

# Synchronized Cycles: An allosteric model of the Kai circadian oscillator

**David K. Lubensky**

*(Univ. of Michigan & VU Amsterdam)*

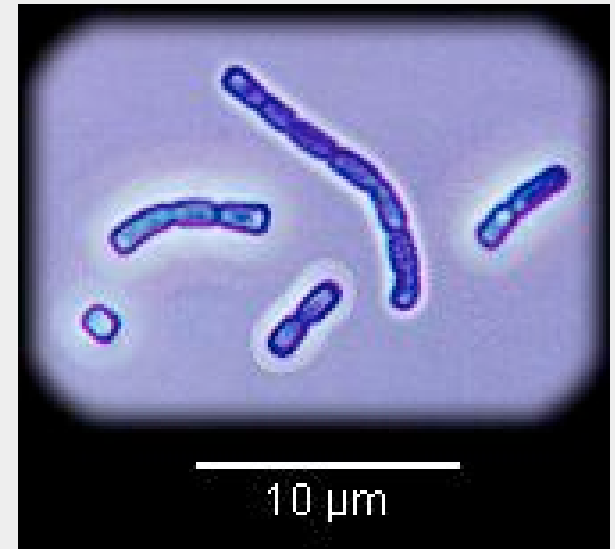
**Jeroen S. van Zon**

*(Imperial College & VU Amsterdam)*

**Pim Altena**

**Pieter Rein ten Wolde**  
*(AMOLF, Amsterdam)*

*S. elongatus*

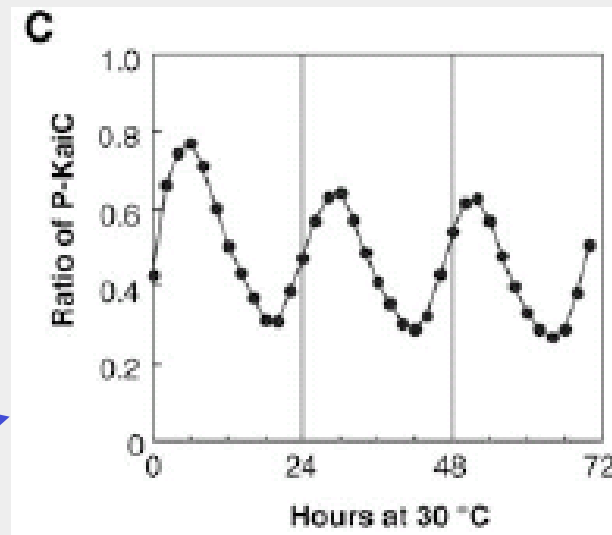
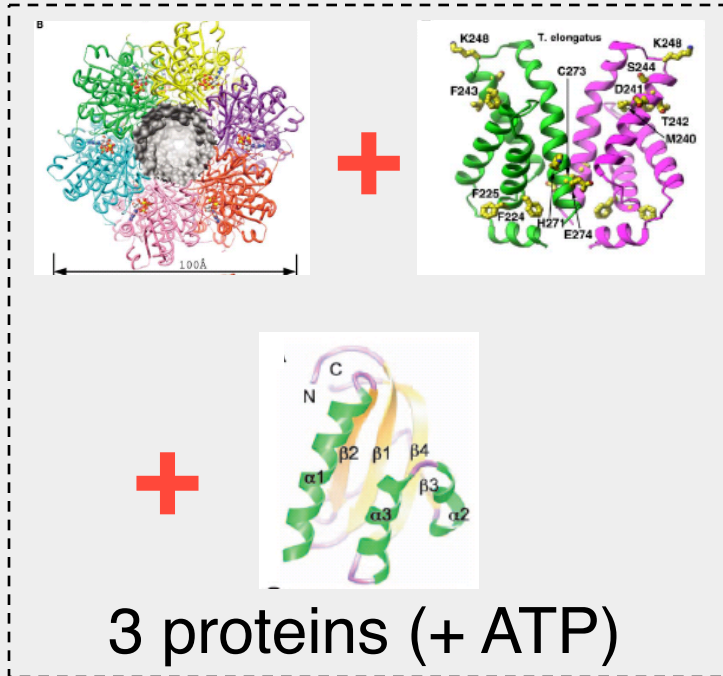
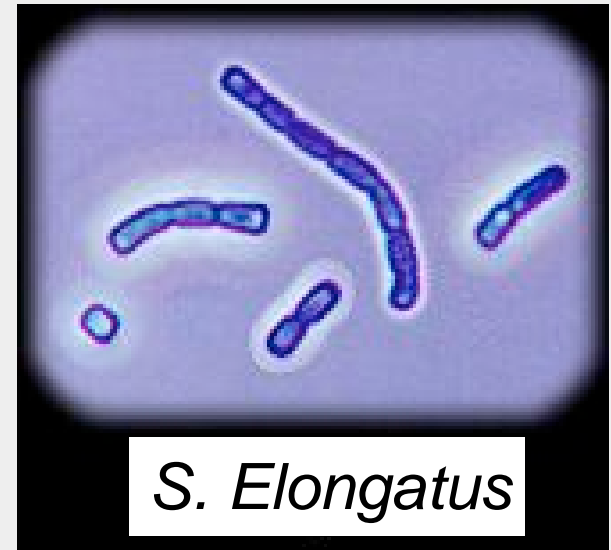


(Image:CIBNOR)

**PNAS 104, 7420 (2007)**

# The Problem...

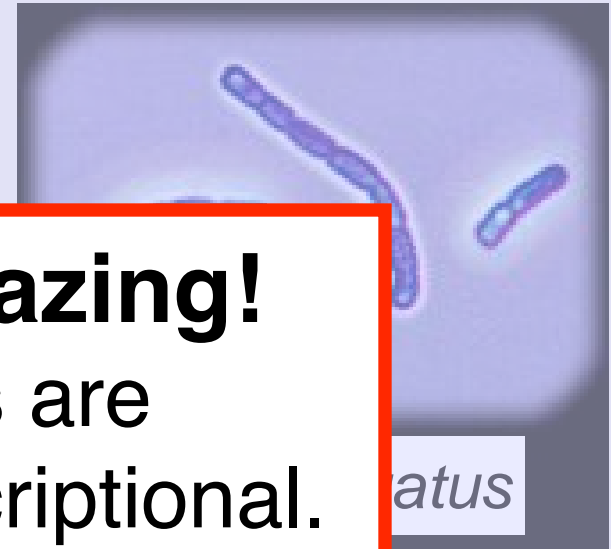
- Circadian rhythm in a test tube:



Oscillatory phosphorylation  
(Nakajima *et al.*, *Science* 2005)

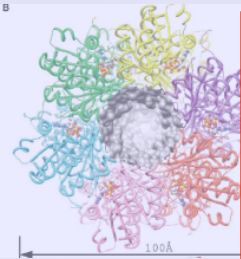
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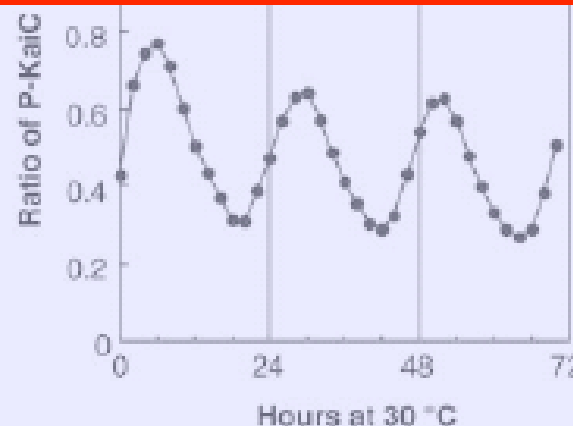
**Wow! That's amazing!**

- Circadian oscillators are supposed to be transcriptional.
- A complete biochemical "circuit" *in vitro*.



+

3 proteins (+ ATP)



Oscillatory phosphorylation  
(Nakajima *et al.*,  
*Science* 2005)

# The Problem...

- Circadian rhythm in a test tube:

## How does this oscillator work?

*Related models:*

Emberly and Wingreen, *PRL* 2006

Clodong *et al.*, *Mol. Sys. Biol.* 2007

Mori *et al.*, *PLoS Biol.* 2007

Yoda *et al.*, *PLoS ONE* 2007

*Other models:*

Mehra *et al.*, *PLoS Comp. Biol.* 2006

Miyoshi *et al.*, *JBR* 2007

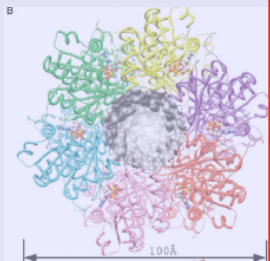
Takigawa-Imamura and Mochizuki, *J. Th. Biol.* 2006

“

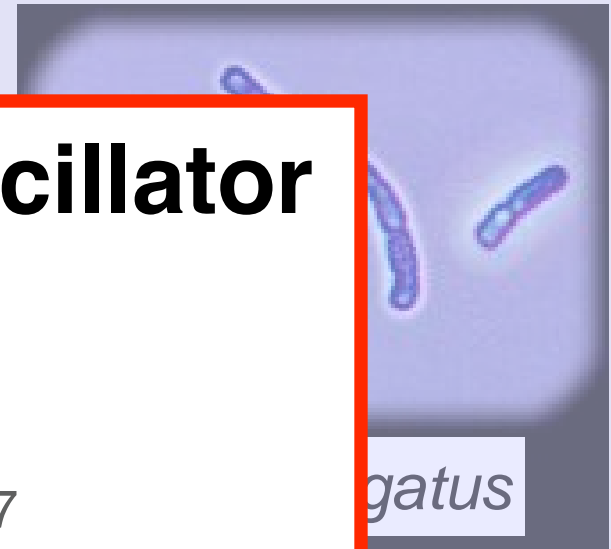
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JBR 2006

akajima *et al.*,  
*Science* 2005)



3 prote



gatus

oscillatory  
phospho-  
ylation

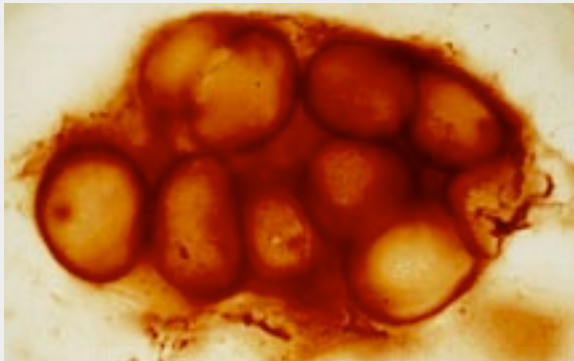


# Outline

- Background
  - Cyanobacteria
  - *In vitro* data
- Allosteric Cycles: Focus on hexamers
  - Thermodynamic consistency
- Synchronization by Differential Affinity
  - Alternative proposals
- Full Model
  - Matching available data
  - “Temperature compensation”
- Conclusions

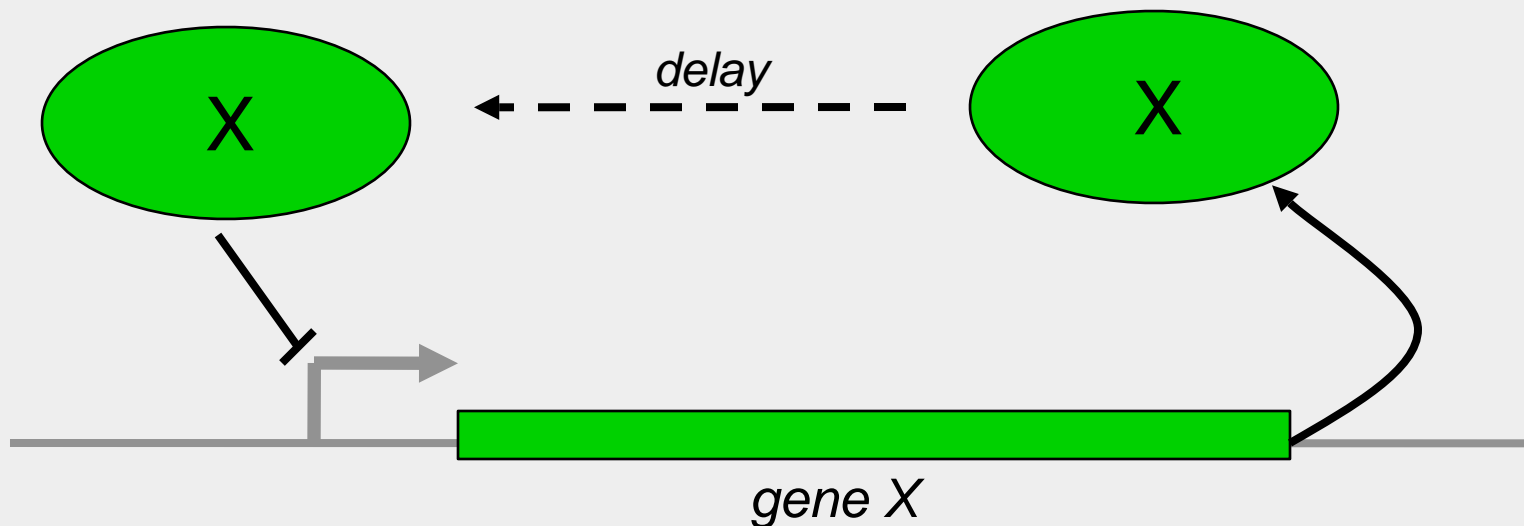
# Cyanobacteria

- Photosynthetic bacteria (O<sub>2</sub> producing)
- Oldest known fossils
  - ~3.5 billion years
  - Responsible for current oxygenic atmosphere
  - Origin of chloroplasts
- “Blue-green algae”
- *S. elongatus*: Genetically tractable model.



