MATRICES, MEMBRANES AND BLACK HOLES

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UCSB, KITP, 05.25.12

THIS IS A TALK ABOUT GEOMETRY

- WHERE ARE OBJECTS LOCATED IN HOLOGRAPHIC MODELS?
- ARE THEY FUZZY?
- HOW DO WE BUILD BLACK HOLES IN HOLOGRAPHIC MODELS?
- WHAT DOES THE INTERIOR OF A BLACK HOLE LOOK LIKE?

PLAN OF TALK

- ☐ BFSS
- ☐ MEMBRANES FROM MATRICES
- ☐ BMN
- ☐ BUILDING BLACK HOLES

BFSS MATRIX MODEL

DIMENSIONAL
REDUCTION OF U(N) SYM IN D=9+1 TO 0+1

$$S_{BFSS} = \frac{1}{2g^2} \int dt \left((D_t X^I)^2 + \frac{1}{2}[X^I, X^J]^2 \right) + \text{fermions}$$

BANKS, FISCHLER, SHENKER, SUSSKIND

THERE ARE 9 DYNAMICAL MATRICES AND
ONE MATRIX CONSTRAINT.

MODULI SPACE

VACUA ARE COMMUTING MATRICES

$$[X^I, X^J] = 0$$

UP TO GAUGE TRANSFORMATIONS THE MATRICES ARE DIAGONAL: EIGENVALUES ARE POSITIONS OF D0-BRANES.

PRODUCES AN $\mathbb{R}^9$

FOR EACH D0 BRANE

- THERE IS AN EFFECTIVE METRIC: OFF DIAGONAL MODES HAVE A MASS THAT MEASURES THE EUCLIDEAN DISTANCE IN 9 FLAT DIMENSIONS.

- WHEN DO BRANES ARE FAR, OFF-DIAGONAL MODES ARE INTEGRATED OUT (HIGH COST IN ENERGY). WHEN THEY ARE NEAR EACH OTHER THEY CAN BECOME ACTIVE.

THE BFSS MATRIX MODEL CONTAINS MEMBRANES

MEMBRANE HAMILTONIAN IN LIGHTCONE (AFTER GAUGE FIXING) IS OF THE FORM

$$(\Pi^I)^2 + \{X^I, X^J\}_{PB}^2$$

J. HOPPE THESIS (1982)

POISSON BRACKETS CAN BE APPROXIMATED BY MATRICES!

SUPERSYMMETRIZED DE WITT, HOPPE, NICOLAI

MEMBRANES

ARE MEMBRANES SHARP GIVEN A CONFIGURATION OF X MATRICES?

OR

ARE THEY FUZZY SO THAT SHAPE CAN ONLY APPEAR IN LARGE N LIMIT FOR SMALL COMMUTATORS?

REMARK

THERE ARE THEOREMS:

$$Diff(\Sigma) = \lim_{N \to \infty} U(N)$$

SUGGESTS THAT AT FINITE N ONE CAN'T TELL ONE RIEMANN SURFACE TOPOLOGY FROM ANOTHER.

MOREOVER

ONE IS SUPPOSED TO BE ABLE TO GET ALL EVEN D-BRANES FROM NON-COMMUTING CONFIGURATIONS.

CONFUSES THE ISSUE OF SHARPNESS OF MEMBRANES (2-BRANES)

SIMPLIFY: WORK IN 3 TRANSVERSE DIMS

CAN BE ARRANGED BY ORBIFOLDING AND ONE CAN PRESERVE SOME SUSY

CLAIM: ONE CAN GET A NONCOMMUTATIVE EMBEDDING (DEFINING SURFACES) OF 3 HERMITIAN RANDOM MATRICES INTO

$$\mathbb{R}^3$$

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WHAT IS ITS GEOMETRY?

TYPICAL IDEA OF MATRIX MODELS:ADD EIGENVALUE.

ONE CAN ALWAYS MAKE THE MATRICES BIGGER.

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$$\begin{pmatrix} X & * \\ *^\dagger & x \end{pmatrix}$$

FERMION MASS MATRIX

$$\sum_i (X^i - x^i) \otimes \sigma^i$$

WHAT MATTERS IS THE SPECTRUM OF THIS ONE MATRIX (PROVIDED BY DYNAMICS)

DEFINES A SPECTRAL DISTANCE:

$$d(X, x) \simeq (\min(\text{Abs}(\text{ Eigenvalues})))$$

D.B. + E. DZIENKOWSKI arXiv:1204.2788

- EIGENVALUES OF FERMION MATRIX CAN CHANGE SIGN

- WE CAN TRACK THE NUMBER OF EIGENVALUES THAT CROSS ZERO (DEFINES AN INDEX FUNCTION FOR EACH POINT IN 3 DIMENSIONS)

$$I(x) \simeq \frac{\dim(V+) - \dim(V-)}{2}$$

INDEX

LOCALLY CONSTANT: COUNTS HOW MANY LAYERS ONE HAS TO CROSS TO GET OUT.

THE LOCUS WHERE INDEX CHANGES ARE SURFACES: THE BEST NOTION OF THE GEOMETRIC EMBEDDING OF THE MATRICES (THEY ARE SHARP).

- THE SURFACES ARE ORIENTED.
- THEY CAN NOT BE CUT OPEN.
- IN STRING THEORY THIS HAS THE INTERPRETATION OF D-BRANE CHARGE AND THAT IT IS CONSERVED.
- ONE CAN BUILD A VECTOR BUNDLE ON SURFACE (PATCH THE NULL VECTORS FOR EACH POINT ON SURFACE)

- LOCAL INDEX PROVIDES A COUNTING OF THE HANANY-WITTEN EFFECT (STRINGS ARE CREATED ON CROSSING THESE SURFACES)
- IT ALSO COUNTS THE U(1) CHARGE OF THE GROUND STATE OF OFF-DIAGONAL FERMION MODES (WRT PROBE BRANE)
- BONUS: WE CAN USE SPECTRAL DISTANCE TO VISUALIZE ANY CONFIGURATION EVERYWHERE.

BMN MODEL

- MASS DEFORMATION OF BFSS
- DESCRIBES DLCQ ON A PLANE WAVE RATHER THAN FLAT SPACE (HAS FLUX)
- NO MODULI SPACE, BUT PRESERVE NUMBER OF SUSY.
- VACUA ARE FUZZY SPHERES.

