Active Smectics – Emergent Stripe Patterns in Simple Active Particle Models

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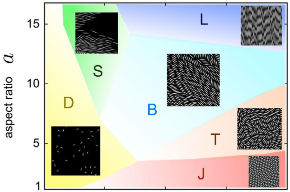
What is an "active" smectic?

- **Active Smectics (AS)**: Non-equilibrium phase of self-propelled objects (polar or nematic) with directional order (ferromagnetic or nematic) **and** positional order along one dimension (2D: "stripes", 3D: "layers")

vibrated rice

Narayan et al, J Stat Mech (2006)

self-propelled rods



Wensink et al, PNAS (2012)

Recent theoretical work:

- active smectic A, nematic SPP
  (Adhyapak et al., PRL, 110, 2013)
  active smectic A, polar SPP
  (Chen & Toner, PRL, 111, 2013)
- long-range order in $d = 3$
- quasi long-range order in $d = 2$
- absence of giant-number fluctuations (normal NF: $\Delta N \sim N^{0.5}$)

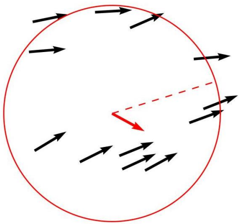
Vicsek-type models with (soft-core) repulsion in 2D

$$\mathbf{r}_i(t+1) = \mathbf{r}_i(t) + v_0 \mathbf{u}_i(t+1)$$
$$\theta_i(t+1) = \arg\left[\epsilon(t)\mathbf{A}_i(t) + \beta\,\mathbf{R}_i(t)\right] + \eta\,\xi(t)$$

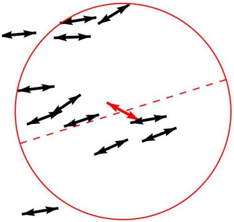
alignment/self-propulsion: ferromagnetic (F) and nematic (N):

$$\mathbf{A}_i = \begin{cases} \frac{1}{N_i}\sum_{j\sim i}\mathbf{u_j} & \epsilon(t) = \ 1 \quad \text{F-model} \\ \frac{1}{N_i}\sum_{j\sim i}\operatorname{sgn}(\cos(\theta_i - \theta_j))\mathbf{u_j} & \epsilon(t) = \pm 1 \quad \text{N-model} \end{cases}$$

F-model



N-model



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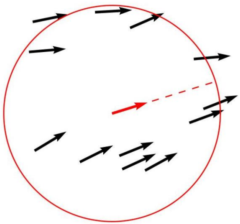
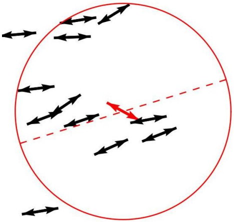


N-model



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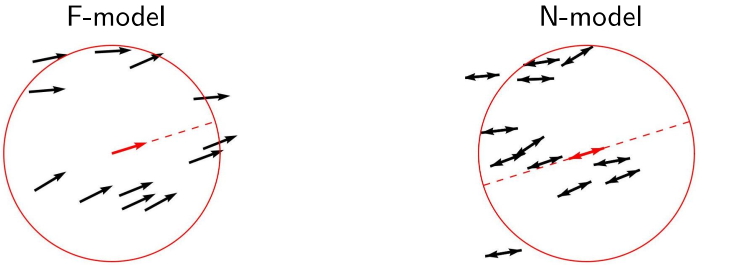
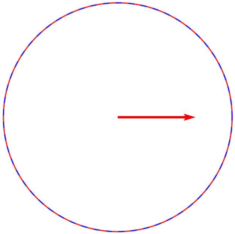


repulsion – different variants

$$\mathbf{R}_i = \frac{1}{N_i - 1} \sum_{j \sim i} g(\phi_{ji}) \hat{\mathbf{r}}_{ji}$$

