

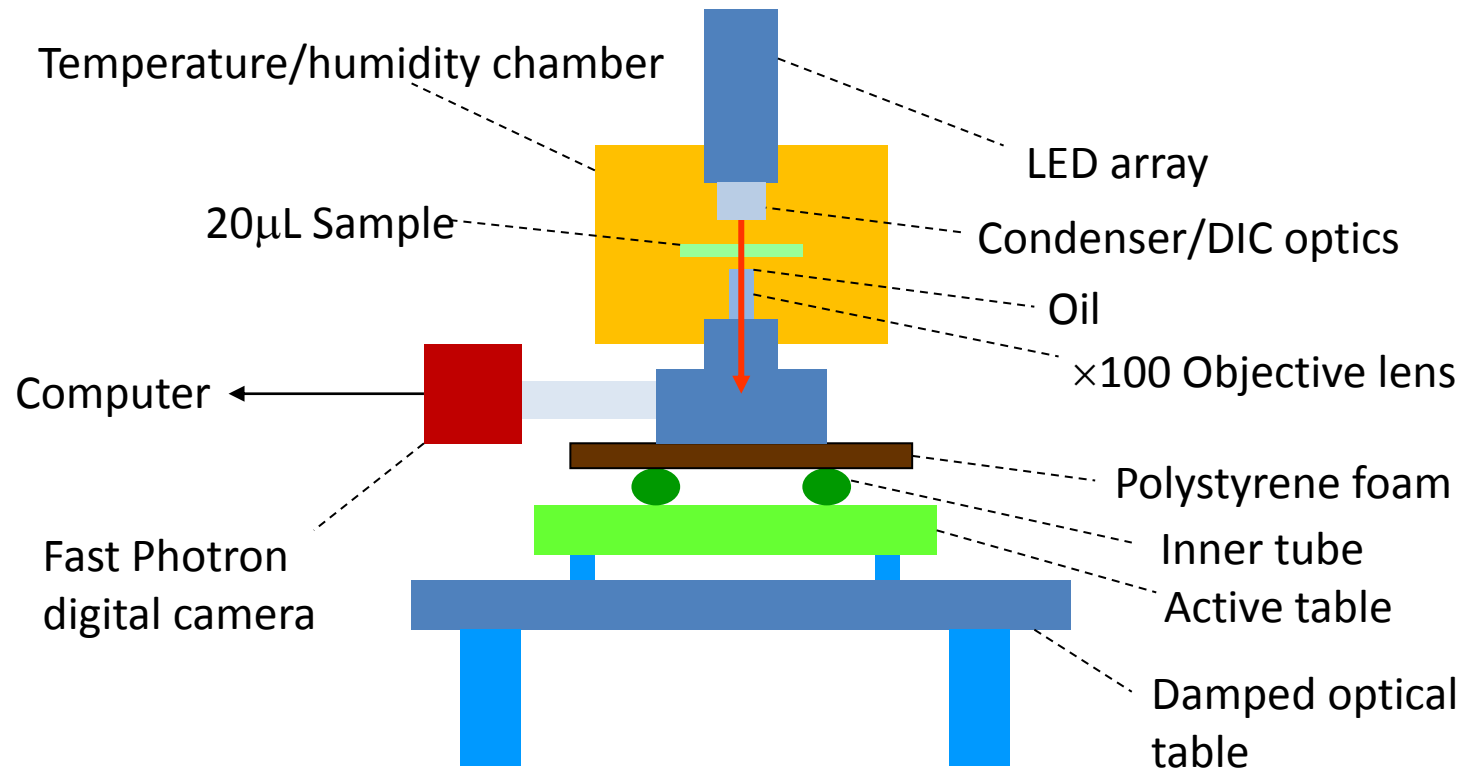
The statistics of particle tracking: a linchpin in countless biophysics experiments

Tom Waigh.

Biological Physics Group and Photon Science Institute,
School of Physics and Astronomy,
University of Manchester.
Email: t.a.waigh@manchester.ac.uk

