



# Defect-Mediated Phase Transitions in Active Soft Matter

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Active Processes in Living and  
Nonliving Matter, KITP, UCSB

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Christopher Bock\*\* and Erwin Frey\*\*

\* MPIPKS Dresden, Germany

\*\* LMU Munich, Germany



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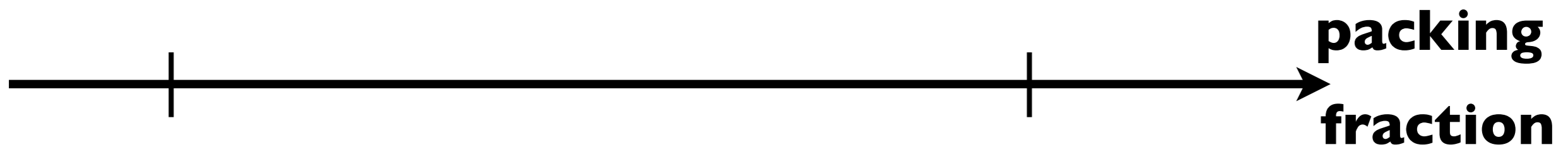
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spherical polar particles with ferromagnetic interactions; in 2d

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Gregoire et al.; Physica D, 157 (2003)

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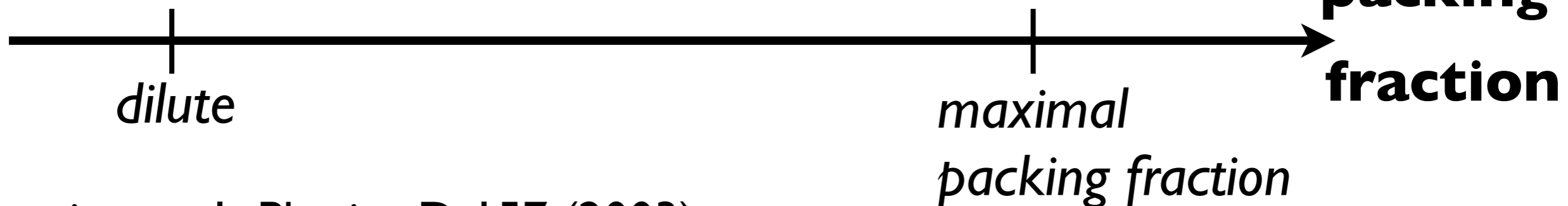


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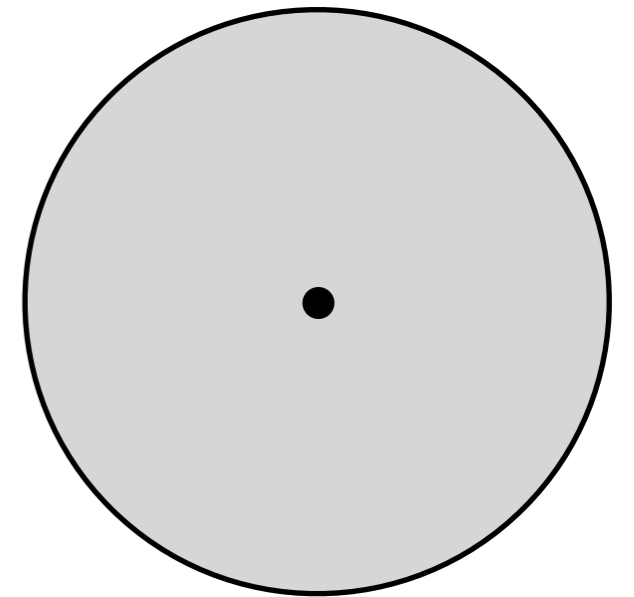
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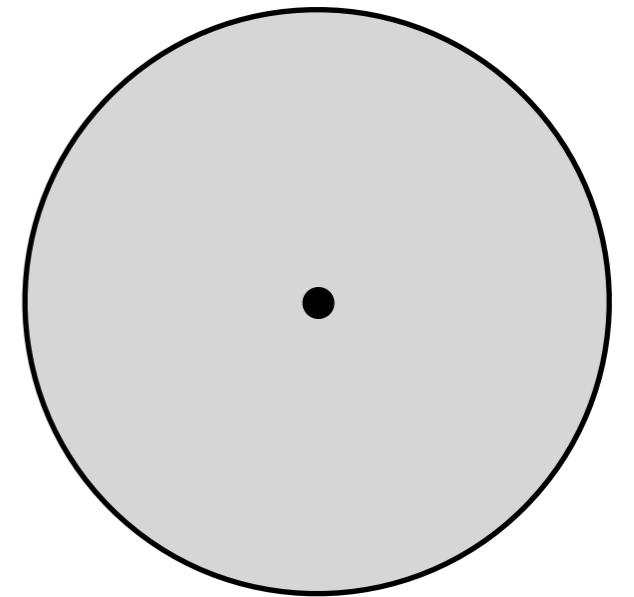
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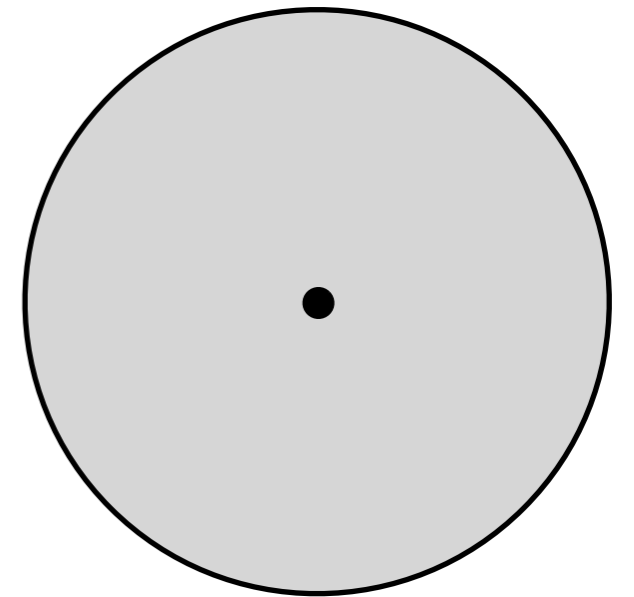
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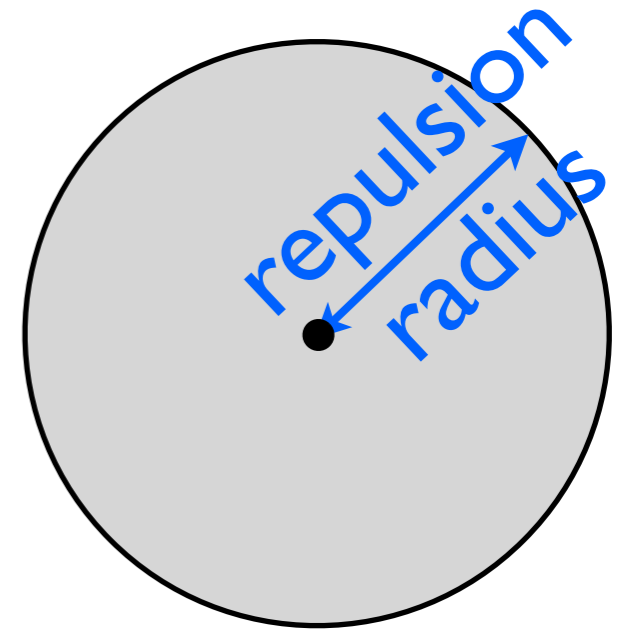
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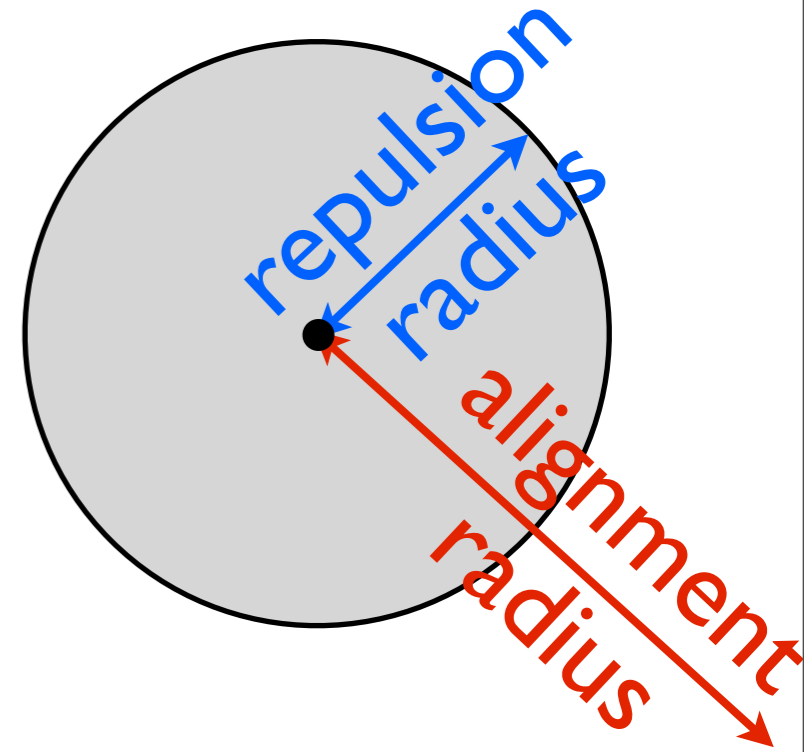
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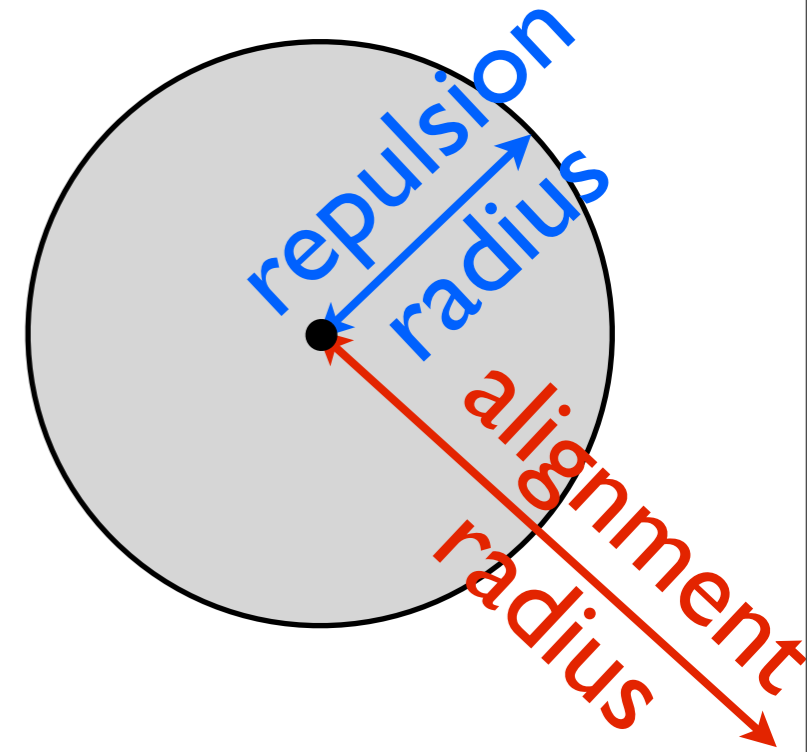
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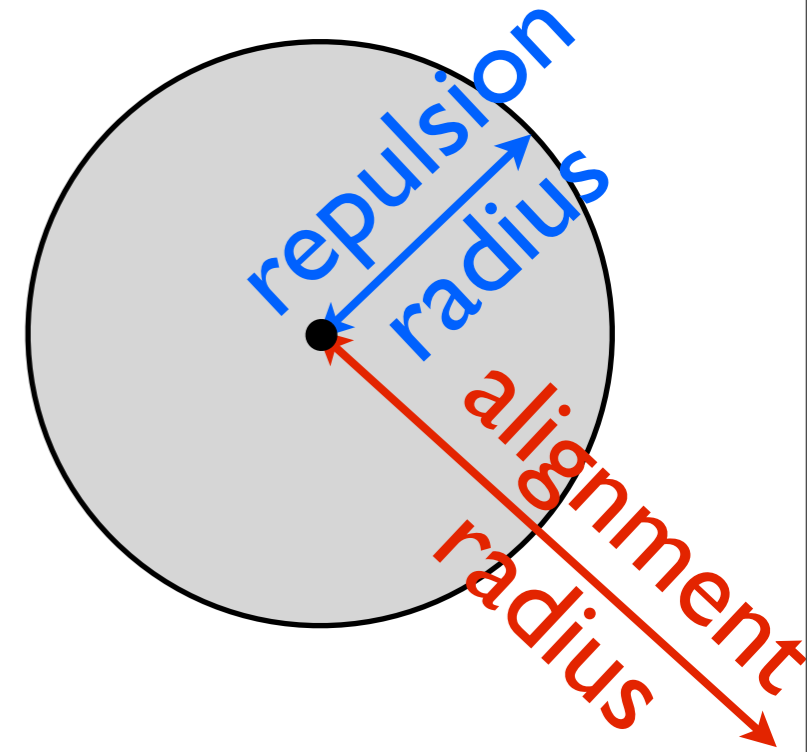
$$\vec{v}_i(t + \Delta t) = v_a \frac{\sum_{j \in \mathcal{A}_i} \vec{n}_j(t)}{|\sum_{j \in \mathcal{A}_i} \vec{n}_j(t)|} + v_r \sum'_{j \in \mathcal{A}_i} \frac{\vec{x}_{ij}(t)}{|\vec{x}_{ij}(t)|}, \quad (1)$$

$$\vec{x}_i(t + \Delta t) = \vec{x}_i(t) + \vec{v}_i(t + \Delta t) \Delta t. \quad (2)$$

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“Vicsek alignment”

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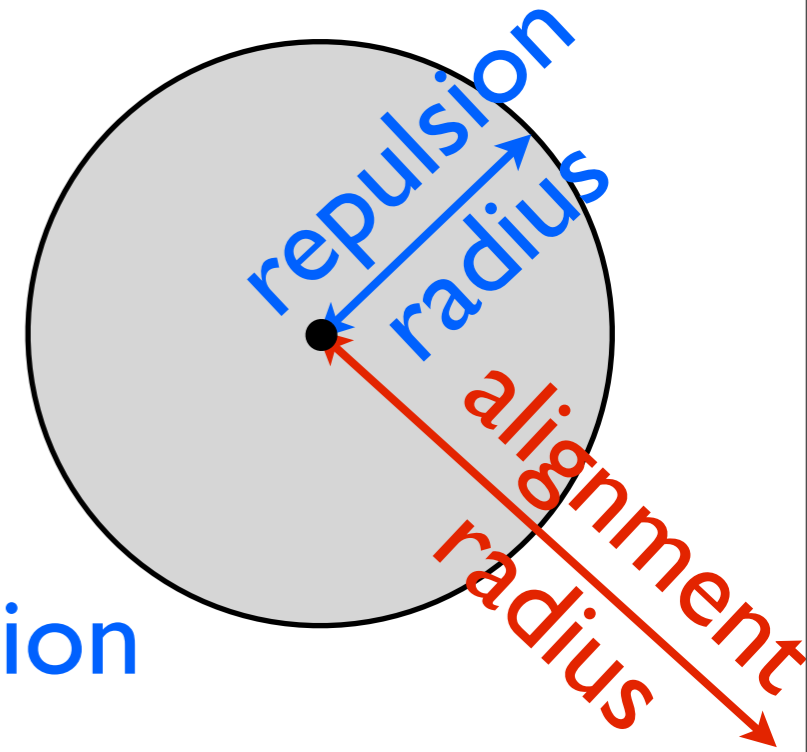
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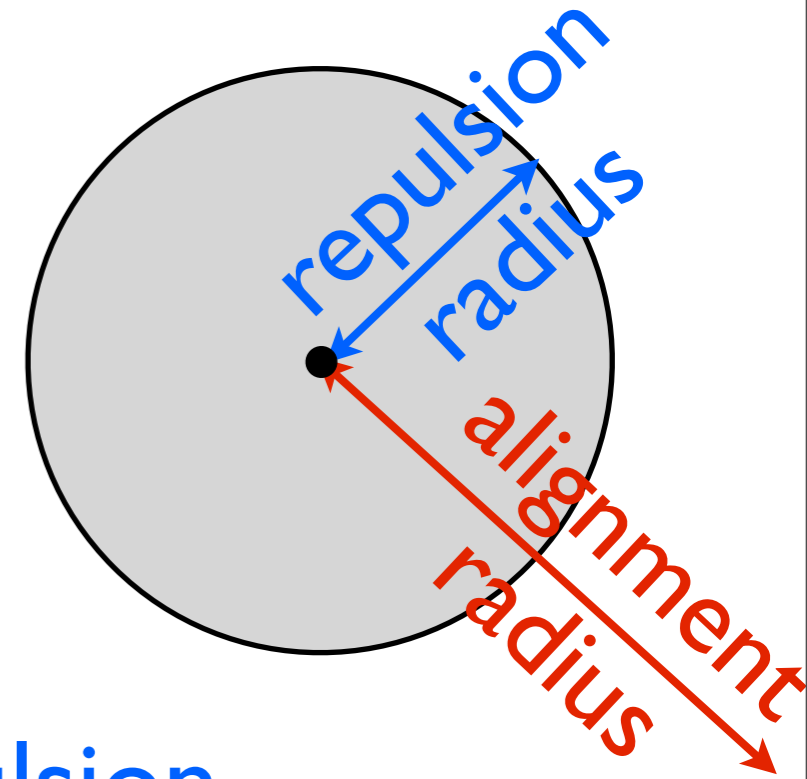
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- > competition between alignment and repulsion



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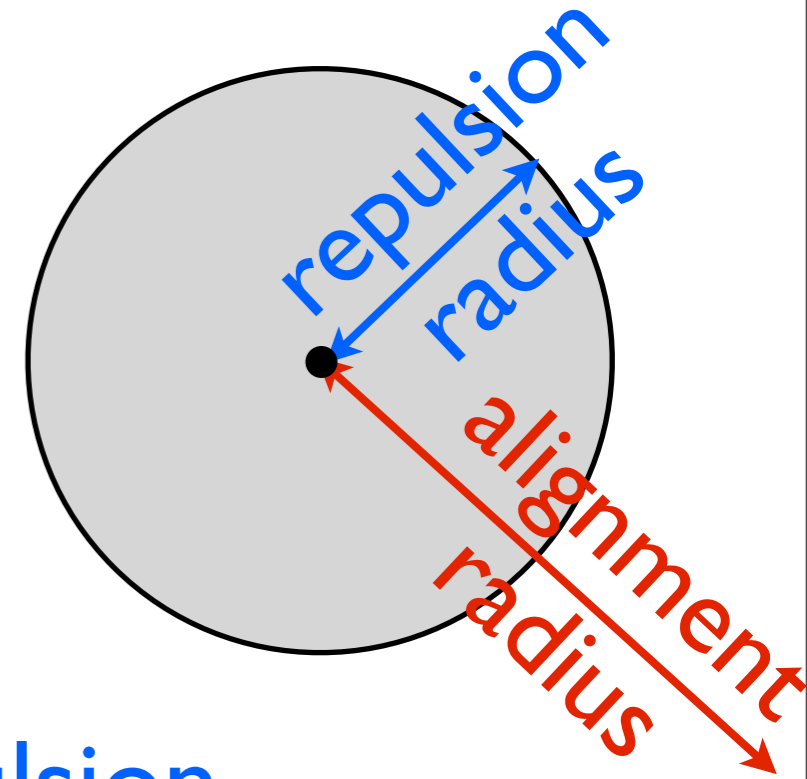
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- choice of radii
- > competition between alignment and repulsion
- small particle overlaps, quasi “soft core”



“Vicsek alignment”      repulsion

$$\vec{v}_i(t + \Delta t) = v_a \frac{\sum_{j \in \mathcal{A}_i} \vec{n}_j(t)}{|\sum_{j \in \mathcal{A}_i} \vec{n}_j(t)|} + v_r \sum_{j \in \mathcal{A}_i}' \frac{\vec{x}_{ij}(t)}{|\vec{x}_{ij}(t)|}, \quad (1)$$

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$$\Psi_6 = \langle \langle \Psi_{6,i} \rangle_i \rangle_t \quad \Psi_{6,i} =$$
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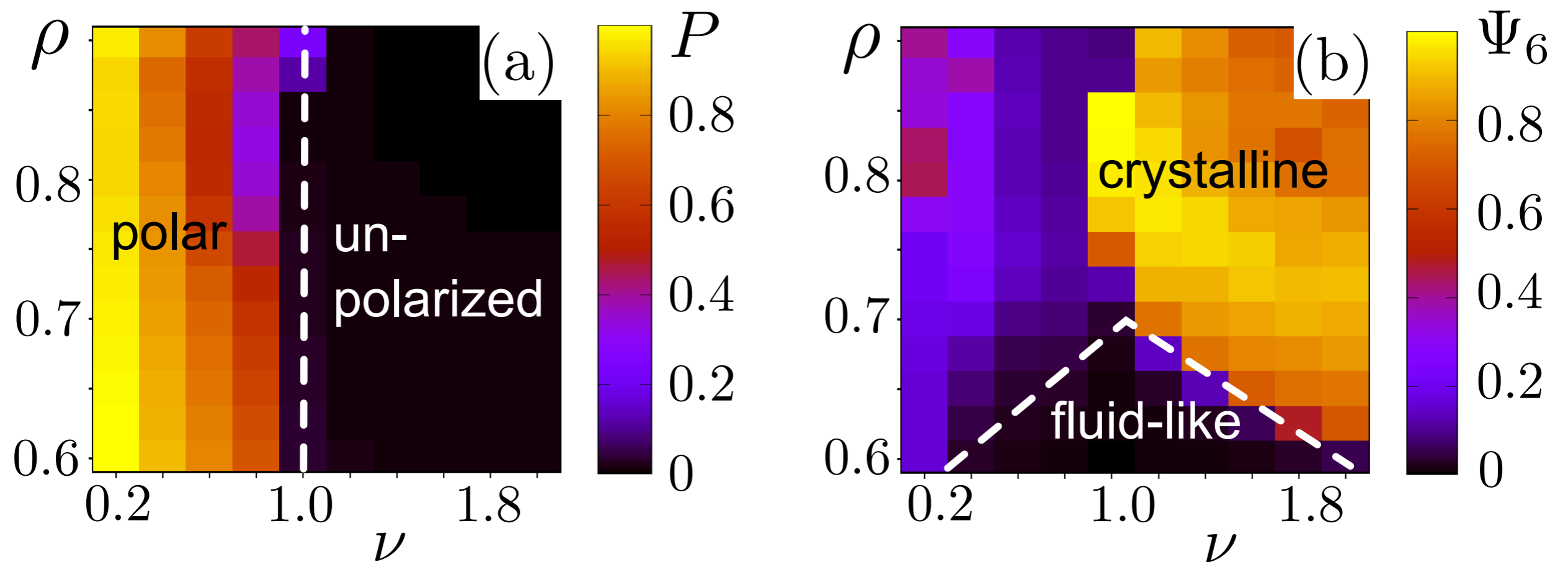
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$P$  mean polarity