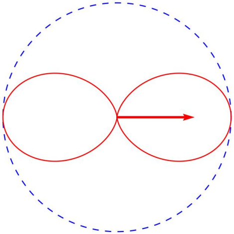
isotropic

$g(\phi_{ji}) = 1$



“ front & back ”

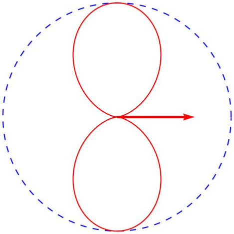
$g(\phi_{ji}) = \cos(\phi_{ji})^2$



$\gamma = 0$

“ left & right ”

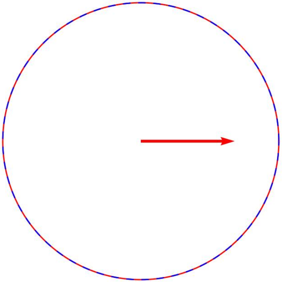
$g(\phi_{ji}) = \cos(\phi_{ji} - \pi/2)^2$



$\gamma = \pi/2$

F-model + isotropic repulsion

$$g(\phi_{ji}) = \cos(\phi_{ji} + \pi/2)^2$$



- **High density behavior for finite repulsion β and low noise η?**

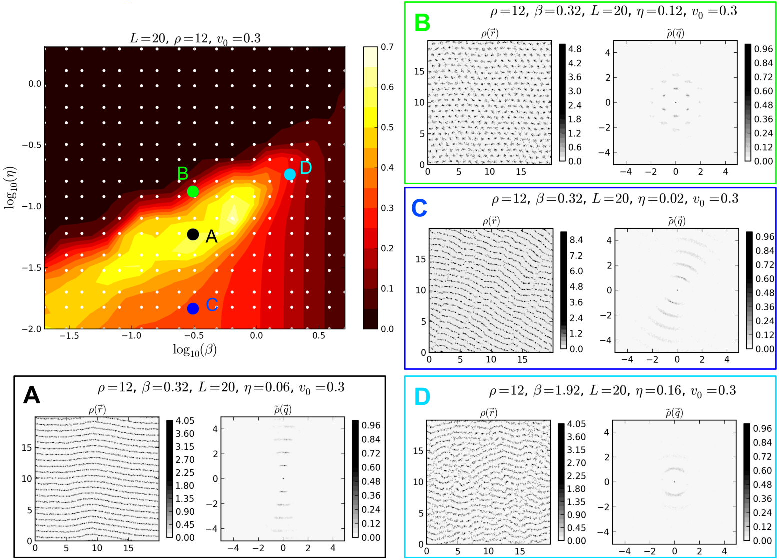
Isotropic Repulsion – Smectic P

Novel, pure non-equilibrium smectic phase

- emergence of stripes oriented parallel to the mean orientation
- N-model: smectic P $\equiv$ smectic A (same symmetries)
- F-model: distinct, pure non-equilibrium smectic phase

