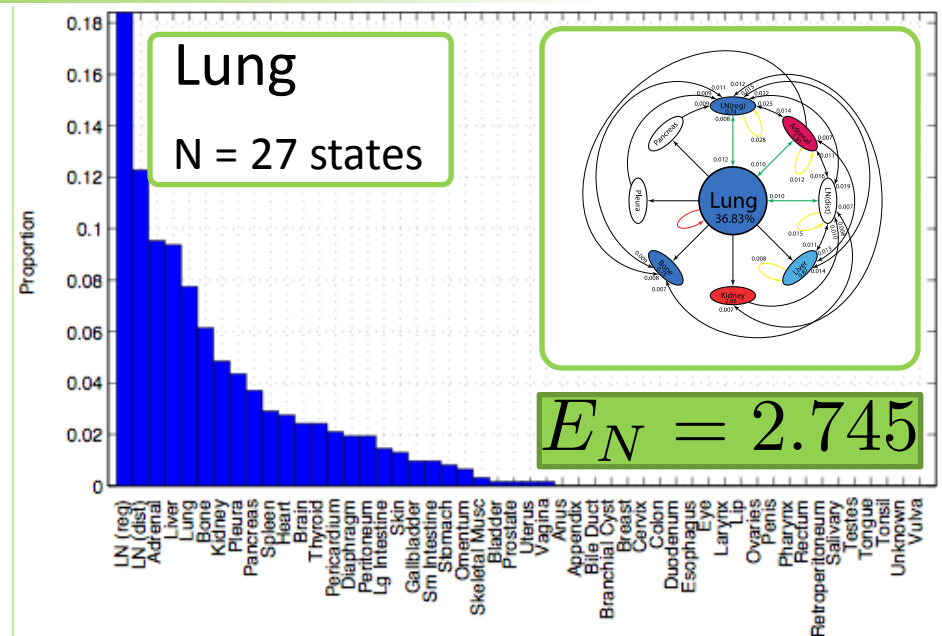
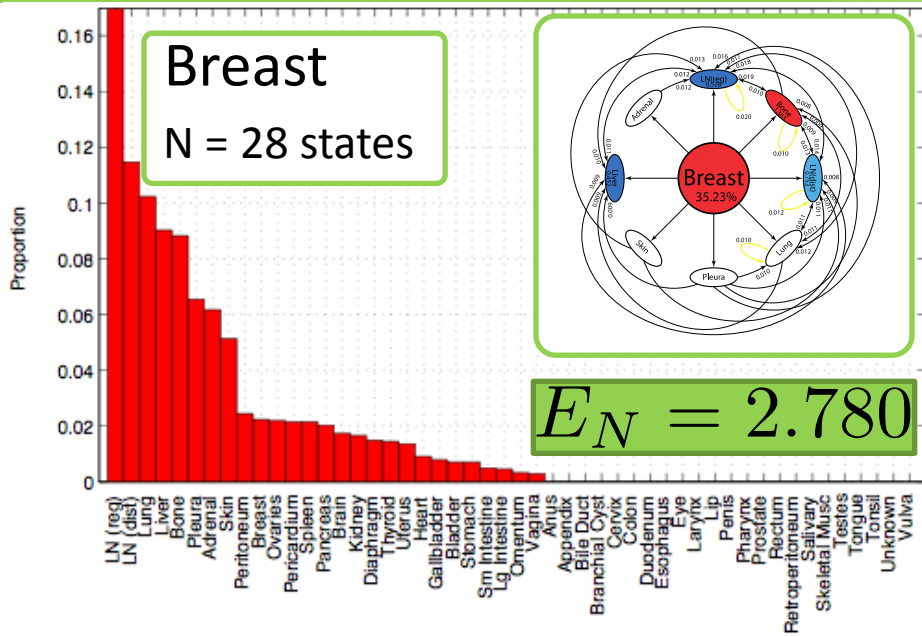
$$0 \leq E_N \leq \log N$$

Initial state: Primary tumor, no mets

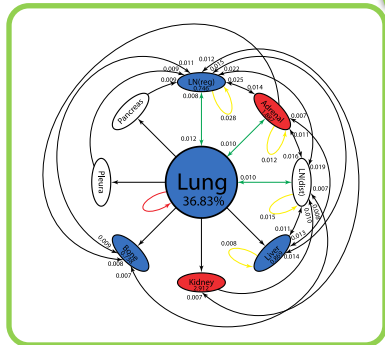
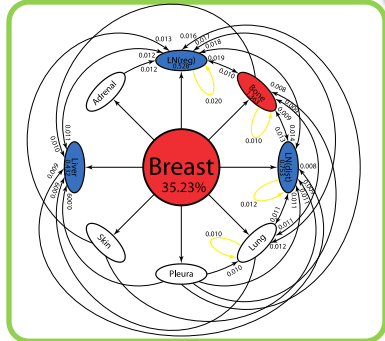
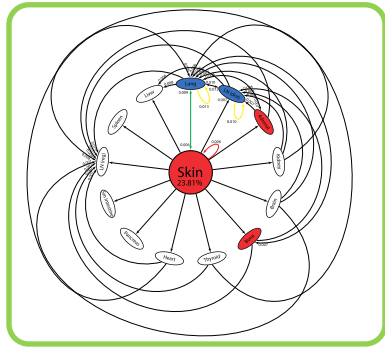
Hypothetical state: Worst case scenario



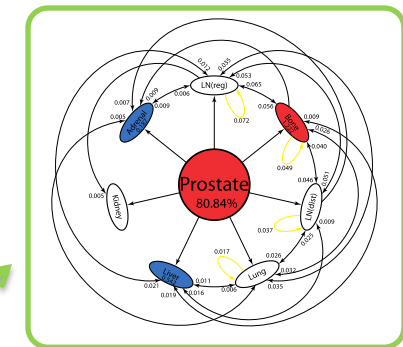
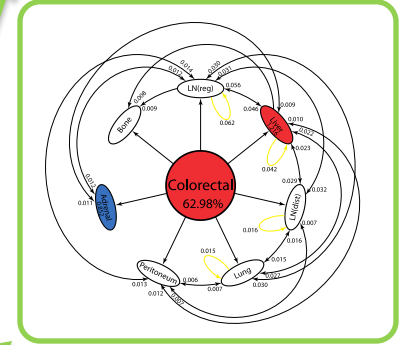
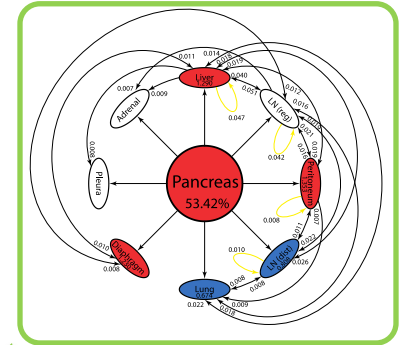
Tumor entropy



Tumor entropy

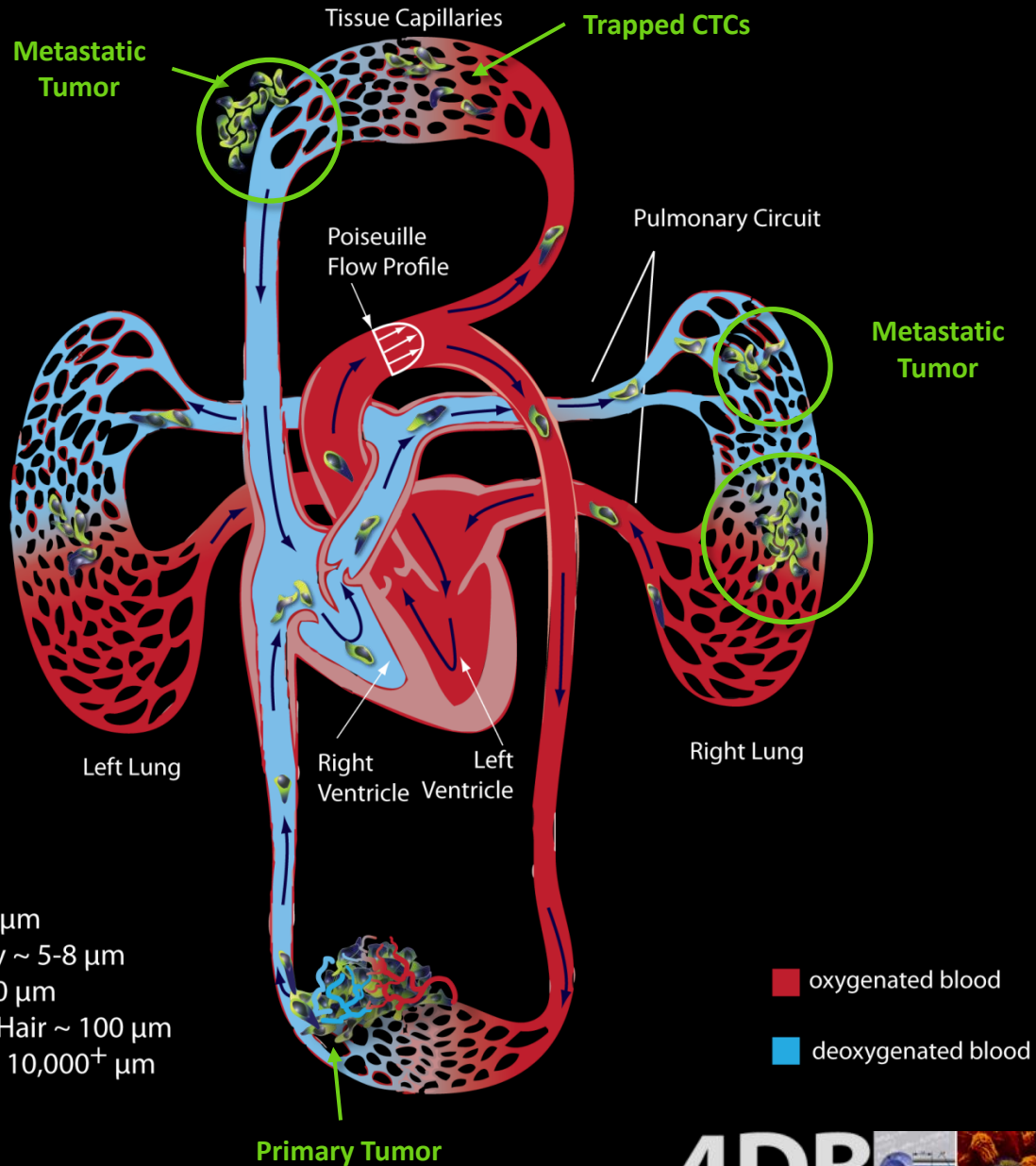


Primary	N	Entropy
Skin	30	2.9945
Breast	27	2.7798
Kidney	27	2.7554
Lung	27	2.7453
All	30	2.7136
Stomach	28	2.6099
Uterine	24	2.5709
Pancreatic	26	2.5540
Colorectal	28	2.4686
Cervical	26	2.3696
Ovarian	21	2.3275
Bladder	22	2.2301
Prostate	21	2.0960



2. Markov chain/random walk models of metastatic progression

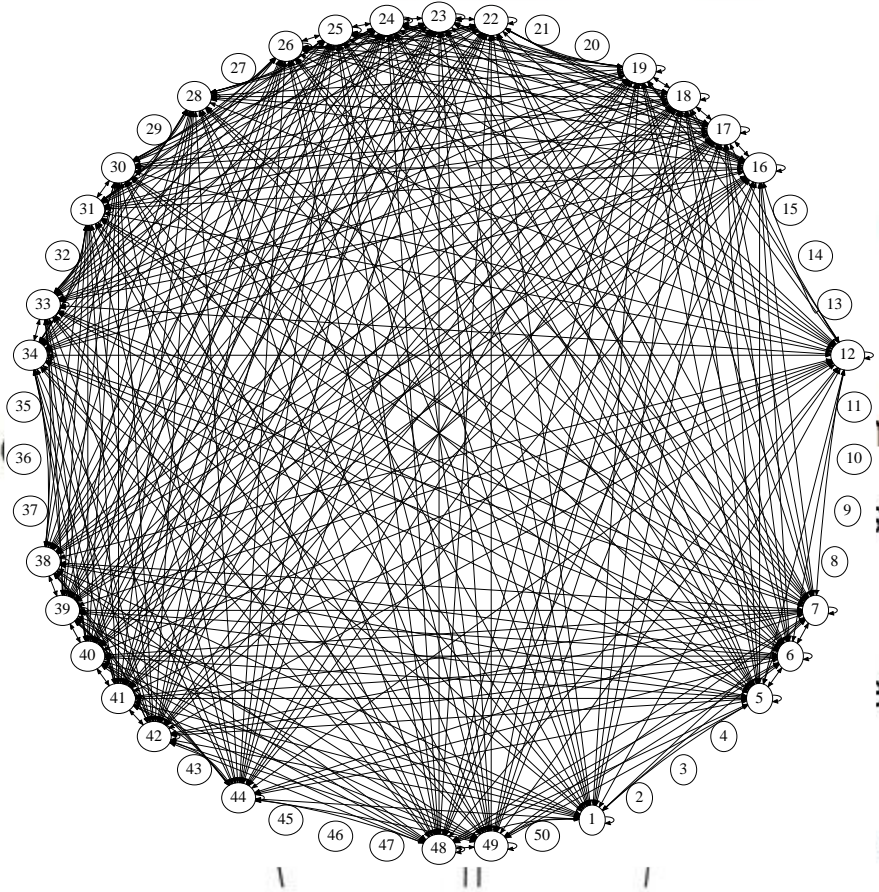
Question: What is the dynamical system that *drives* metastatic progression from a state of low-entropy to a high-entropy state?



Our Cancer Progression Model

Random walkers on an anatomical network

Lung cancer



Main features:

- Nodes are potential tumor sites
- Nodes are linked by directed edges
- Need to construct the edge weights

Similar uses for our model:

- Perform simulated cancer progression:
 - Individual progression with random walks
 - `Ensemble' progression with Monte Carlo simulations
- Run tests under different scenarios
- Calculate `average' number of steps from node i to node j (mean first-passage times)

Markov Chain Basics

$$\vec{v}_{k+1} = \vec{v}_k A, (k = 0, 1, 2, \dots)$$

- States: Lung, Breast, Liver, Adrenal, LN,Deceased
- Initial state: $\vec{v}_0 = (1,0,0,0, \dots)$
- Steady-state: $\vec{v}_\infty = (0.18, 0.12, \dots)$

\vec{v}_k is the state-vector with 50 entries reflecting possible metastatic tumor locations

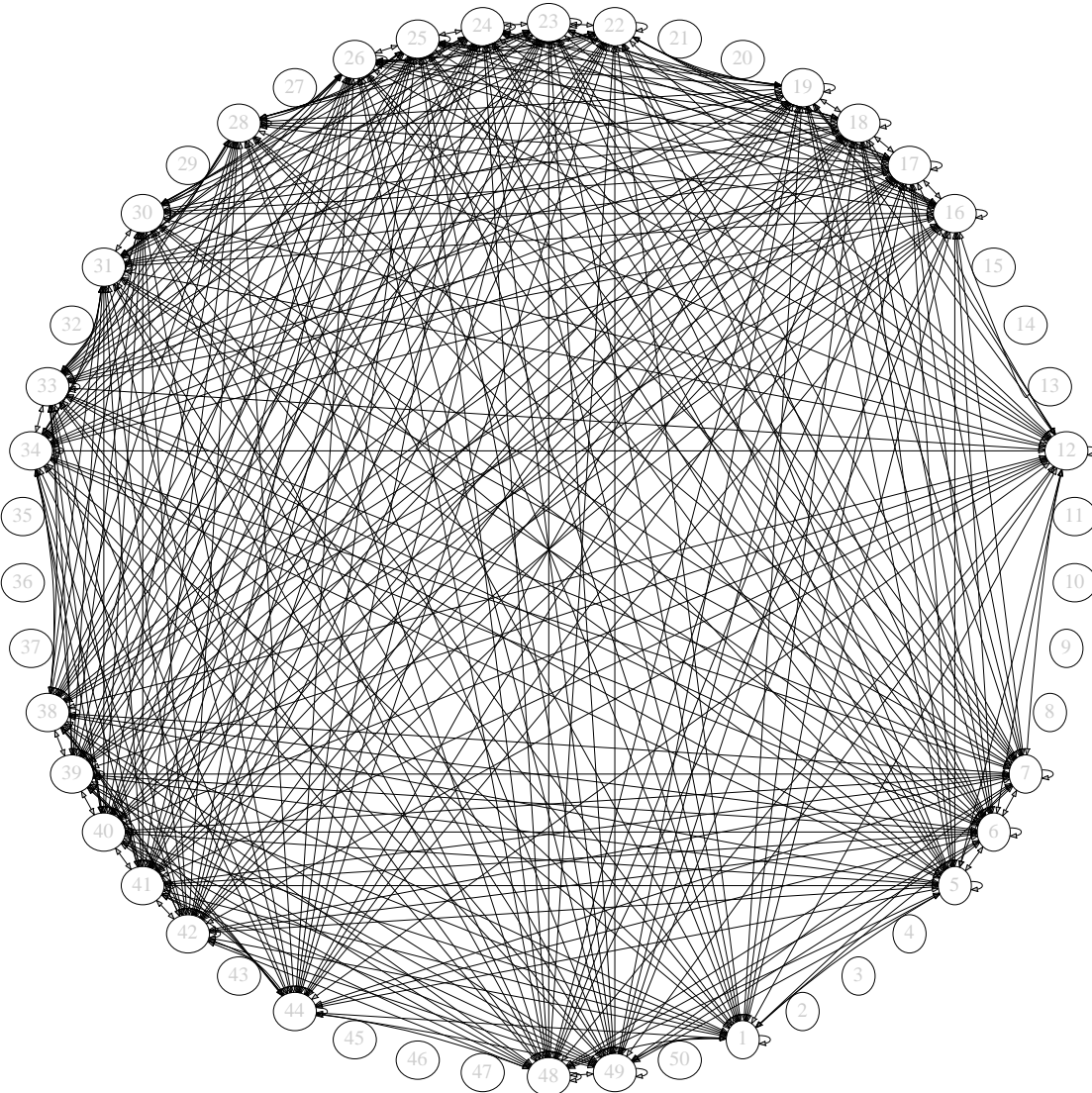
- **A** is the transition matrix governing the transition probabilities from site to site

How to compute (estimate) the entries of the transition matrix A ?