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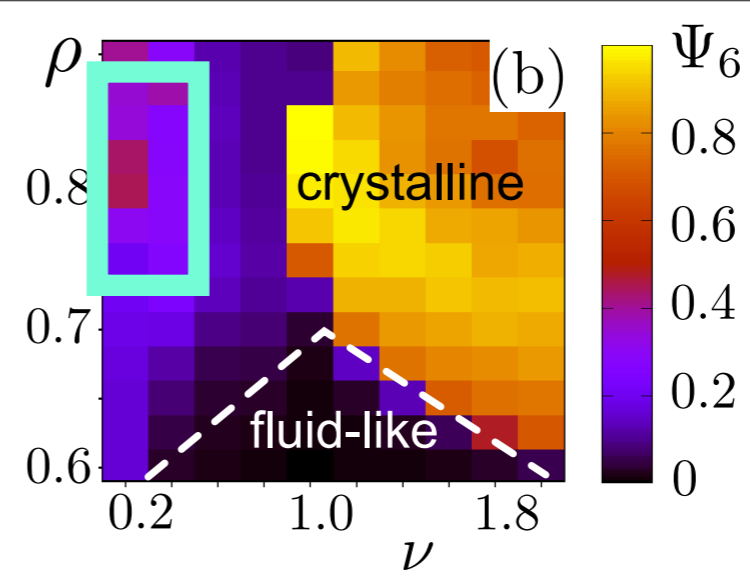
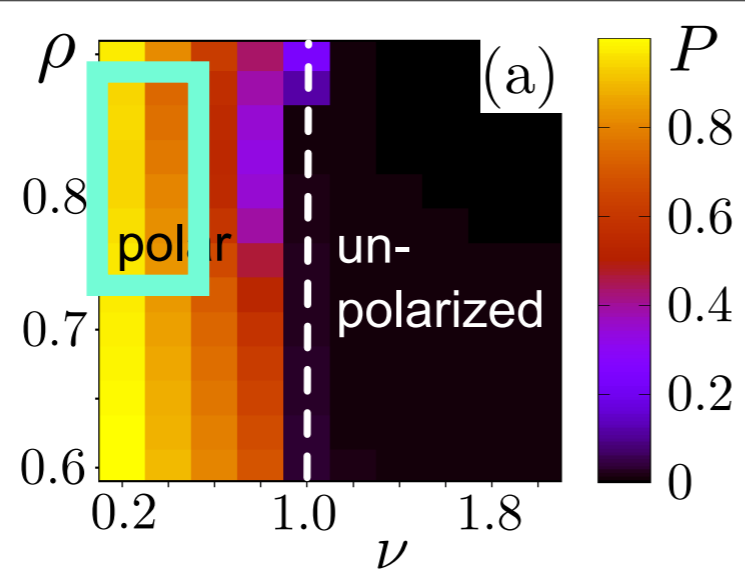
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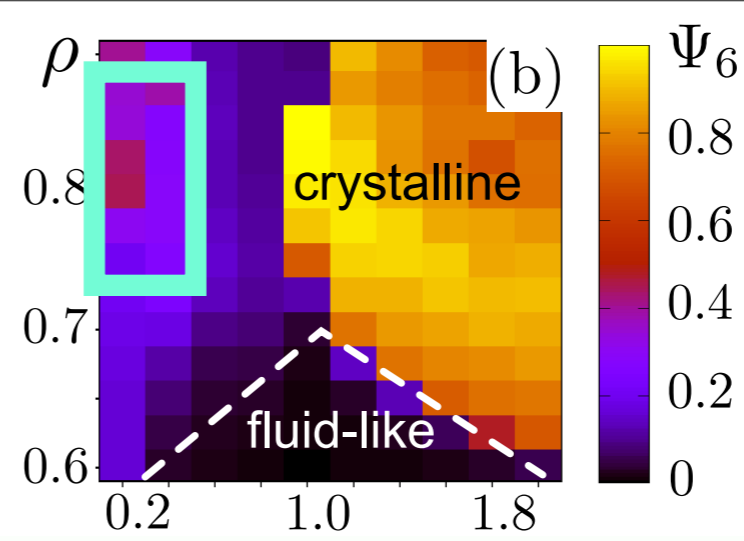
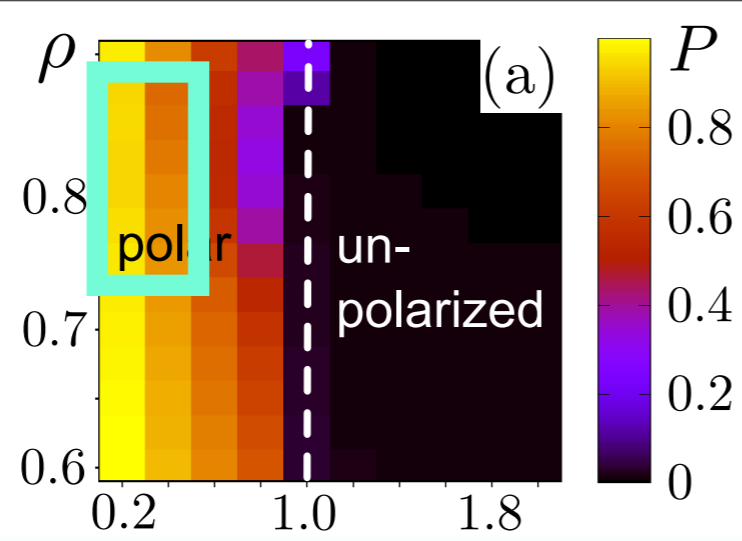
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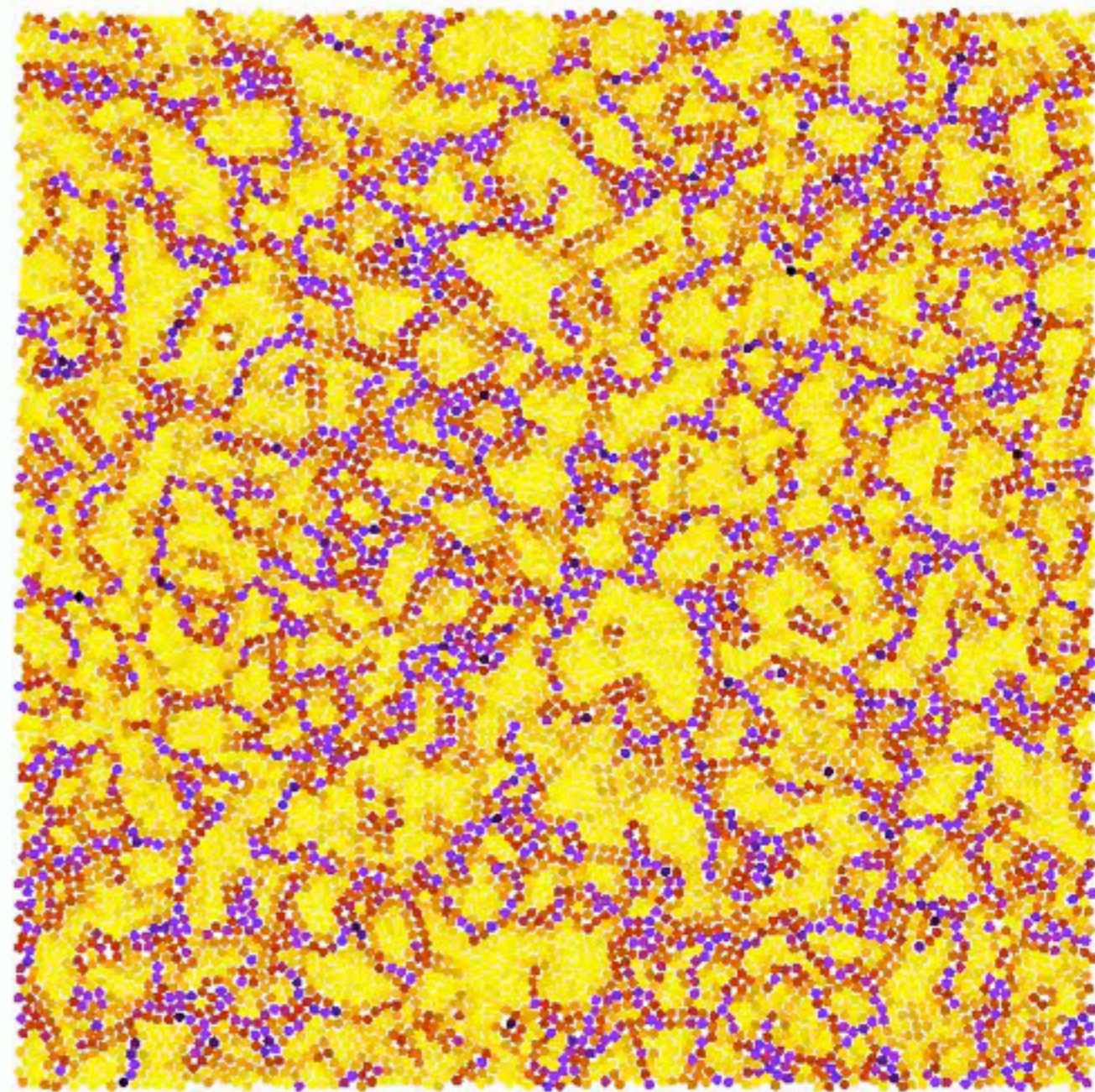
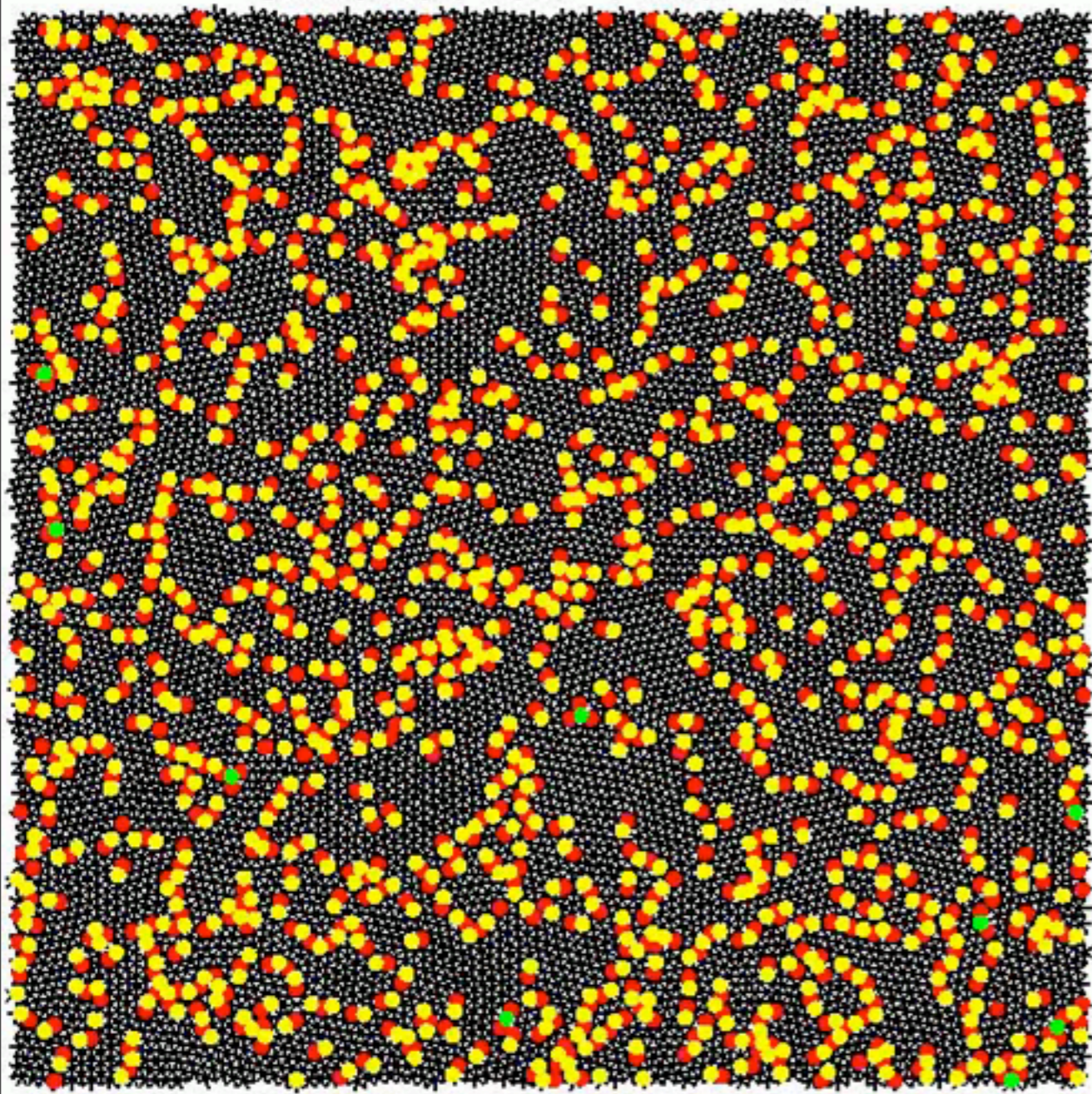
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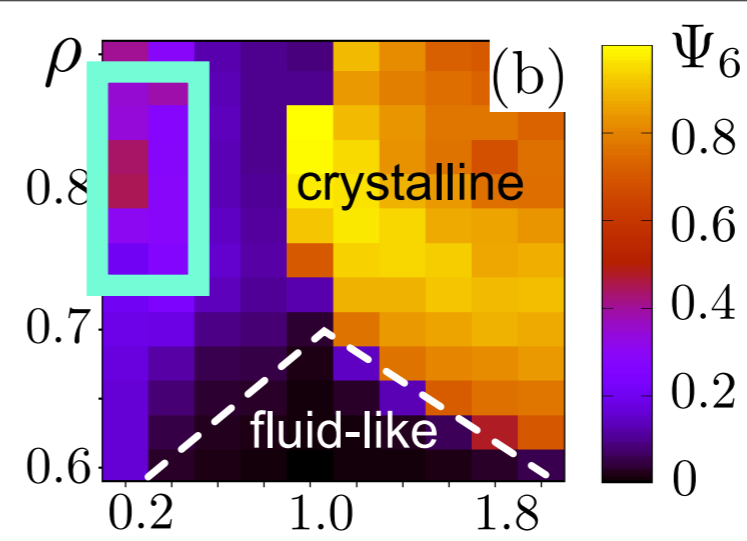
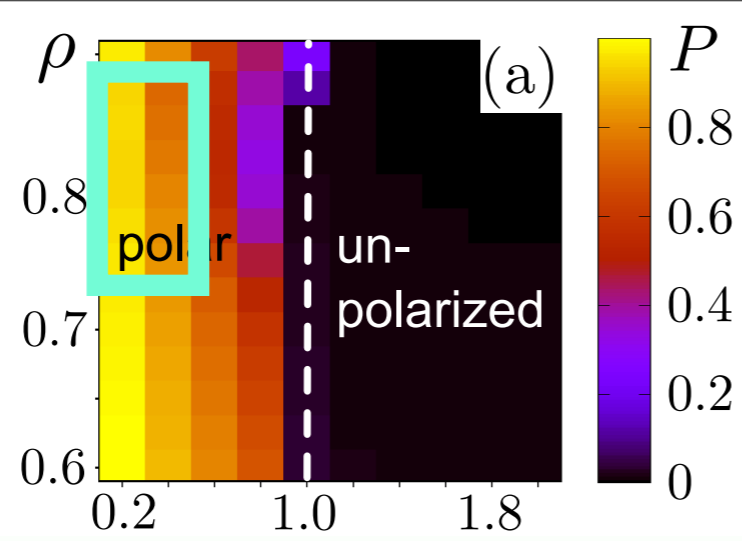
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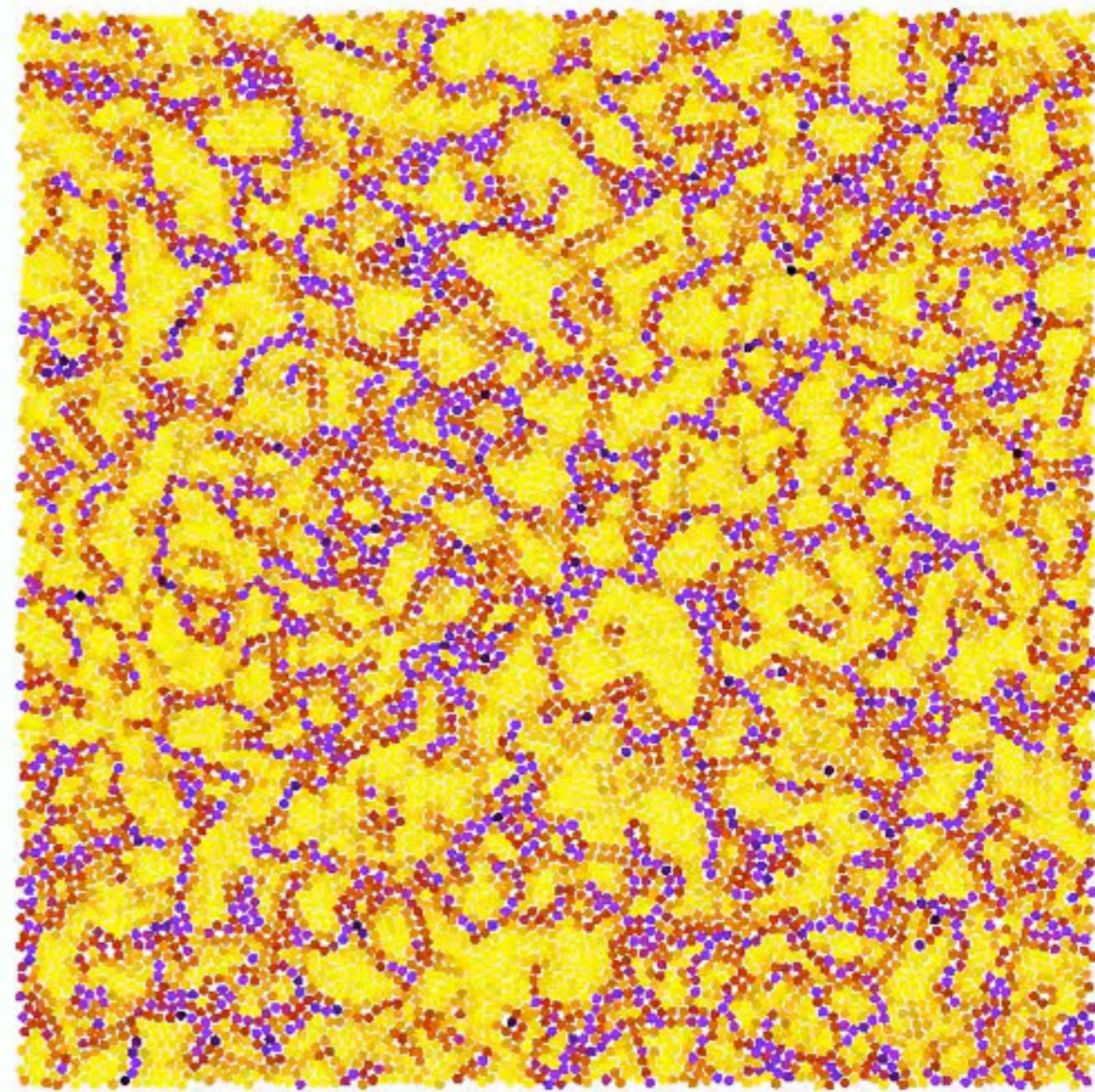
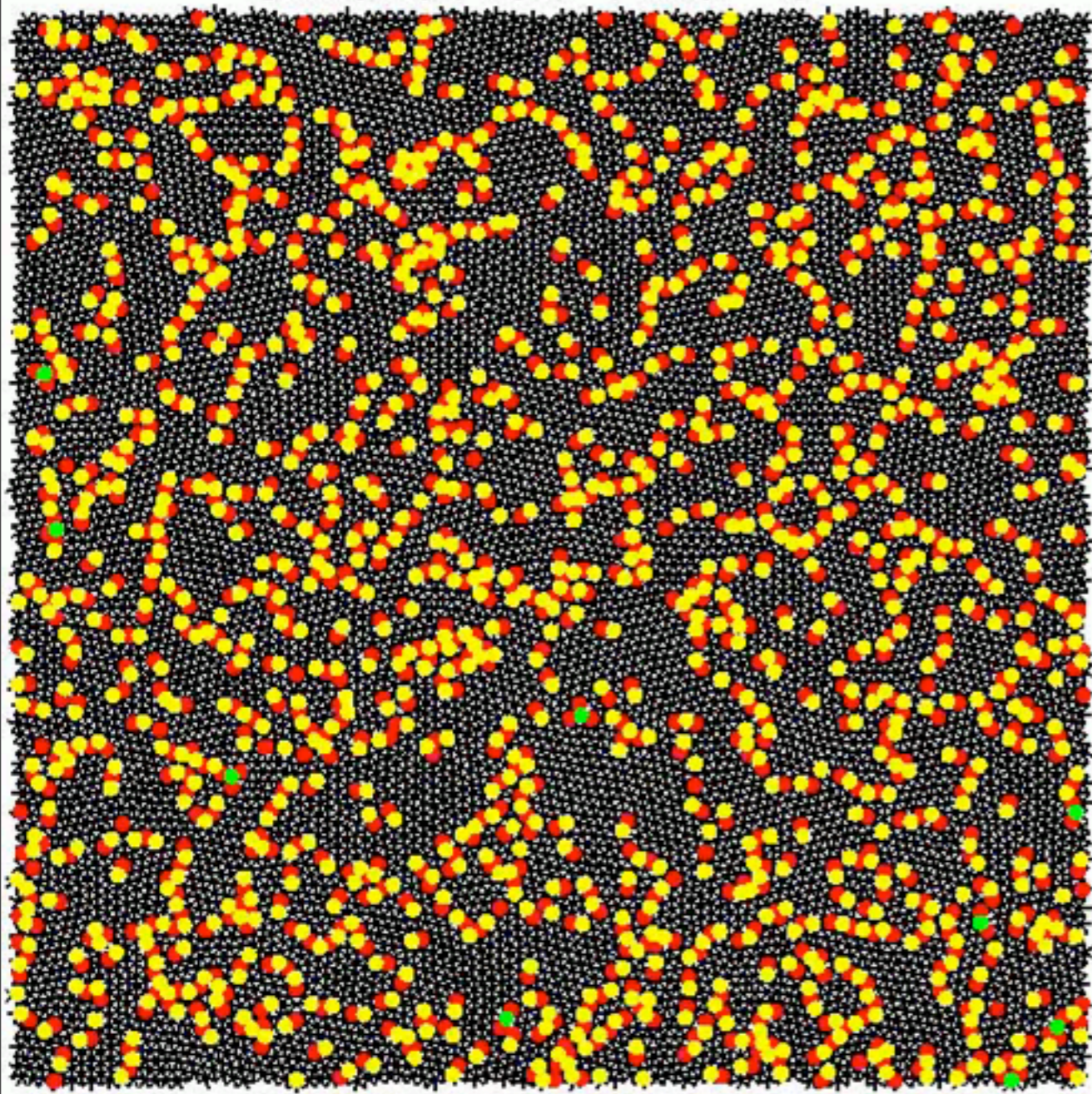


Simulation time = 0

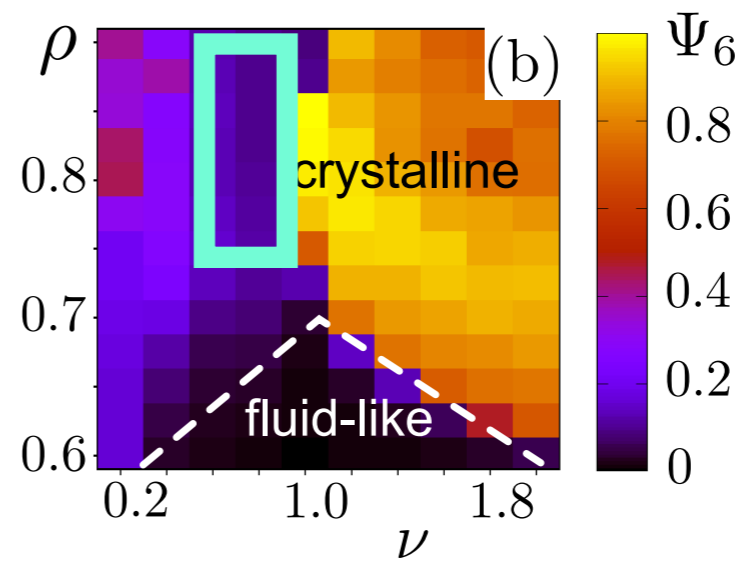
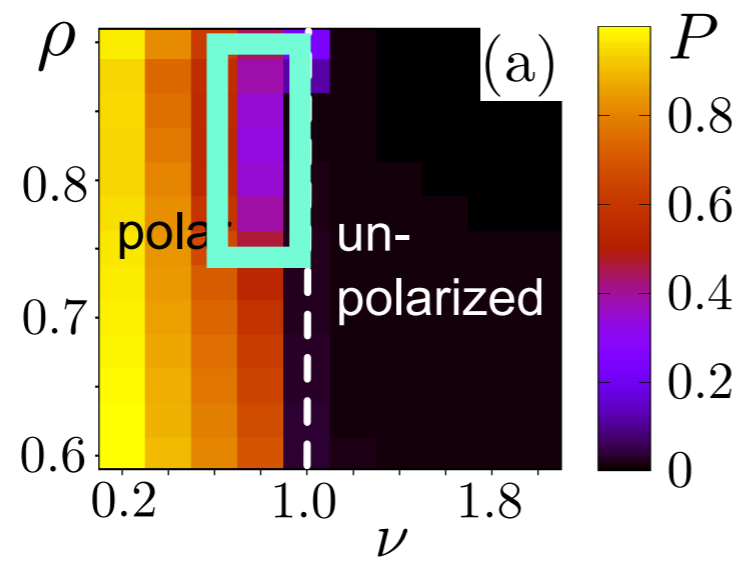


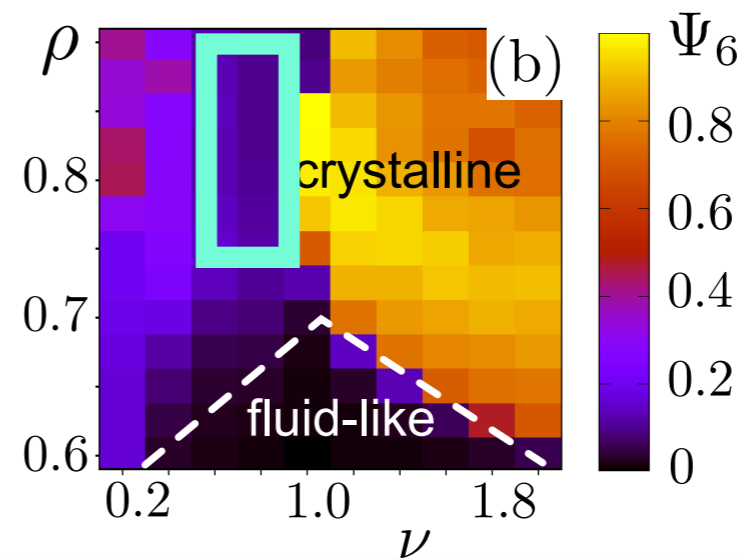
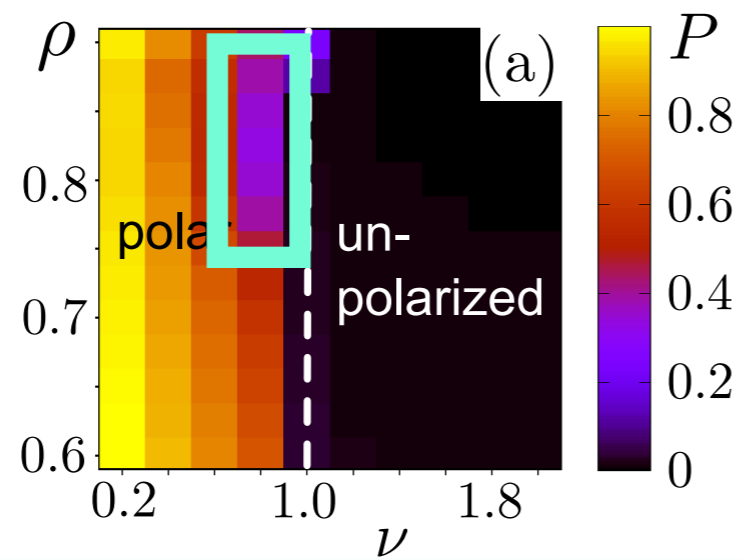


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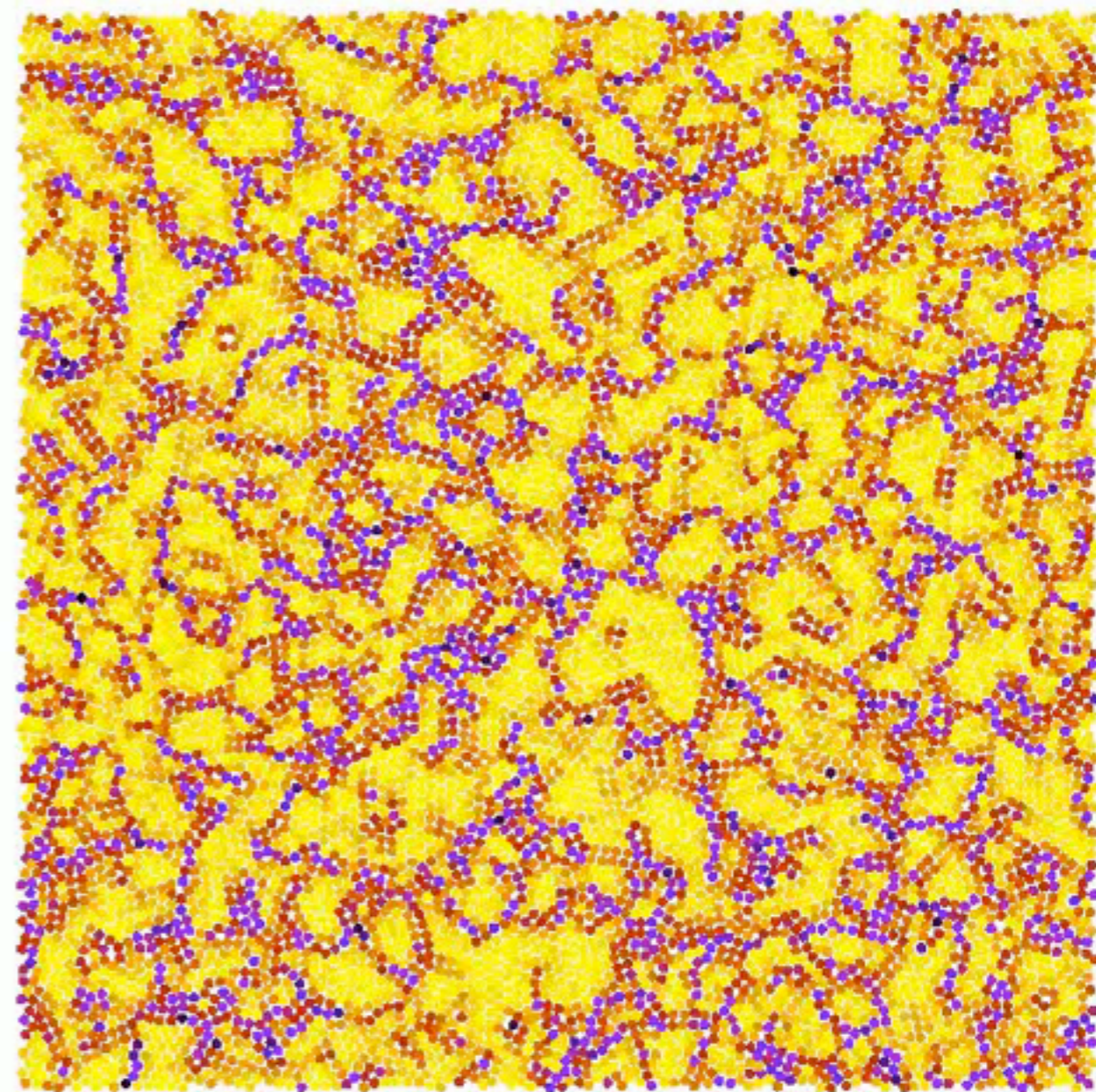
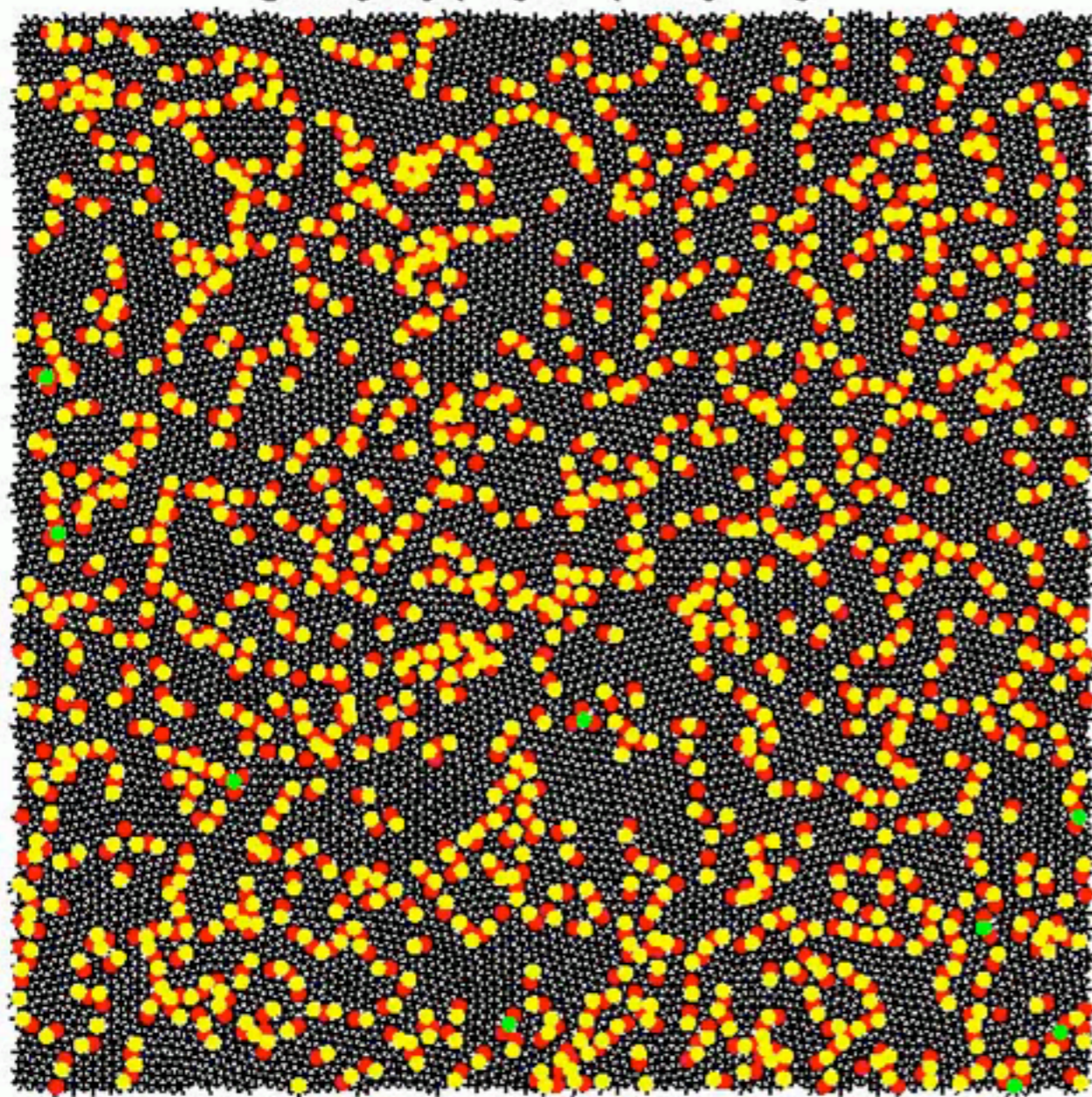


(weakly intermittent) polycrystal

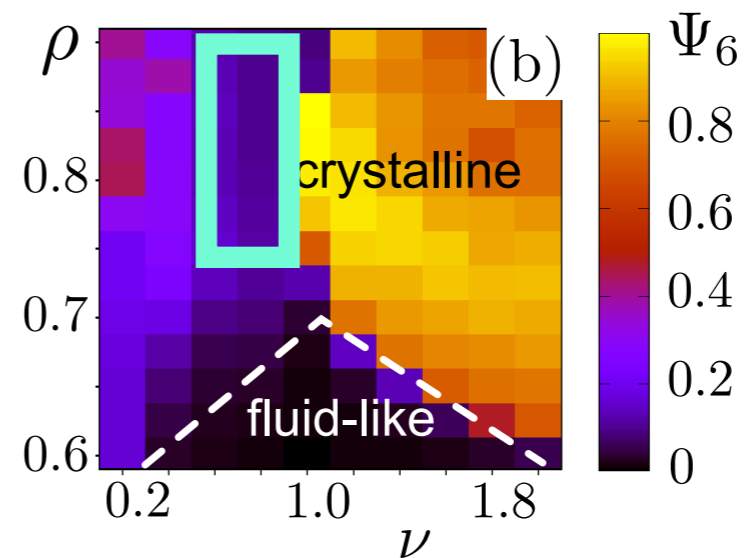
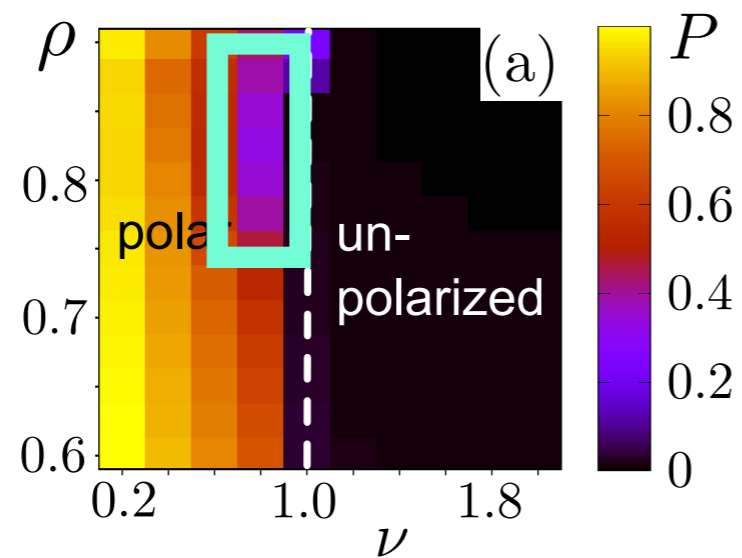




