

# Superthreads and Superstrata:

*New BPS Solutions in Six Dimensions*



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*Black Holes and Information, May 24<sup>th</sup>, 2012*

**Based upon work with**

**I. Bena, J. de Boer, S. Giusto and M. Shigemori  
B. Niehoff and O. Vasilakis**

## The Issue

Find, classify and understand **regular, horizonless solutions** with the same asymptotic structure as a given black hole or black ring  
 $\Leftrightarrow$  **Microstate Geometries** (definition)

- BPS/supersymmetric
- Extremal, non-BPS
- Non-extremal

## Motivation

- What is new about stringy black holes? *Failure of uniqueness ...*
- Semi-classical description of families black hole microstates
  - ★ What sectors of black-hole CFT are captured by diverse microstate geometries?
  - ★ How many microstates can be captured by such geometries
  - ★ **How densely distributed are these microstates?** Good enough for a semi-classical description of black-hole thermodynamics?

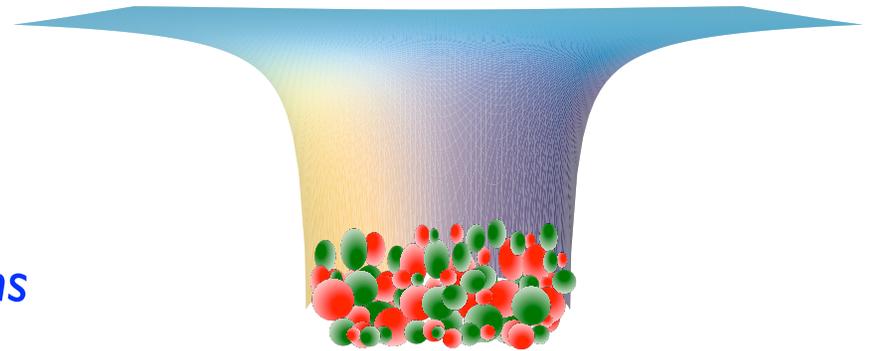
## This Talk:

- ★ Conjecture: there is a completely new class of *smooth BPS* solutions to supergravity, called *Superstrata* with
  - ◆ Fluctuating profiles that depend upon functions of *two variables*.
  - ◆ Carry *three electric charges* and *two independent magnetic dipole charges*.
- ★ Why important?
  - ◆ *Two-charge microstate* geometries characterized by *supertubes* which carry *two electric and one magnetic dipole charge* and depend on functions of *one variable*
  - ◆ Superstrata would represent, for the three-charge BPS black hole, precisely what the two-charge supertube represents for the two-charge BPS black hole: *Its own special solitonic bound state with much larger families of microstate geometries*
- ★ Why we believe the superstratum exists
  - ◆ Constructive argument/algorithm
- ★ How far have we got with the explicit construction

*Context: Highlights of microstate geometries thus far...*

# Three-charge BPS black holes in five dimensions

Horizonless, completely regular bubbled geometries that cap-off at the horizon scale and have the same quantum numbers as BPS black holes with *non-trivial, macroscopic horizons*



- ★ Branes replaced by cohomological fluxes on non-trivial cycles (*bubbles*)
- ★ Bubbles can be made much larger than String/Planck scale  
⇒ Supergravity approximation is valid
- ★ AdS throat ⇒ holography can establish dual CFT states
  - ★ Multi-centered geometries: Coulomb branch of dual field theory. *Denef et al.*
- ★ Depth of throat is limited by quantization of moduli

Analysis of the energy gap of excitations of the deepest allowed deep/scaling microstate geometries yields  $E_{excitations} \sim E_{gap; CFT} \sim (C_{cft})^{-1}$

⇒ **Representative of the “*typical sector*” of CFT**

**Bena, Wang and Warner, arXiv:hep-th/0608217**

**de Boer, El-Showk, Messamah, Van den Bleeken, arXiv:0807.4556**

# Fluctuating Microstate Geometries

Bubbled geometries in five dimensions are completely rigid on internal space ( $T^5$  in IIB)  $\Rightarrow$  Still rather coarse sampling of microstate structure.

Need to consider “fluctuating” microstate geometries that depend upon compactified internal dimensions ... ( $\mathbf{v}$ )

$\Rightarrow$  **BPS solutions in six dimensions** (or more)

★ **Supertubes**: Smooth BPS solutions that depend upon functions of one variable,  $\vec{F}(\mathbf{v})$ .