- Need *the transition probabilities* from site to site:
 - Reconstruct from steady-state info (autopsy data sets)
 - Direct empirical calculation (longitudinal data sets)

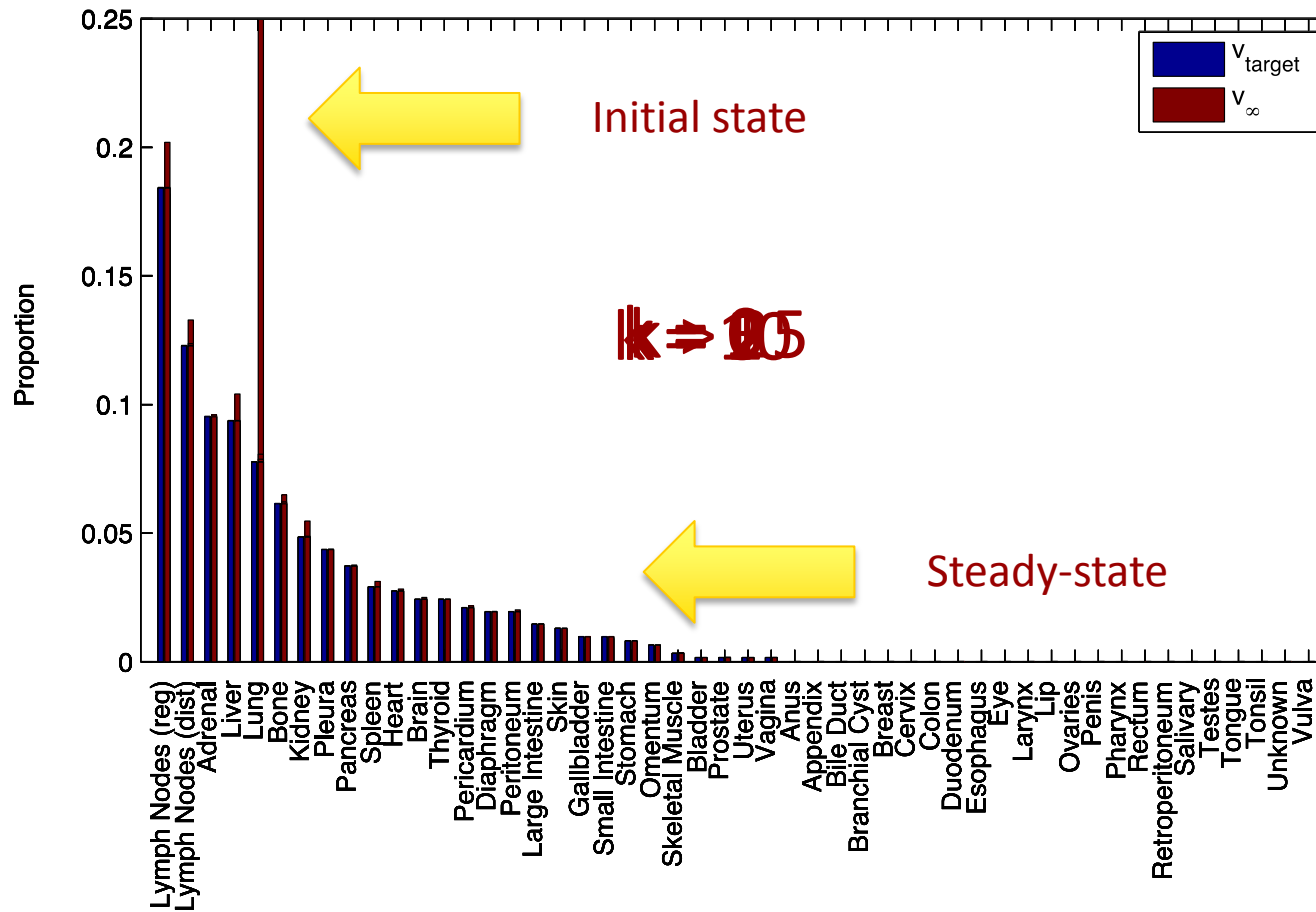
The Lung Cancer Network

- **913 edges**
- Lung: **27 outgoing edges**
- Lung: **49 incoming edges**

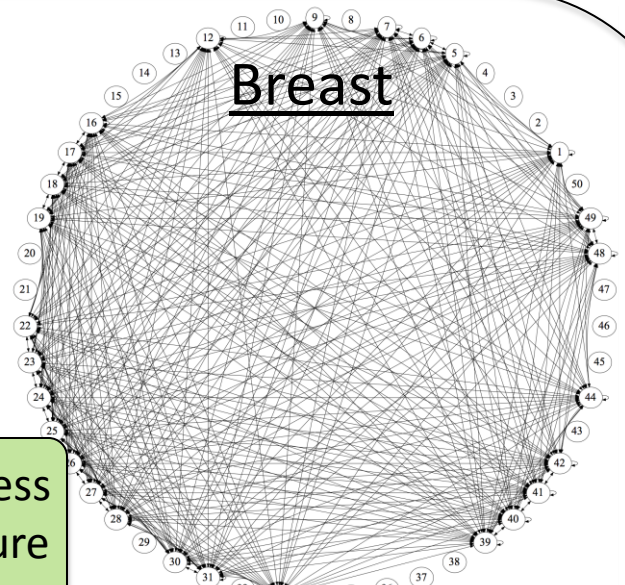
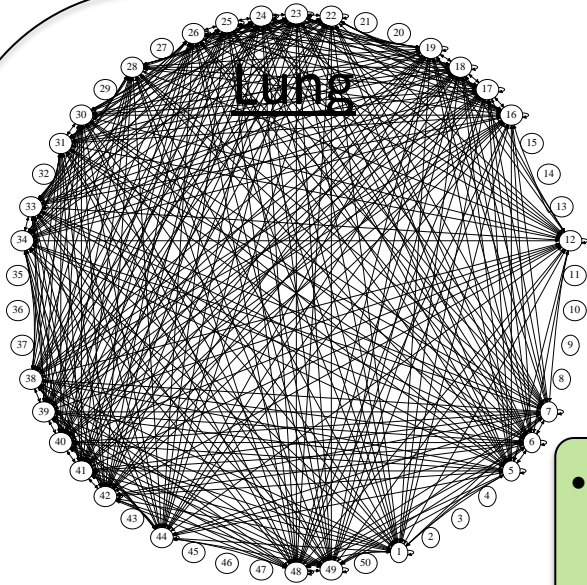


#	Name	#	Name
1	Adrenal	26	Omentum
2	Anus	27	Ovaris
3	Appendix	28	Pancreas
4	Bile Duct	29	Penis
5	Bladder	30	Pericardium
6	Bone	31	Peritoneum
7	Brain	32	Pharynx
8	Branchial Cyst	33	Pleura
9	Breast	34	Prostate
10	Cervix	35	Rectum
11	Colon	36	Retroperitoneum
12	Diaphragm	37	Salivary
13	Duodenum	38	Skeletal Muscle
14	Esophagus	39	Skin
15	Eye	40	Small Intestine
16	Gallbladder	41	Spleen
17	Heart	42	Stomach
18	Kidney	43	Testes
19	Large Intestine	44	Thyroid
20	Larynx	45	Tongue
21	Lip	46	Tonsil
22	Liver	47	Unknown
23	Lung	48	Uteris
24	Lymph Nodes (reg)	49	Vagina
25	Lymph Nodes (dist)	50	Vulva

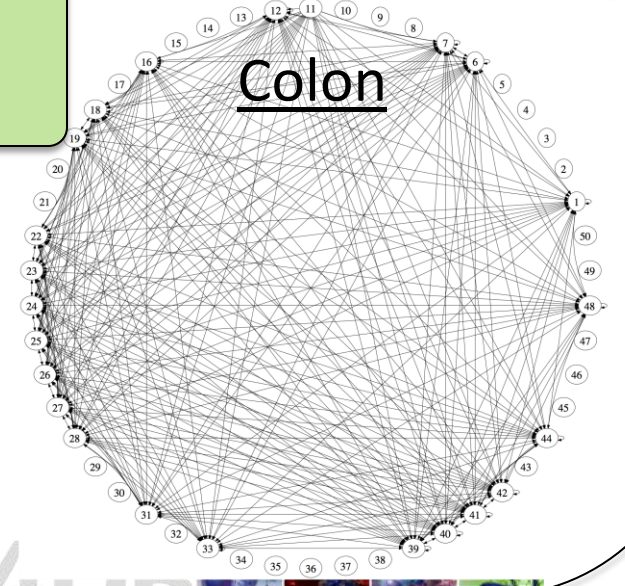
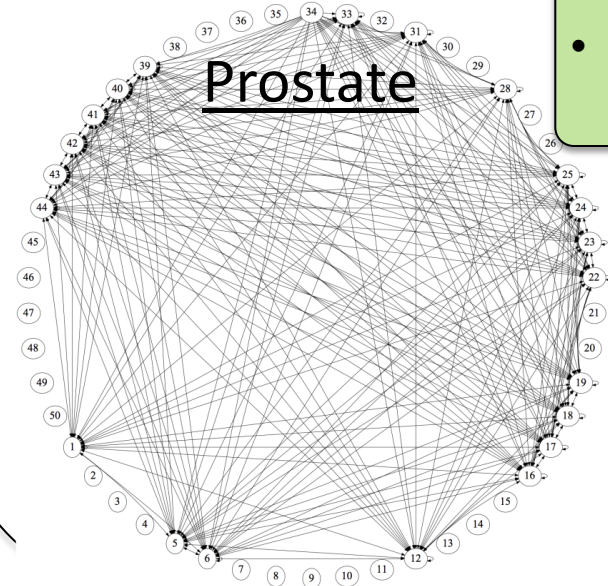
Convergence: Lung cancer



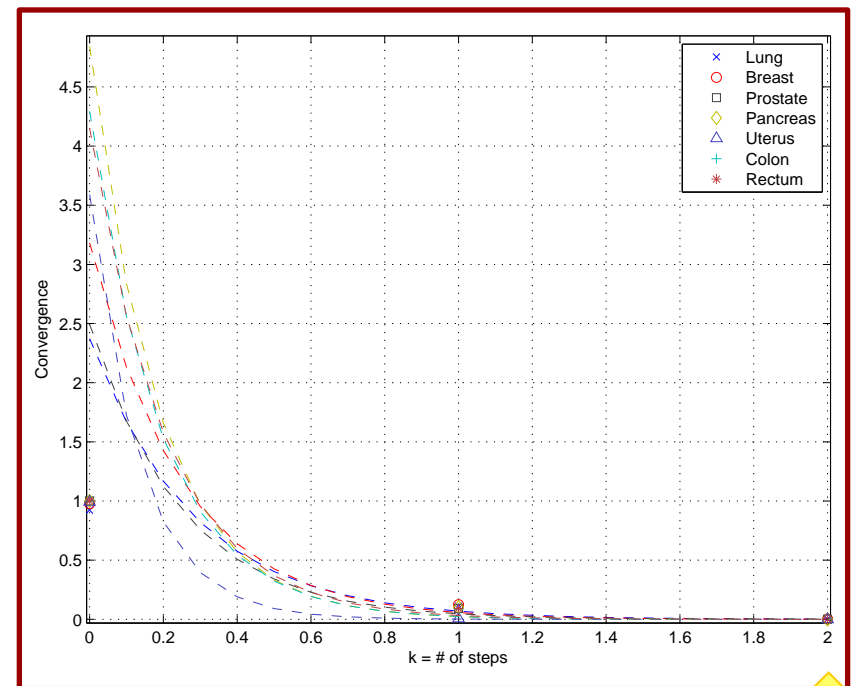
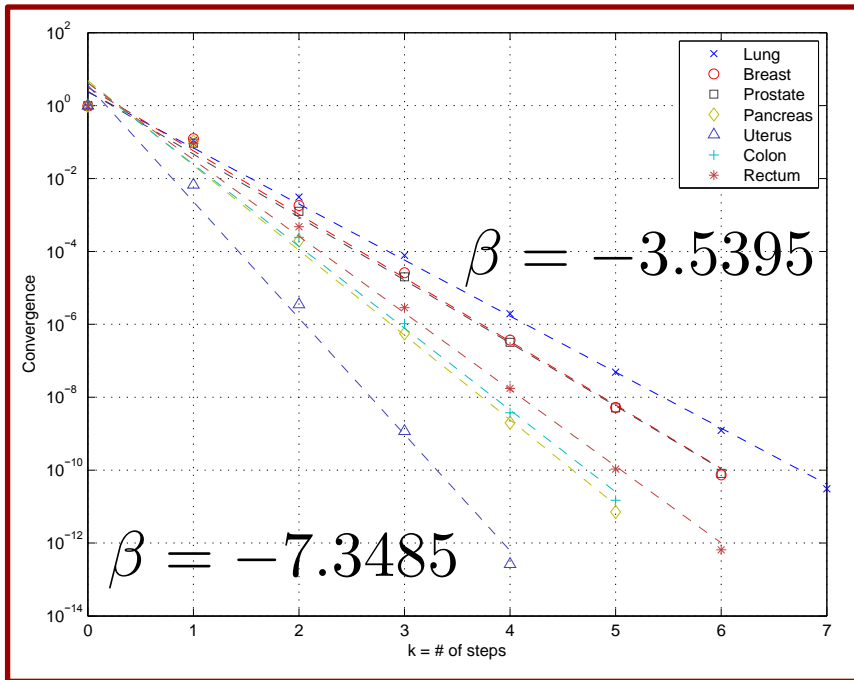
Network diagrams for 4 cancer types



- Network inter-connectedness highlights the *systemic* nature of the disease
- Network *density* reflects relative complexity



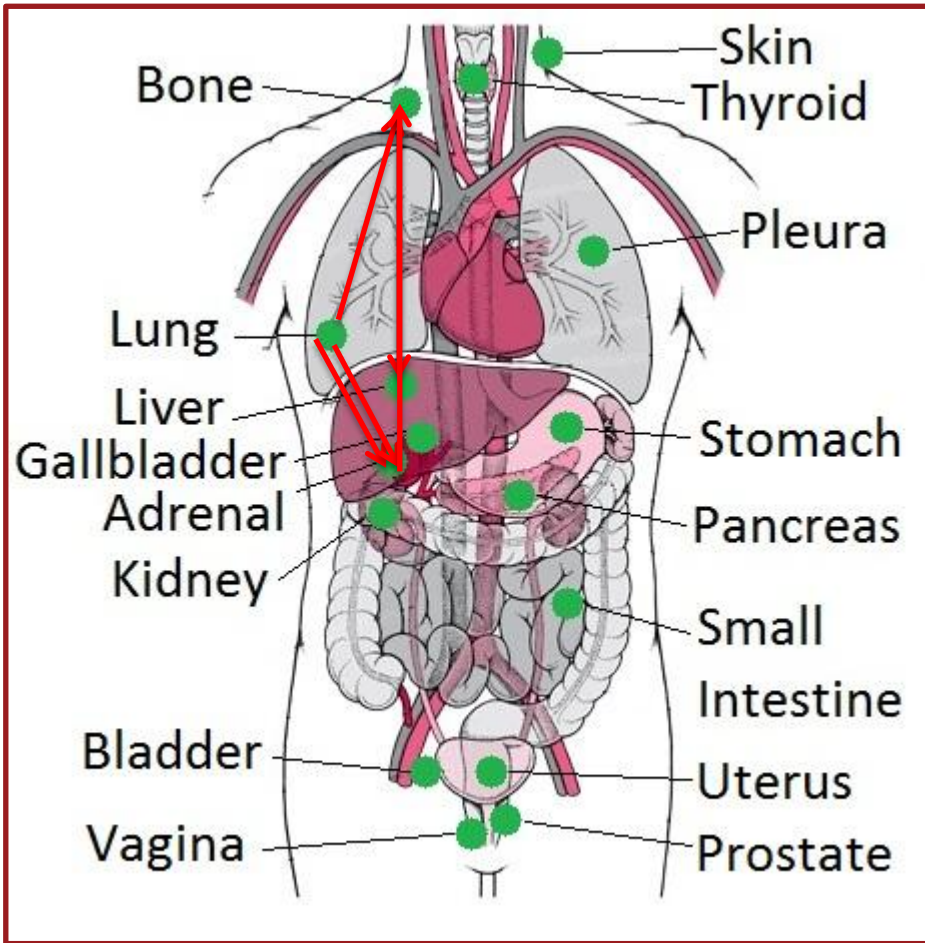
Nearly overlapping eigenvalues steps



$$\|\vec{v}_k - \vec{v}_\infty\| \sim \alpha \exp(-\beta k)$$

2 steps

Constructing metastatic pathways



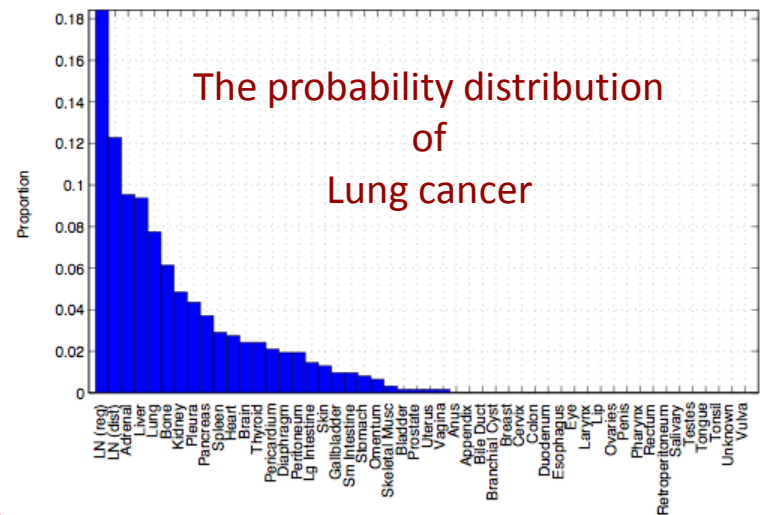
The metastatic pathways

Lung → Bone → Liver → Deceased

Lung → Adrenal → Deceased

Lung → Adrenal → Bone → Deceased

Etc.