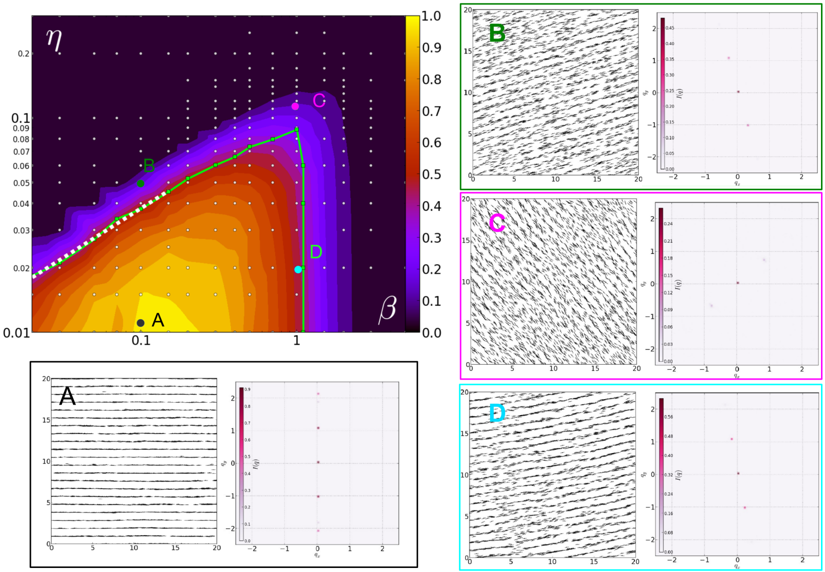
isotropic repulsion, F-model

polar particles + alignment



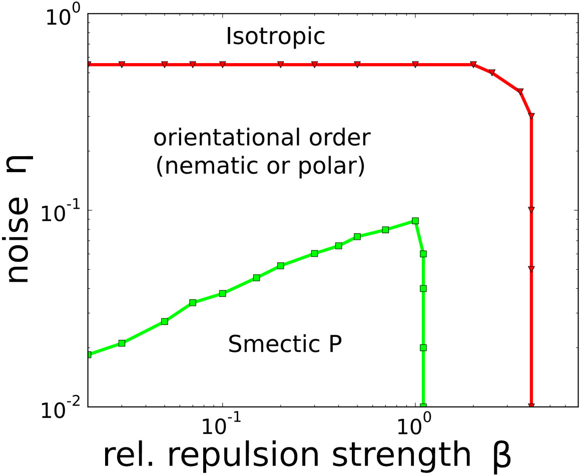
isotropic repulsion, N-model

nematic particles + alignment



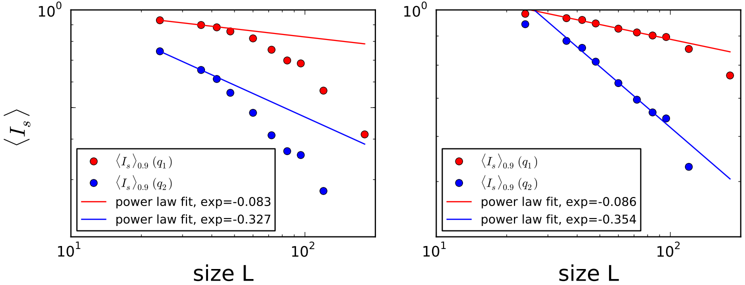
general "phase" diagram

density $\rho_0 = 8$; system size $L = 24$



quasi long-range order?

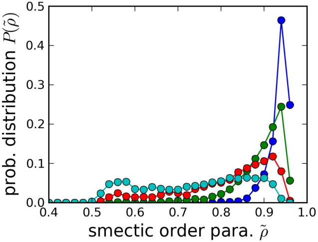
F-model



- theoretical prediction:

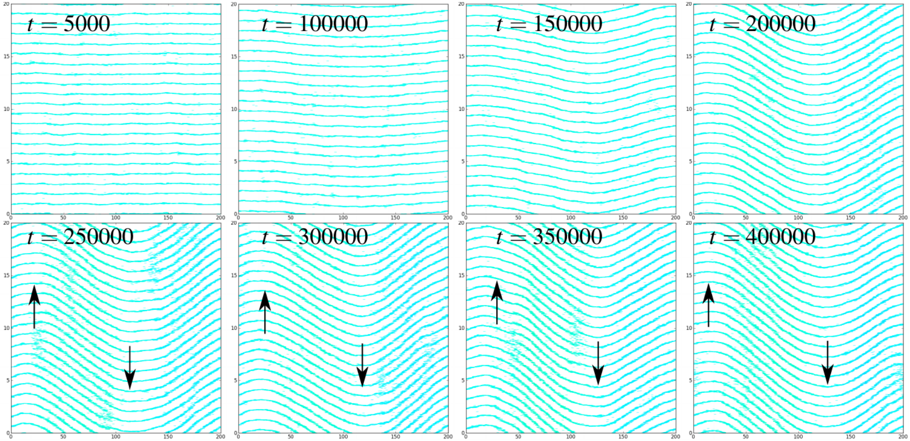
$$I_s \sim L^{-n^2 f(\beta, eta)}$$

- breakdown of power-law scaling at finite system sizes ($L \sim 50 - 100$)



