Black Holes in 3D Higher Spin Gravity

Per Kraus (UCLA)

Work with Ammon, Gutperle, Perlmutter

Introduction

- Higher spin gravity is an (apparently) consistent theory that sits “midway” between low energy field theory and string theory (Vasiliev)
  - infinite towers of fields
  - nonlocal dynamics
  - huge enlargement of gauge symmetry

- Extra symmetry provides soluble examples of AdS/CFT correspondence (Klebanov/Polyakov; Gaberdiel/Gopakumar; …)
  - can we gain insight into the big problems of quantum gravity?

3D HS Gravity

- Extension of Chern-Simons formulation of ordinary gravity with $\Lambda < 0$ (Achucarro, Townsend / Witten)

$$\text{vielbein} \quad e^a_\mu \ , \qquad \text{spin connection} \quad \omega^a_\mu = \frac{1}{2}\epsilon^a{}_{bc}\omega^{bc}_\mu$$

SL(2,R) x SL(2,R) gauge fields $A = (\omega^a + \frac{1}{l}e^a)J_a \ , \quad \overline{A} = (\omega^a - \frac{1}{l}e^a)J_a$

$$[J_a, J_b] = \epsilon_{ab}{}^c J_c \qquad \mathrm{Tr} J_a J_b = \eta_{ab}$$

$$R_{\mu\nu} = \frac{1}{l^2} g_{\mu\nu} \quad \Longleftrightarrow \quad \begin{array}{l} dA + A \wedge A = 0 \\ d\overline{A} + \overline{A} \wedge \overline{A} = 0 \end{array}$$

$$S = \frac{k}{4\pi}\int \mathrm{Tr}(AdA + \frac{2}{3}A^3) - \frac{k}{4\pi}\int \mathrm{Tr}(\overline{A}d\overline{A} + \frac{2}{3}\overline{A}^3) \qquad k = \frac{l}{4G} = \frac{c}{6}$$

- Replacing SL(2) by a larger algebra that contains SL(2) yields a higher spin gravity theory

- Ordinary 3D gravity is a consistent subsector

  - example: SL(3) describes ordinary gravity coupled to a massless spin-3 field (Campoleonii et. al.)

$$g_{\mu\nu} \sim \mathrm{Tr}(\mathrm{e}_\mu \mathrm{e}_\nu) \ , \quad \varphi_{\alpha\beta\gamma} \sim \mathrm{Tr}(\mathrm{e}_\alpha \mathrm{e}_\beta \mathrm{e}_\gamma)$$

$$e \sim A - \overline{A}$$

Gauge symmetry includes coord. transformations under which $g_{\mu\nu}$ and $\varphi_{\alpha\beta\gamma}$ transform as tensors, as well as spin-3 gauge transformations under which $g_{\mu\nu}$ transforms in novel way

e.g. Ricci scalar not gauge invariant

- Just as SL(2) gravity has asymptotic Virasoro symmetry, HS theories have asymptotic W-algebras containing higher spin currents

(Henneaux/Rey; Campoleoni et. al.)

- Pure HS theory contains no propagating degrees of freedom. Adding in propagating matter requires an infinite dimensional gauge algebra.

(Prokushkin/Vasiliev)

hs(λ)

- Introduce $y_{1,2}$ and the Moyal product:

$$f(y_\alpha) * g(y_\beta) = e^{i\epsilon^{\alpha\beta}\partial_\alpha \partial'_\beta} f(y_\alpha) g(y'_\beta)\Big|_{y'=y}$$

$$[y_1, y_2]_* = 2i$$

- Elements of hs(1/2) are symmetric, even degree polynomials
  - SL(2) generated by:

$$L_1 = -\tfrac{i}{4} y_1^2 \ , \quad L_0 = -\tfrac{i}{4} y_1 y_2 \ , \quad L_{-1} = -\tfrac{i}{4} y_2^2$$

- General case of hs(λ) obtained from deformed commutator: $[y_1, y_2]_* = 2i(1+\nu k), \quad \lambda = (1+\nu)/2$

Matter

- Let $C(x^\mu)$ be a hs(λ) valued function obeying

$$dC + A * C - C * \overline{A} = 0$$

  - Evaluated in AdS, the lowest component of C obeys the KG equation with $m^2 = \lambda^2 - 1$

  - More generally, presence of higher spin fields in background leads to higher derivative generalization of KG equation. Nonlocal in general.