Spreaders & Sponges

Question: Which metastatic sites would make the best `targets' for therapy ?

Metastatic site as spreader or sponge

Spreader: $P_{out}/P_{in} > 1$

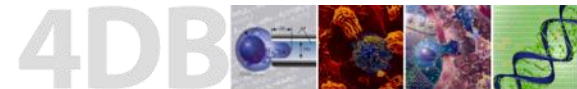
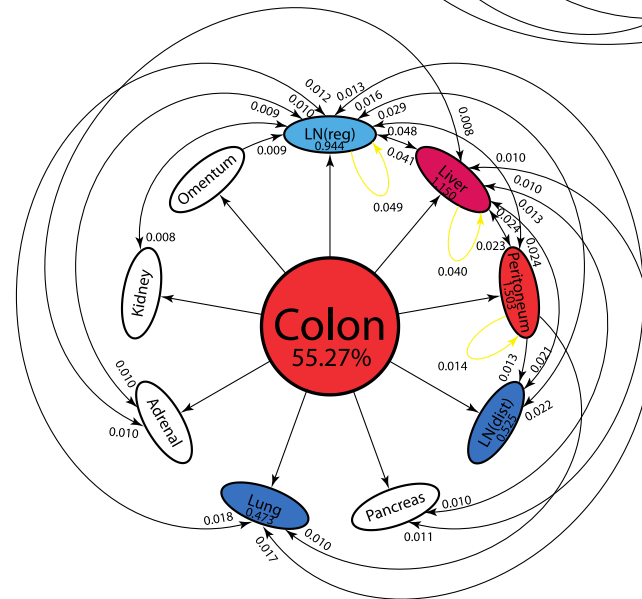
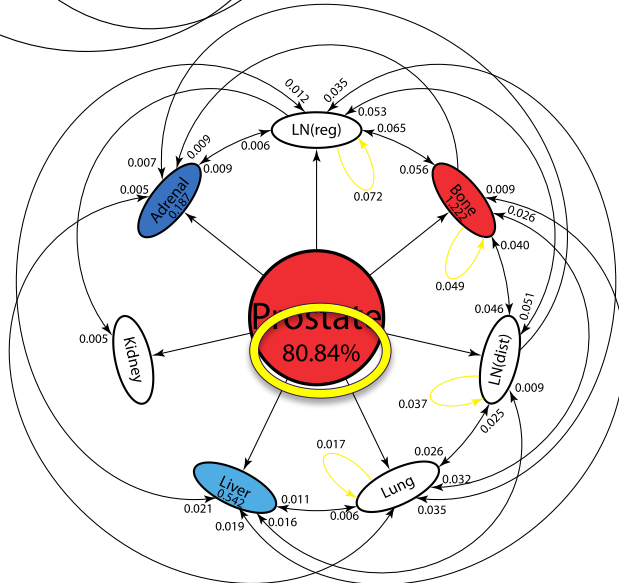
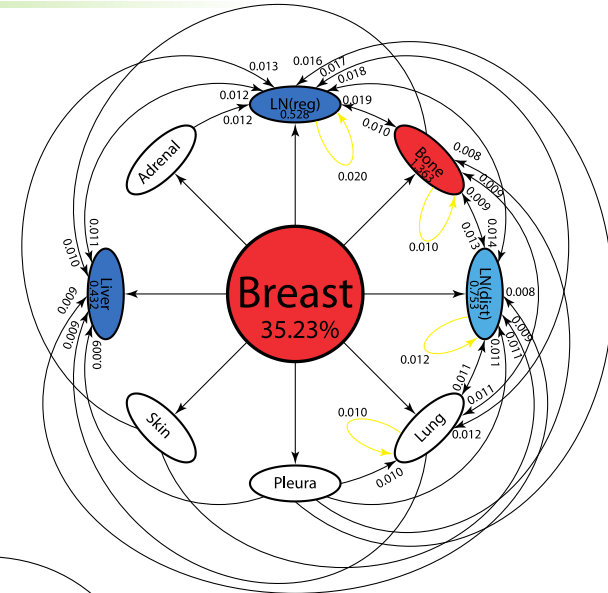
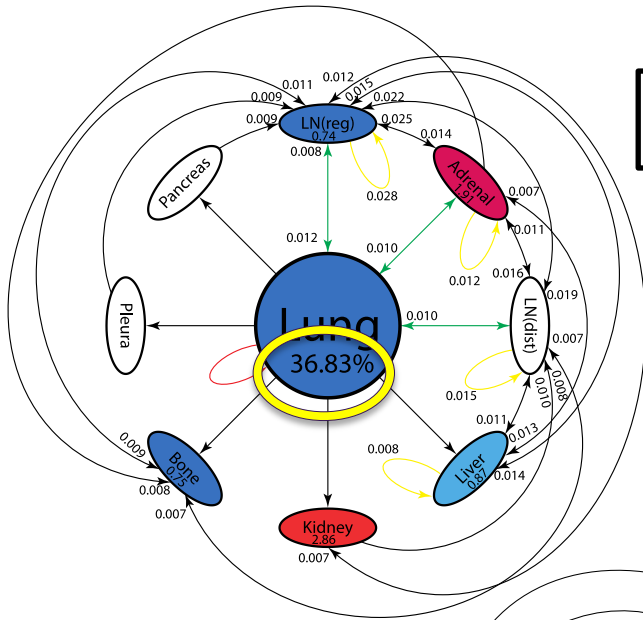
Sponge: $P_{out}/P_{in} < 1$

The spreaders and sponges



Reduced order models

Spreaders
Sponges



Spreader diagrams

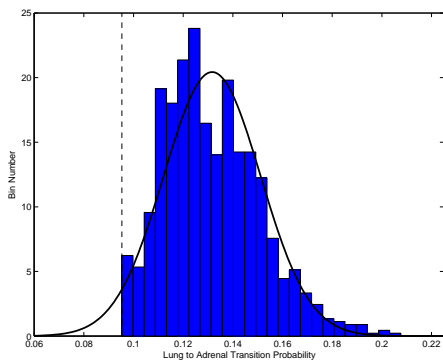
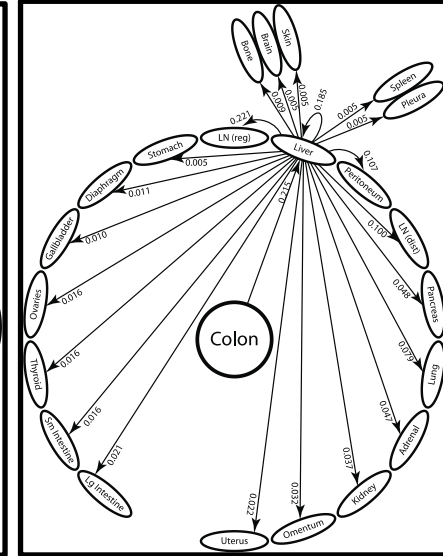
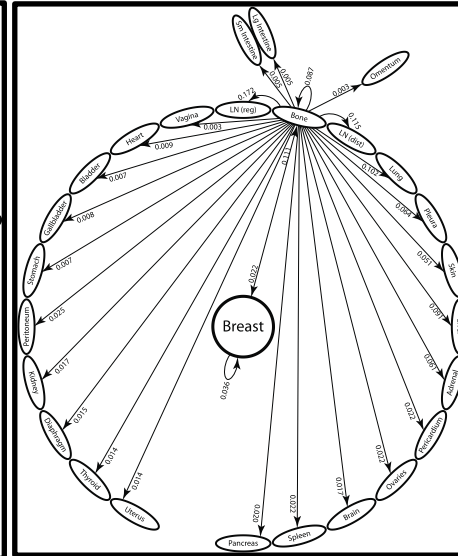
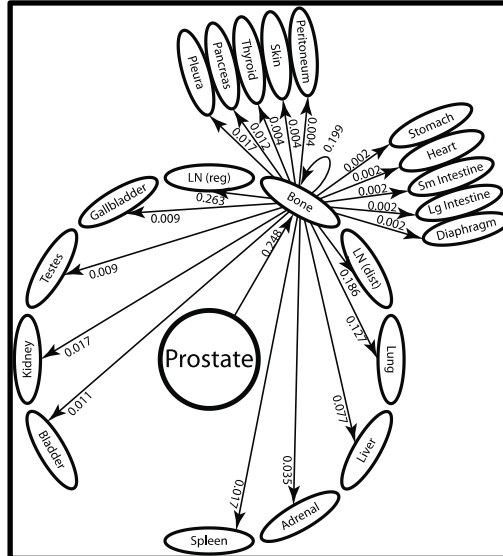
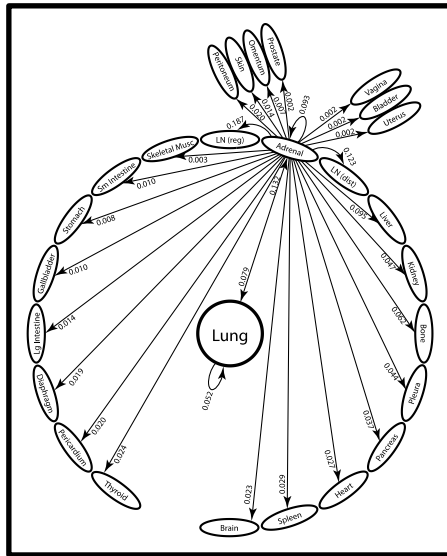


Lung → Adrenal

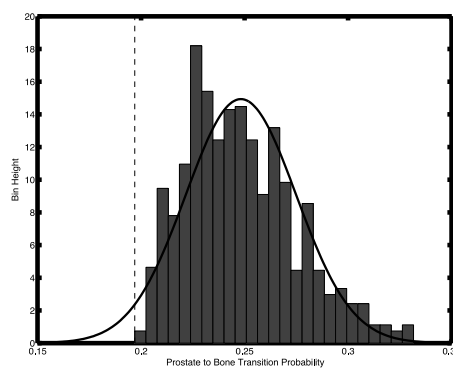
Prostate → Bone

Breast → Bone

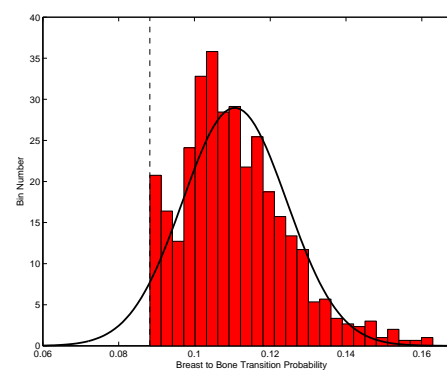
Colon → Liver



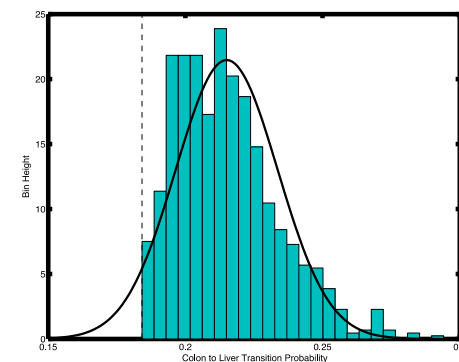
0.132 +/- 0.002



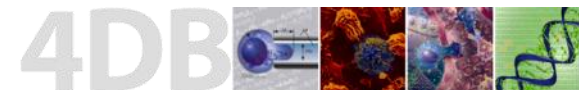
0.248 +/- 0.027



0.111 +/- 0.014



0.215 +/- 0.019

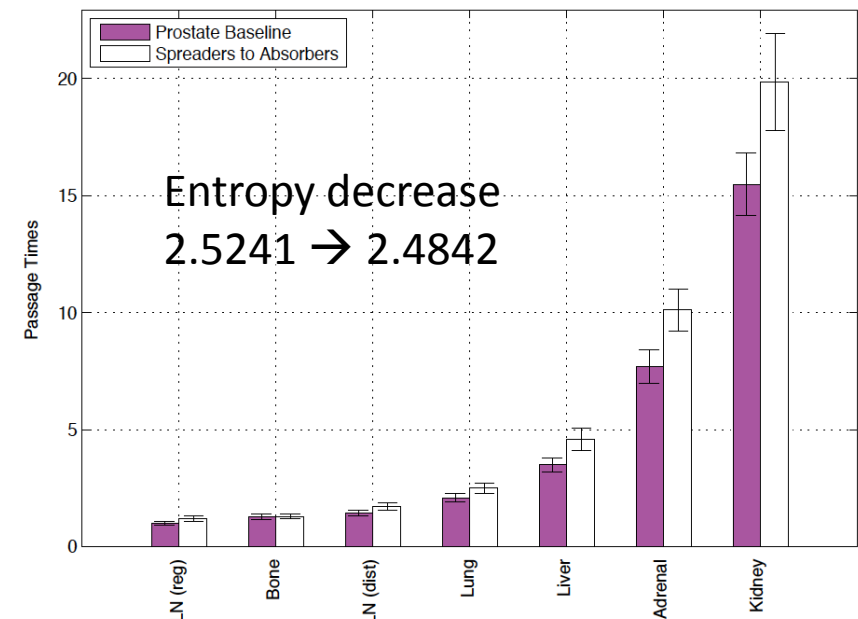
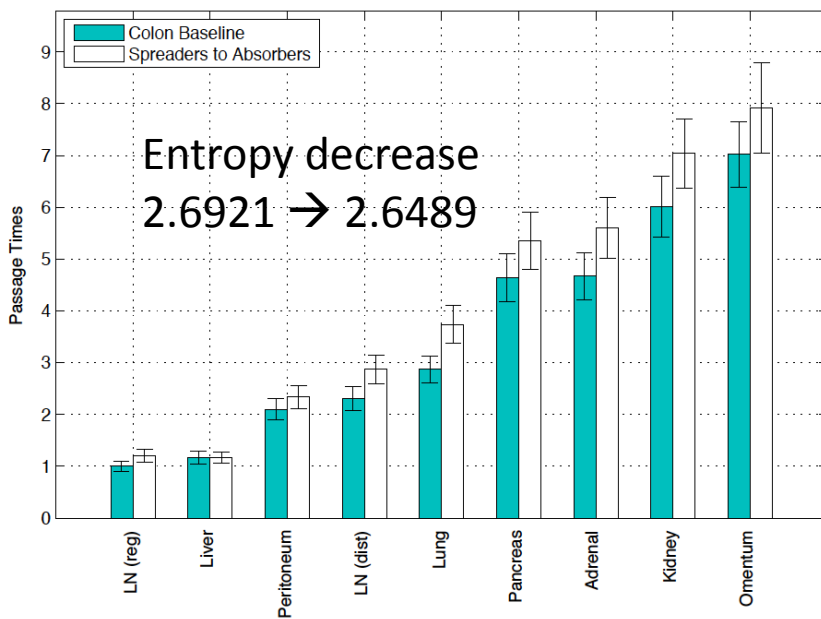
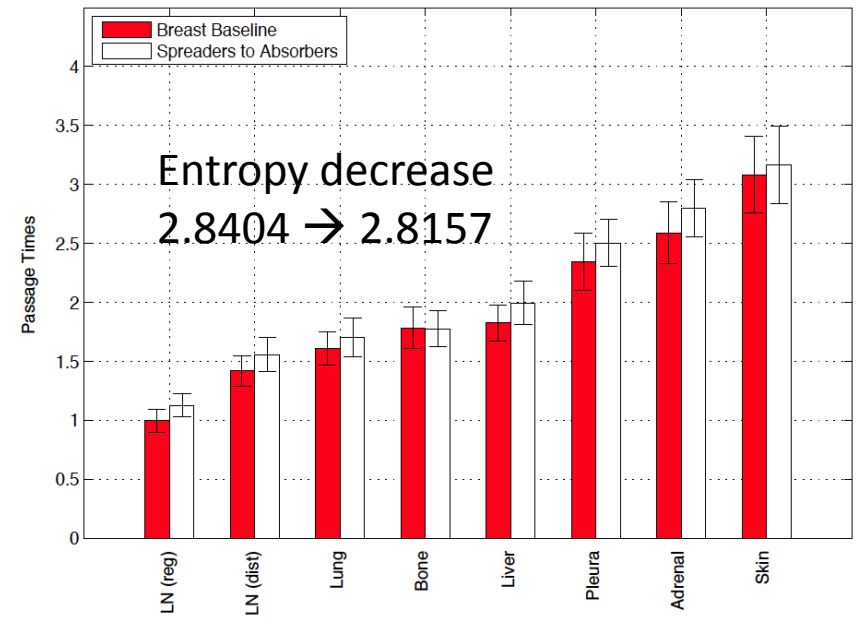
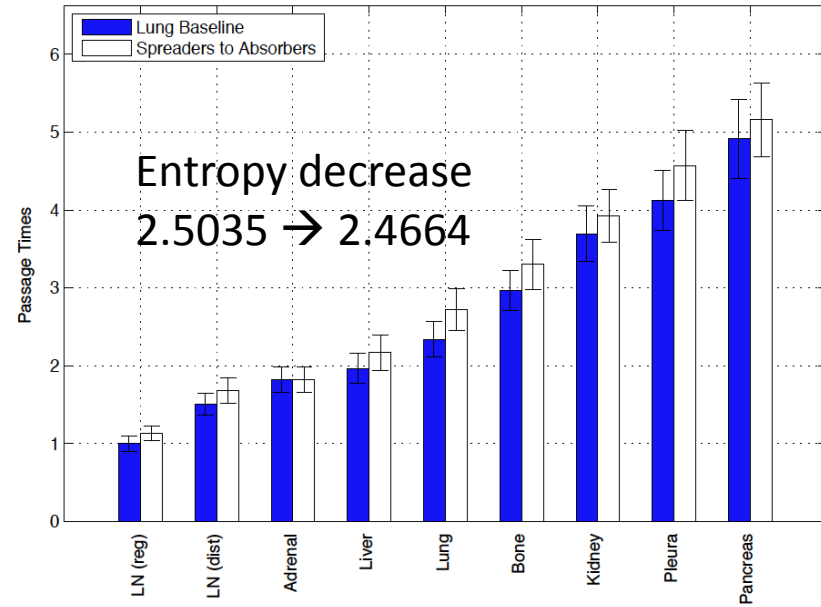