ACTION

SPLIT 9 X INTO 3X + 6 Y

INCLUDES MYERS TERM

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$$S_{BMN} = S_{BFSS} - \frac{1}{2g^2}\int dt \left(\mu^2 (X^i)^2 + \frac{\mu^2}{4}(Y^a)^2 + 2\mu i\, \epsilon_{\ell jk} X^\ell X^j X^k \right)$$

+fermions

INCLUDES MYERS TERM

ONE GETS THE OTHER SIX DIRECTIONS FROM THE Y'S.

ONE CAN GO FROM ONE SOLUTION TO ANOTHER USING ISOMETRIES OF PLANE WAVE

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IT IS NATURAL TO IDENTIFY INITIAL CONDITIONS WITH SPACE.

ONE GETS THE OTHER SIX DIRECTIONS FROM THE Y'S.

ONE CAN GO FROM ONE SOLUTION TO ANOTHER USING ISOMETRIES OF PLANE WAVE

PLAY SAME GAME FOR GEOMETRY

$$\sum_i (X^i - x^i) \otimes \sigma^i + \frac{3}{4}\sigma^{123}$$

FLUX CORRECTION TO FERMION GROUND STATES.

LOCUS FOR FERMION ZERO MODES MOVES:

SURFACES ARE LOCATED ELSEWHERE (MYERS EFFECT)

BONUS

ALL GROUND STATES ARE DESCRIBED BY A DISCRETE SET OF CLASSICAL CONFIGURATIONS (FUZZY SPHERES), SO WE CAN IGNORE QUANTUM WAVE FUNCTIONS TO SETUP INITIAL CONDITIONS

A FUZZY SPHERE CAN ALSO BE INTERPRETED EITHER AS A GRAVITON OR AS A SPHERICAL M2 BRANE.

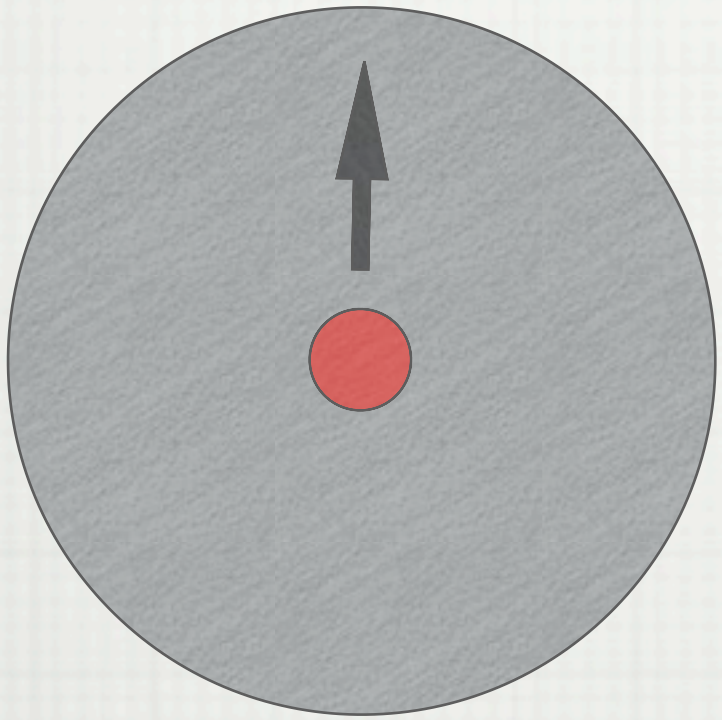
HOW TO LOOK AT IT DEPENDS ON THE STRENGTH OF INTERACTIONS.

$$\hbar$$

MAKING BLACK HOLES

SETUP:
COLLIDE BRANES IN BMN
CHECK THERMALIZATION

- LOOK AT THERMALIZATION IN CLASSICAL DYNAMICS
- INPUT PLANCK'S CONSTANT IN INITIAL CONDITIONS FOR FLUCTUATING FIELDS (SMALL PERTURBATIONS OF A CLASSICAL INITIAL STATE)



TAKE FUZZY SPHERE +

A D0 BRANE AND MAKE THEM COLLIDE.

EXPECTATIONS

OFF DIAGONAL MODES CONNECTING TWO FUZZY SPHERES GROW EXPONENTIALLY CLASSICALLY. ONCE THEY GET LARGE ENOUGH THE REST OF THE SYSTEM BACK-REACTS.

HOPEFULLY ONE ENDS UP WITH AN INTERESTING EVOLUTION THAT THERMALIZES AFTER THAT.

NUMERICS

C. ASPLUND, D.B., D. TRANCANELLI ARXIV:1104.5469

C. ASPLUND, D.B., E. DZIENKOWSKI, D. TRANCANELLI WORK IN PROGRESS

ADD QUANTUM FLUCTUATION SEEDS: GENERATE RANDOMLY FROM GAUSSIAN DISTRIBUTION NORMALIZED TO HARMONIC OSCILLATOR WAVE FUNCTIONS.

$$X^0 = \begin{pmatrix} L_n^0 & 0 \\ 0 & 0 \end{pmatrix}, \quad X^1 = \begin{pmatrix} L_n^1 & \delta x_1 \\ \delta x_1^\dagger & 0 \end{pmatrix}, \quad X^2 = \begin{pmatrix} L_n^2 & \delta x_2 \\ \delta x_2^\dagger & 0 \end{pmatrix},$$

$$P^0 = \begin{pmatrix} 0 & 0 \\ 0 & v \end{pmatrix}, \quad P^{1,2} = 0 = Q^{1,\dots,6}, \quad Y^a = \delta y^a.$$

$$\delta x, \delta y \simeq \sqrt{\hbar/n}$$

INTEREPRETATION

WE MAKE A ZERO BRANE COLLIDE WITH AN M2 BRANE IN THE PLANE WAVE GEOMETRY.

THE COLLISIONS ARE PERIODIC IN TIME UNTIL SYSTEM BACK REACTS.