A tracking experiment

Optical microscopy

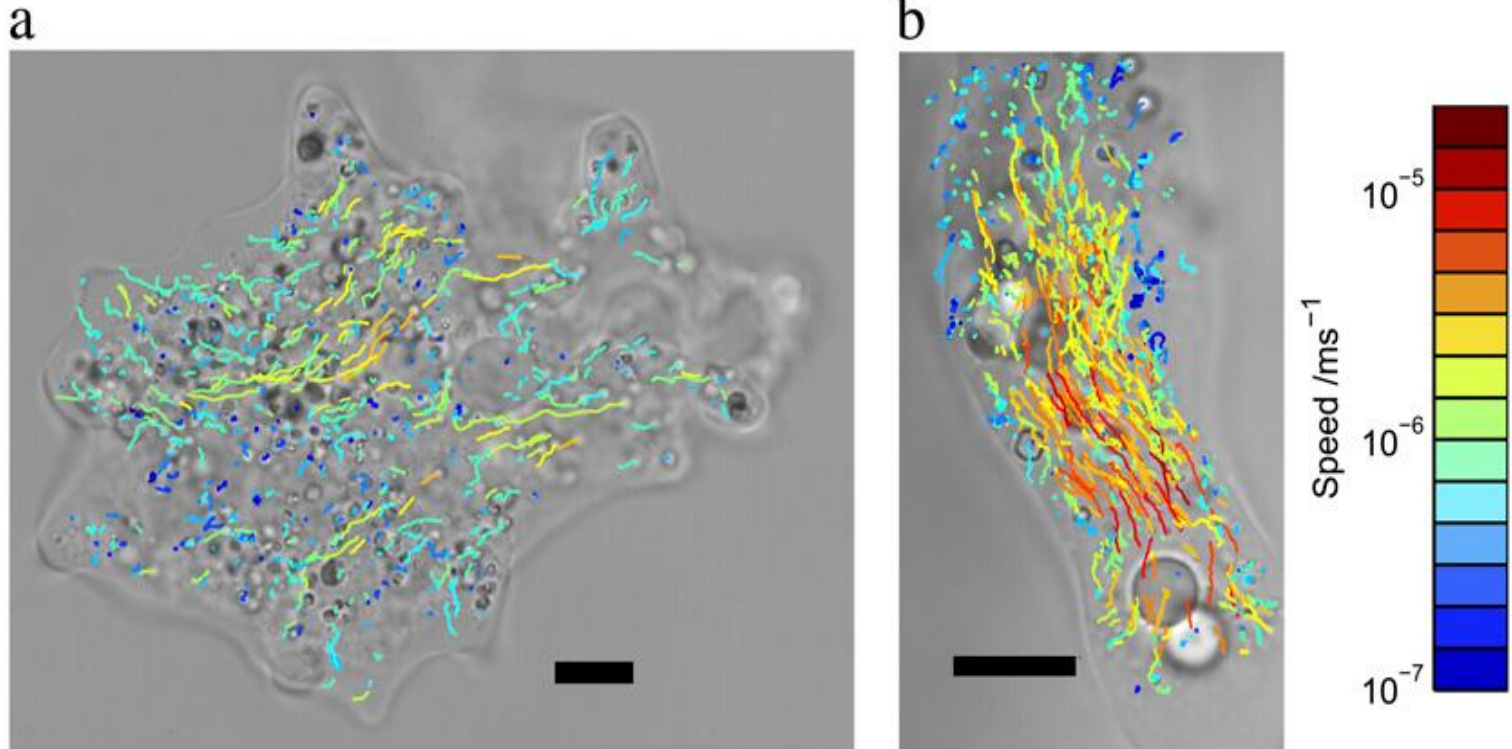


Also: CERN, X-ray imaging, astrophysics (satellites) etc.

A linked list of displacements = a track

Samples

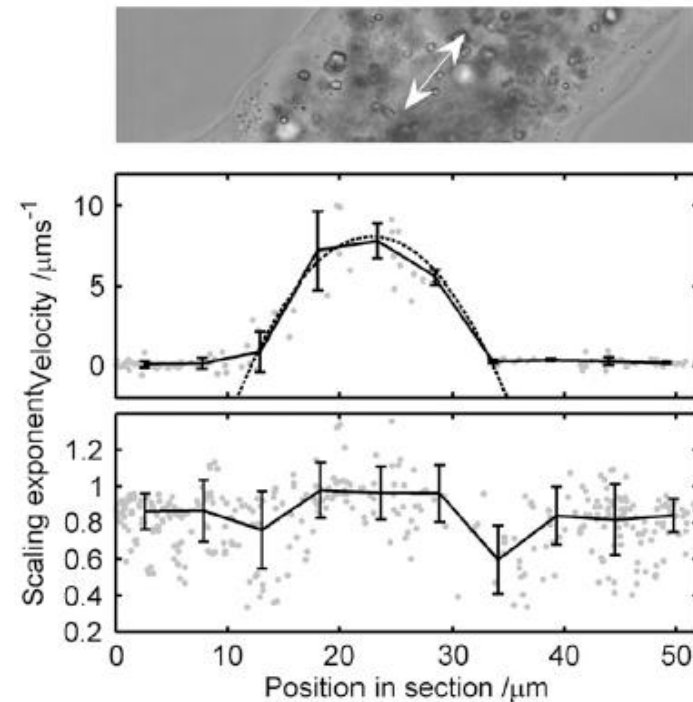
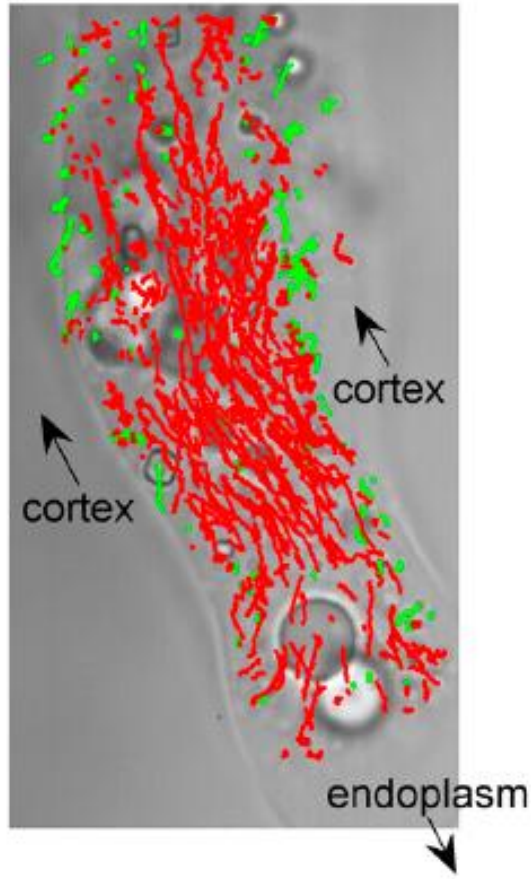
Synthetic soft matter, biomolecules, motor proteins, single molecules, **live cells**



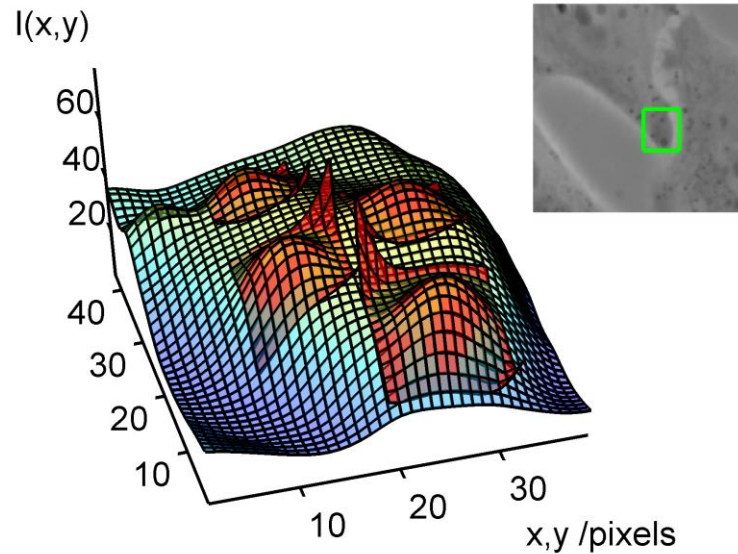
Thousands of organelles moving around at 0.00001s!

S.Rogers, T.A.Waigh, J.Lu, *Biophysical Journal*, 2008, 94, 3313-3322.

