Velocity Distributions in Granular and Active Suspensions

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January 29, 2014

Wealth of Applications

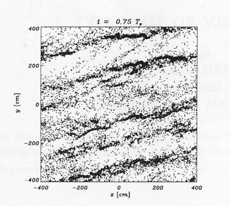
technical

food processing

random close packing

Chaikin et al. 2004

and in nature



ring of Saturn

... of Fundamental Interest

nonequilbrium model system

- grain of sand (diameter d) at room temperature: $\frac{k_B T}{mgd} \sim 10^{-12}$
  T defined by heatbath (environment) is completely irrelevant
  thermodynamics and equil. statistical mechanics not applicable

... of Fundamental Interest

nonequilbrium model system

- T defined by heatbath (environment) is completely irrelevant
- interactions between macroscopic bodies are dissipative
  energy is lost in collision of two grains

... of Fundamental Interest

nonequilbrium model system

- T defined by heatbath (environment) is completely irrelevant
- interactions between macroscopic bodies are dissipative
- decay of an initially agitated state: $E_{kin} = \frac{m}{2} \sum_{i=1}^{N} v_i^2$
  How does the granular fluid cool down?
  of particular interest for dilute systems $\rightarrow$ structure formation

… of Fundamental Interest

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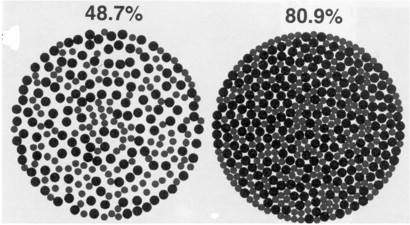
- decay of an initially agitated state: $E_{kin} = \frac{m}{2} \sum_{i=1}^{N} v_i^2$
  $\rightarrow$ spontaneous structure formation
- Grains left to themselves settle into static packing
  How dense can grains be packed?
  Jamming transition as a function of packing fraction

... of Fundamental Interest

nonequilbrium model system

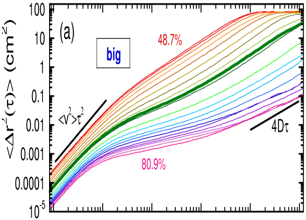
- T defined by heatbath (environment) is completely irrelevant
- interactions between macroscopic bodies are dissipative
- decay of an initially agitated state: $E_{kin} = \frac{m}{2} \sum_{i=1}^{N} v_i^2$
  $\longrightarrow$ spontaneous structure formation
- Grains left to themselves settle into static packing $\longrightarrow$ jamming
- Dynamics due to driving
  shear: rheology of granular particles with and without friction
  gravity, e.g. flow on an inclined plane, out of a hopper ...
  fluidized beds: pumping air or fluid through a granular bed
- stationary state; no detailed balance, no fluctuation dissipation theorem,...

Experiments on Fluidized Beds



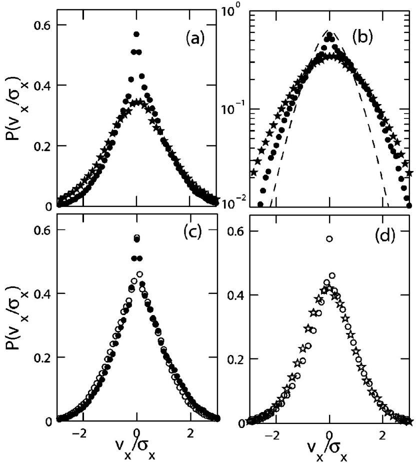
binary mixture of spherical particles in a sieve; driven by uniform upflow of air

Abate and Durian Phys.Rev. E74,031308, 2006



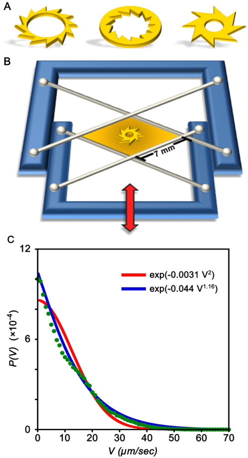
mean square displacement: development of a plateau as volume fraction increases