A 2D SLICE COLORED BY SPECTRAL DISTANCE (21X21 MATRICES)

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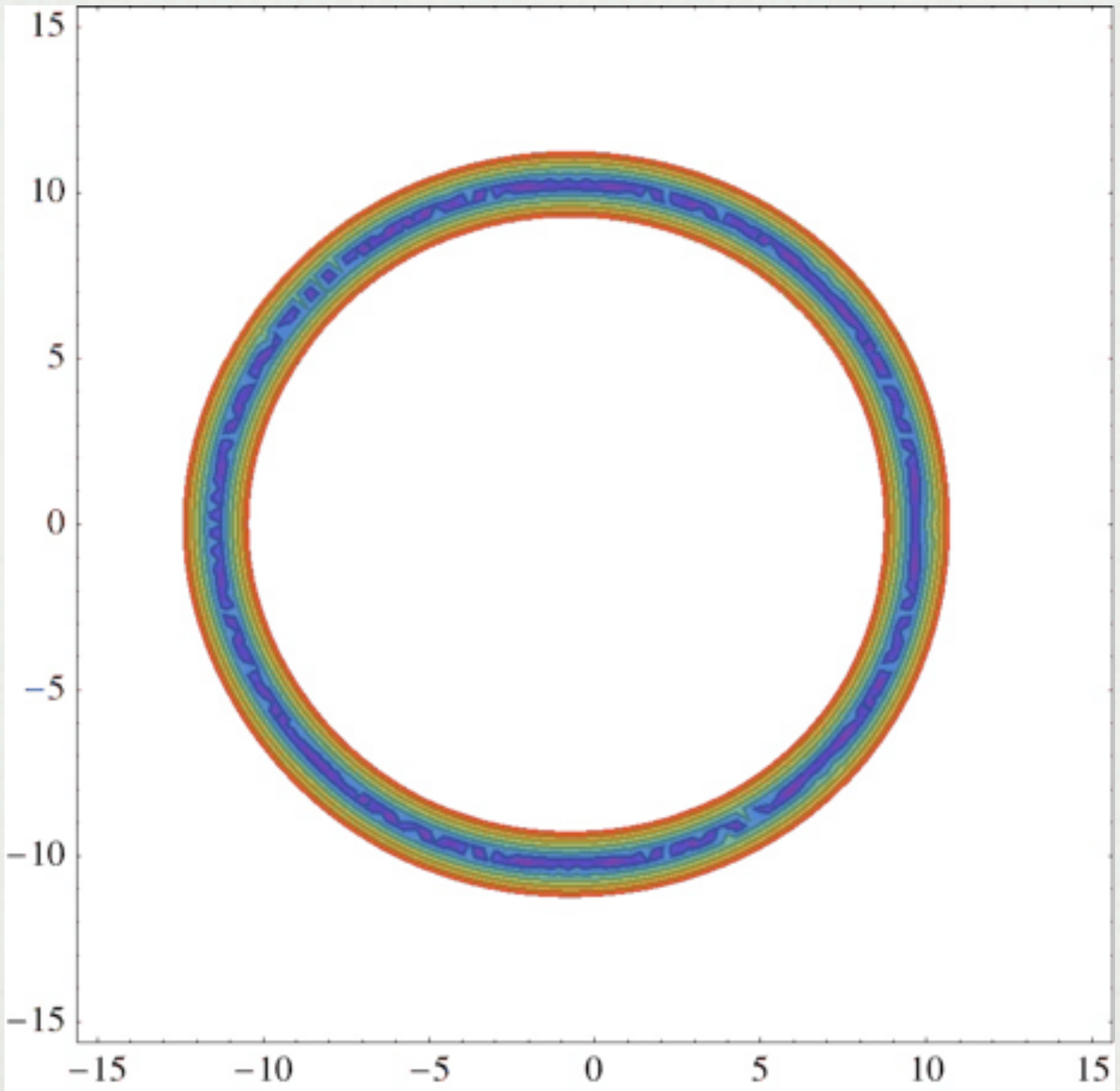
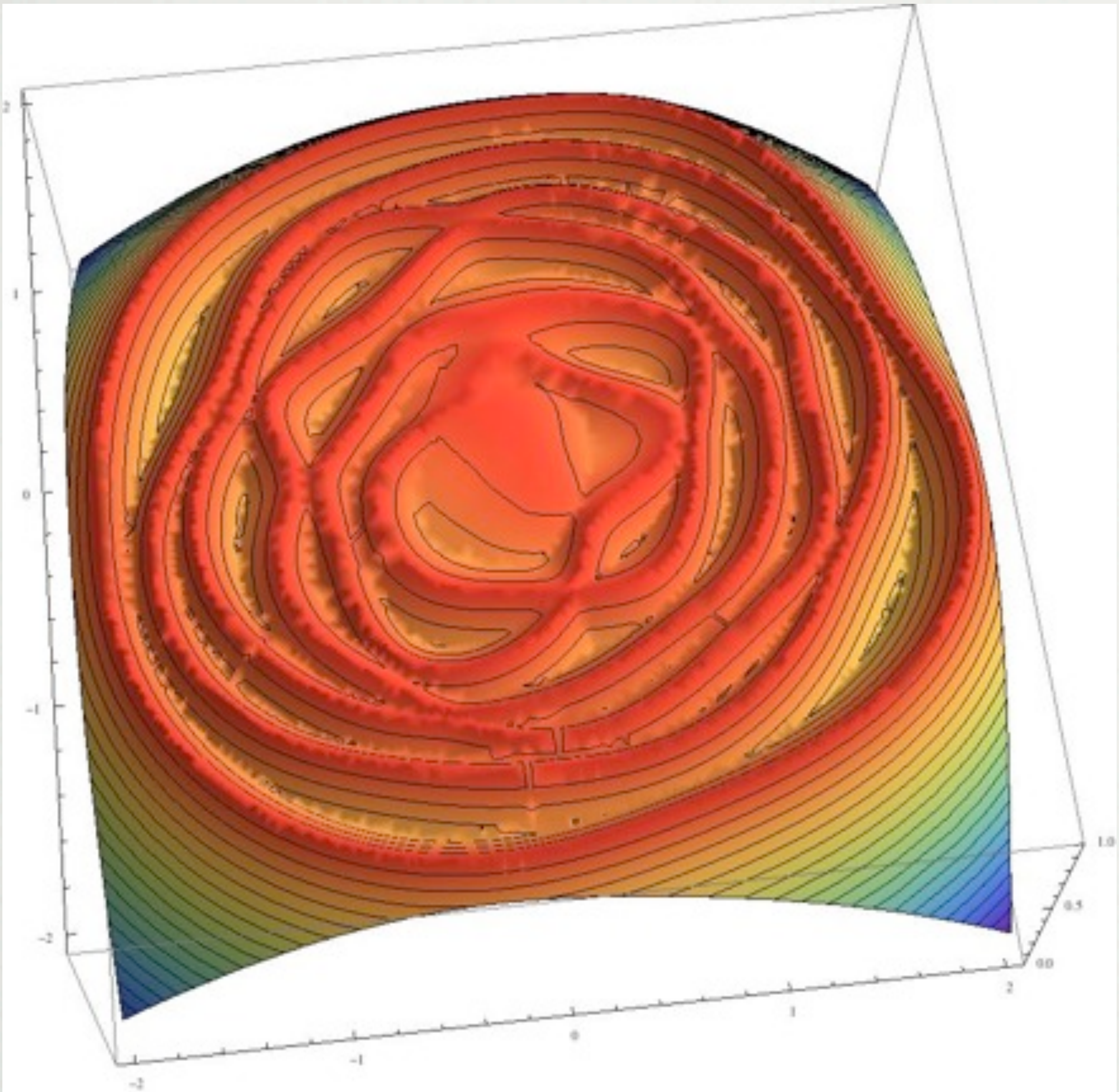