undulation instability – F-model

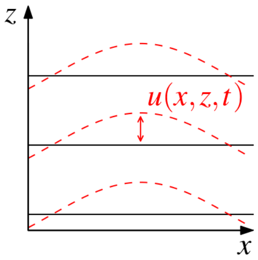
undulation instability – N-model



Undulation instability of smectic P - Theory

N-model: equations of motion for

- density deviations $\delta\rho(x,z,t) = \rho(x,z,t) - \rho_0$
- displacement field $u(x,z,t)$



$$\partial_t u = D_{ux}\partial_x^2 u + B\partial_z^2 u - K\partial_x^4 u + C\partial_z\delta\rho + f_u$$
$$\partial_t \delta\rho = D_{\rho x}\partial_x^2\delta\rho + D_{\rho z}\partial_z^2\delta\rho + C_x\partial_z\partial_x^2 u + C_z\partial_z^3 u + f_\rho$$

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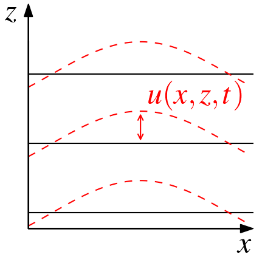

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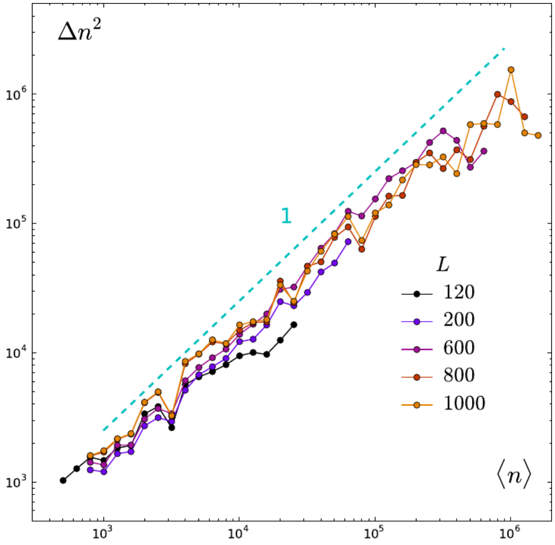
- negative "active tension" $D_{ux} < 0$, induces a finite wavelength instability ➪ emergence of undulations above a critical system size L_c
- stationary undulations for apolar smectic P (N-model)
- travelling undulations for polar smectic P (F-model)

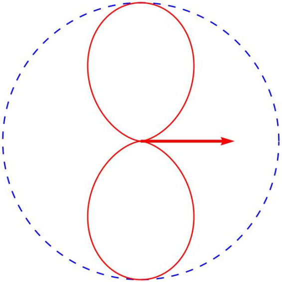
normal number fluctuations

N-model



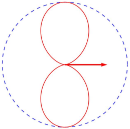
F-model + anisotropic repulsion ($\gamma = \pi/2$)

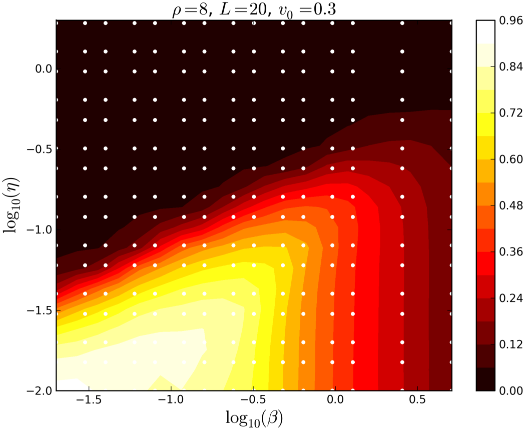
$$g(\phi_{ji}) = \cos(\phi_{ji} + \pi/2)^2$$



