Order and flow in active filament solutions

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Delaware 30/03/2006

Acknowledgements

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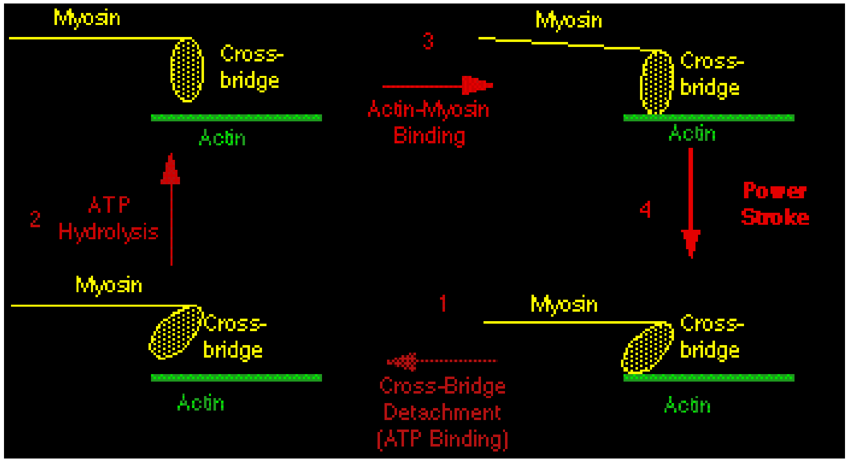
Plan

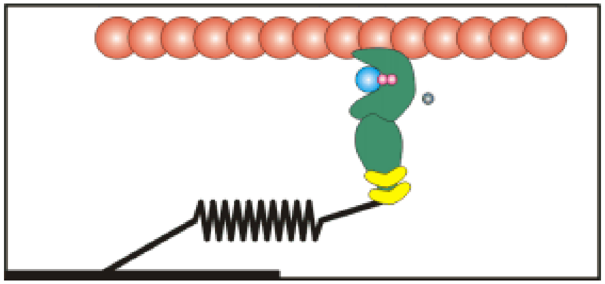
1) Cell Movement and Mechanics
   a) The Cell Cytoskeleton
   b) Filaments and Molecular Motors
2) Dynamics of filament/motor mixtures
3) Self-organisation of homogeneous bulk phases
4) Inhomogeneities and hydrodynamic description
5) Rheology and flow
6) Conclusions and outlook

Acto-myosin molecular motor

- F-actin and Myosin use Adenosine TriPhosphate (ATP) to turn chemical energy into mechanical work



Rigor



muscle contraction

Cell Movement

- A transition from stationary to translation associated with changes in distribution of actin and myosin

QuickTime™ and a
Cinepak Codec by Radius decompressor
are needed to see this picture.

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Cinepak Codec by Radius decompressor
are needed to see this picture.

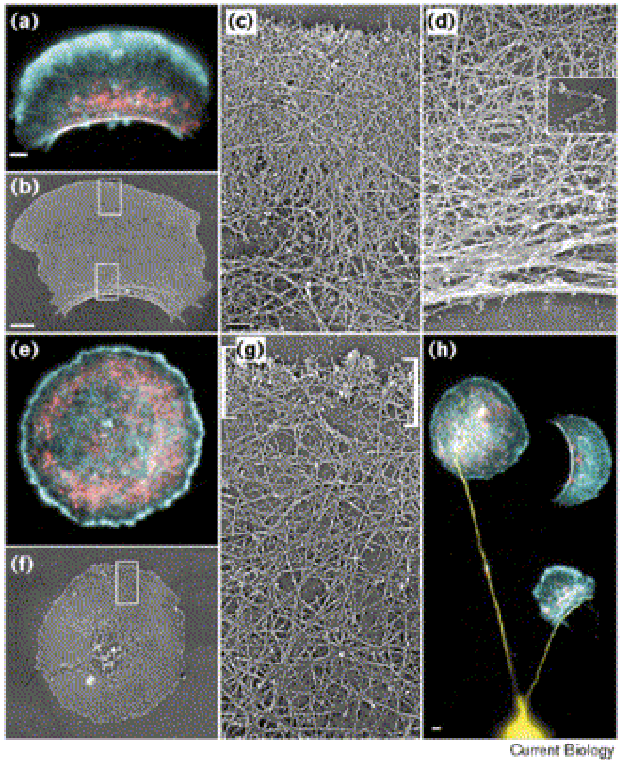
- Dynamics of myosin II concentrations in stationary cell

- Myosin

Borisy Lab http://www.borisylab.nwu.edu/pages/movies.html

Cell Movement

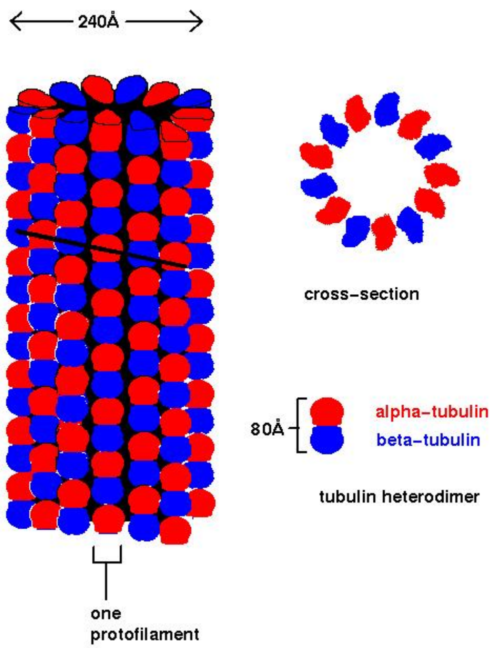
- Actin (cyan)
- Myosin (red)



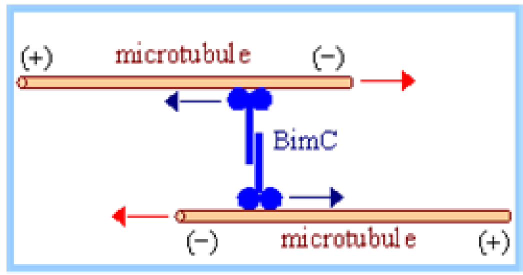
Verkhovsky,Svitkina,Borisy, *Current Biology* **9**, 11-20 (1999)

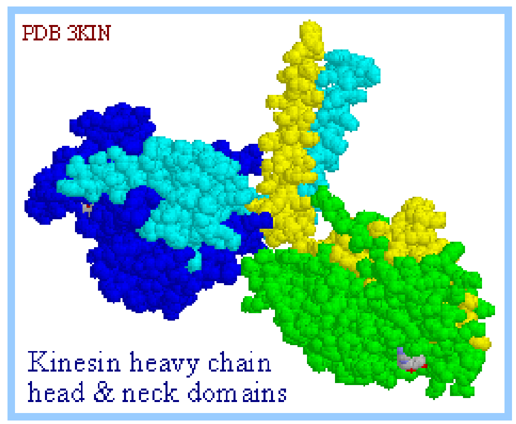
Kinesin-Tubulin molecular motor

- Another ATP-driven active filament/motor pair
- Microtubules and Kinesin turn chemical energy into mechanical work



Structure of a singlet microtubule, indicating slant



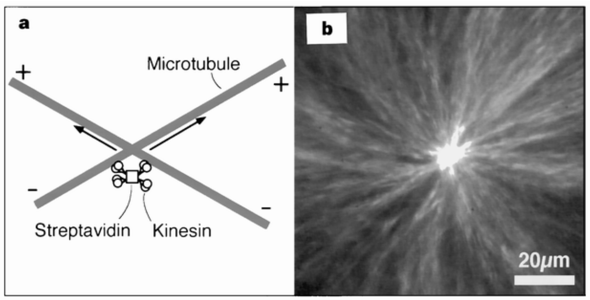


http://www.dentistry.leeds.ac.uk/biochem/MBWeb/mb2/part1/kinesin.htm

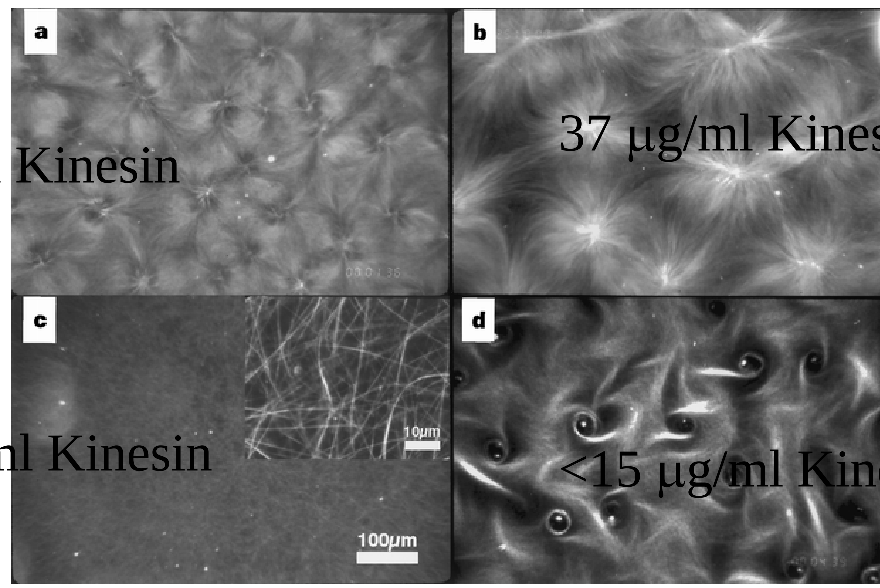
Model Systems

- Microtubles and Kinesin/Ncd show self-organisation on mesoscopic lengthscales. In-vitro experiments on simplified ‘cell’ extracts of filaments and motors
- Motor constructs & Taxol-stablilised microtubule mixture
- ACTIVE temporary cross-links