HIGH- DEFINITION GRAPH SHOWS A LOT OF ZERO DISTANCE SURFACES: RIDGES

BRANE-ANTIBRANE POLARIZATION.

THE ONION

THIS IS A SLICE OF A TRUE ONION: NOT COMPUTER GENERATED.

THIS IS THE IMAGE WE GET OF THE "INSIDE THE BLACK HOLE"

THERMALIZATION

TESTS OF THERMALITY

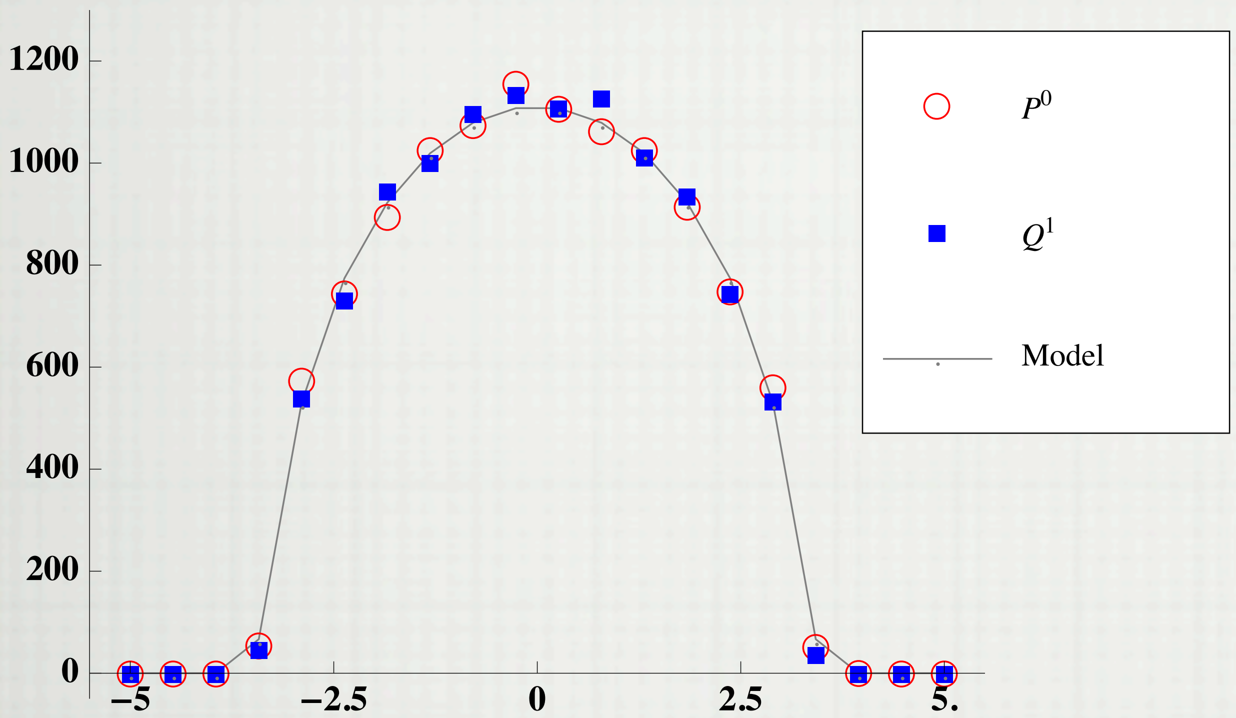
$$H \simeq \frac{P^2}{2} + V(X)$$

THERMAL IMPLIES TIME AVERAGED DISTRIBUTION OF SOME QUANTITIES SHOULD MATCH THE GIBBS ENSEMBLE.

$$\mathcal{P}(P) \simeq \exp(-\beta \frac{P^2}{2})$$

THIS IS THE STANDARD GAUSSIAN MATRIX MODEL ENSEMBLE.