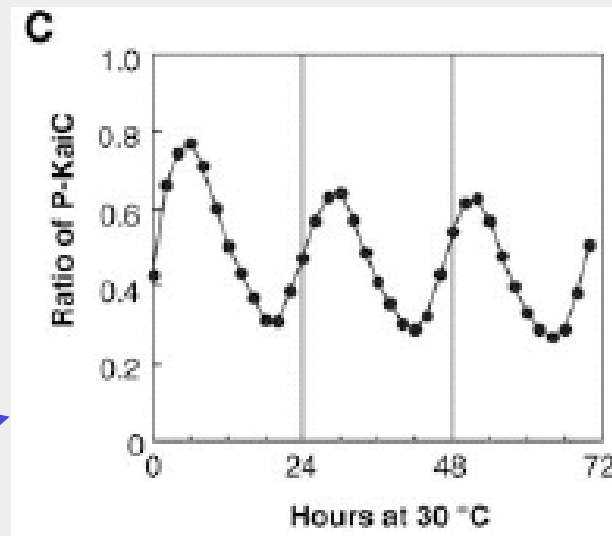
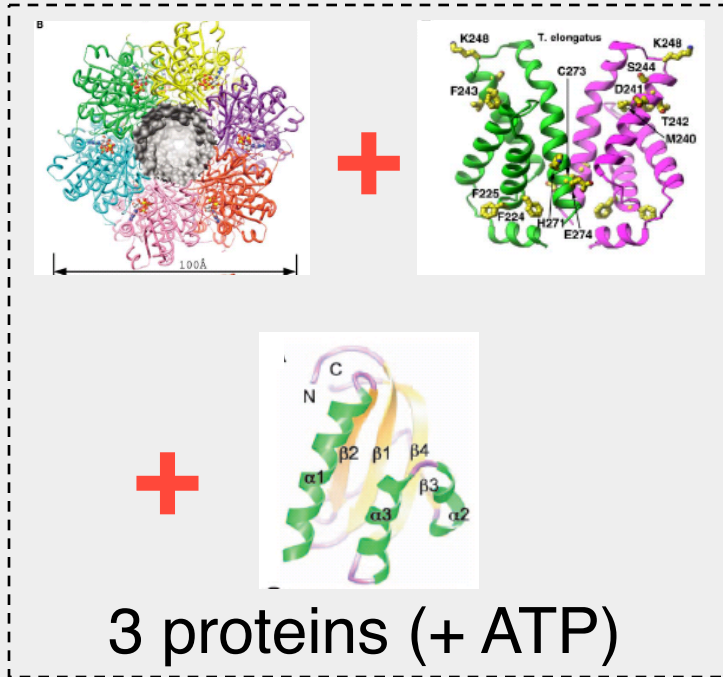
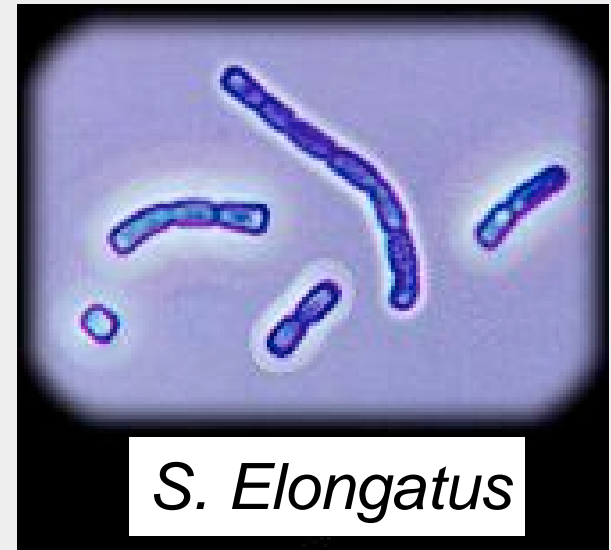
# Circadian Rhythms

- Most eukaryotes, cyanobacteria,...
- Free-running oscillation, ~24 hours.
- Entrained by light, temperature, etc.
- Temperature-compensated.
- Textbook model: Negative transcriptional feedback



# ...but no transcription needed in *S. Elongatus*

- Circadian rhythm in a test tube:



Oscillatory phosphorylation  
(Nakajima *et al.*,  
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# The Players

- KaiC

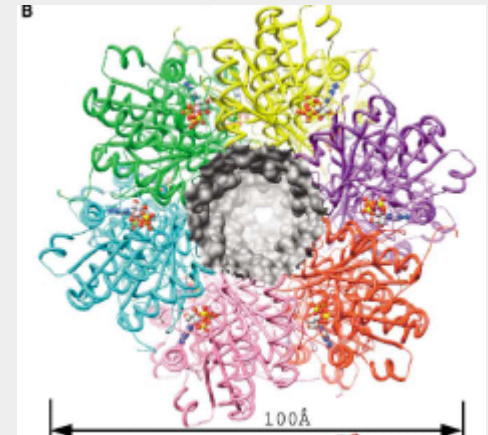
- Hexamer (AAA+ ATPase).
- Auto (de)phosphorylation activity.
- Transcriptional repressor *in vivo*.
- Phosphorylation level oscillates with 24 hour period.

- KaiA

- Dimer
- Stimulates KaiC phosphorylation.
- Complexes w/KaiC and KaiB + KaiC.

- KaiB

- Dimer or tetramer
- Attenuates KaiA's effects.
- Complexes w/KaiC and KaiA + KaiC.



Pattanayek *et al.*, *Mol. Cell* 2004

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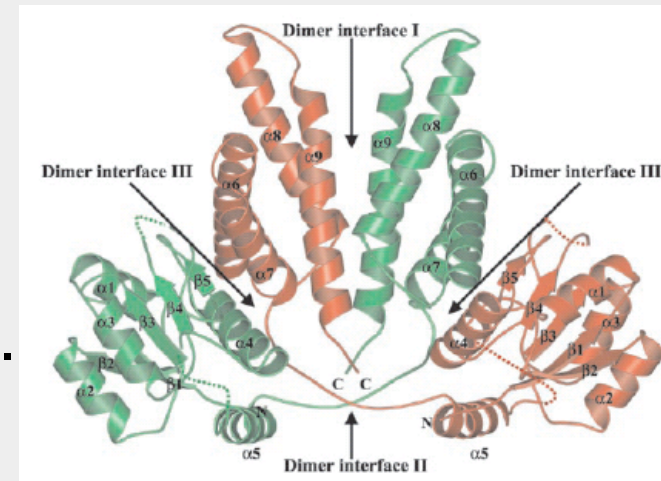
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Ye et al., *J Biol. Chem.* 2004

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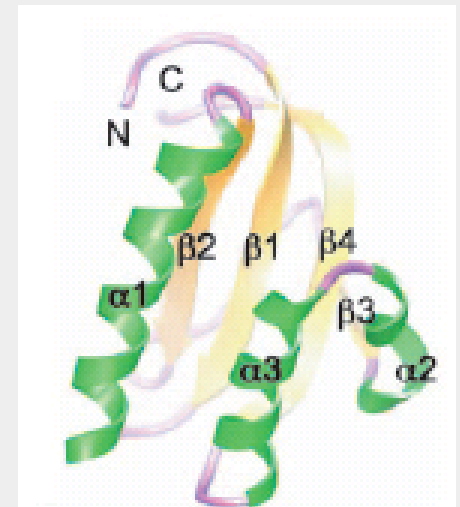
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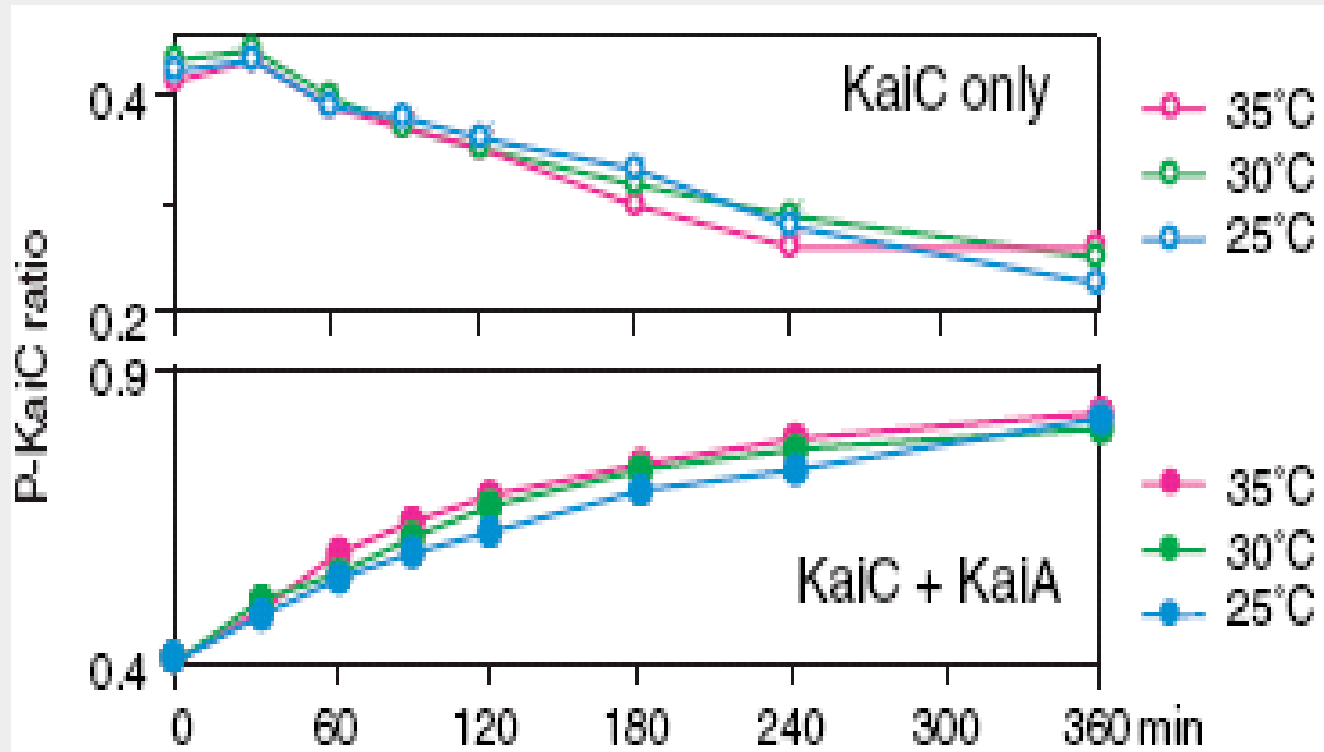
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# In Vitro Data

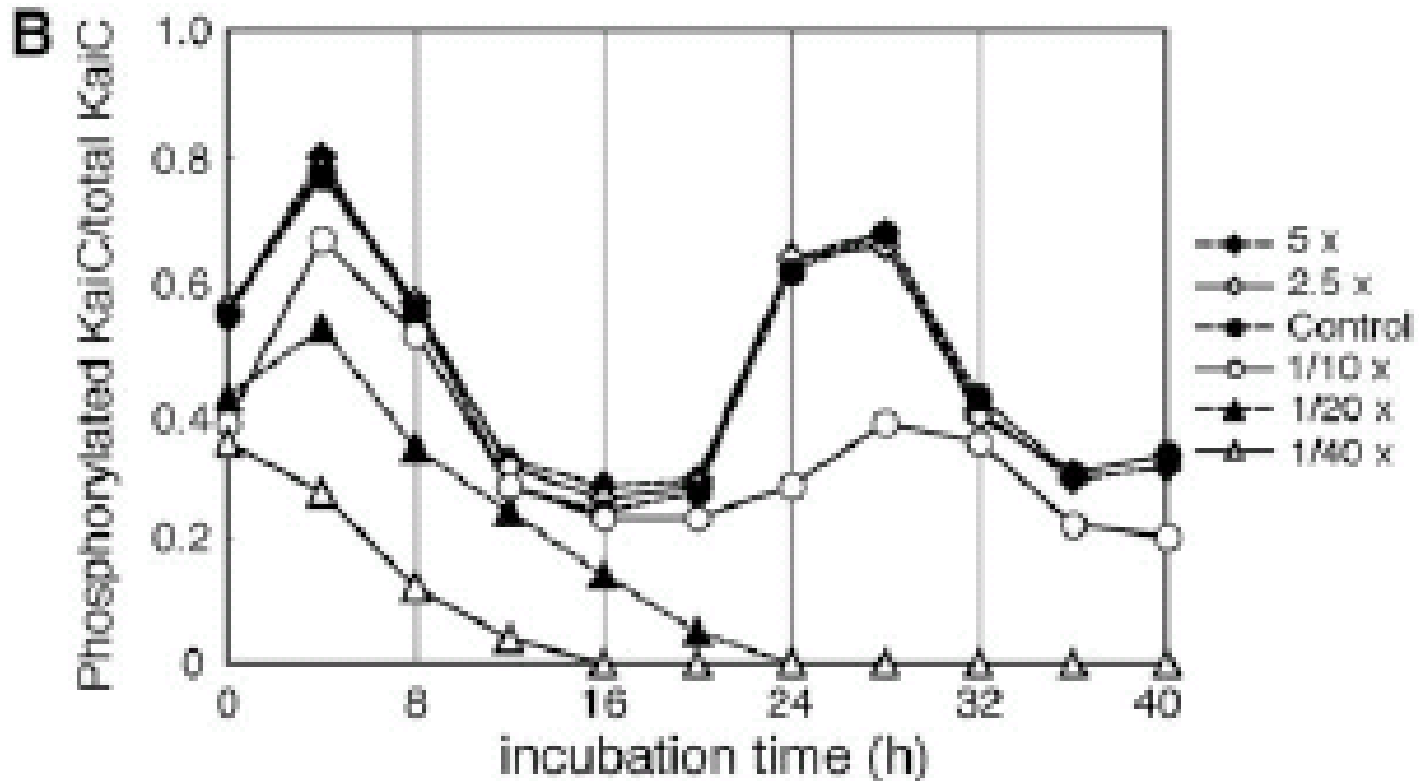
- Different combinations of proteins
  - Phosphorylation and dephosphorylation each slow and temperature-compensated.