Targeting the spreaders

Question: What would be the response if we could therapeutically target the spreaders?

Spreader → Absorbing state

Entropy decrease, mfpt increase



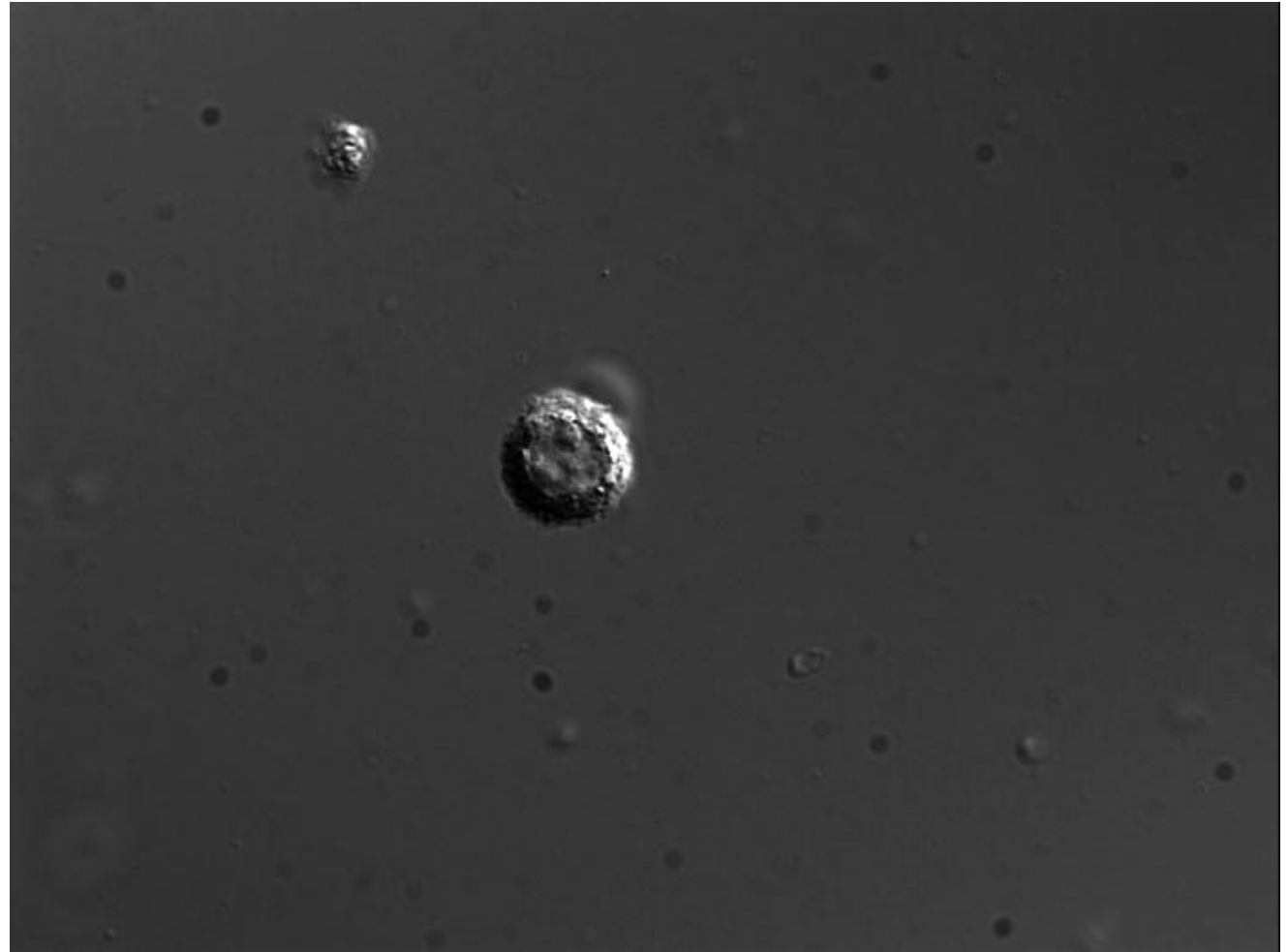
3. CFD models of procoagulant CTCs in the bloodstream

- Cancer patients have a higher than average incidence of blood clot formation leading to stroke.
- CTCs express TF that triggers a complex chain of chemical reactions leading to thrombosis.

Experiment of fibrin formation

Coagulation process of fibrin formation by SW480 cell
(McCarty Lab, OHSU)

Cell is ~12 um
60x real time
DIC microscopy



Mathematical infrastructure

Blood flow velocity field

$$\nabla \cdot \vec{u} = 0$$

$$\rho \left(\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} \right) = -\nabla p + \mu \Delta \vec{u}$$

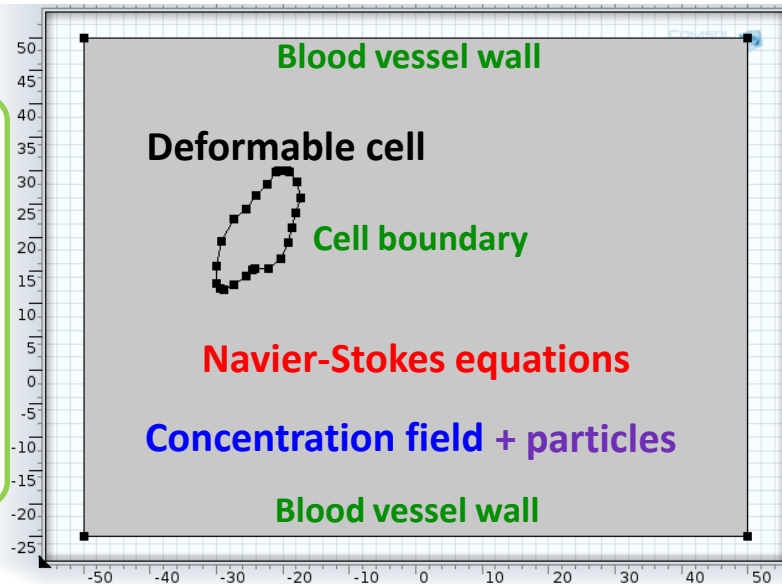
$$\vec{u}(\vec{x}, 0) = \vec{f}(\vec{x})$$

Boundary conditions

$$\left. \frac{\partial C}{\partial n} \right|_{\text{wall, cell}} = 0$$

$$\vec{u} \Big|_{\text{wall}} = 0$$

$$\vec{u} \Big|_{\text{cell}} = \text{velocity of cell boundary}$$



Particles that follow concentration gradients

$$\frac{\partial \vec{q}}{\partial t} = \frac{1}{c_0(C + \epsilon)} \left(\frac{\nabla C}{\|\nabla C + \epsilon\|} \right)$$

where c_0 is a velocity scaling coefficient

Concentration fields of thrombin associated with each CTC

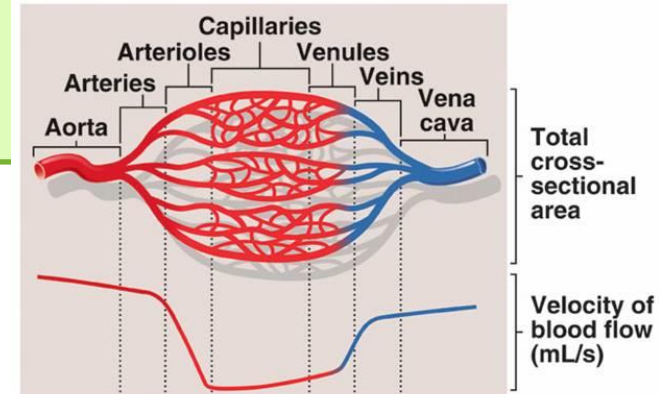
$$\frac{\partial C^{(i)}}{\partial t} + \vec{u} \cdot \nabla C^{(i)} = \alpha_i \Delta C^{(i)}$$

$\alpha_i (i = 1, \dots, n)$: Diffusion constants associated with coagulation factors

Blood parameters in venous system

- whole blood: density = 1060 kg/m^3 at 37°C
dynamic viscosity = $3 \cdot 10^{-3} \text{ Pa s}$ at 37°C
- venules: flow rate: $v = 0.03\text{-}0.1 \text{ cm/s}$
diameter: $d = 7\text{-}50 \text{ }\mu\text{m}$

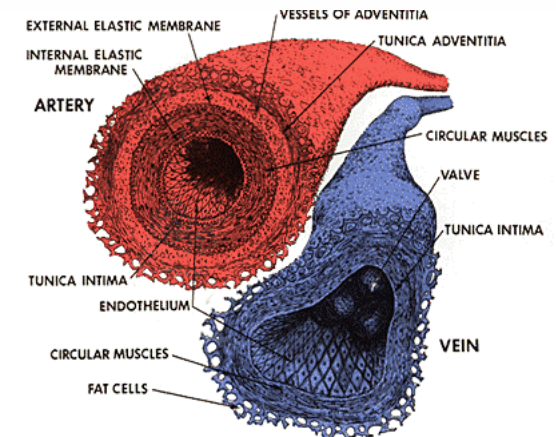
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Diffusion coefficients for coagulation factors

Factor	Stokes Radius (nm)	C (nM)	$D^{20, \text{plasma}} (10^{-7} \text{ cm}^2 \text{ s}^{-1})$	J/J ^{VII}
VII	3.5 (Gladhaug et al. 1970)	7-13 (Fair 1983)	5.1	1
IX	4.1 (Suomela 1976)	40-142 (Yang 1978)*	4.4	3-22
VIII	8.8 (Hoyer et al. 1981)	0.4-1 (Hoyer 1994)*	2	<<1
X		101-162 (Epstein 1984)	5	8-23
V	9.5 (Esmon 1979)	12-42 (Kamphuisen 2000)	1.9	2-10
II	4.1 (Stenflo 1972)	1200-1730 (Legnani 2003)	4.4	100-266

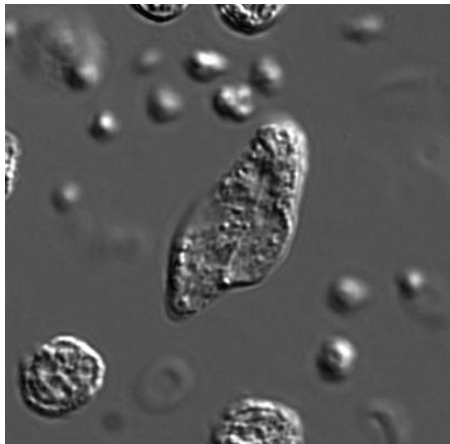
* calculated (FIX: 100% = $4.5 \mu\text{g mL}^{-1}$; FVIII: 100% = $0.2 \mu\text{g mL}^{-1}$)



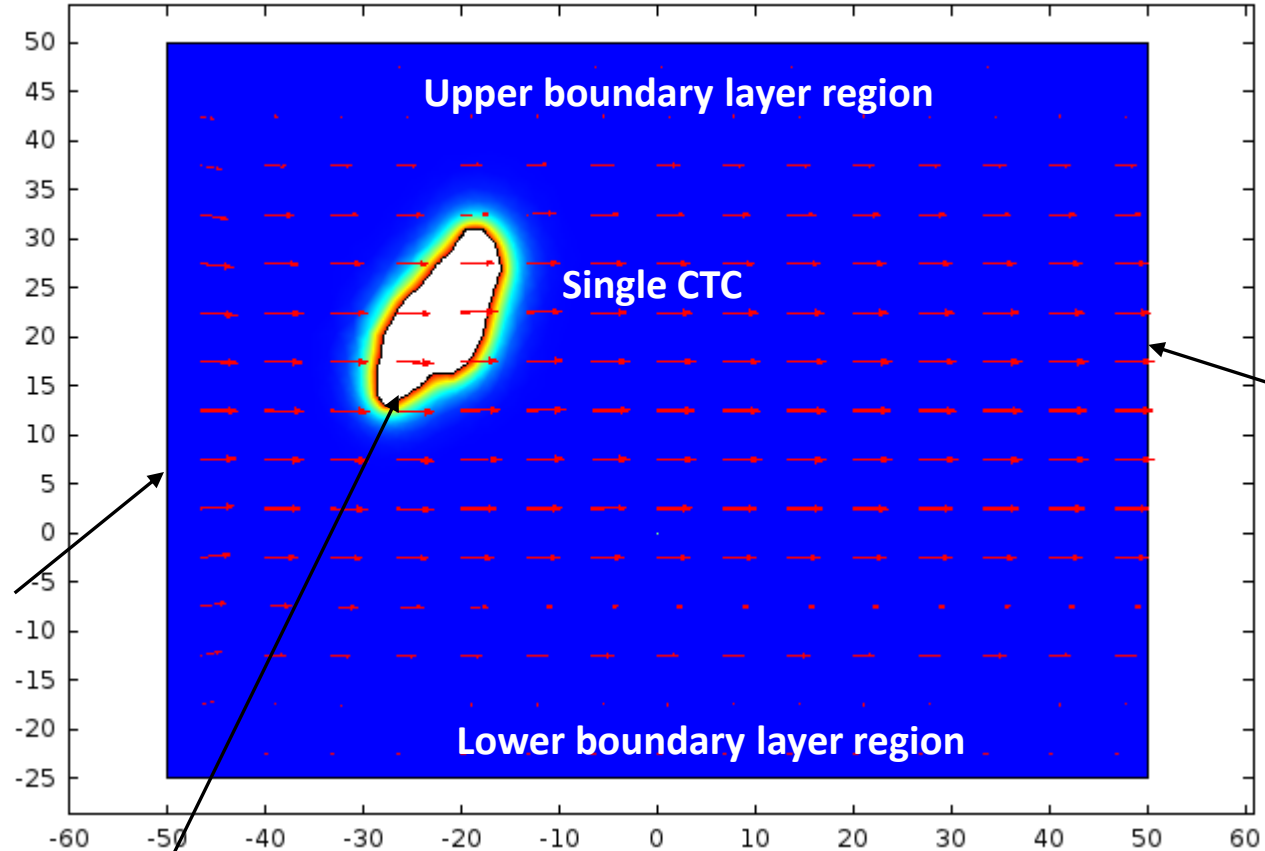
CFD model of flow in channel

Breast cancer cell imaged from Kuhn Lab

DIC image



Time=0 Surface: Concentration (mol/m³) Arrow Surface: Velocity field (Spatial)

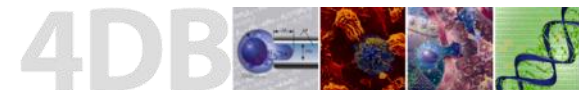


Inflow
(flow profile set
up by pressure
gradient)