SEMICIRCLE TESTS

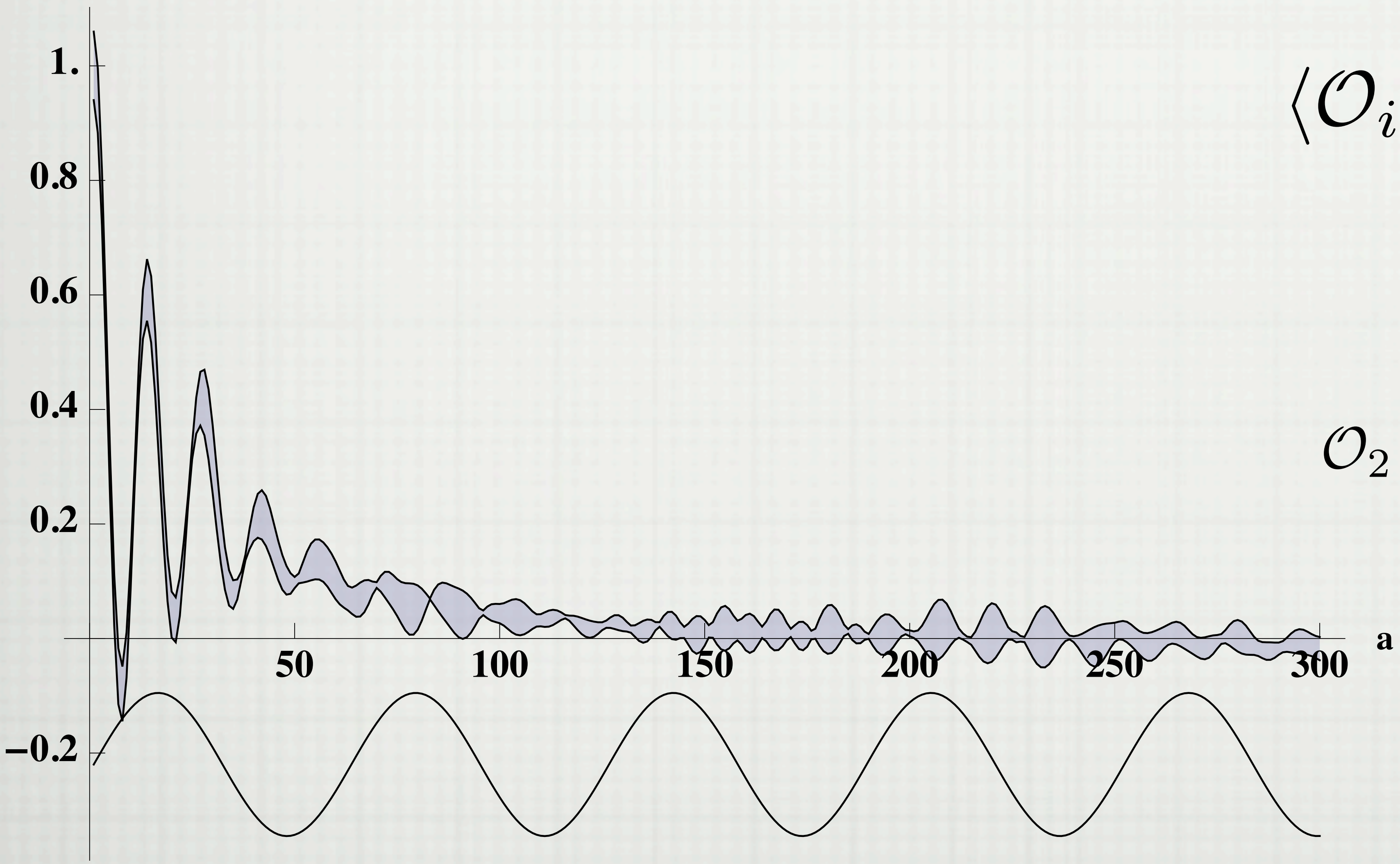


SEMICIRCLE DISTRIBUTION FOR MOMENTA EIGENVALUES: AVERAGE OVER TIME.

TEMPERATURE IN X AND Y MATCH

FAST THERMALIZATION?