(to 0 at long times)



# In Vitro Data

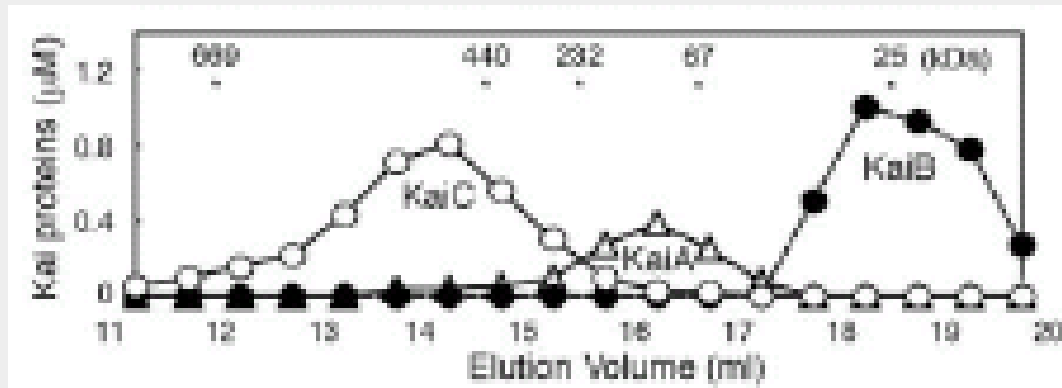
- Varying concentrations
  - Increasing all concentrations by same factor  $\Rightarrow$  No change.



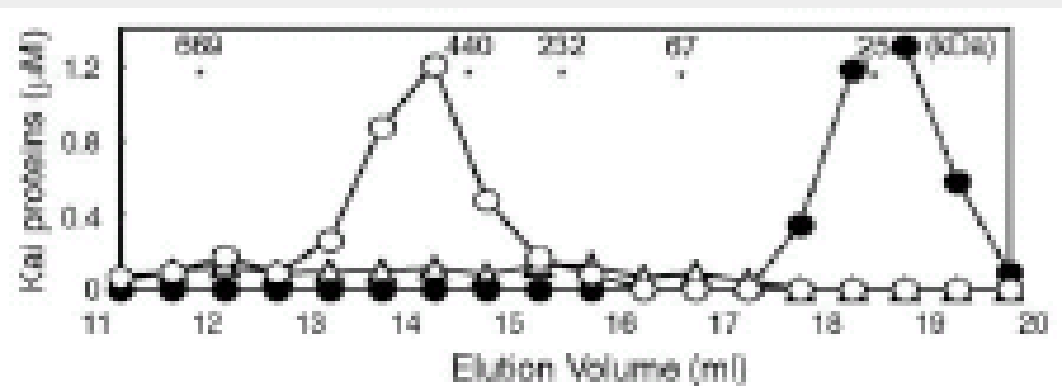
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- Sizes of Complexes

- No evidence for interactions between KaiC hexamers.
- Almost no free KaiA.



t = 0 (reference)

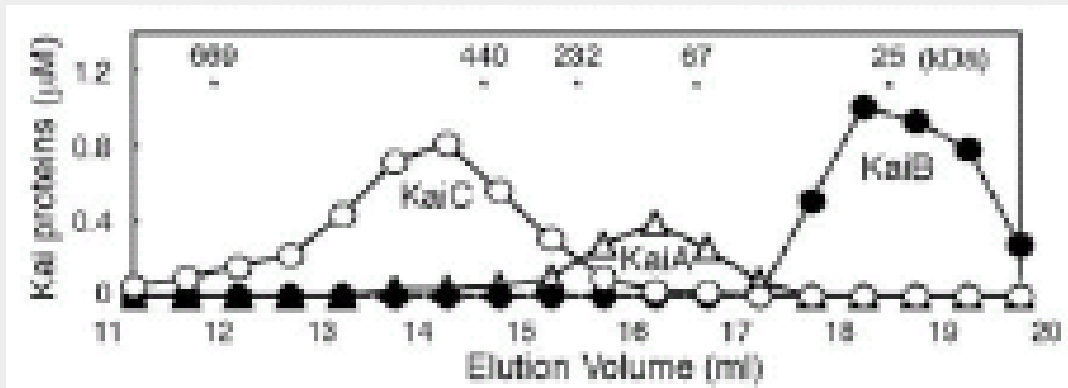


t = 24 hrs.

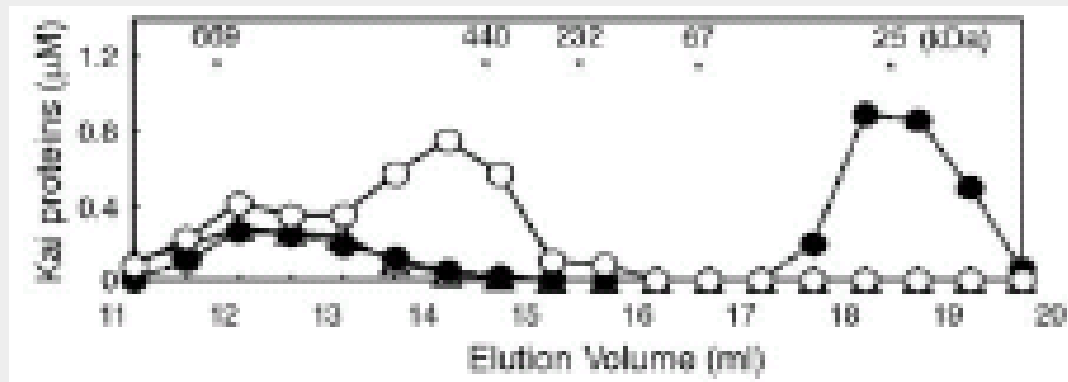
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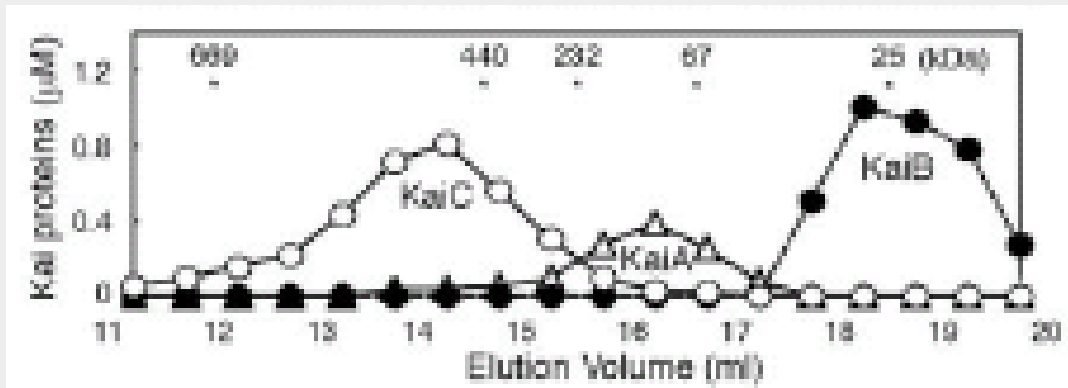


t = 30 hrs.

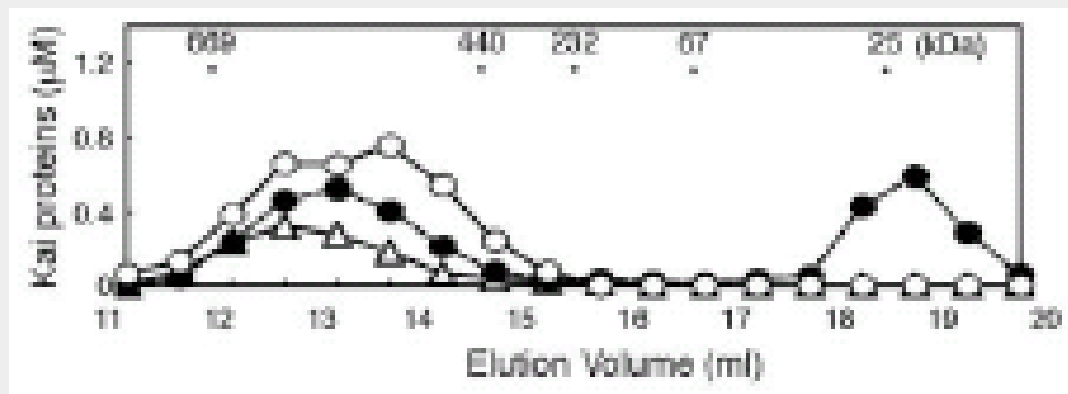
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$t = 0$  (reference)



$t = 36$  hrs.

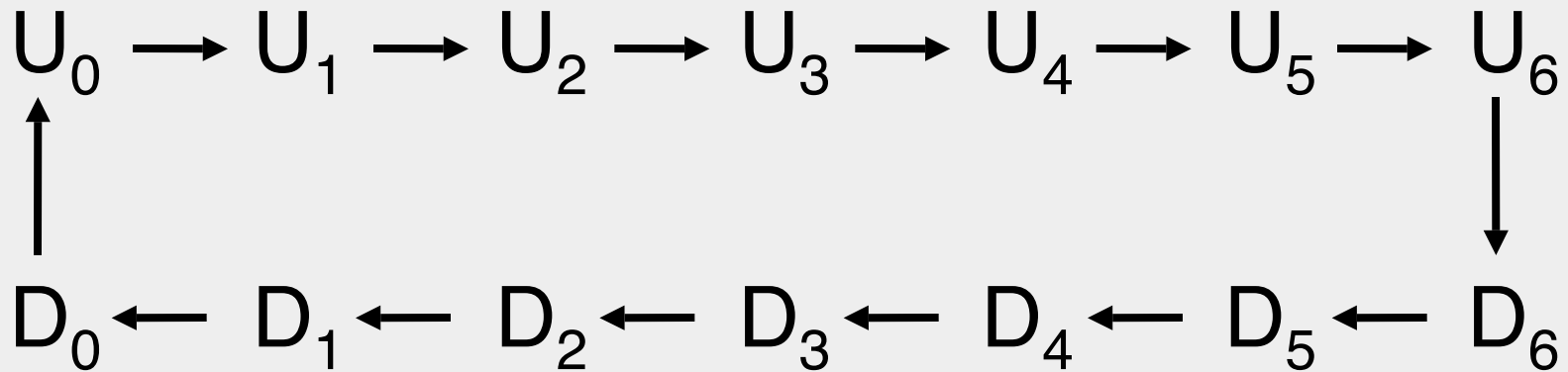


# Modeling Challenges

- Kai proteins neither created nor destroyed.
- KaiC (de)phosphorylation = only driven (energy-consuming) reactions.
  - No other covalent modifications or enzymatic activities.
  - Other reactions obey detailed balance (unless tightly coupled to phosphorylation cycle).
- KaiC hexamers don't interact directly.
  - Single KaiC hexamer can't oscillate coherently.
  - KaiC's coupled only indirectly through KaiA and KaiB.
- Seriously constrained by biochemical data.

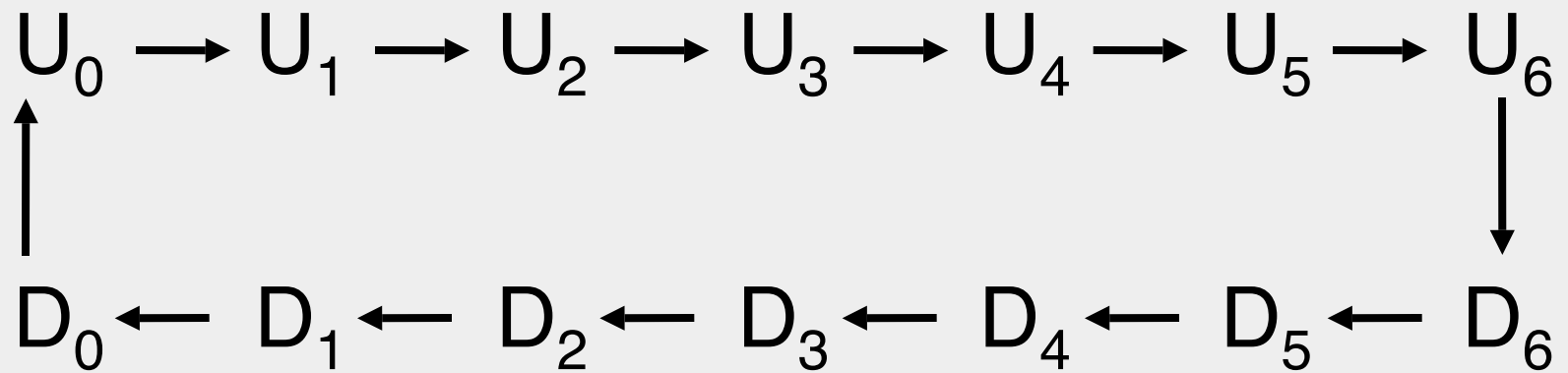
# Model, Part 1: Allosteric Cycles

- 2 KaiC conformations:
  - “U” favors phosphorylation
  - “D” favors dephosphorylation
- KaiC alone cyclically adds & removes P’s:



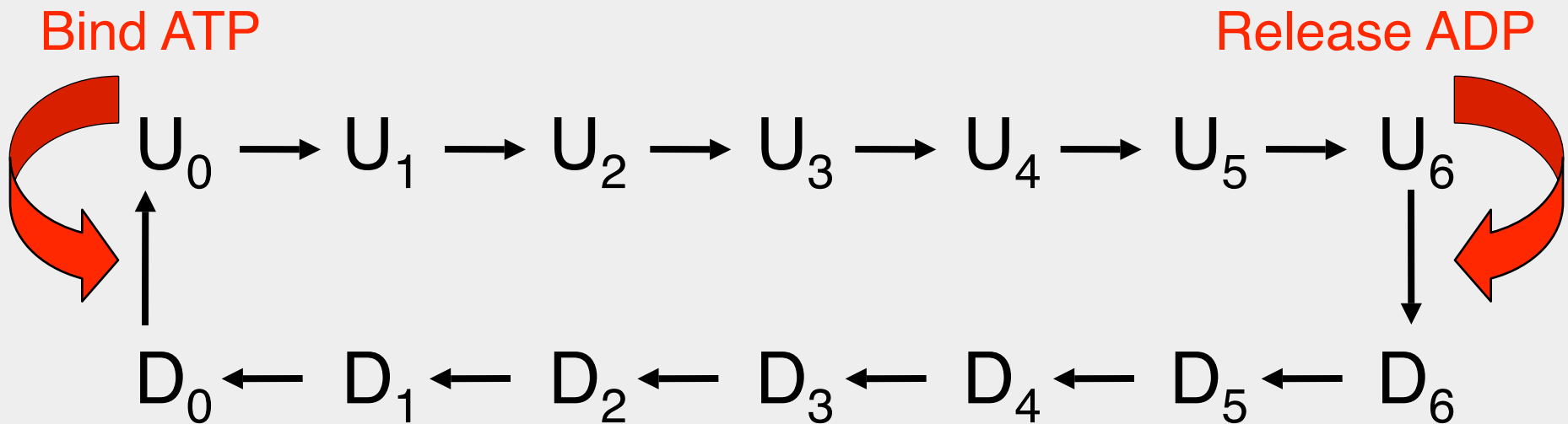
(subscript indicates # phosphates)

