Would Schrödinger's Cat Have Collapsed Its Own Wavefunction? A Search for Gravitational Decoherence

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Abstract

Despite the lack of a complete theory of quantum gravity, one can still extract information from simple general relativistic systems for which approximate quantum treatments can be given. This work focuses on a minisuperspace model of a spherical perfect fluid shell, and we consider whether or not this idealized system can exhibit the phenomenon known as "gravitational decoherence". Originally suggested by Feynman in the '60s and notoriously advocated by Roger Penrose since the '80s, gravitational decoherence is claimed to result from taking the effects of spacetime curvature into account when forming quantum superpositions. The model considered here is a generalization of the dust shell model that Kraus and Wilczek used in the mid 90s in an attempt to correct the standard Hawking radiation to include gravitational back-reaction. For our purposes, the spherical fluid shell (with classical 4-velocity u^{μ}) will be a self-consistent interferometer, designed for splitting apart quantum wave-packets to create "Schrödinger cat" states. The split-apart wave-packets are then brought back together to interfere with themselves, and examined for traces of decoherence. If present, this type of decoherence could provide a mechanism for the universe to effectively "observe" itself, i.e., for Schrödinger's cat to "collapse" its own wavefunction.

Penrose's Argument from Geometry

Standard (Degenerate) Quantum Stationary States

$$i\hbar\partial\left|\psi\right\rangle/\partial t=\hat{H}\left|\psi\right\rangle=E\left|\psi\right\rangle,\ \hat{H}\left|\chi\right\rangle=E\left|\chi\right\rangle,\ \hat{H}\left(\lambda\left|\psi\right\rangle+\mu\left|\chi\right\rangle\right)=E\left(\lambda\left|\psi\right\rangle+\mu\left|\chi\right\rangle\right)$$

- What about gravity? In a stationary spacetime, one can define stationary quantum states as eigenstates of the timelike Killing vector $T=\partial/\partial t$ (when viewed as a differential operator)
- How does T act on superpositions of states corresponding to spacetimes with different timelike Killing vectors?
- Penrose argues that this inherent ambiguity leads to an instability, when superposing two gravitational fields $\vec{\Phi}$ and $\vec{\Phi}'$ (in the Newtonian limit):

$$\Delta t\simeq\hbar/E_{\Delta},\qquad E_{\Delta}\sim\int d^3x\left(\vec{\Phi}-\vec{\Phi}'\right)^2$$

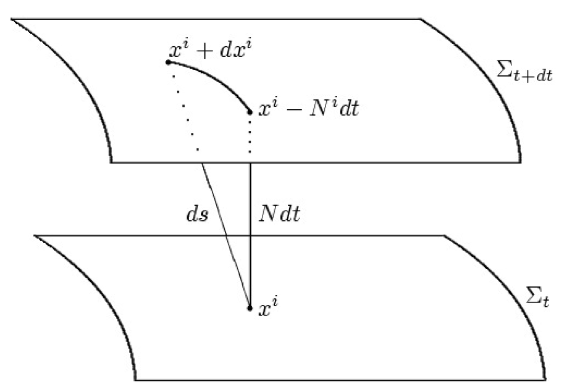
Assume a Spherical Cat...

Action for a Spherical Self-Gravitating Perfect Fluid Shell

$$I=\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,\mathscr{R}-\int d\lambda\sqrt{-\hat{g}_{\mu\nu}\frac{d\hat{x}^{\mu}}{d\lambda}\frac{d\hat{x}^{\nu}}{d\lambda}}\,\widehat{M(R)}$$

(Hereafter, hats indicate quantities evaluated on the Shell History)

Spacetime Metric in ADM Form



$$ds^2=-N^2dt^2+L^2\left(dr+N^r dt\right)^2+R^2d\Omega^2$$

Action In Hamiltonian Form

$$I=\int dt(P\dot{X}-m_{ADM})+\int dt\,dr\left(\pi_R\dot{R}+\pi_L\dot{L}-NH_0-N^rH_r\right)$$

Hamiltonian and Momentum Constraint Equations

$$H_0=\frac{L\pi_L^2}{2R^2}-\frac{\pi_L\pi_R}{R}+\left(\frac{RR'}{L}\right)'-\frac{(R')^2}{2L}-\frac{L}{2}+\sqrt{L^{-2}P^2+M(R)^2}\,\delta(r-X)=0$$
$$H_r=R'\pi_R-L\pi_L'-P\delta(r-X)=0$$

