Swarming in the dirt: Flocking in the Presence of Quenched Disorder

With: Nicholas Guttenberg,
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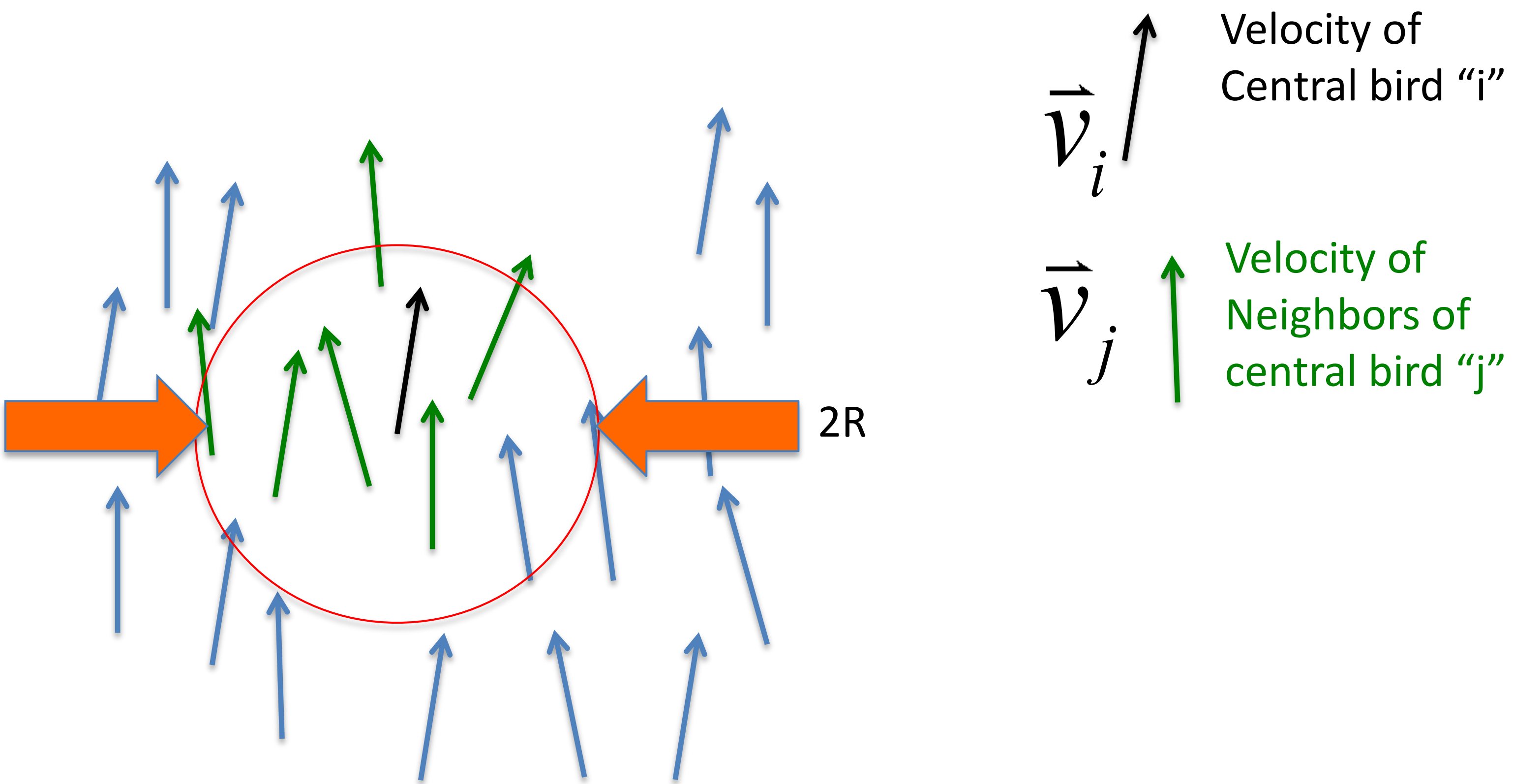
What do I mean by Flocking?

In Vivo: All moving in the same direction

1: Several hundred thousand wildbeest in a moving front, grazing on the Serengeti. .R.E. Sinclair (1977), reproduced by permission of the author and publisher.

the photograph of wildbeest grazing on the Serengeti Plains of Africa, hundreds of

In silica: Vicsek algorithm (e.g.)



What do I mean by Flocking?

- Spontaneous (no road signs (“no external fields”) telling critters which way to go)
- Short ranged interactions
- (no e.g., chemotaxis)

Flocking in two dimensions is ASTOUNDING!

Why? In equilbrium, lower critical dimension

$$d_{LC} = 2$$

Long-ranged order (LRO) impossible

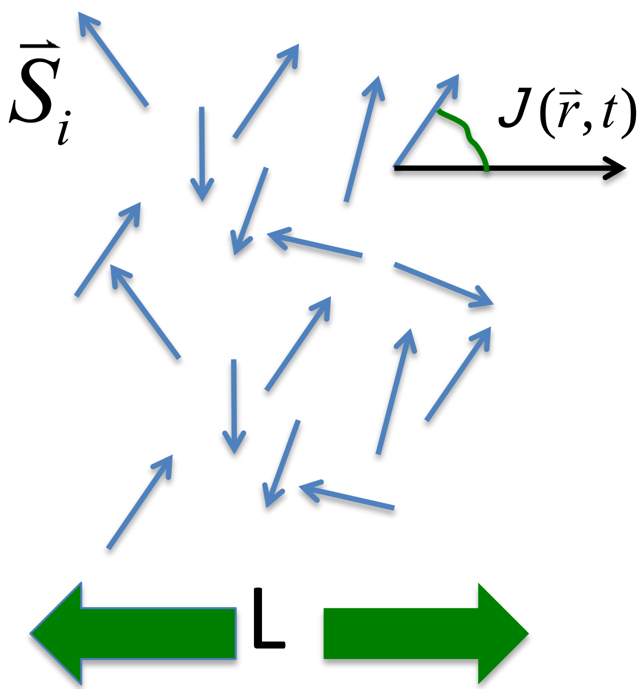
For $d \pounds d_{LC} = 2$

(Mermin-Wagner Theorem)

Mermin-Wagner theorem:

Pointers vs. Flockers

- APS pointers ("XY model"):



Error size

CAN NOT all POINT in same direction in d=2,

$$< J^2 > = D\ln(L) \xrightarrow[L\to\infty]{} \infty$$

"Open up to infinity and become infinity"

but → No Long Ranged order

(JT's Yogi teabag while preparing this slide)