# Thermodynamic Constraints



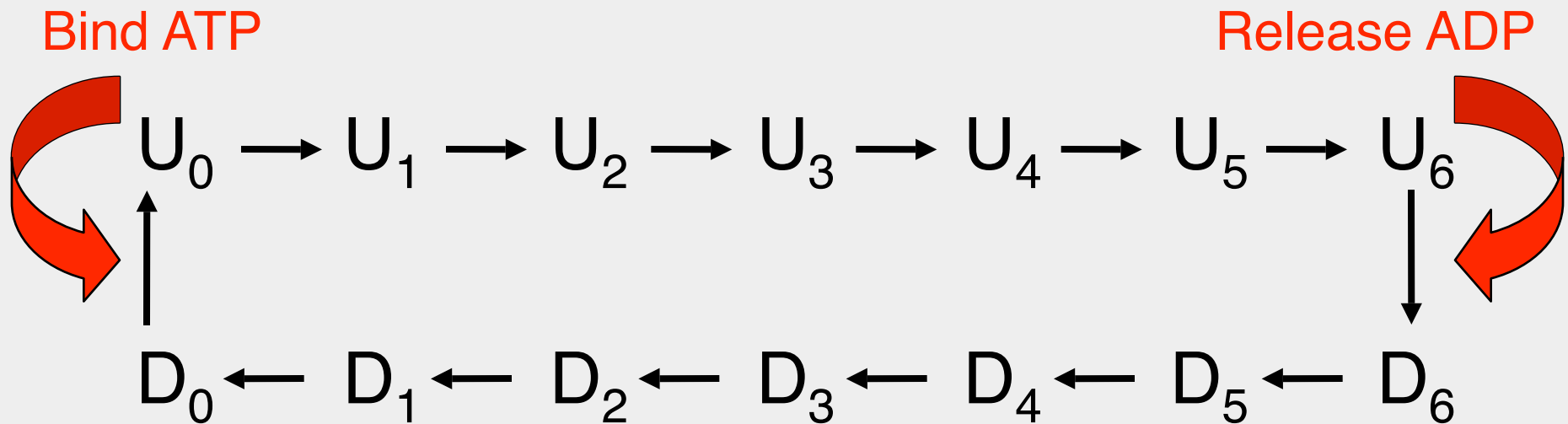
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Nucleotide exchange.

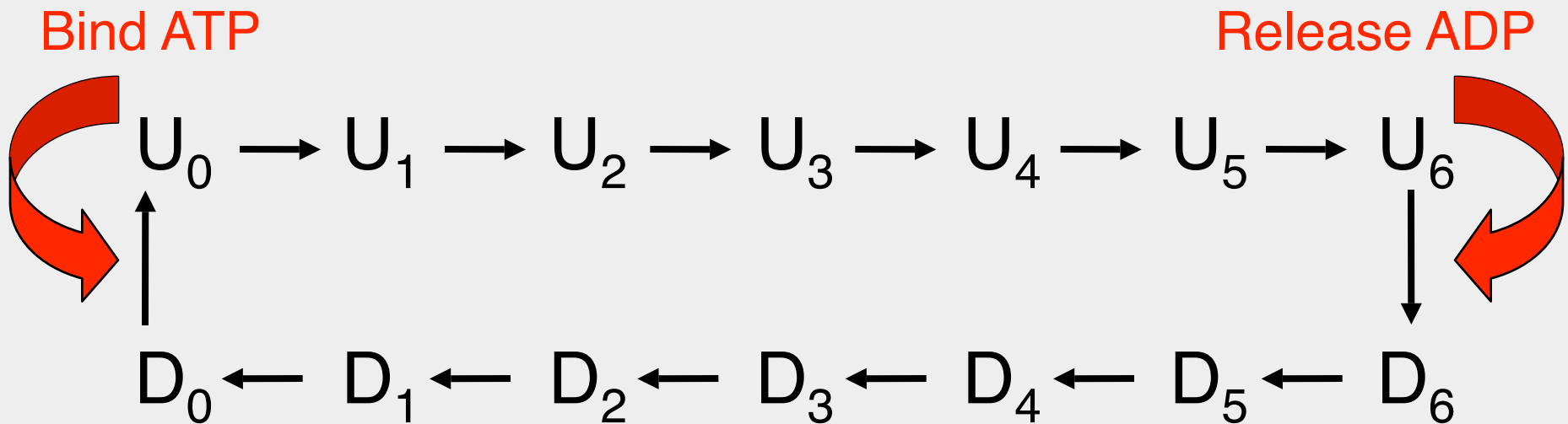


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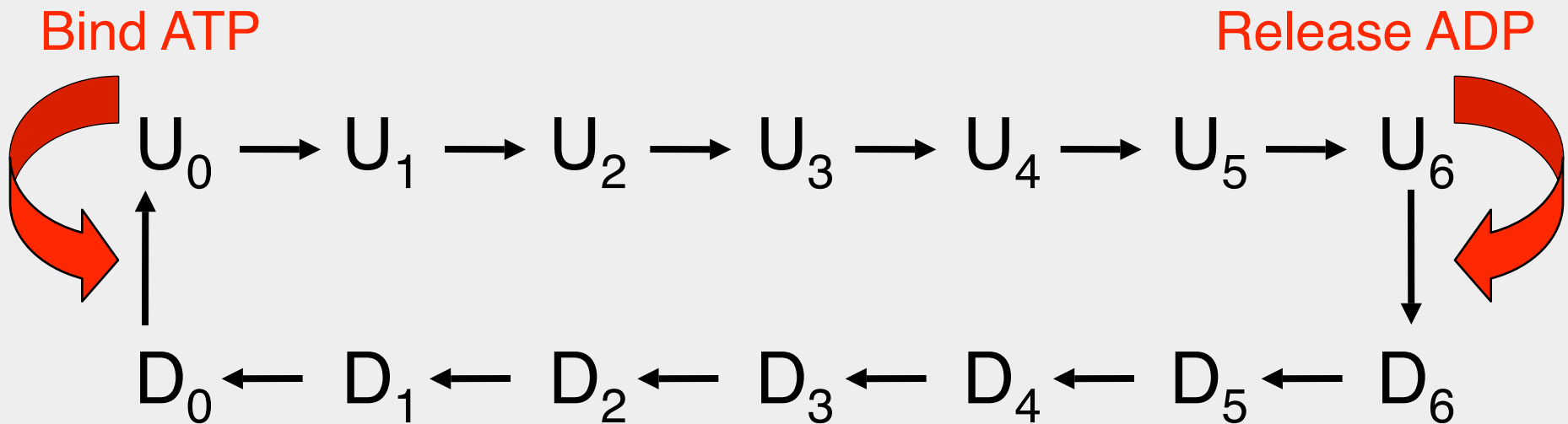


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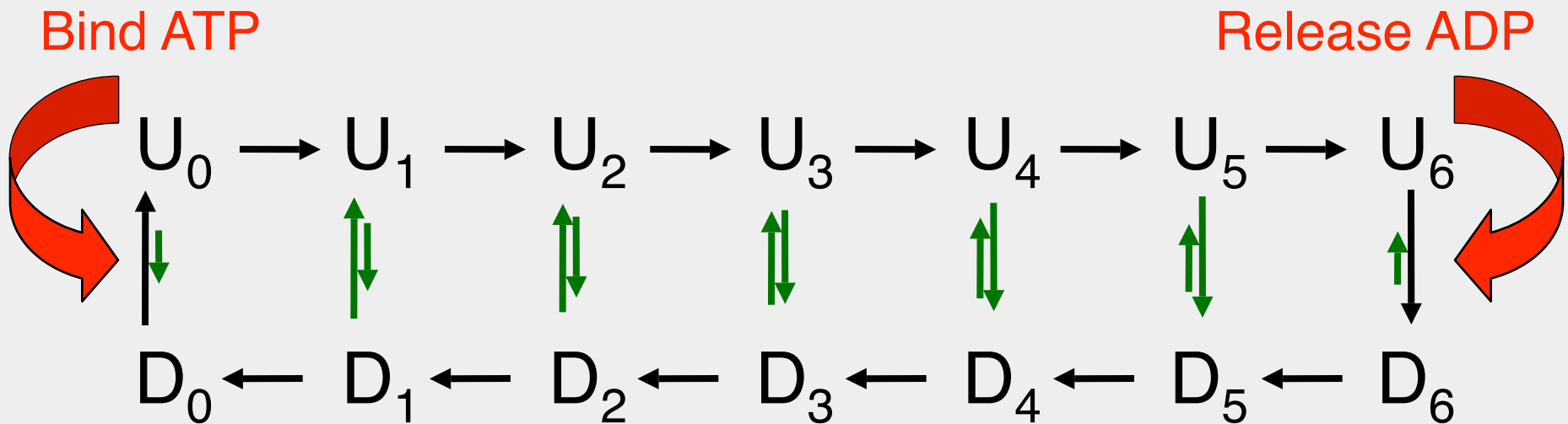


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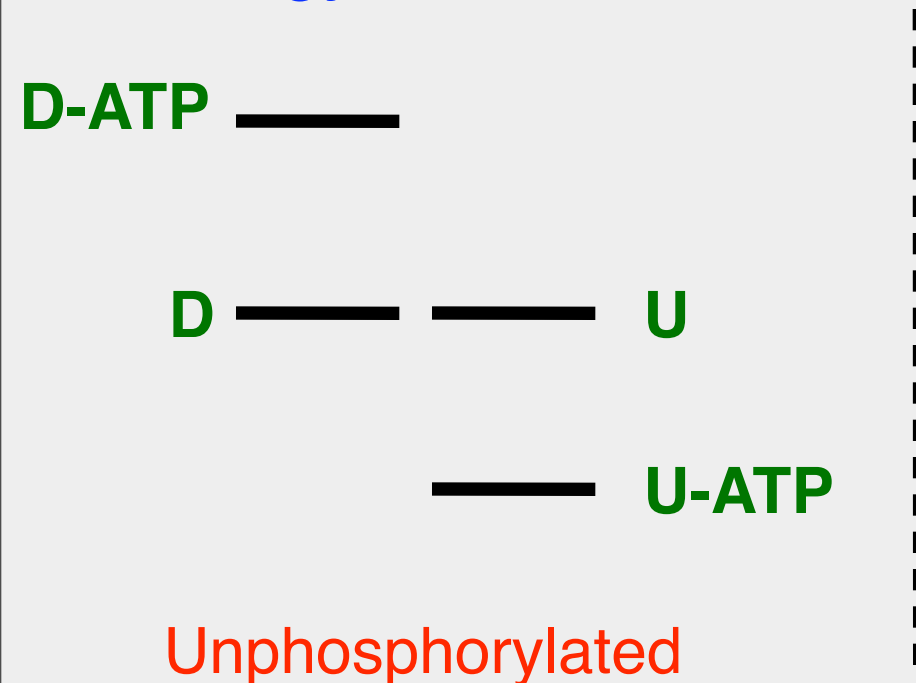
- All obvious driven steps now “used up”.
- Further reactions should obey detailed balance.
- Must allow reverse, intermediate reactions.





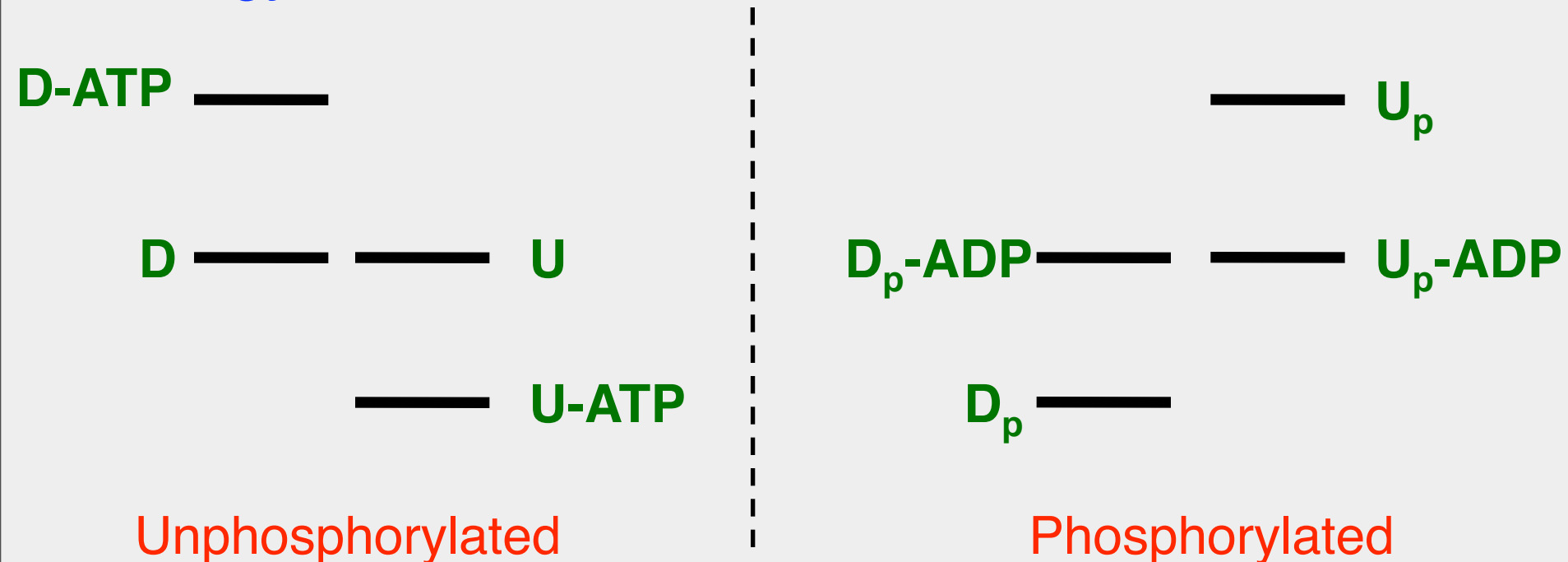
# Monomer States

- 8 KaiC monomer states:
  - U or D conformation
  - Phosphorylated or not
  - Nucleotide (ATP/ADP) bound or not
- Energy levels