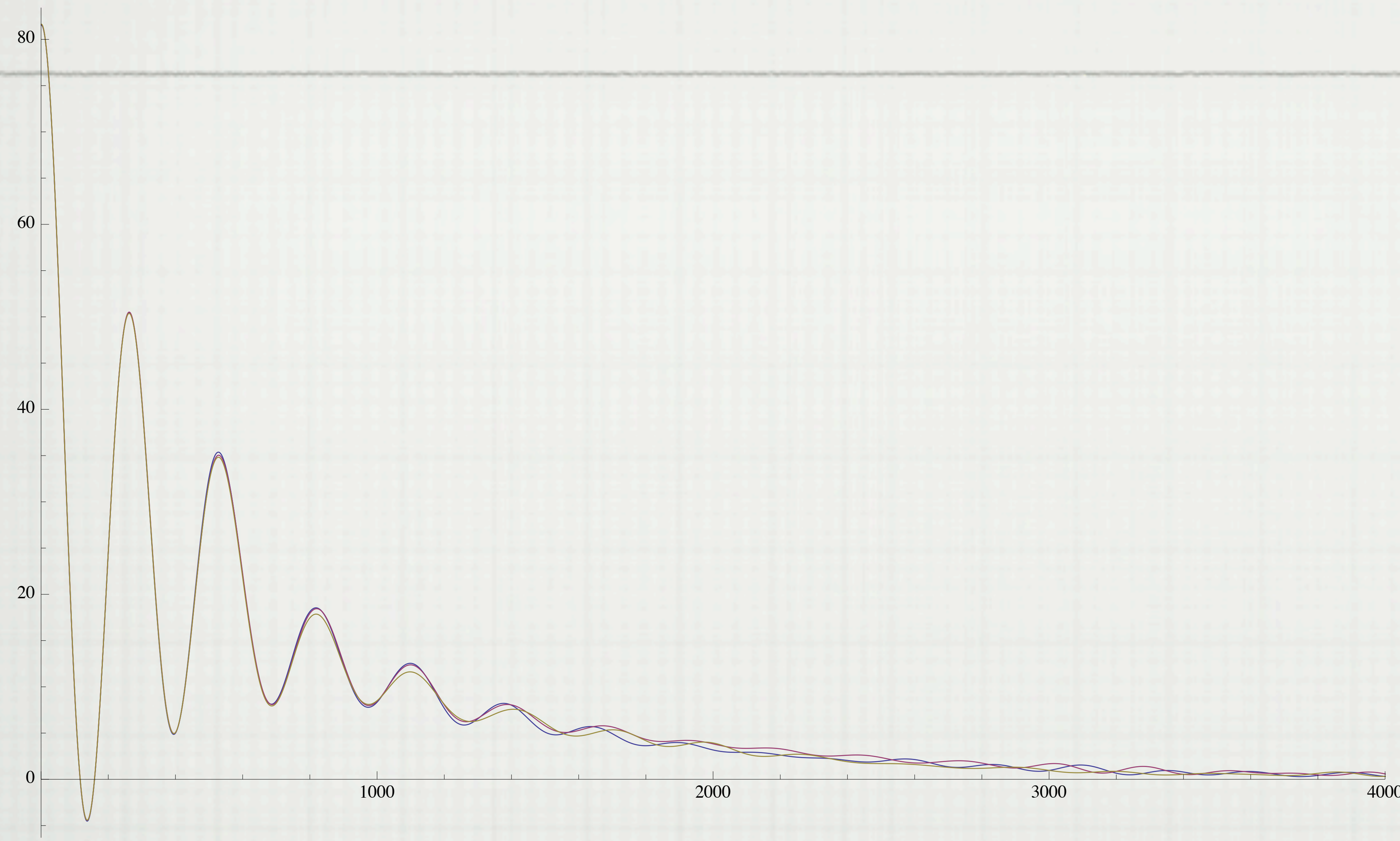
TEST VIA NORMALIZED AUTOCORRELATION FUNCTIONS



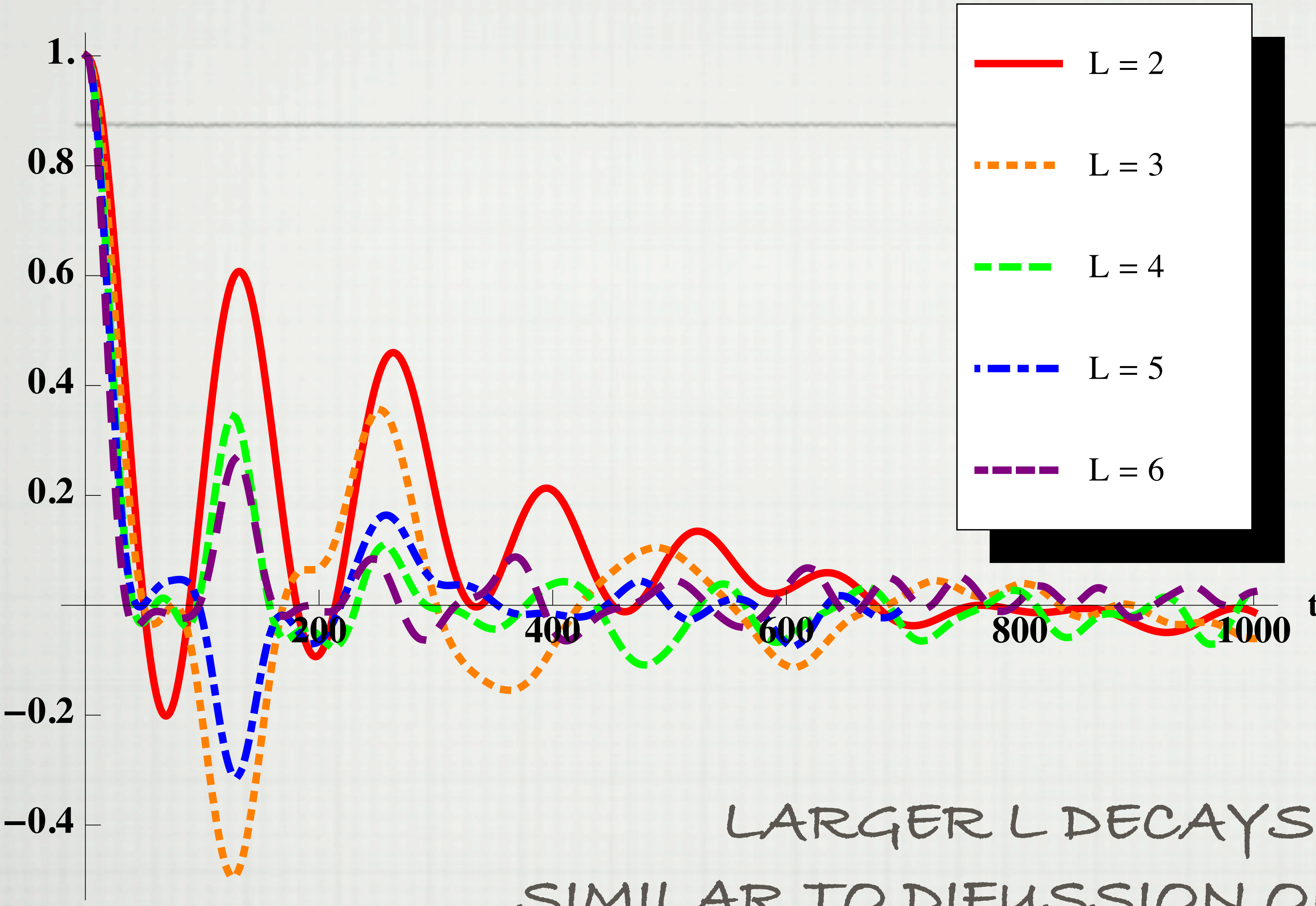
$$\langle \mathcal{O}_i(t)\mathcal{O}_i^\dagger(t+a)\rangle$$