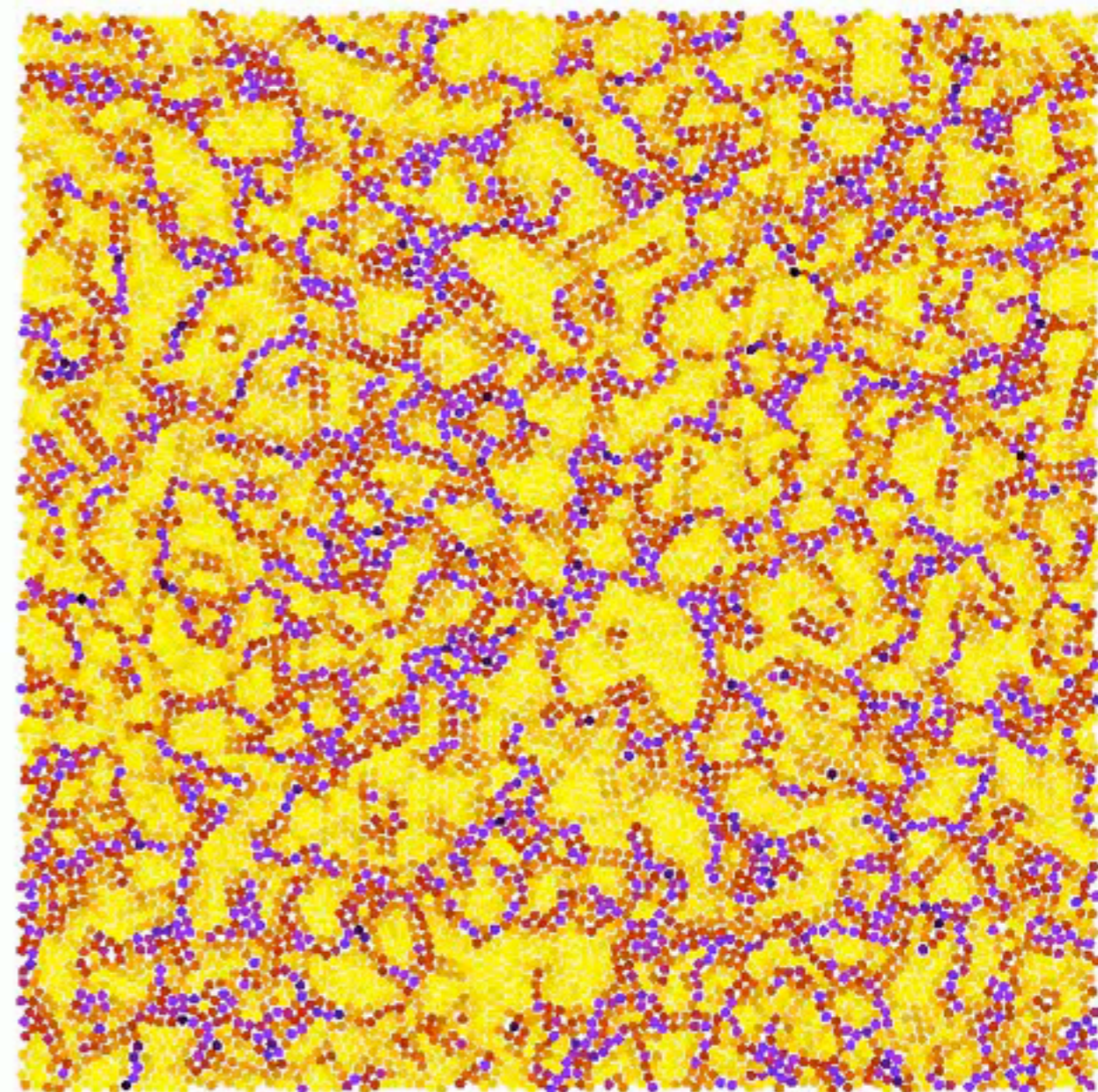
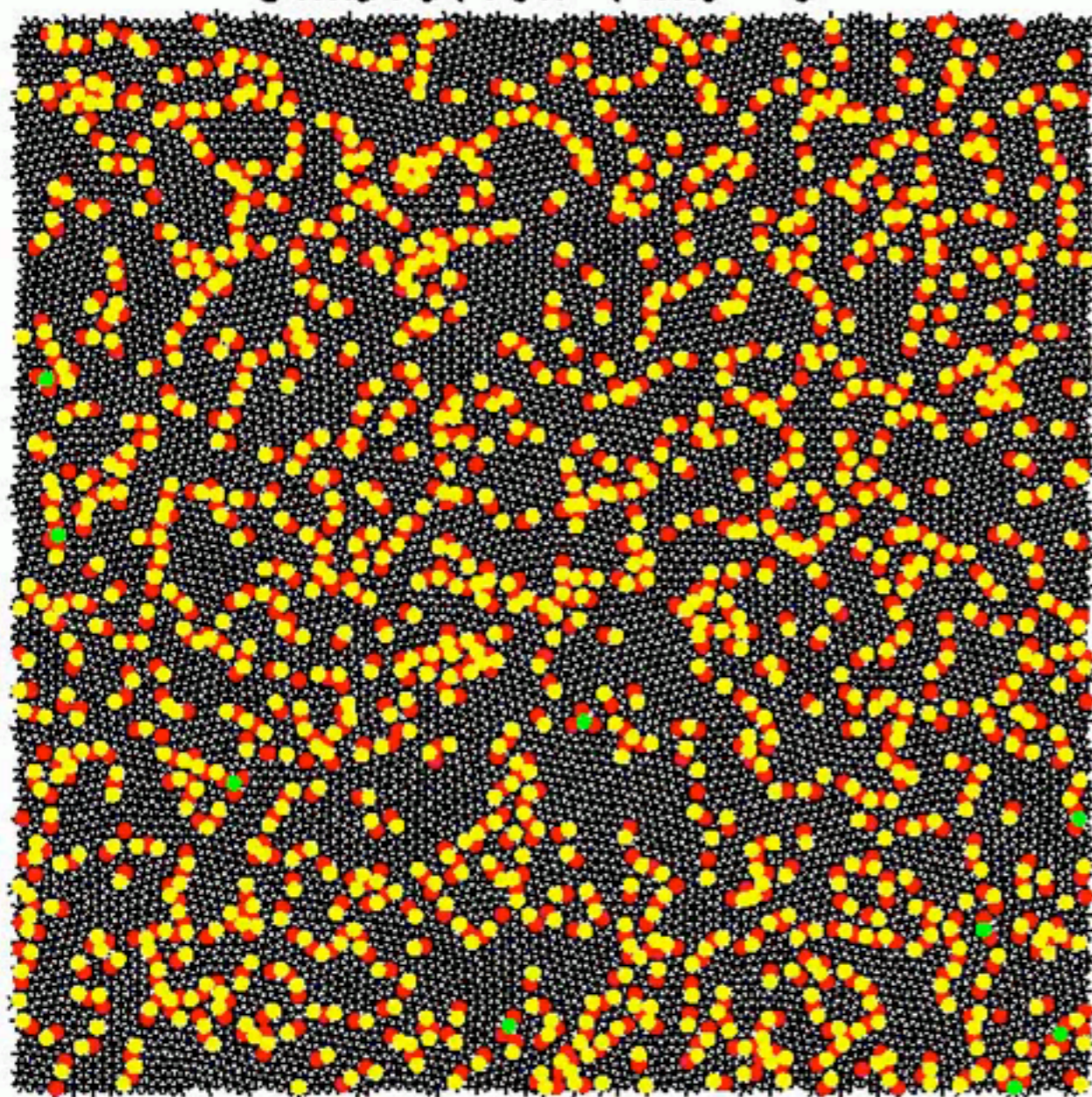
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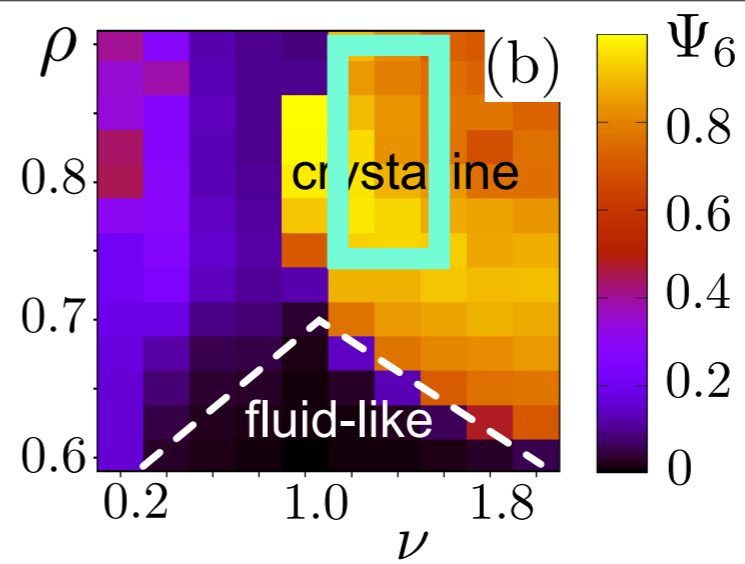
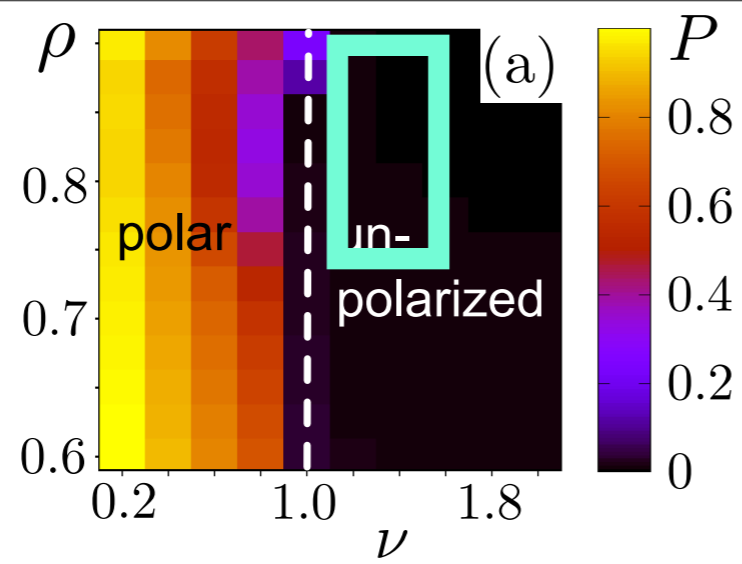


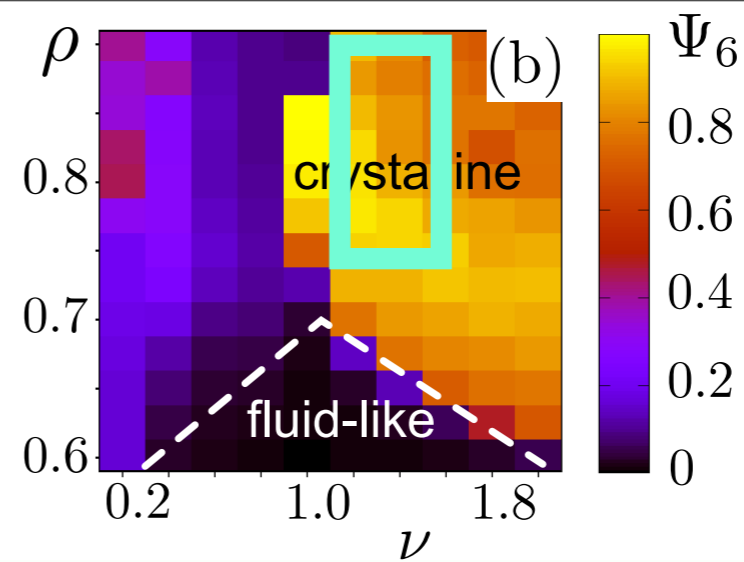
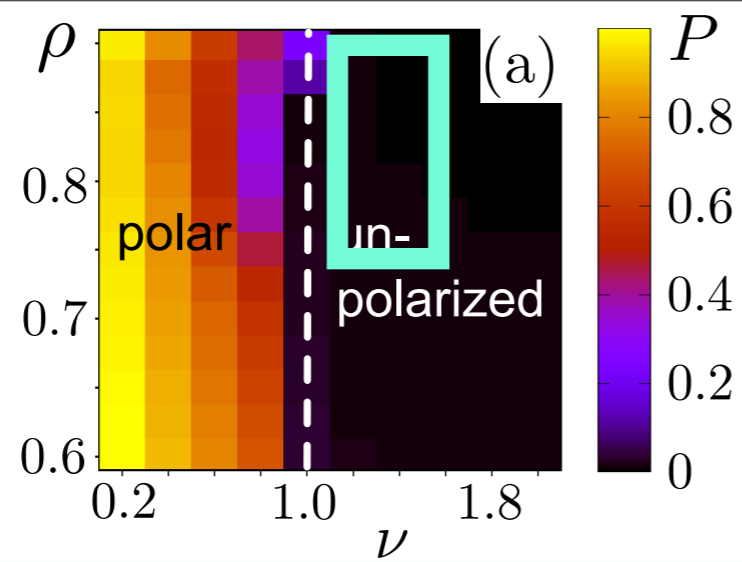


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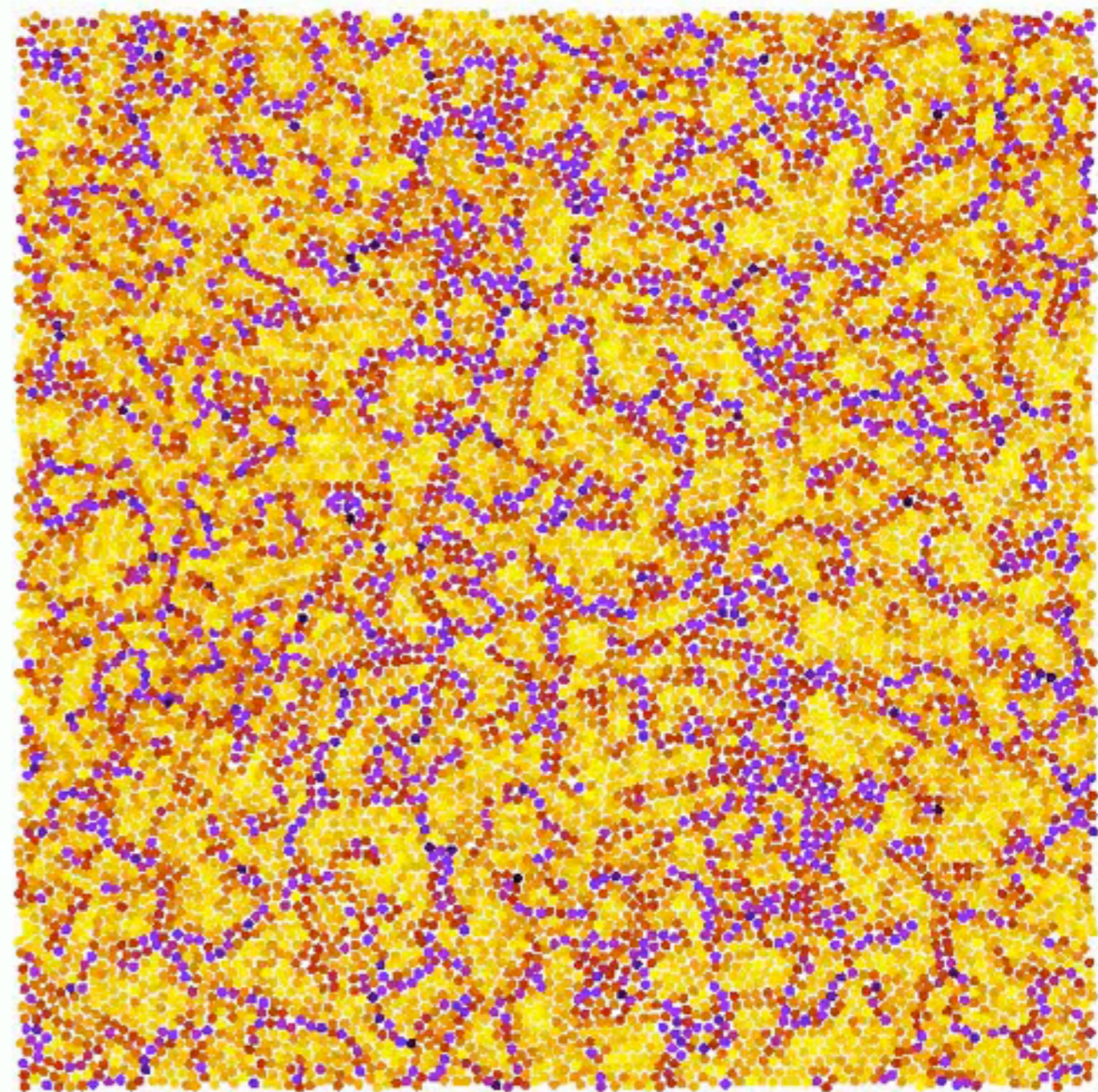
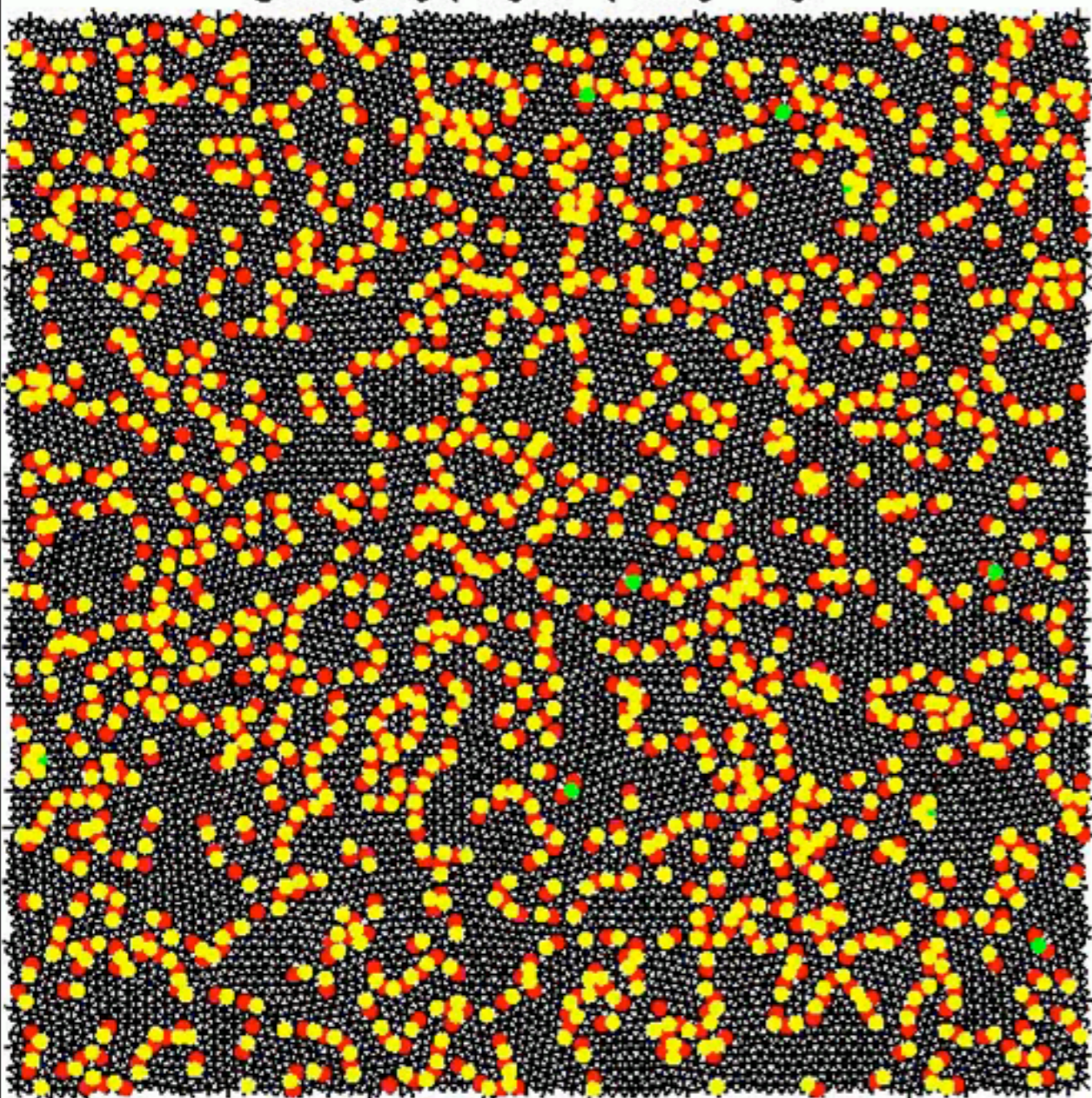


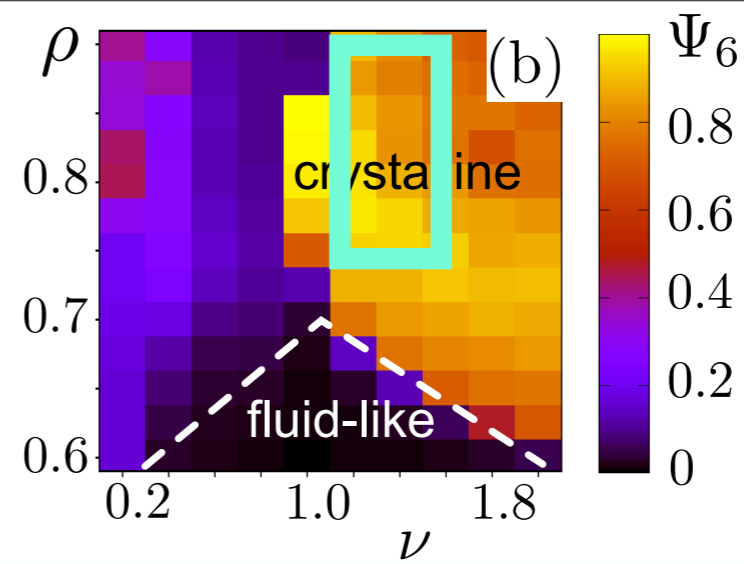
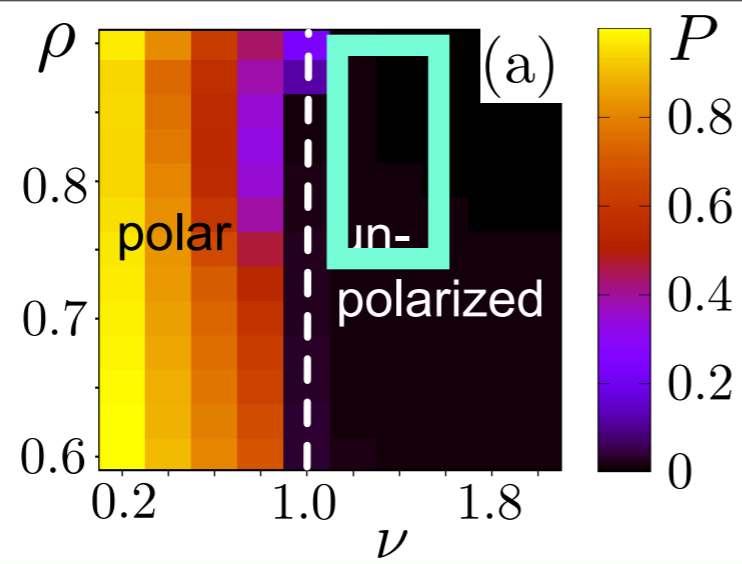
intermittent polycrystal



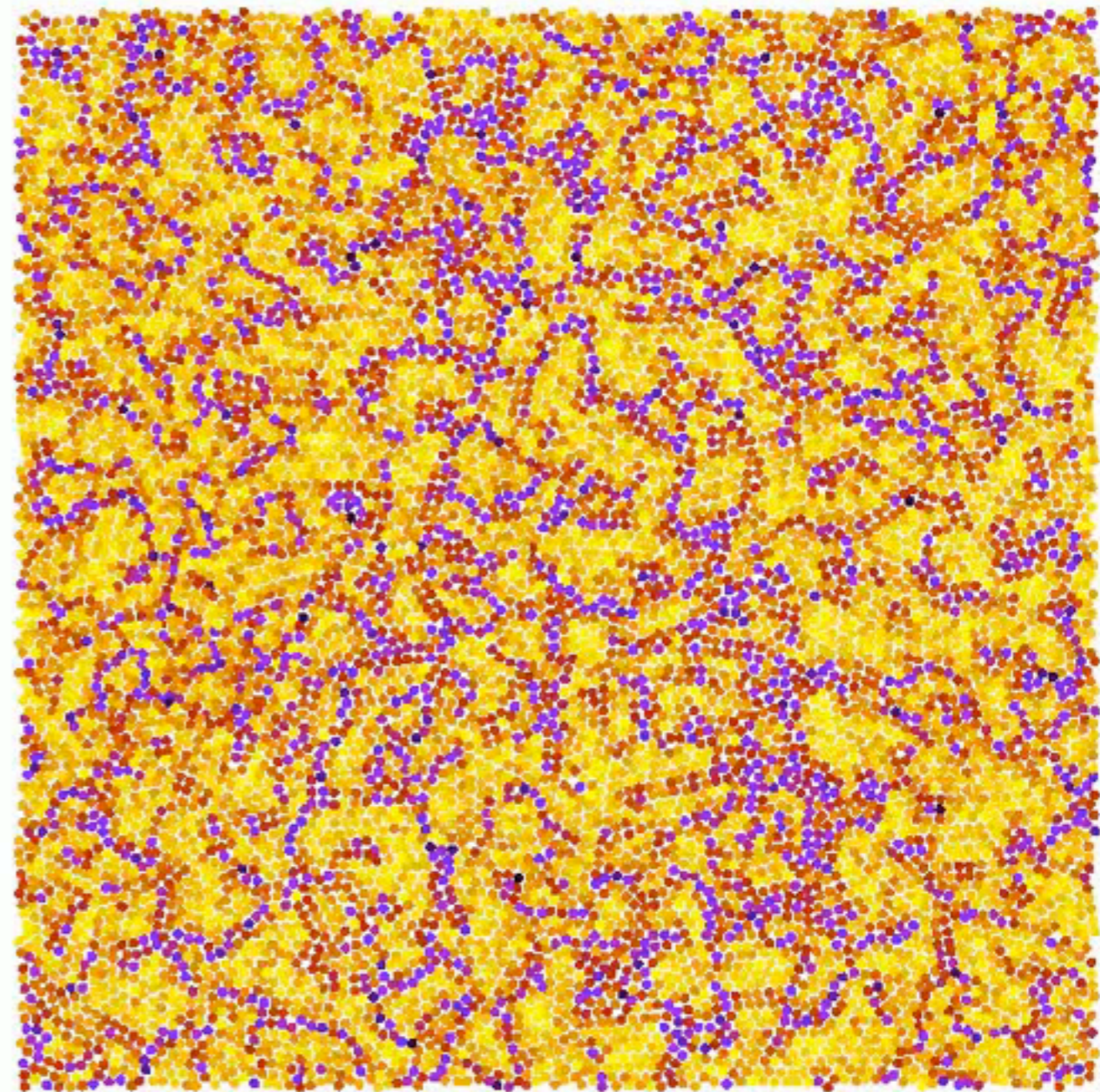
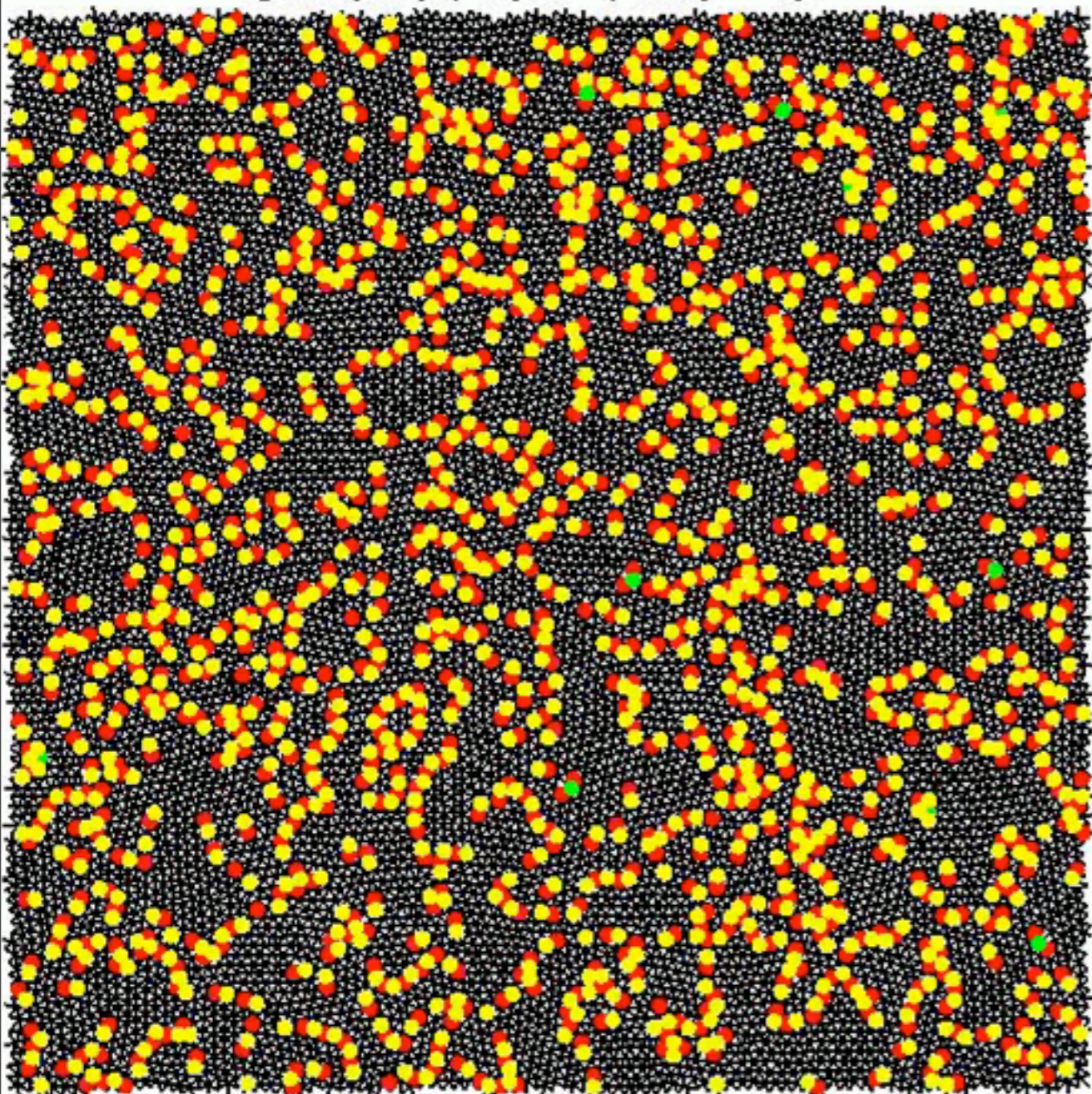