- Full interacting extension is known; fixed by gauge symmetry (Prokushkin, Vasiliev)

Duality

- Asymptotic symmetry algebra is $W_\infty(\lambda)$
- This and other properties matches up with those of W_N minimal model CFTs

$$\frac{SU(N)_k \oplus SU(N)_1}{SU(N)_{k+1}} \qquad k, N \to \infty \;, \qquad \lambda = \frac{N}{k+N} \quad \text{fixed}$$

$$c \sim N(1-\lambda^2)$$

Gaberdiel and Gopakumar conjecture an AdS/CFT duality

- much of the challenge in proving this involves defining the bulk theory at the fully quantum, non-perturbative level

Black Holes

- Questions:
  - Can we find and understand black hole solutions in HS gravity, including those with higher spin charge?
  - Can we match black hole entropy to CFT entropy?
  - Can we study info. loss paradox, by creating a black hole and letting it evaporate, or by studying AdS/CFT correlators in the black hole background?
- In principle, everything is computable due to large amount of symmetry

Building HS Black Holes

- BTZ is trivially a solution, and its entropy matches CFT
- More interesting are BHs carrying higher spin charge. Focus on solutions with spin-3 charge
  - These can be embedded in either SL(3) or hs(λ) theories. In latter case we can compare with minimal model CFT result.
- Main challenge: due to enhanced spacetime symmetries, definition of black hole is not obvious

Building HS Black Holes

- BTZ:

$$A = (e^\rho L_1 - \tfrac{2\pi}{k} e^{-\rho} \mathcal{L} L_{-1}) dx^+ + L_0 d\rho$$
$$\overline{A} = -(e^\rho L_{-1} - \tfrac{2\pi}{k} e^{-\rho} \overline{\mathcal{L}} L_1) dx^- - L_0 d\rho$$

$\Rightarrow$ BTZ

$$\mathcal{L} = \frac{M-J}{4\pi} \quad \overline{\mathcal{L}} = \frac{M+J}{4\pi}$$

- Now add in spin-3 chemical potential. Ward identity analysis establishes:

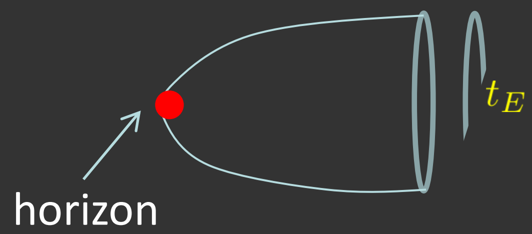
$$A_- \sim \mu e^{2\rho} W_2 + \cdots$$

chiral spin-3 chemical potential → μ; spin-3 generator → W_2

- Expect this to induce spin-3 charge: $A_+ \sim e^{-2\rho} \mathcal{W} W_{-2}$
  - In hs(λ) case expect infinite number of charges to be induced, due to nonlinear symmetry algebra

Smoothness conditions

- ordinary gravity: relation between (M,Q) and their conjugate potentials (T,μ) fixed by smoothness at Euclidean horizon



- Inapplicable for HS black holes, since even existence of event horizon is a gauge dependent statement. Need a new gauge invariant condition

- gauge invariant information captured by holonomies of CS gauge fields
- We demand that holonomy around Euclidean time circle should be trivial (as it is for BTZ)
- Gives precisely enough information to fix all charges in terms of the potentials

Thermodynamics

- Think of black hole as contribution to

$$Z(\tau,\alpha) = \mathrm{Tr}\left[e^{4\pi^2 i(\tau\mathcal{L}+\alpha\mathcal{W})}\right]$$

$$\tau = \text{modulus of boundary torus}$$
$$\alpha = \bar{\tau}\mu = \text{spin}-3 \text{ chemical potential}$$

- Existence of Z requires that we obey integrability condition:

$$\frac{\partial\mathcal{L}}{\partial\alpha} = \frac{\partial\mathcal{W}}{\partial\tau}$$

- Turns out that holonomy conditions imply integrability of charges

- We can now integrate to find black hole entropy. For SL(3) this can be done exactly:

$$S = 2\pi\sqrt{2\pi k\mathcal{L}}\ f\left(\frac{27k\mathcal{W}^2}{64\pi\mathcal{L}^3}\right)$$

$$f(x) = \cos\left[\frac{1}{6}\arctan\left(\frac{\sqrt{x(2-x)}}{1-x}\right)\right] = 1 - \frac{1}{36}x - \frac{35}{776}x^2 + \cdots$$

- This is the spin-3 generalization of Cardy's formula. Should apply to any CFT with $\mathcal{W}_3$ symmetry and c >> 1

complementary approaches: (Castro, Hijano, Lepage-Jutier, Maloney) (Banados, Canto, Theisen)

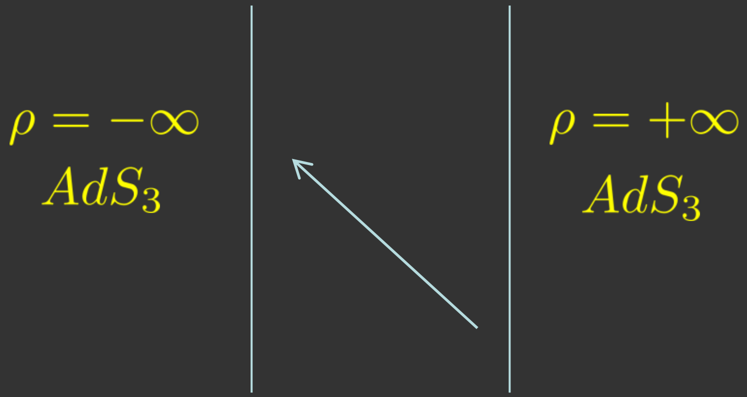
Causal structure

- Metric for non-rotating case takes form

$$ds^2 = d\rho^2 - F(\rho)dt^2 + G(\rho)d\phi^2$$

$$F(\rho),\ G(\rho)\ >\ 0$$ no event horizon!