Nédélec et al, Nature **389**, 305 (1997)
Surrey et al, Science **292**, 1167 (2001)



•Simulations got similar patterns



25 μg/ml Kinesin

37 μg/ml Kinesin

50 μg/ml Kinesin

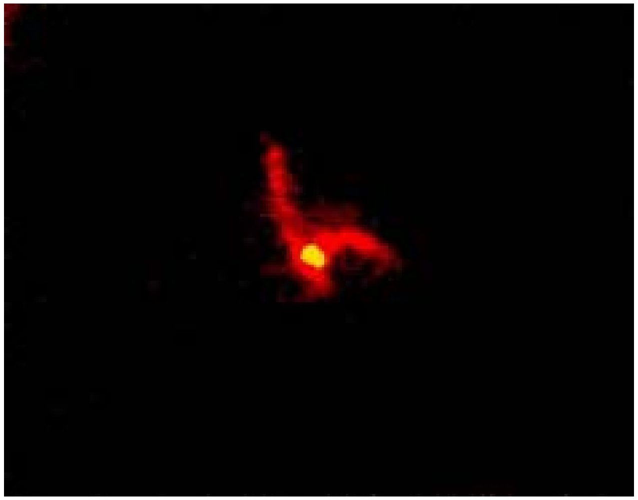
<15 μg/ml Kinesin

QuickTime™ and a
H.263 decompressor
are needed to see this picture.

http://www.embl-heidelberg.de/ExternalInfo/**nedelec**/

Model Systems

- In-vitro experiments on simplified ‘cell’ extracts of filaments and motors
- Myosin II & length-stablilised F-Actin mixture (J. Käs group)
- Myosin tail is hydrophobic and forms proto-thick filaments or ‘micelles’ with heads on the surface (N_c ~ 10)
- ACTIVE temporary cross-links



Humphrey et al,
Nature **416**, 413 (2002)

Questions ?

- How does the effects of the motor affect the states of a solution of filaments.
- What is the phase behaviour of a motor/filament mixture
- What are the changes to the mechanical properties (viscoelasticity) of a isotropic solution of filaments (e.g F-actin)due to the presence of motors (e.g. Myosin II) and ATP
- Can we predict (from a microscopic model) what happens when we add the motors?
- What role does the flexibility of the filaments play?

Describing Active Filaments

"Microscopic" simulations

Explicit filaments - expensive computation

Very successful at models for particular structure e.g. mitotic spindle.

Nedelec et al (2004)

↓

Mean-field (effective theory)

"Hydrodynamic" equations

Polymer Physics - Doi model

Dynamics of order parameter fields.

Huge number of terms allowed on symmetry grounds.

Non-equilibrium so there is no 'variational' principle.

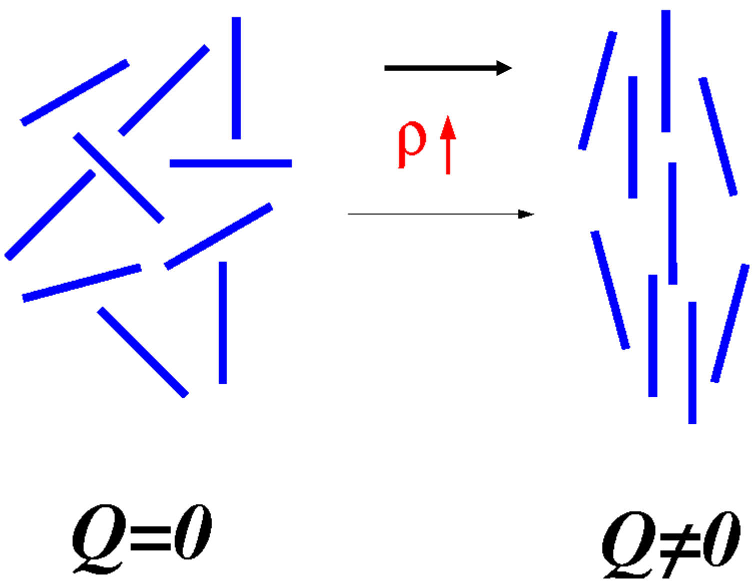
Lee & Kardar (2001), Simha & Ramaswamy (2002), Kruse et al (2004), Sankararaman, Menon and Kumar (2003), Hatalwane et al (2004).

Equations give pattern formation - asters, vortices…

How does one parametrise them ? …

Lyotropic Liquid Crystals

Recall the isotropic (I) to nematic (N) transition of a solution of thin *(l >> b)* hard rods.
Onsager - I-N transition upon increasing density.



Orientation Distribution function $\Psi(\hat{n})$

$$\overleftrightarrow{Q} = \int d\hat{n}\left(\hat{n}\hat{n} - \tfrac{1}{d}\overleftrightarrow{\delta}\right)\Psi(\hat{n})$$

$$\rho_{IN} \approx \frac{4}{\ell^2 b}$$

Competition between rotational and translational entropy.

Active Polar Rods

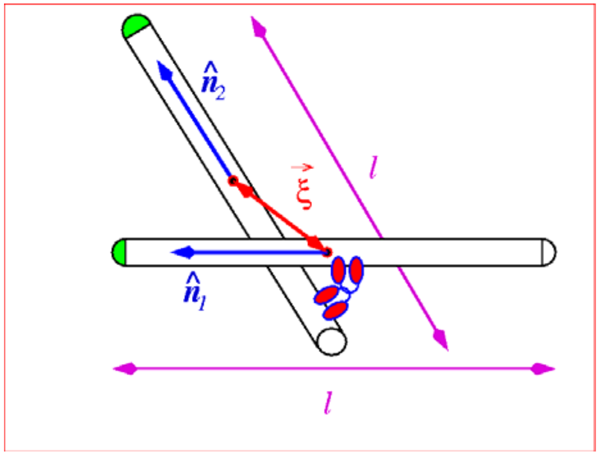
- Take into account rod excluded volume
- Approximately take into account confining effect of other chains , ‘entanglement’
- Ignore semi-flexibility (valid for $l << L_p$ and for timescales longer than those required to relax internal modes)
- Rigid rods of length l and diameter $b << l$.
- Position, $\boldsymbol{r}$. Orientation $\hat{\boldsymbol{n}}$
- Filament Probability distribution $\Psi(\boldsymbol{r},\hat{\boldsymbol{n}},t)$ $\Rightarrow$ continuum model

(1) Extension of the Doi model of rods in solution Doi and Edwards (1986)

(2) Motor induced velocities like Kruse/Julicher sliding filaments

Kruse/Julicher (2000), Kruse et al (2001)

$d=2,3$



TBL, MC Marchetti (2003)

Evolution of Rod distribution

Conservation of Probability $\Psi(\boldsymbol{r},\hat{\boldsymbol{n}},t)$ Distribution function

$$\partial_t \Psi + \nabla \bullet \boldsymbol{J} + \boldsymbol{R} \bullet \boldsymbol{J} = 0 \qquad \boldsymbol{R} = \hat{\boldsymbol{n}} \times \partial_{\hat{\boldsymbol{n}}}$$

$$J_i = -D_{ij}\partial_j \Psi - \frac{D_{ij}}{T} \Psi \, \partial_j V_{ex} + J_i^{act}$$

$$\boldsymbol{J}_i = -D_r R_i \Psi - \frac{D_r}{T} \Psi R_j V_{ex} + \boldsymbol{J}_i^{act}$$

$$D_{ij} = D_\perp \left(\delta_{ij} - \hat{n}_i \hat{n}_j\right) + D_\parallel \hat{n}_i \hat{n}_j$$

Active Currents

Potential
"excluded volume"
$V_{ex}(\boldsymbol{r},\hat{\boldsymbol{n}},t)$

Diffusion of hard rods (excl. vol., entangle.) and local driving from motors

TBL,MCMarchetti, PRL 90, 138102 (2003)

Active Interactions

Active translation currents

$$\boldsymbol{J}^{act}(\boldsymbol{r},\hat{\boldsymbol{n}}_1) = \Psi(\boldsymbol{r},\hat{\boldsymbol{n}}_1)\boldsymbol{v}_{\boldsymbol{act}}(\boldsymbol{r},\hat{\boldsymbol{n}}_1)$$

$$\boldsymbol{v}_{\boldsymbol{act}}(\boldsymbol{r},\hat{\boldsymbol{n}}_1) = \int_{n_2}\int_{\Omega_{\text{int}}} d^d\xi\, \boldsymbol{v}(\xi,\hat{\boldsymbol{n}}_1,\hat{\boldsymbol{n}}_2)\Psi(\boldsymbol{r}+\xi,\hat{\boldsymbol{n}}_2)$$

Active rotation currents

$$\boldsymbol{J}^{r/act}(\boldsymbol{r},\hat{\boldsymbol{n}}_1) = \Psi(\boldsymbol{r},\hat{\boldsymbol{n}}_1)\boldsymbol{\omega}_{act}(\boldsymbol{r},\hat{\boldsymbol{n}}_1)$$

