Fun with Infinities at Future Infinity

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The motivation is to "turn AdS/CFT on the side", or more precisely to "analytically continue" from AdS to dS. In fact this continuation is valid order by order in perturbation theory (Maldacena, Harlow+Stanford), so the idea isn't immediately crazy.

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Today I will present some concrete calculations in a real example, and along the way we will see how several of these complaints are addressed.

(work with Anninos and Denef)

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The χ^a's are Grassman scalars transforming in the N dimensional representation of $Sp(N)$. (N is even).

Properties of the $Sp(N)$ Theory

- It is conformal, with a whole set of primary conserved currents, of the form

$$J_{\mu_1 \ldots \mu_s} \equiv \Omega_{ab} \chi^a (\partial_{\mu_1} \ldots \partial_{\mu_s}) \chi^b. \tag{3}$$

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- The spectrum of operators matches Vassiliev's only if we impose an $Sp(N)$ singlet constraint; we can do this for example by weakly gauging the $Sp(N)$ symmetry by coupling it to Chern-Simons with $k \to \infty$ (Shenker, Yin).

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- It is free!

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- The thing we wish to compute then is

$$Z_{CFT}[g,\sigma] \equiv \lim_{k\to\infty} \int \mathcal{D}A\,\mathcal{D}\chi \exp\Big[- S_{CS}[A] - S_{CFT}[g,\chi,A] \\ - \frac{1}{2}\int d^3x\sqrt{g}\sigma(x)\chi^2\Big].$$

An Important Subtlety

Before proceeding to describe our results, I want to emphasize an important point. Dio previously defined the *Critical* $Sp(N)$ theory as the IR limit of a "double-trace" deformation of the free $Sp(N)$ theory by $(\chi^2)^2$. What is the deal with this?

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- Consider a general double-trace flow:

$$Z_{IR}[\tilde{\sigma}] = \int \mathcal{D}\mathcal{M} \exp\left[-S_{CFT} + \rho f \tilde{\sigma}\mathcal{O} - \frac{f}{2}\int d^d x \mathcal{O}^2\right]. \quad (4)$$

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- The usual thing to do here is to introduce an auxilliary field, in terms of which the generating functional of the IR critical theory is related to the UV theory by a Hubbard/Stratonovich transformation. Heuristically

$$Z_{IR}[\tilde{\sigma}] = \int \mathcal{D}\sigma e^{A\sigma^2 - B\sigma\tilde{\sigma}} Z_{UV}[\sigma]. \quad (5)$$

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- This is simpler than in AdS/CFT, where the UV theory is interpreted as a different quantization of the bulk theory than the IR theory.
- More specifically, in the critical $Sp(N)$ theory we have

$$\sqrt{N}\tilde{\sigma} = \epsilon^{\Delta - d}\phi \tag{6}$$

while in the free $Sp(N)$ theory we have

$$\sqrt{N}\sigma = 4\left(T\partial_T\phi - \phi\right). \tag{7}$$

Our Calculations

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- A constant mass deformation on S^3 in the free theory.
- Zero mass deformation on $S^2 \times S^1$ in the free theory.
- Zero mass deformation on a squashed S^3 in the free theory.
- Constant mass deformation on S^3 in the critical theory.

Constant Mass on S^3

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- Some renormalization is necessary in computing this determinant; we fix this by insisting that on a round S^3

$$Z_{CFT}[0] = 1 \tag{9}$$

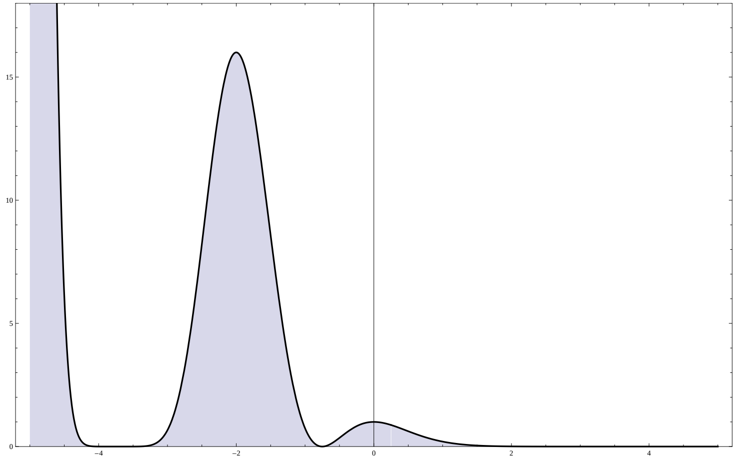
$$\left.\frac{\delta Z_{CFT}}{\delta\sigma}\right|_{\sigma=0} = 0. \tag{10}$$

S^3 continued

Now choosing $\sigma(x) = \sigma_0$, we have

$$\log Z_{CFT} = \frac{N}{2} \sum_{\ell=0}^{\infty} (\ell+1)^2 \left[\log \left(1 + \frac{\sigma_0}{\ell(\ell+2)+3/4} \right) - \frac{\sigma_0}{\ell(\ell+2)+3/4} \right]. \tag{11}$$

This sum can be evaluated analytically in terms of Polylogarithms, but a picture is more illuminating-



This shows a plot of $|\Psi|^2$ as a function of the zero mode of the mass. Two interesting points

- It has zeroes at $\sigma = -\frac{3}{4} - \ell(\ell + 2)$ - these are fermion zero modes.
- It is non-normalizeable at large negative σ!