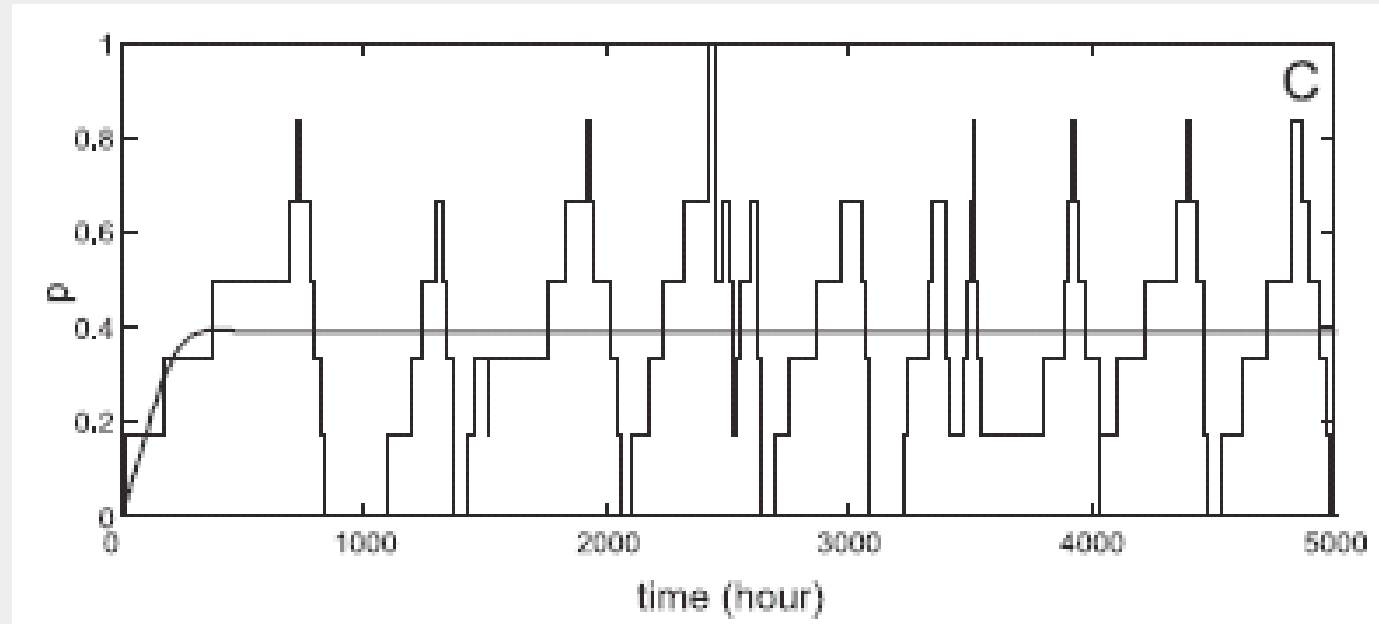


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# Single Hexamer: Noisy Oscillations

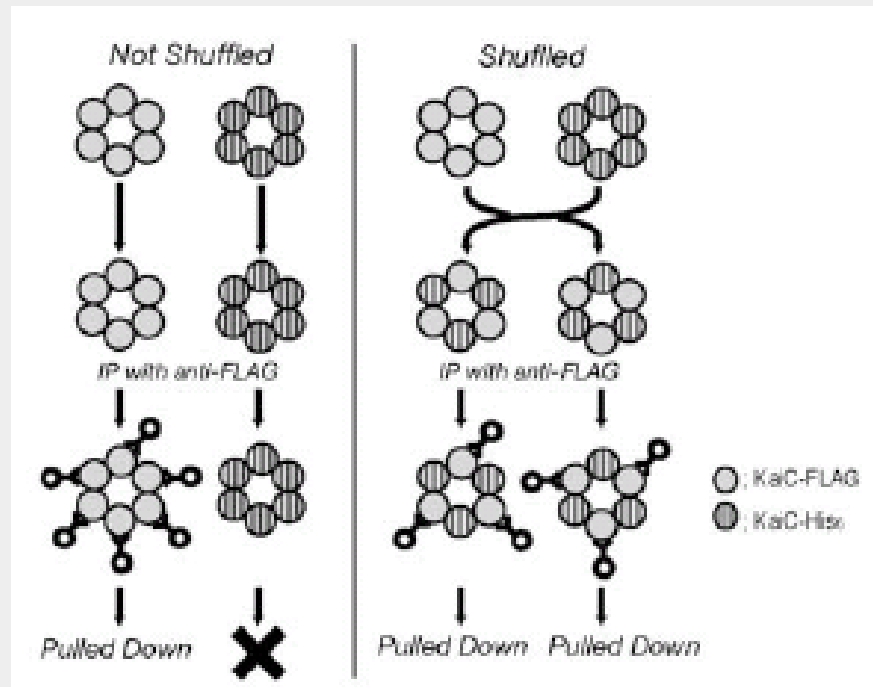


Must **synchronize** the different hexamers.

# Monomer Exchange

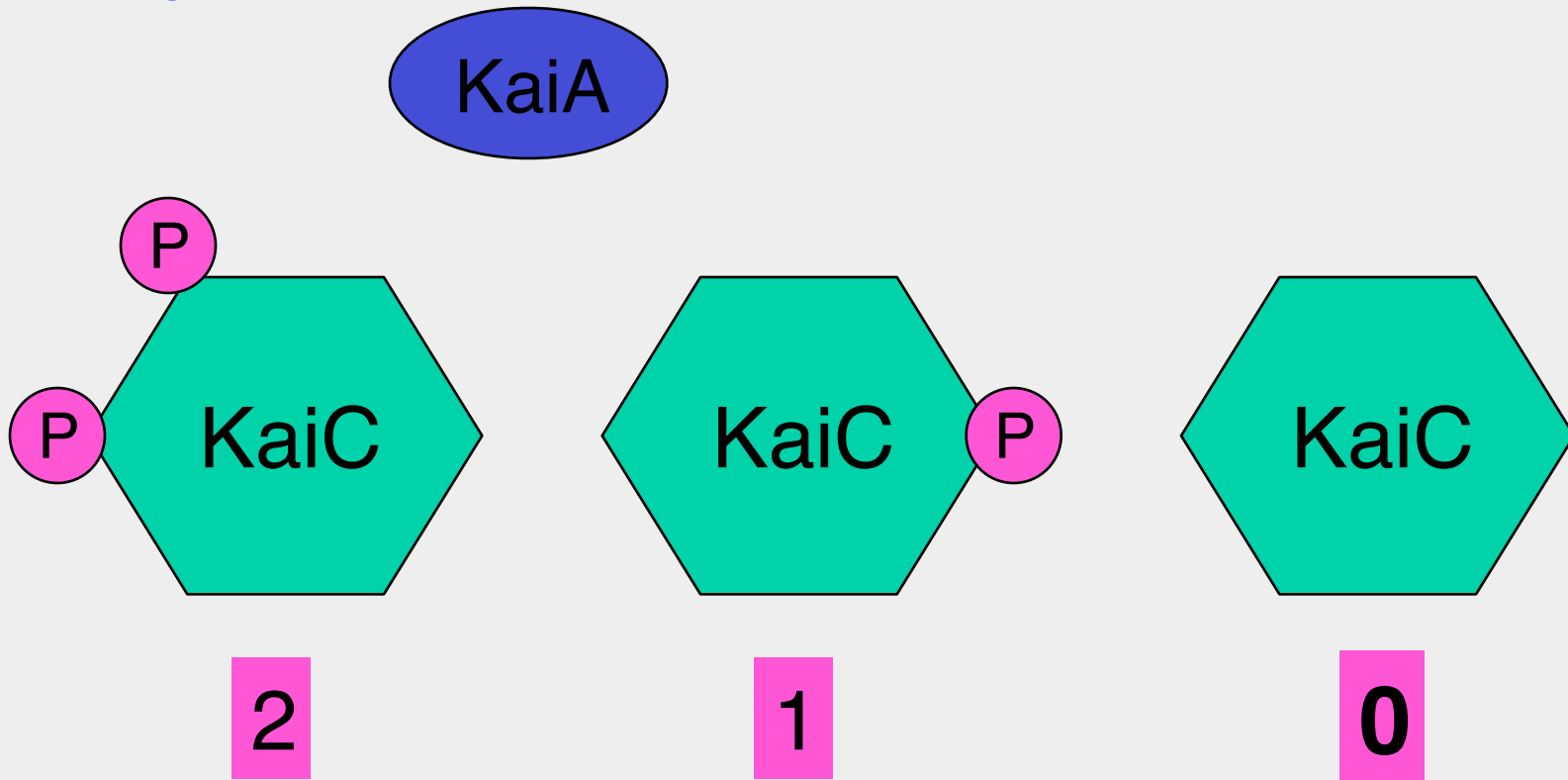
Emberly & Wingreen  
*PRL 2006*

- It happens during dephosphorylation phase.
- It is (probably) not enough to explain synchronization



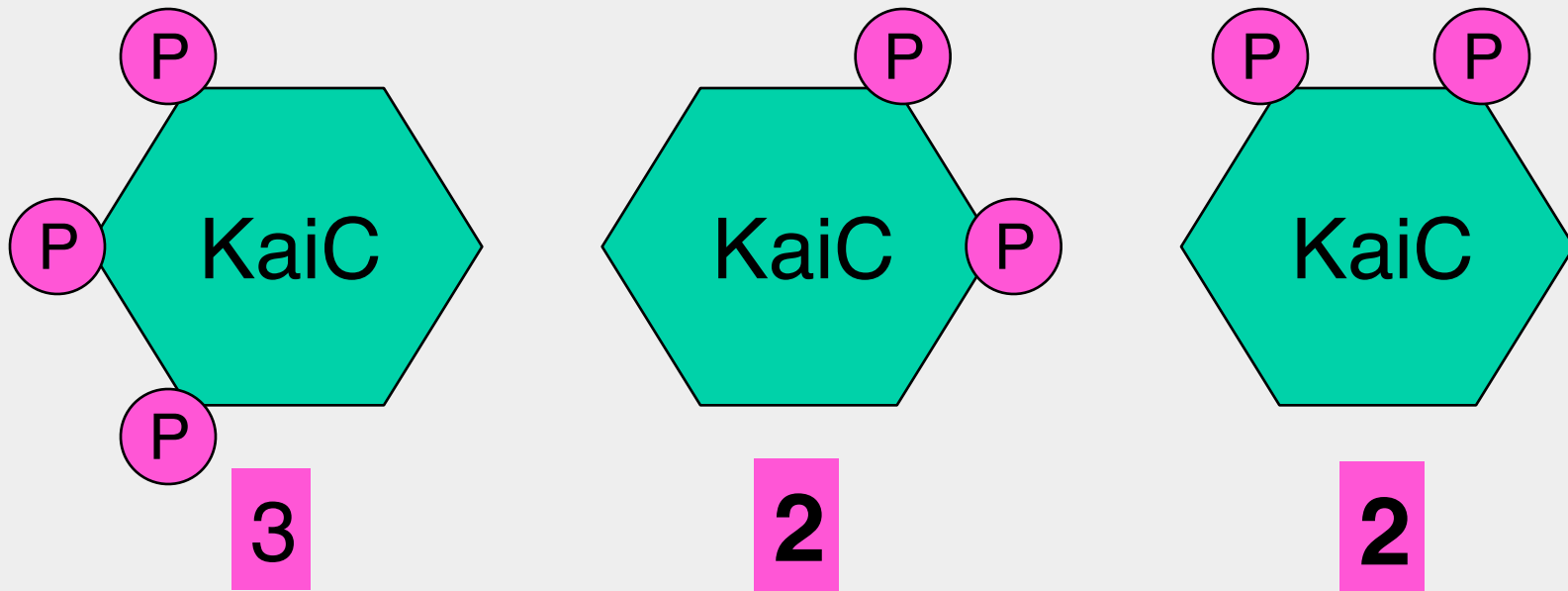
# Model, Part 2: Differential Affinity

- KaiA catalyzes KaiC phosphorylation.
- [KaiA] limiting.
- KaiA binds laggards (fewer phosphates) more strongly than leaders (more phosphates).