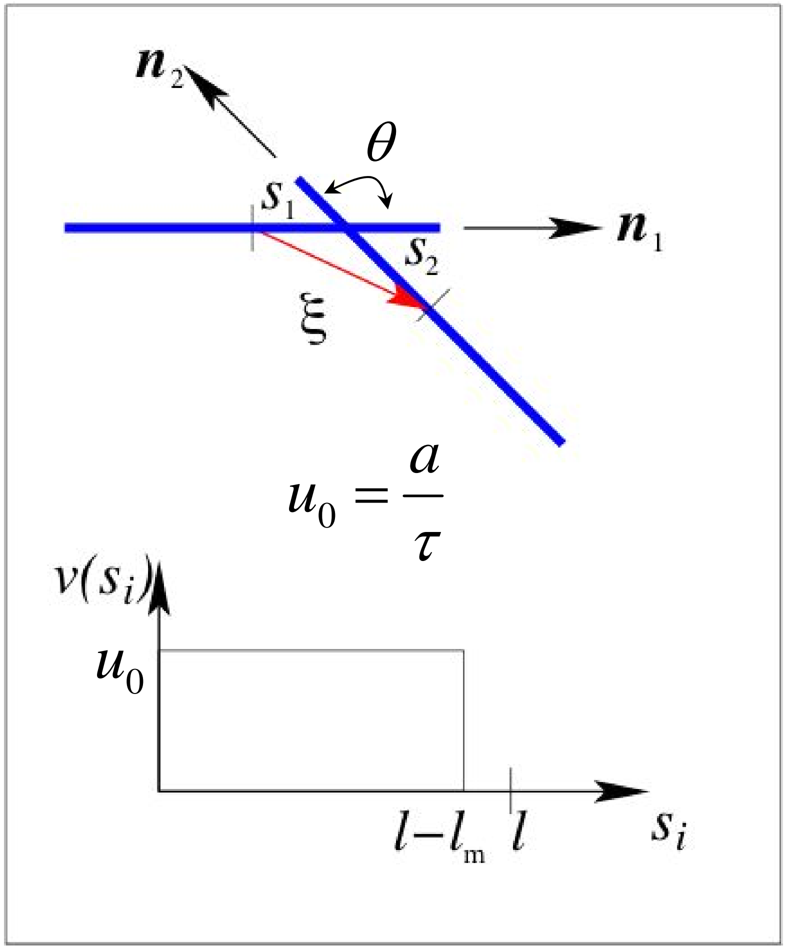
$$\boldsymbol{\omega}_{act}(\boldsymbol{r},\hat{\boldsymbol{n}}_1) = \int_{n_2}\int_{\Omega_{\text{int}}} d^d\xi\, \omega(\xi,\hat{\boldsymbol{n}}_1,\hat{\boldsymbol{n}}_2)\Psi(\boldsymbol{r}+\xi,\hat{\boldsymbol{n}}_2)$$

Model for motor induced active 'velocities'

$$\boldsymbol{v}(\xi,\hat{\boldsymbol{n}}_1,\hat{\boldsymbol{n}}_2),\ \omega(\xi,\hat{\boldsymbol{n}}_1,\hat{\boldsymbol{n}}_2)$$

Microscopic Model for velocities

- How does the motor-induced velocity depend on the separation of centres and relative orientations?
- Motors walk with inhomogenous velocity profile as a function of s_i the distance (along filament) active cross-link is from filament centre ↓ Could be that motors stall at the end / Crowding ?
- Motor velocity profile is a step function which goes to zero at some distance l_m from the end.



TBL, MC Marchetti, Europhysics Letters (2005)

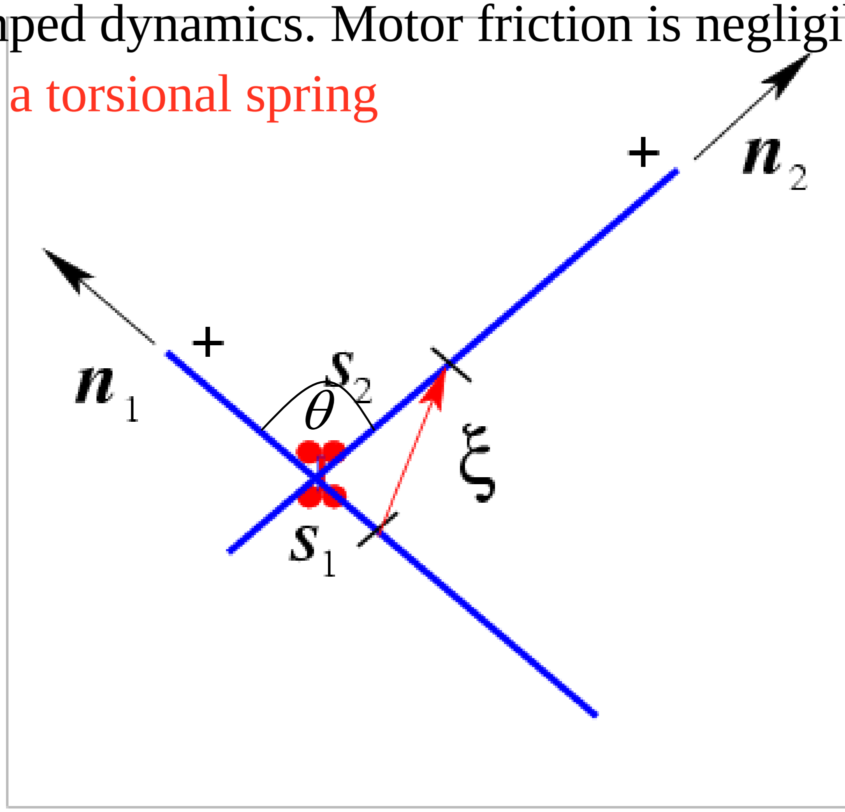
Microscopic Model

- Start with rigid motor clusters. Dimer made up of two heads walking along each filament at speed *u(s)* (separation of rod centres)
- Cross-link position $\boldsymbol{r}_1^{\times} = \boldsymbol{r}_2^{\times}$

$$\boldsymbol{r}_2 - \boldsymbol{r}_1 = \vec{\xi} = s_1\hat{\boldsymbol{n}}_1 - s_2\hat{\boldsymbol{n}}_2$$

$$\boldsymbol{r}_a^{\times} = \boldsymbol{r}_a + \hat{\boldsymbol{n}}_a s_a \,; \quad -l/2 < s_a < l/2 \,; \quad a \in \{1,2\}$$

- Over-damped dynamics. Motor friction is negligible.
- Cluster is a torsional spring



Microscopic Model

- Anisotropic friction tensor of rods.

$$\zeta(\hat{\boldsymbol{n}}_a) = \zeta_\perp(\delta - \hat{\boldsymbol{n}}_a\hat{\boldsymbol{n}}_a) + \zeta_\parallel \hat{\boldsymbol{n}}_a\hat{\boldsymbol{n}}_a \; ; a \in \{1,2\}$$

- Velocity of filament a , $\boldsymbol{v}_a(s_1, \hat{\boldsymbol{n}}_1, s_2, \hat{\boldsymbol{n}}_2)$
- Angular velocity of filament a , $\boldsymbol{\omega}_a(s_1, \hat{\boldsymbol{n}}_1, s_2, \hat{\boldsymbol{n}}_2)$
- No external forces - any force/torque applied on rod 1 by an active cross-link is balanced by equal and opposite force on rod 2.

$$\zeta(\hat{\boldsymbol{n}}_1) \bullet \boldsymbol{v}_1 + \zeta(\hat{\boldsymbol{n}}_2) \bullet \boldsymbol{v}_2 = 0$$

$$\zeta_{\mathrm{r}}\boldsymbol{\omega}_1 + \zeta_{\mathrm{r}}\boldsymbol{\omega}_2 = 0$$

Viscous force/torque balance

- Torsional spring

$$\dot{\theta} = \frac{\kappa}{\zeta_{\mathrm{r}}}\theta \approx \frac{\kappa}{\zeta_{\mathrm{r}}}\sin\theta = \gamma\sin\theta$$

TBL, MC Marchetti, Europhys. Lett. (2005)

Microscopic Model

- Velocity of the motor head attached to filament

$$\boldsymbol{v}_a^m = \frac{d\boldsymbol{r}_a^{\times}}{dt} = \boldsymbol{v}_a + \hat{\boldsymbol{n}}_a u(s_a) + s_a \boldsymbol{\omega}_a \times \hat{\boldsymbol{n}}_a \; ; \quad a \in \{1,2\}$$

- Angular velocity of motor cluster

$$\boldsymbol{\omega}_a^m = \boldsymbol{\omega}_a + (-1)^a \dot{\theta} \frac{(\hat{\boldsymbol{n}}_1 \times \hat{\boldsymbol{n}}_2)}{|\hat{\boldsymbol{n}}_1 \times \hat{\boldsymbol{n}}_2|} (\hat{\boldsymbol{n}}_1 \bullet \hat{\boldsymbol{n}}_2); \quad a \in \{1,2\}$$

$$\dot{\theta} = \frac{\kappa}{\zeta_r}\theta \approx \frac{\kappa}{\zeta_r}\sin\theta = \gamma \sin\theta$$

- Motors in cluster rigidly attached to each other

$$\boldsymbol{v}_1^m = \boldsymbol{v}_2^m \qquad \boldsymbol{\omega}_1^m = \boldsymbol{\omega}_2^m$$