Reduced Spherical Cat Action

- Gravitational constraints are solved in flat-slice coordinates such that $R=X$ holds at the shell location
- The system is reduced to having one physical degree of freedom, X, which gives the shell's reduced area
- In these coordinates, asymptotic time evolution is generated by the ADM mass: $H=m_{ADM}$

$$I_{reduced}=\int dt\left(P_c\dot{X}-H(X,P_c)\right)$$

Implicit Definition of the Reduced Hamiltonian $H(X,P_c)$

$$P_c=-\sqrt{2HX}-X\ln\left(\frac{X+\beta-\sqrt{2HX}}{X}\right),\quad \beta=\frac{H-\frac{\hat{M}^2}{2X}\mp\sqrt{\left(H-\frac{\hat{M}^2}{2X}\right)^2-\hat{M}^2\left(1-\frac{2H}{X}\right)}}{1+\sqrt{\frac{2H}{X}}}$$

A Perfect Fluid on the Shell

Density, σ, and Tangential Pressure, p

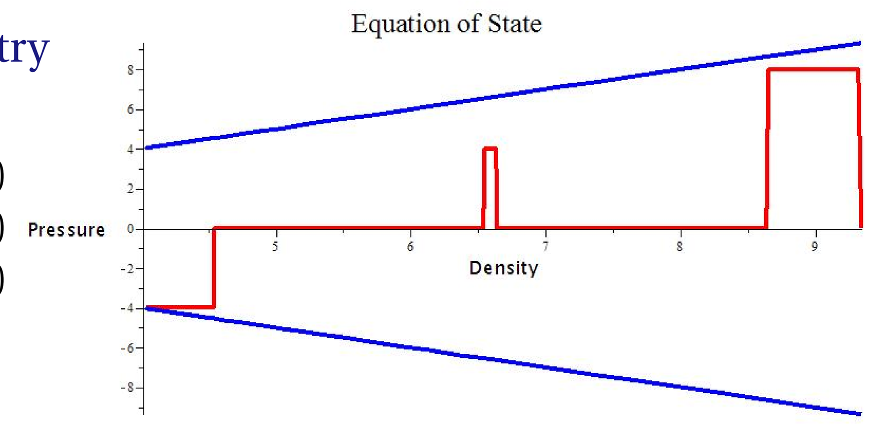
$$\sigma=M(X)/4\pi X^2\equiv\hat{M}/4\pi X^2,\quad p=-M'(X)/8\pi X,\quad T^{\mu\nu}=\left(\sigma u^{\mu}u^{\nu}+p\,g^{\Omega\Omega}\delta^{\mu}_{\Omega}\delta^{\nu}_{\Omega}\right)\delta(\chi)$$

Perfect Fluid Stress-Energy Tensor in Coordinates Intrinsic to the Shell History

$$S^{ab}=\sigma u^a u^b+p(\gamma^{ab}+u^a u^b)\quad\text{(Gaussian Normal Coordinate }\chi=0)$$
$$T^{\mu\nu}=S^{ab}e^{\mu}_a e^{\nu}_b\delta(\chi),\quad \gamma_{ab}=e^{\mu}_a e^{\nu}_b g_{\mu\nu},\quad e^{\mu}_a=u^{\mu}\delta^{\tau}_a+\delta^{\mu}_{\Omega}\delta^{\Omega}_a$$

Equation of State for Interferometry

$$\begin{aligned}p=\ &p_1\ (\Theta(\sigma-\sigma_1)-\Theta(\sigma-\sigma_2))\\ +\ &p_2\ (\Theta(\sigma-\sigma_3)-\Theta(\sigma-\sigma_4))\\ +\ &p_3\ (\Theta(\sigma-\sigma_5)-\Theta(\sigma-\sigma_6))\end{aligned}$$



Parametrization

$$M(X)=M_0+4\pi\sum_i\tilde{p}_i\left(X_i^2-X^2\right)\Theta(X-X_i)$$
$$\tilde{p}_2=-\tilde{p}_1=p_1<0,\quad \tilde{p}_4=-\tilde{p}_3=p_2>0,\quad \tilde{p}_6=-\tilde{p}_5=p_3>0$$

- Constraint for negative pressure peak: $\sigma_1+p_1>0$
- Shell is bound between two regions ($X_5<X<X_4$ and $X_3<X<X_2$) with constant 'masses' $M_{(\mp)}$, and one region $X_4<X<X_3$ with a quadratic 'mass' $M_{(0)}$:

$$M_{(-)}=\frac{M_0\sigma_5(\sigma_6+p_3)}{\sigma_6(\sigma_5+p_3)}<M_0$$
$$M_{(+)}=\frac{M_{(-)}\sigma_3(\sigma_4+p_2)}{\sigma_4(\sigma_3+p_2)}<M_{(-)}$$
$$M_{(0)}=M_{(-)}+4\pi\left(X_4^2-X^2\right)$$

