Non-perturbative Effects in Higher Spin Theories

Alejandra Castro
McGill University

R. Gopakumar, M. Gutperle, J. Raeymaekers, AC arXiv 1111.3381
E. Hijano, A. Lepage-Jutier, A. Maloney, AC arXiv 1110.4117

Precision Holography

AdS

CFT

Gauge symmetries

Global symmetries

Vasiliev's Theory
Massless higher spin fields

Solvable theory:
Free,
Minimal models

$$Z_{\mathrm{CFT}}(\beta) \quad = \quad \mathrm{Tr}_{\mathcal{H}}(e^{-\beta H})$$

$$
\begin{aligned}
Z_{\mathrm{CFT}}(\beta) &= \mathrm{Tr}_{\mathcal{H}}(e^{-\beta H}) \\
&= Z_{\mathrm{HS}}(\beta) \\
&= \int_{\partial\mathcal{M}} [\mathcal{D}g \cdots] \; e^{-S_{\mathrm{E}}}
\end{aligned}
$$

$$
\begin{aligned}
Z_{\mathrm{CFT}}(\beta) &= \mathrm{Tr}_{\mathcal{H}}(e^{-\beta H}) \\
&= Z_{\mathrm{HS}}(\beta) \\
&= \int_{\partial\mathcal{M}} [\mathcal{D}g \cdots] \; e^{-S_{\mathrm{E}}}
\end{aligned}
$$

Can we re-write the partition function as a sum over geometries?

Does a higher spin theory resemble at all Einstein gravity?

What do we expect (assume) a gravitational path integral looks like?

- ✦ Include changes in topology
- ✦ Admit a saddle point approximation

$$Z_{\mathrm{HS}} = \sum_{\phi_{cl}} \exp\left(-\frac{1}{\hbar} S_E^{(0)} + S_E^{(1)} + \hbar S_E^{(2)} + \cdots\right)$$

Non-perturbative
(e.g. black holes)

Loop corrections
(field fluctuations)

Overview

AdS_3 ⟷ CFT_2

state duality

↓ ↓

Classical phase space

Spectrum states

Overview

AdS_3 ⟷ CFT_2

state duality

↓ ↓

Classical phase space | Spectrum states

↘ ↙

What did we learn?

Chern-Simons and Higher Spin

why is it easy to construct hs thys in 3d?
Eqns are bacground independent

$$F = dA + A^2 = 0$$

$$\bar{F} = d\bar{A} + \bar{A}^2 = 0$$

$$A, \bar{A} \in SL(N, \mathbb{R})$$

$$A = \omega + \frac{1}{\ell} e$$

$$\bar{A} = \omega - \frac{1}{\ell} e$$

Linearized eom's describe spin-s fields

$$s = 2, 3, \ldots, N$$

$$g_{\mu\nu} \sim \mathrm{Tr}(e_\mu e_\nu)$$

$$\psi_{\mu\nu\rho} \sim \mathrm{Tr}(e_\mu e_\nu e_\rho)$$

Chern-Simons and Higher Spin

why is it easy to construct hs thys in 3d?
Eqns are bacground independent

$$F = dA + A^2 = 0$$

$$\bar{F} = d\bar{A} + \bar{A}^2 = 0$$

$$A, \bar{A} \in SL(N, \mathbb{R})$$

$$A = \omega + \frac{1}{\ell} e$$

$$\bar{A} = \omega - \frac{1}{\ell} e$$

Linearized eom's describe spin-s fields

$$s = 2, 3, \ldots, N$$

~~$g_{\mu\nu} \sim \mathrm{Tr}(e_\mu e_\nu)$~~

~~$\psi_{\mu\nu\rho} \sim \mathrm{Tr}(e_\mu e_\nu e_\rho)$~~

Not gauge invariant!

A generalization to include infinite number of fields

$$SL(N) \to hs[\lambda]$$

$$s = 2, 3, \cdots, \infty$$

$$\lambda \in \mathbb{R}$$

Can also add propagating d.o.f. : massive scalar field

$$m^2 = -1 + \lambda^2$$

AdS3/CFT2

AdS$_3$

Vasiliev's theory

$$hs[\lambda]$$

& one scalar

$$m^2 = -1 + \lambda^2$$

CFT$_2$

W$_N$ minimal model

$$c \leq N - 1$$

$$\frac{SU(N)_k \otimes SU(N)_1}{SU(N)_{k+1}}$$

$$N, k \to \infty$$

$$\lambda = \frac{N}{k+N} \leq 1$$

Gaberdiel, Gopakumar 1011.2986 & 1205.2472

Evidence... so far...