- Solve for the 'velocities' of the filaments

$$\boldsymbol{\omega}_1 = -\boldsymbol{\omega}_2 = \gamma(\hat{\boldsymbol{n}}_1 \times \hat{\boldsymbol{n}}_2)(\hat{\boldsymbol{n}}_1 \bullet \hat{\boldsymbol{n}}_2) \; ; \quad \boldsymbol{v}_1 = \boldsymbol{v}/2 + \boldsymbol{V} \; ; \quad \boldsymbol{v}_2 = -\boldsymbol{v}/2 + \boldsymbol{V}$$

$$\boldsymbol{v} = (\alpha_0 + \alpha_1(\hat{\boldsymbol{n}}_1 \bullet \hat{\boldsymbol{n}}_2))\frac{\vec{\xi}}{\ell} + (\beta_0 + \beta_1(\hat{\boldsymbol{n}}_1 \bullet \hat{\boldsymbol{n}}_2))(\hat{\boldsymbol{n}}_1 - \hat{\boldsymbol{n}}_2)$$

$$\boldsymbol{V} = (\lambda_0 + \lambda_1(\hat{\boldsymbol{n}}_1 \bullet \hat{\boldsymbol{n}}_2))(\hat{\boldsymbol{n}}_1 + \hat{\boldsymbol{n}}_2)$$

$$\lambda_{0,1} \approx \beta_{0,1} \approx v_0 \quad ; \quad \alpha_{0,1} \approx \frac{\ell_m}{\ell} v_0$$

TBL, MC Marchetti, Europhysics Letters (2005)

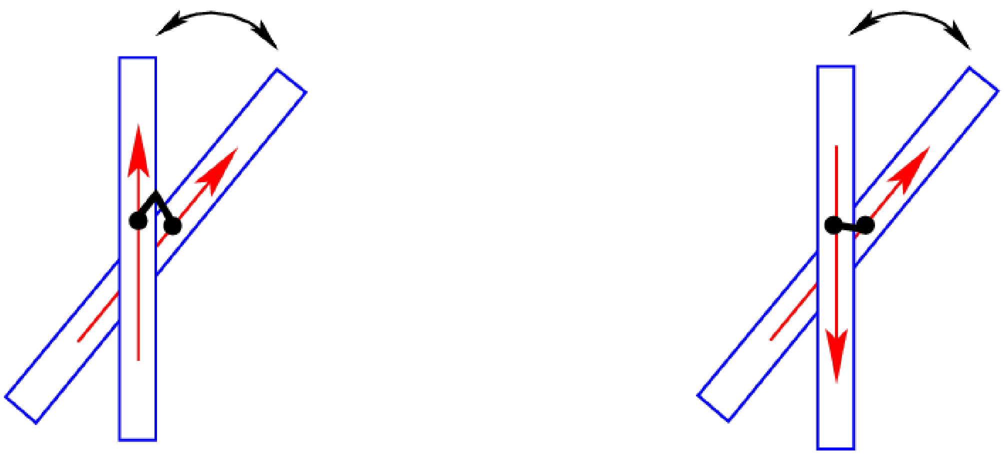
Polar Clusters

Does the spring differentiate between parallel and antiparallel configurations

$$\omega_1(\xi, \hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2) = -\omega_2 = (\gamma_0 + \gamma_1(\hat{\boldsymbol{n}}_1 \bullet \hat{\boldsymbol{n}}_2))(\hat{\boldsymbol{n}}_1 \times \hat{\boldsymbol{n}}_2)$$

$$g = \frac{\gamma_0}{\gamma_1}$$

Cluster 'polarity'



(a) (b)

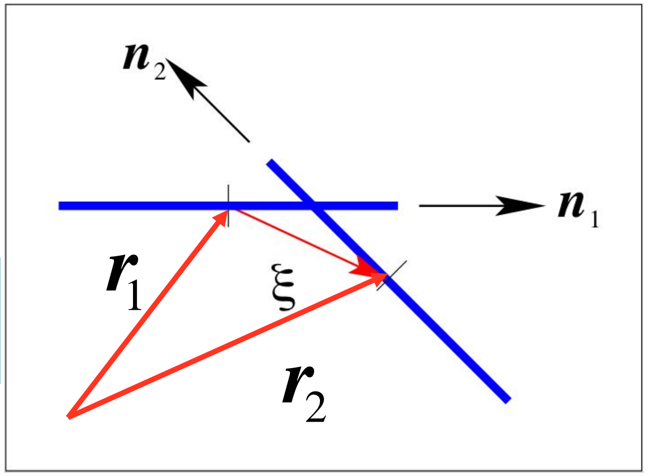
A Ahmadi, TBL, MC Marchetti, PRE **72** 060901(R) (2005)

Excluded Volume Interactions

Filaments separated by a distance ξ

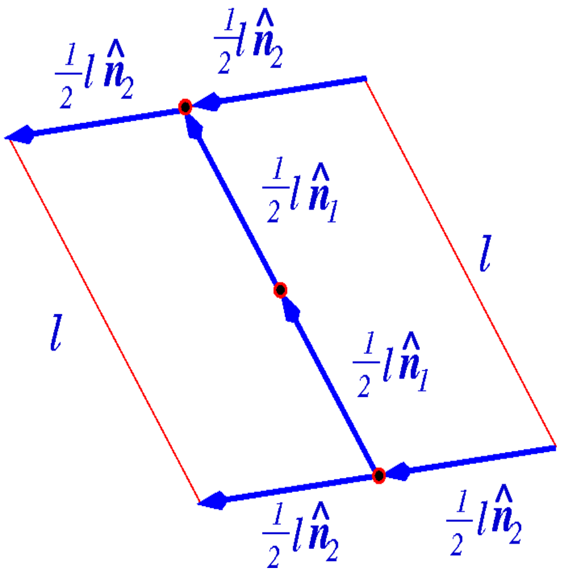
Filaments interact only if ξ is in interaction volume

$$\Omega_{\text{int}} = v_0 \left| \hat{\boldsymbol{n}}_1 \times \hat{\boldsymbol{n}}_2 \right| = v_0 \sqrt{1 - \left(\hat{\boldsymbol{n}}_1 \bullet \hat{\boldsymbol{n}}_2 \right)^2} \quad ; v_0 = l^2 b^{d-2}$$



Excluded volume interaction

$$V_{ex}\left(\boldsymbol{r}_1, \hat{\boldsymbol{n}}_1\right) = k_B T \int_{n_2} \int_{\Omega_{\text{int}}}^{\text{restrict'}} d^d r_2 \, \Psi\left(\boldsymbol{r}_2, \hat{\boldsymbol{n}}_2\right)$$
$$= k_B T \int_{n_2} \int_{\Omega_{\text{int}}} d^d \xi \, \Psi\left(\boldsymbol{r}_1 + \xi, \hat{\boldsymbol{n}}_2\right)$$



Hydrodynamic description

Active currents

$$\boldsymbol{J}^{act}(\boldsymbol{r},\hat{\boldsymbol{n}}_1) \propto \Psi(\boldsymbol{r},\hat{\boldsymbol{n}}_1)\int_{n_2}\int_{\Omega_{\text{int}}} d^d\xi\, \boldsymbol{v}(\xi,\hat{\boldsymbol{n}}_1,\hat{\boldsymbol{n}}_2)\Psi(\boldsymbol{r}+\xi,\hat{\boldsymbol{n}}_2)$$

Expansion of `local' distribution, integration over Ω_{int}

$$\Psi\left(\boldsymbol{r}+\vec{\xi},\hat{\boldsymbol{n}}_2\right)=\Psi(\boldsymbol{r},\hat{\boldsymbol{n}}_2)+\vec{\xi}\bullet\nabla\,\Psi(\boldsymbol{r},\hat{\boldsymbol{n}}_2)+\tfrac{1}{2}\left(\vec{\xi}\bullet\nabla\right)^2\Psi(\boldsymbol{r},\hat{\boldsymbol{n}}_2)+\cdots$$

Coordinate system for integration over Ω_{int}

$$\vec{\xi}=\xi_1\frac{(\hat{\boldsymbol{n}}_1+\hat{\boldsymbol{n}}_2)}{|\hat{\boldsymbol{n}}_1+\hat{\boldsymbol{n}}_2|}+\xi_2\frac{(\hat{\boldsymbol{n}}_1-\hat{\boldsymbol{n}}_2)}{|\hat{\boldsymbol{n}}_1-\hat{\boldsymbol{n}}_2|}\quad;\quad\int_{\Omega_{\text{int}}} d^3\xi = b\int_{\Omega_{\text{int}}} d\xi_1 d\xi_2$$

Project on to local density, local orientation (polarisation) - to obtain hydrodynamic equations ↓

$$\begin{pmatrix}\rho(\boldsymbol{r},t)\\ \boldsymbol{p}(\boldsymbol{r},t)\\ \overleftrightarrow{Q}(\boldsymbol{r},t)\end{pmatrix}=\int d\hat{\boldsymbol{n}}_1\begin{pmatrix}\boldsymbol{1}\\ \hat{\boldsymbol{n}}_1\\ \hat{\boldsymbol{n}}_1\hat{\boldsymbol{n}}_1-\frac{1}{d}\overleftrightarrow{\delta}\end{pmatrix}\Psi(\boldsymbol{r},\hat{\boldsymbol{n}}_1,t)$$

Bulk Phases

Equations for homogeneous order parameters

$$\begin{aligned}
\rho_0 &= \text{constant} \\
\partial_t p_i &= -\Big(D_r - m_0\rho_0\gamma_0\Big)p_i + \Big[\frac{8D_r}{3\pi} - m_0(2\gamma_0 - \gamma_1)\Big]\rho_0 S_{ij}p_j \ , \\
\partial_t S_{ij} &= -\Big[4D_r - \frac{8D_r\rho_0}{3\pi} - m_0\rho_0\gamma_1\Big]S_{ij} + 2m_0\rho_0\gamma_0\Big(p_ip_j - \frac{1}{2}\delta_{ij}p^2\Big) \ .
\end{aligned}$$

