Toy Models and Fast Scrambling (II)

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Duck-billed Platypus

- lays eggs
- has a beak
- has fur

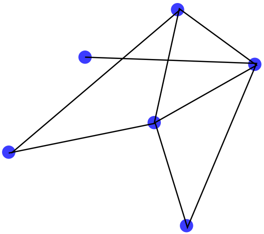
Fast Scrambler

- Hamiltonian
- scrambles all local perturbations
- in log time

Plan

1. eggs + fur (Hamiltonian + scrambles in log time)
2. eggs + beak (Hamiltonian + scrambles all local perturbations)
3. A Lieb-Robinson-type lower bound
4. Scrambling and AdS/CFT

Ising



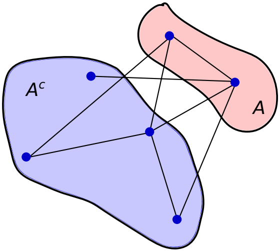
Ising model on a (nonlocal) graph $G = (V, E)$:

$$H = \frac{|V|}{|E|} \sum_{(i,j)\in \text{ edges}} \sigma_z^{(i)} \sigma_z^{(j)}.$$

System is integrable, but can still scramble in the σ_x eigenbasis. Consider an initial state

$$|\Psi(0)\rangle = |i_1^x\rangle |i_2^x\rangle ... |i_n^x\rangle.$$

Ising (2)



$$M = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$

The time evolution operator is periodic with period $\pi|E|/|V|$. At time $\pi|E|/2|V|$, the state $|\Psi(t)\rangle$ is as entangled as it is going to get. Moreover

$$S(\rho_A) = \text{rank}_{Z_2} M$$

where M is an $|A|$ by $|A^c|$ matrix, with $M_{ij} = 1$ if $i \in A$ is connected to $j \in A^c$, 0 otherwise.

Ising (3)

Math problem: minimize $|E|/|V|$ subject to constraint that M be full rank for almost all subsystems.

Our solution: take a random graph of connectivity

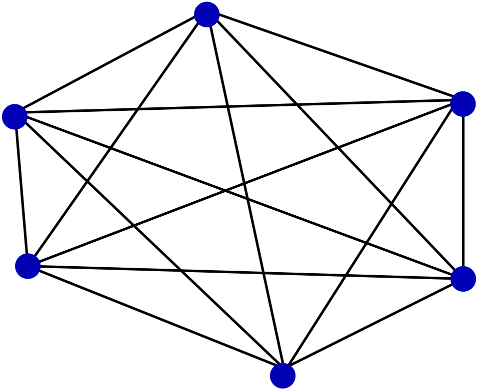
$$\left\langle \frac{|E|}{|V|} \right\rangle = \langle \# \text{ neighbors} \rangle = \log n.$$

For these graphs, the states $|i_1^x\rangle|i_2^x\rangle...|i_n^x\rangle$ get scrambled within a time

$$t_* = \frac{\pi}{2} \log n.$$

A numerical cautionary tale

$$H = \frac{1}{n} \sum_{\alpha,\beta,i,j} J_{\alpha\beta}^{(ij)} \sigma_\alpha^{(i)} \sigma_\beta^{(j)}$$



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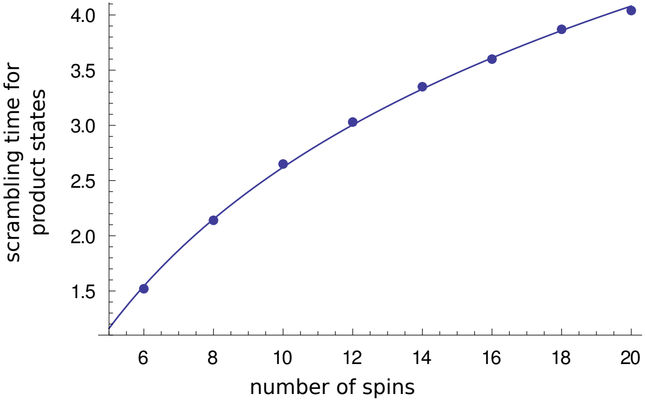

A numerical cautionary tale (2)

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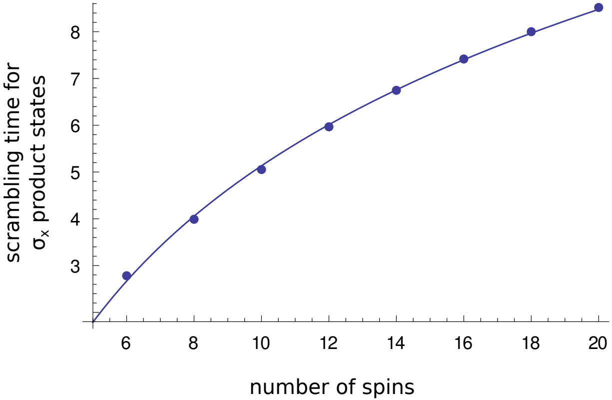

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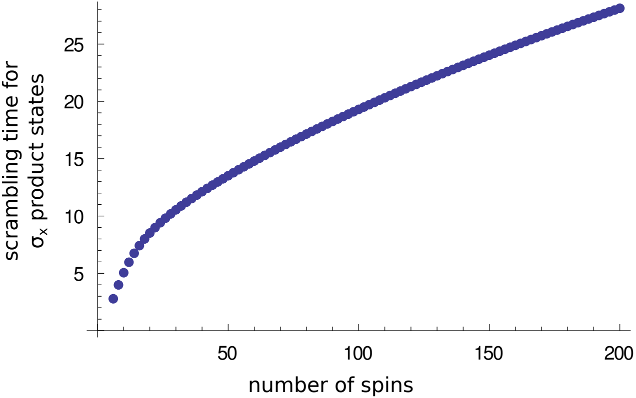
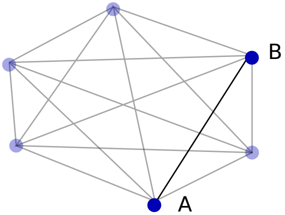


General bounds?