Asymptotic symmetries

Henneaux, Rey 1008.4579
Campoleoni, Fredenhagen, Pfenninger, Theisen 1008.744
Gaberdiel, Hartman 1101.2910
Campoleoni, Fredenhagen, Pfenninger 1107.0290

HS black holes

Gutperle, Kraus 1103.4304
Kraus, Perlmutter 1108.2567
Gaberdiel, Hartman, Jin 1203.0015

Correlation functions

Chang, Yin 1106.2580, 1112.5459
Papadodimas, Raju 1108.3077
Ammon, Kraus, Perlmutter 1111.3926

Perturbative spectrum

Gaberdiel, Gopakumar, Hartman, Raju 1101.2910
Gaberdiel, Gopakumar, Saha 1009.6087

Boundary Spectrum

Easy to compute in the CFT dimensions of primaries

$$h(\Lambda_+, \Lambda_-)$$

$$h(f;0) \underset{N,k\to\infty}{\longrightarrow} \frac{1}{2}(1+\lambda) \qquad h(0;f) \underset{N,k\to\infty}{\longrightarrow} \frac{1}{2}(1-\lambda)$$

$$h(\Lambda,\Lambda) \underset{N,k\to\infty}{\longrightarrow} \frac{\lambda^2}{N}$$

What is the bulk interpretation?

3D Higher Spin Gravity

Note: Gravity $\neq$ Chern-Simons

3D Higher Spin Gravity

Note: Gravity $\neq$ Chern-Simons

Gravitational theory requires:

✦ Picking embedding of SL(2) in SL(N)

✦ Imposing boundary conditions

✦ Allowing the topology to vary

3D Higher Spin Gravity

Note: Gravity ≠ Chern-Simons

Gravitational theory requires:

- ✦ Picking embedding of SL(2) in SL(N)
- ✦ Imposing boundary conditions

→ Asymptotic AdS
Exclude A=0

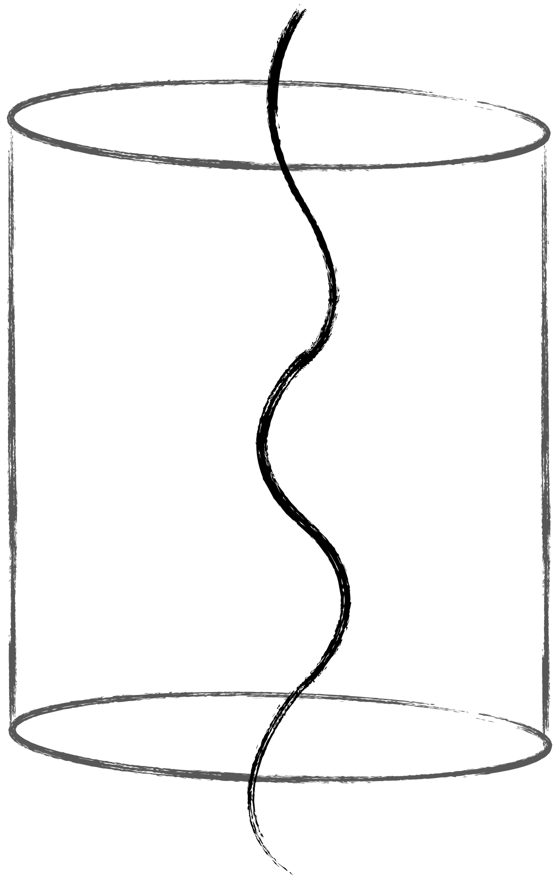
- ✦ Allowing the topology to vary

→ Black holes
are welcomed

Observable

Goal: To construct smooth solutions

What does that mean?