$$\mathcal{O}_2 = \mathrm{tr}[(X^1 + iX^2)^2]$$

IMPROVING STATISTICS



MORE $\mathcal{O}_L = \mathrm{tr}[(X^1 + iX^2)^L]$



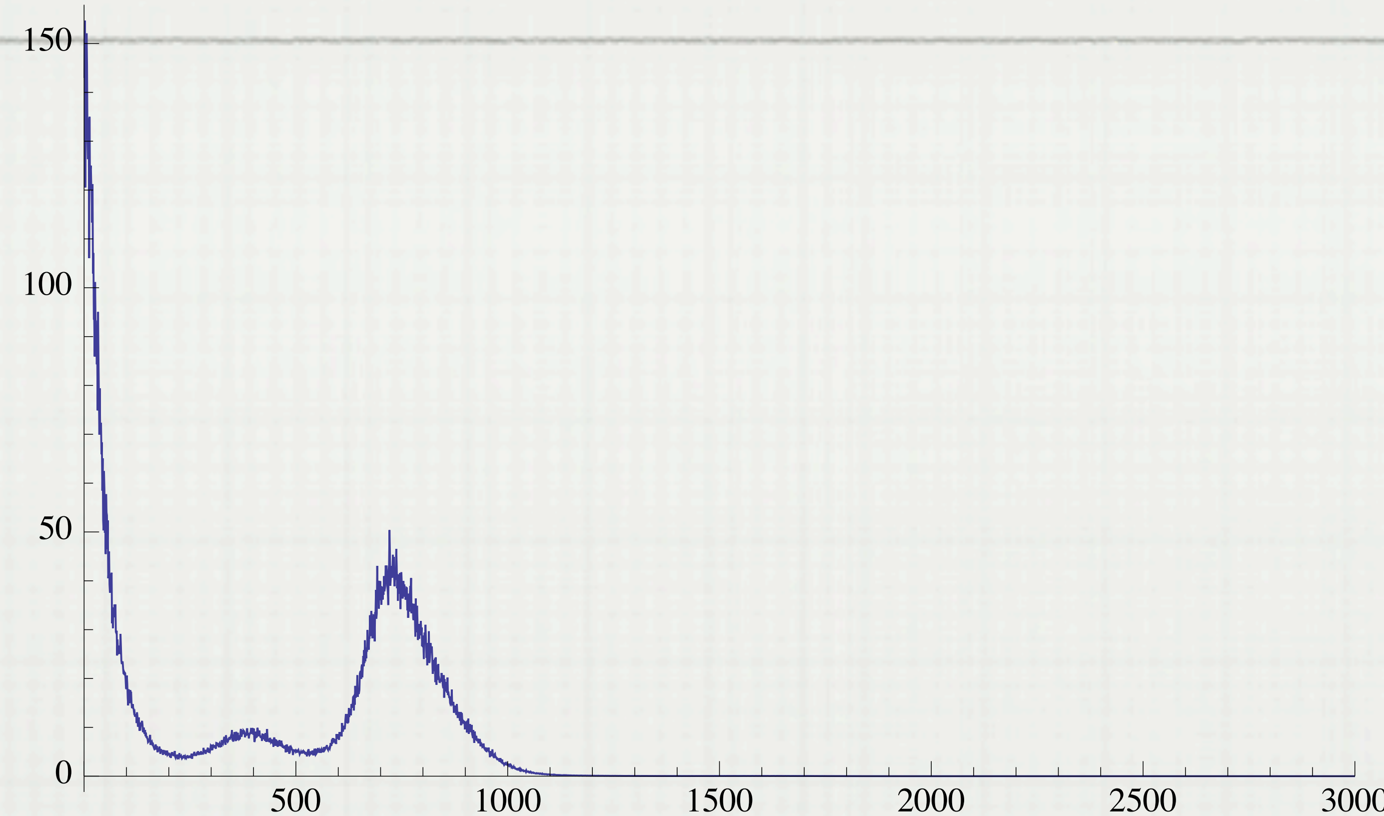
LARGER L DECAYS FASTER:
SIMILAR TO DIFUSSION ON BH HORIZON.

AUTOCORRELATIONS

BETTER IN FOURIER SPACE.

AUTOCORRELATION FUNCTION IS
FOURIER TRANSFORM OF POWER SPECTRUM

HIGH QUALITY L=2.