$$X_6^2=\frac{M_0}{4\pi\sigma_6}$$
$$X_5^2=\frac{M_0}{4\pi\sigma_6}\frac{(\sigma_6+p_3)}{(\sigma_5+p_3)}$$
$$X_4^2=\frac{M_0\sigma_5}{4\pi\sigma_4\sigma_6}\frac{(\sigma_6+p_3)}{(\sigma_5+p_3)}$$
$$X_3^2=\frac{M_0\sigma_5}{4\pi\sigma_4\sigma_6}\frac{(\sigma_6+p_3)}{(\sigma_5+p_3)}\frac{(\sigma_4+p_2)}{(\sigma_3+p_2)}$$
$$X_2^2=\frac{M_0\sigma_3\sigma_5}{4\pi\sigma_2\sigma_4\sigma_6}\frac{(\sigma_6+p_3)}{(\sigma_5+p_3)}\frac{(\sigma_4+p_2)}{(\sigma_3+p_2)}$$
$$X_1^2=\frac{M_0\sigma_3\sigma_5}{4\pi\sigma_2\sigma_4\sigma_6}\frac{(\sigma_6+p_3)}{(\sigma_5+p_3)}\frac{(\sigma_4+p_2)}{(\sigma_3+p_2)}\frac{(\sigma_2+p_1)}{(\sigma_1+p_1)}$$

- Parameters: $M_0>0, p_1<0, p_2>0, p_3>0, 0<\sigma_1<\cdots<\sigma_6$ (constrained only by $\sigma_1+p_1>0$)

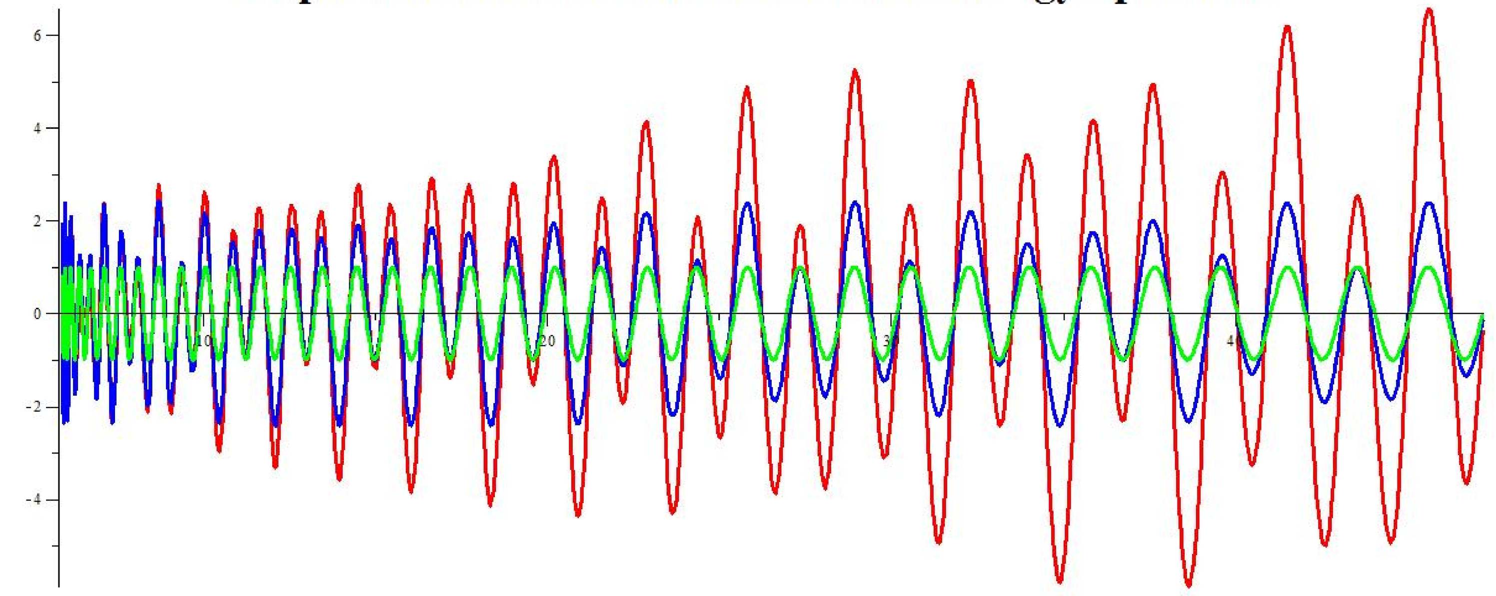
Quantum Coherence?