Look for non-equilibrium steady states

Define (dimensionless) cluster activity

$$\mu = \frac{\rho_N m_0 \gamma_1}{D_r}$$

Cluster polarity ($g=0 \Rightarrow$ non-polar)

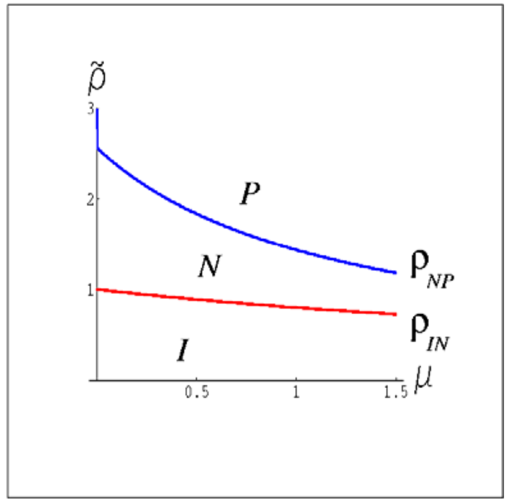
$$g = \frac{\gamma_0}{\gamma_1}$$

Bulk Phases

Phase diagram for *g<1/4*

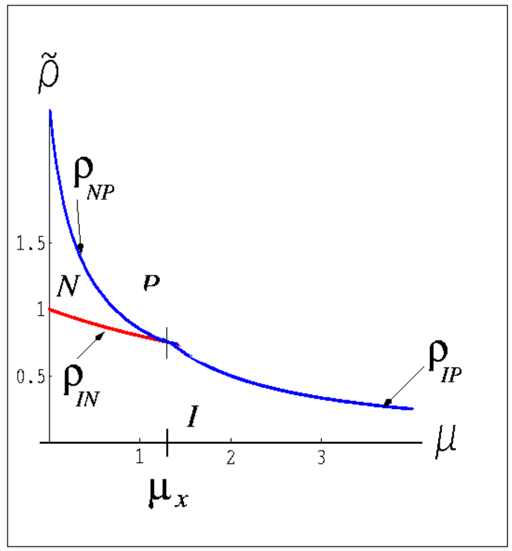
$$g = \frac{\gamma_0}{\gamma_1}$$

$$\mu = \frac{\rho_N m_0 \gamma_1}{D_r}$$



Phase diagram for *g>1/4*

Critical activity, μ_x



Hydrodynamic modes

Examine fluctuations about the homogenous states

$$\rho(r,t) = \rho_0 + \delta\rho(r,t), \quad \boldsymbol{p}(r,t) = \boldsymbol{p}_0 + \delta\boldsymbol{p}(r,t), \quad \boldsymbol{Q}(r,t) = \boldsymbol{Q}_0 + \delta\boldsymbol{Q}(r,t)$$

Long lived, long wavelength hydrodynamic modes
(conserved quantities, broken symmetry)

Isotropic phase ↓ density $\delta\rho \sim e^{z_\rho(k)t}, \quad z_\rho = -k^2\left[\frac{1}{6} - 2\mu\tilde{\alpha}\right].$

Polarized phase ↓ density, polarization director $\boldsymbol{p}(r,t) = p(r,t)\hat{\boldsymbol{n}}(r,t)$

In broken symmetry phase we choose wlg polarization director along y axis

$$\delta\mathbf{n} = \mathbf{n} - \hat{\mathbf{y}} = \hat{\mathbf{x}}\delta n_x \quad \delta\rho \sim e^{z_\rho(k)t}, \delta n_x \sim e^{z_n(k)t}$$

$$z_\rho = ikc_1\mu\tilde{\beta} - \frac{k^2}{8}\left[1 - \frac{g\mu}{6} - 20\mu\tilde{\alpha}\right],$$

$$z_n = -ikc_2\mu\tilde{\beta} - \frac{5k^2}{48}\left[1 + \frac{2}{5}\mu(g - 6\tilde{\alpha})\right],$$

$$\boldsymbol{k} = k\hat{\boldsymbol{y}}$$

In general coupled travelling density and polarization modes

Hydrodynamic modes

Long lived, long wavelength hydrodynamic modes
(conserved quantities, broken symmetry)

Nematic phase ➔ density, nematic director

$$\boldsymbol{Q}(r,t) = \boldsymbol{Q}_0 + \delta\boldsymbol{Q}(r,t) \qquad \boldsymbol{Q}(r,t) = S(r,t)\left(\hat{\boldsymbol{n}}(r,t)\hat{\boldsymbol{n}}(r,t) - \tfrac{1}{2}\delta\right)$$

In broken symmetry phase we choose w.l.g, nematic director along y axis

$$\hat{\boldsymbol{n}} = \hat{\boldsymbol{y}}$$

$$\delta\mathbf{n} = \mathbf{n} - \hat{\mathbf{y}} = \hat{\mathbf{x}}\delta n_x \quad \delta\rho \sim e^{z_\rho(k)t}, \delta n_x \sim e^{z_n(k)t}$$

$$z_\rho = -k^2\left[\frac{1}{6} - 2\mu\tilde{\alpha}\right], \quad z_n = -\frac{k^2}{8}\left[1 + \frac{19}{36}\mu\right].$$

$$\boldsymbol{k} = k\hat{\boldsymbol{y}}$$

In general coupled diffusive density and nematic director modes

Linear stability - modes

Orientation modes, density mode

$$\delta \boldsymbol{n}(r,t) = \sum_{\boldsymbol{k}} \boldsymbol{n}_k \, e^{i\boldsymbol{k}\bullet\boldsymbol{r}} \quad , \quad \delta\rho(r,t) = \sum_{\boldsymbol{k}} \rho_k \, e^{i\boldsymbol{k}\bullet\boldsymbol{r}}$$

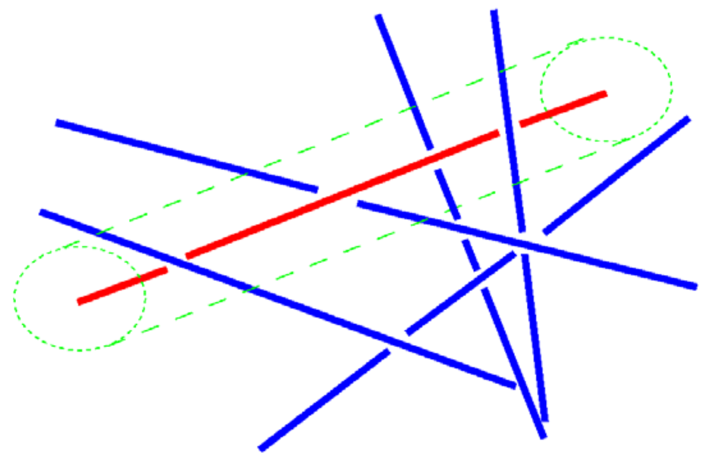
$(d-1)$ Degenerate Transverse modes $\boldsymbol{n}_k^T = \hat{\boldsymbol{k}} \times \boldsymbol{n}_k$

Longitudinal modes $\boldsymbol{n}_k^L = \hat{\boldsymbol{k}} \bullet \boldsymbol{n}_k$

Eigenvalues $\lambda_{\pm}(k)$

$$\partial_t \begin{pmatrix} \boldsymbol{n}_k^L \\ \rho_k \end{pmatrix} = \mathbf{M} \bullet \begin{pmatrix} \boldsymbol{n}_k^L \\ \rho_k \end{pmatrix}$$

$$\partial_t \, \boldsymbol{n}_k^T = \lambda^T(\boldsymbol{k})\, \boldsymbol{n}_k^T$$



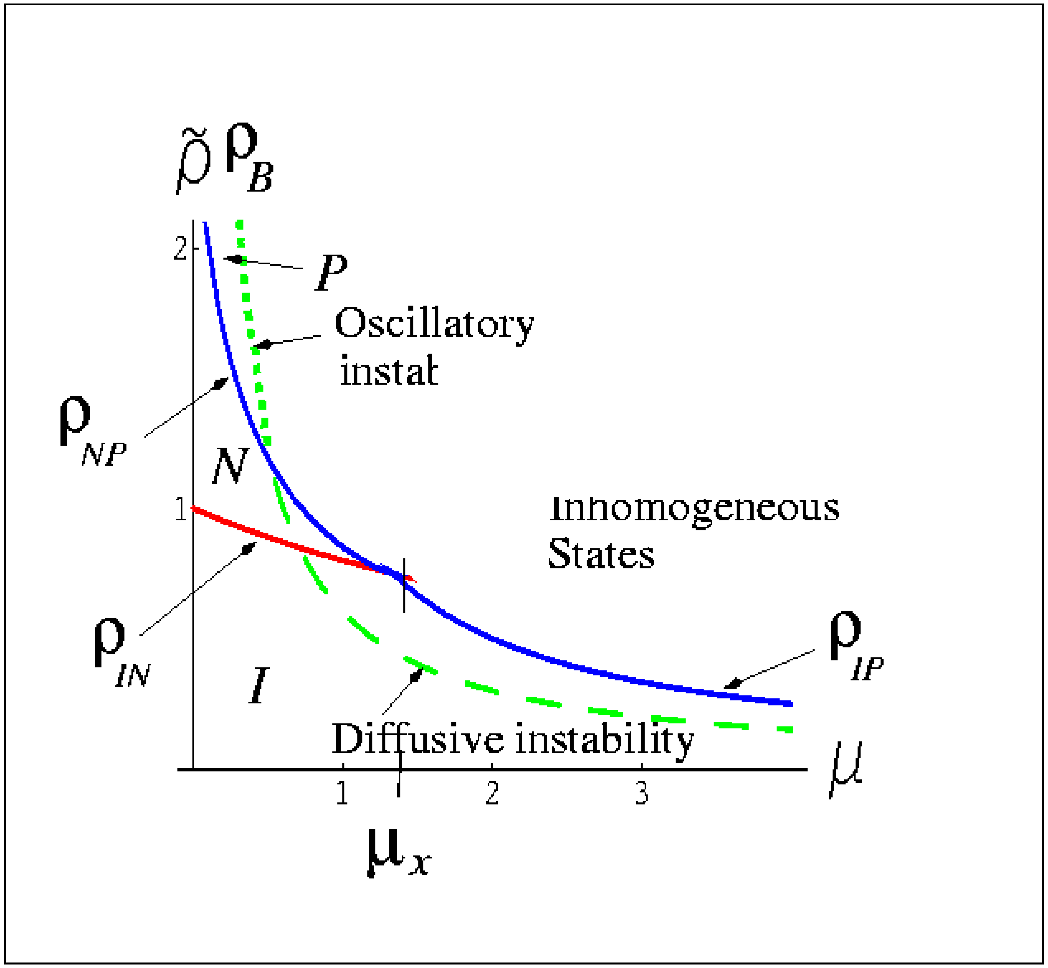
'Entanglement' (tube) estimated in diffusion constants