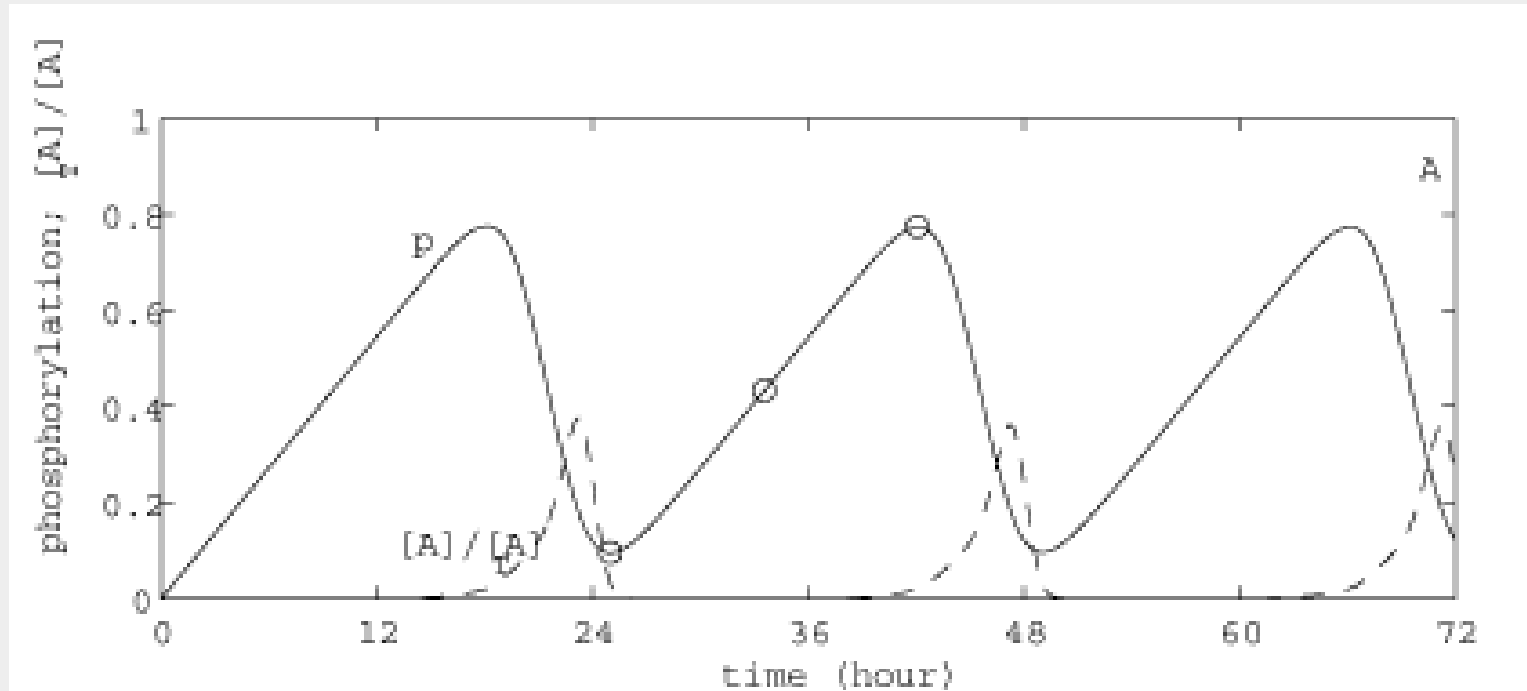
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# Generic Differential Affinity

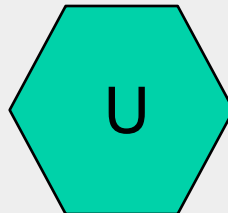
- KaiA and KaiC only.
- Oscillates:



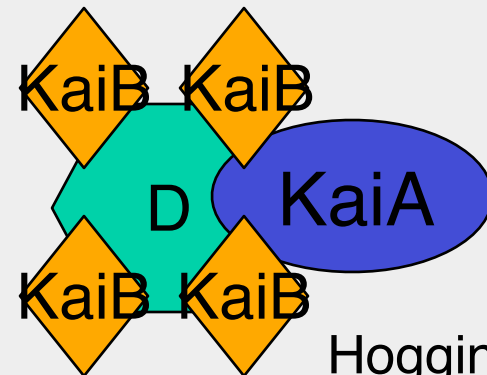
- Does not agree quantitatively with experiments.

# Full Model

- Should include KaiB.
- Should agree with data on KaiC alone, KaiA + KaiC, abundance of different complexes,...
- Changes/Additions:
  - Weak dephosphorylation in U conformation.
  - U more stable than D: AVOID OVERSHOOTS.
  - KaiB binds to, stabilizes D conformation.
  - KaiB-KaiC complexes sequester KaiA (differential affinity).



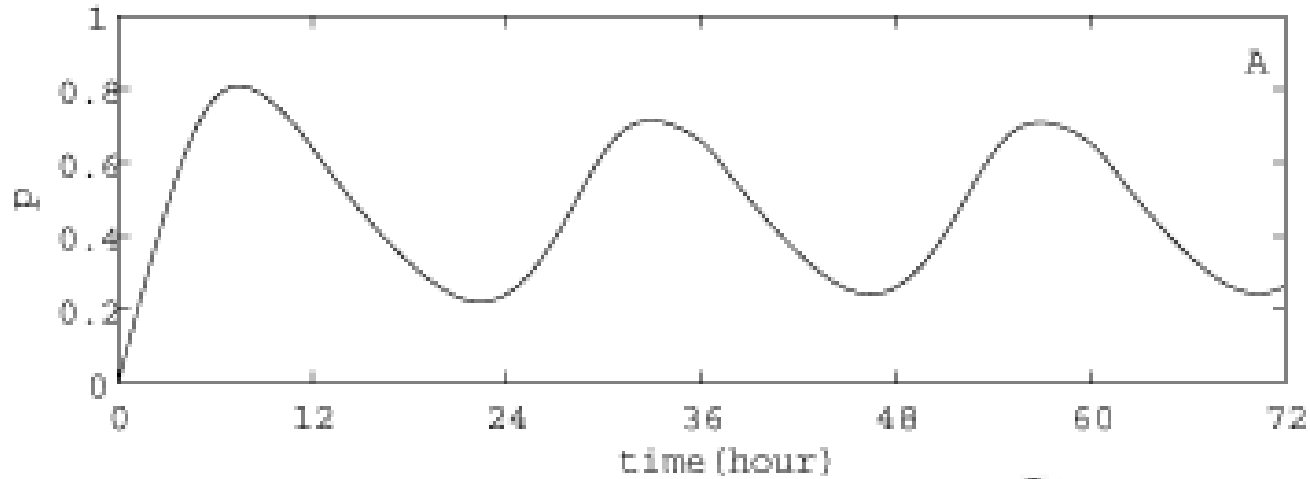
No KaiA, can't add P's



Hogging KaiA



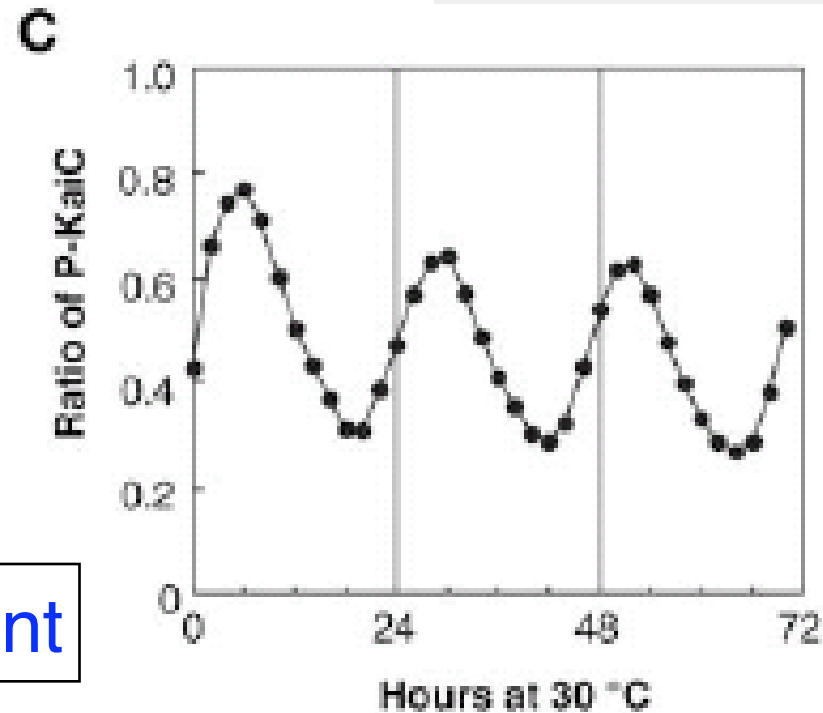
# Full Model vs. Experiment



Model

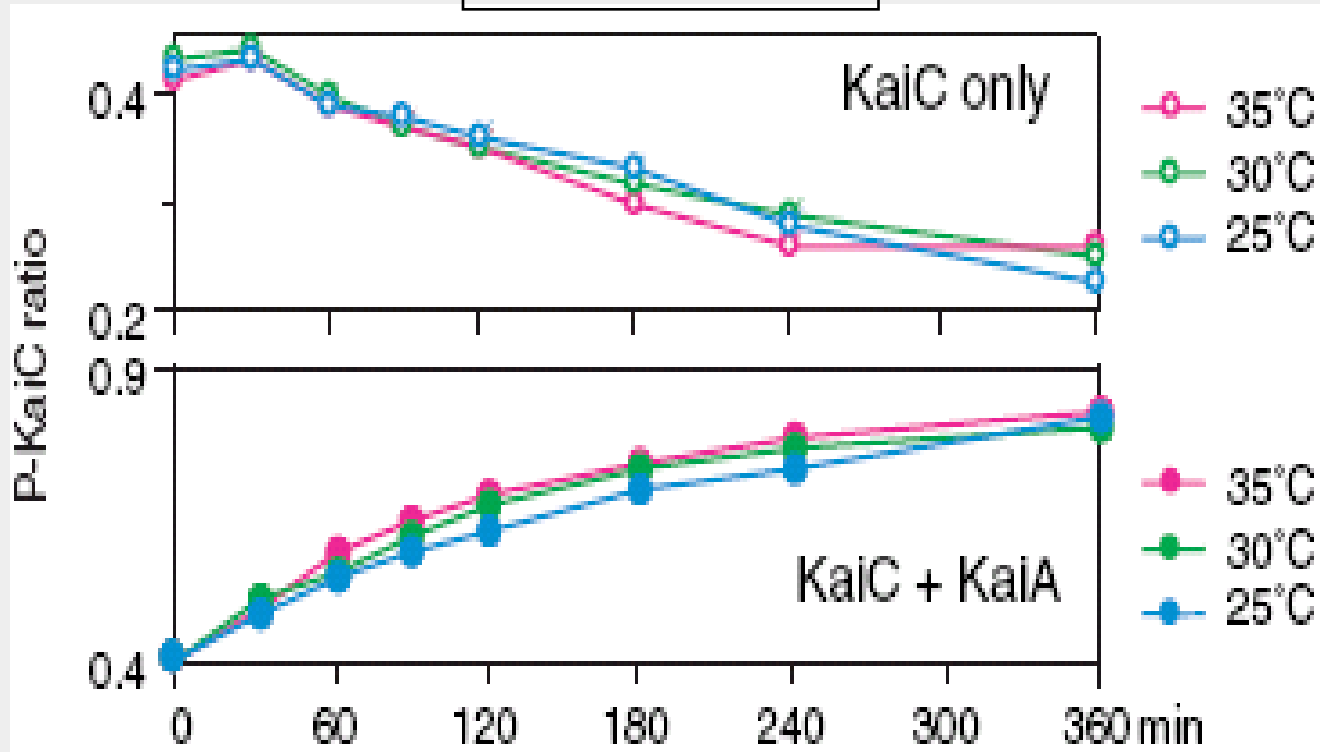
## Oscillations

Experiment



# Full Model vs. Experiment

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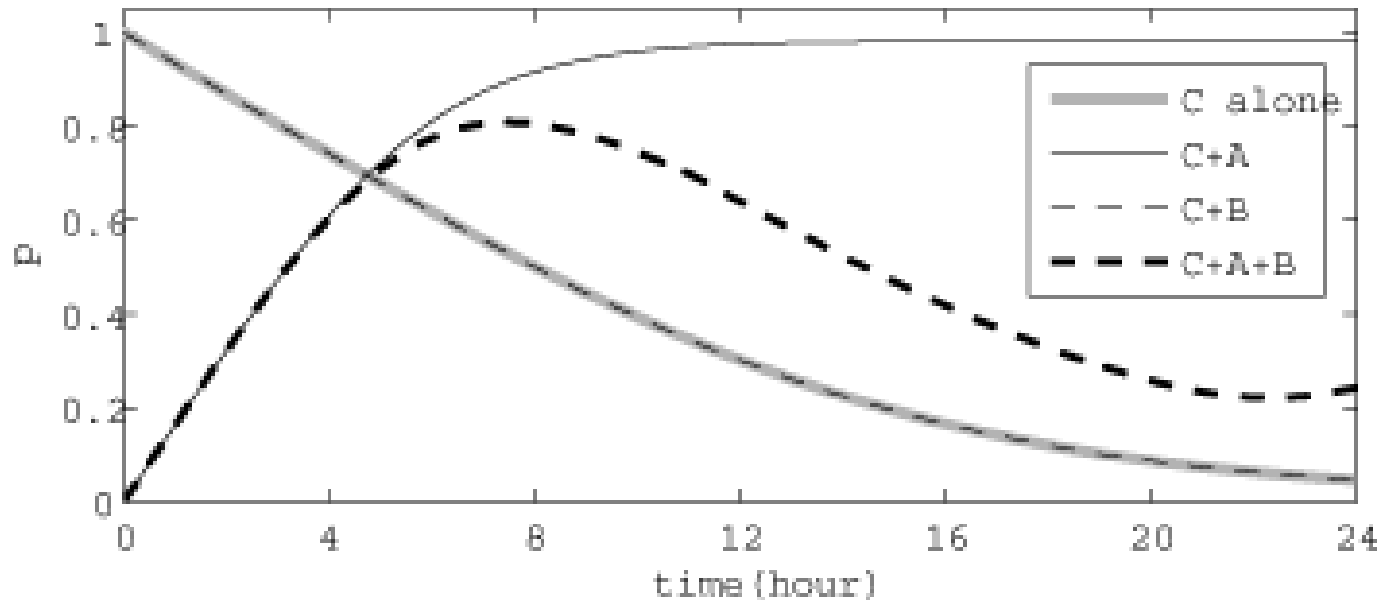