Cytoplasmic streaming



Software – polyparticletracker

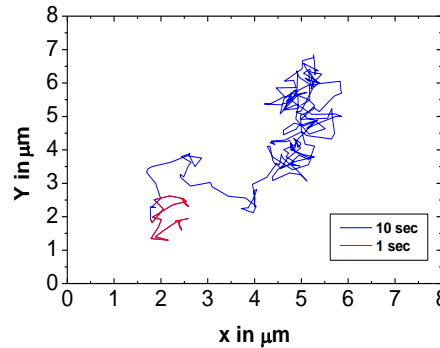


Advantages:

Good for cells,
Works well with fluorescence at low copy numbers,
Can be used with huge data sets from fast cameras (100,000 fps),
Parallel versions now exist,
It is freely available.

S.S.Rogers, et al, *Physical Biology*, 2007, 4, 220-227.

Analysis: What do we do with the tracks?



For Brownian motion

$$\langle \Delta r(t) \rangle = 0$$

Mean square displacement

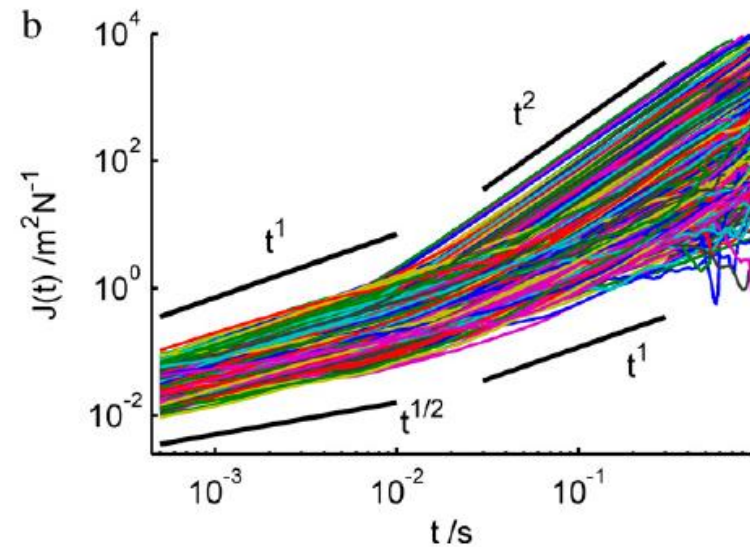
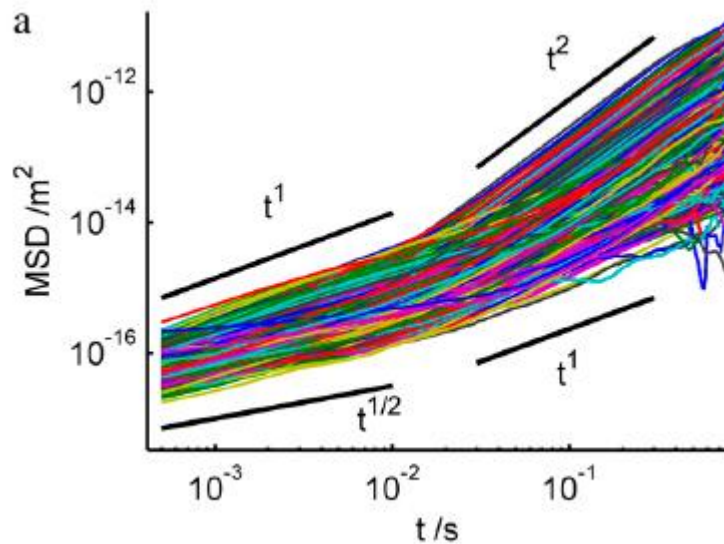
$$\langle \Delta r^2(t) \rangle = 4Dt$$

Einstein 1905

Perrin 1926 – Nobel Prize

Beware of defining a velocity $v=r/t$ for stochastic processes!

The many uses of MSDs



Useful measure of stochastic motion.

Linearly proportional to the compliance ($J(t) = \frac{\pi a}{kT} \langle \Delta r^2(t) \rangle = \frac{e(t)}{\sigma(t)}$). Leads to **microrheology**. Huge range of uses e.g. examine mucus in the intestines, antibiotic resistant bacteria, heart muscle etc.

What else is possible?

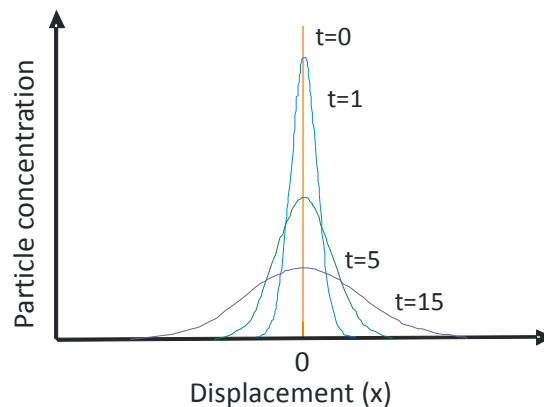
Calculate **higher moments**,

$\langle r^3 \rangle$, how skew the distribution is.

$\langle r^4 \rangle$, kurtosis, how non-Gaussian the distribution is.

..... Moment generating function.

Full probability distribution function.



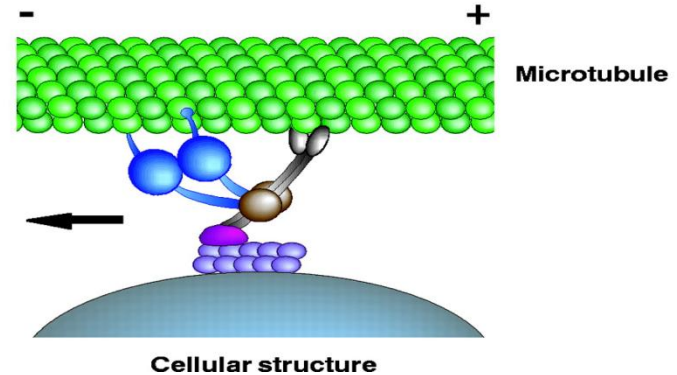
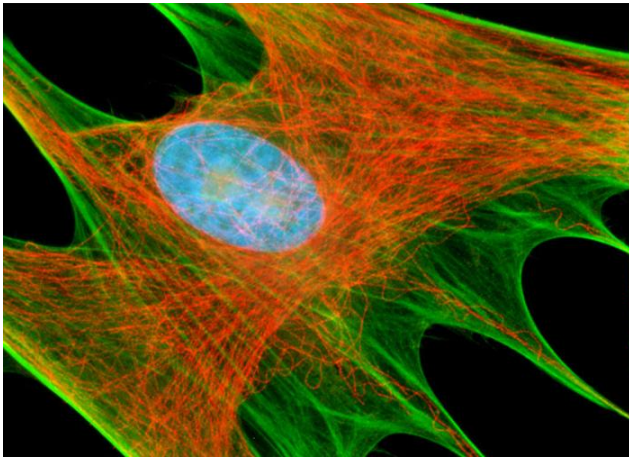
Or its Fourier Transform, **intermediate scattering function**.