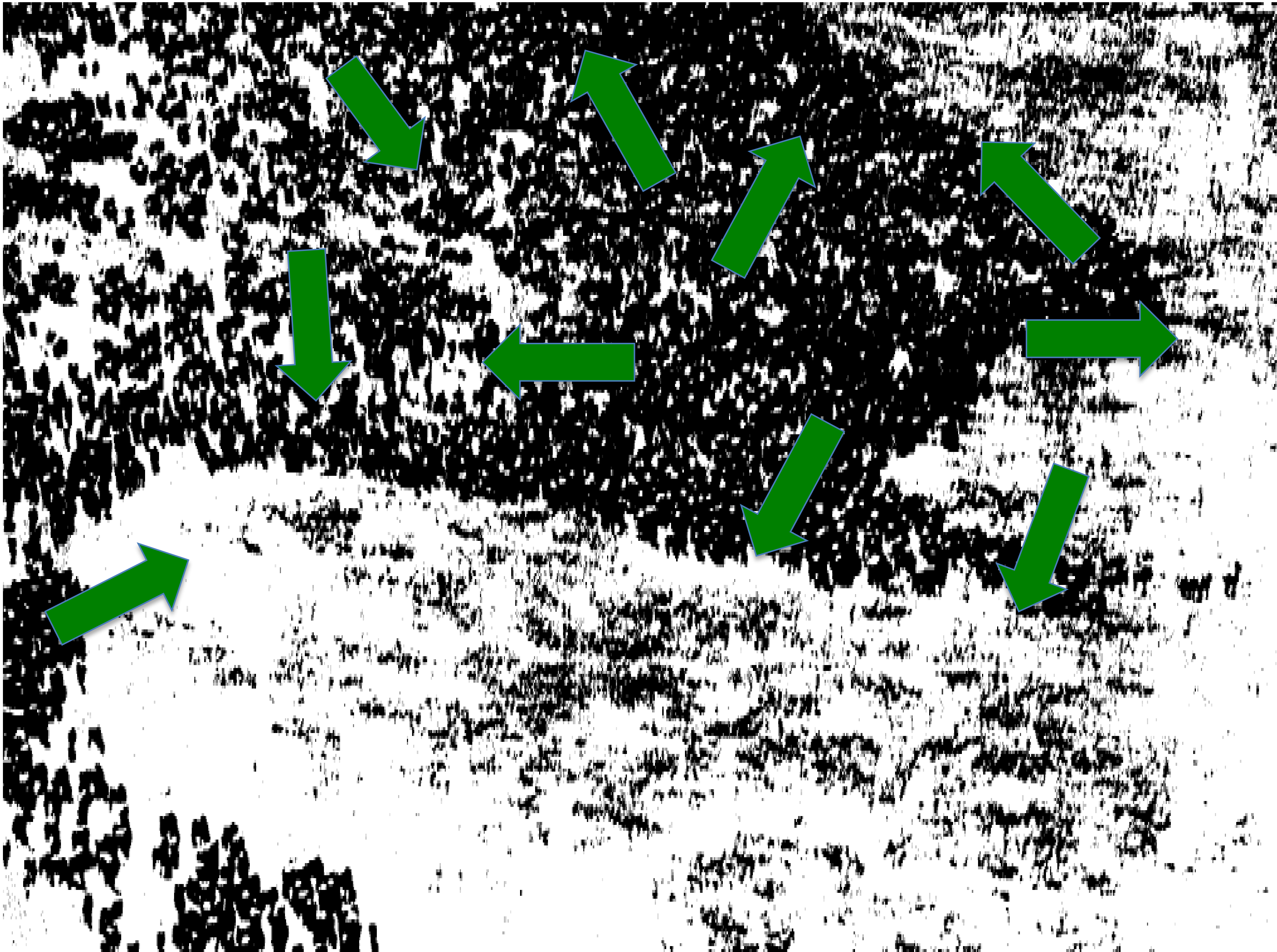
CAN all MOVE in same direction

$$< \vec{S}(\vec{r},t) > = \vec{0}$$

Equilibrium result

This is without signs (“external fields”)

- With signs, easy to order in ANY dimension d
- IF the signs are accurate (consistent)
- What if the signs point in random directions?
- “Quenched disorder”
- (“Quenched” means the signs never change)



With quenched disorder, Flocking in ANY physical dimension (d=2 AND d=3) is even MORE Astounding!

$d_{LC} = 4$ in equilibrium with quenched disorder

(Grinstein and Luther, 1979)

Yet it also happens!

Outline

- I) Review of flocks with “annealed disorder” (i.e., Langevin (time-dependent) noise (with Yu-hai Tu, IBM Watson)
- (J. Toner, Y.-h. Tu, and S. Ramaswamy, Ann. Phys. 318, 170 (2005))
- Surprising result: LRO in d=2 (IMPOSSIBLE in equilibrium)

- II)Flocks with Quenched (i.e., time-independent) disorder (with Nicholas Guttenberg, UO)
- Even more surprising results:
- LONG RANGED ORDER in d=3 (IMPOSSIBLE in equilibrium)
- QUASI-LONG RANGED ORDER in d=2 (vs Short ranged order in equilibrium)

Flocks with annealed disorder
(i.e., Langevin (time-dependent) noise)

I) Microscopic models (Vicsek)

Important points: rotation invariance
locality

II) Mermin-Wagner Theorem: Are birds smarter than nerds?

III) Continuum theory: analogy with fluid mechanics

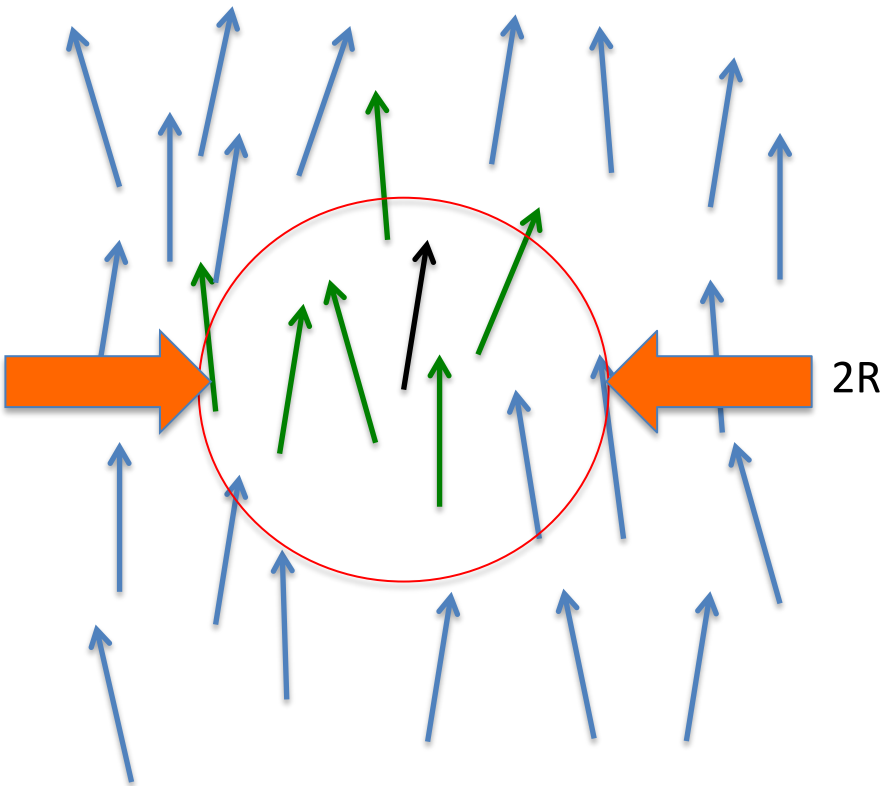
IV) Predictions:

A) Sound modes, whose speeds vanish in certain directions: crucial for quenched disorder

B) How motion beats Mermin-Wagner ("anomalous hydrodynamics")

I) Microscopic Models

Vicsek algorithm:



$\vec{v}_i$ Velocity of Central bird "i"

$\vec{v}_j$ Velocity of Neighbors of central bird "j"

Update rule:

Errors

$$\vec{v}\ (t+1) = v_0 \left(\frac{\sum_j \vec{v}_j(t) + \vec{f}_i(t+1)}{\left| \sum_j \vec{v}_j(t) + \vec{f}_i(t+1) \right|} \right)$$

Speed of birds

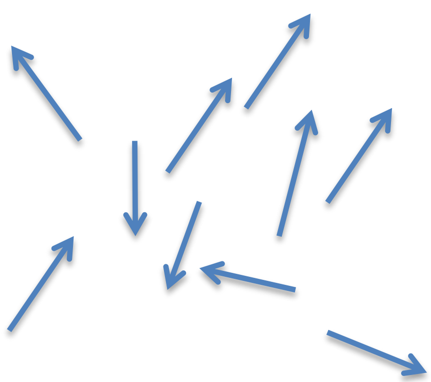
Essential Features of Algorithm

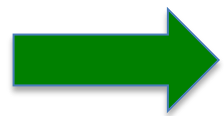
- Only Local interactions: short ranged in space and time
- Ferromagnetic interactions (favor alignment)
- “Birds” keep moving ($\vec{v} \neq \vec{0}$) and making errors

Symmetries:

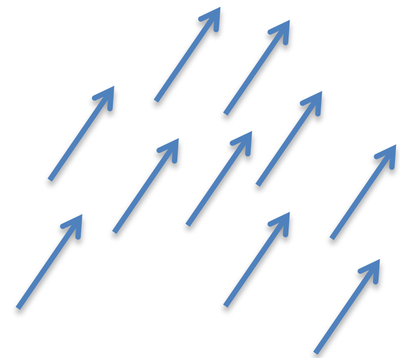
	Dynamics	Phase
Translation Invariance	YES	YES $<\rho(\vec{r},t)> \equiv \rho_0$ $= CONSTANT$
Rotation Invariance	YES	NO $<\vec{v}(\vec{r},t)> \equiv \vec{v}_0 \neq \vec{0}$
Galilean Invariance	NO	NO

Dynamics produces order:





Run algorithm
Many steps (t>>1)



t=0

$$<\vec{v}(\vec{r},t)>=\vec{0}$$

t>>1

$$<\vec{v}(\vec{r},t)>=\vec{v}_0 \neq \vec{0}$$

However…..

This should be **Impossible!**

Why?

Violates **Mermin-Wagner** theorem

Are Birds smarter than nerds?