Outflow

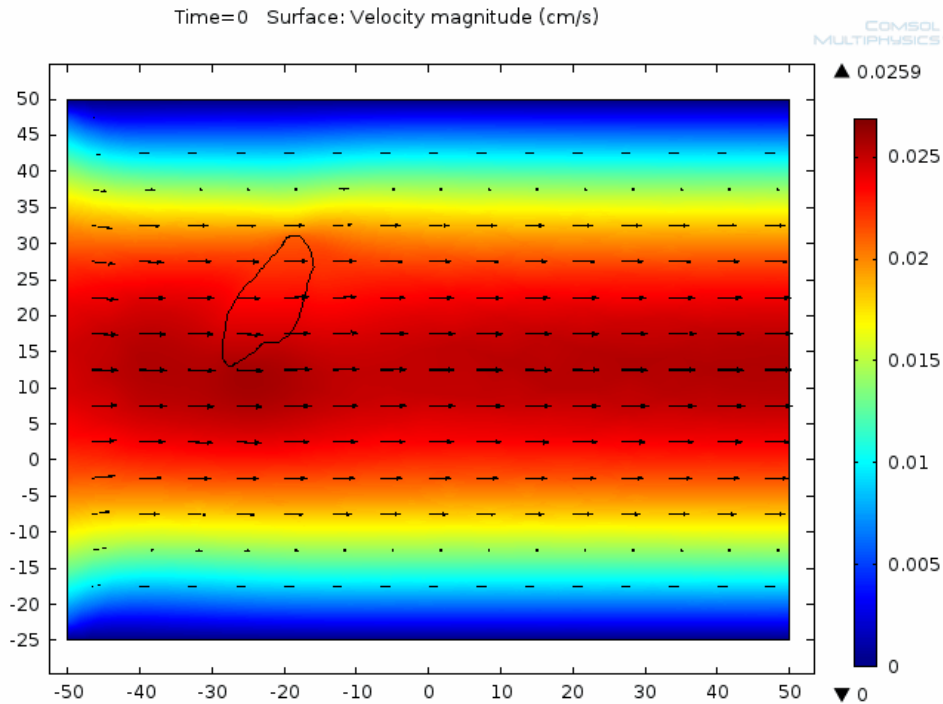
Concentration field of
thrombin on surface of CTC

Viscous boundary conditions
along vessel walls



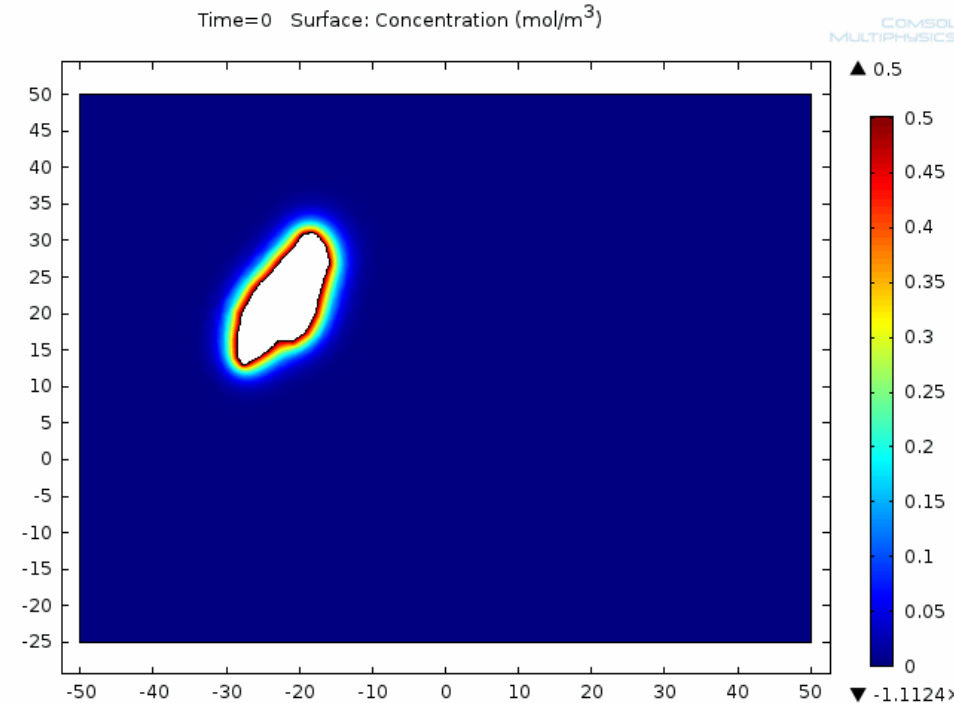
Simulation of CTC under flow

Blood velocity field



Max velocity = 0.026 cm/s

Thrombin concentration field



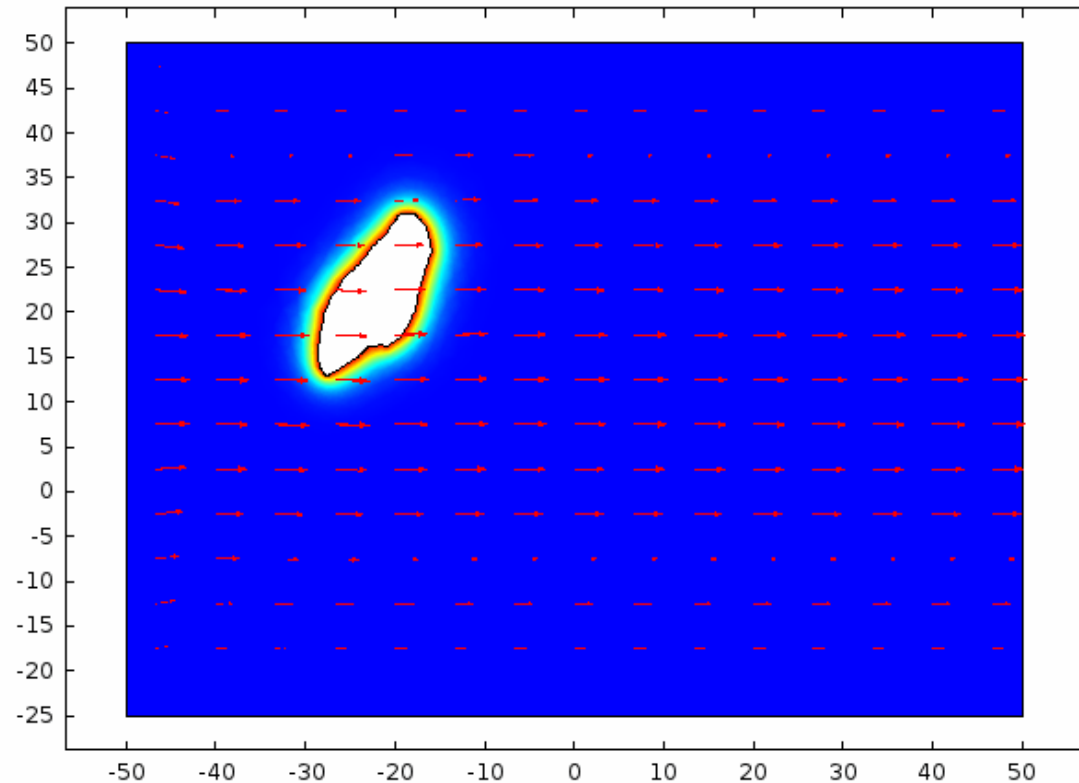
Diffusion coefficient = $1e-7$ cm²/s

Simulation of CTC under flow

Thrombin field and velocity vectors

Time=0 Surface: Concentration (mol/m³)
Arrow Surface: Velocity field (Spatial)

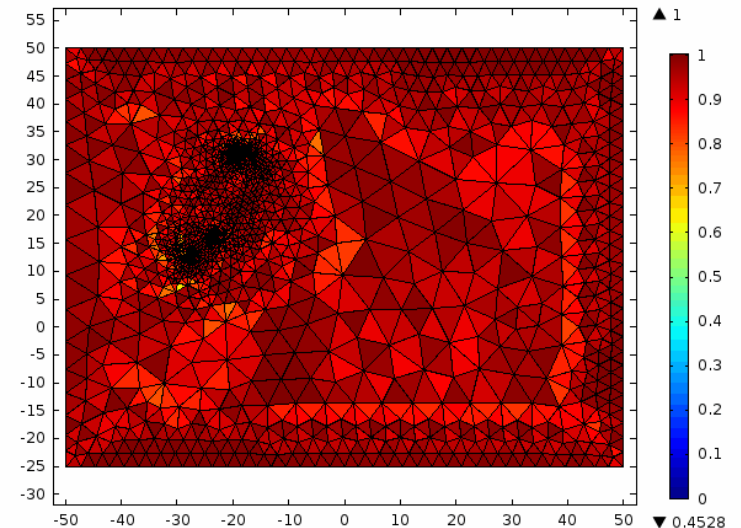
COMSOL
MULTIPHYSICS



Adaptive mesh generation

Time=0 Mesh

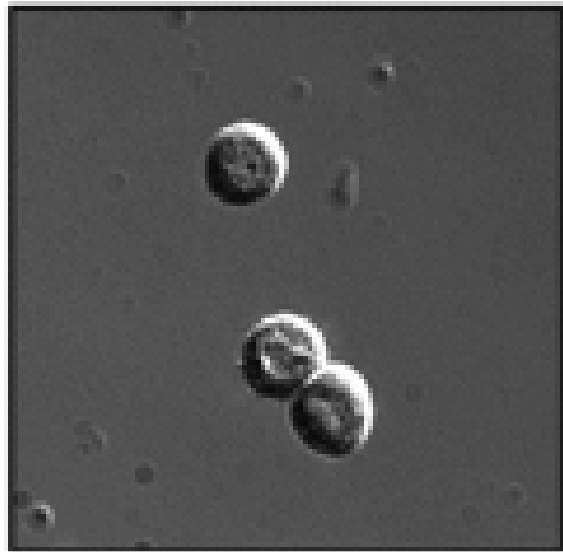
COMSOL
MULTIPHYSICS



Complete mesh consists of
2552 domain elements and
183 boundary elements

CFD model of branching venules

DIC image

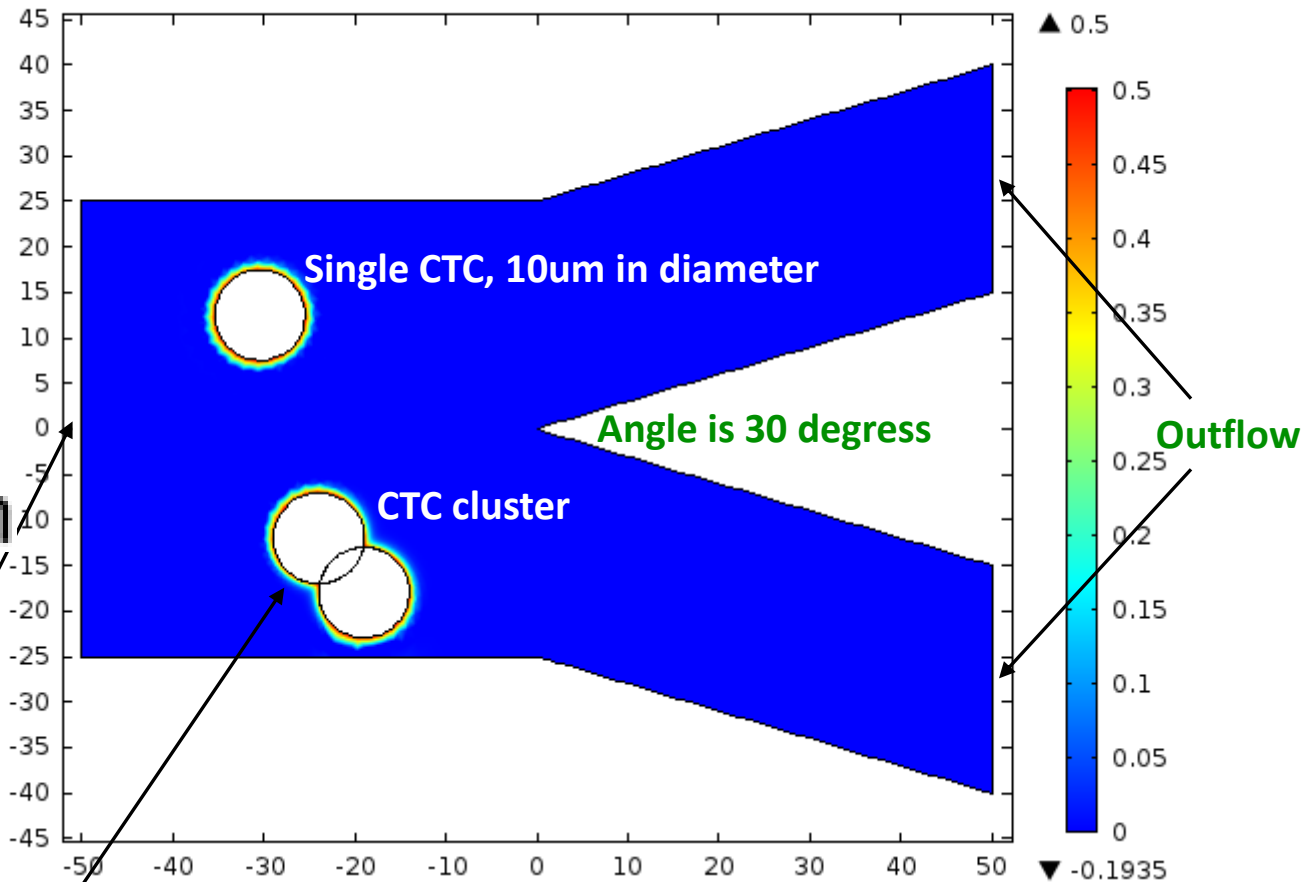


20 μm

Inflow
(flow profile set
up by pressure
gradient)

SW480 colon adenocarcinoma cells

Time=0 Surface: Concentration (mol/m³)



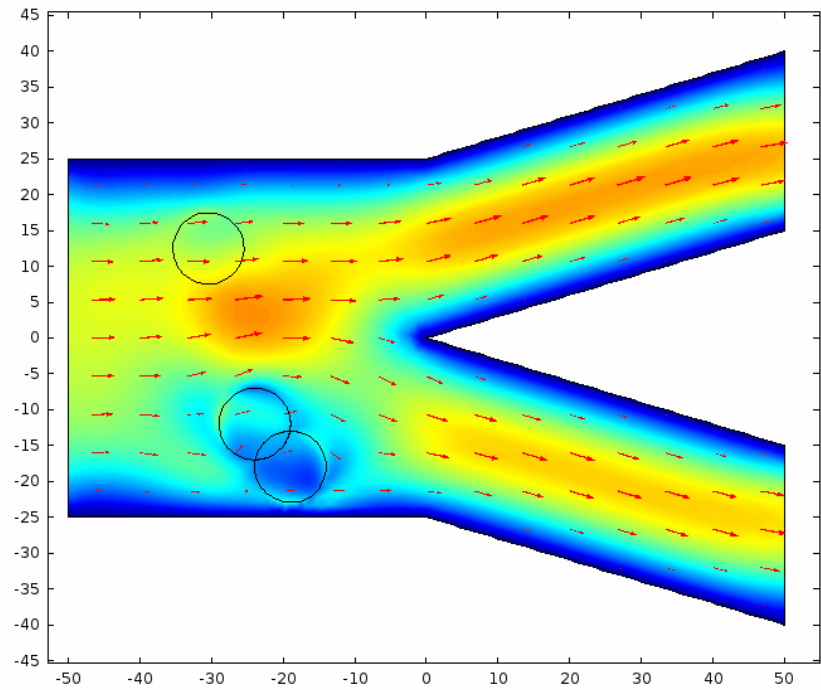
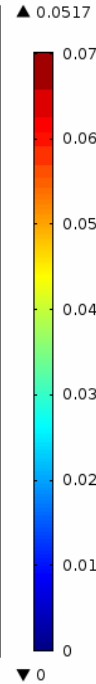
Concentration field of
thrombin on surface of CTC

Simulation of CTC cluster under flow

Blood velocity field

Time=0 Surface: Velocity magnitude (cm/s) Arrow Surface: Velocity field (Spatial)

COMSOL MULTIPHYSICS

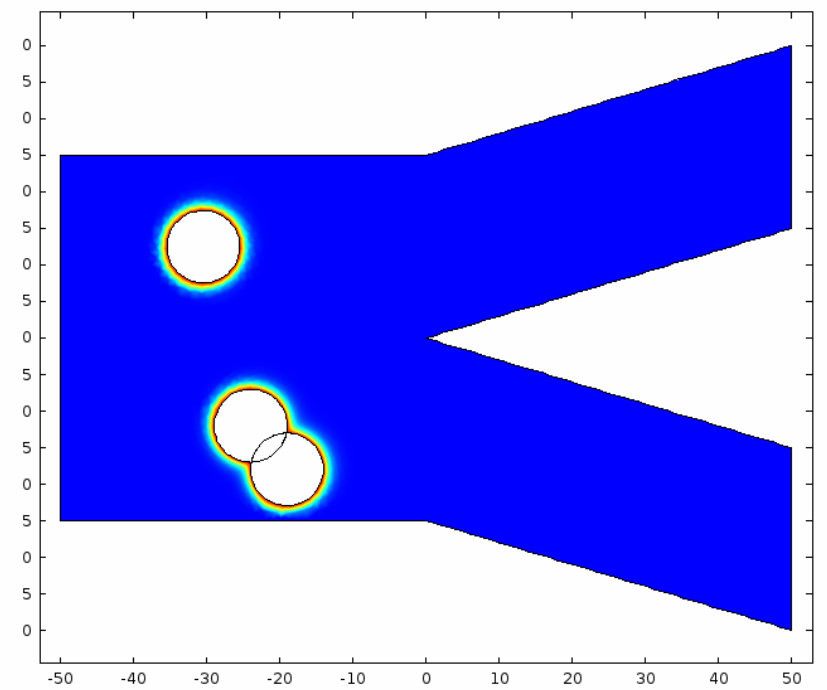
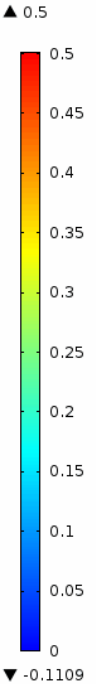