Take a completely nonlocal Hamiltonian

$$H = \frac{1}{n} \sum_{j,k} H_{jk}$$

and consider the evolution of the commutator $[A(t), B]$.



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$$[A(t), B] = [A, B] + it[[H, A], B] - \frac{t^2}{2}[[H, [H, A]], B] + \ldots$$

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$$\big\|[A(t), B]\big\| \leq \big\|[A, B]\big\| + \big\|t[[H, A], B]\big\| + \frac{t^2}{2}\big\|[[H, [H, A]], B]\big\| + \dots$$

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$$\|[A(t), B]\| \leq \|[A, B]\| + \|t[[H, A], B]\| + \frac{t^2}{2}\|[[H, [H, A]], B]\| + \ldots$$

$$\Big(|\sin(x)| \leq |x| + |x^3/3!| + |x^5/5!| + \ldots\Big)$$

Lieb-Robinson

There's a better way!

$$[A(t), B] = [A, B] + \sum_{j=1}^{m} [A(t_{j+1}), B] - [A(t_j), B]$$

Lieb-Robinson

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Lieb-Robinson

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$$\implies \big\|[A(t),B]\big\| \leq \big\|[A,B]\big\| + 2\big\|A\big\| \int_0^t ds \big\|[h(s),B]\big\|$$

where h is the part of H that doesn't commute with A.

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where h is the part of H that doesn't commute with A. Similarly,

$$\big\|[H_{jk}(s),B]\big\| \le \big\|[H_{jk},B]\big\| + 2\big\|H_{jk}\big\| \int_0^s ds' \big\|[h'(s'),B]\big\|$$

where h' is the part of H that doesn't commute with H_{jk}.

Lieb-Robinson (2)

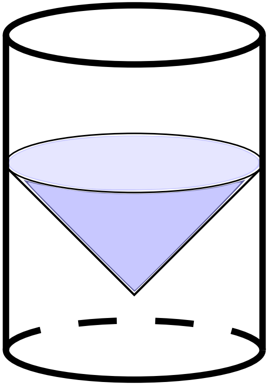
Iterate this inequality to get a bound. Roughly, one gets a sum over paths through the graph, starting at the vertex of A and ending at the vertex of B, weighted by $(t/n)^{\ell}/\ell!$. The number of such paths is $n^{\ell-1}$, so the sum is

$$\|[A(t), B]\| \leq \sum_{\ell=1}^{\infty} \left(\frac{t}{n}\right)^{\ell} \frac{n^{\ell-1}}{\ell!} \leq \frac{1}{n} \exp t.$$

So that

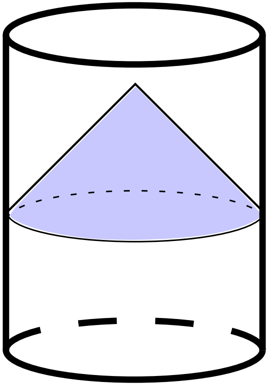
$$t_* \geq \log n.$$

Scrambling and AdS/CFT



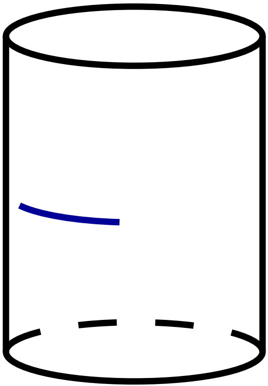
Polchinski, Susskind, Toumbas 1999. Probed *only* by nonlocal precursors of decreasing nonlocality. “Unscrambling”.

Scrambling and AdS/CFT



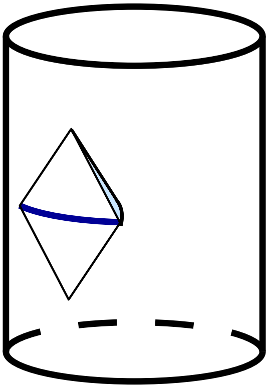
Probed *only* by nonlocal precursors of increasing nonlocality.
Radial causality $\sim$ scrambling.

Scrambling and AdS/CFT (2)



- (Susskind, Witten 1998)
- Bousso, Leichenauer, Rosenhaus 2012
- Hubeny, Rangamani 2012
- Czech, Karczmarek, Nogueira, Van Raamsdonk 2012

Scrambling and AdS/CFT (2)



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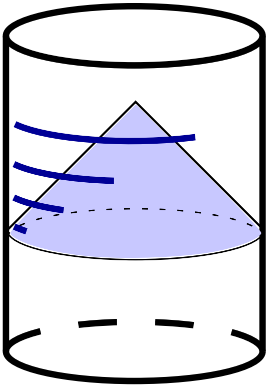
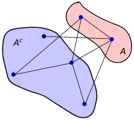
Scrambling and AdS/CFT (3)

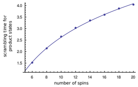


$$\ell \sim t$$

These perturbations scramble ballistically, not diffusively!

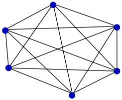
Conclusions



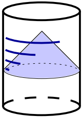


Ising: simple systems can scramble certain states fast

Numerics: robust scrambling, but can't find log



L-R: even complete graphs have speed limits



AdS/CFT: "ballistic" scrambling $\sim$ radial causality

Future?

- Find a real fast scrambler?
- Attack the matrix Hamiltonian directly?
- Relate this work to other approaches: Barbon/Magan hyperbolic diffusion, Asplund/Berenstein/Trancanelli numerical work, holographic thermalization.