2 crucial unanswered questions in cellular biology

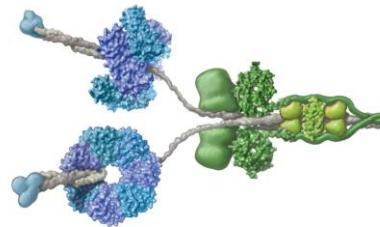
: vesicle transport dynamics

1) Tug of war model.

2) Nature of subdiffusive motion at short times in cells: many models - *ant in a fractal labyrinth, fractional Brownian motion, continuous time random walk.*



Dynein

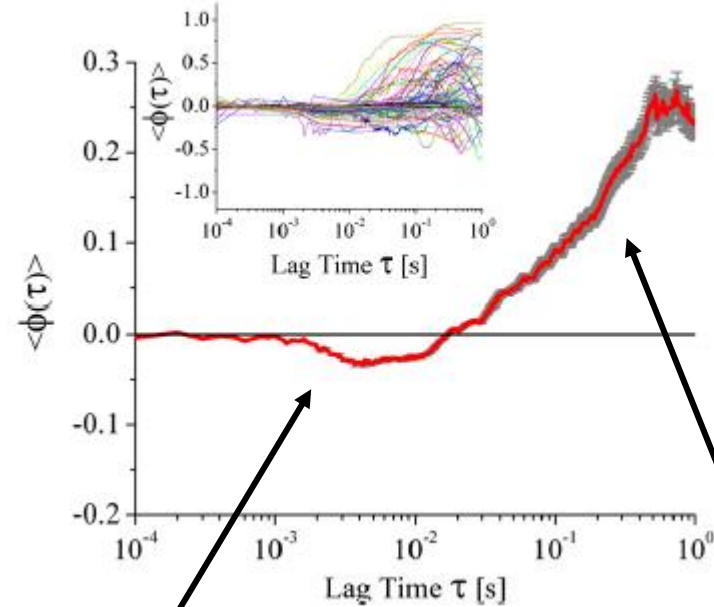
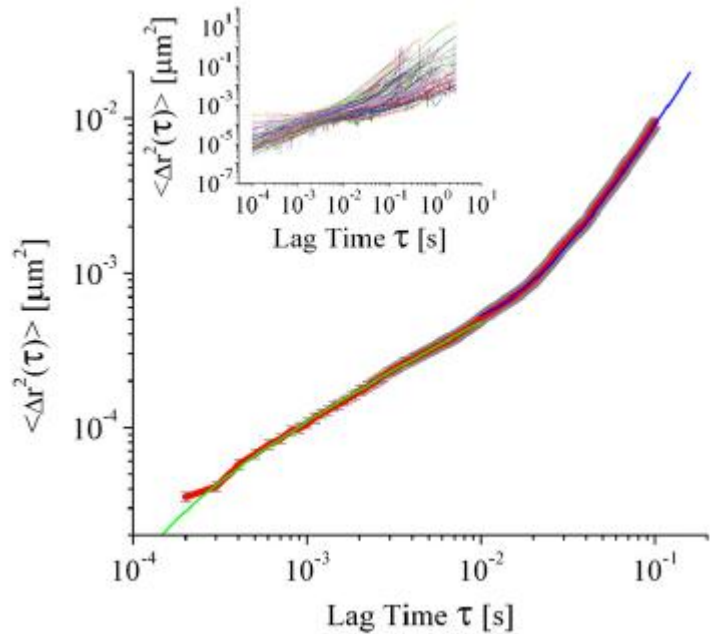


Kinesin



Angular corrections

$$\langle \phi(\tau) \rangle = \langle \cos \theta(\tau) \rangle = \frac{r_1(\tau) \cdot r_2(\tau)}{|r_1(\tau)| |r_2(\tau)|}$$



Anti-persistent motion

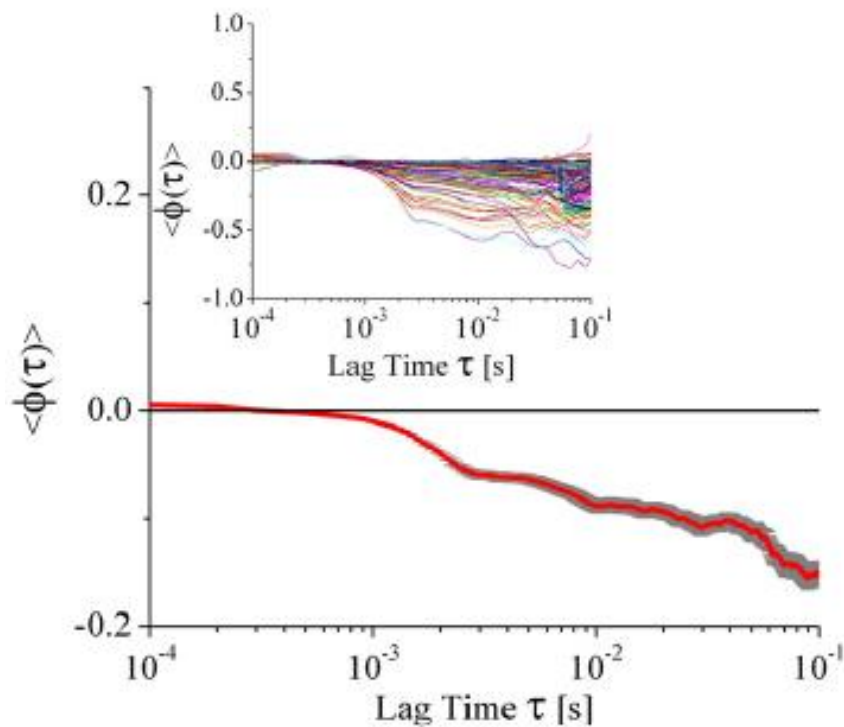
Persistent motion

A.W.Harrison, et al, *Physical Biology*, 2013, 10, 36002.