Approximate Energy Eigenfunctions

$$\Psi_E(X,t)\approx e^{-iEt/\hbar}\sum_{\pm}\frac{C_{\pm}}{\sqrt{\partial E/\partial P_c^{\pm}}}\exp\left(\frac{i}{\hbar}\int dX\,P_c^{\pm}(E,X)\right)$$

- Using these WKB approximations, along with boundary conditions (assuming perfect reflection on the inner and outer pressure peaks, p_3 and p_1), we can obtain a discrete energy spectrum, which enables us to form wavepackets

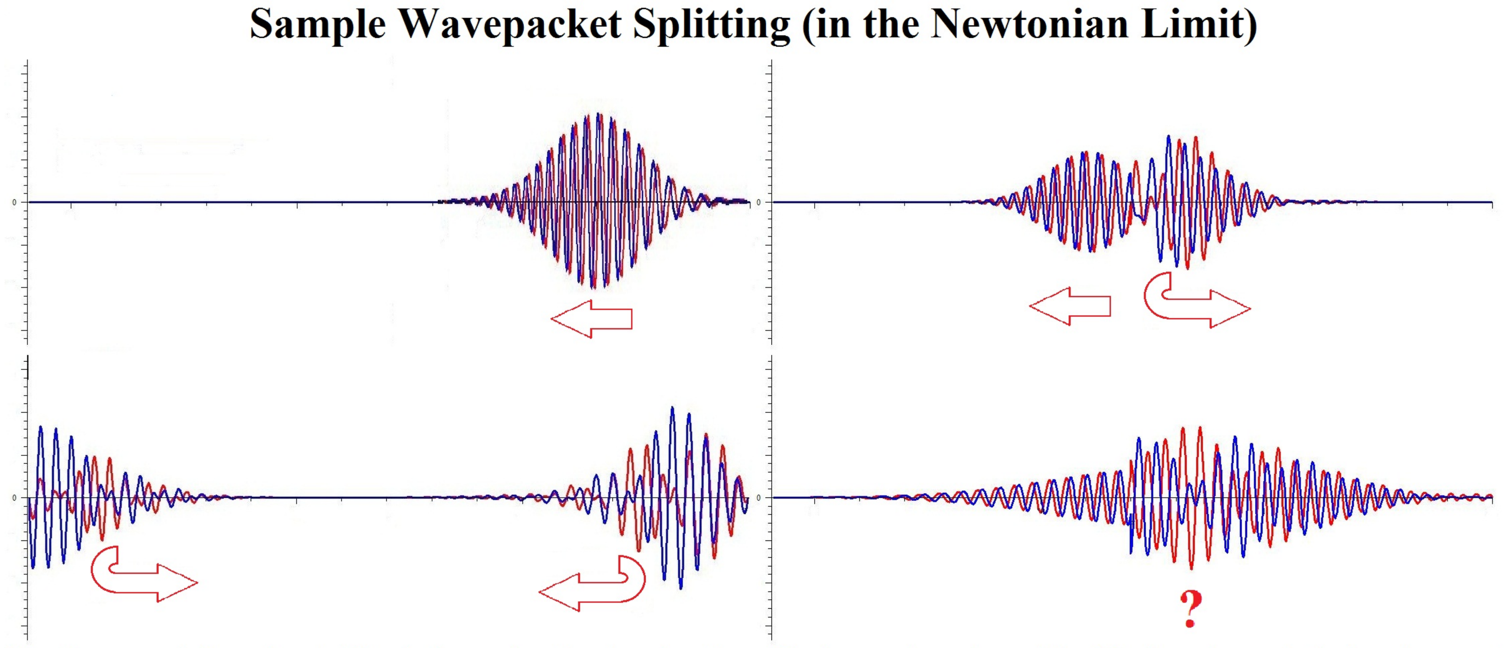
Sample Determination of the Discrete Energy Spectrum



Energy eigenvalues are zeros of the red function, while the blue/green zeros represent different analytical approximations

- If we set up an initial pulse in the outer region travelling inward, it will continue inward until the central pressure peak (p_2). At this point the pulse splits apart, as part of the wavepacket reflects off of the pressure peak
- The split-apart portions of the wavepacket reflect off of the inner and outer peaks p_3 and p_1, and are brought back together again

Sample Wavepacket Splitting (in the Newtonian Limit)



The blue curve is the real part of the shell wavefunction (as a function of X at successive values of t), while the red curve is the imaginary part

- General relativistic simulations currently in progress
- What happens if we use Schwarzschild-like time, $t_S=t-2\sqrt{2XE}+2E\ln\left(\frac{\sqrt{X}+\sqrt{2E}}{\sqrt{X}-\sqrt{2E}}\right)$?
- Can this framework be used to demonstrate that gravity allows our "Schrödinger cat" states to continuously observe themselves?

Acknowledgments

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