$$D_\perp \approx \frac{D}{\left(1+\tilde{\rho}_0\left(\frac{l}{b}\right)^{d-2}\right)^2} \; , D_r \approx \frac{6D}{l^2\left(1+\tilde{\rho}_0\left(\frac{l}{b}\right)^{d-2}\right)^2} , D_\parallel \approx D$$

Teraoka et al, J. Chem. Phys. **89**, 6989 (1988) ; **91** 2643 (1989).

Combined Phase diagram

Inhomogeneous states



A Ahmadi, TBL and MC Marchetti (2005)

Flow

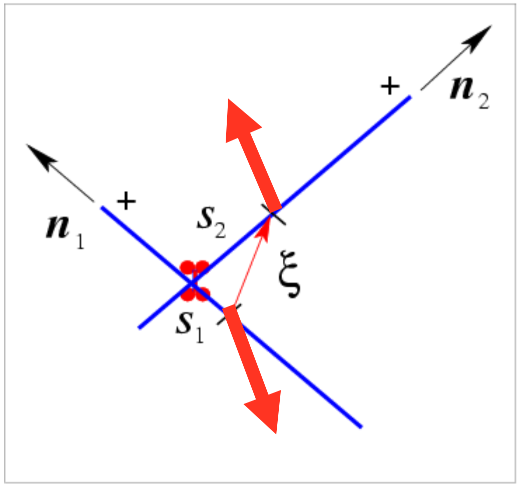
- Momentum - Stokes Equation at low Re

$$\eta\nabla^2\mathbf{v}(\mathbf{r},t) - \nabla p = \left\langle \sum_i \int_{-\ell/2}^{\ell/2} ds\, \mathbf{f}_i(s)\, \delta\left(\mathbf{r} - \mathbf{r}_i(s,t)\right) \right\rangle = -\nabla\cdot\sigma^{rods}$$

- Evolution of probability $\Psi(\boldsymbol{r},\hat{\boldsymbol{n}},t)$

$$\partial_t\Psi + \nabla\bullet\left(\Psi\boldsymbol{v} + \boldsymbol{J}\right) + \boldsymbol{R}\bullet\left(\Psi\left[\hat{\boldsymbol{n}}\times\left(\hat{\boldsymbol{n}}\bullet\nabla\right)\boldsymbol{v}\right] + \boldsymbol{J}\right) = 0$$

- Force and torque dipoles due to motors
- Derive a microscopic expression for the average (filament) stress tensor



Stress Tensor

- Microscopic expression for the stress tensor

$$\sigma_{ij}^{p}(r) = A\,Q_{ij}(r) + B\left(p_i(r)\,p_j(r) - \tfrac{1}{2}p^2\delta_{ij}\right) + C\,\delta_{ij} + O(\nabla)$$

$$A = \left(2 - \rho/\rho_N\right)k_B T + \tfrac{1}{16}\zeta\rho\alpha_0\mu\ell^3$$

$$B = \tfrac{1}{32}\zeta\rho\alpha_1\ell^3$$

$$C = \left(2 + \rho/\rho_N\right)\rho k_B T + \tfrac{1}{32}\zeta\rho^2\alpha_0\mu\ell^3 + \tfrac{1}{16}\zeta p^2\alpha_1\mu\ell^3$$

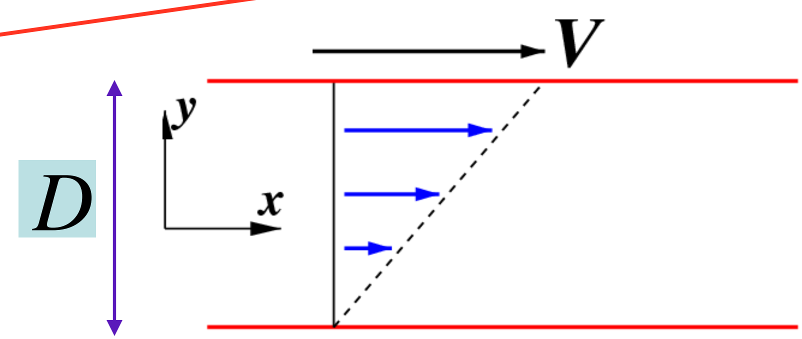
Mechanical properties - Rheology

Strain rate

- Linear Rheology-Apply small shear strain and measure shear stress
- Shear Modulus

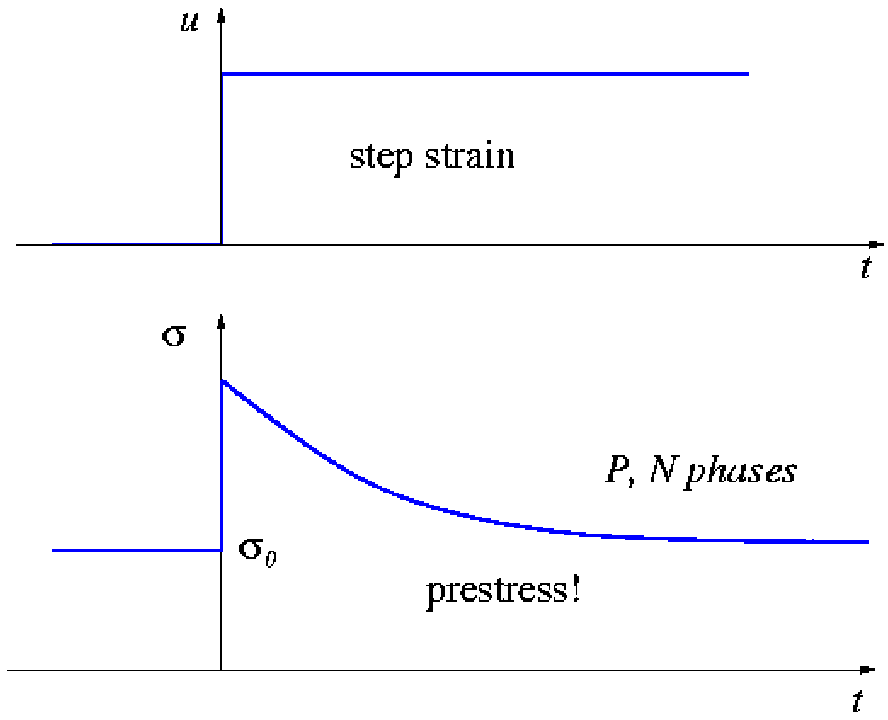
$$\sigma_{xy}(t) = \int_{-\infty}^{t} dt' \, G(t-t') \, \dot{\gamma}_{xy}(t')$$

$$\dot{\gamma}_{xy} = V / D$$



Linear Rheology

- Active contributions to stress even without deformation.