Some comments on the non-normalizeability

- This does NOT happen for a free massive scalar in a *dS* background. In that calculation one finds

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- We suspect that the interpretation of this result is that de Sitter space is not stable in Vassiliev theory; this is consistent with general arguments about the impossibility of a stable theory of dS space. (Goheer, Kleban, Susskind)

$S^2 \times S^1$

- We can also study the theory on $S^2 \times S^1$ as a function of β, the relative size of S^1 and S^2 - the machinery for this calculation was developed by many of the people in this room: (Gross, Witten, Aharony, Marsano, Minwalla, Papadodimas, Van Raamsdonk, Shenker, Yin).

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- This turns out to be annoying to deal with in the $Sp(N)$ model because $Sp(N)$ is a real group, so for this example we will change the symmetry group to $U(N)$. The action now is based on terms like $\chi^\dagger\chi$ instead of $\chi^T\Omega\chi$, but the only real difference is that when we integrate out χ's we need to square the determinant.

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- We now need to choose boundary conditions for the fermions around the S^1. It turns out that in the singlet sector the choice does not matter, as I will point out in a moment.

After integrating out the fermions and gauge fixing, one finds (people I mentioned above)

$$Z = \frac{1}{N!}\int \prod_i d\alpha_i \exp\Big[\sum_{i<j}\log\sin^2[\frac{\alpha_i-\alpha_j}{2}] \\ -2\sum_{m=1}^{\infty}\frac{1}{m}z_S(m\beta)\sum_{i=1}^{N}\cos(m\alpha_i)\Big] \qquad (13)$$

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This expression is exact, but at large N we can approximate it by looking for a saddle point in the integral over α_i (Gross/Witten).

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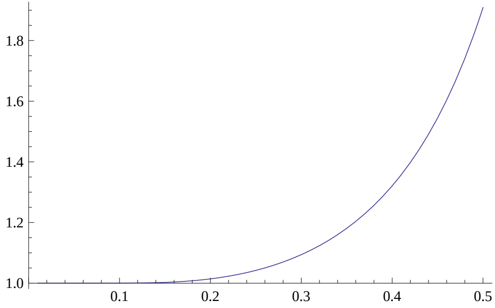

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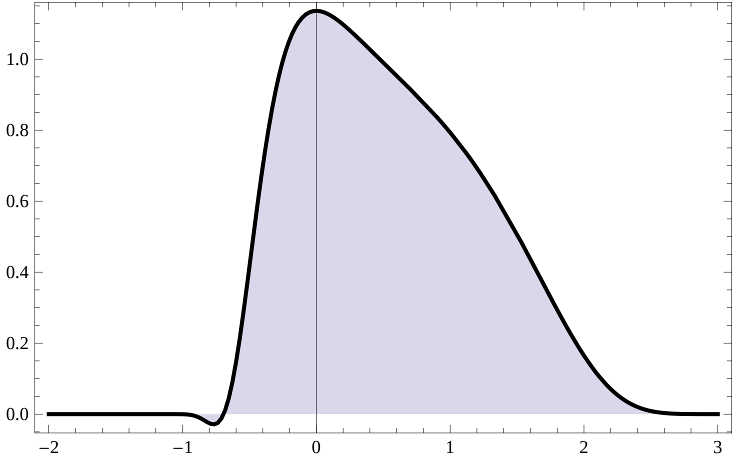
- Moreoever in Einstein gravity we have found that we can actually reproduce this divergence via a Hawking-Page type semiclassical calculation, but perhaps surprisingly it is dimension dependent. It happens in bulk dimensions $d = 6 \bmod 4$, so for four dimensions it actually doesn't happen!

Squashed Sphere

We also computed the wave function for a squashed sphere:



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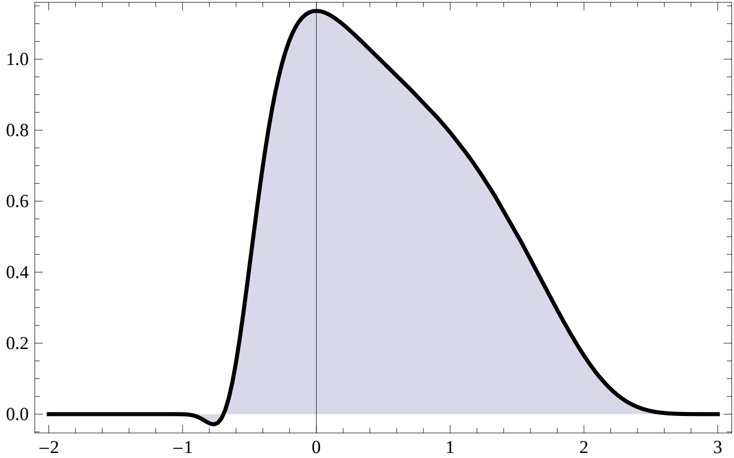

Note this one is normalizeable!

Critical Mass

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- This means that the perturbative fixed point one computes by summing cactus diagrams is not what the double trace flow from the free theory actually produces! The dominant saddle point is far off at negative σ, and it goes off to infinity at late times. The reason is that probability is constantly flowing out to negative σ, and going back to field space the wave function is very time dependent.

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- Our suspicion is that this prescription is an unnatural conditioning and that it is unstable to runaways with the slightest perturbation of the initial conditions, but we have not been able to say this decisively.

Conclusion

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Things to work on:

- Understand the bulk better.
- Understand the irrelevant operators.
- How is this connected to string theory?
- How is this framework affected by eternal inflation? ¡- crucial for understanding phenomenology!
- How can we think about the static patch - in particular how is this related to dS/dS and/or conformal quantum mechanics?