traversable wormhole:



- But when holonomy conditions are obeyed, one can find a true black hole metric somewhere on this gauge orbit

Black holes in hs[λ]

- The spin-3 chemical potential α now sources an infinite number of charges. System can be solved perturbatively in α

Partition function

$$\ln Z(\tau,\alpha) = \frac{i\pi k}{2\tau}\left[1 - \frac{4}{3}\frac{\alpha^2}{\tau^4} + \frac{400}{27}\frac{\lambda^2-7}{\lambda^2-4}\frac{\alpha^4}{\tau^8} - \frac{1600}{27}\frac{5\lambda^4-85\lambda^2+377}{(\lambda^2-4)^2}\frac{\alpha^6}{\tau^{12}} + \cdots\right]$$

valid for: $\tau \to 0, \quad \alpha \to 0, \quad \frac{\alpha}{\tau^2} \quad \text{fixed}$

- should agree with CFT partition function

Comparison with CFT

λ=1: free bosons

D=3k complex bosons: $\mathcal{W} = ia(\partial^2 \overline{\phi}^i \partial \phi_i - \partial \overline{\phi}^i \partial^2 \phi_i)$

$$a = \sqrt{\frac{5}{12\pi^2}}$$

- expand in modes and compute partition function in presence of spin-3 chemical potential

$$\ln Z(\tau,\alpha) = -\frac{3ik}{2\pi\tau}\int_0^\infty \left[\ln\left(1 - e^{-x + \frac{2ia\alpha}{\tau^2}x^2}\right) + \ln\left(1 - e^{-x - \frac{2ia\alpha}{\tau^2}x^2}\right)\right]$$

expansion in α matches black hole result at λ=1

- similar story at λ=0 in terms of free fermions

Comparison with CFT

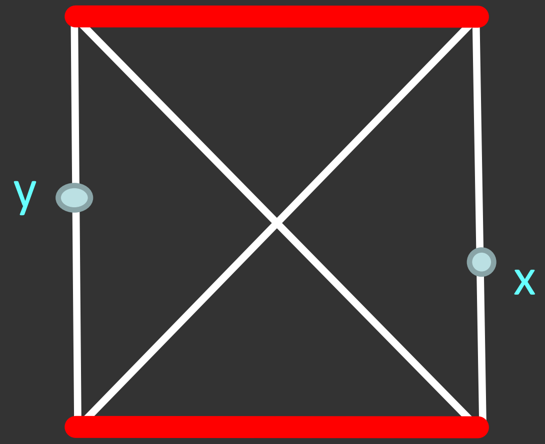
- For general λ, direct evaluation of CFT partition function yields agreement with black hole at first few orders (Gaberdiel, Hartman, Jin)

- As expected, only need to use symmetry algebra of theory to demonstrate agreement

Probing causual structure

- Our "black hole" metrics either look like traversable wormholes or black holes, depending on choice of gauge

- To map out physical causal structure we can compute AdS/CFT two-point functions of probe scalars, and look for lightcone singularities



- Black hole causal structure: G(x,y) nonsingular

Scalar two-point function

- Elegant approach to obtaining scalar bulk-boundary propagator: start from propagator at $A = \overline{A} = 0$, then gauge transform to physical solution c.f. (Giombi/Yin)

λ=1/2:

highest weight states: $\hat{C} = e^{-iy_1 y_2}$ or $y_1 * e^{-iy_1 y_2} * y_2$

gauge transform: $C = g^{-1}(\rho, z, \overline{z}) * \hat{C} * \overline{g}(\rho, z, \overline{z})$

bulk-bound. prop: $G(\rho, z, \overline{z}) = \mathrm{Tr}(\mathrm{C})$

- purely algebraic procedure

e.g. pure AdS: $g^{-1} = e^{-\rho L_0} * e^{-L_1 z}, \quad \overline{g} = e^{L_{-1}\overline{z}} e^{-\rho L_0}$

- Higher spin black hole viewed as perturbation of BTZ will presumably yield correlator that exhibits no singularities for operators on opposite boundaries. Would like to extend this to all orders.
- Similar approach can be used to efficiently compute scalar-scalar-current correlators, in agreement with CFT result

Open issues

- subleading corrections to entropy
- phase structure
- effect of light states/conical defects
- black holes formed from collapse?
- …