Continuum Theory of Flocks

- Hard (impossible) to solve microscopic model with ~10^5 birds
- Harder to figure out what happens if you change model (universal vs system-specific)
- Historical analog: Fluid mechanics (Navier, Stokes, 1822):

    No theory of atoms and molecules

    No statistical physics

    No computers, ipad, ipod, etc

- So, how'd they do it?

Continuum Approach

Replace $\vec{r}_i(t) \rightarrow$ Continuous fields:

$r(\vec{r}, t)$: Coarse grained number density

$\vec{v}(\vec{r}, t)$: Coarse grained velocity

Valid for: Length scales L >> interatomic distance

Time scales t >> collision time

Equations of motion for $r(\vec{r},t), \vec{v}(\vec{r},t)$

Make 'em up!

Rules: -Lowest order in space, time derivatives
-Lowest order in fluctuations

$$dr(\vec{r},t)^{o}\ r(\vec{r},t) - < r(\vec{r},t) >$$

$$d\vec{v}(\vec{r},t)^{o}\ \vec{v}(\vec{r},t) - < \vec{v}(\vec{r},t) >$$

Respect Symmetries (for flocks, Rotation invariance)

Worked for fluids, should work for flocks

Our (Yu-hai Tu and JT) idea: same approach, different symmetry

- No Galilean invariance (birds move through a Special “rest frame” (e.g., air, water, surface of Serengeti. Etc.…))

Hydrodynamic equations for Flocks:

Connection to Equilibrium Ferromagnet (Pointers)
(and Mermin-Wagner Theorem):
(“convective
Derivative”)

New terms (forbidden
in NS equations due to
Galilean invariance)

Velocity EOM:

Other “2 v, one grad” terms

“move faster,”

“but not TOO Fast!”

Slowpoke

$$\partial_t \vec{v} + \lambda_1 (\vec{v}\cdot\vec{\nabla})\vec{v} + \lambda_2 \vec{v}(\vec{\nabla}\cdot\vec{v}) + \lambda_3 (\vec{\nabla}|\vec{v}|^2) = a\vec{v} - b|\vec{v}|^2\,\vec{v}$$

$$-\vec{\nabla}P(\rho) - \vec{v}(\vec{v}\cdot\vec{\nabla}P_2(\rho)) + D_B\vec{\nabla}(\vec{\nabla}\cdot\vec{v}) + D_T\nabla^2\vec{v} + D_2(\vec{v}\cdot\vec{\nabla})^2\vec{v} + \vec{f}$$

Anisotropic pressure

Anisotropic
Viscosity

Noise (errors)

Density EOM:

$$\partial_t \rho + \vec{\nabla}\cdot(\rho\vec{v}) = 0$$

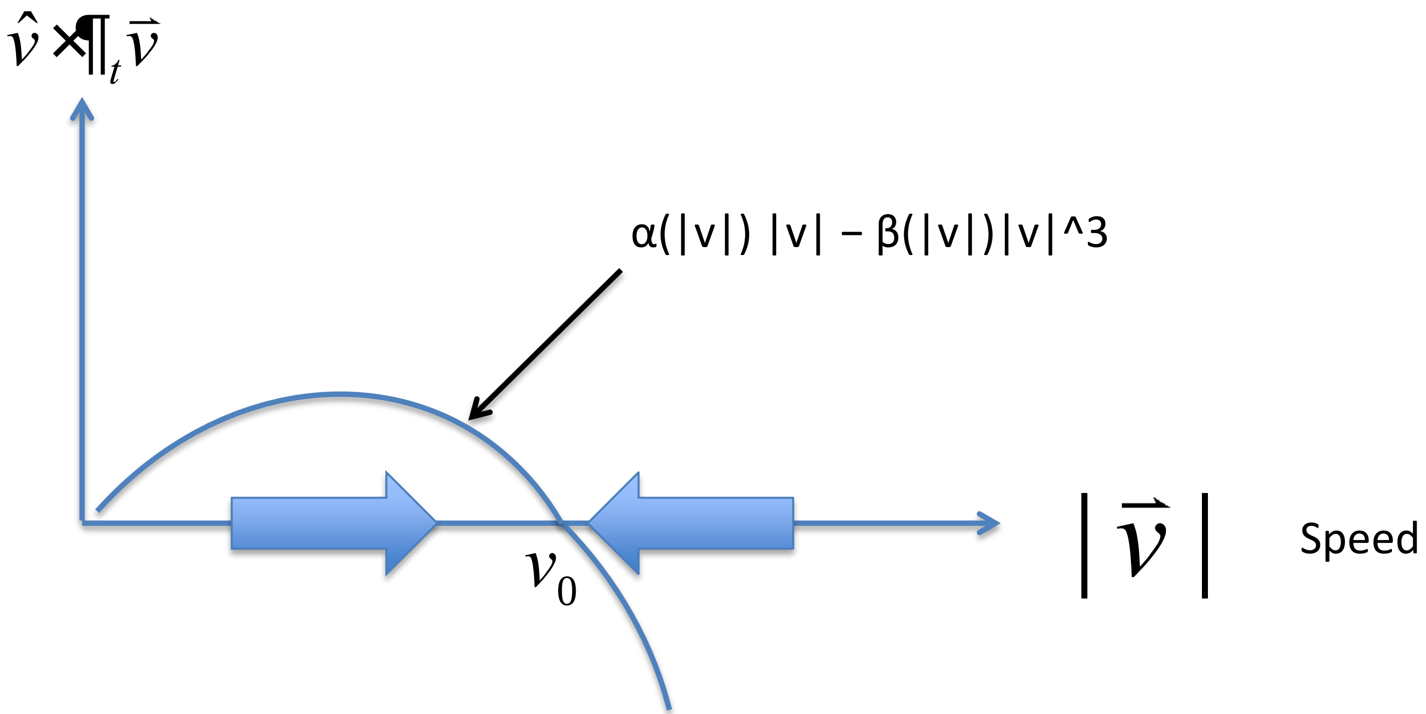
Number conservation
(“immortal” flock)

$\vec{f}(\vec{r}, t)$: **Langevin** white noise (i.e., short range correlated in **TIME** (and space)

$$< f_i(\vec{r}, t) f_j(\vec{r}\,', t') >= D d_{ij} d^d(\vec{r} - \vec{r}\,') d(t - t')$$

Noise strength

Acceleration in direction of motion:



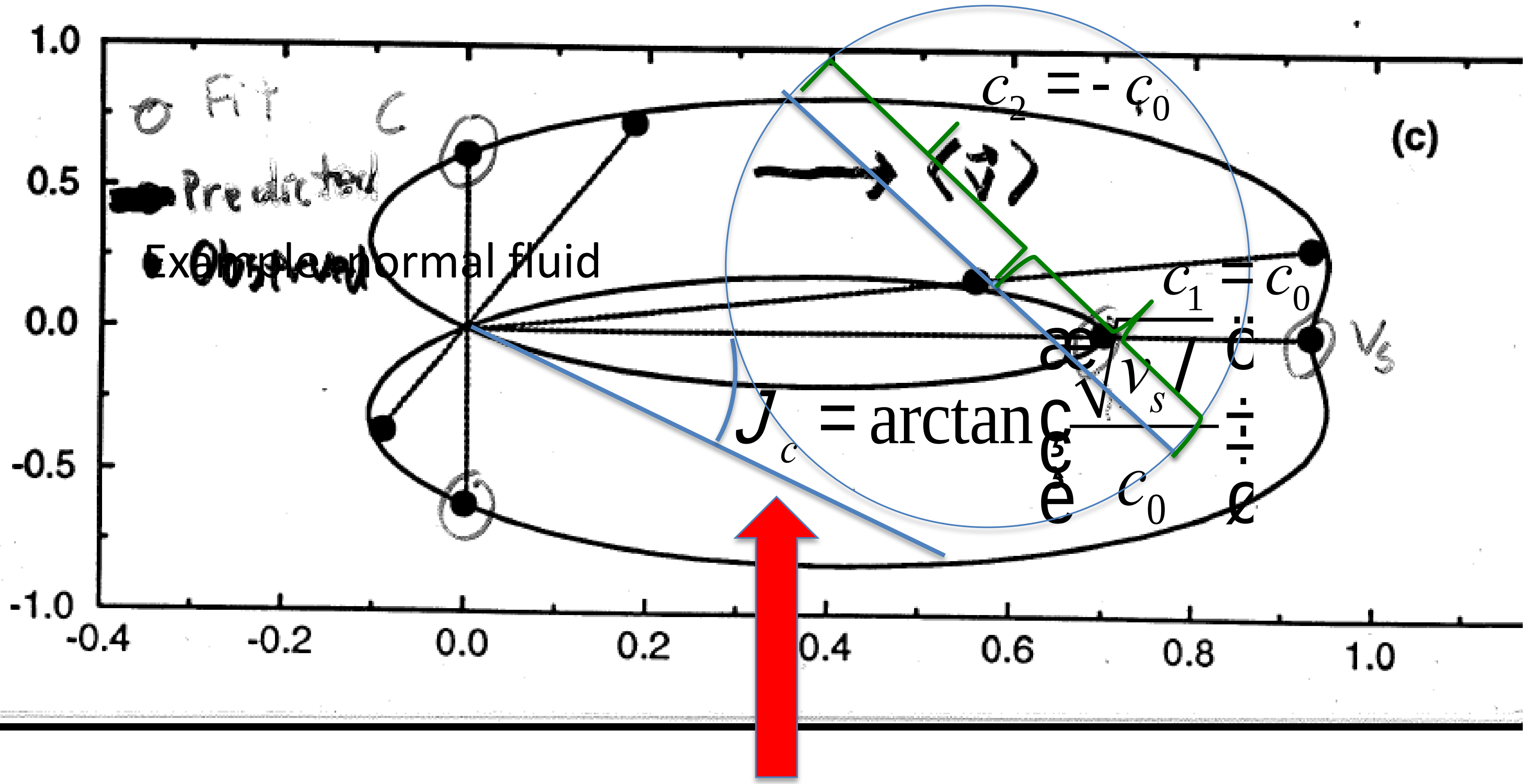
$$\Rightarrow \langle \vec{v}(\vec{r},t) \rangle = v_0 \hat{x}$$