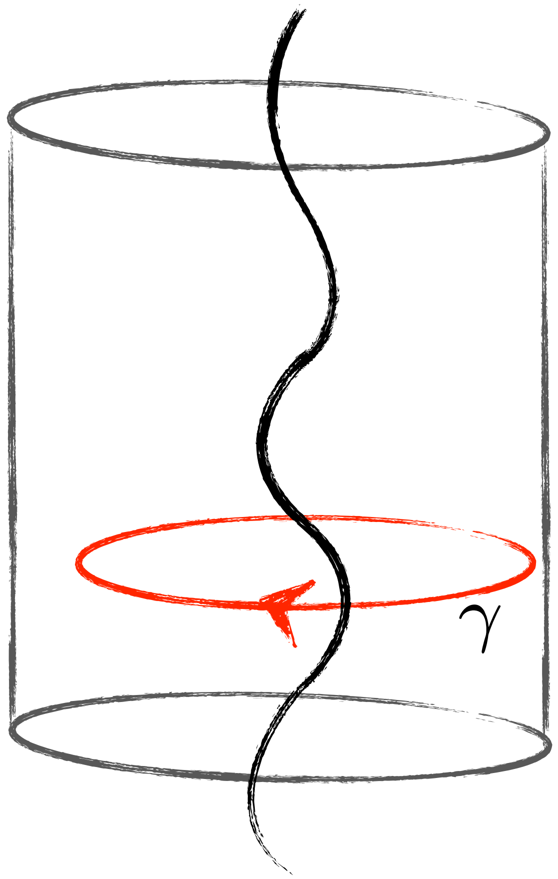


Observable

Goal: To construct smooth solutions

What does that mean?

$$\mathrm{Hol}_\gamma(A) = \mathcal{P}\exp\left(\oint_\gamma A\right)$$

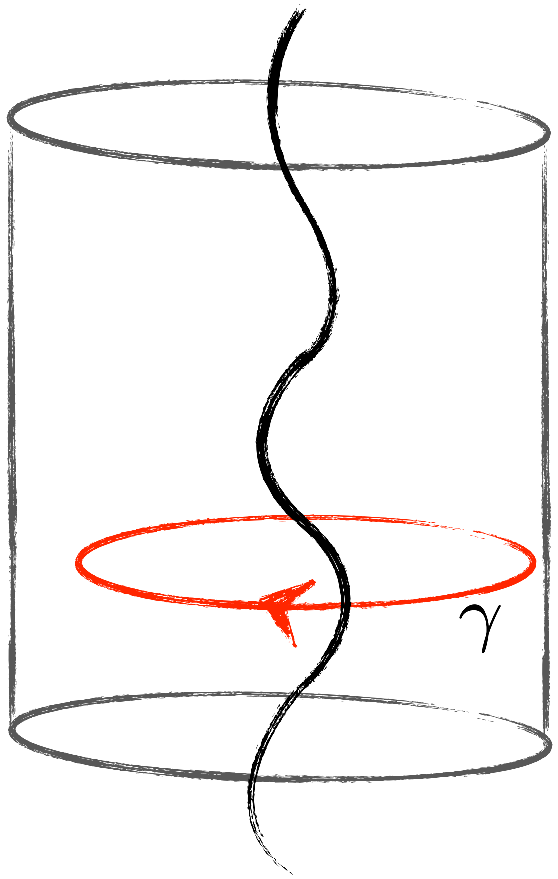


Observable

Goal: To construct smooth solutions

What does that mean?

$$\mathrm{Hol}_\gamma(A) = \mathcal{P}\exp\left(\oint_\gamma A\right)$$



Perfect features:

- ✦ Independent of metric
- ✦ Topological invariant
- ✦ Traces are gauge invariant!

$$\mathrm{Hol}_\gamma(A) = \mathcal{P}\exp\left(\oint_\gamma A\right)$$

Rule: If holonomy is trivial around contractible cycle
⇒ solution is smooth

$$\mathrm{Hol}_\gamma(A) = \mathcal{P}\exp\left(\oint_\gamma A\right)$$

Rule: If holonomy is trivial around contractible cycle
⇒ solution is smooth

Note: Easy to implement for SL(N); hard to compute for hs[λ]

In the following, study SL(N) HS gravity which corresponds to

$$\mathrm{Hol}_\gamma(A) = \mathcal{P}\exp\left(\oint_\gamma A\right)$$

Rule: If holonomy is trivial around contractible cycle
⇒ solution is smooth

Note: Easy to implement for SL(N); hard to compute for hs[λ]