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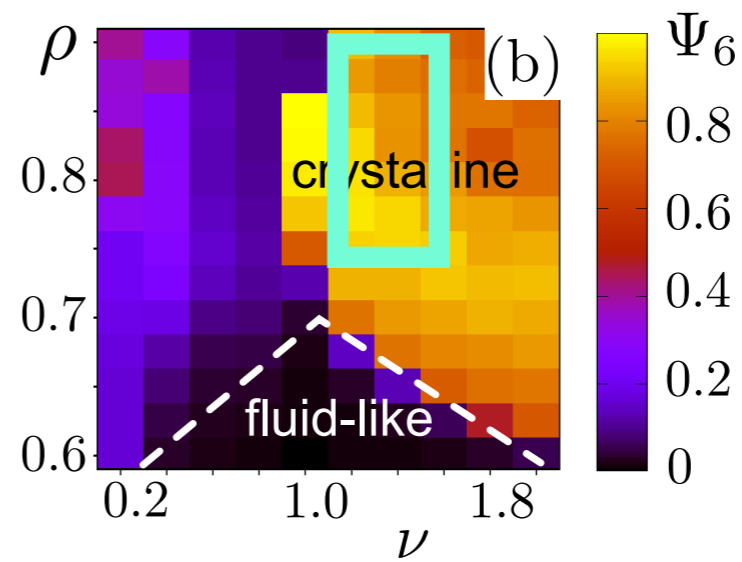
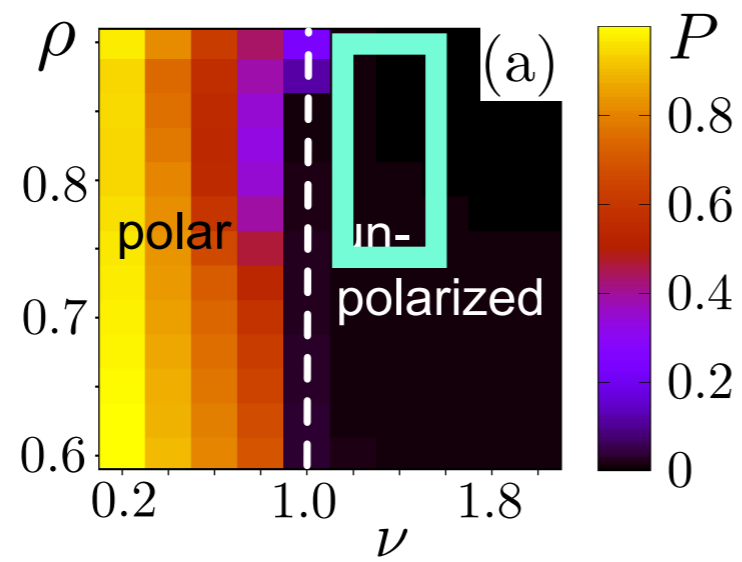


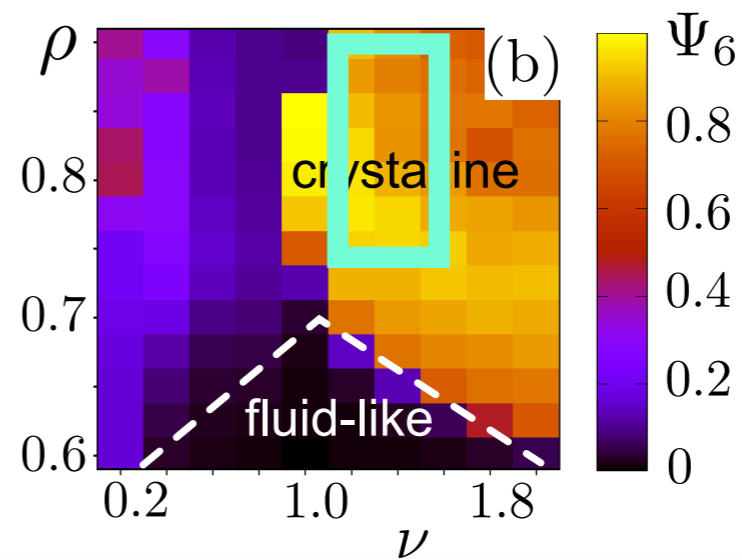
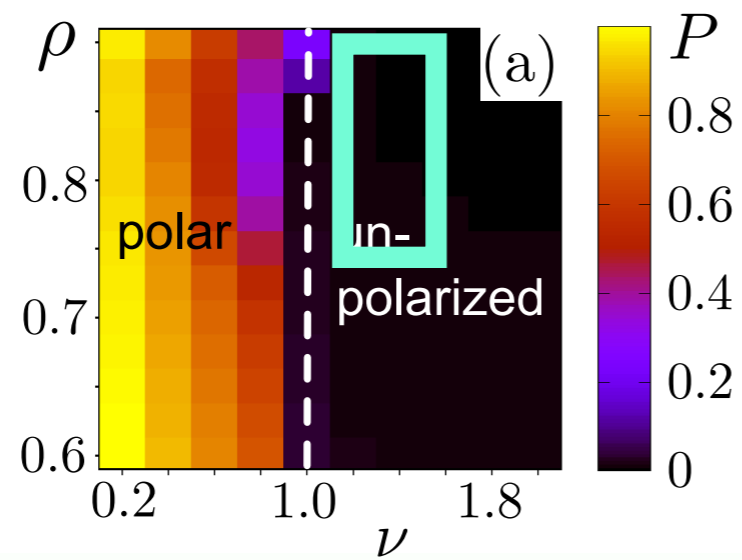


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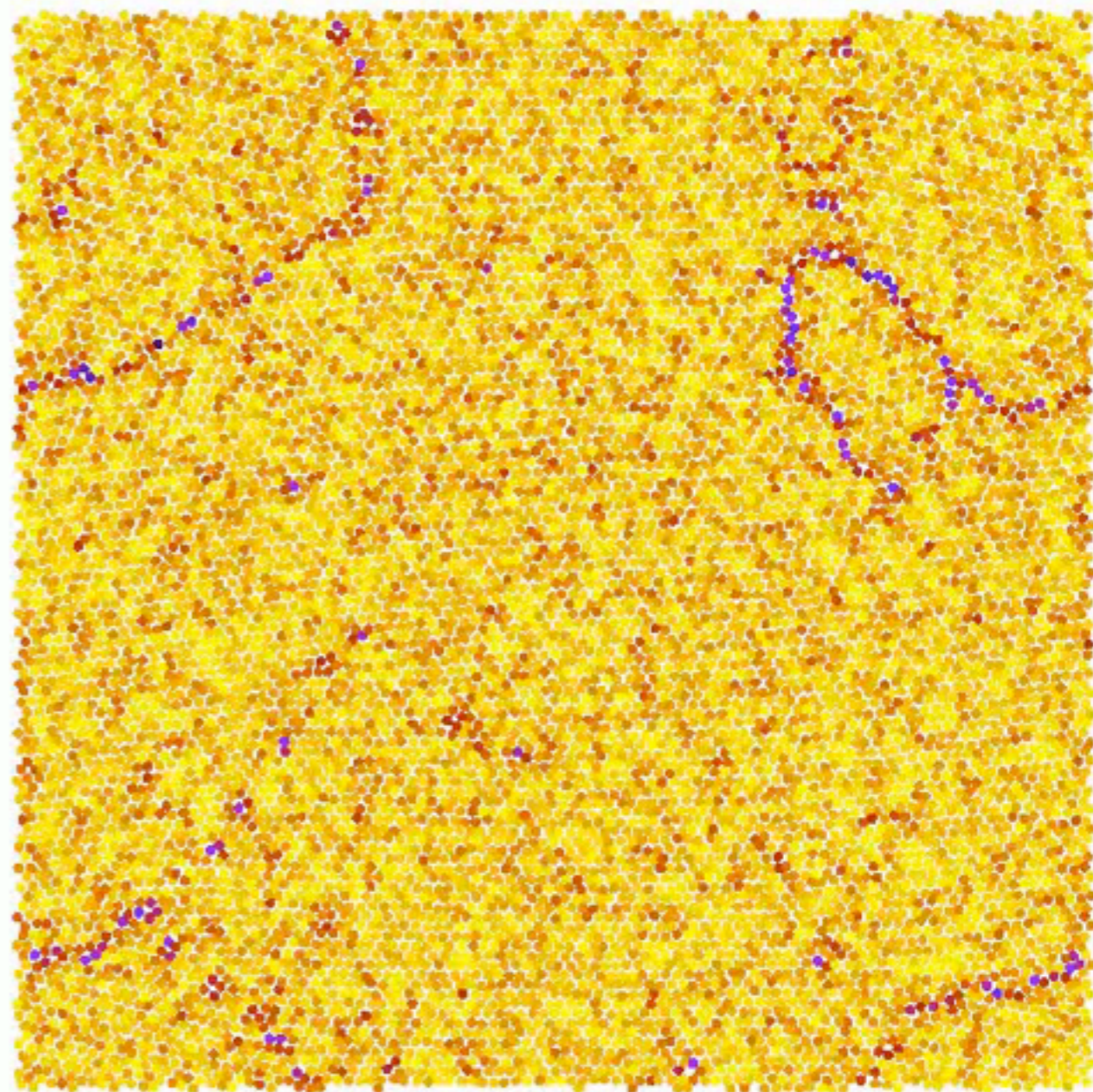
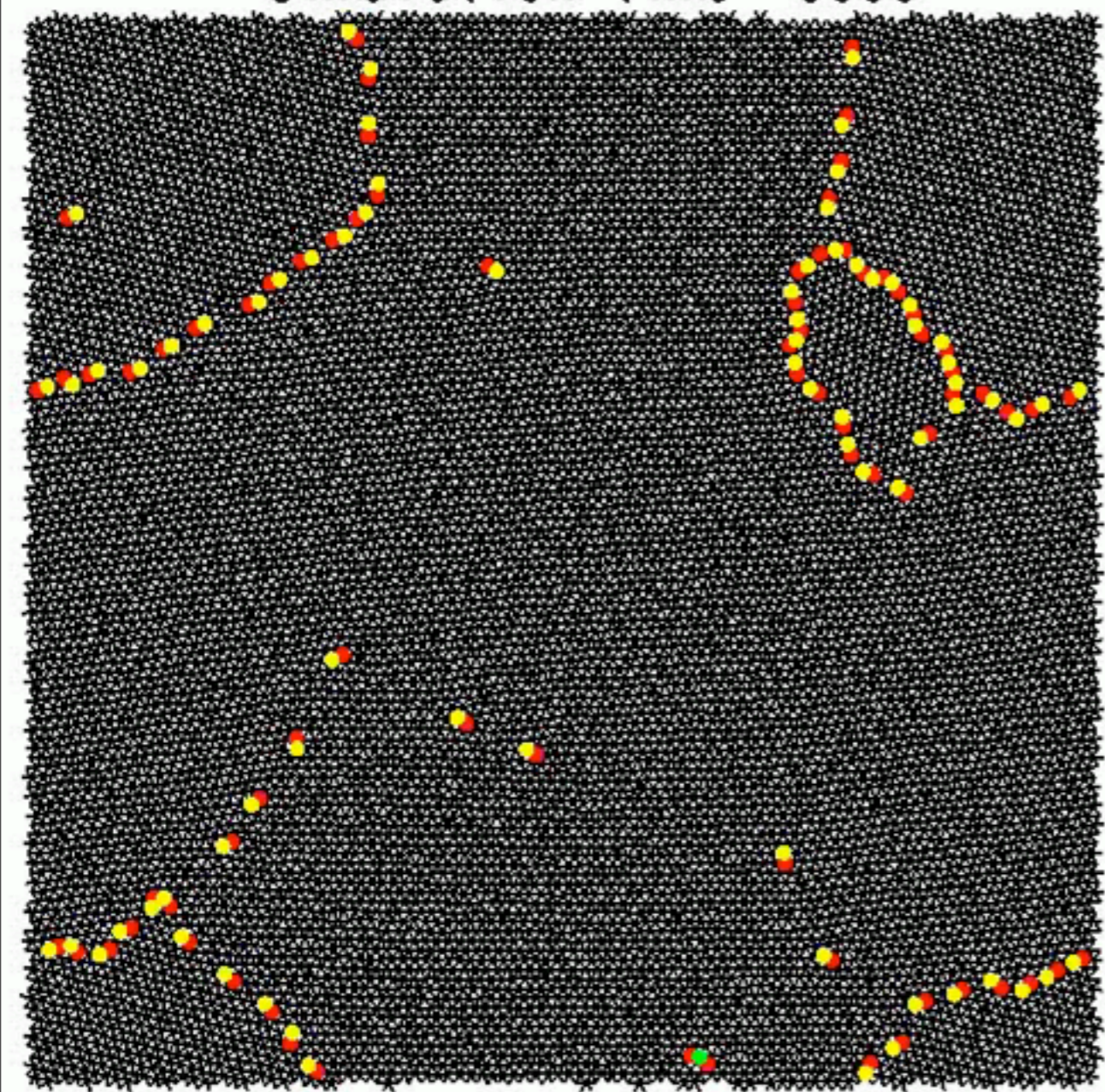


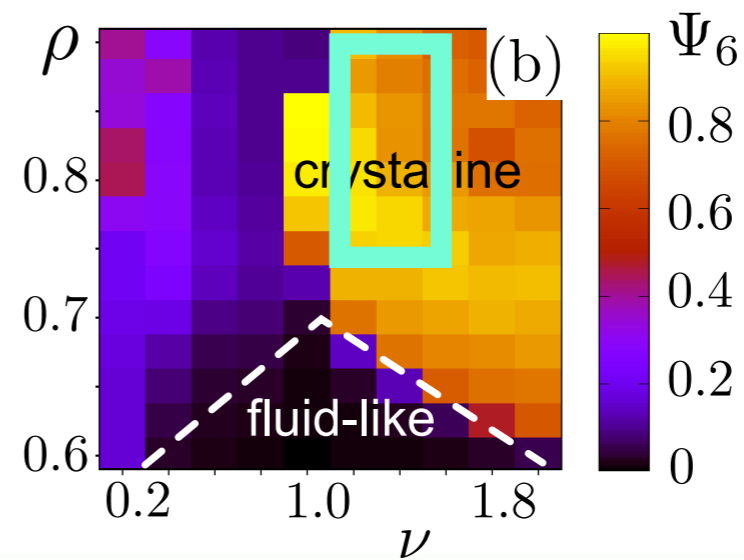
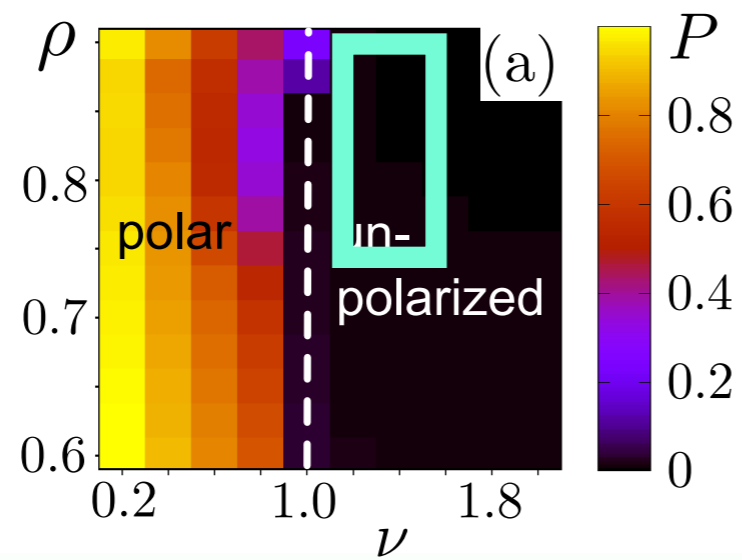
Do the grain boundaries vanish?



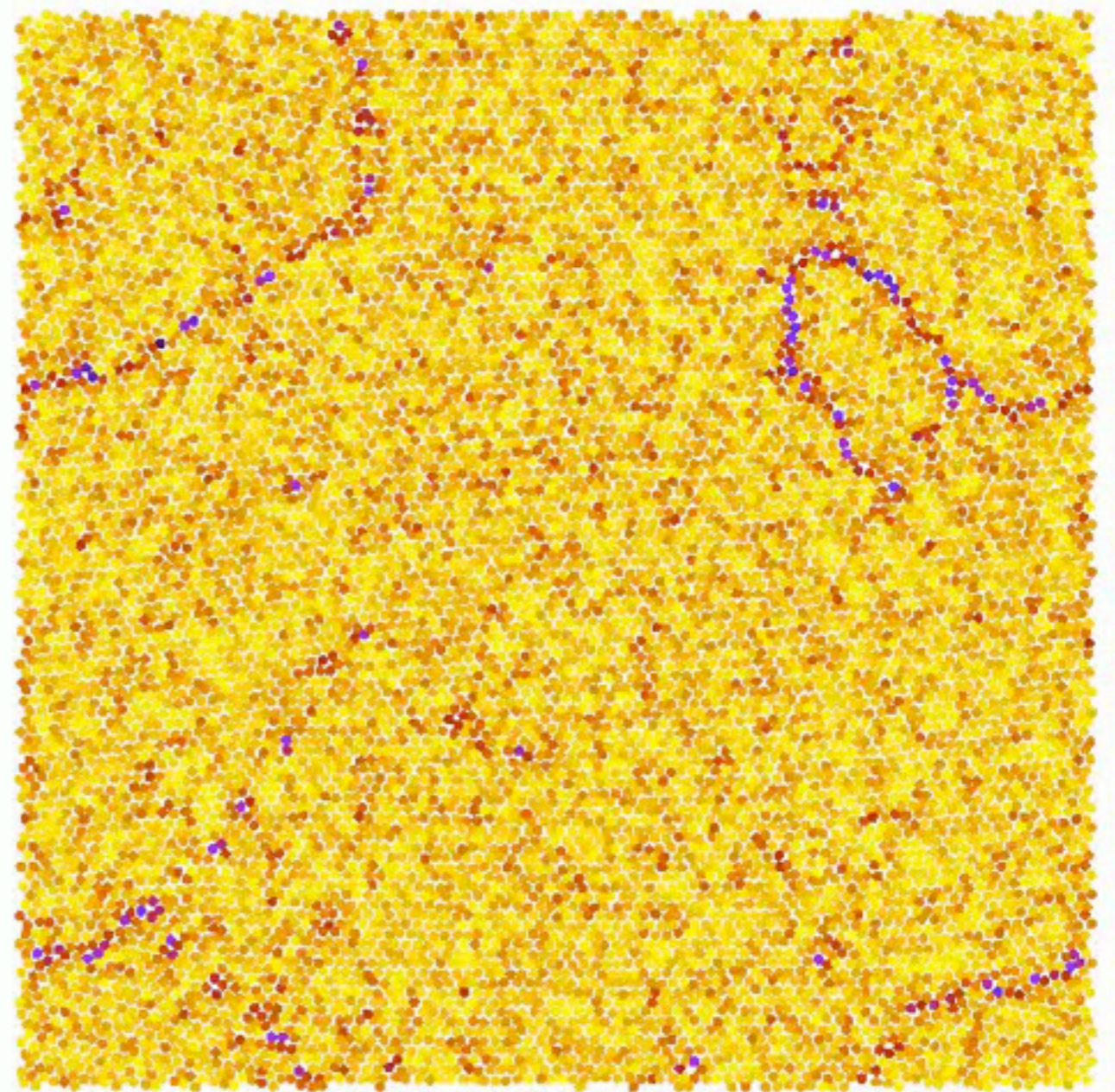
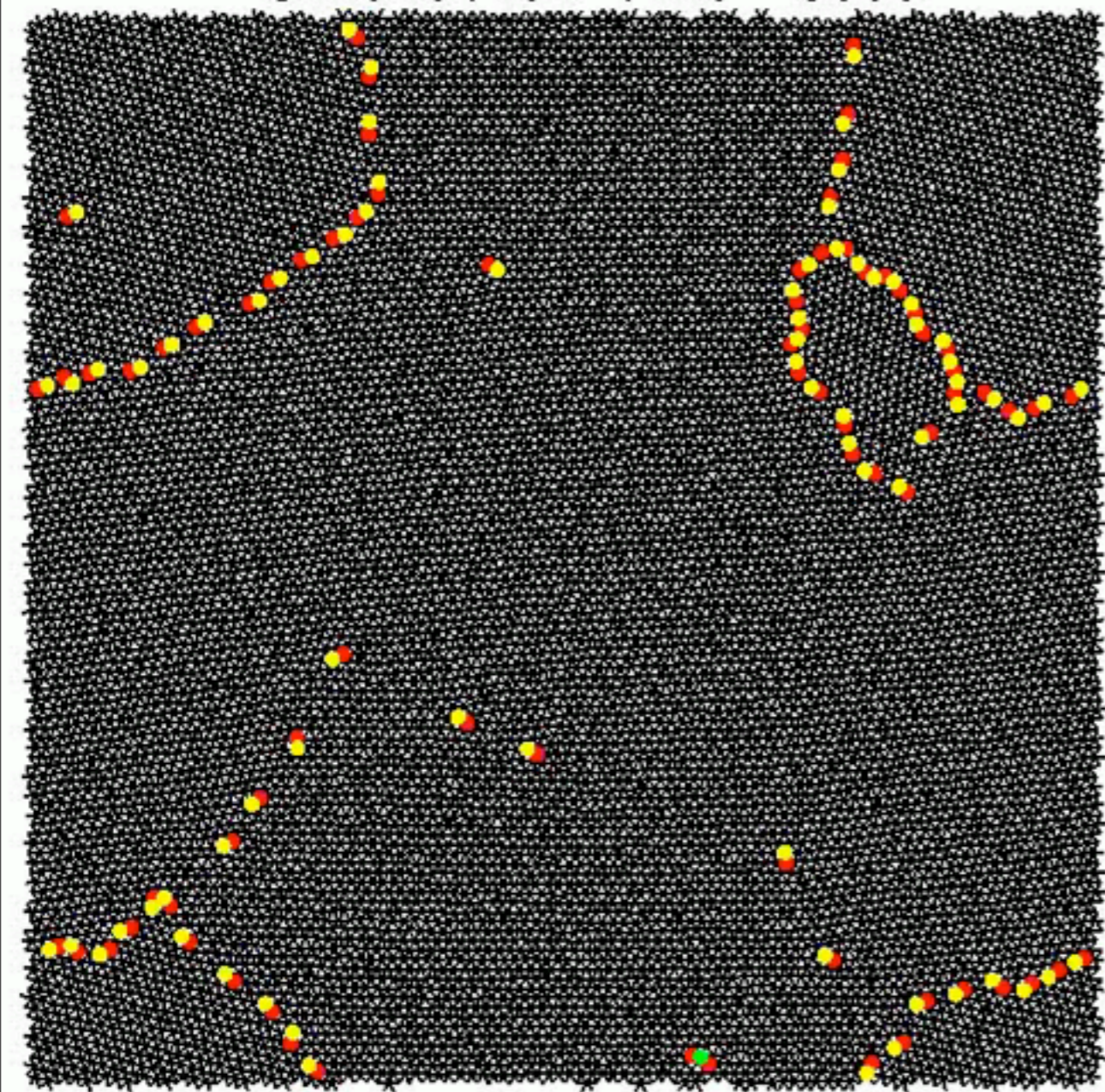


Simulation time = 5000



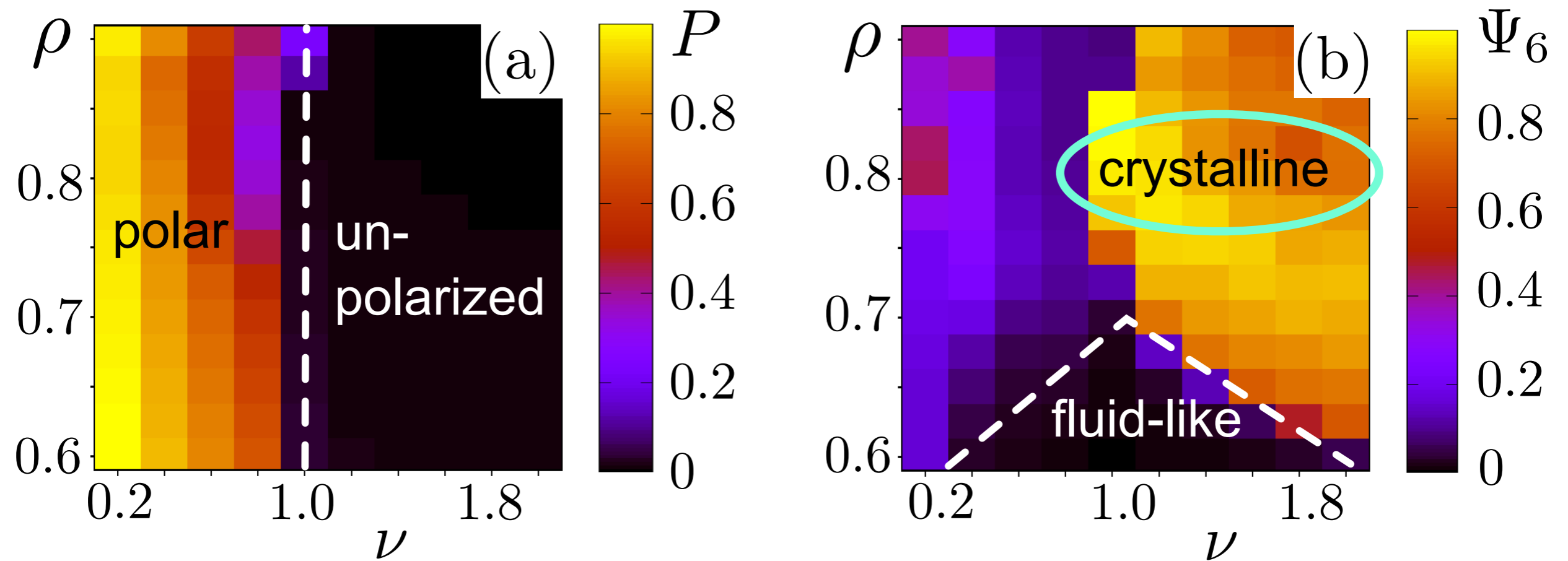


Simulation time = 5000



yes, but it take a looong time ...

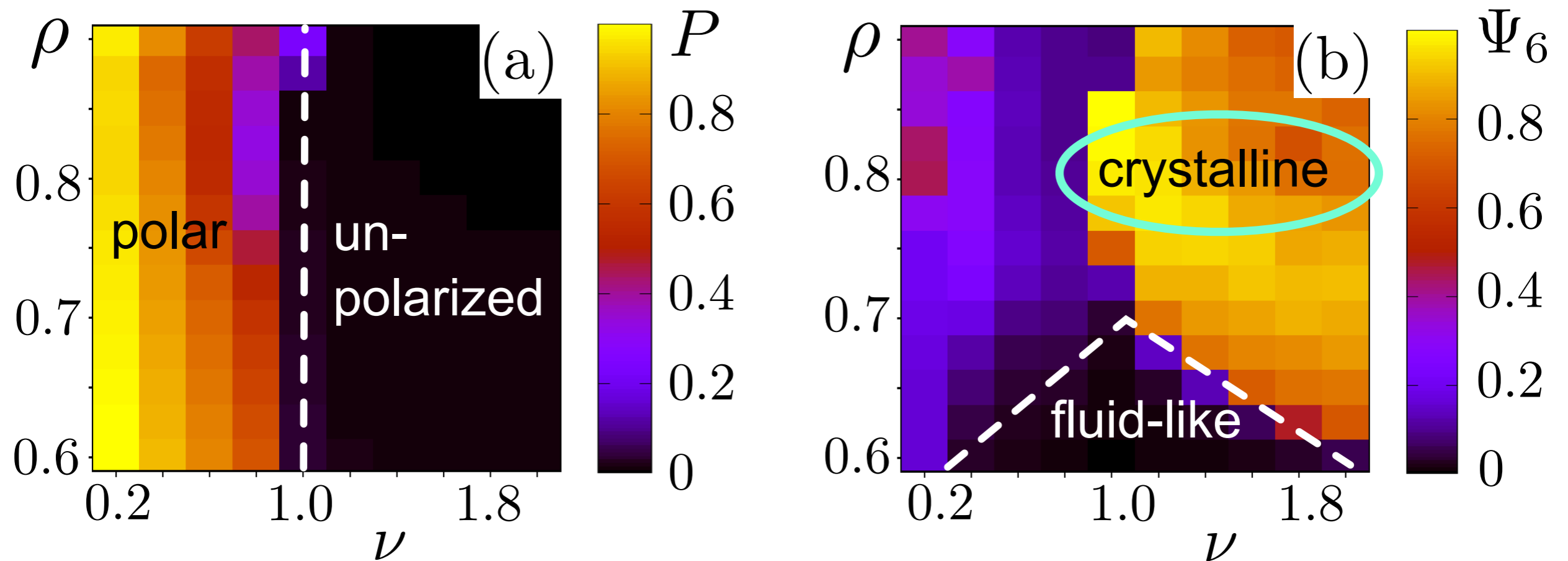
# Defect-Mediated Phase Transitions in Active Soft Matter



**Questions:**



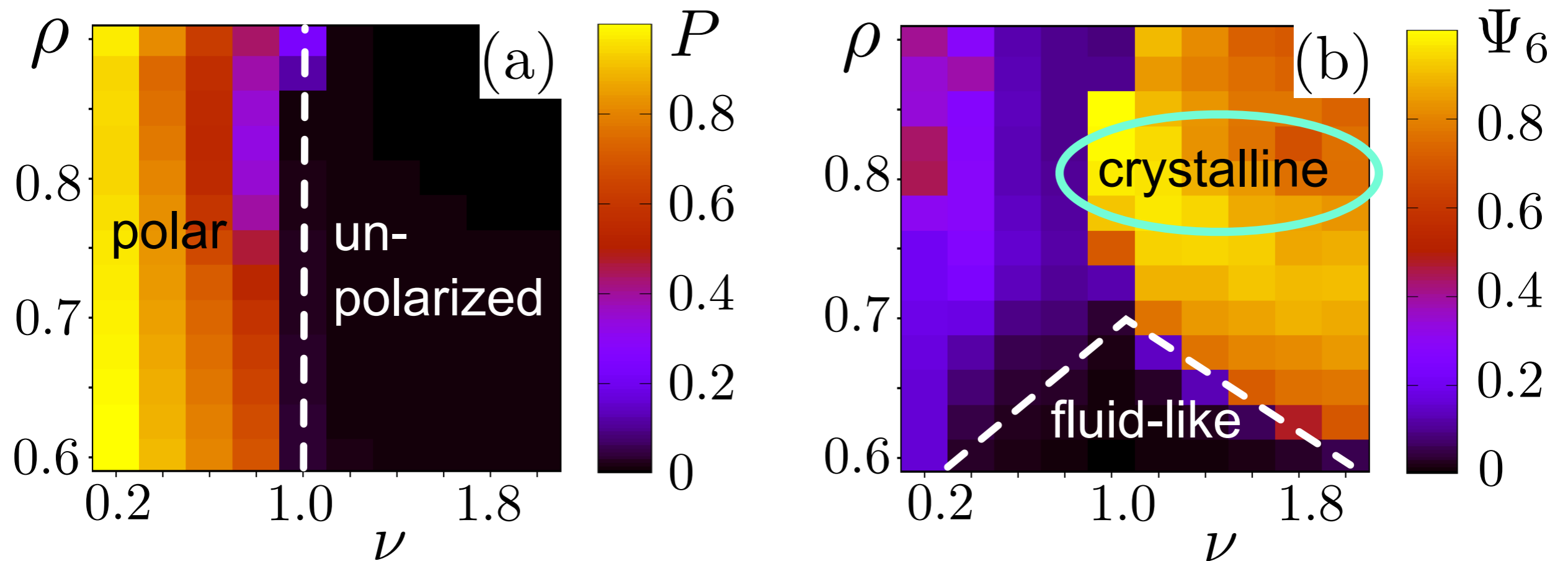
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What is the degree of translational order in the “crystalline” regime?