Velocity Distributions in Experiment



vibrated granular medium

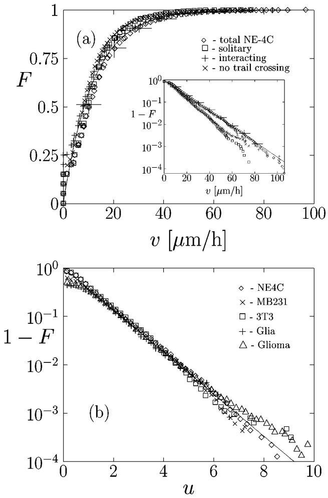
van Zon, Swinney et al. PRE 2004



swimming bacteria

(bacillus subtilis)

Sokolov, Aranson et al. PNAS 2010



in vitro cells

Czirok, Vicsek et al. PRL 1998

Simple Model

hard spheres in a fluid
with viscous drag γ:

$$\partial_t \mathbf{v}_i = -\gamma \mathbf{v}_i + \left.\frac{\Delta \mathbf{v}_i}{\Delta t}\right|_{coll} + \left.\frac{\Delta \mathbf{v}_i}{\Delta t}\right|_{Dr}$$

collisions: $(\mathbf{v}_i - \mathbf{v}_j)\mathbf{n} = -\epsilon(\mathbf{v}'_i - \mathbf{v}'_j)\mathbf{n}$

elastic collisions $\epsilon = 1$
incomplete normal restitution: $\epsilon < 1$

discrete kicks: $\mathbf{u}$ with frquency f_{Dr}
crude approximation to run-and-tumble behavior of bacteria;
in time interval $\Delta t = 1/f_{Dr}$ particle is accelerated,
subsequently randomized by surrounding fluid and interactions
with other particles

stationary state: $2m\gamma < v^2 >= m < u^2 > f_{Dr}$ elastic case

3 parameters: γ, f_{Dr} and volume fraction Φ

Event Driven Simulations

ballistic motion
idea: in between collison (or kicks) particles move freely

$$\mathbf{r}_i(t) - \mathbf{r}_j(t) \equiv \mathbf{r}_{i,j}(t) = \mathbf{r}_{i,j}(t_0) + \mathbf{v}_{i,j}(t_0)(t - t_0) \tag{1}$$

compute time t_{coll} for next collision to happen $R_i + R_j = |\mathbf{r}_{i,j}(t_{coll})|$

icluding viscous drag

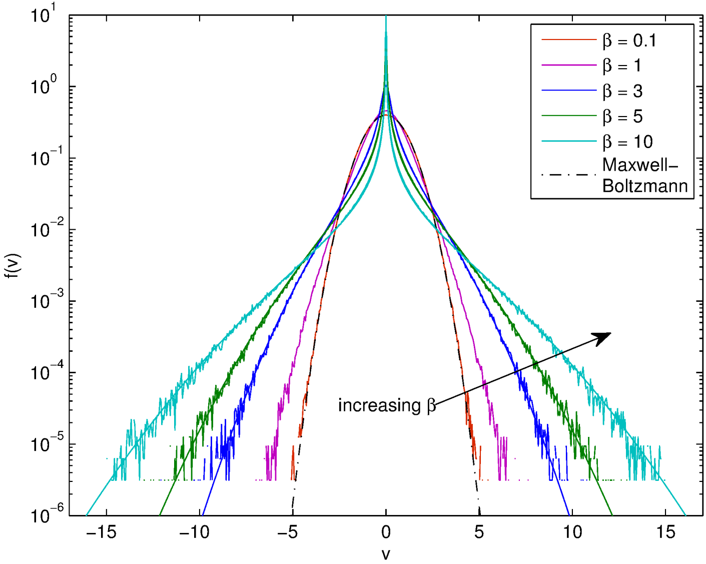
$$\mathbf{r}_{i,j}(t) = \mathbf{r}_{i,j}(t_0) + \mathbf{v}_{i,j}(t_0)\frac{1 - e^{-\gamma(t-t_0)}}{\gamma} \tag{2}$$