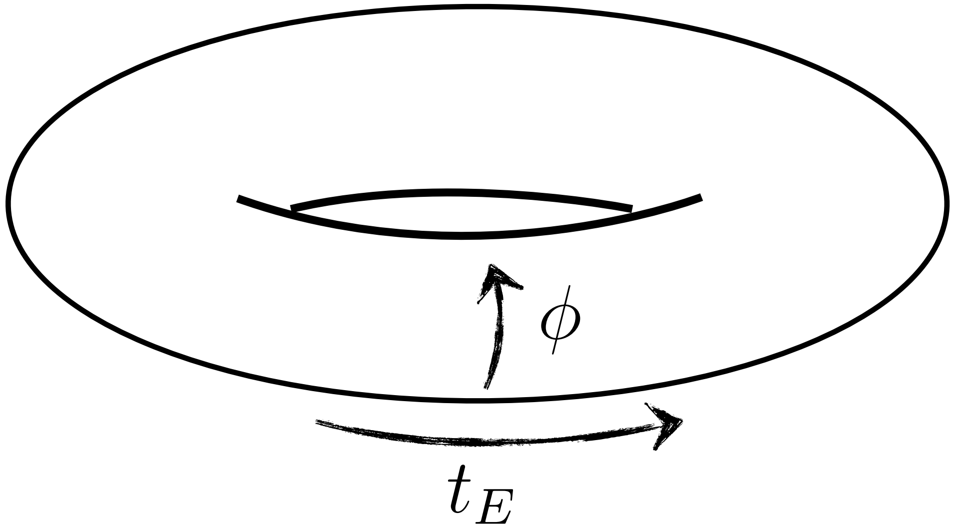
In the following, study SL(N) HS gravity which corresponds to