Arbitrary direction (Spontaneously Broken symmetry)

Predictions of hydrodynamic theory

- Sound waves
- Anomalous Hydrodynamics for d<4.
- Long-ranged order in d=2

Polar plot of sound speeds: Flock



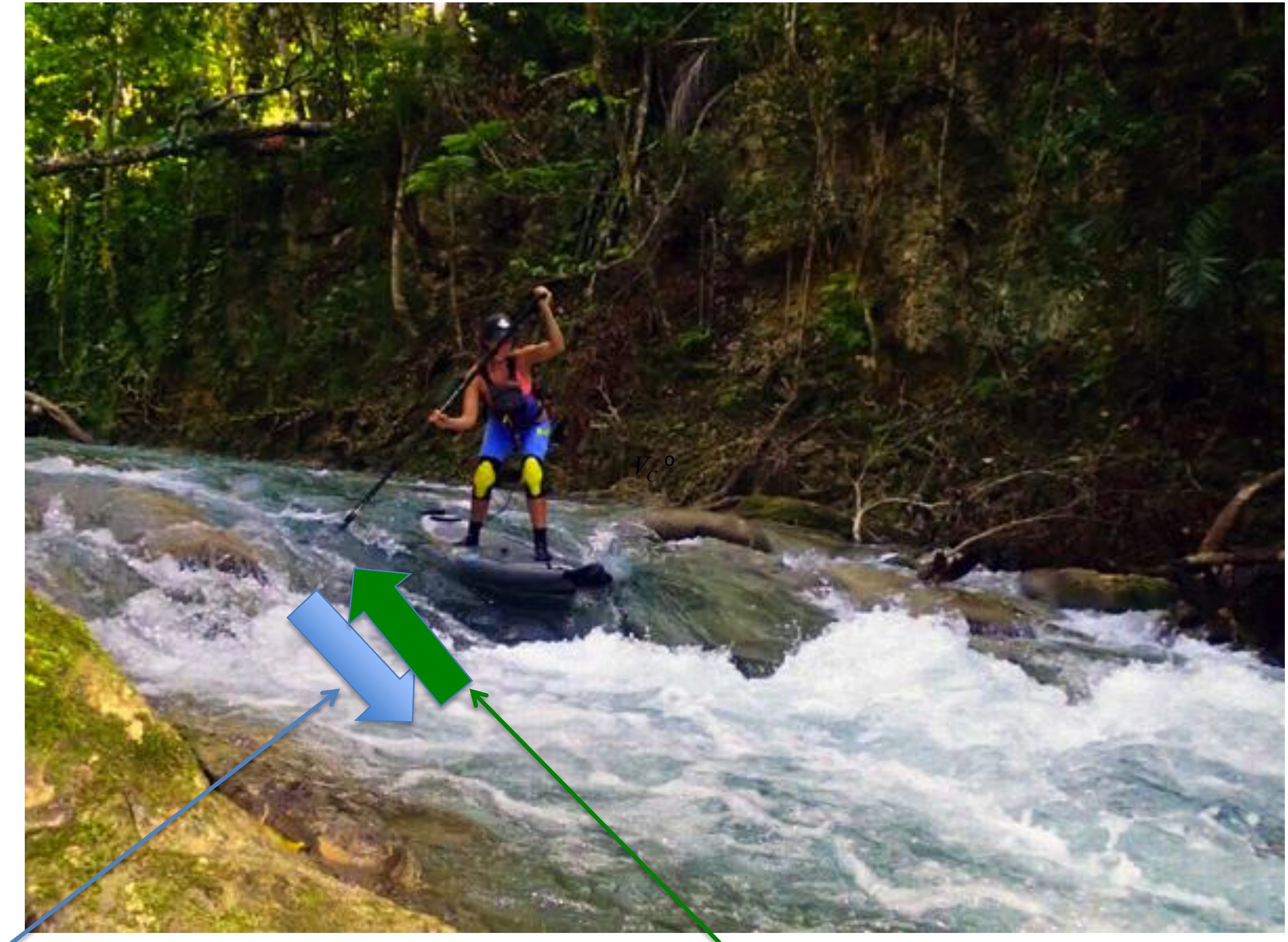
$J = \pm J_c \pm p$

One sound speed vanishes: crucial for quenched disorder problem

Why does sound speed vanish in certain directions?