Max velocity = 0.08 cm/s

Thrombin concentration field

Time=0 Surface: Concentration (mol/m³)

COMSOL MULTIPHYSICS



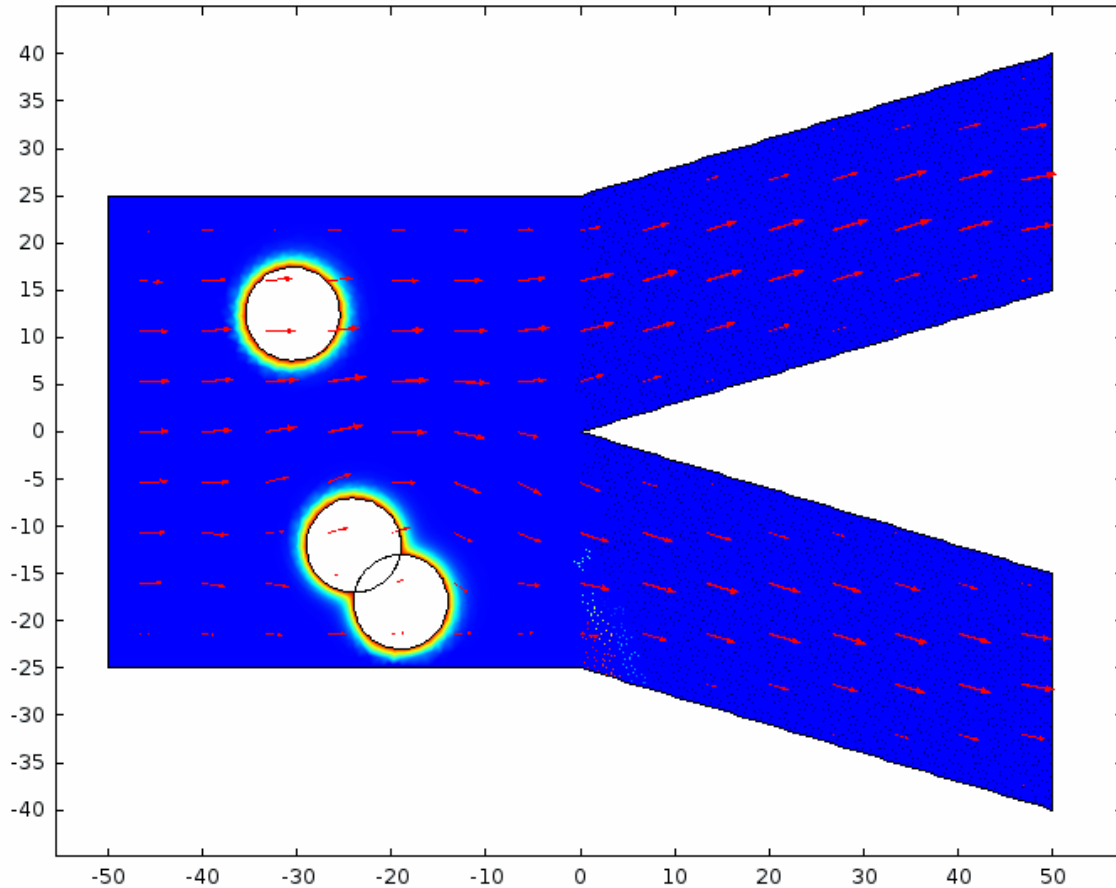
Diffusion coefficient = $3e-7$ cm²/s

Simulation of CTC cluster under flow

Particle tracking for thrombin field

Time=0 Surface: Concentration (mol/m³)

COMSOL
MULTIPHYSICS

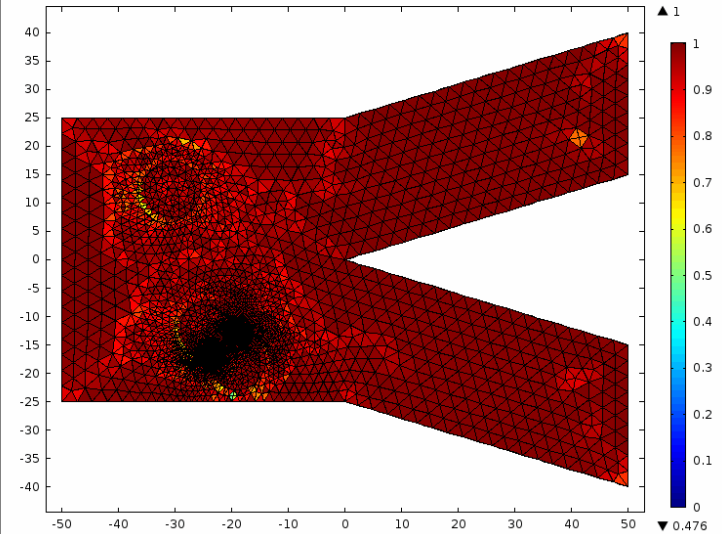


1500 particles in simulation

Adaptive mesh generation

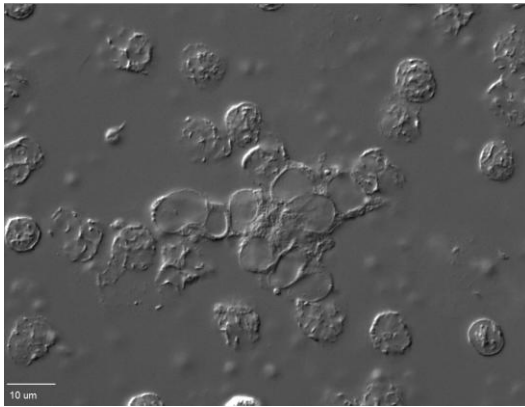
Time=0 Mesh

COMSOL
MULTIPHYSICS

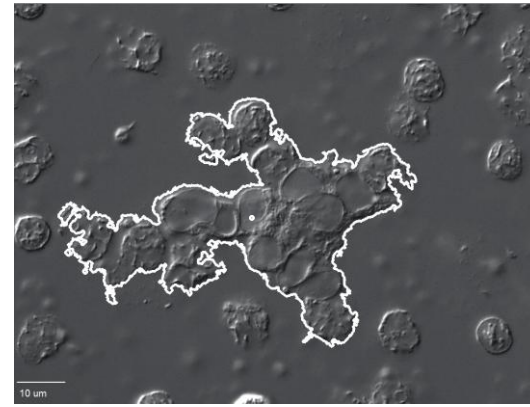


Complete mesh consists of
5696 domain elements and
334 boundary elements

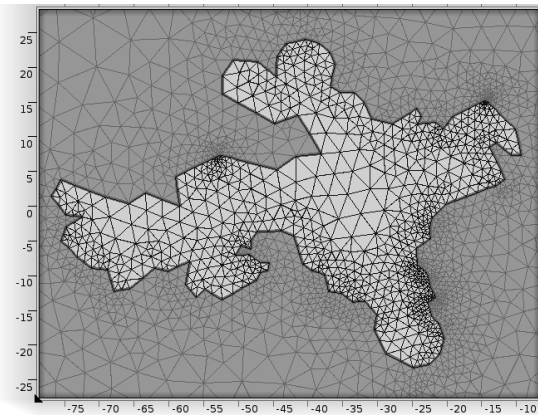
Patient sample to computational simulation



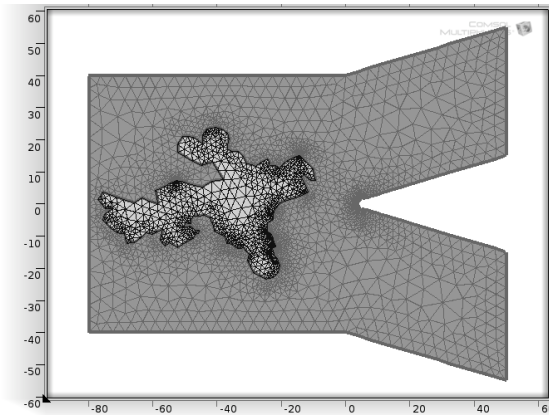
(a) DIC image of lung cancer cluster.



(b) Outline of cluster is obtained using Cell Profiler. Center of cluster is marked by white dot.

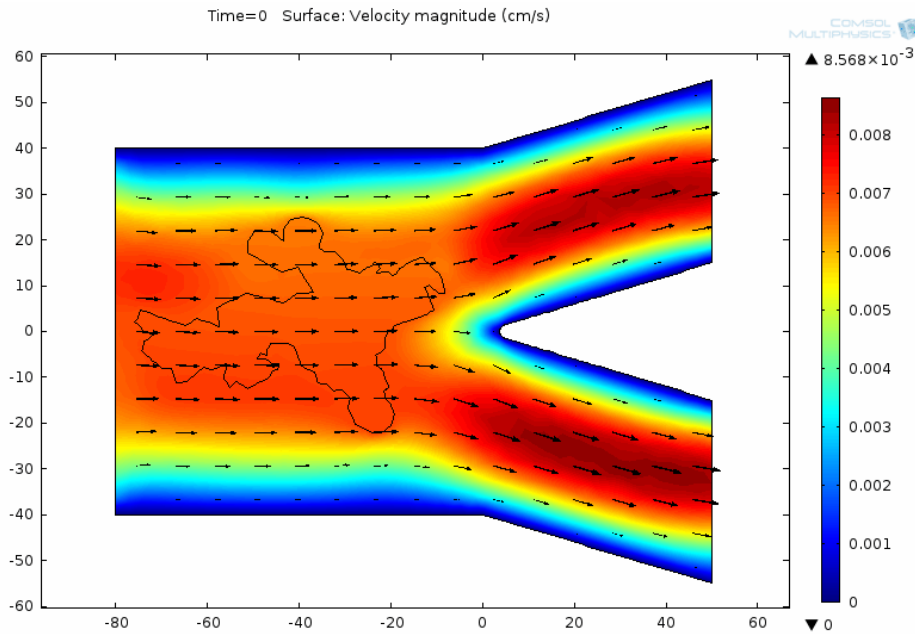


(c) Shape of cluster is generated in computational domain (COMSOL).



(d) Cluster is placed in a branching venule, with the cluster centered vertically in the middle of the venule.

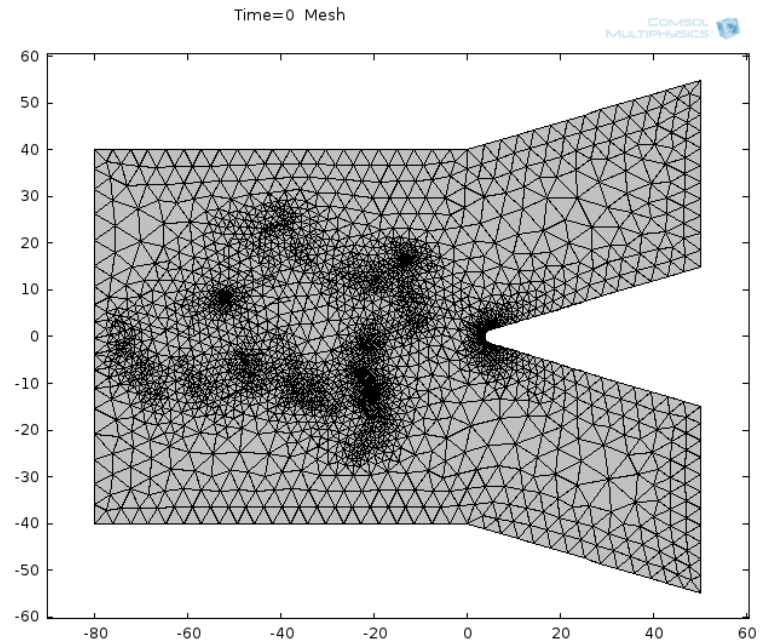
Blood velocity field



Maximum velocity $\sim 0.008 \text{ cm}^2/\text{s}$

Upon reaching the “fork in the vessel”, the cluster begins to deform and significantly obstructs the blood flow.

Mesh generation

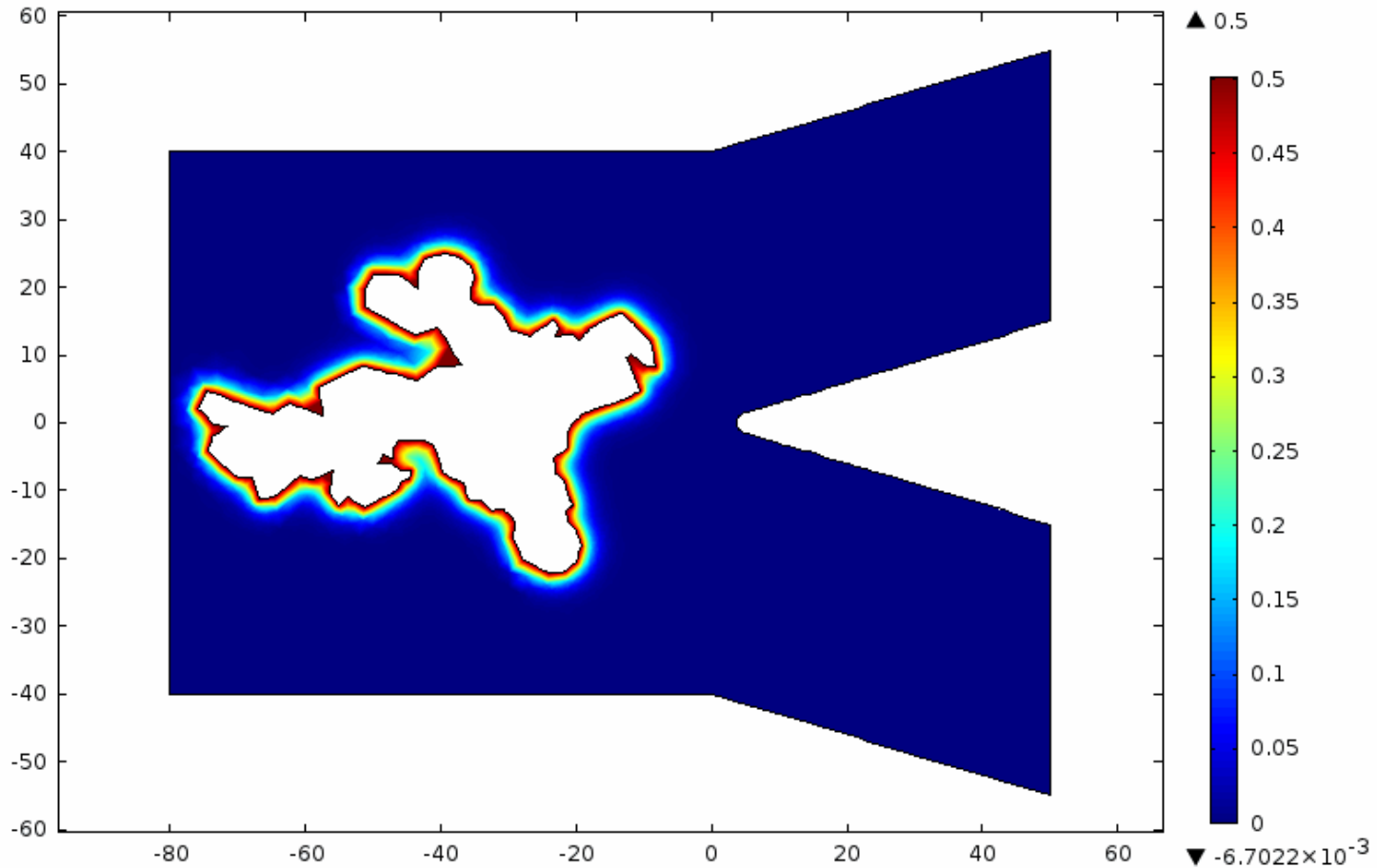


Complete mesh consists of 4872 domain elements and 346 boundary elements.