$$c \longrightarrow \infty$$

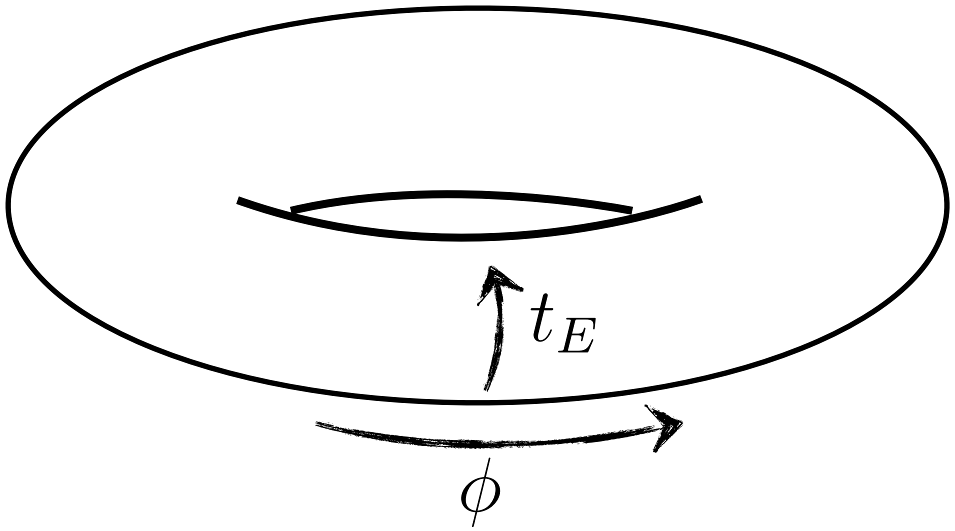
N fixed

Consider the topology of a solid torus

thermal AdS



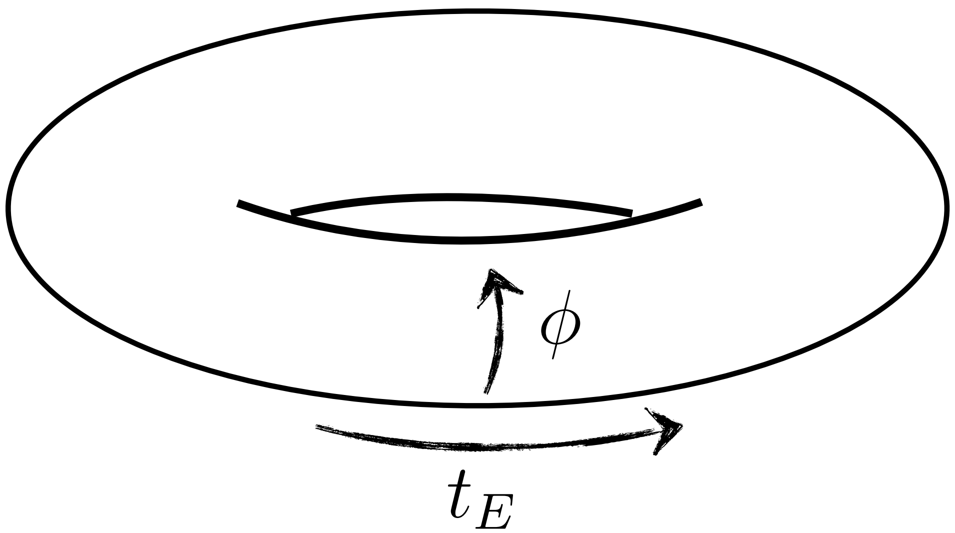
BTZ BH



Bulk Spectrum

Consider the topology of a solid torus

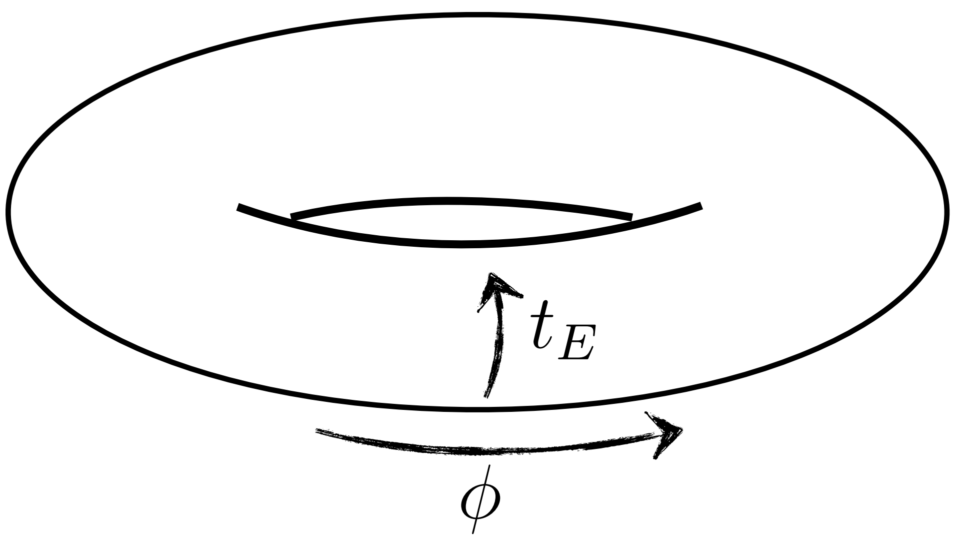
thermal AdS



$$\gamma_{\rm AdS} : \phi \;\rightarrow\; \phi + 2\pi$$

$$\mathrm{Hol}_\gamma(A) = 1$$

BTZ BH

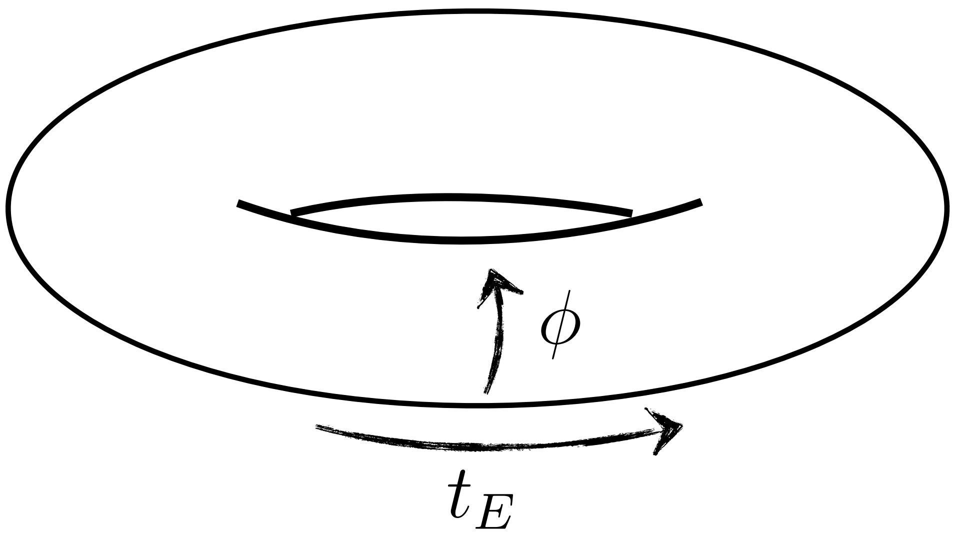


$$\gamma_{\rm BTZ} : t_E \;\rightarrow\; t_E + 2\pi\beta$$

Bulk Spectrum

Consider the topology of a solid torus

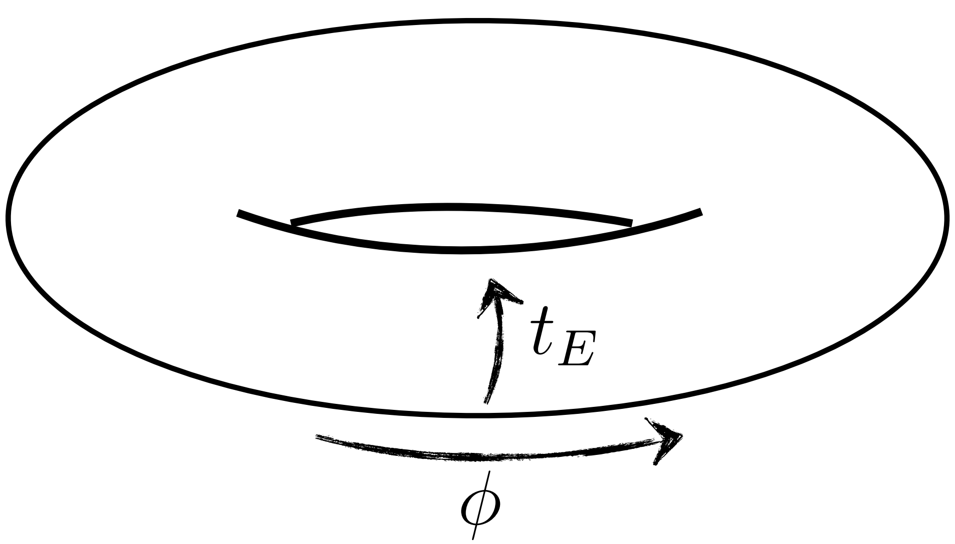
thermal AdS



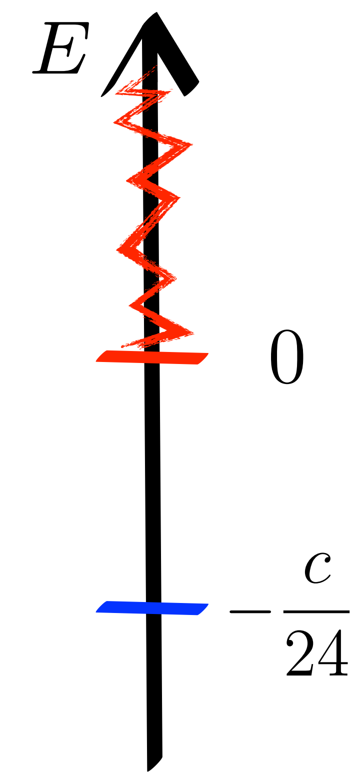
$$\gamma_{\text{AdS}} : \phi \ \rightarrow \ \phi + 2\pi$$

$$\text{Hol}_\gamma(A) = 1$$

BTZ BH



$$\gamma_{\text{BTZ}} : t_E \ \rightarrow \ t_E + 2\pi\beta$$