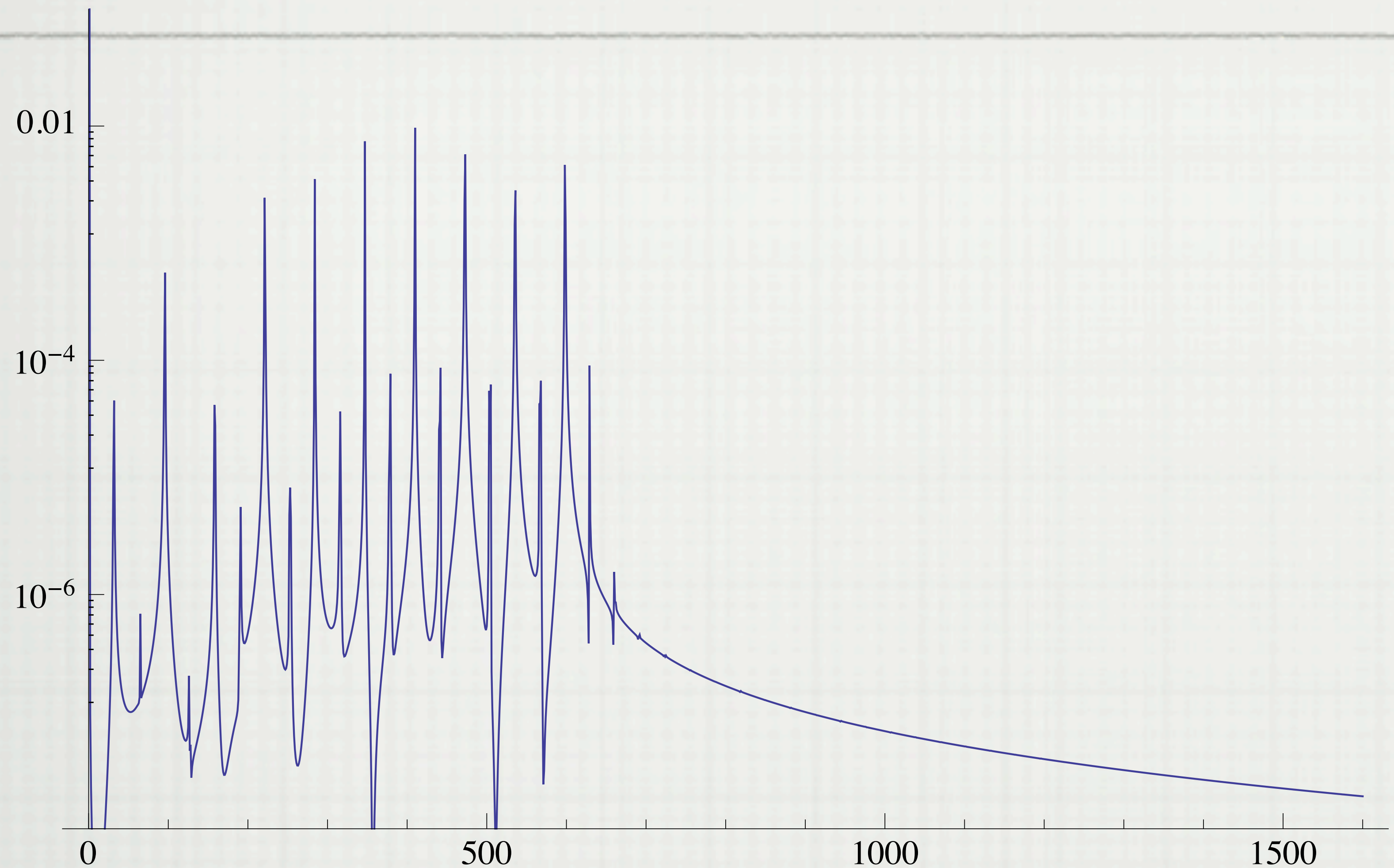
POWER

FREQUENCY

BROADBAND NOISE INDICATES CHAOS: VERY BROAD INDICATES FAST THERMALIZATION (NO NARROW RESONANCE).

INTEGRABILITY WOULD SHOW AS DELTA FUNCTION PEAKS

SMALL PERT. ON FUZZY



INTERESTING IR

- POWER SPECTRUM SEEMS ALMOST SINGULAR AT ZERO.
- THE LOG OF POWER SPECTRUM SEEMS TO HAVE AN ABSOLUTE VALUE SINGULARITY. SUCH SINGULARITY WOULD IMPLY POLYNOMIAL DECAY OF AUTOCORRELATION FUNCTIONS FOR ASYMPTOTICALLY LONG TIMES.
- STILL LOOKING FOR INTERPRETATION: HYDRODYNAMICS?

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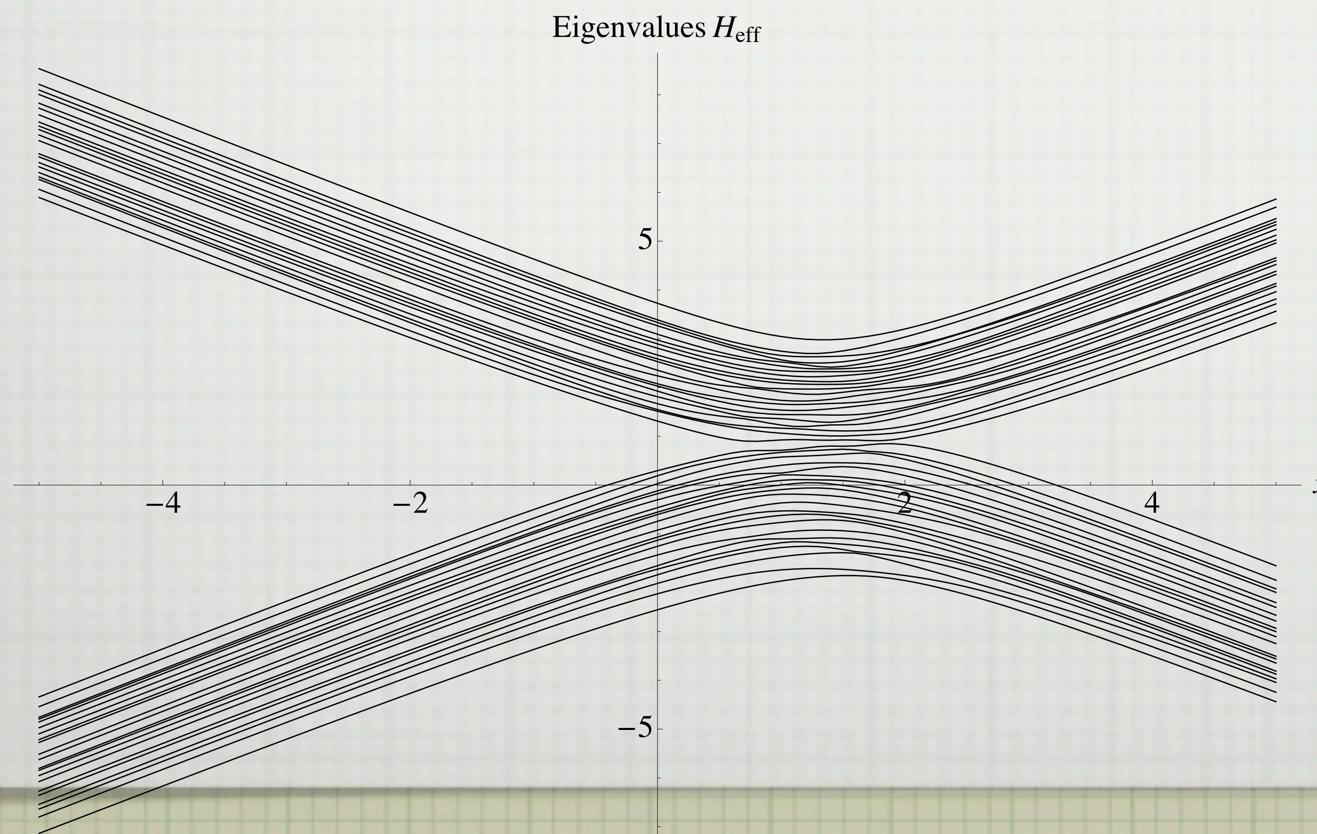
THESE STOP THE STUFF THAT IS FALLING IN BLACK HOLE LIKE A SPIDERWEB.

INSIDE

LOOK AT SPECTRAL DENSITY OF FERMIONIC MODES:
DEFINES A NOTION OF DIMENSION

NATURAL NOTION OF
DIMENSION INSIDE A BH IS 2 (1+1) AND
SPECTRUM GOES ALL THE WAY TO ZERO (NO GAP)



EMERGENT 2D CFT?

CONCLUSIONS

- ☐ GEOMETRIC CONFIGURATIONS OF MATRICES DEFINE SHARP D-BRANES. FERMIONS REALLY MATTER.
- ☐ FAST THERMALIZATION (EVIDENCE)
- ☐ INTERESTING PATTERN OF AUTOCORRELATIONS WHEN SYSTEM THERMALIZES (LOTS OF QUESTIONS HERE)
- ☐ EMERGENT 1+1 CFT? (SAME TYPICAL DENSITY OF STATES)