- Stiffening seen in actin-myosin networks

J Uhde et al, PRL **93** 268101 (2004)

Linear Rheology

- Shear flow $\kappa_{ij} = \nabla_i v_j = \varepsilon \delta_{ix} \delta_{jy}$
- Isotropic regime, linear order in ε

$$\left[\partial_t + \frac{1}{\tau}\right] Q_{xy} = \frac{\varepsilon}{4}\rho \quad ; \quad \frac{1}{\tau} = 4 D_r \left(1 - \rho/\rho_N\right) - m_0 \gamma_1 \ell^2 \rho$$

- Step strain

$$\dot{\gamma}_{xy}(t) = \kappa_0\, \delta(t) \longrightarrow \sigma^p_{xy}(t) = G(t)\,\kappa_0 + G^a \mu$$

- Maxwell model

$$G(t) = \frac{A}{4} \exp\left[-t/\tau\right] \quad ; \quad D_r = \frac{k_B T_a}{\ell^3 \eta}$$

Semiflexible Polymers

- Wormlike Chain Model (WLC)
- Conformations have Bending Energy

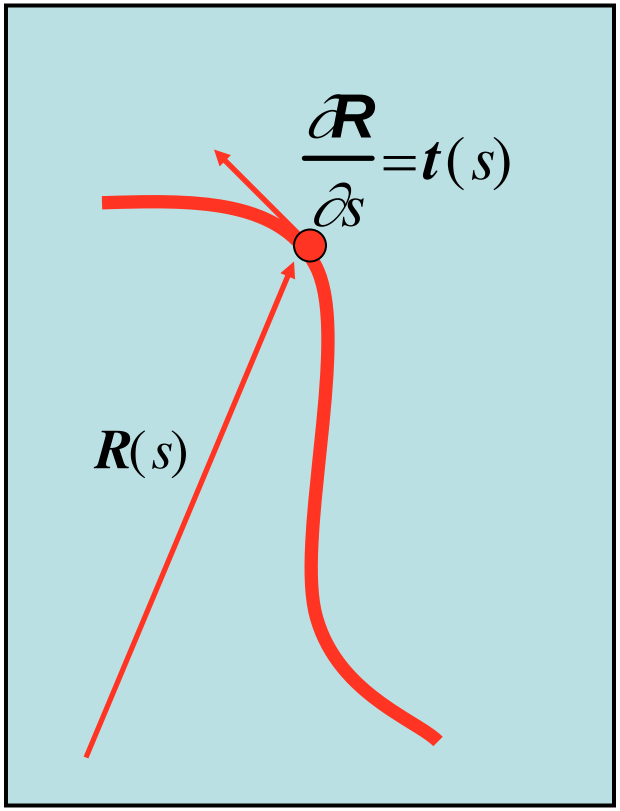
$$H_{bend}[\boldsymbol{R}(s)] = \frac{A}{2}\int ds \left(\frac{\partial^2 \boldsymbol{R}}{\partial s^2}\right)^2$$

- Inextensible $\left(\frac{\partial \boldsymbol{R}}{\partial s}\right)^2 = 1$
- Tangent correlation function

$$\langle \boldsymbol{t}(s) \bullet \boldsymbol{t}(0) \rangle = \exp(-s/L_p)$$

- Persistence length

$$L_p = \frac{A}{k_B T}$$



Anisotropic Dynamics

$$\partial_t \boldsymbol{r}_\perp(s,t) - \dot{\gamma}\bullet\boldsymbol{r}_\perp = \frac{1}{\zeta_\perp}\left[-A\,\partial_s^4 \boldsymbol{r}_\perp + \partial_s\left(\Lambda(s,t)\,\partial_s \boldsymbol{r}_\perp\right)\right] + \boldsymbol{f}_\perp(s,t)$$

$$\partial_t r_\parallel(s,t) - \dot{\gamma}:\hat{\boldsymbol{n}}\hat{\boldsymbol{n}}\left(r_\parallel - s\right) = \frac{1}{\zeta_\parallel}\left[-A\,\partial_s^4 r_\parallel - \partial_s\Lambda\right] + f_\parallel(s,t) + \hat{\boldsymbol{n}}\bullet\boldsymbol{f}_{act}(s,t)$$

$$\partial_s r_\parallel = \frac{1}{2}\left|\partial_s \boldsymbol{r}_\perp\right|^2 + \dots$$

- Self-consistent solution
- Average over transverse fluctuations and use inextensibility

Solve for fluctuating $\boldsymbol{v}(\boldsymbol{r})$ with $\langle \boldsymbol{v}(\boldsymbol{r})\rangle$ given by the prescribed flow $\dot{\gamma}\bullet\boldsymbol{r}$. Step strain $\dot{\gamma}(t) = \varepsilon\delta(t)$

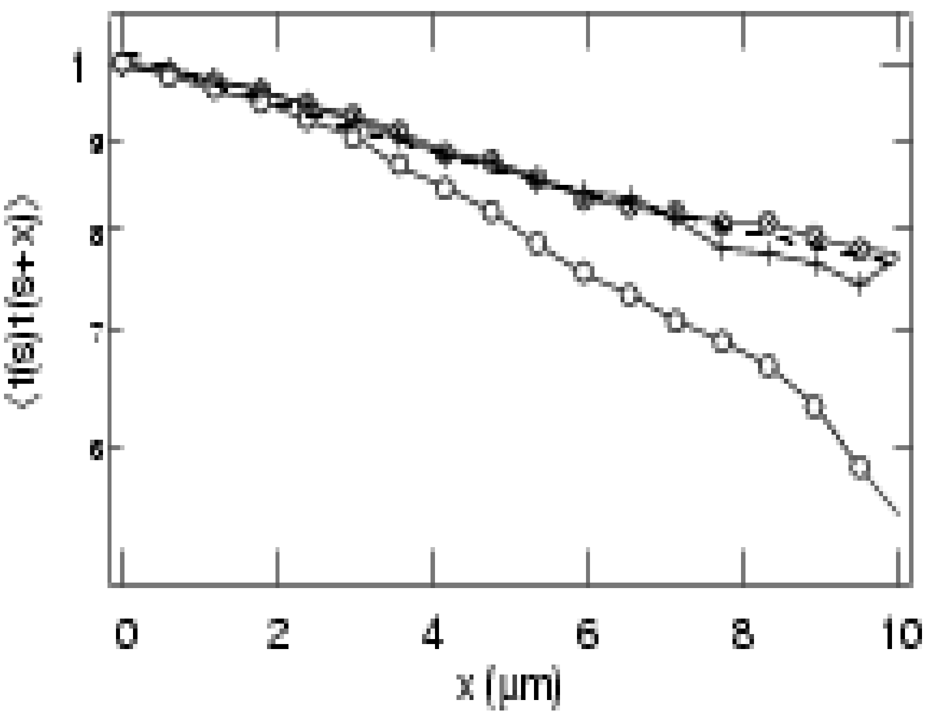
Microscopic calculation of stress - configurational average

$$\eta\nabla^2\boldsymbol{v} + \nabla p = \sum_i \int \frac{ds}{a}\boldsymbol{F}_i(s,t)\,\delta\left(\boldsymbol{r} - \boldsymbol{R}_i(s,t)\right) \quad ; \quad \nabla\bullet\boldsymbol{v} = \boldsymbol{0}$$

TBL,Maggs & Ajdari, PRL **86**, 4171, 2001

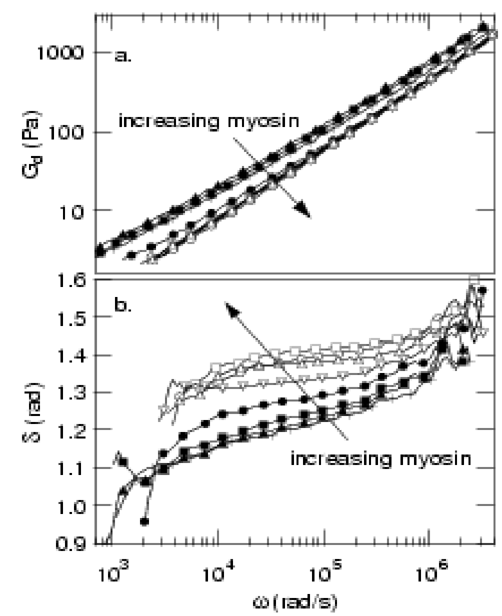
S1-Myosin/F-Actin Solution

Experiments on F-Actin and Myosin S1 fragments



Tangent correlation function

DWS micro-rheology



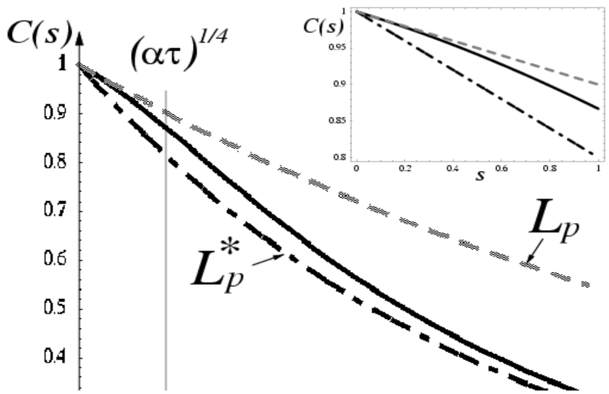
$$\langle x^2(\omega)\rangle = \frac{2k_B T G''(\omega)}{6\pi R\omega |G(\omega)|^2}$$

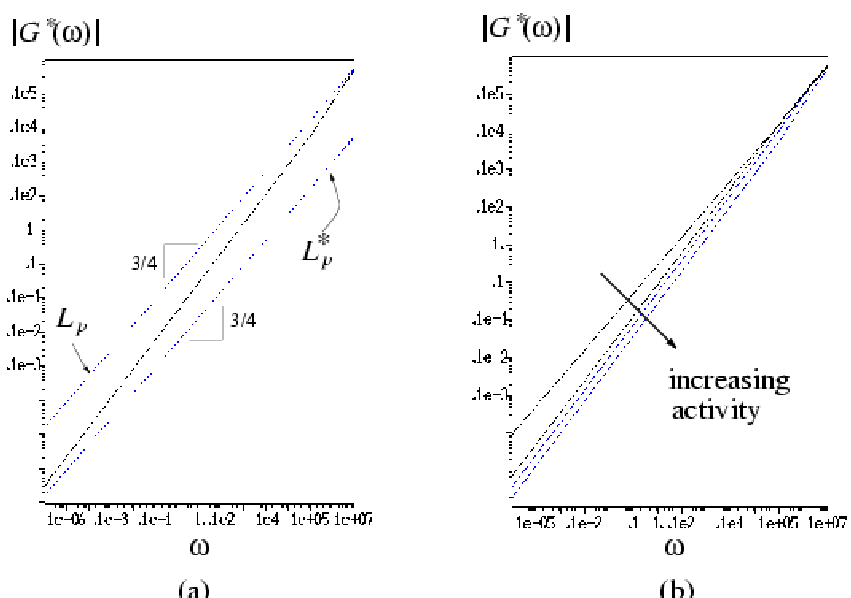
$$G_{eq}(\omega) \propto \omega^{3/4}$$

$$G_{act}(\omega) \propto \omega^{7/8} \quad ?$$

Le Goff et al , PRL 88, 018101 (2001)