Tomita *et al.*, *Science* 2005

1 or 2 Kai proteins

# Full Model vs. Experiment

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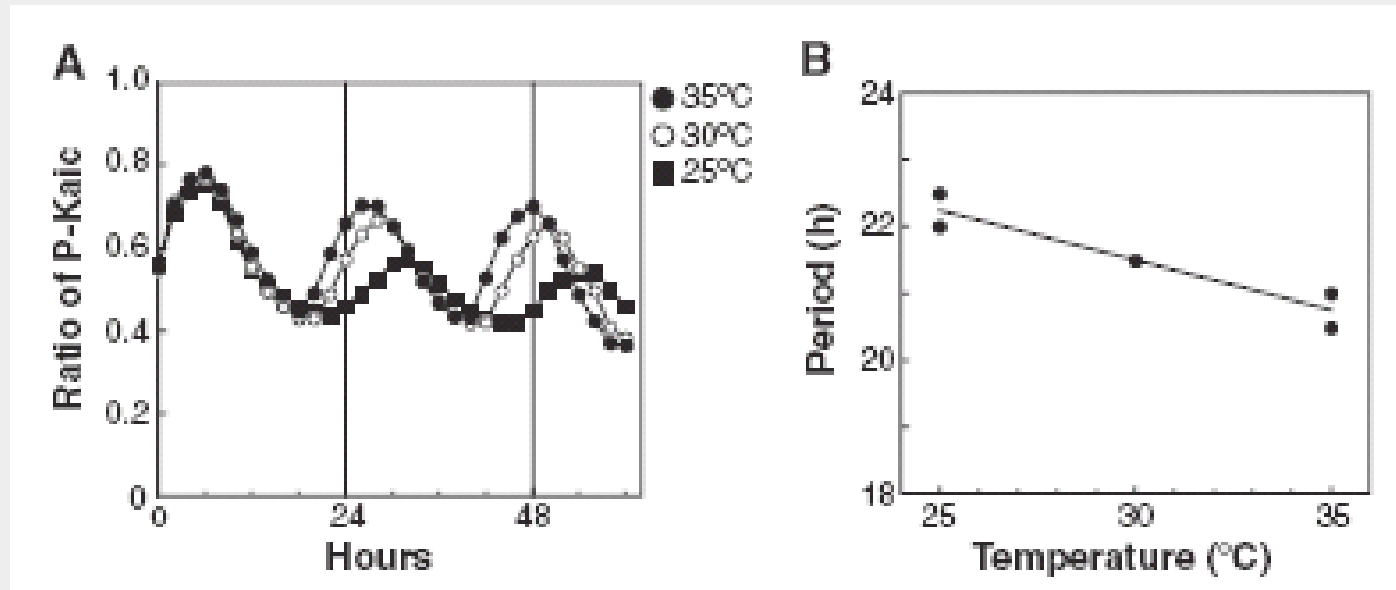


1 or 2 Kai proteins

# Temperature Compensation

- Recall

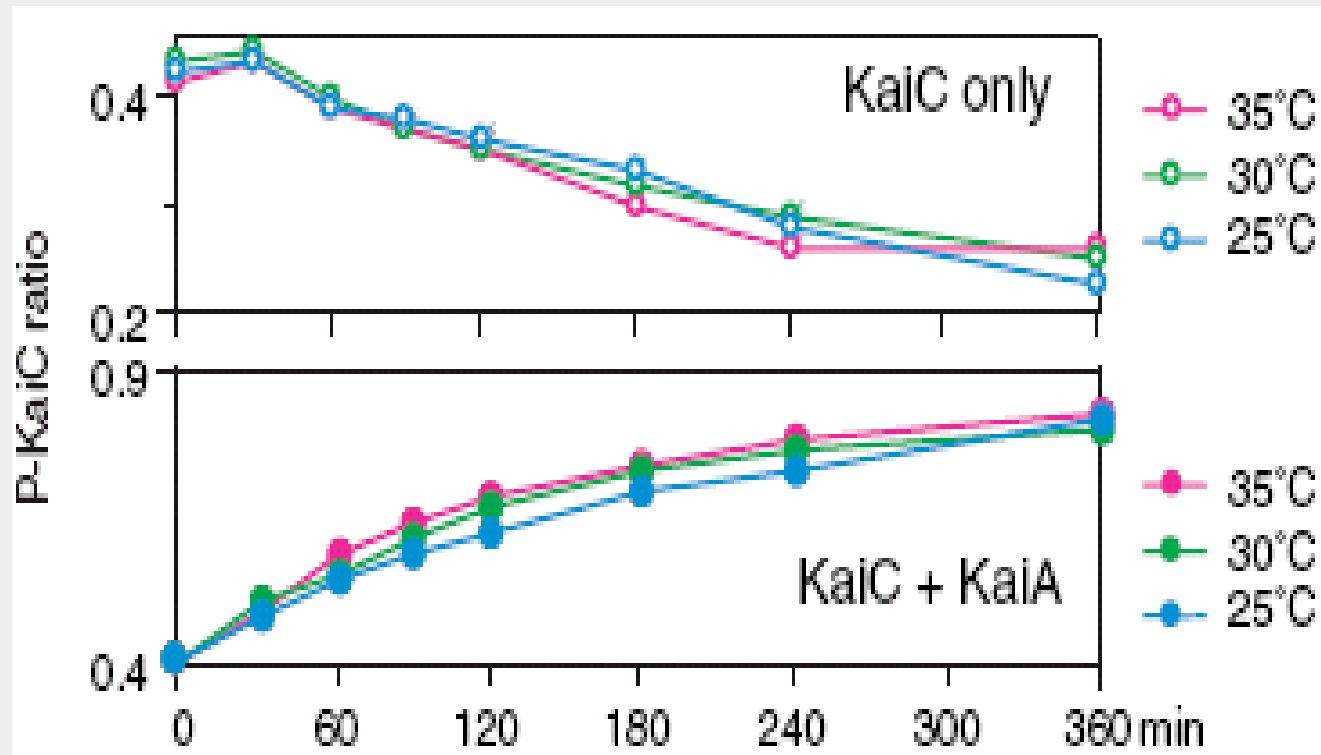
- Period insensitive to temperature.
- (De)phosphorylation rates separately insensitive to temperature.



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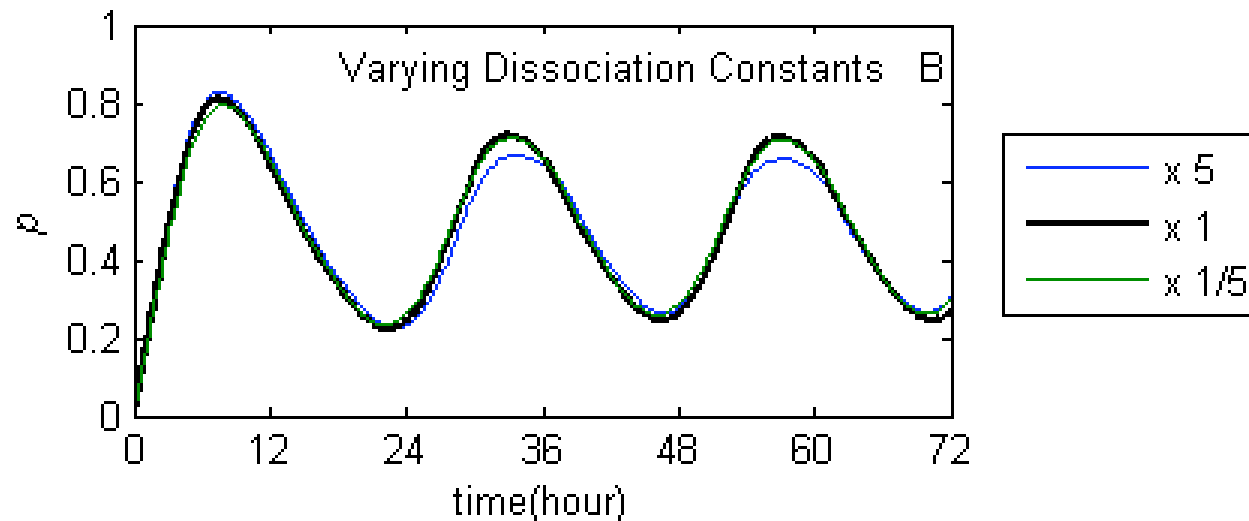
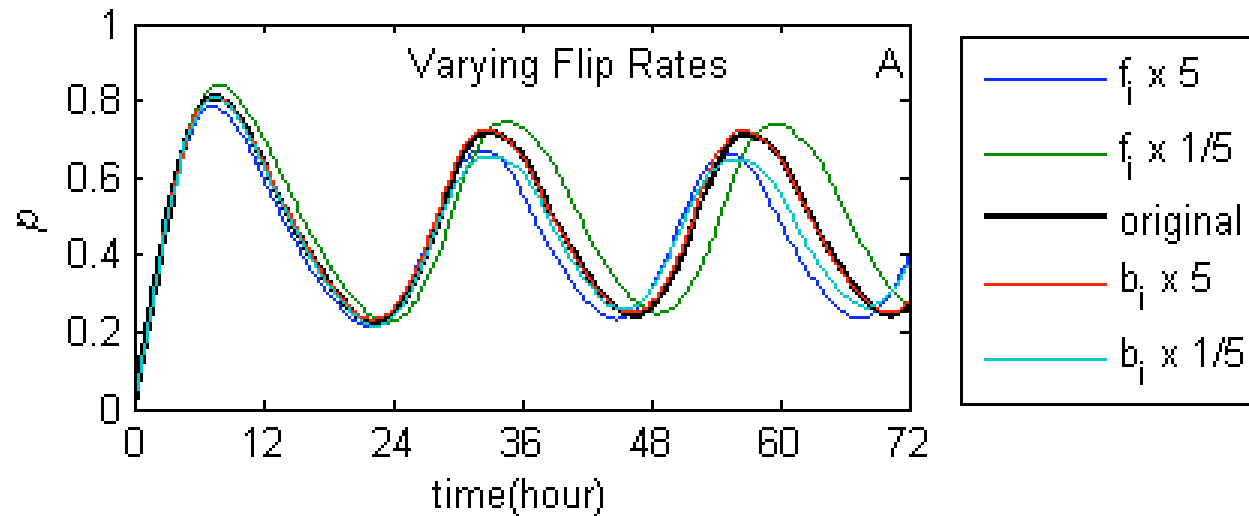
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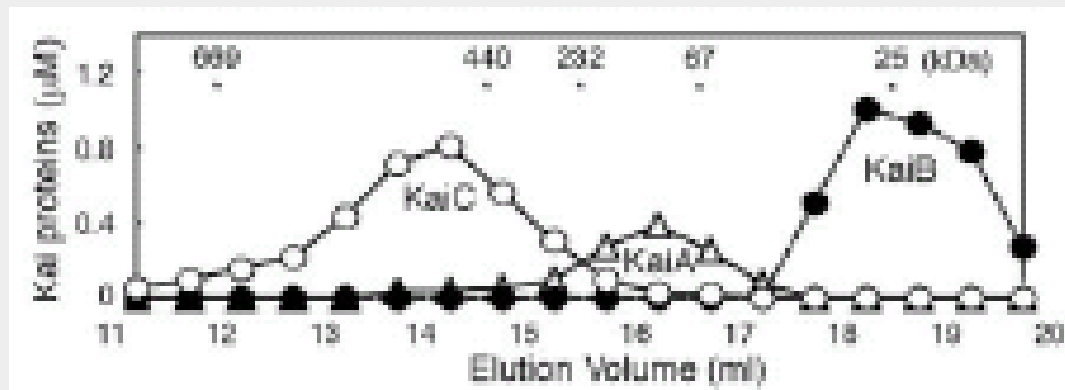
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$$K_d \ll 1 \Rightarrow \frac{[A]}{K_d + [A]} \approx 1$$

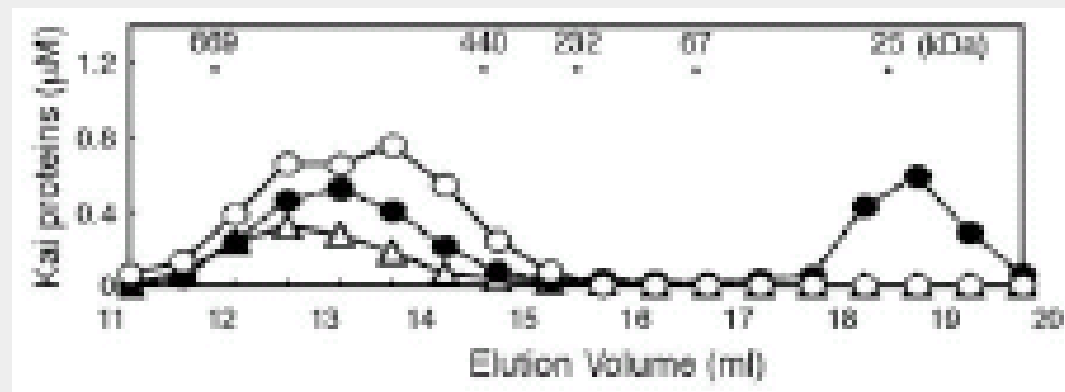
# T.C. Model



# Temperature Compensation



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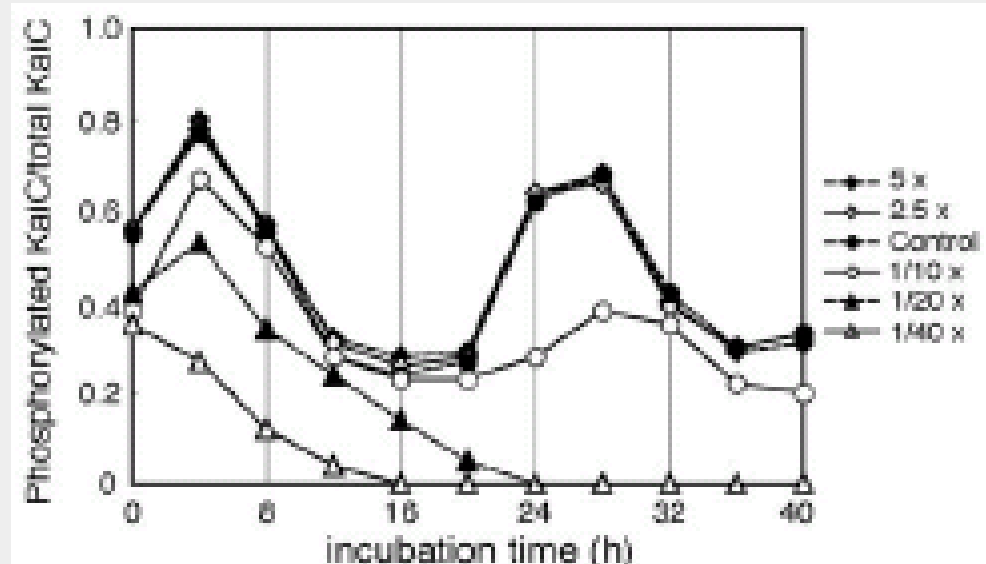
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# Temperature Compensation

- Mechanism makes predictions
  - No free KaiA
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Kageyama *et al.*,  
*Mol. Cell* 2006