collision time known from ballistic simulation, replace:

$$(t_{coll} - t_0) \to (1 - e^{-\gamma(t_{coll}-t_0)})/\gamma \tag{3}$$

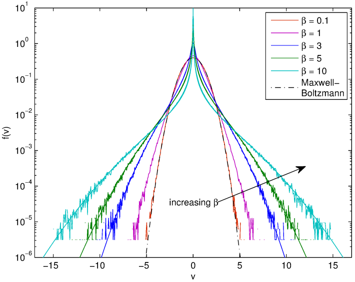
allows to simulate several millions of particles

Velocity Distributions in stationary State



important parameter $\beta := \gamma / f_{Dr}$; $\Phi = 0.35$

Velocity Distributions in Stationary State



$\beta := \gamma / f_{Dr}$; $\Phi = 0.35$

- Gaussian only in the limit $\beta \rightarrow 0$
- overpopulated at small v, velocities decay with rate γ singularity for $\beta \rightarrow \infty$
- overpopulated at large v due to kicks
- independent of volume fraction $0.05 \leq \Phi \leq 0.4$
- depends only on ratio $\beta := \gamma / f_{Dr}$

Single-Particle-Model

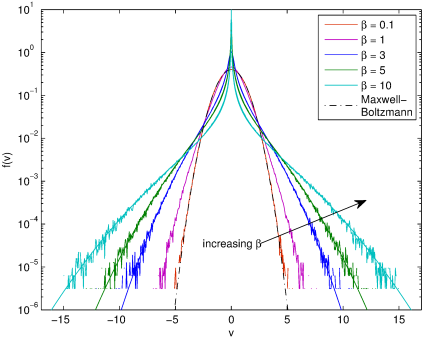
$$f(v) = \int_{-\infty}^{\infty} dx\ p_k(v-x)\ f(xe^{\beta})\ e^{\beta}$$

$\beta = \gamma / f_{Dr} \to 0$: recover Maxwell-Boltzmann distribution

$\beta \gg 1$: nontrivial distribution

solve by iteration; fast convergence

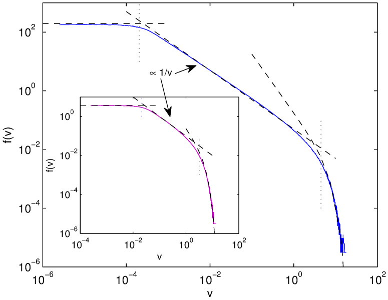
good agreement with simulations



Single-Particle-Model

$$f(v) = \int_{-\infty}^{\infty} dx\ p_k(v-x)\ f(xe^{\beta})\ e^{\beta}$$

$\beta \gg 1$:



$$f(v) \approx \begin{cases} \dfrac{e^{\beta}-1}{2\sqrt{\pi\beta^3}} & \frac{|v|}{\sqrt{2\beta}} \ll e^{-\beta} \\ \dfrac{1}{2\beta|v|} & e^{-\beta} \ll \frac{|v|}{\sqrt{2\beta}} \ll 1 \\ \dfrac{1}{\sqrt{\pi\beta}}\dfrac{1}{v^2}e^{-v^2/4\beta} & \frac{|v|}{\sqrt{2\beta}} \gg 1 \end{cases}$$

$\beta = 5$ (lower), $\beta = 10$ (upper),
$\Phi = 0.35$

Conclusion and Generalisations

Largely universal velocity distributions
depending only on $\beta = \gamma / f_{Dr}$, independent of volume fraction, particle interactions
$\beta \to 0$: Maxwell- Boltzmann is recovered
$\beta \gg 1$: Distribution is divergent for small v, falls off as $1/v$ for intermediate v and is Gaussian for the largest v

- distribution of kick amplitudes and waiting times in betwen kicks
- particles with orientation, include rotational degrees of freedom
- anisotropic particles, simplest case: needles