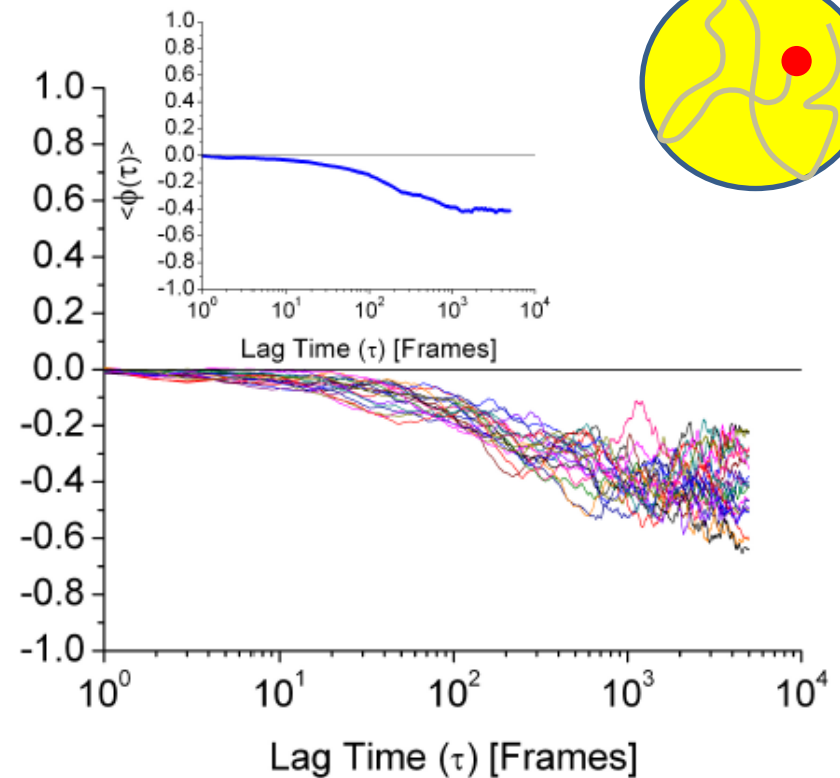
B.Stanislav et al, *PNAS*, 2013, 110, 49, 19689.

Angular correlations without microtubules

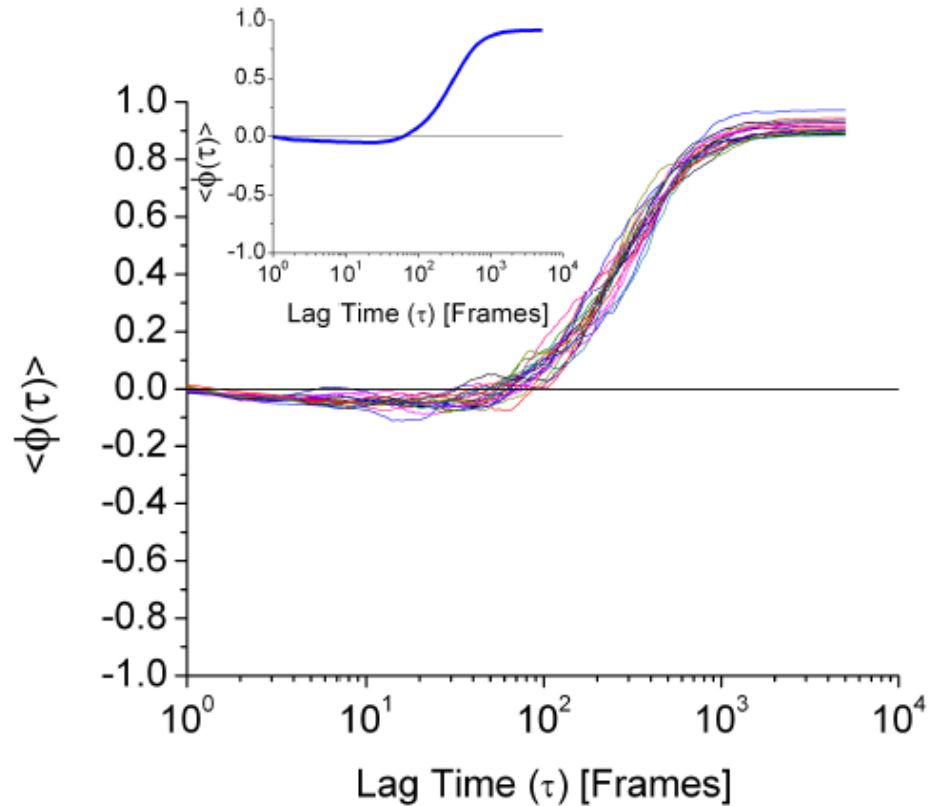
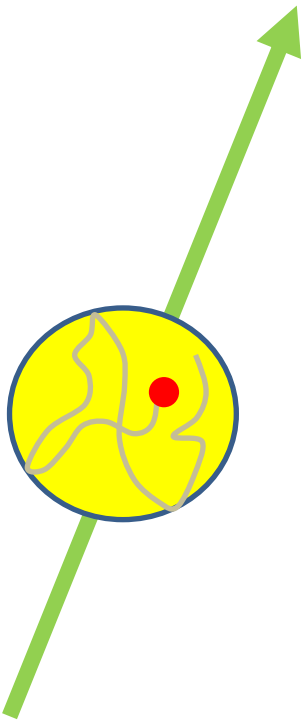
Data



Simulation

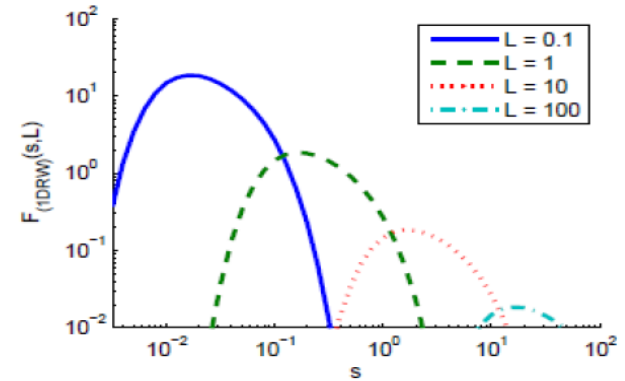


Angular correlations with microtubules: simulations



First passage probability

$$F(s, L) = \frac{\pi D}{L} \sum_{k=0}^{\infty} (-1)^k (2k + 1) e^{-\left(\frac{(2k+1)\pi}{2L}\right) \frac{Ds}{L}}$$



Advantages:

Sensitive to a small number of tracks that move over long distances.