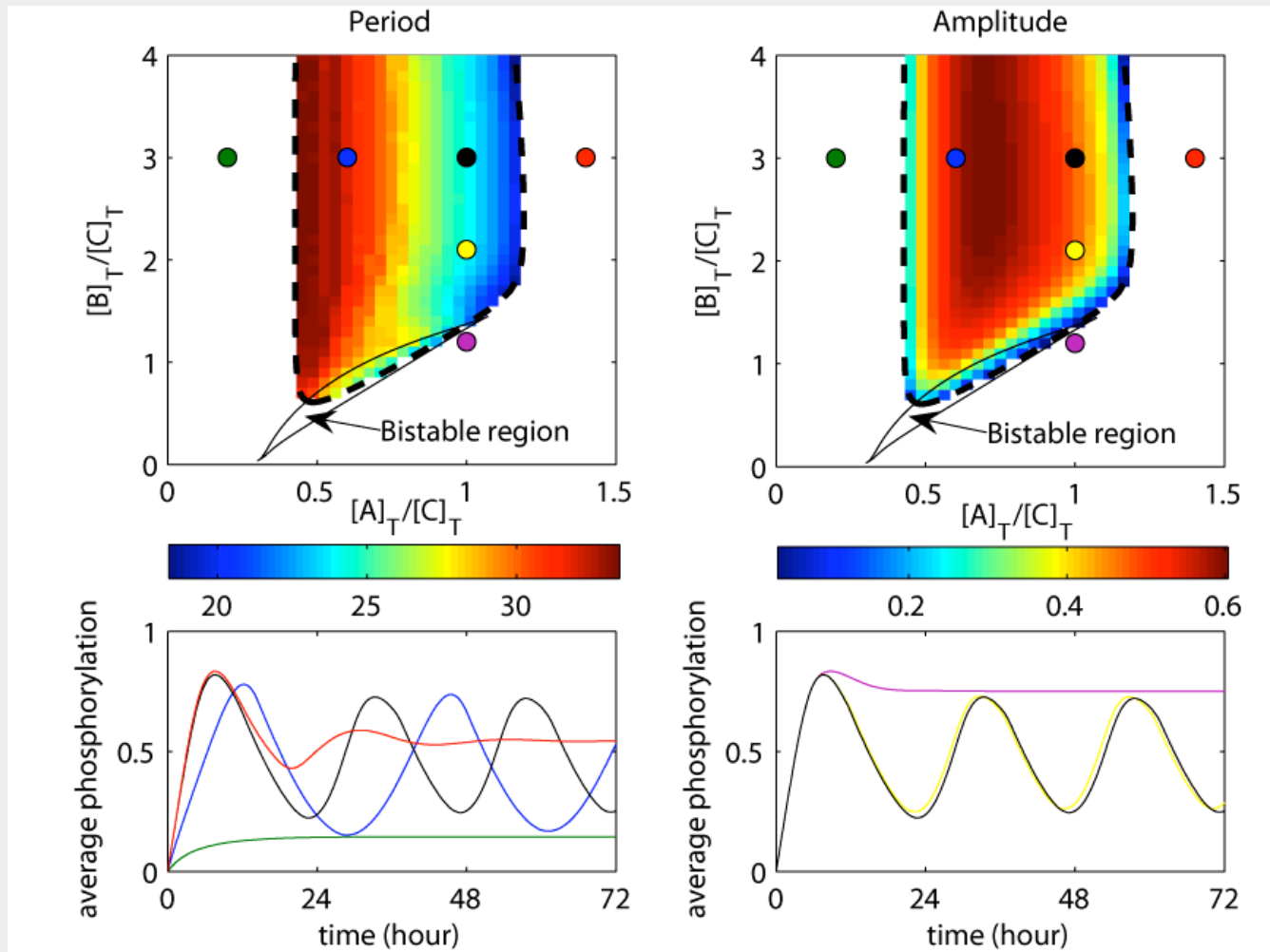


# Predictions

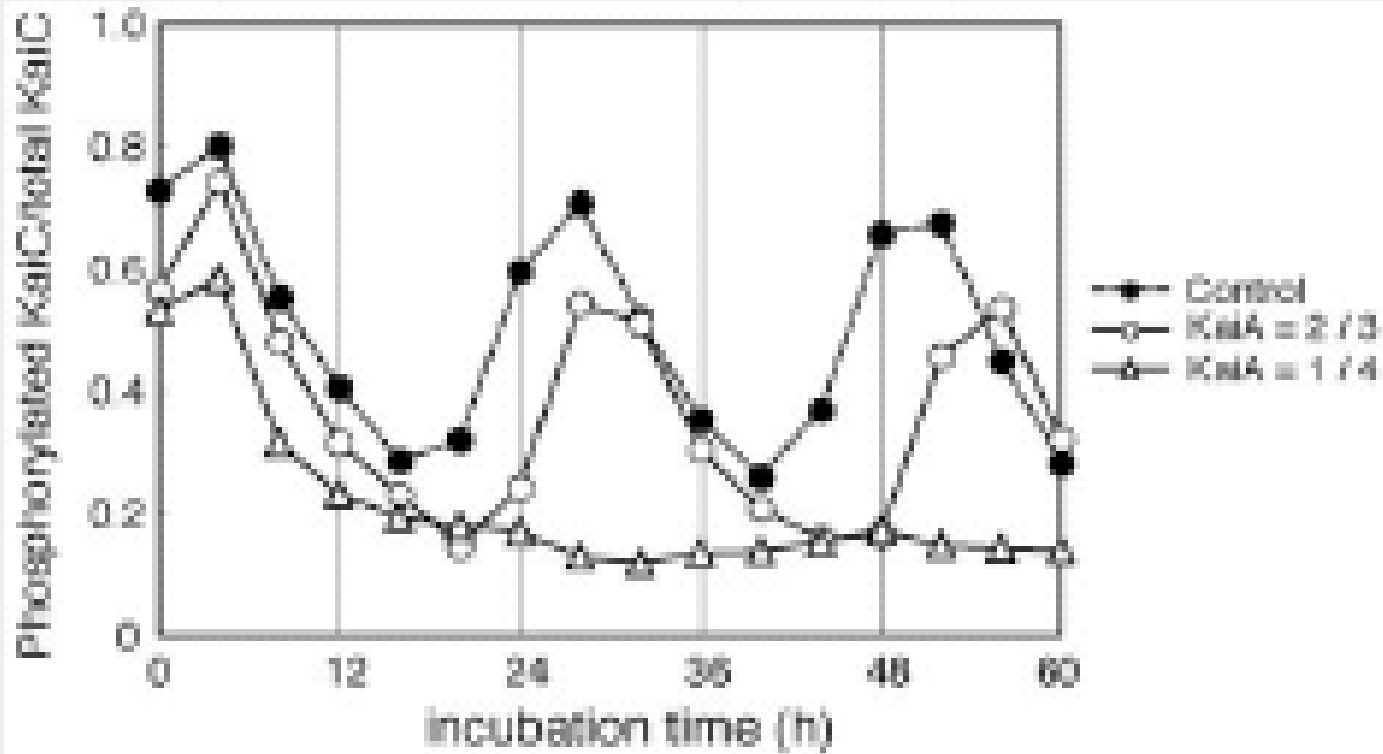
- KaiC has 2 conformations
  - Transitions between coupled to ATP hydrolysis/ phosphorylation.
  - This limits choice of rates, irreversibility,...
- Binding reactions driven to completion.

# Predictions

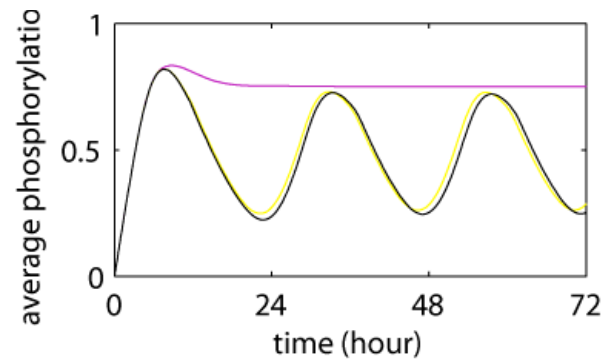
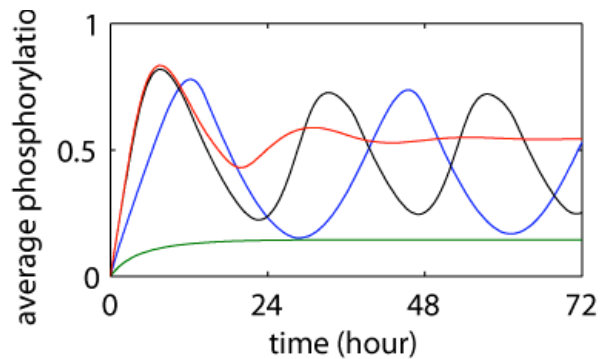
- Increasing  $[\text{KaiB}]$  leaves oscillations unaffected.
- Increasing  $[\text{KaiA}]$  destroys oscillations.



# Predictions



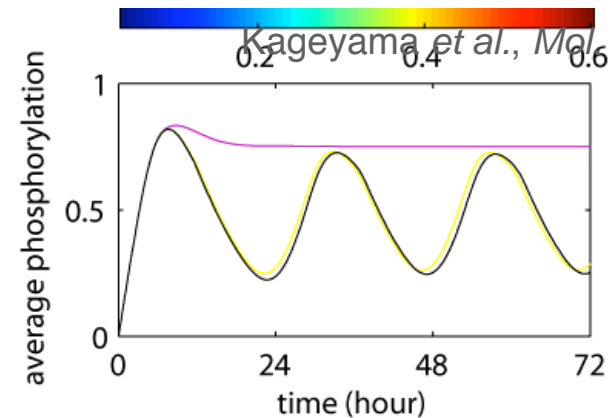
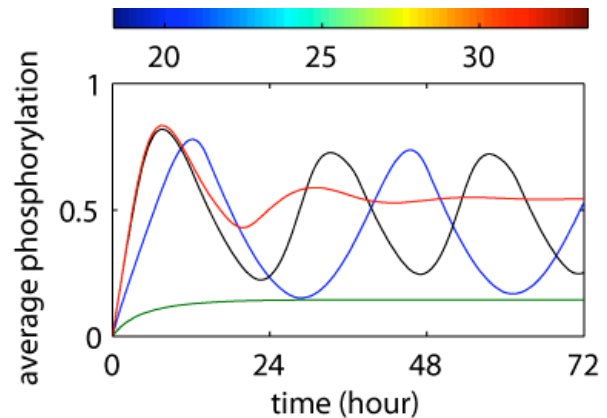
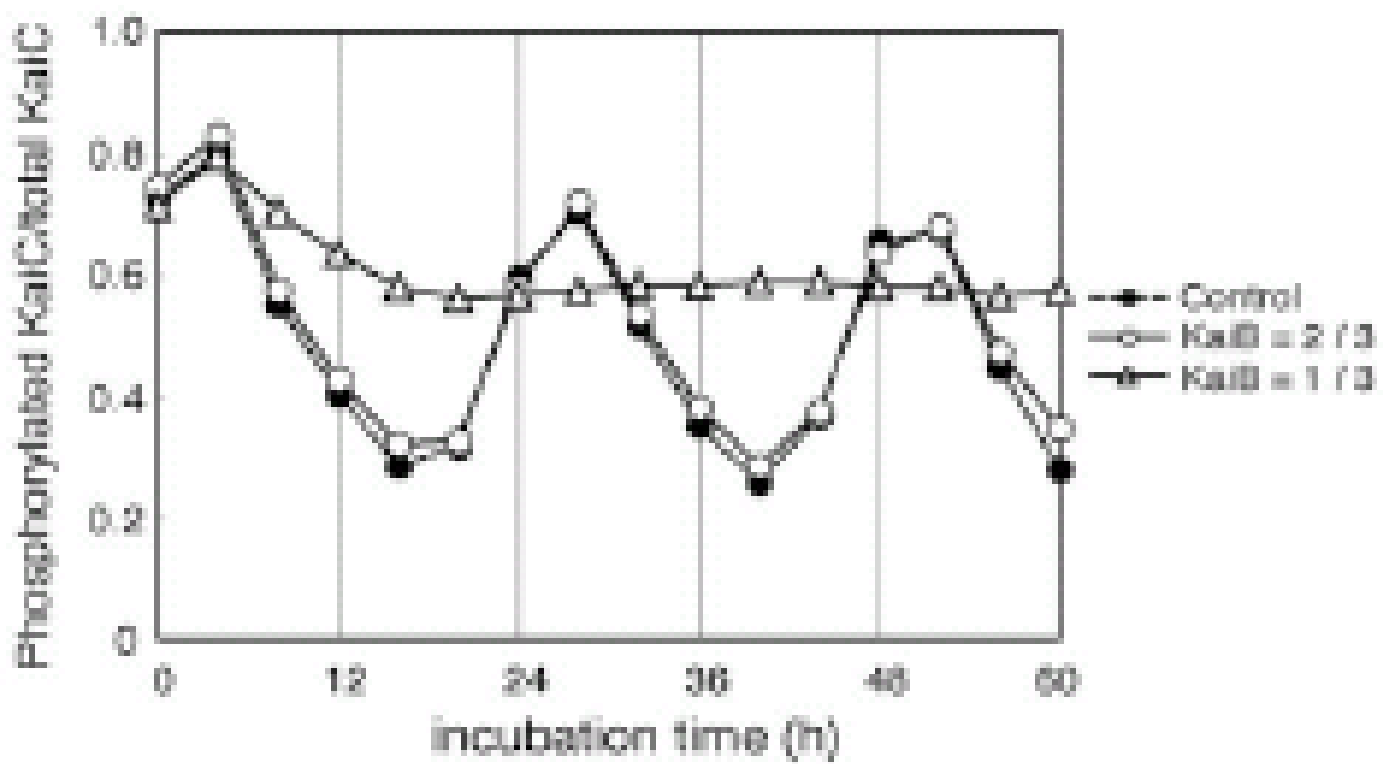
Kageyama *et al.*, *Mol. Cell* 2006



# Predictions

- Inc
- Inc

d.



Kageyama et al., Mol Cell 2006

# Summary

- Challenge: Mechanism for “minimal” protein oscillator.
  - Only 3 purified proteins.
  - Only 1 reaction cycle driven out of equilibrium.
- Proposal: Synchronization of molecular cycles via **differential affinity**.
- Predict: Increasing [KaiA], but not [KaiB], destroys oscillations.
- Outlook.
  - *In vivo*? Lower [KaiA], transcriptional feedback.
  - Evolution: No KaiA in ~50% of cyanobacteria!