Are there more smooth solutions?
(with the same boundary conditions)

$$\mathrm{Hol}_\phi(A_{\mathrm{AdS}}) \sim \exp(2\pi i \lambda_{\mathrm{AdS}})$$

$$\lambda_{\mathrm{AdS}} = \left(-\frac{N-1}{2}, \cdots, \frac{N-1}{2}\right)$$

Are there more smooth solutions?
(with the same boundary conditions)

$$\mathrm{Hol}_\phi(A_{\mathrm{AdS}}) \sim \exp(2\pi i \lambda_{\mathrm{AdS}})$$

$$\lambda_{\mathrm{AdS}} = \left(-\frac{N-1}{2}, \cdots, \frac{N-1}{2}\right)$$

$$\mathrm{Hol}_\phi(A_{\mathrm{new}}) \sim \exp(2\pi i \lambda_{\mathrm{n}})$$

$$\lambda_{\mathrm{n}} = (\lambda_1, \cdots, \lambda_N)$$

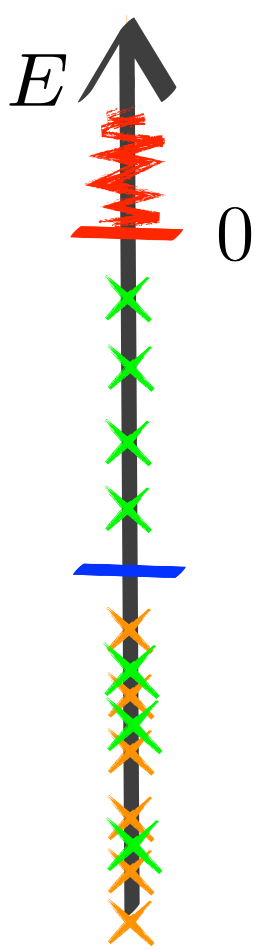
$$\lambda_i = m_i - \frac{m}{N} \qquad m_i \in \mathbb{Z}$$

$$m = \sum_i m_i$$

Are there more smooth solutions? (with the same boundary conditions)