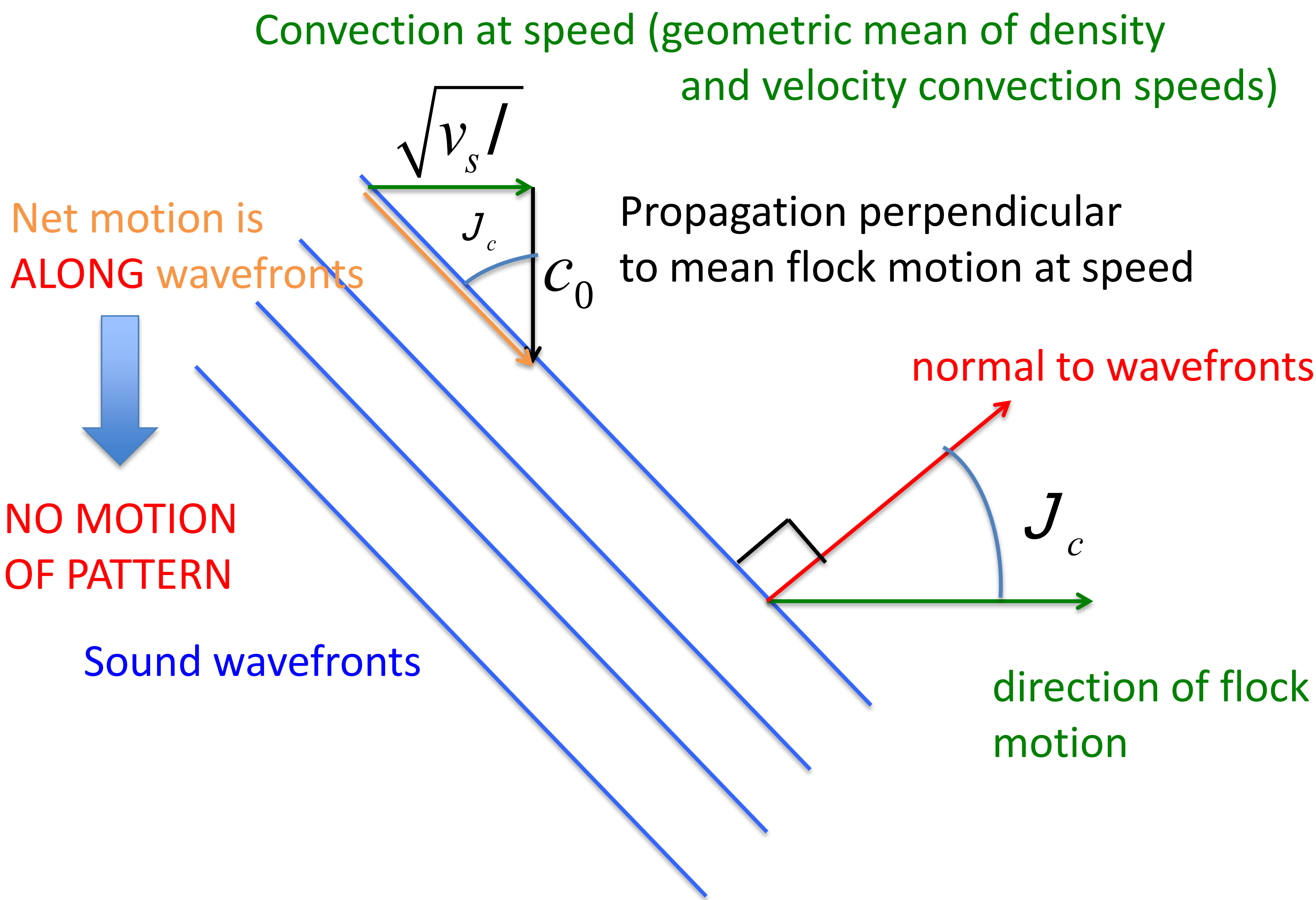
$$J = \pm J_c \pm p$$

Competition between CONVECTION and PROPAGATION



V_C o Velocity of current (i.e., convection)

V_P o Propagation velocity of wave


Convection at speed (geometric mean of density
and velocity convection speeds)
$\sqrt{v_s l}$
Net motion is
ALONG wavefronts
J_c
c_0
Propagation perpendicular
to mean flock motion at speed
normal to wavefronts
NO MOTION
OF PATTERN
J_c
Sound wavefronts
direction of flock
motion

Anomalous Hydrodynamics

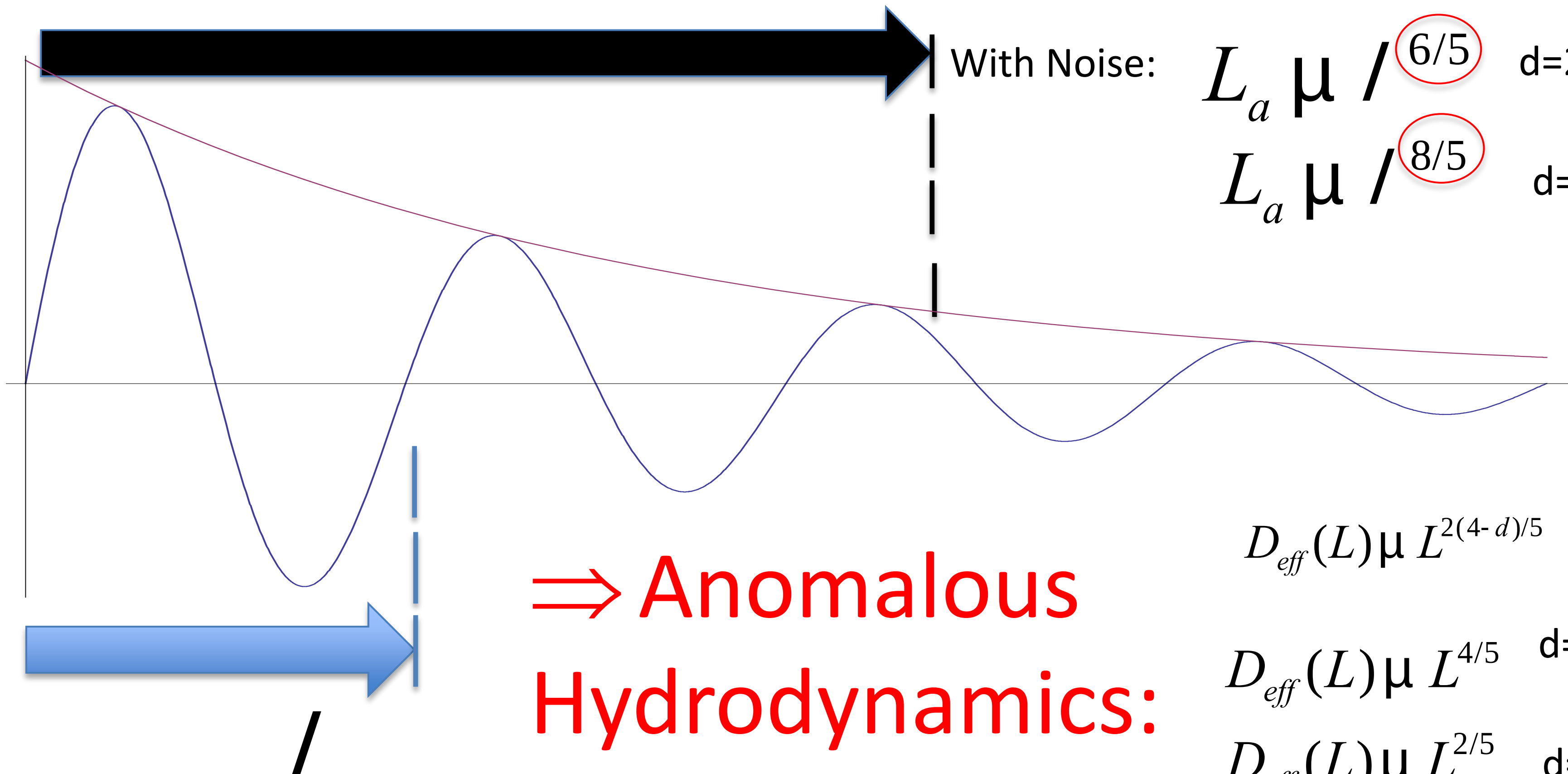
Hydrodynamics With noise	≠	Hydrodynamics Without noise

In ALL d £ 4

Example: Sound damping in flocks:

Without Noise: $L_a \mu /^{2} / D$

Attenuation length L_a



With Noise: $L_a \mu /^{6/5}$ d=2

$L_a \mu /^{8/5}$ d=3

$D_{eff}(L) \mu L^{2(4-d)/5}$

$\Rightarrow$ Anomalous Hydrodynamics:

$D_{eff}(L) \mu L^{4/5}$ d=2

$D_{eff}(L) \mu L^{2/5}$ d=3

Why does this happen?

Fluctuations (waves) interact due to

Convective nonlinearity

$$\partial_t \vec{v} + /_1 (\vec{v} \cdot \vec{\nabla}) \vec{v} + /_2 \vec{v} (\vec{\nabla} \cdot \vec{v}) + /_3 (\vec{\nabla} |\vec{v}|^2) = a \vec{v} - b |\vec{v}|^2 \vec{v}$$

$$- \vec{\nabla} P(r) - \vec{v} (\vec{v} \cdot \vec{\nabla} P_2(r)) + D_B \vec{\nabla} (\vec{\nabla} \cdot \vec{v}) + D_T \nabla^2 \vec{v} + D_2 (\vec{v} \cdot \vec{\nabla})^2 \vec{v} + \vec{f}$$

Long-ranged order in d=2

- Stabilized by this breakdown of hydrodynamics (damps out noise induced fluctuations (negative feedback: fluctuations create divergence of D, which suppresses fluctuations)