- **Improvement of Smectic P via repulsion mainly to the sides?**

F-model + anisotropic repulsion ($\gamma = \pi/2$)

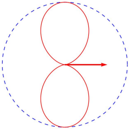




- anisotropic repulsion enhances strongly (local) smectic P order;

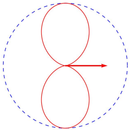
F-model + anisotropic repulsion ($\gamma = \pi/2$)

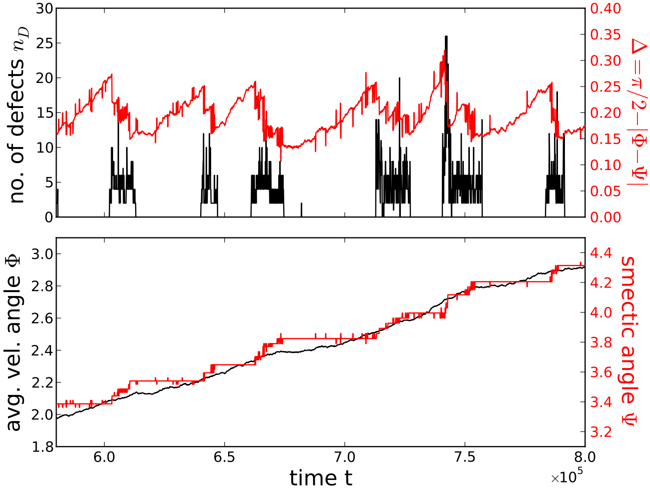
Emergence of global rotation

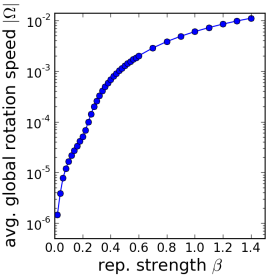


- emergence of global rotation via spontaneous symmetry breaking
  $\rightarrow$ defect nucleation, breakdown of QLRO-scaling

orientation drift drives the rotation of stripes

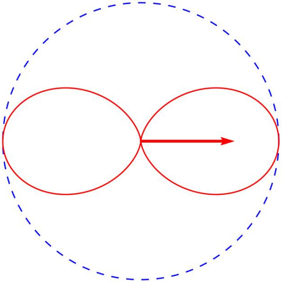






F-model + anisotropic repulsion ($\gamma = 0$)

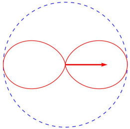
$$g(\phi_{ji}) = \cos(\phi_{ji})^2$$

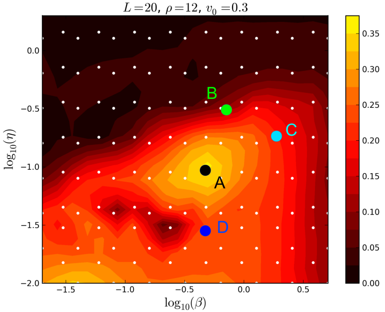


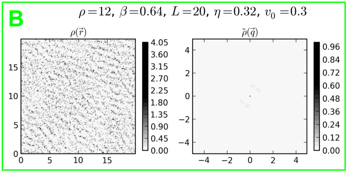
- **Smectic A due to repulsion mainly to the front and back?**

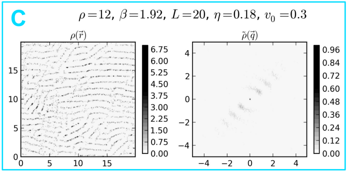
F-model + anisotropic repulsion ($\gamma = 0$)

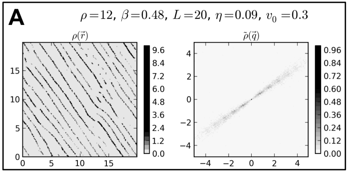
Smectic A due to repulsion to front and back?