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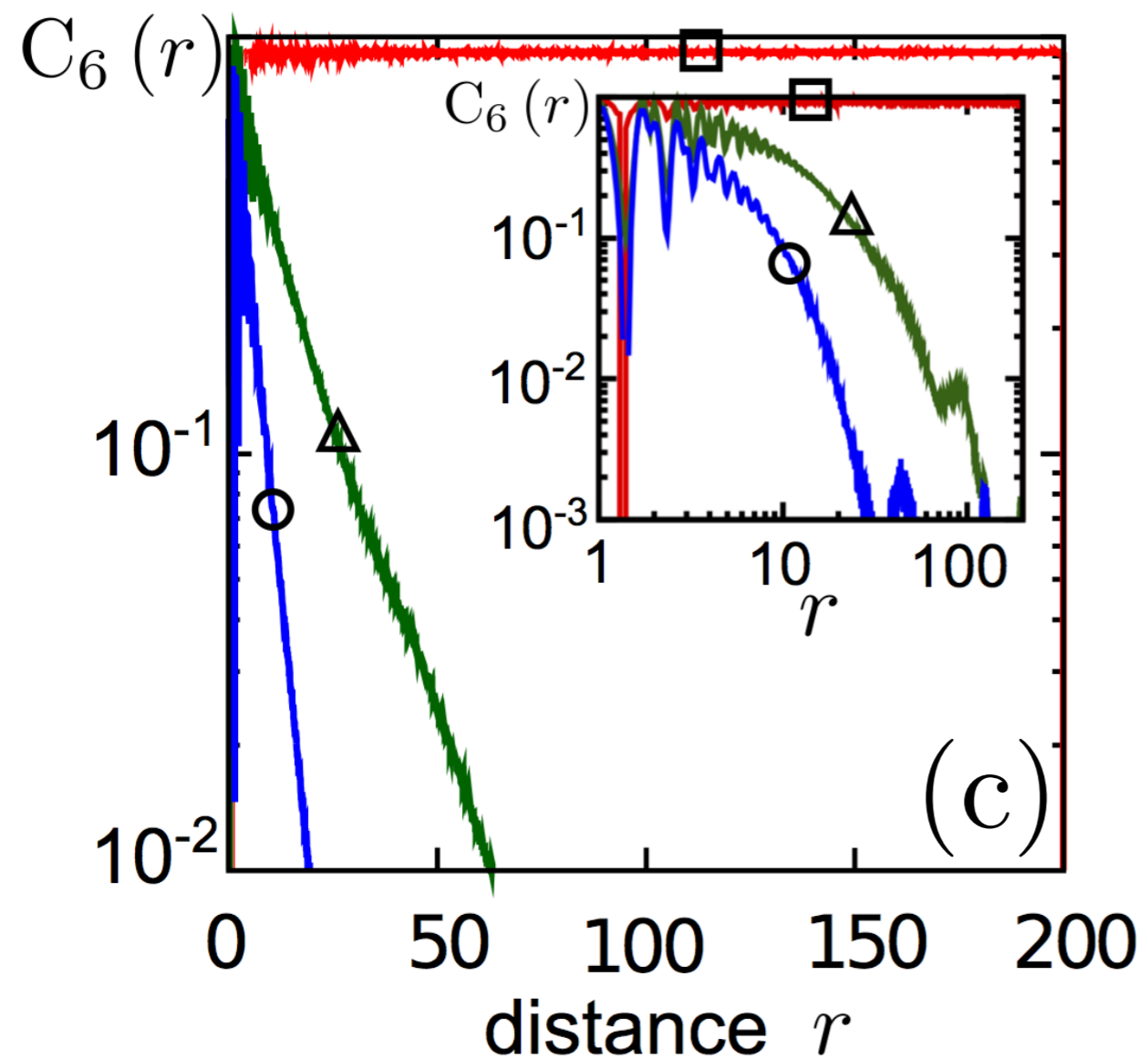
What is the degree of translational order in the “crystalline” regime?

What is the role of defects for the selection of states?

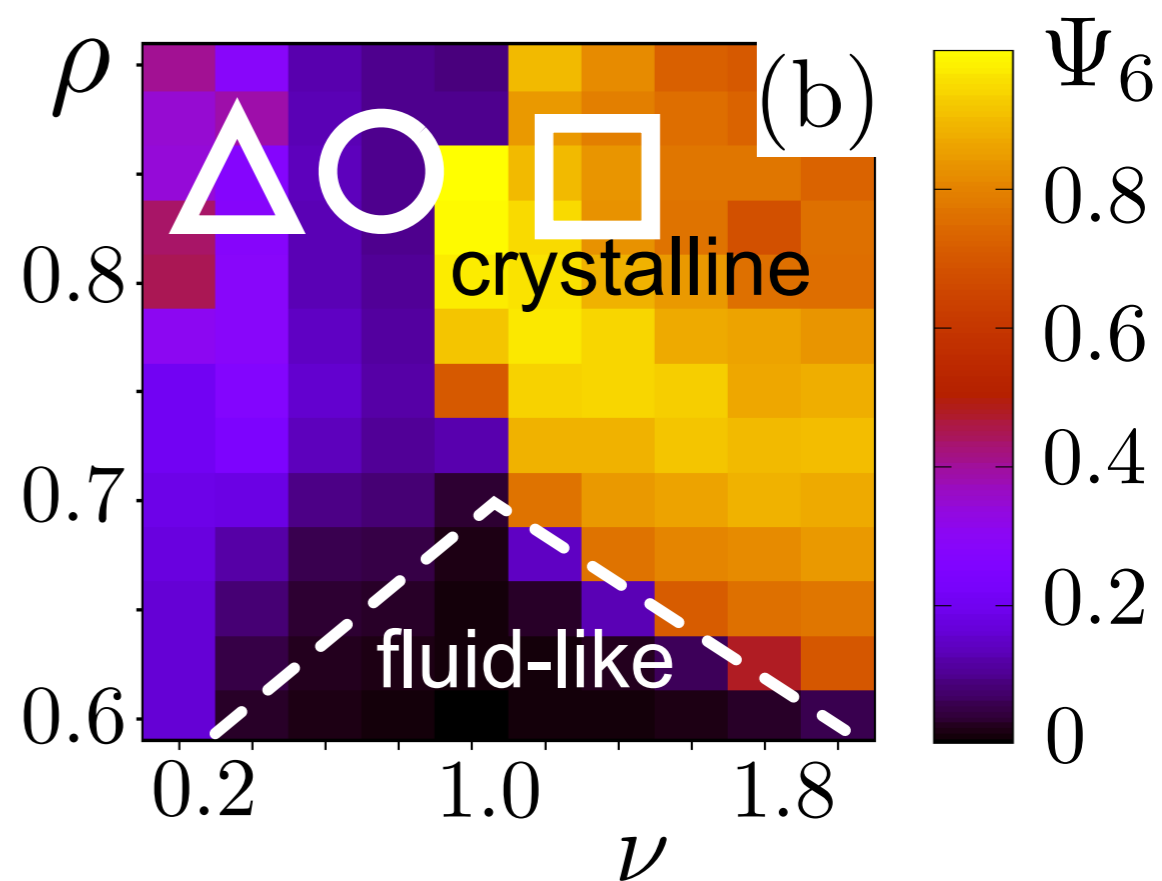
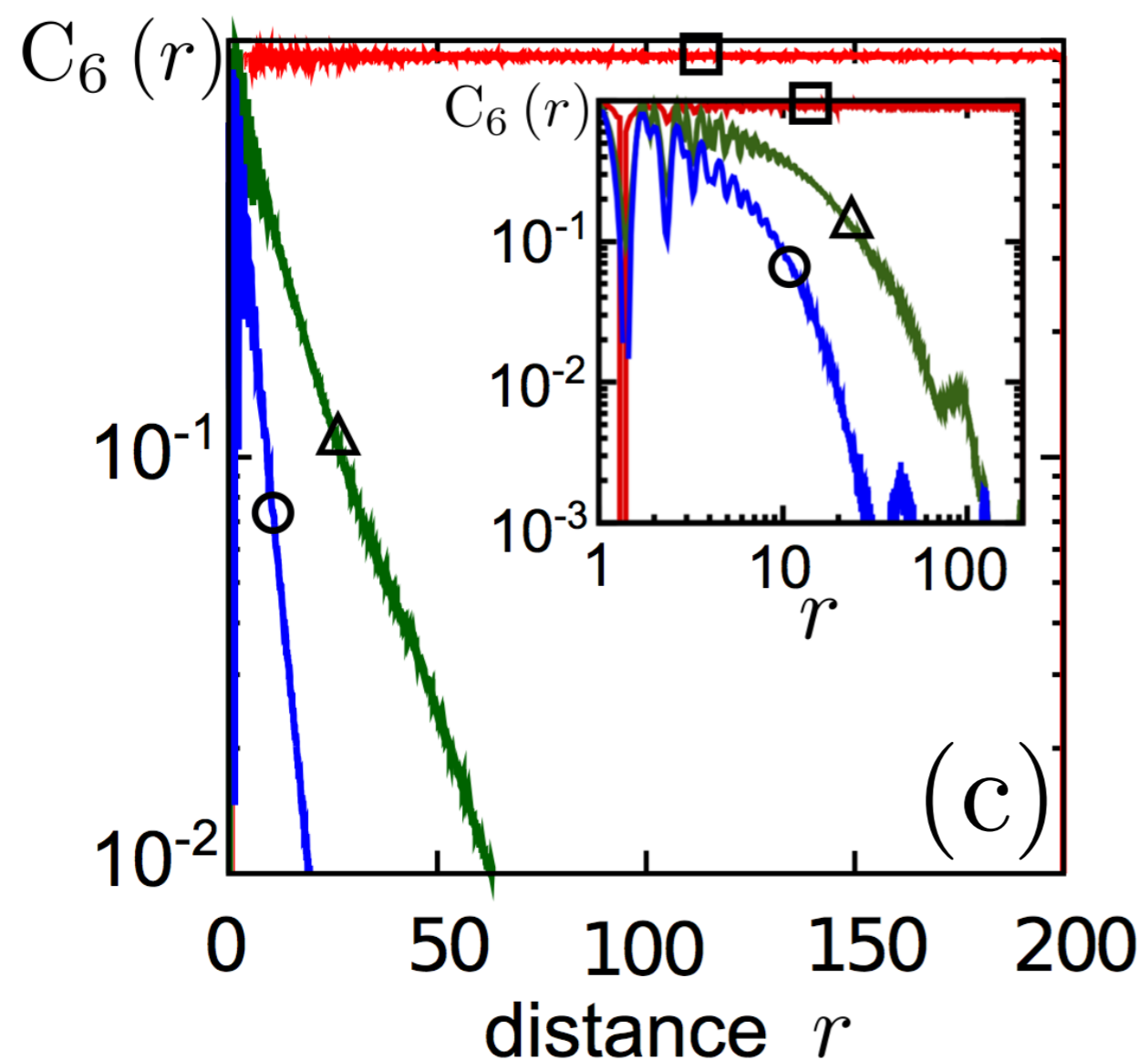


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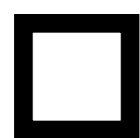
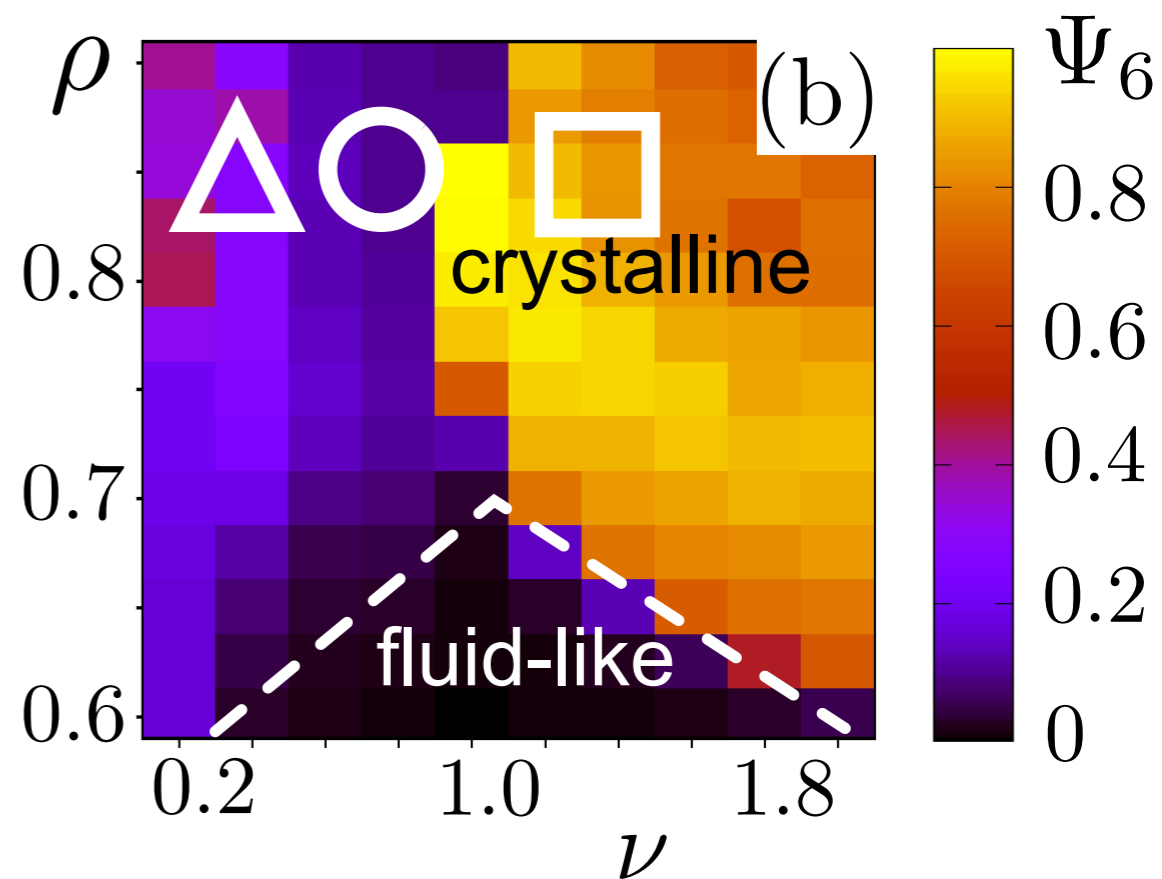
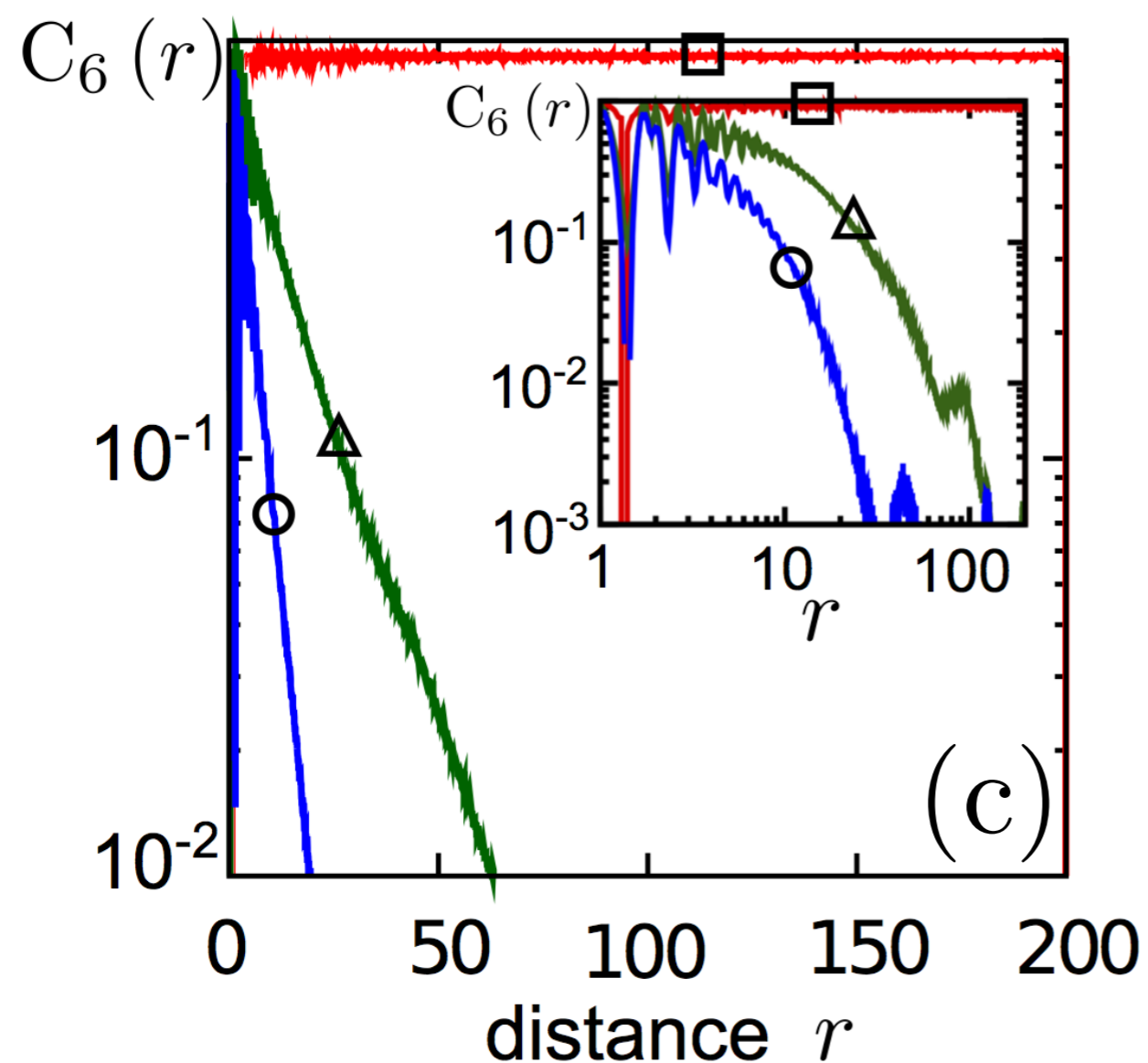
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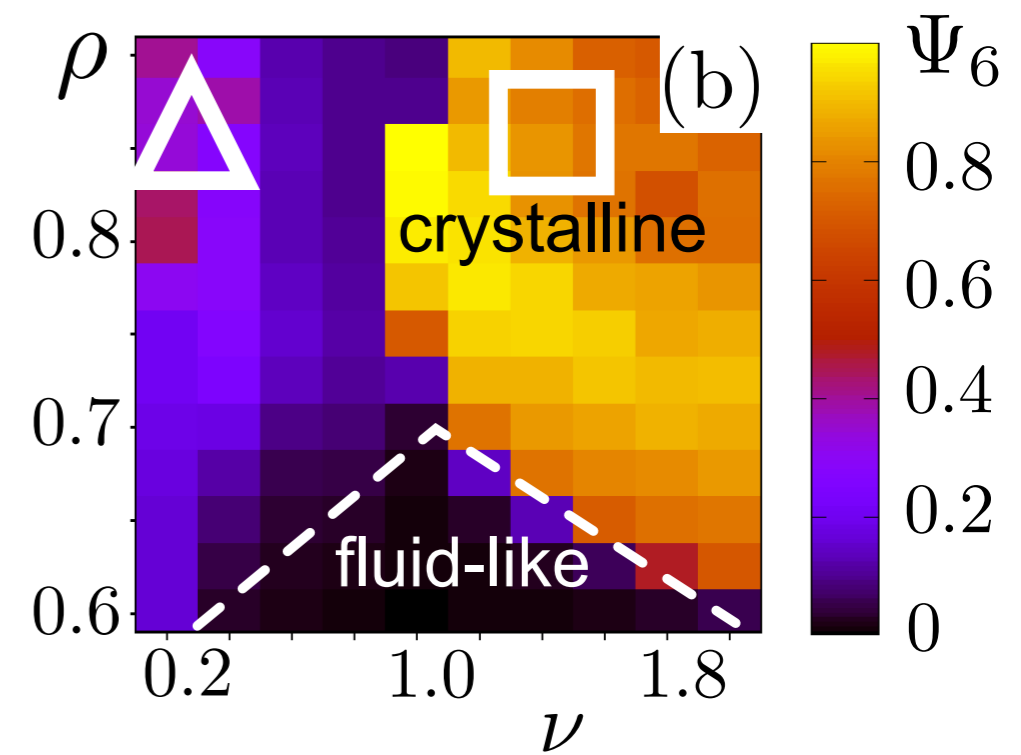
long-range hexatic order



short-range hexatic order in polycrystalline state(s)

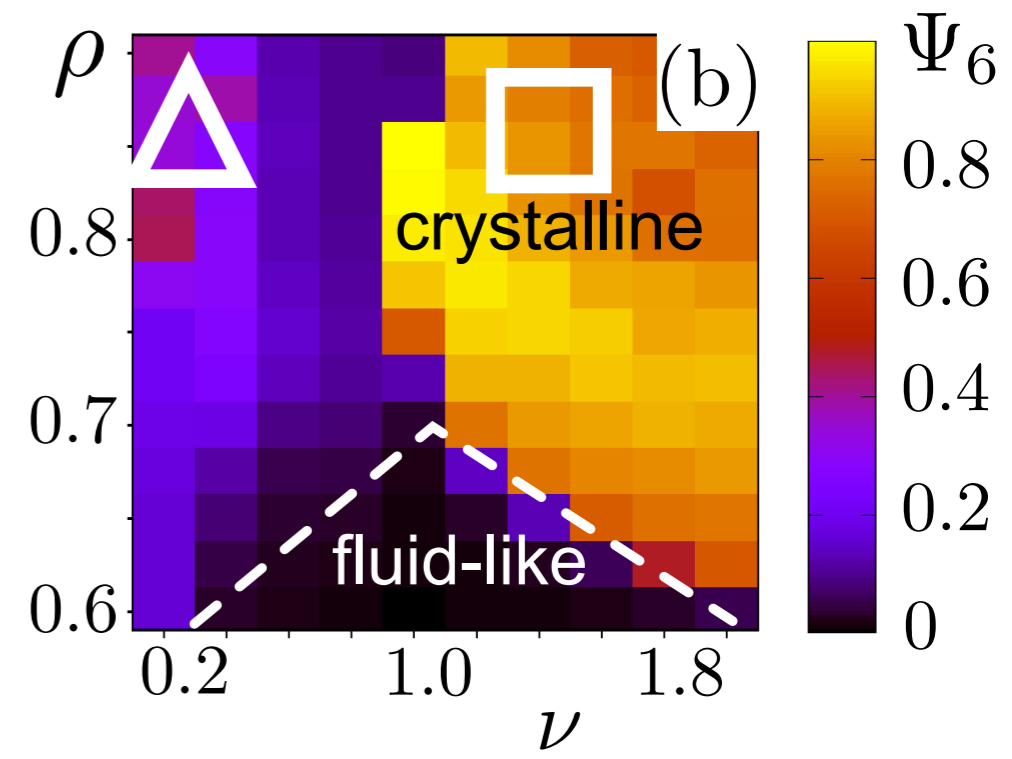
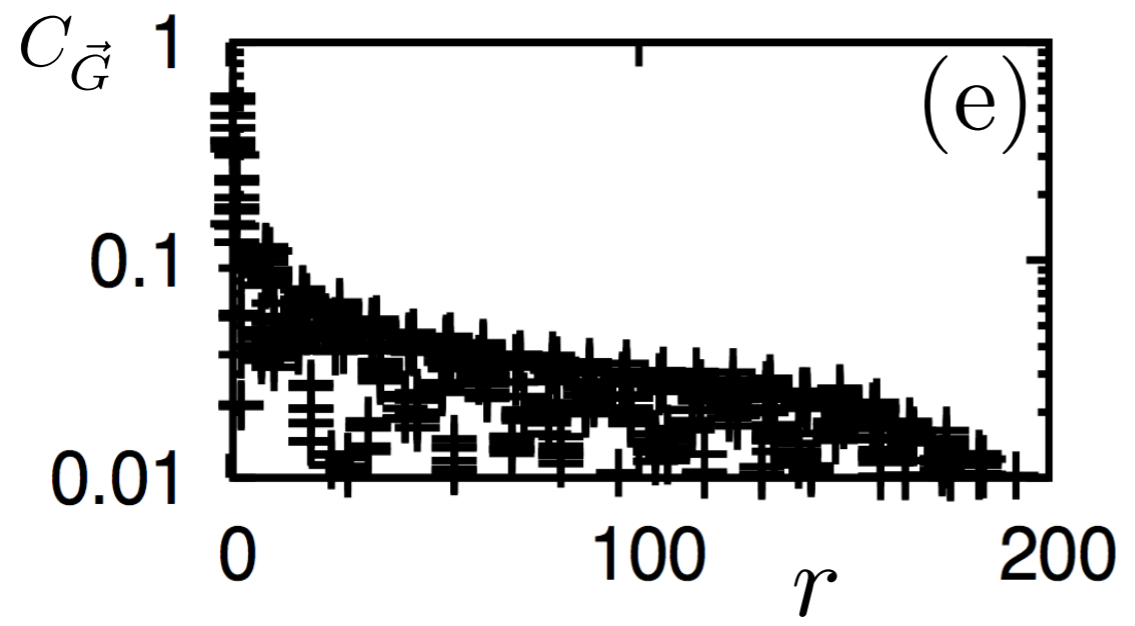
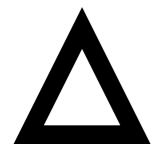
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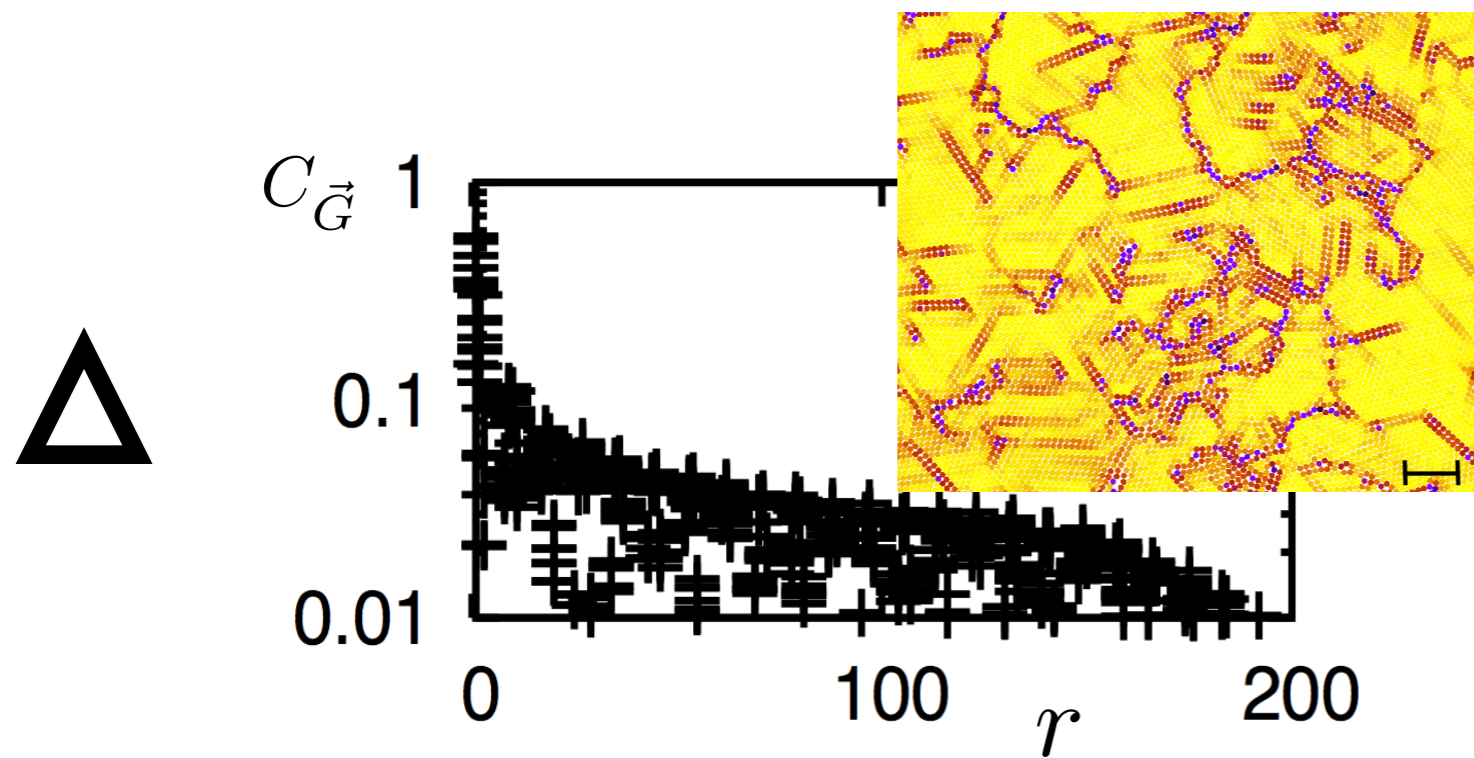
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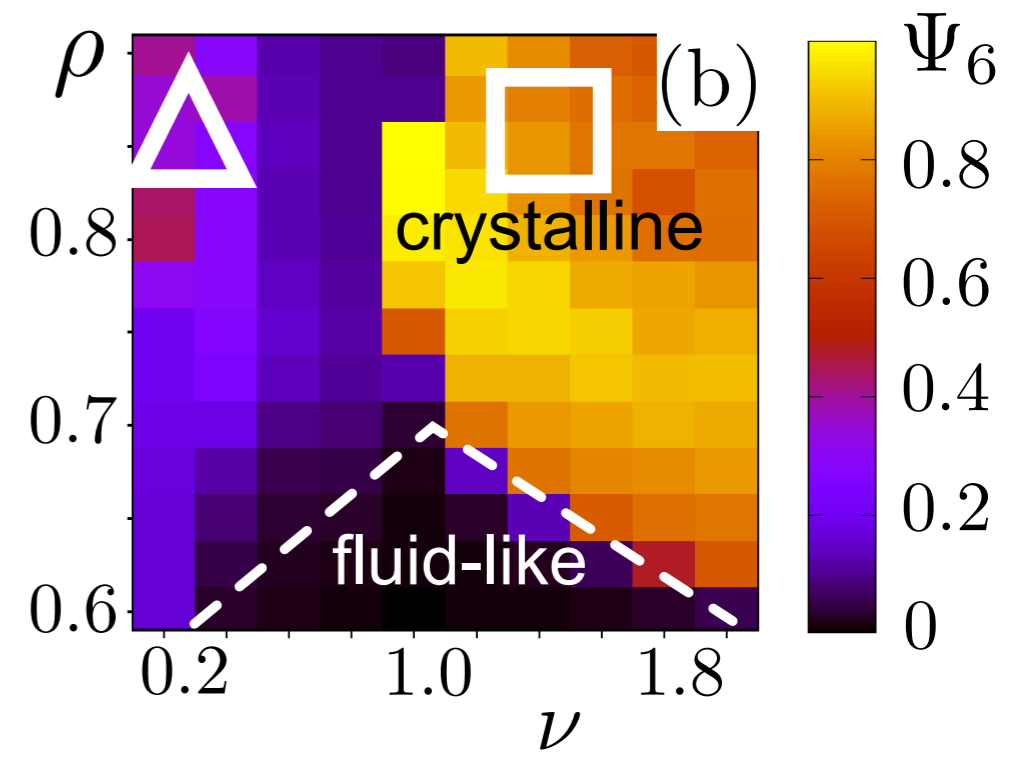