Model for flock with quenched disorder: Same as before, except random forcing now has quenched piece, as well as Langevin piece:

$$\partial_t \vec{v} + /_1 (\vec{v} \cdot \vec{\nabla}) \vec{v} + /_2 \vec{v} (\vec{\nabla} \cdot \vec{v}) + /_3 (\vec{\nabla} | \vec{v} |^2) = a \vec{v} - b | \vec{v} |^2 \, \vec{v}$$

$$- \vec{\nabla} P(r) - \vec{v} (\vec{v} \cdot \vec{\nabla} P_2(r)) + D_B \vec{\nabla} (\vec{\nabla} \cdot \vec{v}) + D_T \nabla^2 \vec{v} + D_2 (\vec{v} \cdot \vec{\nabla})^2 \vec{v} + \vec{f}$$

$$+ f_Q(\vec{r})$$

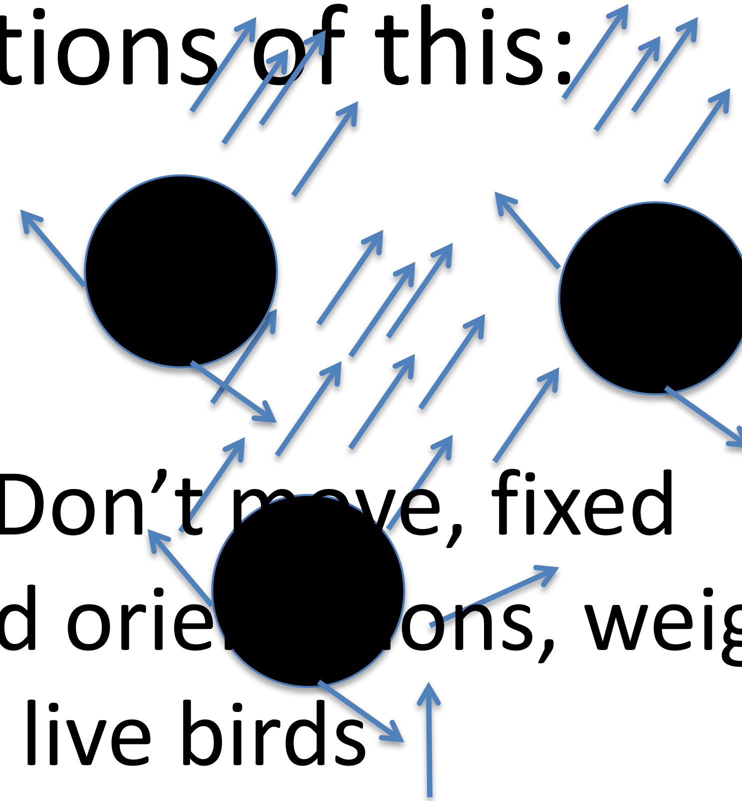
Quenched disorder=Static random forcing

Note: No delta fn in t-t’

$$< f_i^Q(\vec{r},t) f_j^Q(\vec{r}\,',t') >= \Delta \delta_{ij} \delta^d(\vec{r} - \vec{r}\,')$$

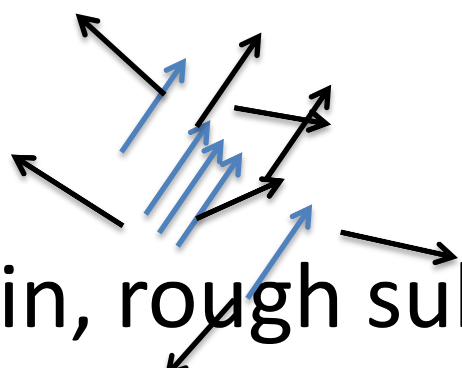
Infinitely long correlations in time

Physical realizations of this:



1) Obstacles:

2) "Dead birds": ↑ Don't move, fixed random positions and orientations, weight W<1 in averaging for live birds



3) random terrain, rough substrate, etc

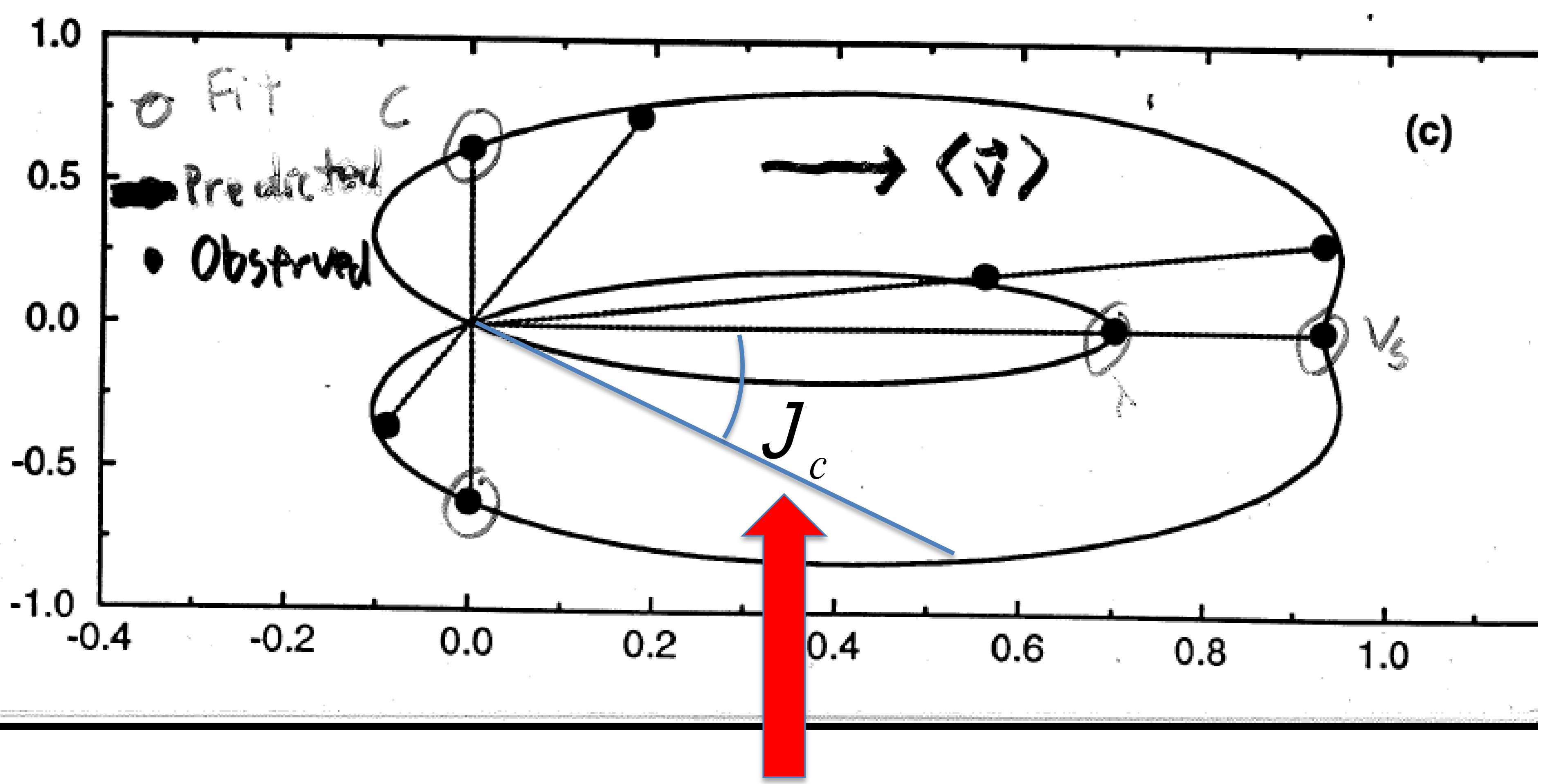
What is the effect of quenched disorder IN EQUILIBRIUM?

- HUGE: $d_{LC} = 4$ vs $d_{LC} = 2$ for annealed disorder (see, e.g. Grinstein and Luther, 1979)
- => No LRO in d=2 OR d=3!

What is the effect of quenched disorder on FLOCKS?

- You might think: not much, because
- In co-moving frame of flock, disorder looks time dependent
- But: what matters is not how flock moves, but how sound moves (sound carries fluctuations)
- And some sound waves DON'T move!

Polar plot of sound speeds:



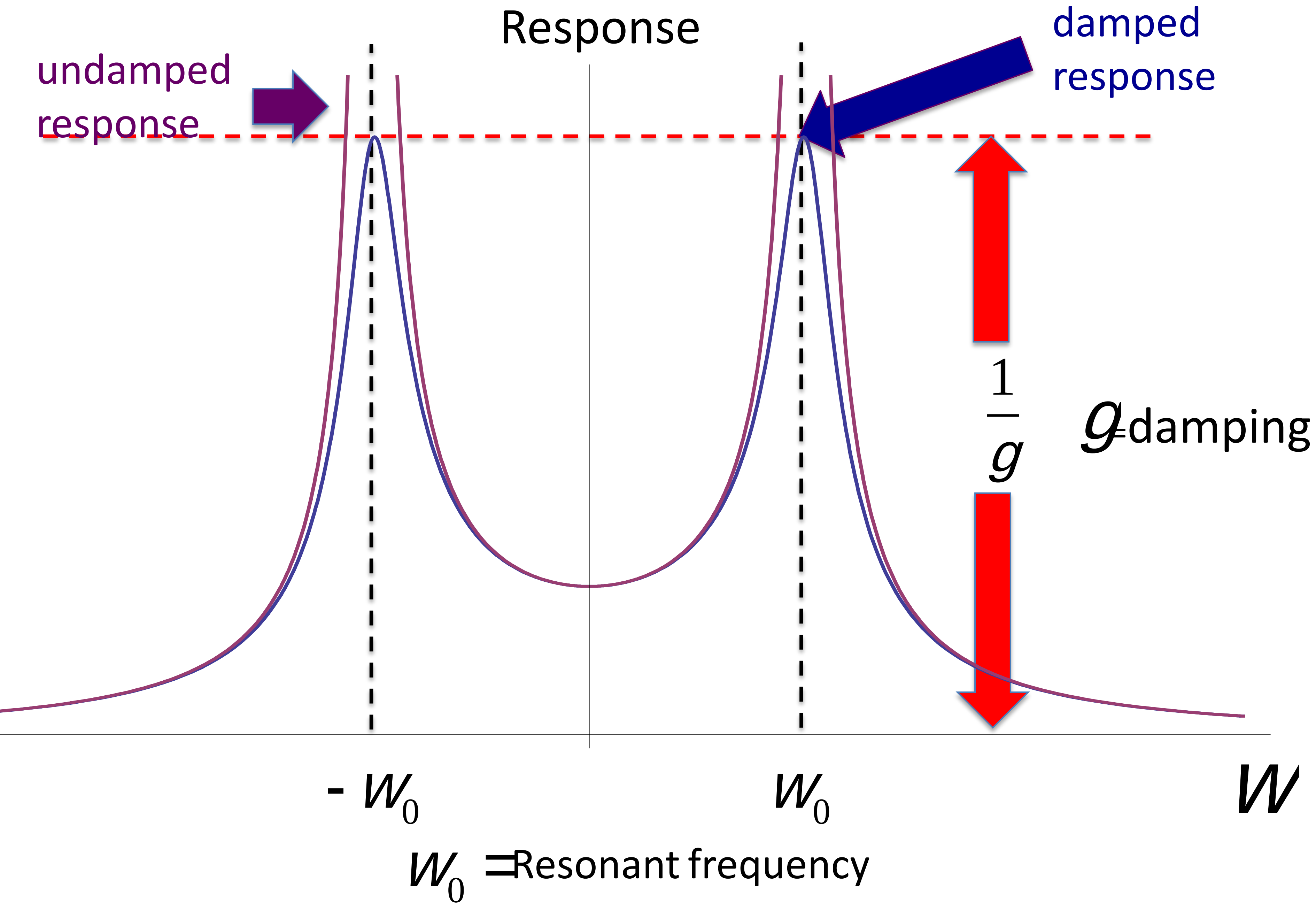
One sound speed vanishes: crucial for quenched disorder problem

Analogy: forced, very underdamped oscillator

- Off resonance: response small, limited by spring constant of oscillator (independent of damping)

$$|Z(\omega)|^2 = \frac{1}{(\omega^2 - \omega_0^2)^2 + \gamma^2 \omega^2}$$

- O
  n
  r



Response
undamped
response
damped
response
$\frac{1}{g}$
g =damping
$-w_0$
w_0
w
w_0 = Resonant frequency

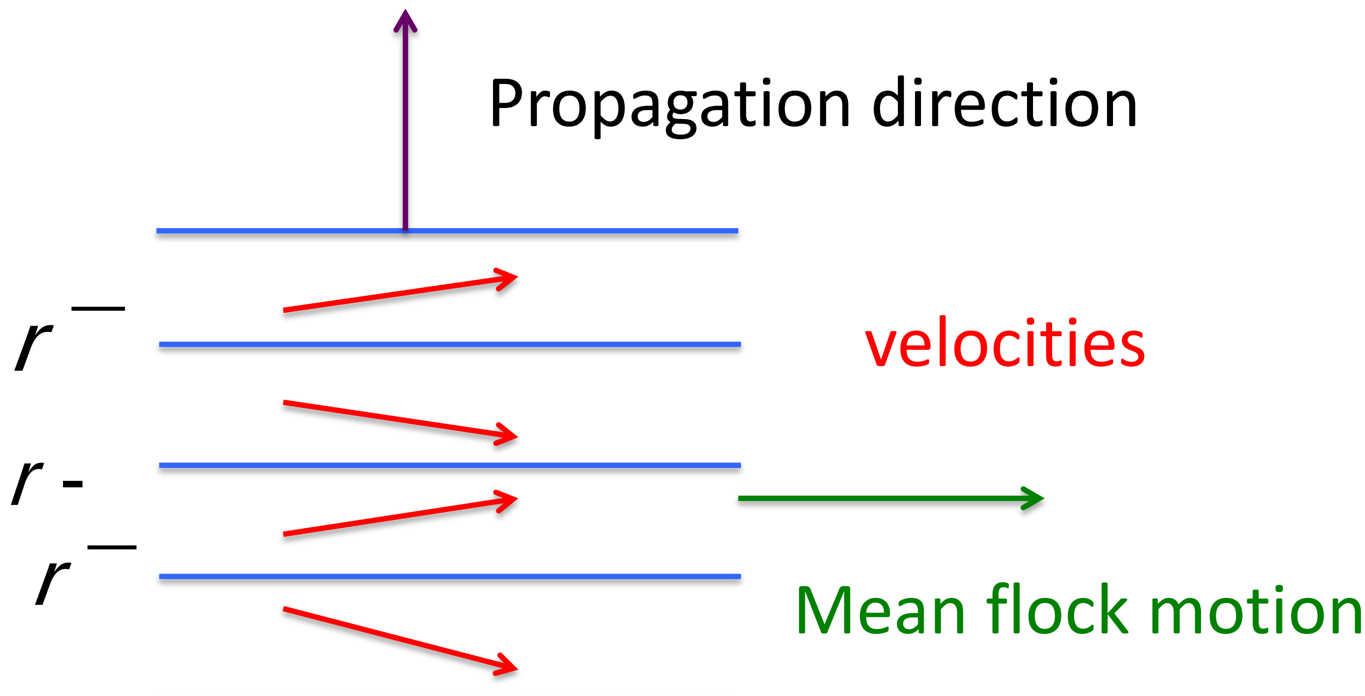
Driven underdamped oscillator	Flock with Quenched disorder
External driving force f	Quenched disorder $f_Q(\vec{r})$
Driving frequency w	$w = 0$ (Disorder is static)
Resonant frequency w_0	$c_{1,2}(J)q$
Damping g	Dq^2

So, when $c_{1,2}(J)=0$, quenched disorder is "on resonance" at $w=0$

=> Biggest fluctuations at $J=\pm J_c \pm p$

One other weird wrinkle: fluctuations vanish at $J = \pm p / 2$

Why?