$$\mathrm{Hol}_\phi(A_{\mathrm{AdS}}) \sim \exp(2\pi i \lambda_{\mathrm{AdS}})$$

$$\lambda_{\mathrm{AdS}} = \left(-\frac{N-1}{2}, \cdots, \frac{N-1}{2}\right)$$



$$\mathrm{Hol}_\phi(A_{\mathrm{new}}) \sim \exp(2\pi i \lambda_{\mathrm{n}})$$

$$\lambda_{\mathrm{n}} = (\lambda_1, \cdots, \lambda_N)$$

$$\lambda_i = m_i - \frac{m}{N} \qquad m_i \in \mathbb{Z}$$

$$m = \sum_i m_i$$

× : States with degenerate eigenvalues

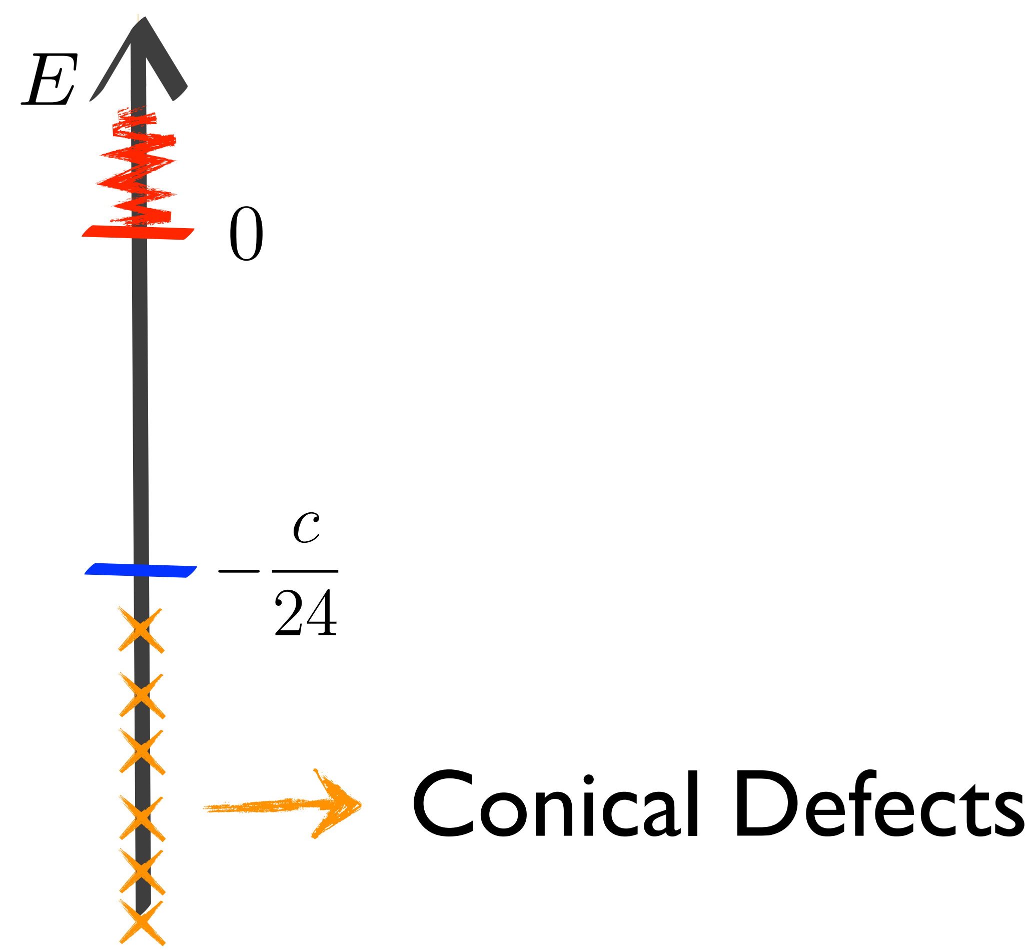
× : States with non-degenerate eigenvalues

Don't forget to impose boundary conditions

$$(A - A_{\mathrm{AdS}})_{|\rho \to \infty} = \mathcal{O}(1)$$

The fall off of the connection restricts eigenvalues to be non-degenerate.

States ✕ we throw. States ✕ we keep.



Interpretation

Where does × fit in the CFT spectrum?