Activity ↓ anomalous fluctuations





- Active force correlations

$$\left\langle f^i_{act}(s,t) f^j_{act}(s',t')\right\rangle = \Theta \zeta_\perp^{-2} \delta^{ij} \delta(s-s')\Phi(t-t')$$

$$\Phi(t) = \exp(-|t|/\tau)$$

- Tangent correlation function

$$\Theta\tau\zeta = k_B T_{act} \quad L_p^{>} \approx L_p\left(1+\frac{T_{act}}{T}\right)^{-1}$$

- High Frequency shear modulus

- **WHY?**

TBL, PRE **67**, 031909, (2003)

Numbers

- Assume motors arrive at random times with a constant rate
- Fraction of bound motors
- Distance between aggregates
- Activity Parameter

$$\phi = \rho_a / (k_a + \rho_a)$$

Actomyosin eq. binding const. ~500nM

Actin concentration

$$\ell_m \approx \left(\phi \rho_m \xi^2\right)^{-1}$$

Motor concentration

$$\Theta \approx (f_0 a)^2 / \ell_m$$

Motor stall force ~ 5pN

$$\rho_a a \approx \xi^{-2}$$

Actin diameter ~ 7nm

Meshsize

F-actin ρ_a= 14μM, S1 myosin ρ_m =5μM

$$\Theta \tau \zeta_\perp \approx k_B T \Rightarrow \quad L_p^* \approx \tfrac{1}{2} L_p \;\&\; \quad \ell_C \approx 0.1 \mu m$$

Motor cycle time, ~5ms

$$\ell_m \approx 2a \quad \& \quad \phi \approx 0.9$$

Almost full coverage

Tangent correlation

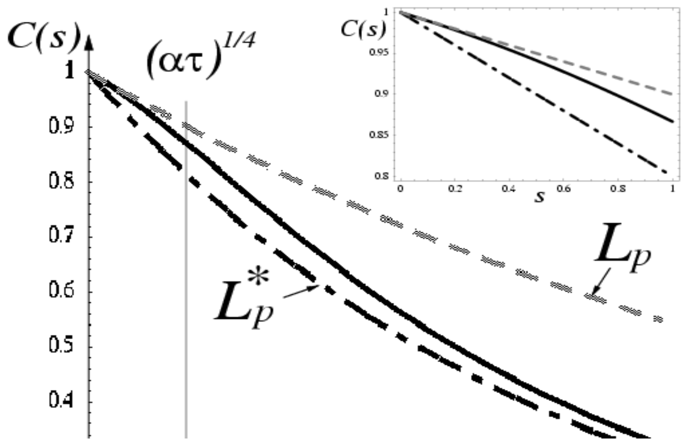
$$\langle \boldsymbol{t}(s)\bullet\boldsymbol{t}(0)\rangle \approx \exp\left(-\frac{s}{L_p}-\frac{2\Theta}{\alpha^2}\int_q \frac{1-\cos qs}{q^2\left(q^4+(A\tau/\zeta_\perp)^{-1}\right)}\right)$$

Length-scale dependent rigidity

Crossover length-scale

Tangent correlation function

$$\ell_C \approx (\tau A/\zeta_\perp)^{1/4}$$



TBL, PRE **67**, 031909 (2003)

Active 'temperature' scale

$$\Theta\tau\zeta = k_B T_{act}$$

$$L_p^< \approx L_p$$

$$L_p^> \approx L_p\left(1+\frac{T_{act}}{T}\right)^{-1}$$

High frequency viscoelasticity

Tension in filament

$$\Lambda(s,\omega) \approx \frac{\dot{\gamma}(\omega) : \hat{n}\hat{n}}{i\omega K(\omega)}$$

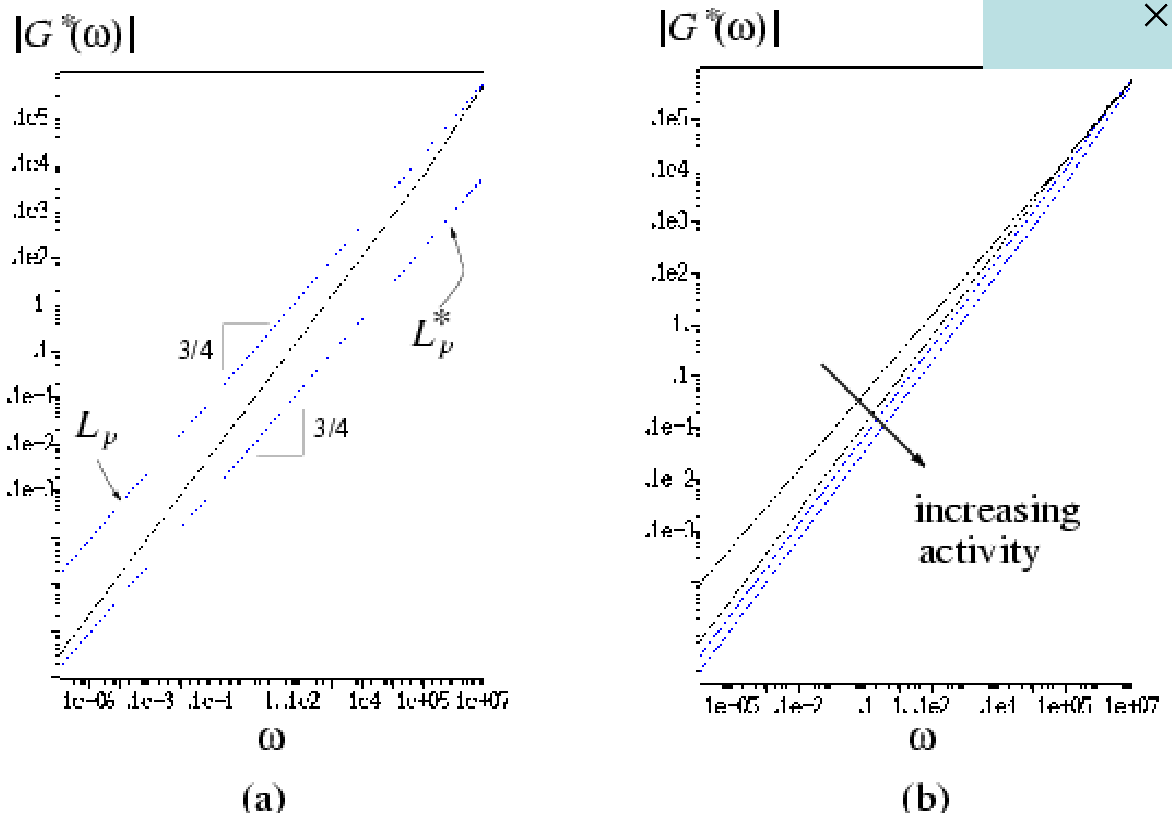
$$K_{eq}(\omega) = 2^{-3/4}\left(k_B T\right)^{1/4} L_p^{5/4}\left(i\omega\zeta_\perp\right)^{-3/4}$$

Morse, Gittes/MacKintosh (1998)

Stress

$$\sigma_{ij} = \rho\langle \Lambda n_i n_j \rangle \Rightarrow G(\omega) = \frac{2\rho}{15K(\omega)}$$

$$K_{act}(\omega) \sim 2^{-3/4}\,\Theta\,\tau\zeta_\perp\left(k_B T L_p\right)^{-5/4} \times \left[\left(i\omega\zeta_\perp\right)^{-3/4} - \tfrac{1}{4}\left(\zeta_\perp/2\tau\right)^{-3/4}\right]$$



Active compliance crosses over from high effective persistence length at high frequencies to low effective persistence length at lower frequencies

Conclusions

- Active filament solutions are significantly different from passive ones
- Non-equilibrium analogues of bulk phase transitions
- Development of inhomogeneous states can also be driven by motor activity. Type of defect structures depend on homogeneous states.
- From a simple microscopic model, bundling is due to mean motor velocity which decreases close to + end.
- Explicit expression for the stress tensor in terms of microscopic parameters.

Perspectives

ONGOING WORK

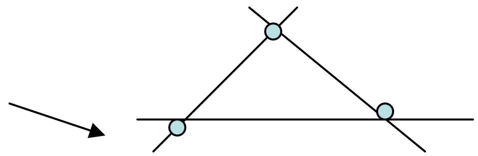
- Non-linear analysis of inhomogenous states
- Detailed study of dynamics of motor density

$$\Rightarrow \partial_t n_m = \boldsymbol{F}(n_m, \boldsymbol{Q}, \boldsymbol{u}, \rho)$$

- Effect of (shear) flow on instabilities.
- Many body interactions, frustration

FUTURE WORK

- Polydispersity, treadmilling $\Rightarrow$



- Hydrodynamic interactions
- Include passive cross-linkers
- Couple semiflexiblity to self-organisation

$$\Psi(\boldsymbol{r}, \hat{\boldsymbol{n}}, \ell, t) \quad ; \quad \frac{d\langle \ell \rangle}{dt}$$