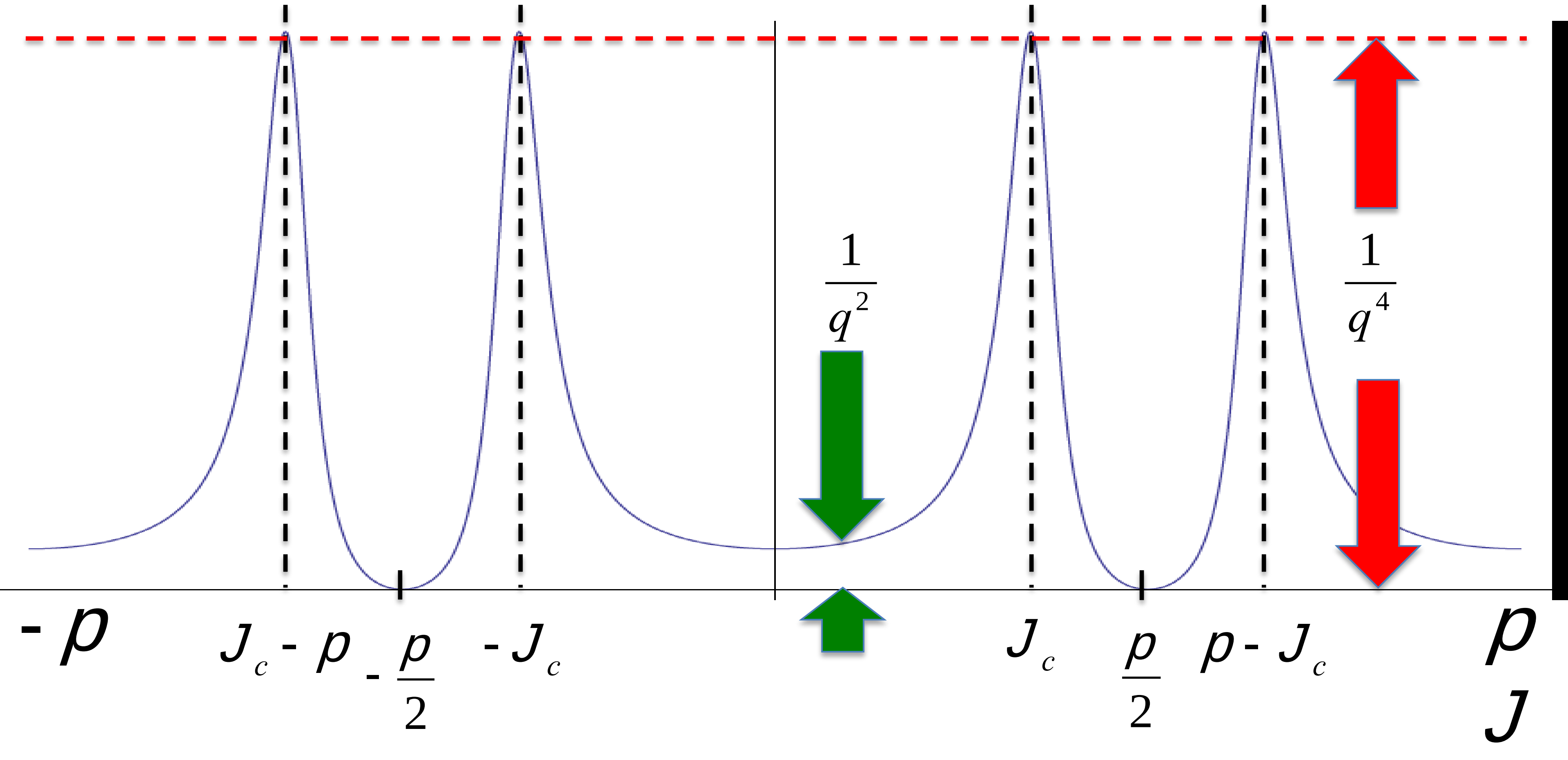


No convection
To relieve
Density buildup

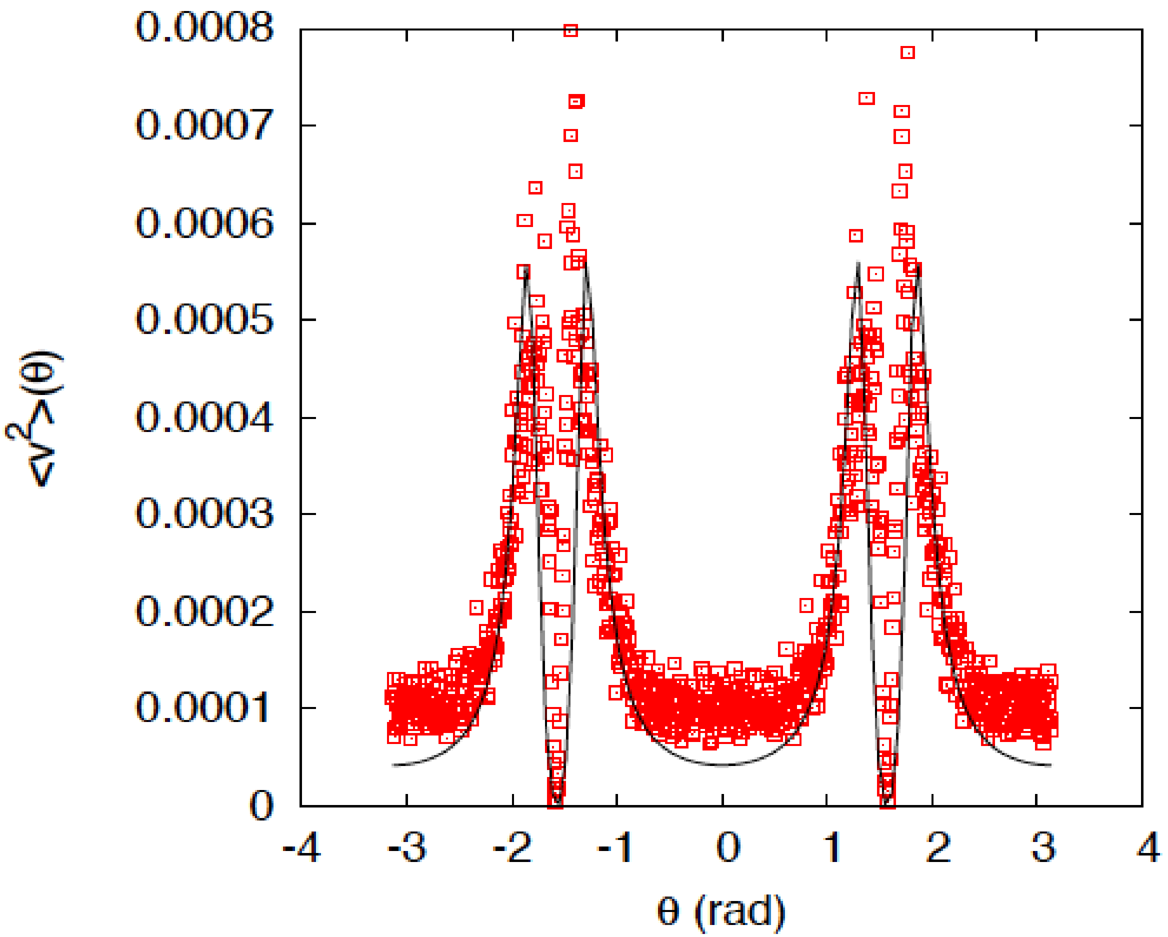
Pressure builds up, suppresses these fluctutions

Summary of velocity fluctuations
(Fourier space $\vec{q}$)

$$\overline{|v(\vec{q})|^2} = \frac{\tilde{D}\cos^2 J}{q^2[D^2q^2 + (\sin^2 J - \tan^2 J_c \cos^2 J)^2]}$$



Result from “dead bird” simulation:



Great, but….

- Plot looks the SAME for smaller q’s!
- What’s happening?!
- Anomalous Hydrodynamics (again!)

Anomalous Hydrodynamics (again!)

$D_{eff}(q)\,\mu\, q^{(d-5)/3}$ (vs $D_{eff}(q)\,\mu\, q^{2(d-4)/5}$ for annealed problem)

=> in d=2, $D_{eff}(q)\,\mu\, q^{-1}$

=> $\overline{|v(\vec{q})|^2} = \dfrac{\tilde{D}\cos^2 J}{q^2[D_{eff}^2(q)q^2 + (\sin^2 J - \tan^2 J_c \cos^2 J)^2]} \propto \dfrac{1}{q^2}$

For ALL J (Even $J = J_c$)

$$D_{eff}(q) = Aq^{-1}$$

- If noise is small, this coefficient will be small
- => Big peak at J_c, but q-independent
- EXACTLY what we see

Back to real space (d=2):

$$\overline{|v(\vec{q})|^2}\ \mu\ \frac{1}{q^2} \Rightarrow |<v(\vec{r})>| \propto L^{-h(D)}$$

$h(D)$ Non-universal exponent, grows as noise D grows

"Quasi-long-ranged order"
Exactly what's seen by Peruani et al in simulations of flocks with obstacles

d=3

$$D(q) \propto q^{(d-5)/3} = q^{-2/3}$$

Strong enough to
Stabilize long-ranged order

$$< \vec{v}(\vec{r}) > \neq \vec{0}$$

Summary:

- Flocking is robust against quenched disorder,
- as well as annealed disorder
- Long ranged order in d=3
- Quasi-long-ranged order d=2
- Very detailed predictions of hydrodynamic
- theory verified by simulations (ours and others)

Thanks for your attention, and happy St Patrick's Day!

An actual pub in Dublin, IRE

$$\partial_t \vec{v} + /_1 (\vec{v} \cdot \vec{\nabla}) \vec{v} + /_2 \vec{v} (\vec{\nabla} \cdot \vec{v}) + /_3 \vec{\nabla} |\vec{v}|^2 = a \vec{v} - b |\vec{v}|^2 \vec{v}$$

$$- \vec{\nabla} P(r) - \vec{v} (\vec{v} \cdot \vec{\nabla} P_2(r)) + D_B \vec{\nabla} (\vec{\nabla} \cdot \vec{v}) + D_T \nabla^2 \vec{v} + D_2 (\vec{v} \cdot \vec{\nabla})^2 \vec{v}$$

$$+ f_Q(\vec{r})$$