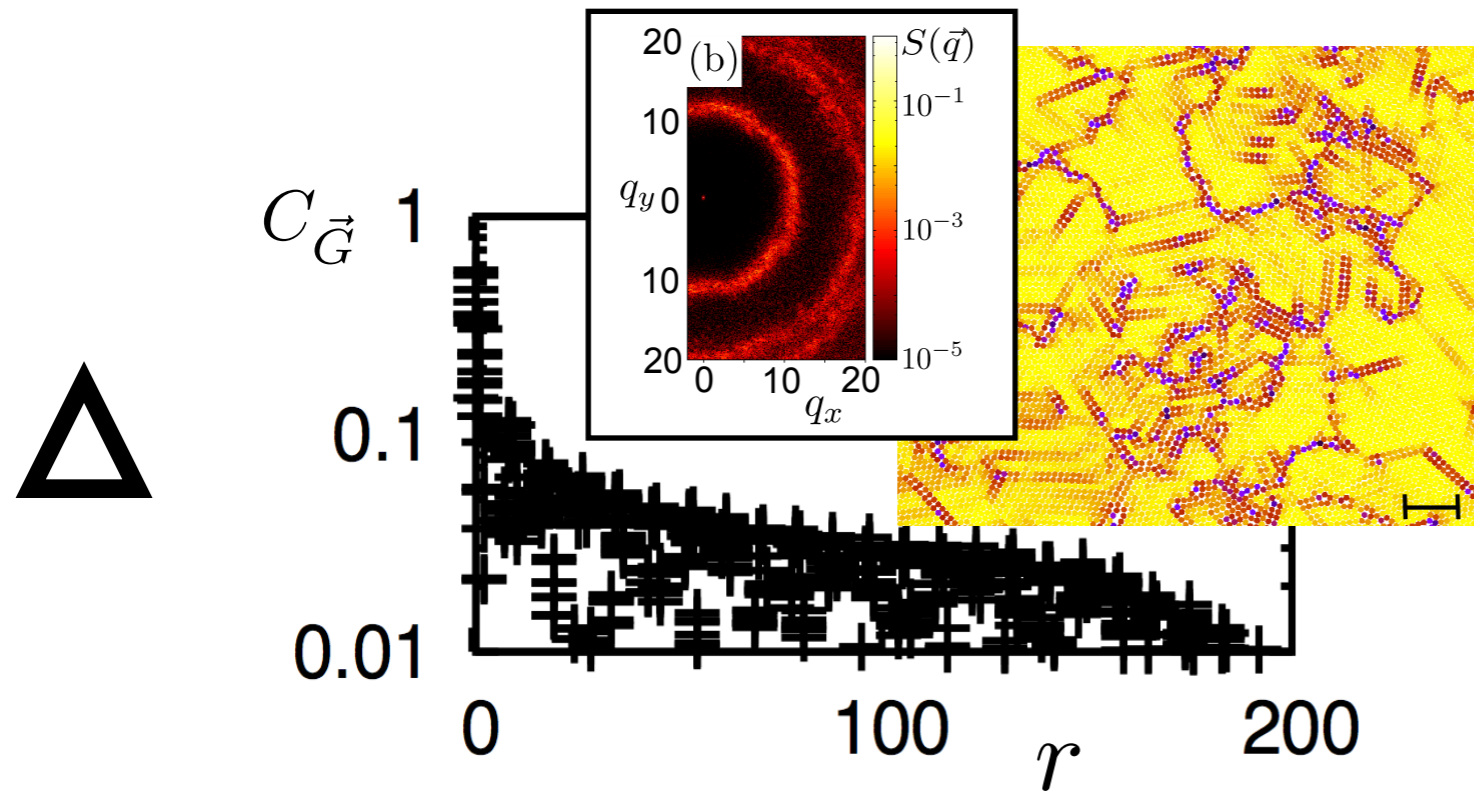
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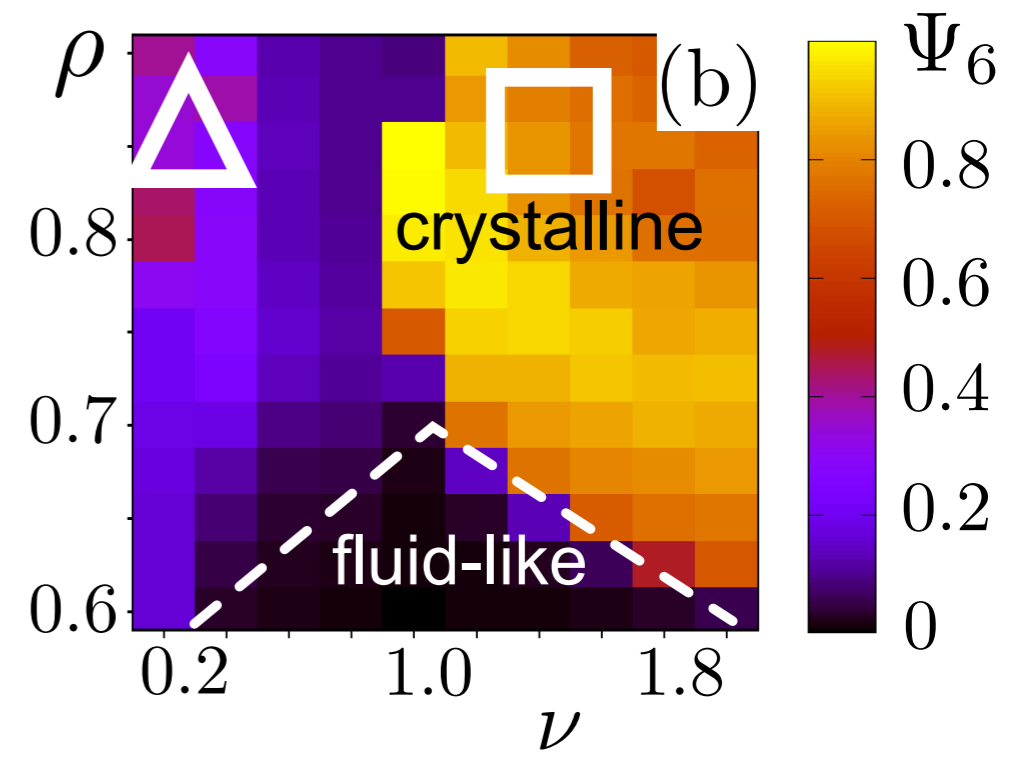
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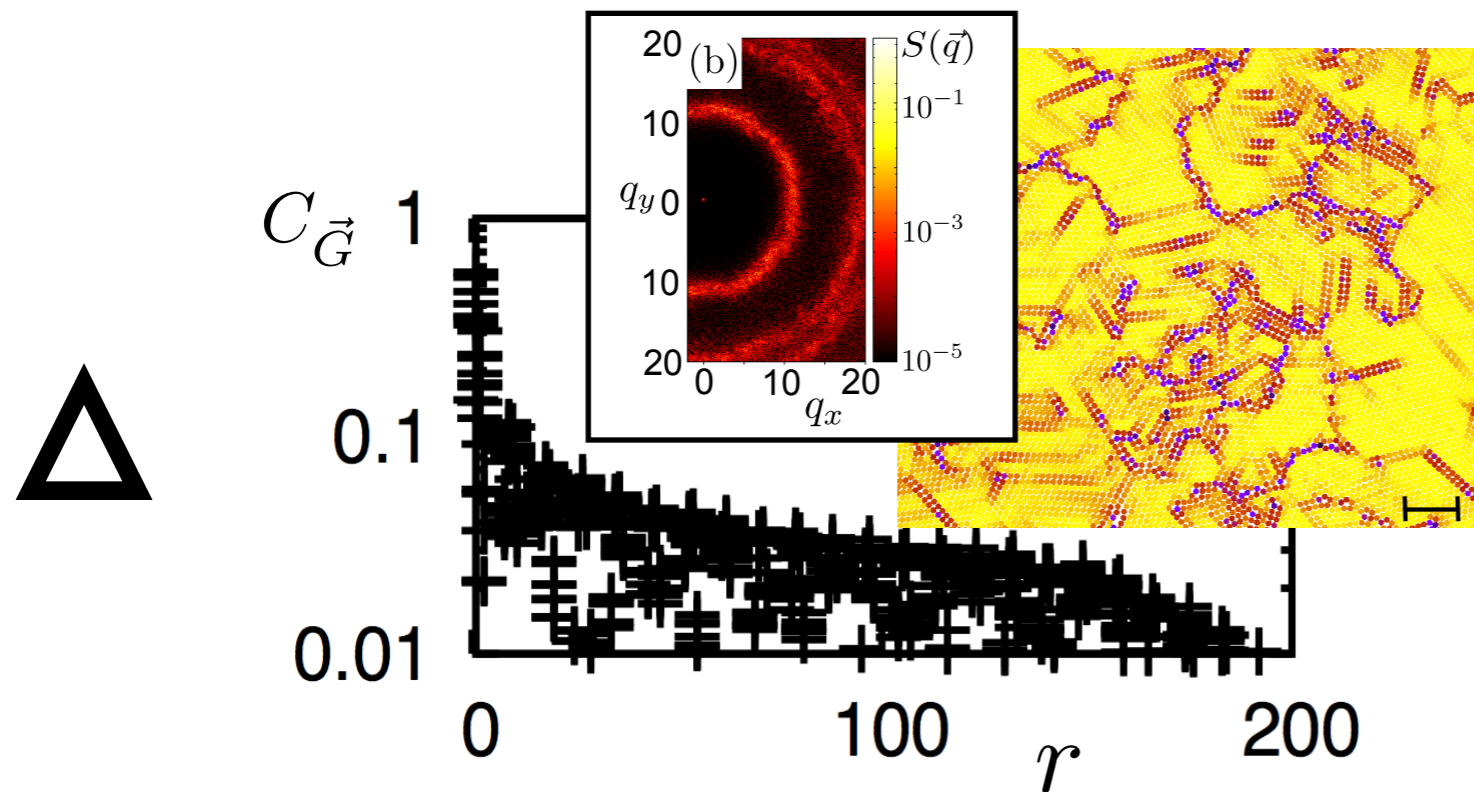
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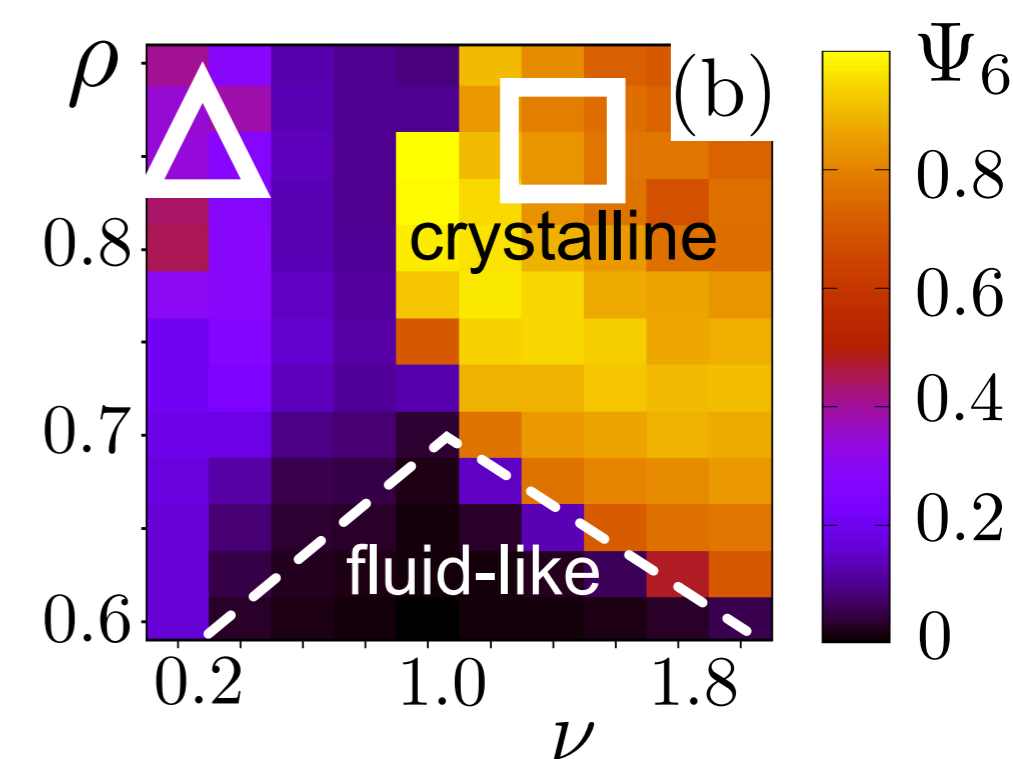


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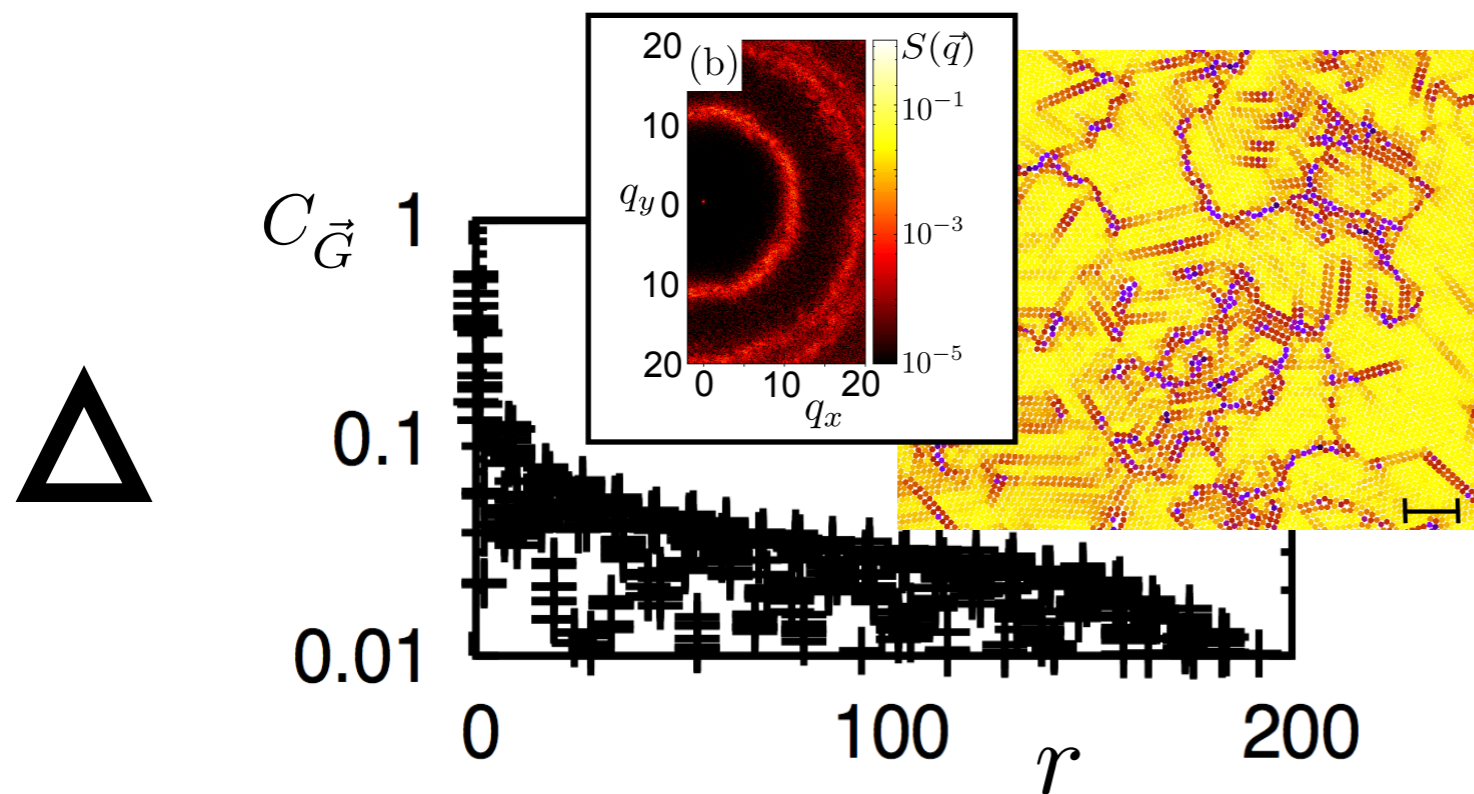


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 short-range translational order

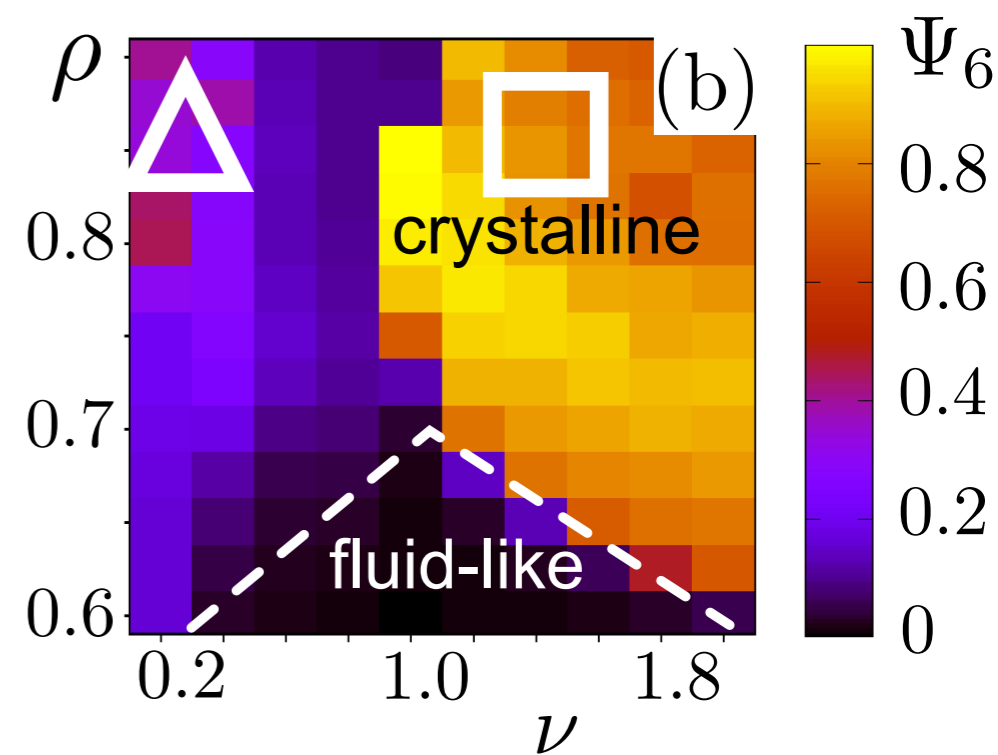
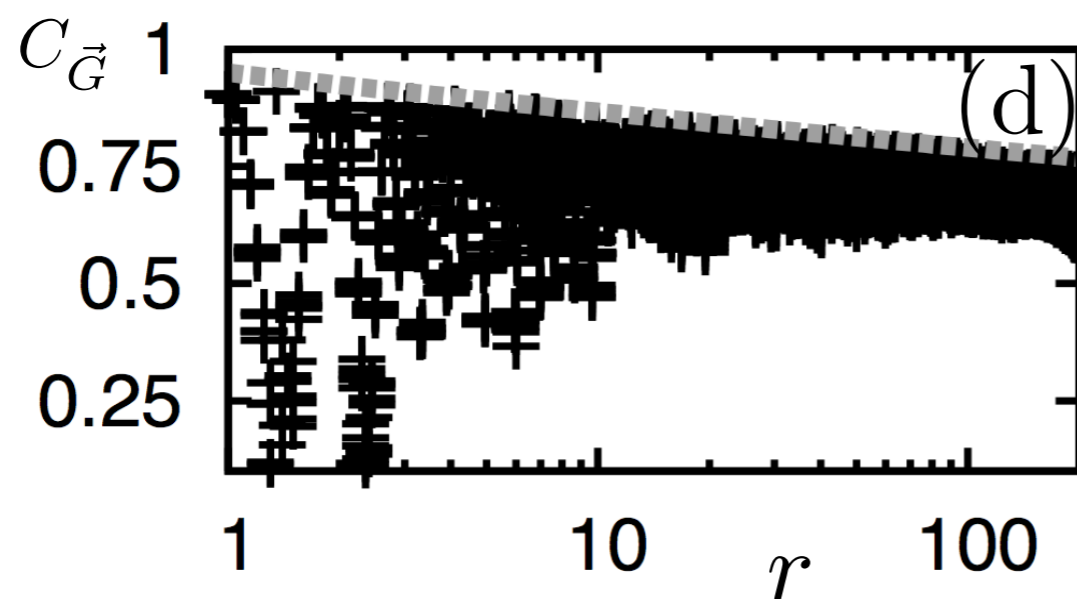
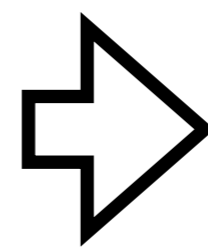
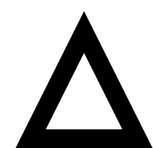


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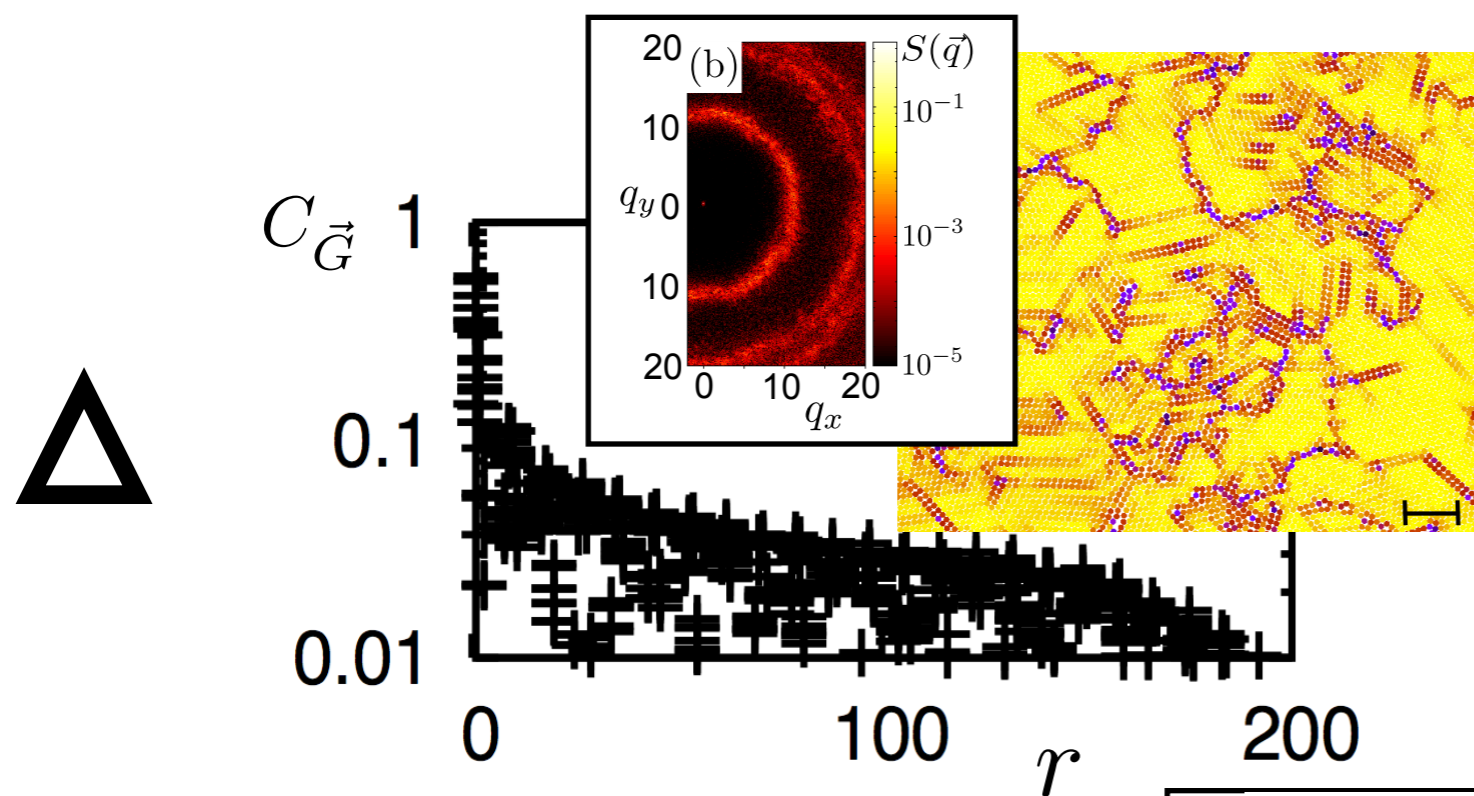