Concentration field of thrombin

Time=0 Surface: Concentration (mol/m³)

COMSOL
MULTIPHYSICS

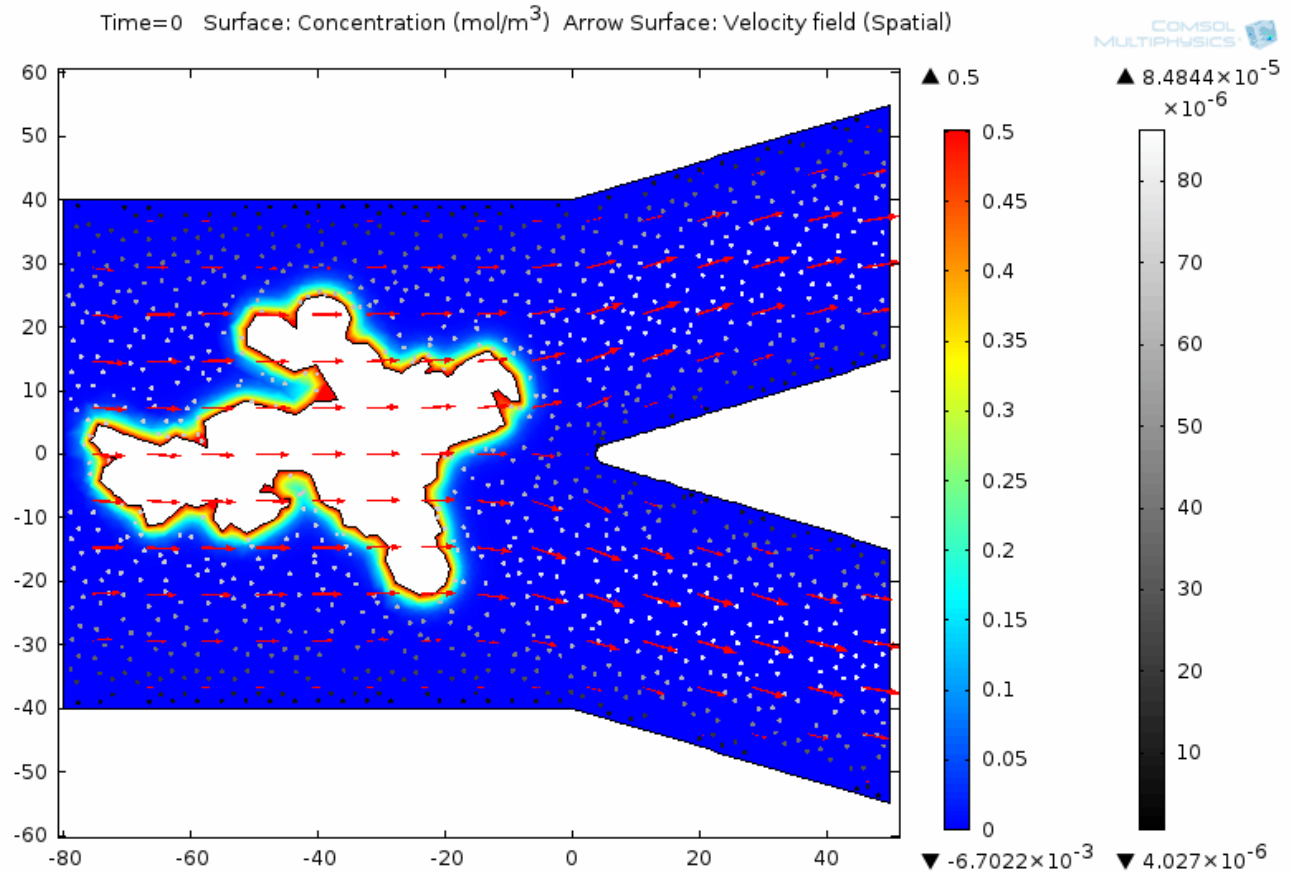


Diffusion coefficient = $1e-7$ cm²/s

Particles are tracking velocity field with concentration field shown

$$\frac{\partial \vec{q}}{\partial t} = \vec{u}$$

1000 particles are released at t = 0 s

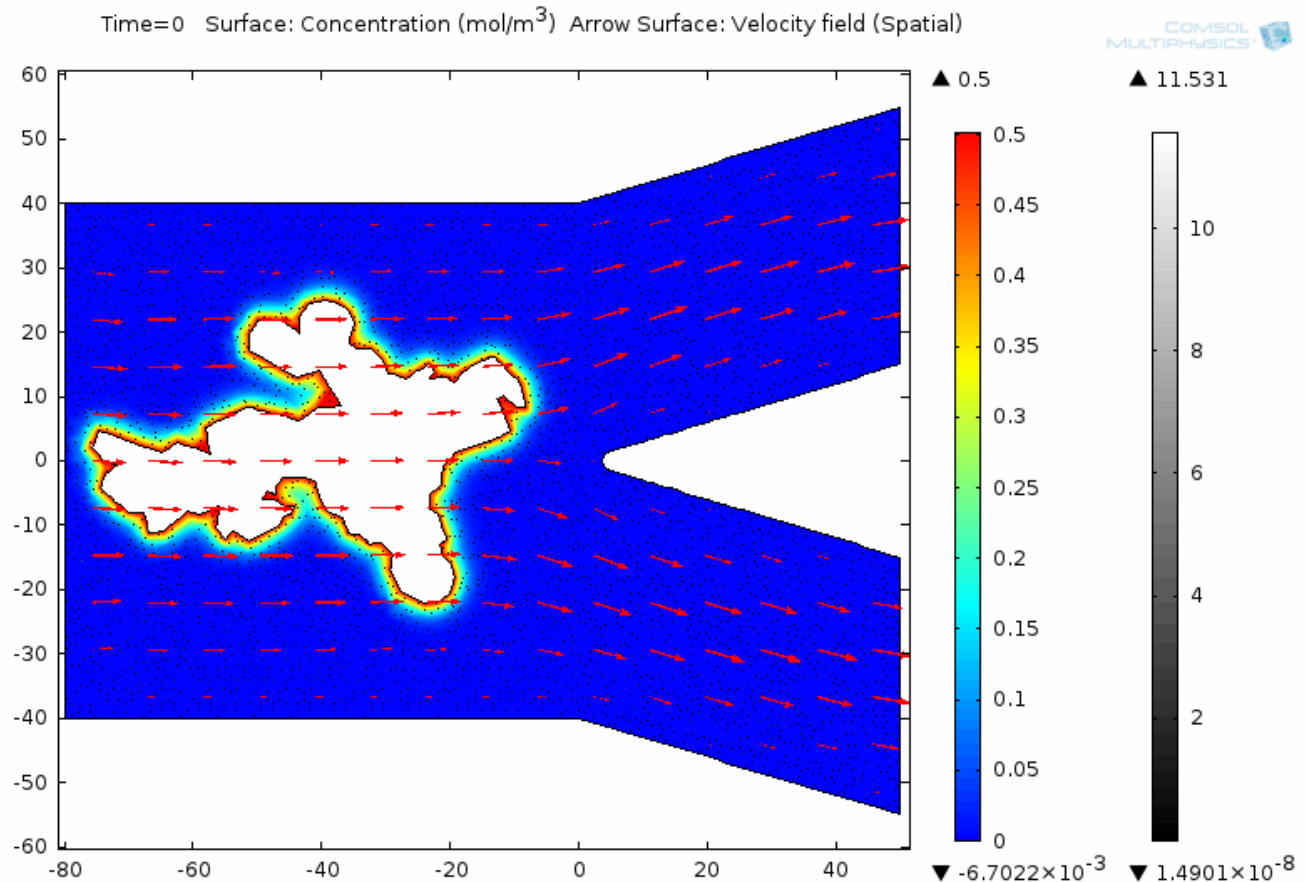


The particles clearly display the parabolic flow profile of the blood, and show that blood flow in the lower branch is slightly faster than in the upper branch.

Particles are tracking concentration field

$$\frac{\partial \vec{q}}{\partial t} = \frac{1}{c_0(C + \epsilon)} \left(\frac{\nabla C}{\|\nabla C + \epsilon\|} \right) \text{ where } C_0 \text{ is a velocity scaling coefficient}$$

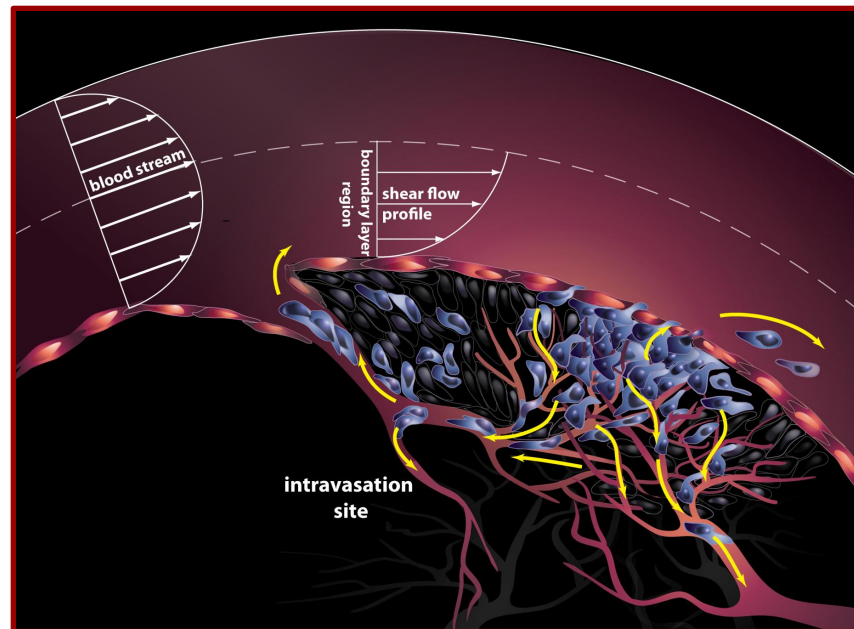
5000 particles are released four times at $t = 0, 0.15, 0.3, 0.45$ s



The particles show that early in the simulation, the thrombin field closely resembles the shape of the cluster, whereas later in the simulation, the thrombin field is dictated by the blood vessel geometry.

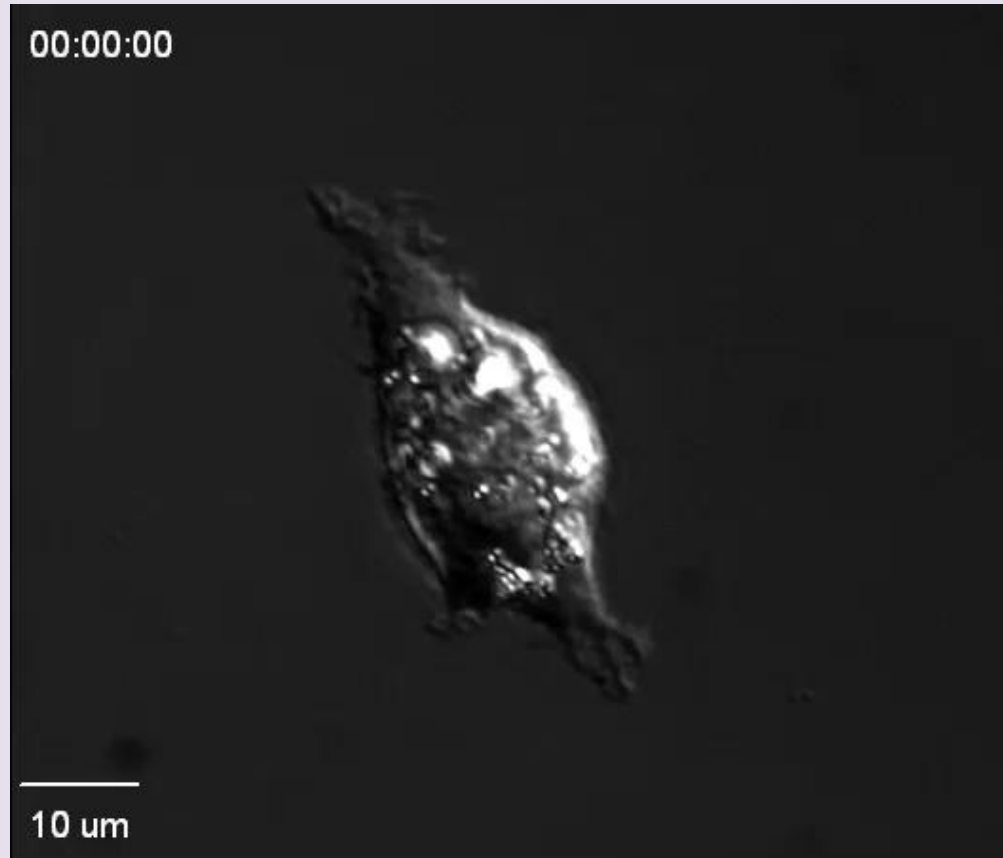
4. Building low-dimensional deformation models of CTCs using 'Active Shape Algorithms'

- Bridging the gap between experiment and computations



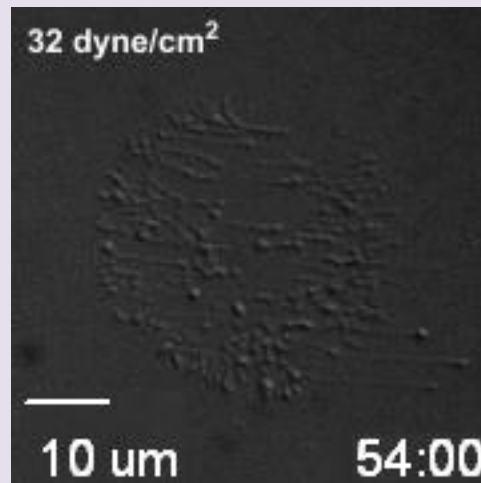
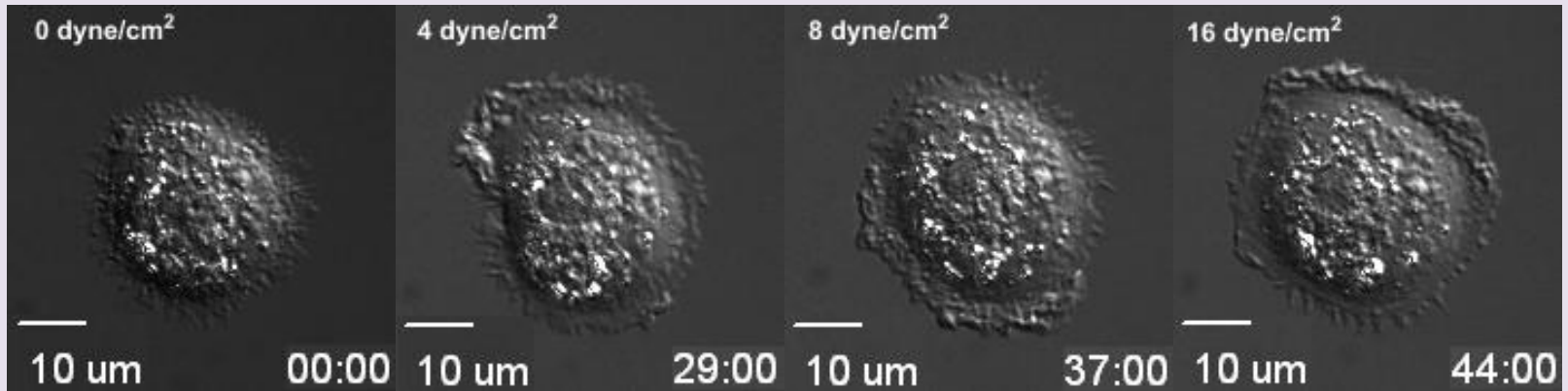
MDA-MB-231 breast cancer cell in flow

(McCarty Lab OHSU)



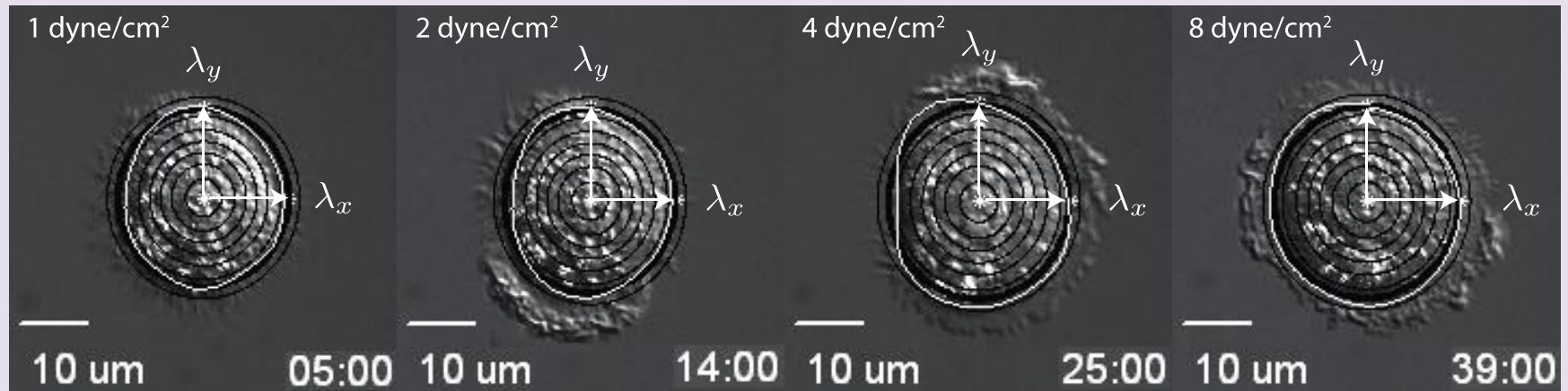
Low-d deformation model of cancer cell release

(McCarty Lab OHSU)