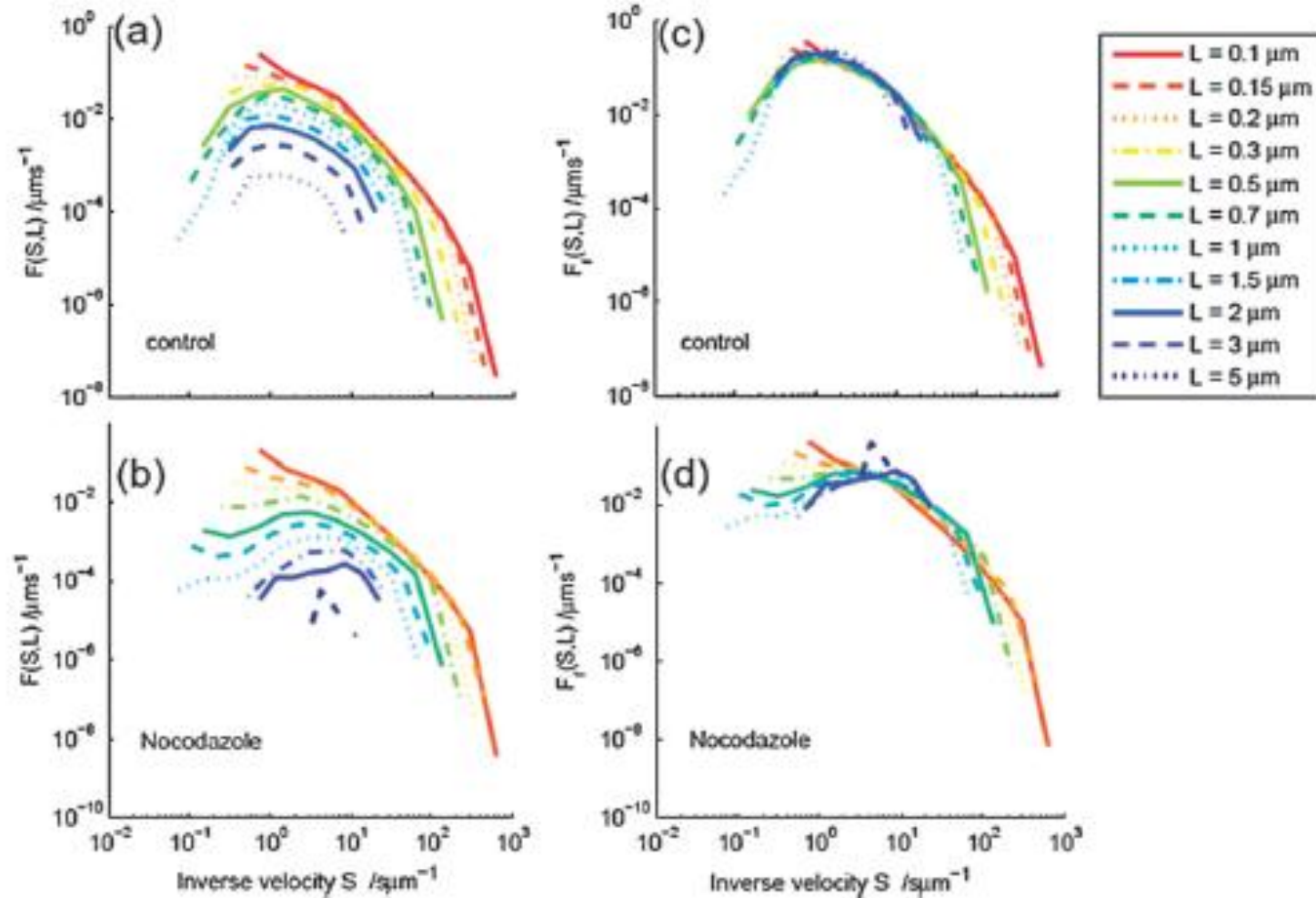
Robust representation of data. No smoothing assumptions are made.

Fits in well with diffusion to reaction type behaviour.