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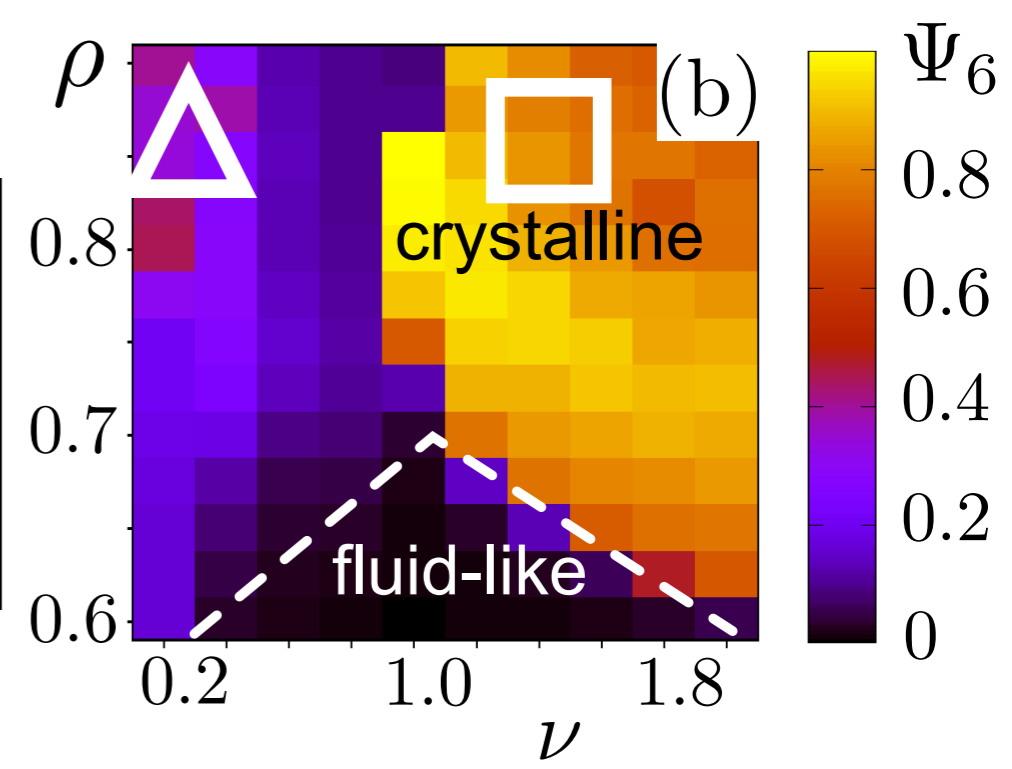
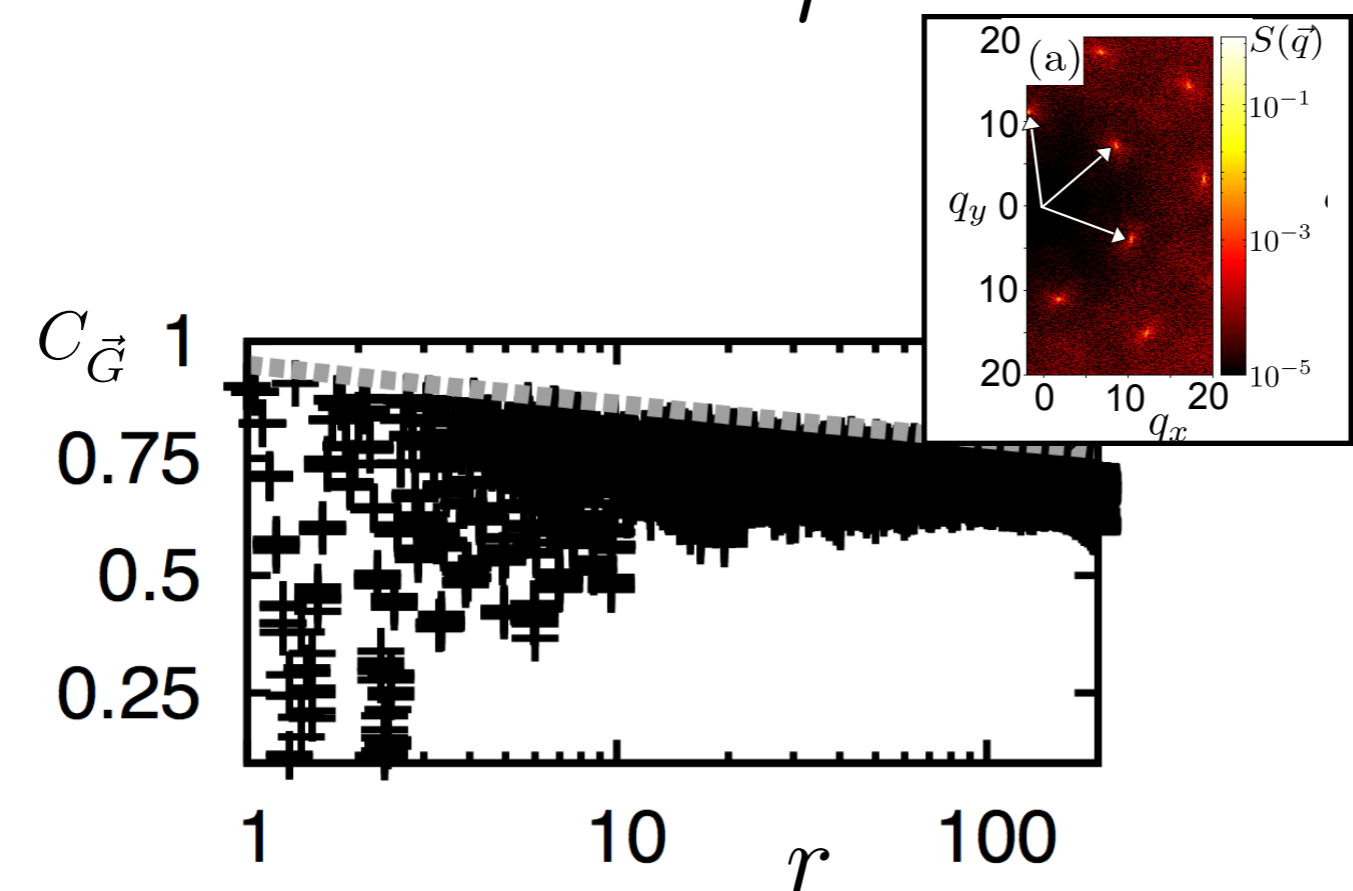
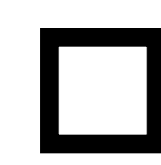
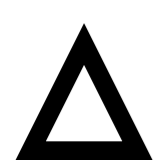


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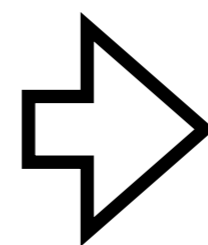
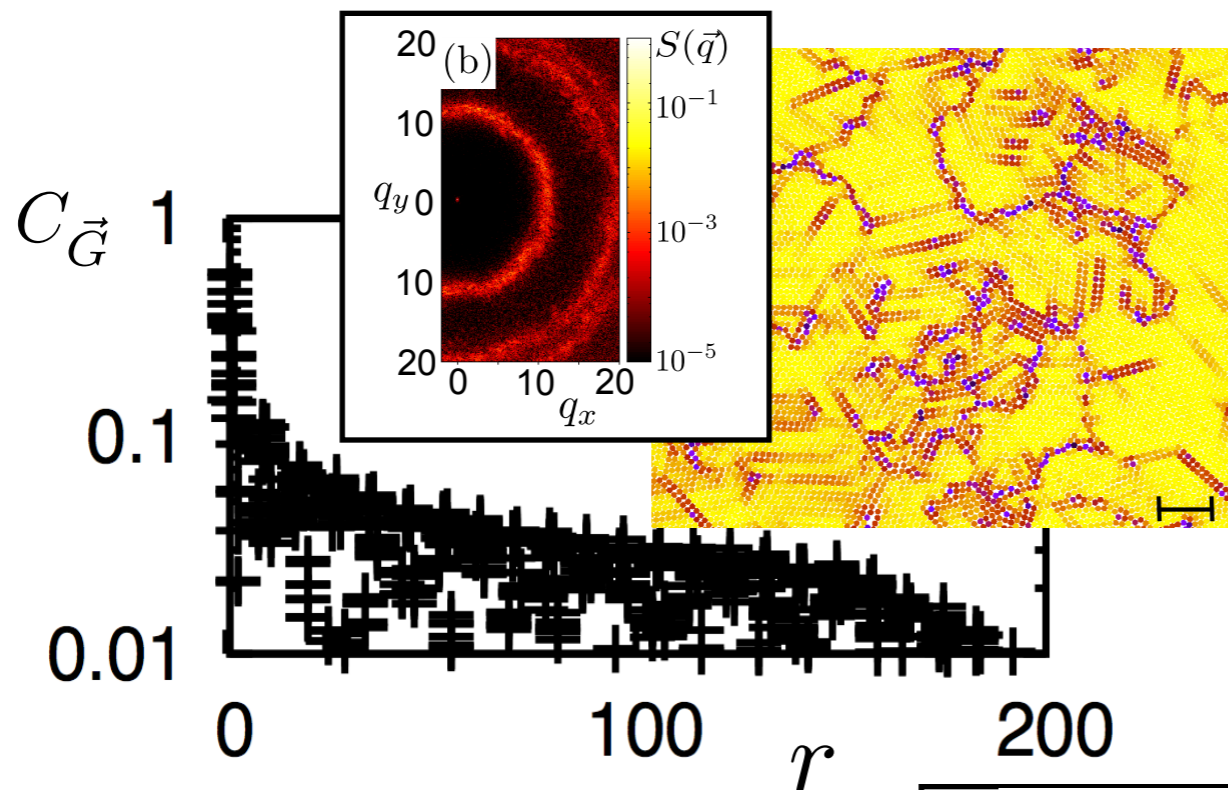
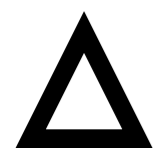
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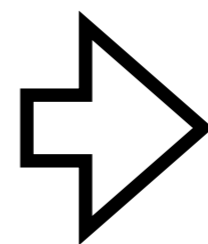
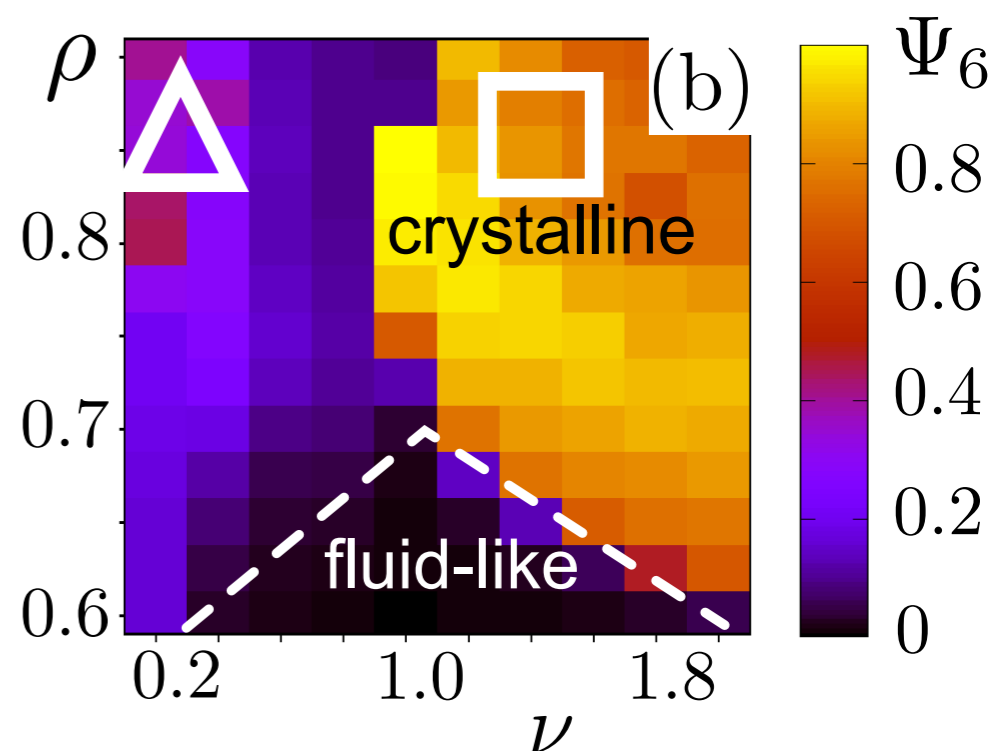
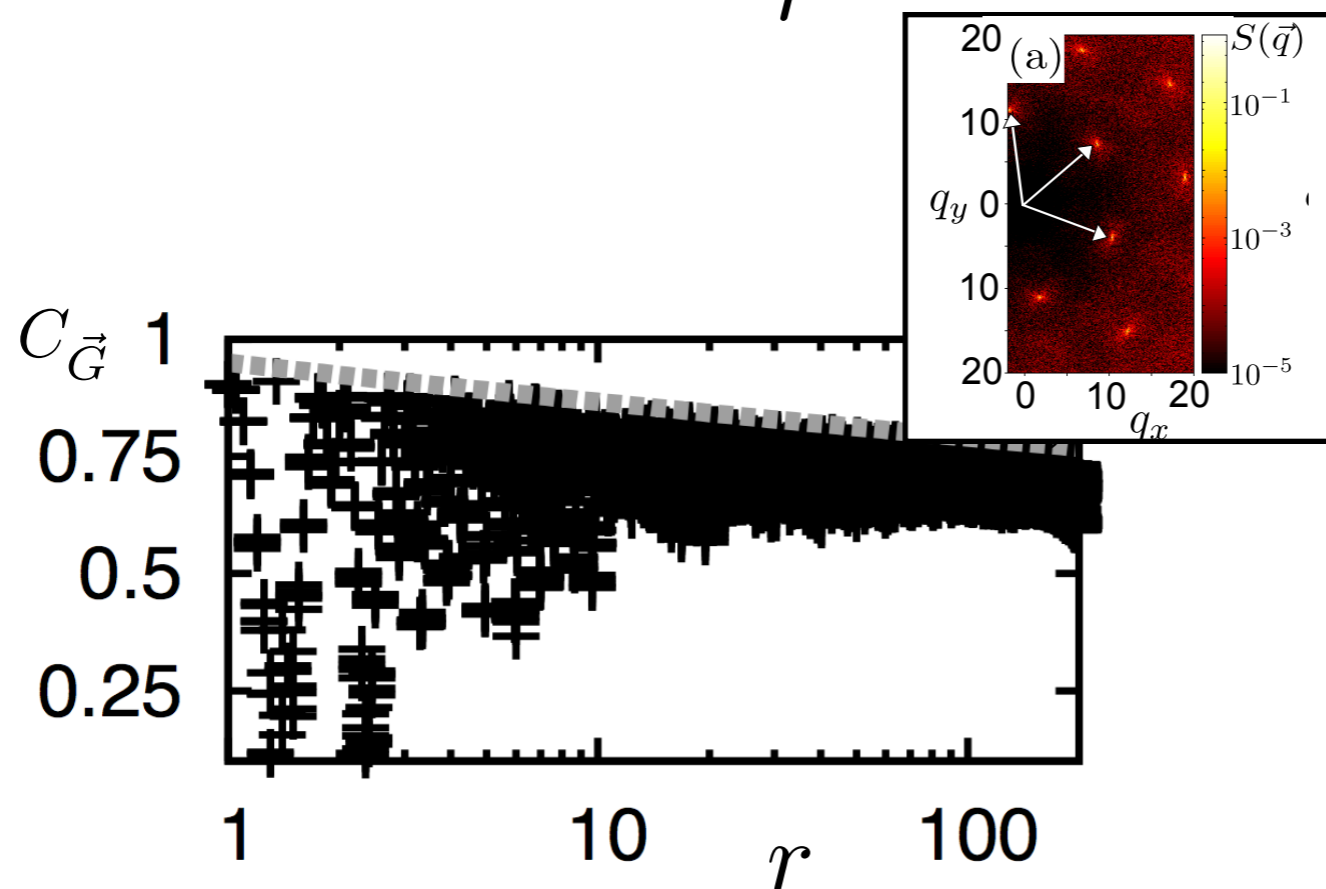
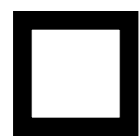


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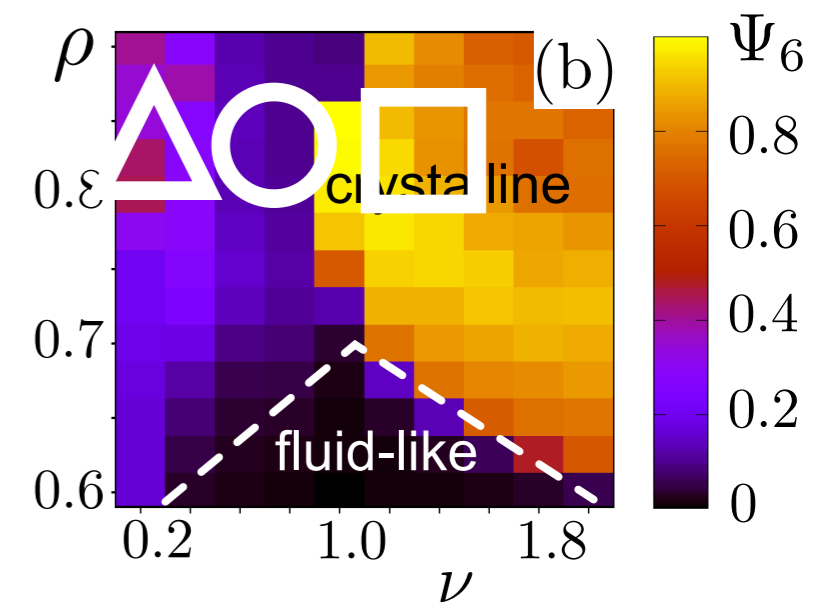
short-range translational order



quasi long-range translational order; “active crystal”

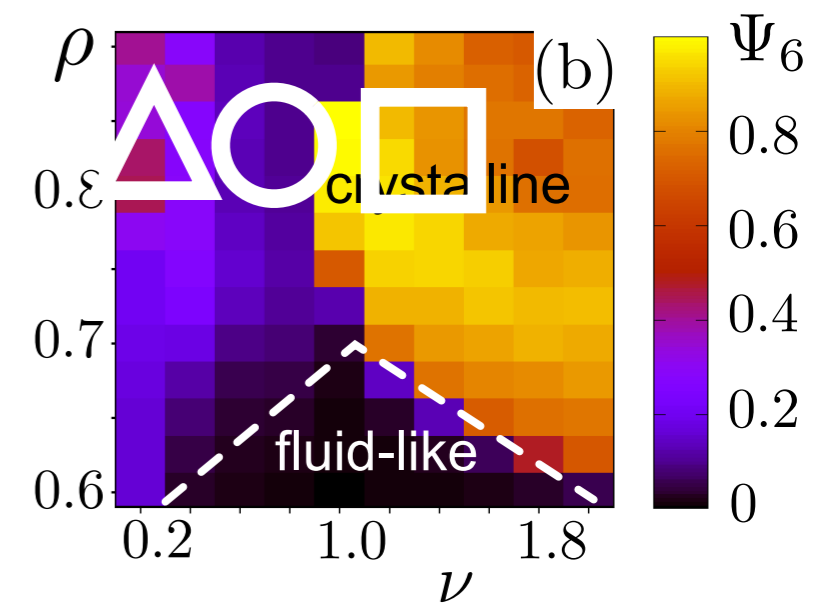
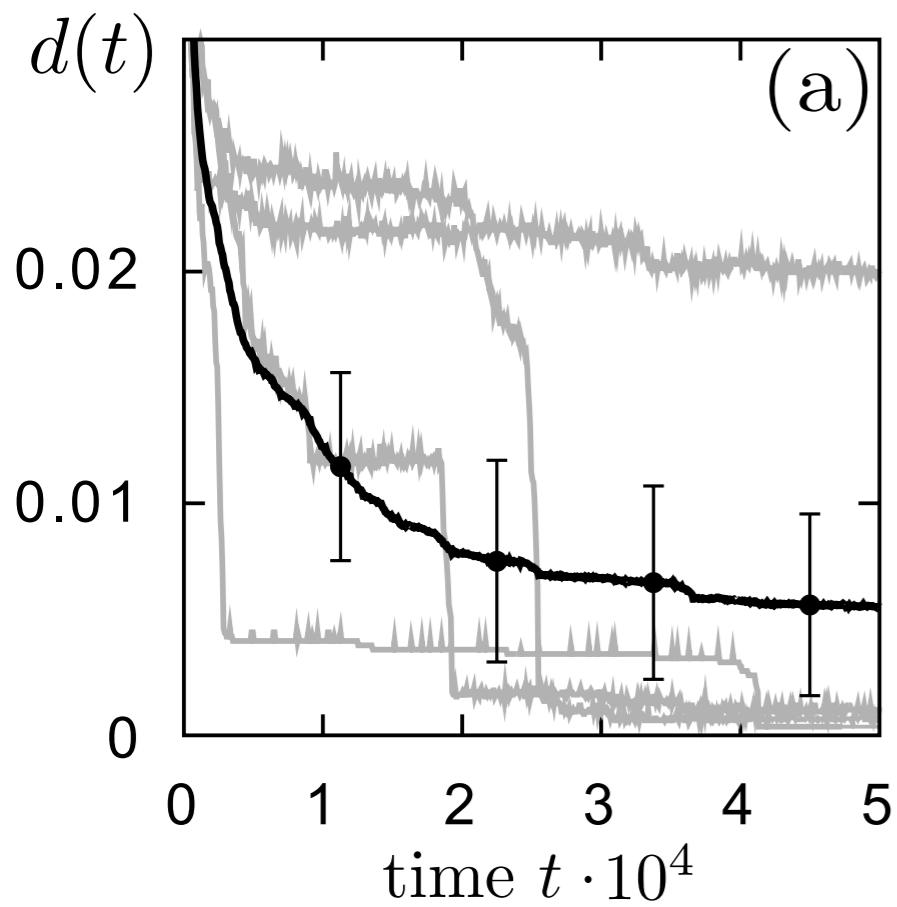
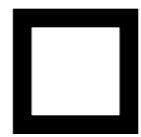
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$d =$  defect fraction



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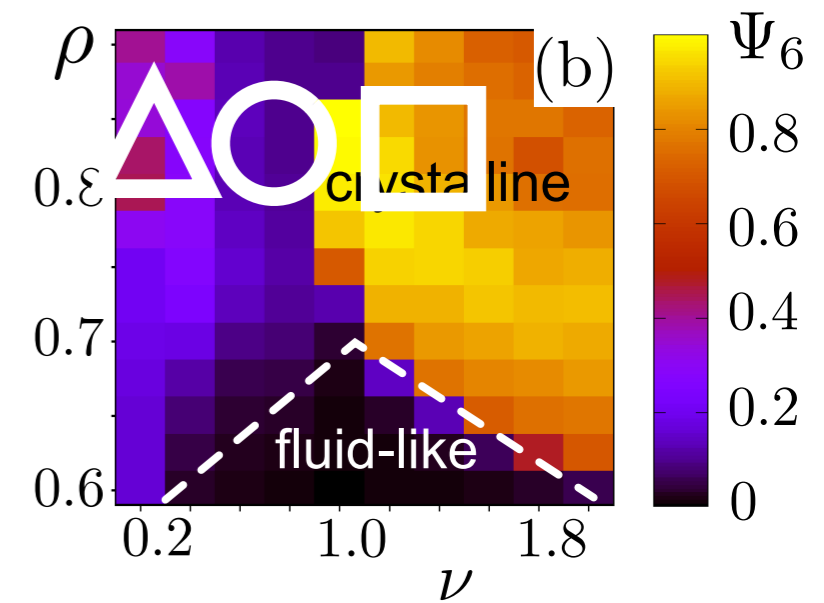
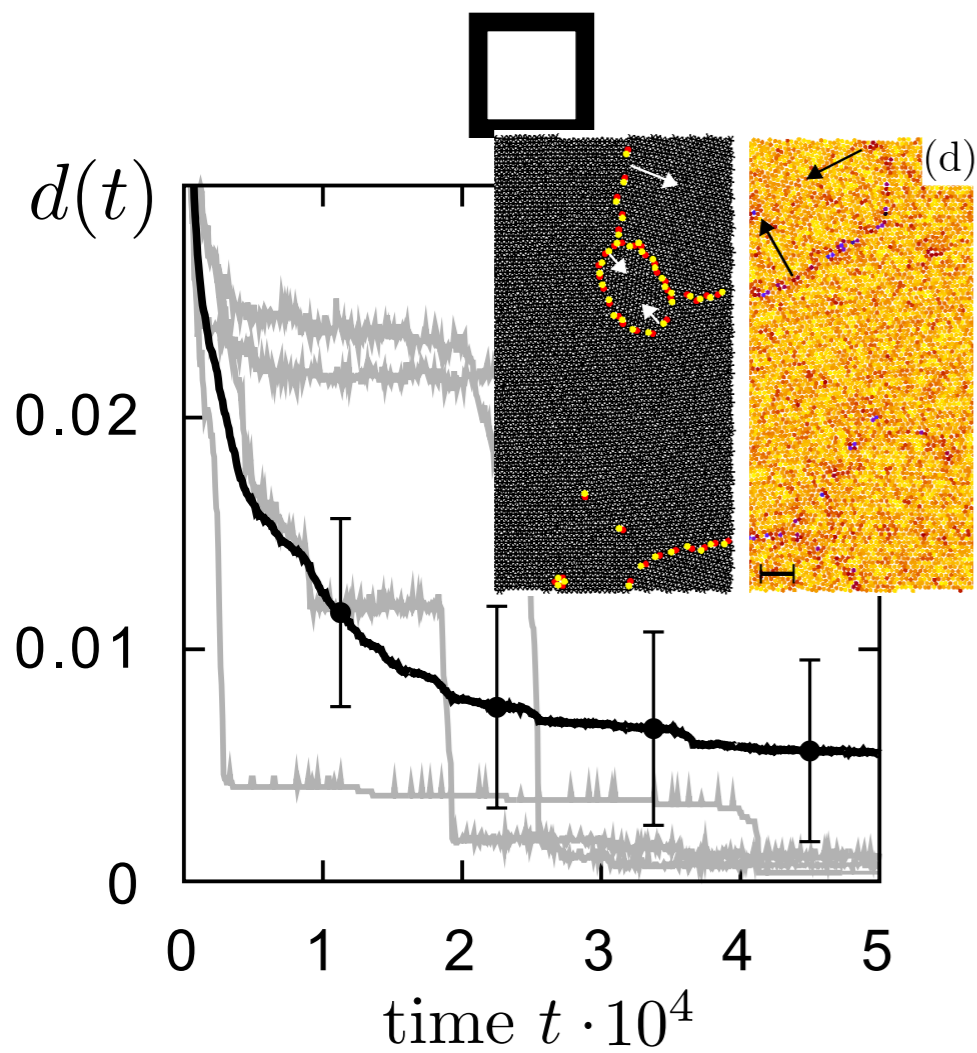
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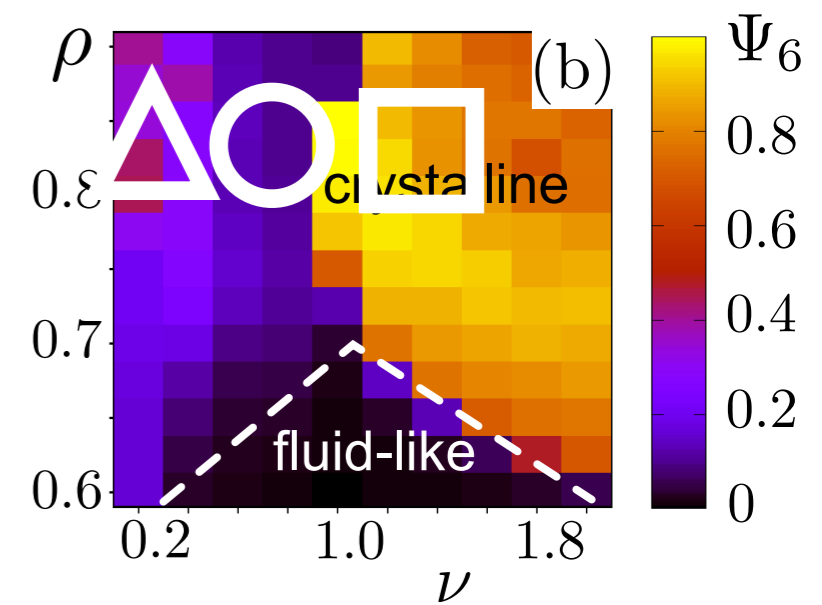
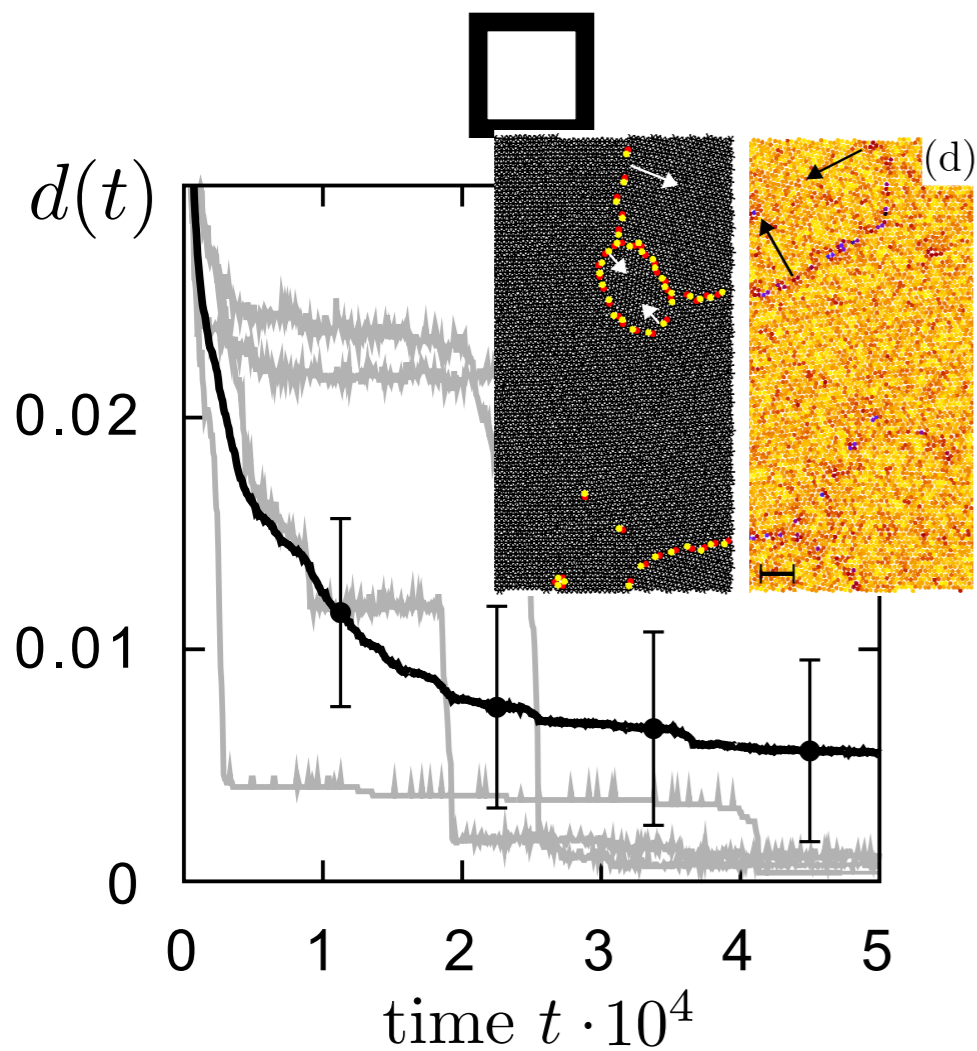
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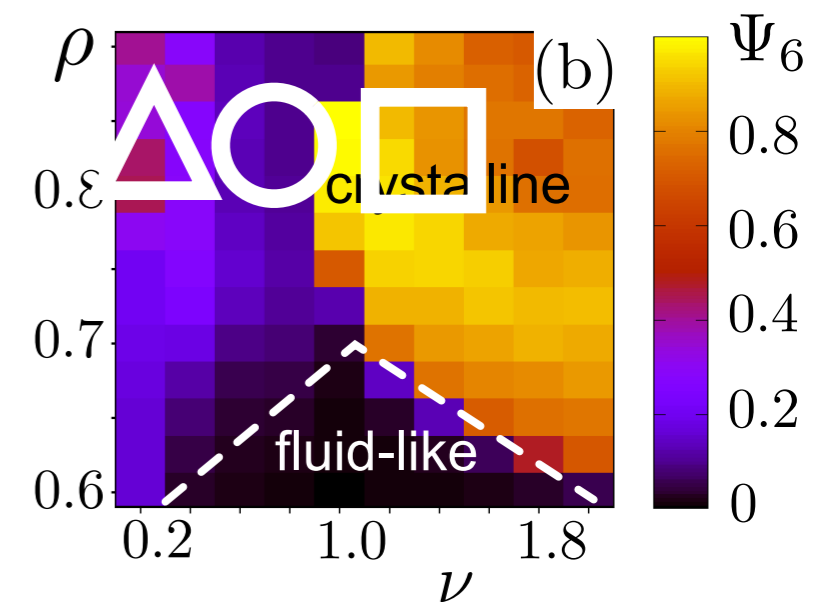
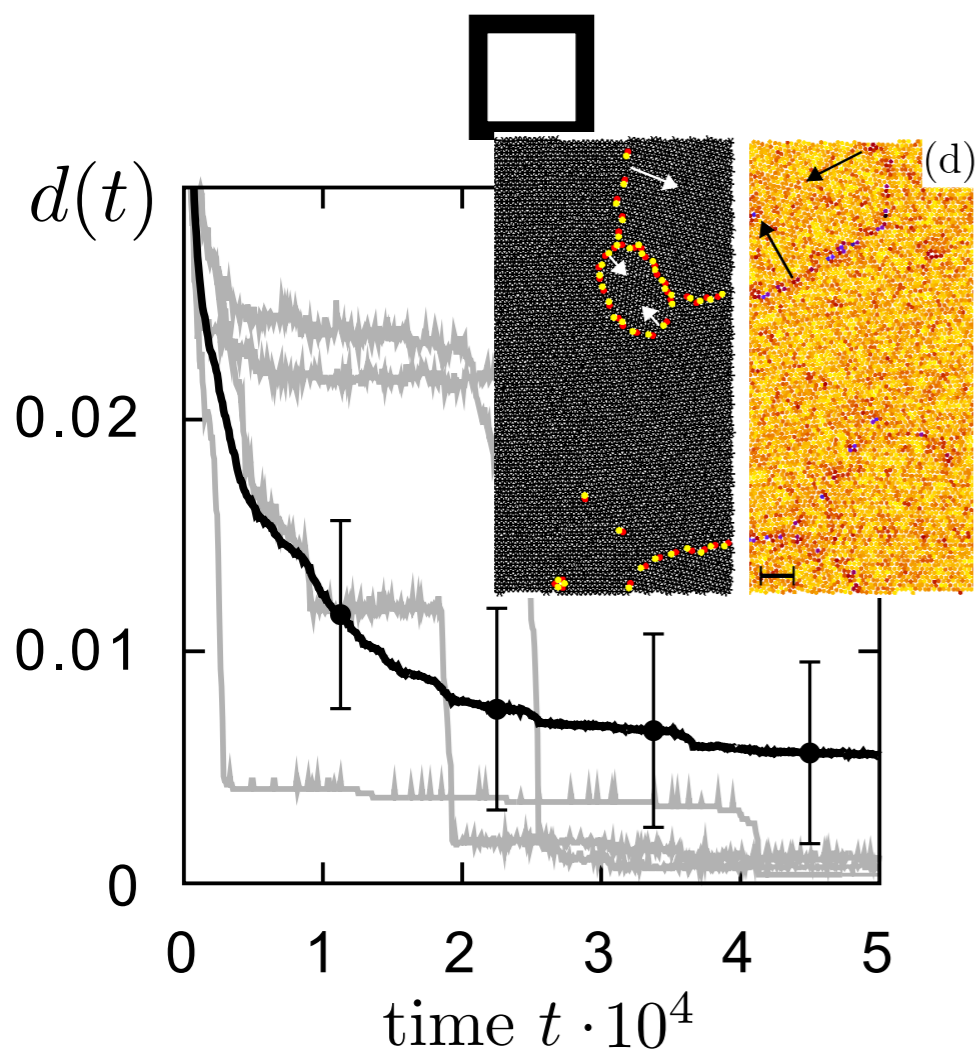
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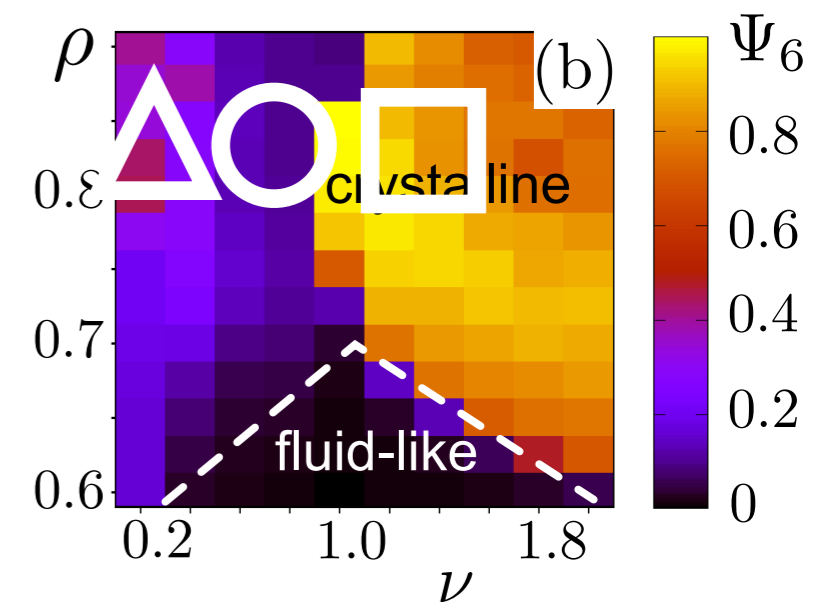
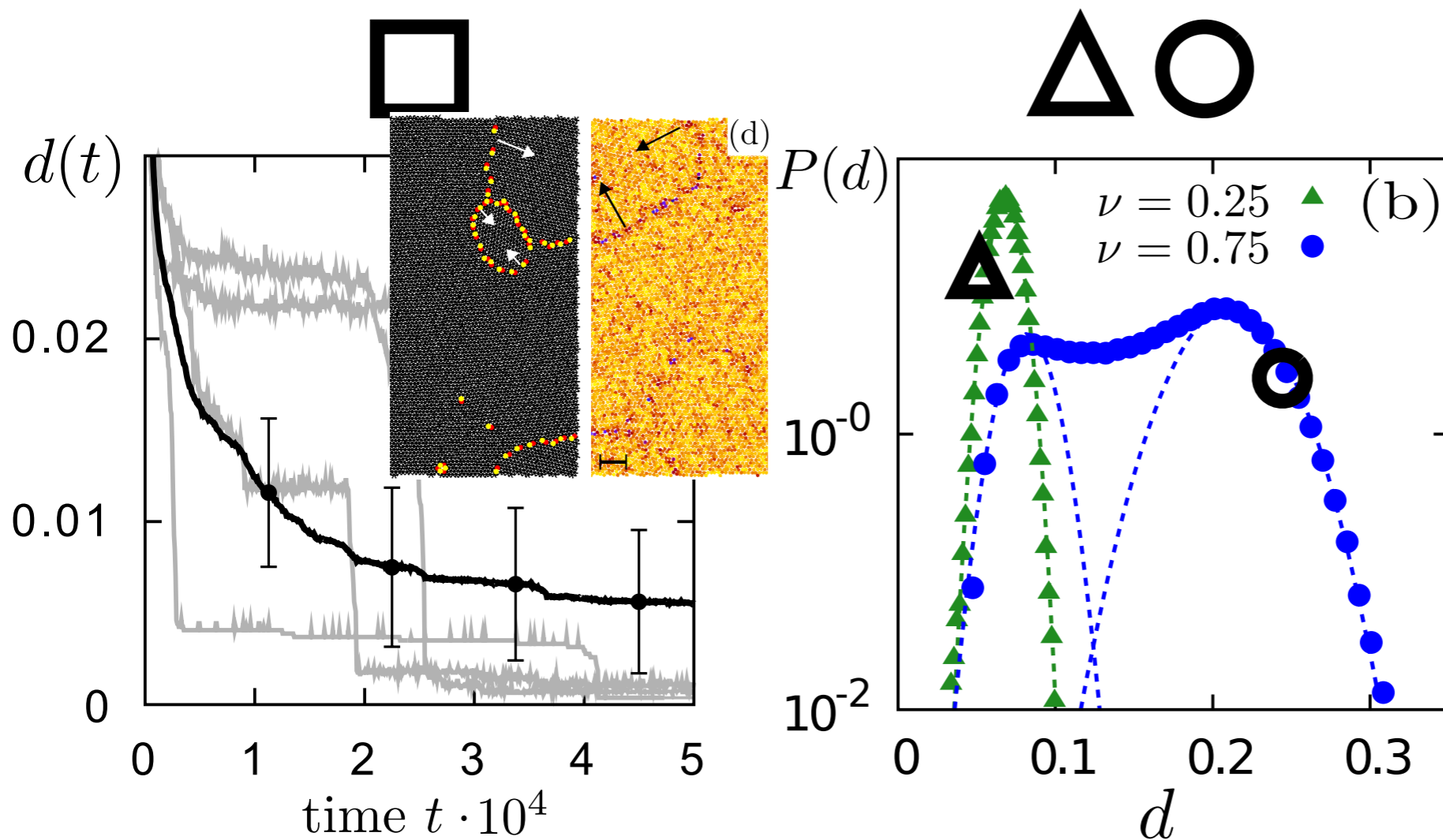