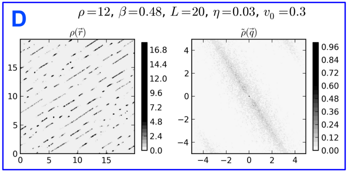






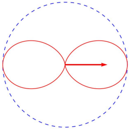






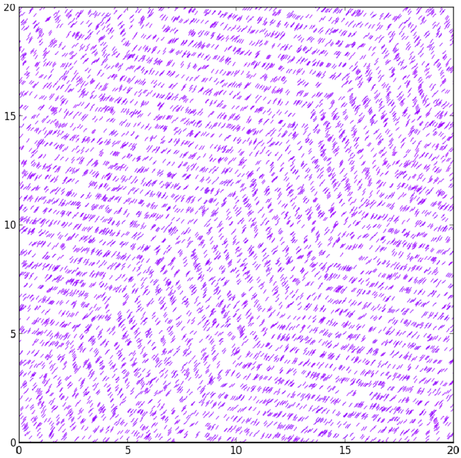
F-Model + anisotropic repulsion ($\gamma = 0$)

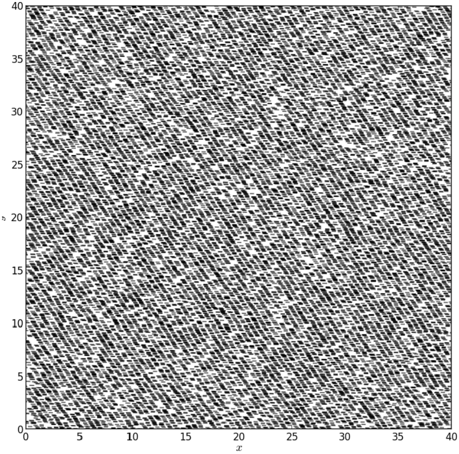
competition of self-propulsion (→SmP) and repulsion (→SmA)



N-Model + anisotropic repulsion ($\gamma = 0$)

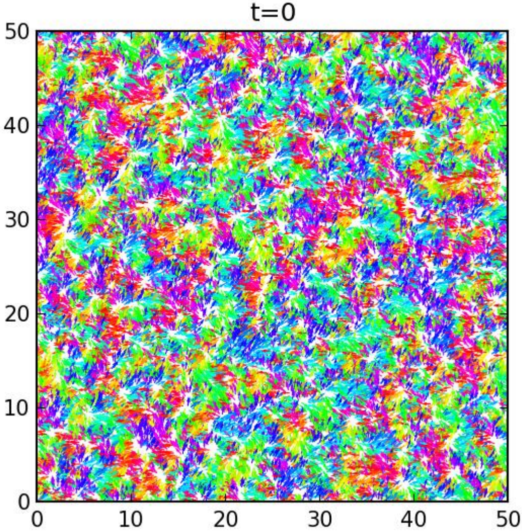
emergence of smectic C order

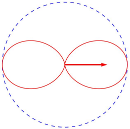




F-model + anisotropic repulsion ($\gamma = 0$)

spontaneous formation of large-scale "swirls"





summary

- emergent (local) smectic order in simple self-propelled particle model
- confirmed also for Langevin-type models with continuous time (not shown)
- systematic classification of observed patterns

Repulsion	isotropic	$\gamma = \frac{\pi}{2}$	$\gamma = 0$
F-model	P, travel. undul.	P, rotation	A, swirls
N-model	P, osc. undul.	P, undulations	C

- breakdown of quasi long-ranged smectic order due to undulations/active nucleation of defects
- absence of giant number fluctuations
- emergence of new rotating phases

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