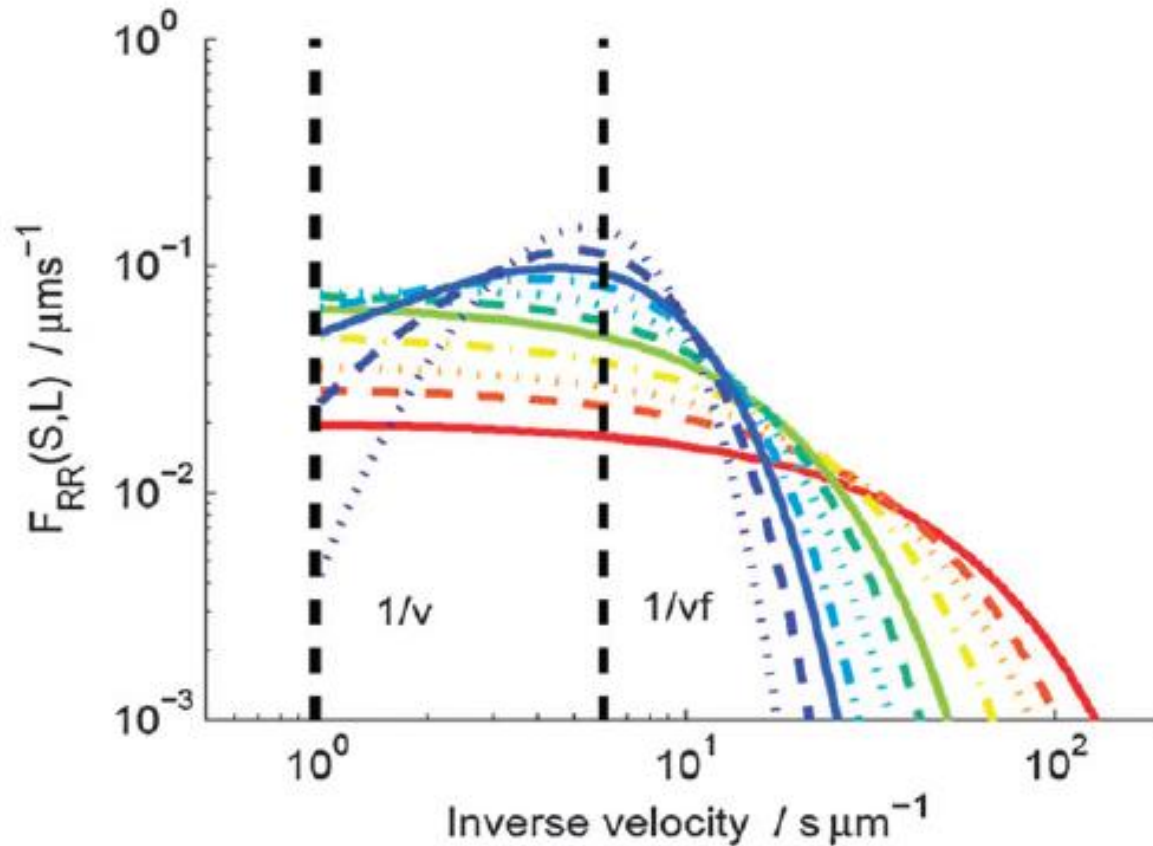
Provides two new results – stretched exponential distribution of vesicle transport lengths and the dependence of vesicle velocity on transport length (different motors are used for different transit lengths?).

Provides a spatially dependent rate constant that can be used with systems biology modelling.

FPP for Rab5 transport in live cells



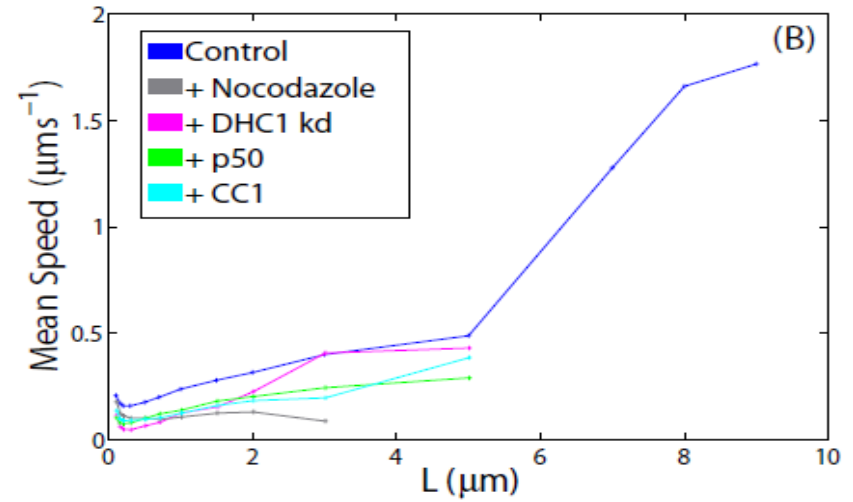
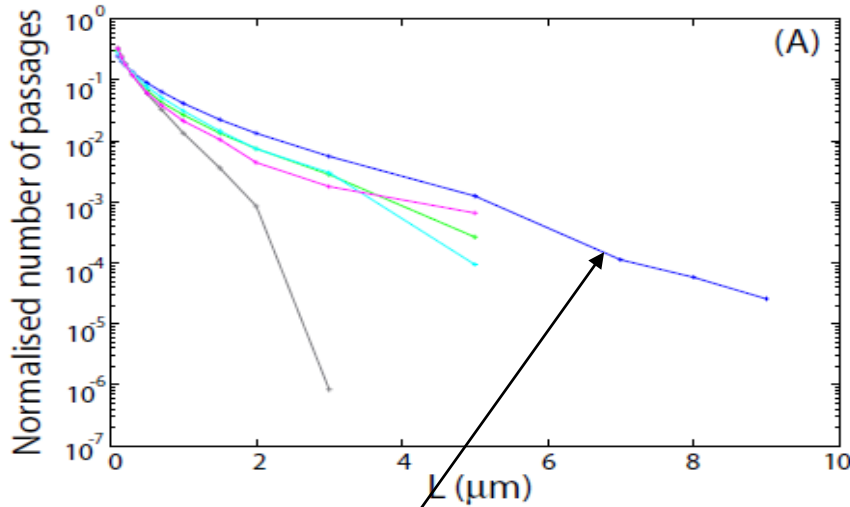
Analytic model for slow FPP



Unidirectional run-rest model.

f is the fraction of the time spent running and v is the average velocity.

First passage probability



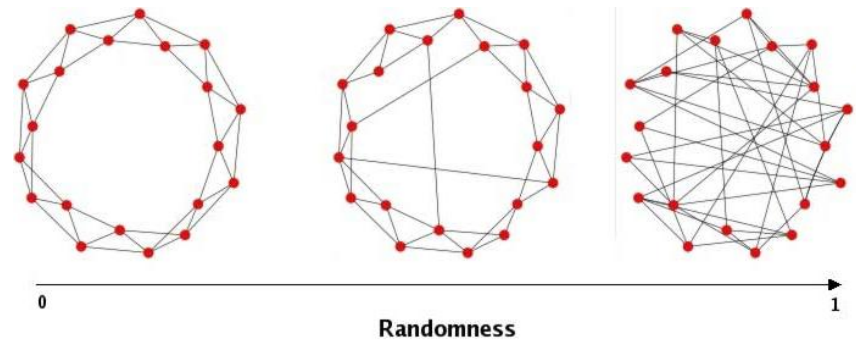
Fat-tailed distribution

French rail system model:

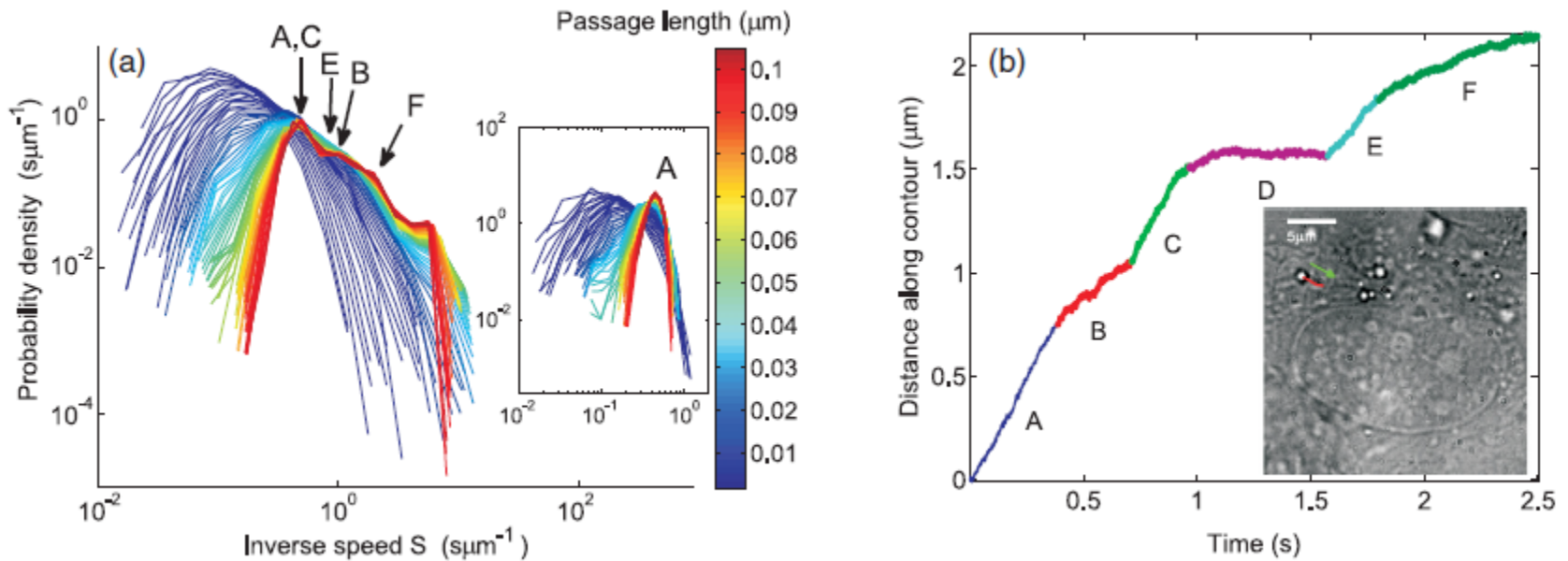
Local transport is slow,

Global transport is fast.

Small world network dynamics?

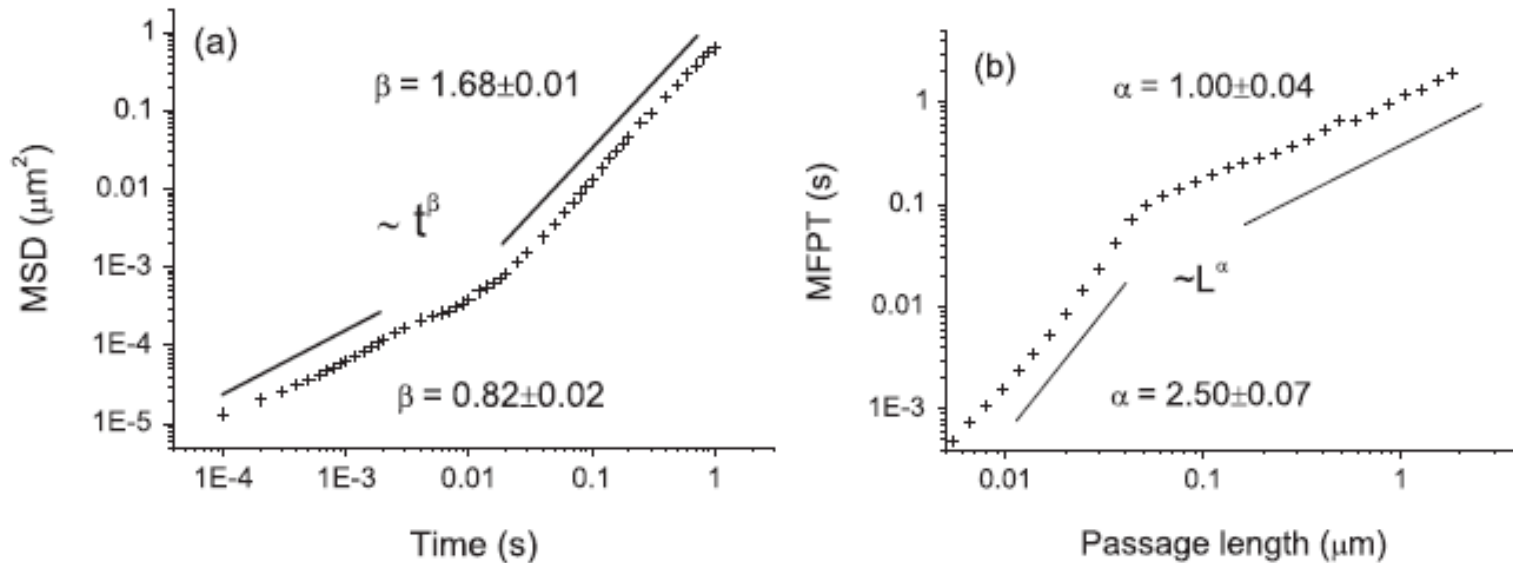


Fast first passage probability



Extract unbiased velocity of runs.

Mean first passage time



Fractal Brownian Motion

$$\langle r^2 \rangle = MSD \sim t^{1/\nu}$$

$$MFPT \sim L^{2\nu}$$

Reasonable, but not perfect agreement, walk dimension $\nu=1.21$ versus 1.25.

Conclusions

Angular correlation and first passage probabilities are useful underused measures of active motion.

Future work

- Super-resolution fluorescence techniques.
- Stepping motion with a Kalman filter (Bayesian method – need a reasonable guess of the transport dynamics).