Compare with $h(\Lambda, \Lambda)$

Interpretation

AdS

$$\begin{aligned} w_0^{(2)} &= -\beta_0 C_2(\lambda_{\mathrm{n}}) \\ w_0^{(3)} &= i\beta_0^{3/2}\, C_3(\lambda_{\mathrm{n}}) \\ &\vdots \end{aligned}$$

$$\beta_0 = \frac{c}{N(N^2-1)}$$

CFT

$$\begin{aligned} w_0^{(2)} &= \alpha_0^2 C_2(\Lambda) \\ w_0^{(3)} &= \alpha_0^3\, C_3(\Lambda) \\ &\vdots \end{aligned}$$

$$\begin{aligned} \alpha_0^2 &= \frac{1}{N(N+1)} - \frac{c}{N(N^2-1)} \\ &\underset{c\to\infty}{\longrightarrow} -\beta_0 \end{aligned}$$

Interpretation

AdS

$$
\begin{aligned}
w_0^{(2)} &= -\beta_0 C_2(\lambda_{\rm n}) \\
w_0^{(3)} &= i\beta_0^{3/2}\, C_3(\lambda_{\rm n}) \\
&\vdots
\end{aligned}
$$

$$\beta_0 = \frac{c}{N(N^2-1)}$$

Reliable for large central charge and fixed N

CFT

$$
\begin{aligned}
w_0^{(2)} &= \alpha_0^2 C_2(\Lambda) \\
w_0^{(3)} &= \alpha_0^3\, C_3(\Lambda) \\
&\vdots
\end{aligned}
$$

$$
\begin{aligned}
\alpha_0^2 &= \frac{1}{N(N+1)} - \frac{c}{N(N^2-1)} \\
&\underset{c\to\infty}{\longrightarrow} -\beta_0
\end{aligned}
$$

Unitary for central charge less than N

SL(N) states map via analytic continuation to light states of W_N minimal model

What happens in this limit to the other primaries?

$$h(f;0) \underset{c\to\infty}{\to} -\frac{1}{2}(N-1) \qquad h(0;f) \underset{c\to\infty}{\to} -\frac{c}{2N^2}$$

What happens in this limit to the other primaries?

$$h(f;0) \underset{c\to\infty}{\longrightarrow} -\frac{1}{2}(N-1) \qquad h(0;f) \underset{c\to\infty}{\longrightarrow} -\frac{c}{2N^2}$$

Compare to

$$h(f;0) \underset{N,k\to\infty}{\longrightarrow} \frac{1}{2}(1+\lambda) \qquad h(0;f) \underset{N,k\to\infty}{\longrightarrow} \frac{1}{2}(1-\lambda)$$

What happens in this limit to the other primaries?

$$h(f;0) \underset{c\to\infty}{\to} -\frac{1}{2}(N-1) \qquad h(0;f) \underset{c\to\infty}{\to} -\frac{c}{2N^2}$$

Compare to

$$h(f;0) \underset{N,k\to\infty}{\to} \frac{1}{2}(1+\lambda) \qquad h(0;f) \underset{N,k\to\infty}{\to} \frac{1}{2}(1-\lambda)$$

Bulk interpretation

$$h(f;f) = h(f;0) + h(0;f) - \frac{N-1}{N}$$

$h(f;f)$ → Conical

$h(f;0)$ → Scalar

$h(0;f)$ → Conical defect + sprinkle of scalar

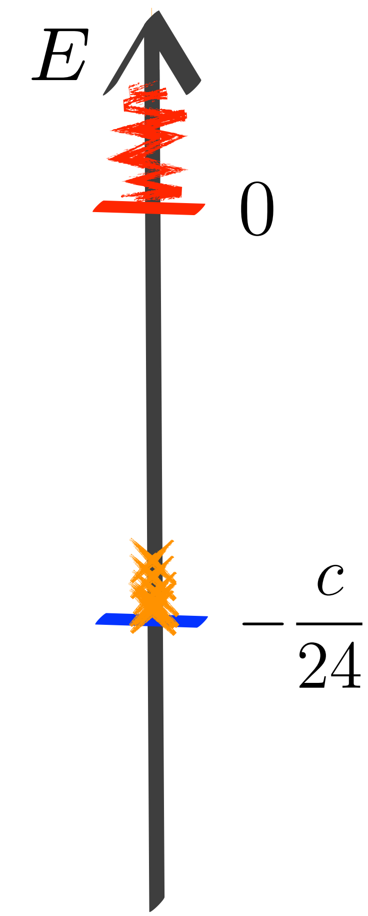
Is the black hole still the king (or queen)?

Is the black hole still the king (or queen)?

Black dominates at high temperature,
but what are we counting?

$$S_{\mathrm{BH}} = S_{\mathrm{Cardy}}$$

"Typical" gravity behavior:
Hawking-Page transition



Conclusions

RG flow equations in AdS/CFT

Are black holes truly big in HS thy?

What is Cardy's formula counting?

Did we learn anything new?