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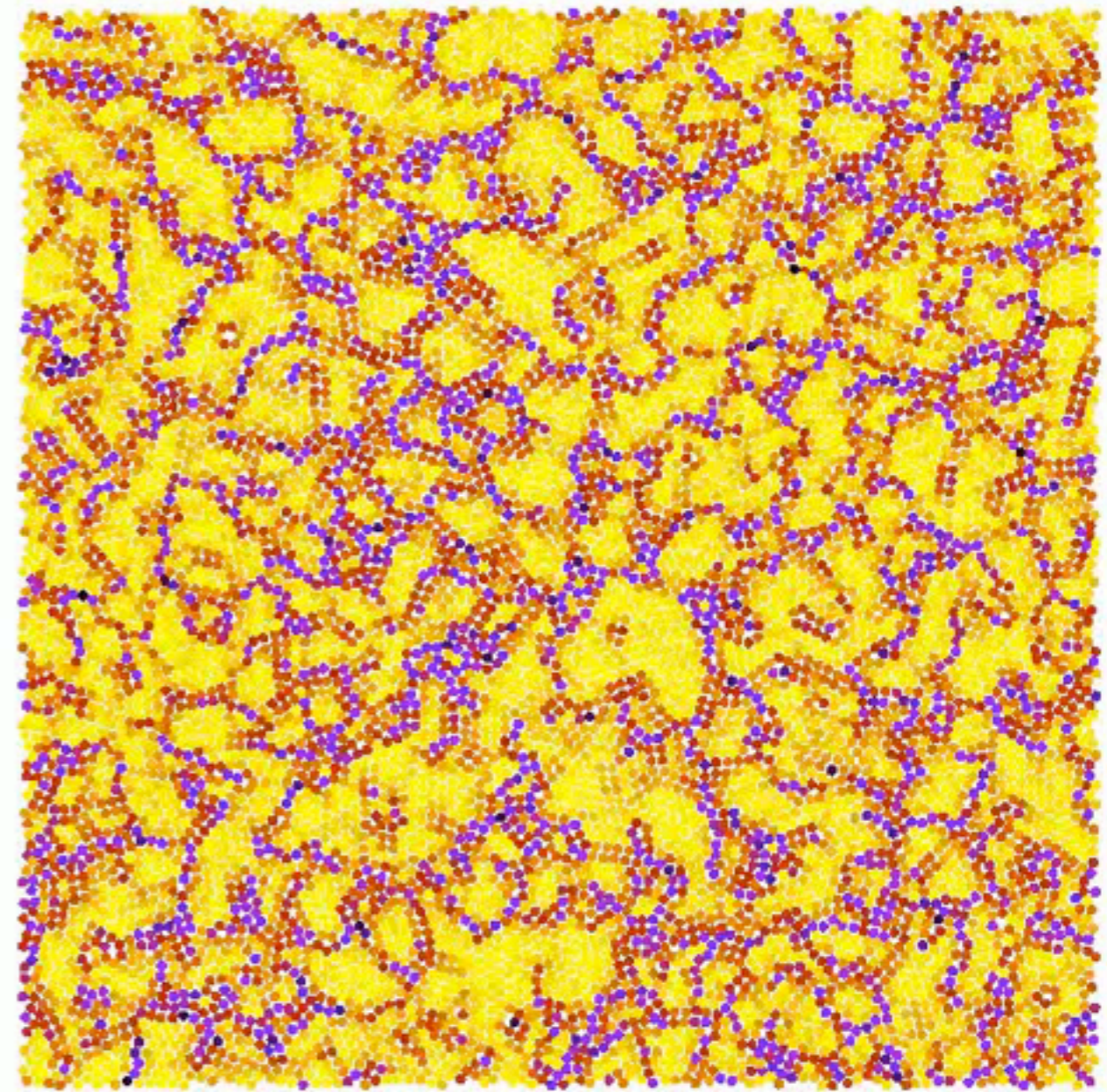
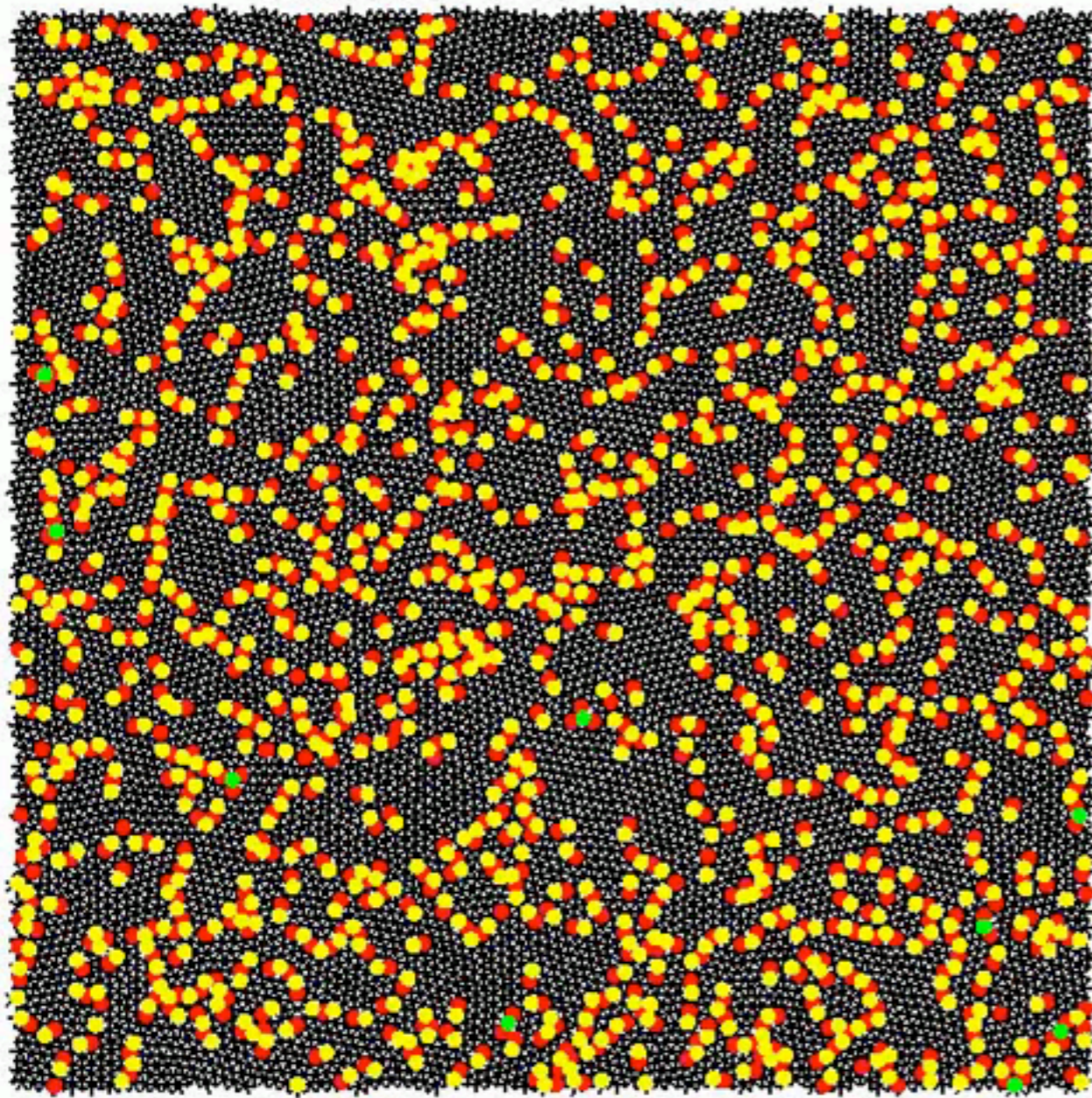
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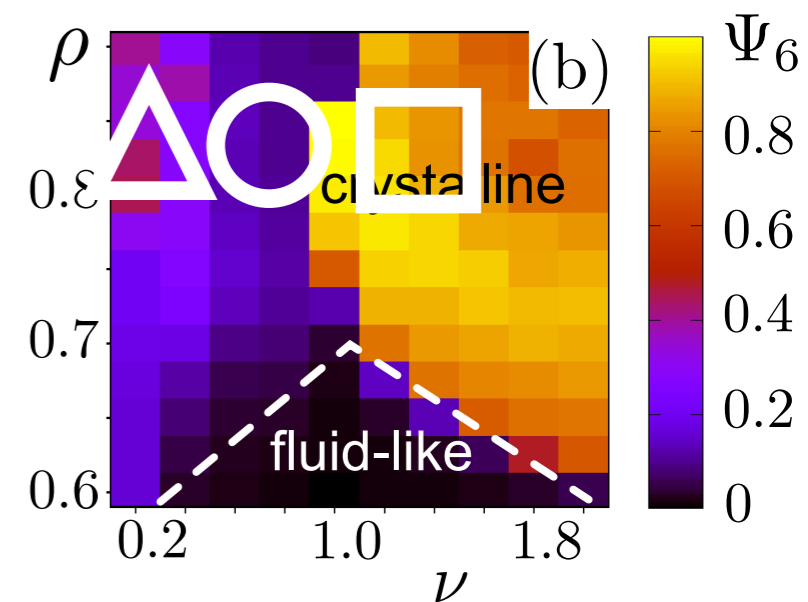
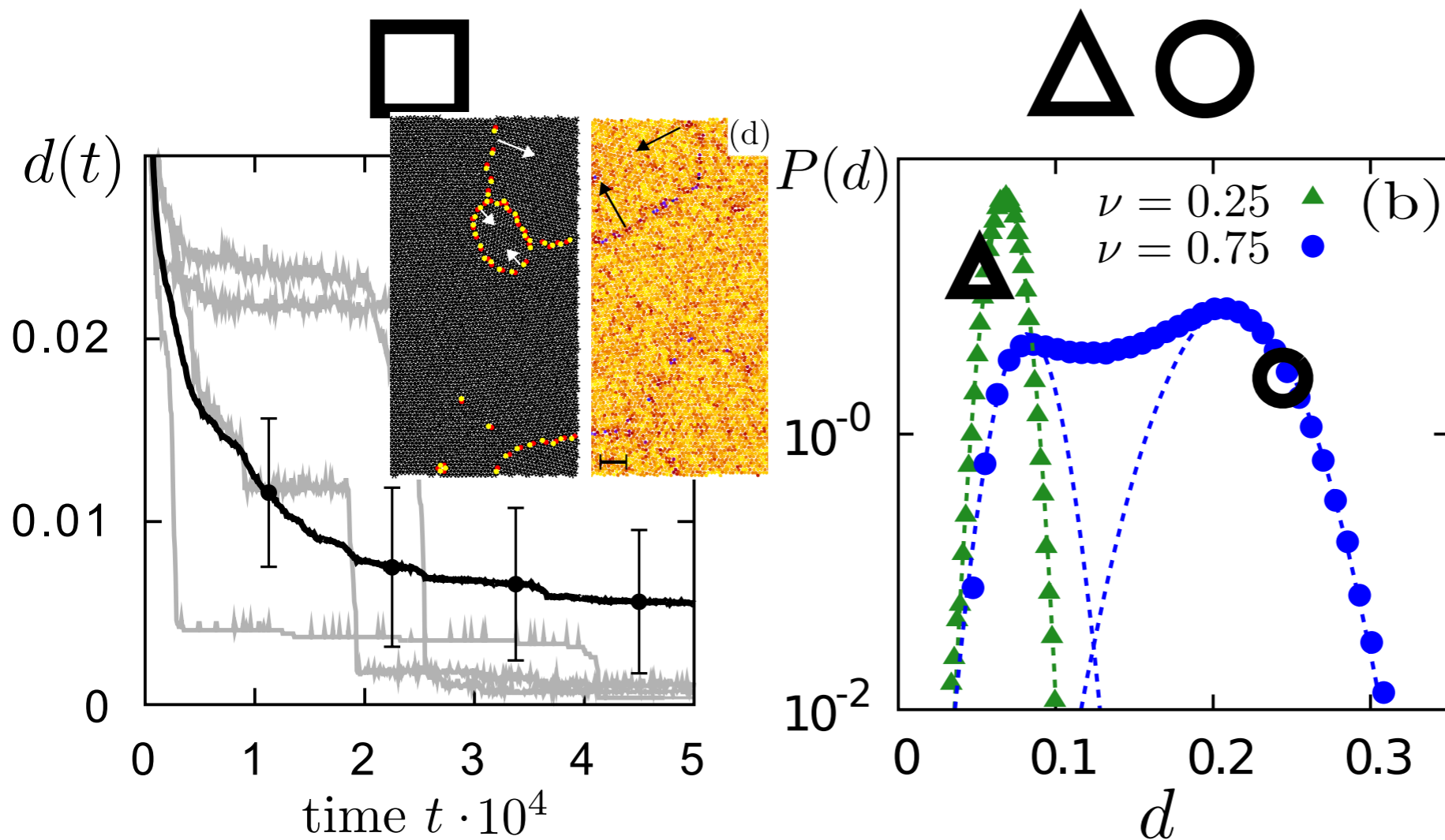
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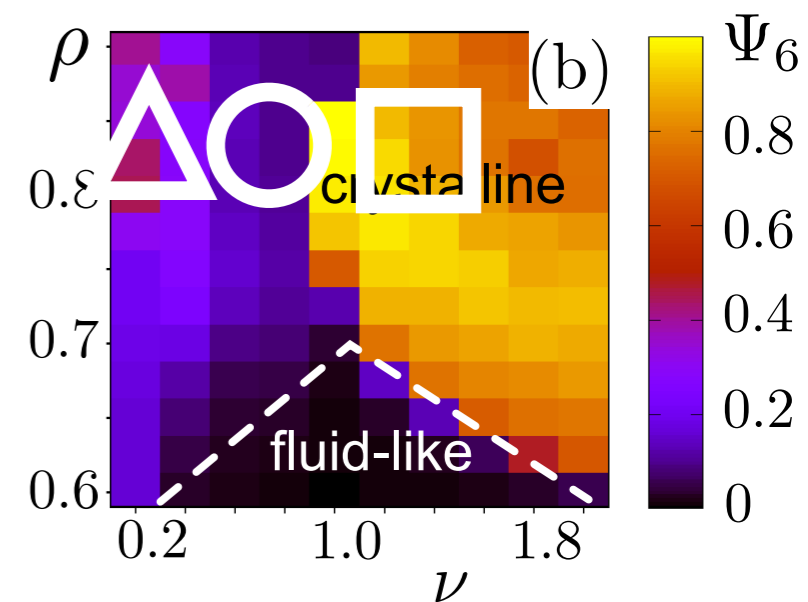
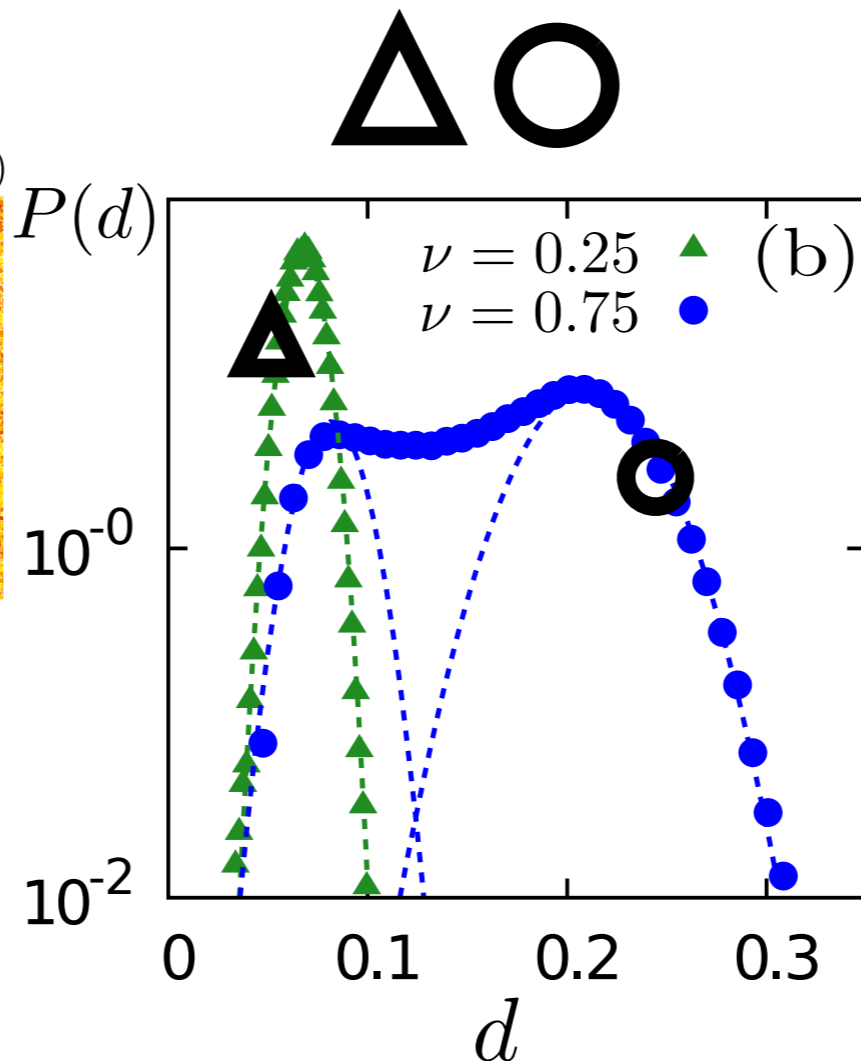
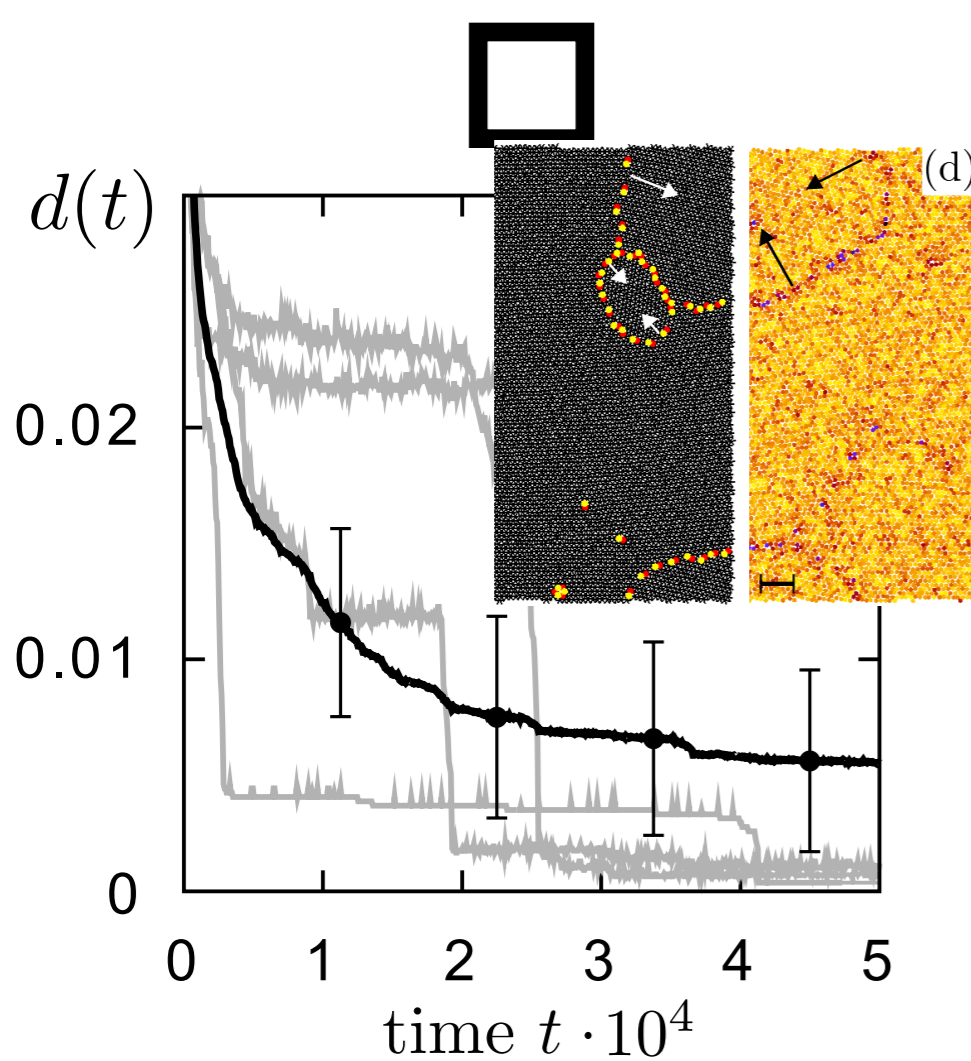


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competition between sound waves and local polar ordering  
 -> intermittency



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Summary & short facts:



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  - > “*polarized (active) polycrystal*” and “*active crystal*”

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Thanks for listening!