Low-d deformation model of cancer cell release

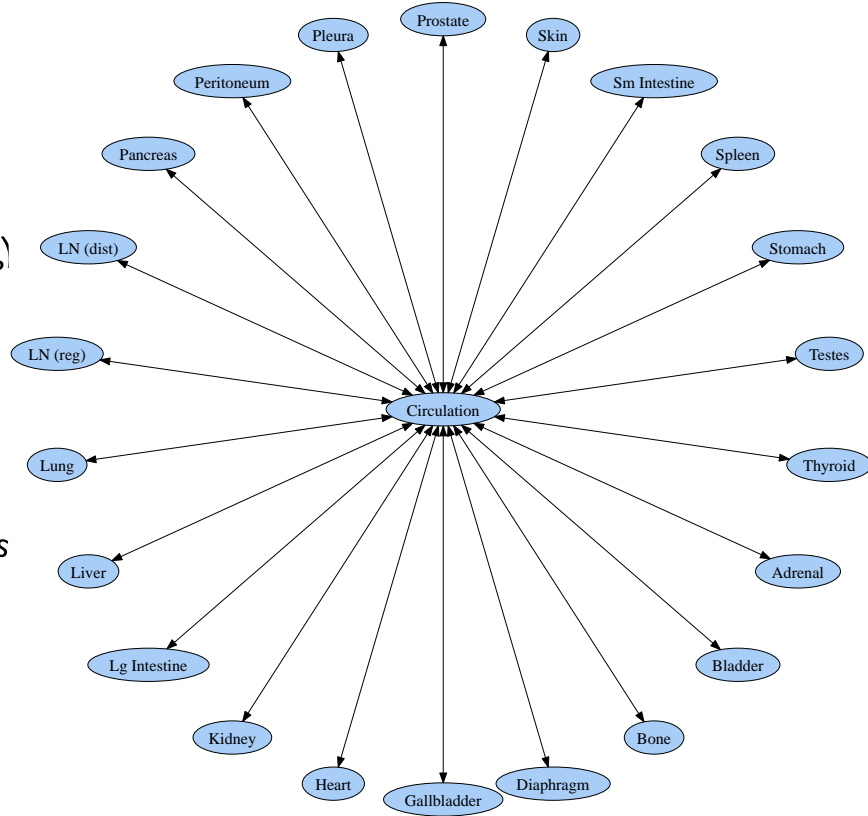
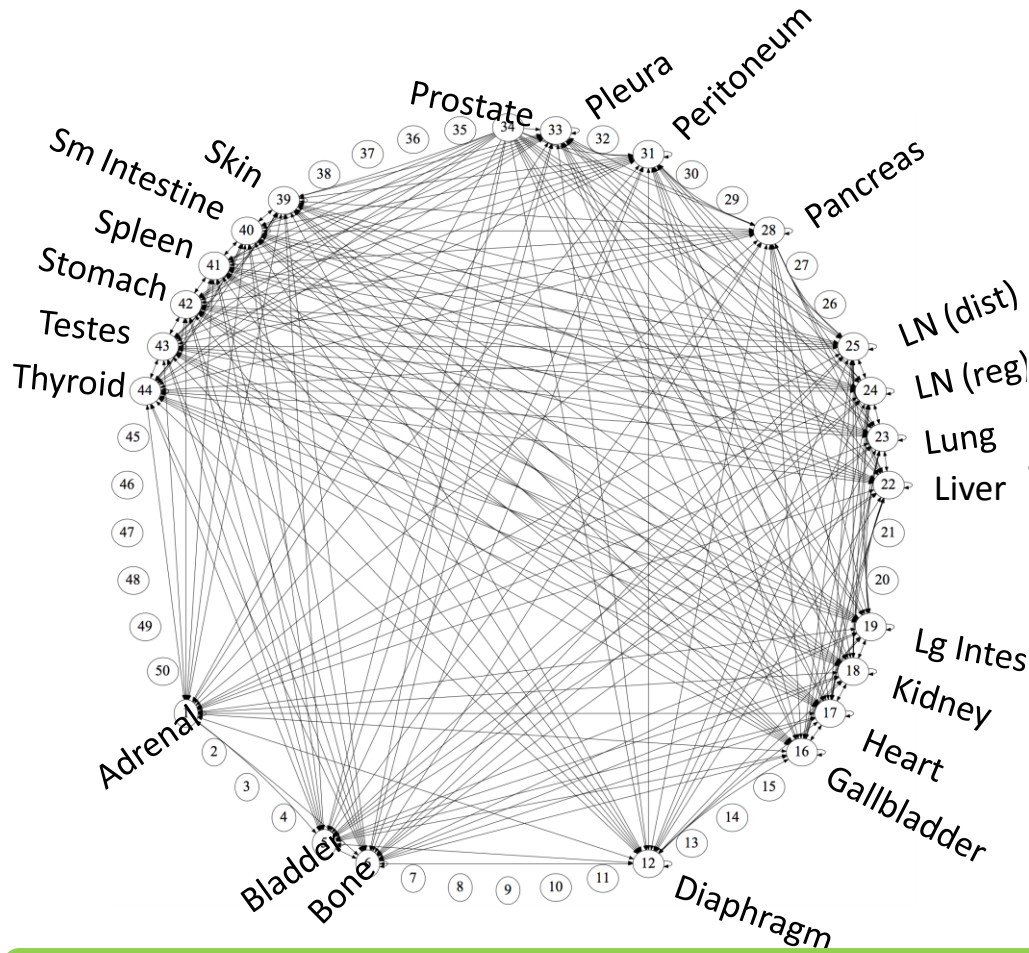
- Active shape modeling gives principal modes of deformation
- Used to 'train' parameters in an 'empirical constitutive equation' for a low-dimensional model
- The model is then used in a stochastic Stokes flow simulation of cell release



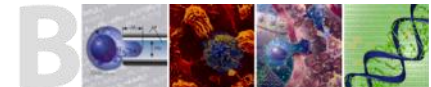
5. Cell-trafficking models based on evolutionary game theory

- Building a dynamical system on the anatomical network that tracks the evolutionary properties of different cell populations

22 anatomical sites for prostate cancer



- 22 cell types
- Preoncogenic/Postoncogenic cancer Boolean (**True** or **False**)
- Each anatomical site has indigenous cell type
- Prostate is initially populated with cancerous & healthy cells



Cell Attributes

Location 1,...23 (including circulation)
Type 1,...22
Cell ID 1,...,n
Preonc/Postonc Boolean: true/false

Birth time
Death time
Circulation time
Parent Cell
Previous Locations [1,23,2,23]

Primary Tumor Site: 1

[Primary tumor cancer cells]

Birth Rate $\mu^{(1)}$

Death Rate $\beta^{(1)}$

Mutation Rate $\gamma^{(1)}$

Circulation Rate $\lambda^{(1)}$

Secondary Sites: $i = 2, \dots, 22$

[Healthy cells]

Birth Rate $\mu^{(i)}$

Death Rate $\beta^{(i)}$

Mutation Rate $\gamma^{(i)}$

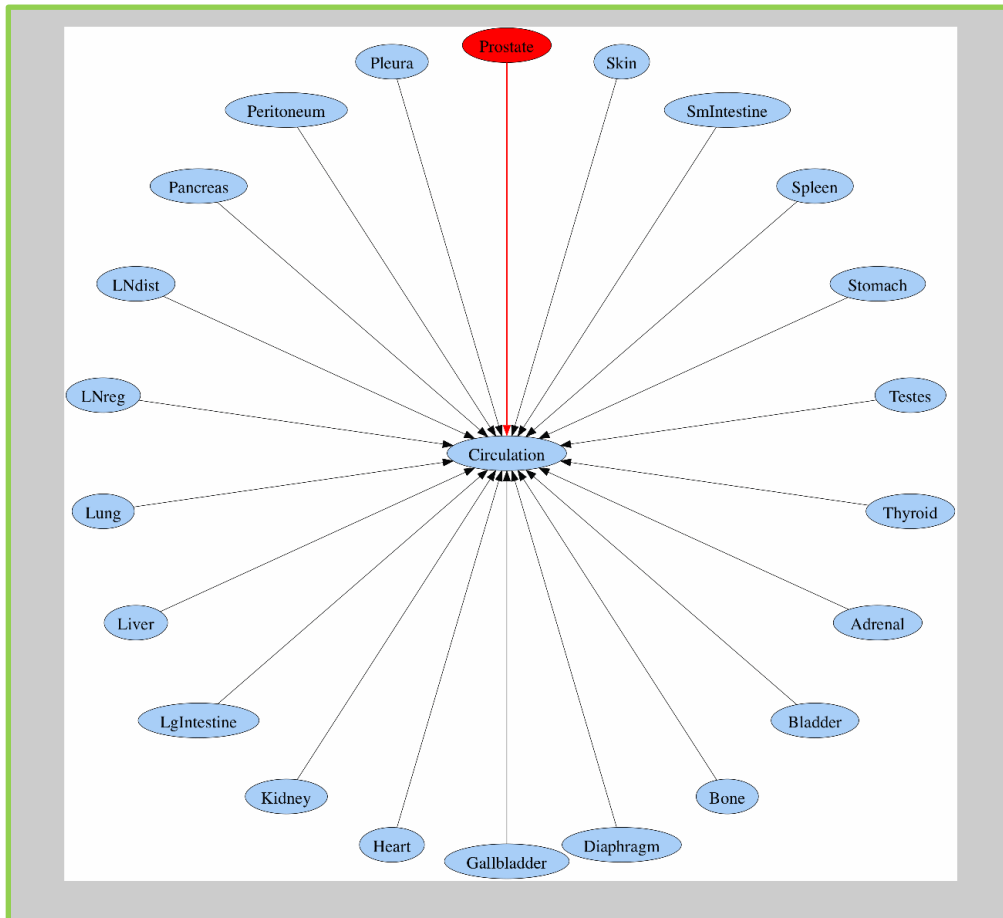
Circulation Rate $\lambda^{(i)}$

23rd location is circulation

Entering Circulation

- Probability of entering entering site i

$$p(i) = 1 - e^{-\lambda^{(i)} t}$$



Dynamics in Circulation

- Holds many cell types at each time step
- Cell death can occur, but not birth
- No mutations
- No replicator equations

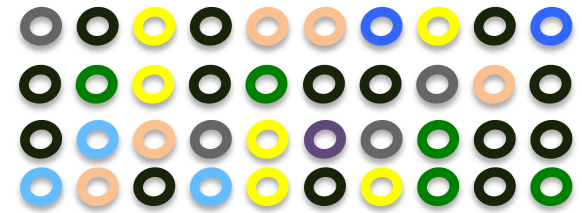
- Entering Circulation

$$p_{(i)} = 1 - e^{-\lambda^{(i)} t}$$

- Exiting Circulation
 - Based on Markov transition probabilities

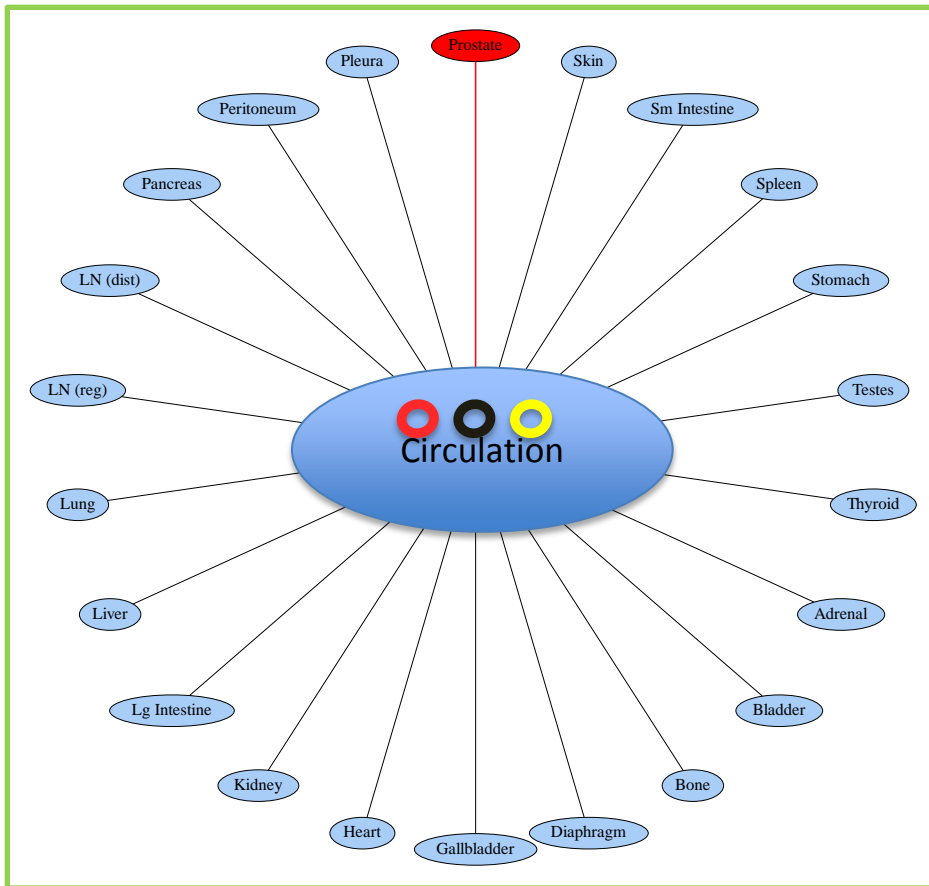
- Preoncogenic Prostate Cell
- Postoncogenic Prostate Cell
- Healthy Prostate Cell
- Liver Cell

Circulation



Other colors indicate healthy cells from various anatomical sites

Circulation



- Prostate

$$p_1 = 1 - e^{-\lambda^{(i)}t}$$

- Each anatomical site has indigenous cell type

- Healthy Prostate Cell
- Postoncogenic Prostate Cell
- Liver Cell

Replicator Equations

- Large cell population of n types, proportions given by the state vector, whose components sum to 1:

$$\vec{x} = (x_1, x_2, \dots, x_n) \in S_n$$

$$\sum_{i=1}^n x_i = 1$$

- Fitness landscape of cell type i given by:

$$f_i = (Ax)_i = a_{i1}x_1 + \dots + a_{in}x_n$$

A = payoff matrix

Ex: $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$

- Average fitness of the cell population:

$$\phi = \vec{x} \cdot \vec{f}$$

- Replicator equation describing cell population dynamics:

$$\dot{x}_i = \sum_{j=1}^n x_j f_j(\vec{x}) q_{ji} - \phi(\vec{x}) x_i$$

- Q is the mutation matrix
- Controls all mutation rates from type i to j .
- Each row must sum to 1.

Ex: Q allows no mutations. $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Neutral Payoff Matrix

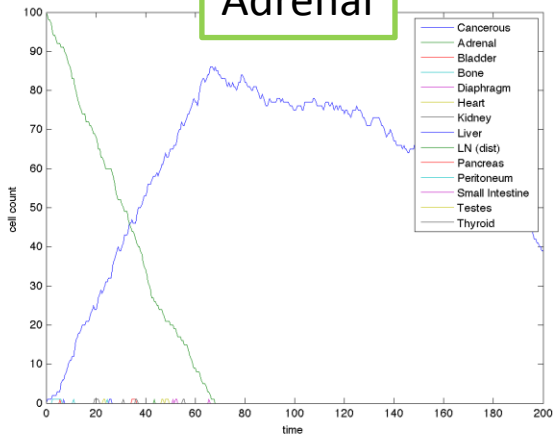
$$A = \begin{matrix} & \begin{matrix} \text{Preoncogenic} \\ \text{Postoncogenic} \\ \text{Adrenal} \\ \text{Bone} \\ \text{Liver} \\ \text{Prostate (Healthy)} \end{matrix} \\ \begin{matrix} \text{Preoncogenic} \\ \text{Postoncogenic} \\ \text{Adrenal} \\ \text{Bone} \\ \text{Liver} \\ \text{Prostate (Healthy)} \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{matrix}$$

Annotations: Blue arrows point to the 'Adrenal', 'Liver', and 'Prostate (Healthy)' columns. A red arrow points to the 'Bone' column. Yellow circles highlight the '1' in the top-right cell and the '-1' in the bottom-left cell.

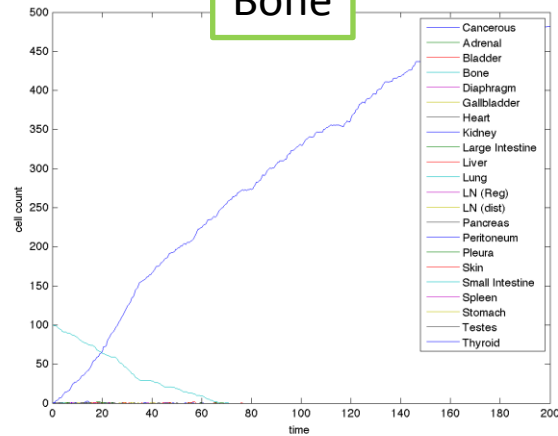
Sample Monte Carlo simulation

Circulation and mutation in sponges and spreaders

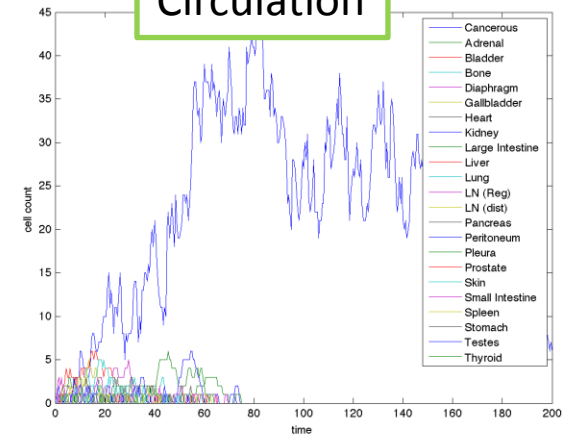
Adrenal



Bone



Circulation



Parameter Values

- Mutation: 1.5
- Circulation: 1.0
- Remove from circulation: 1.5
- Birth/Death: 0
- Cancerous Birth: 0

Questions?