*Semi-classical quantization*  $\Rightarrow$   $S \sim Q$

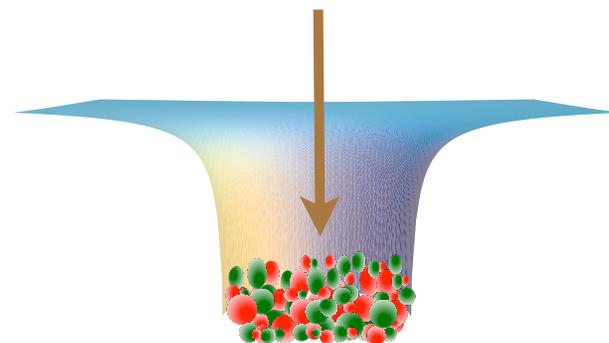
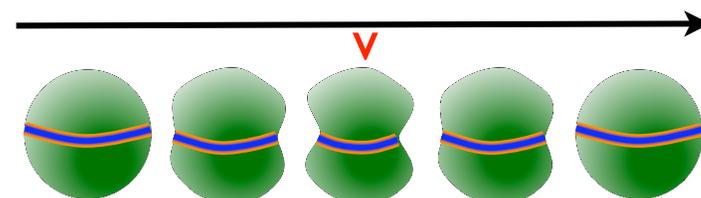
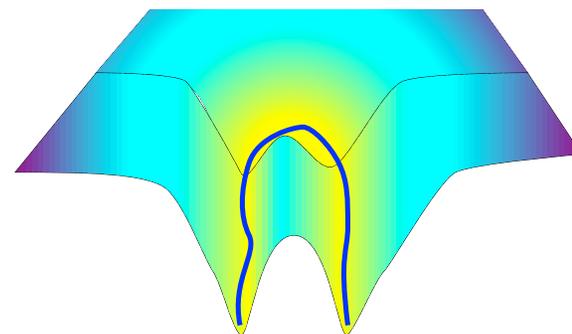
★ Simple *fluctuating bubbled geometries* described functions of one variable

*Semi-classical quantization*  $\Rightarrow$   $S \sim Q$

★ **Entropy enhancement**

Fluctuating supertubes and bubbles in deep scaling geometry become much floppier due to strong *dipole-dipole interactions*

*Semi-classical quantization*  $\Rightarrow$   $S \sim Q^{5/4}$



## So far...

- ★ Singularity resolved and capped off at (macroscopic) horizon scale
- ★ Solutions sample typical sector of underlying CFT
- ★ *Fluctuating* microstate geometries sample vast numbers of states in CFT, and not just the Coulomb branch states  $S \sim Q^{5/4}$
- ★ *Fluctuating* microstate geometries are based on (non-perturbative) solitons

So far we have really only used the degrees of freedom motivated by supertubes: Functions of one variable  $F(v)$ .

*Supertubes* lie at the heart of Mathur's proposal for the *two-charge system*.  
Supertubes carry two charges and one magnetic dipole charge.  $S \sim Q$ .

**Can one do more and better for the three charge system?**

### **Superstrata:**

Implement a doubled supertube transition.

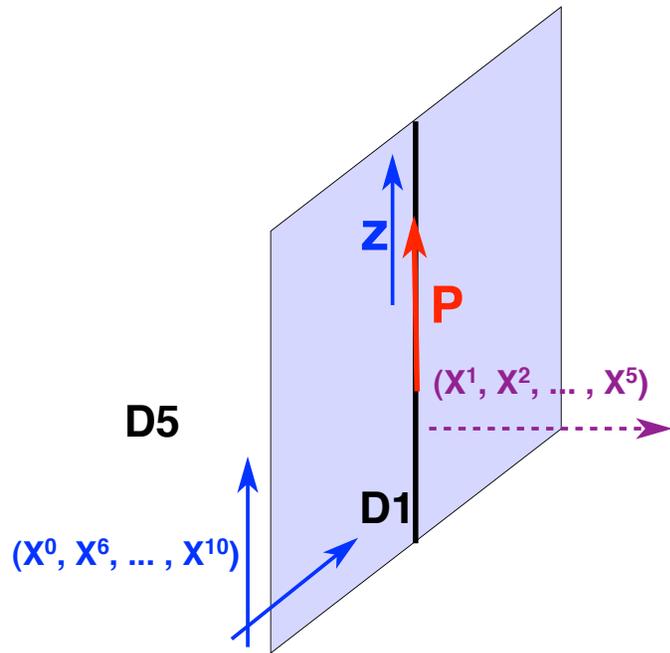
de Boer and Shigemori arXiv:1004.2521

# D-Branes, Charges and Supersymmetry

Simple stacks of  $D_p$ -branes are  $\frac{1}{2}$  BPS objects

Stacks of  $D1$ -branes and  $D5$ -branes are compatible  $(\frac{1}{2})^2$  BPS objects

+ momentum charge,  $P$ , compatible  $(\frac{1}{2})^3$  BPS objects



$$\Pi_{D1} = \frac{1}{2} (\mathbb{1} + \Gamma^{0z} \sigma_1)$$

$$\Pi_{D5} = \frac{1}{2} (\mathbb{1} + \Gamma^{0z6789} \sigma_1)$$

$$\Pi_P = \frac{1}{2} (\mathbb{1} + \Gamma^{0z})$$

$$\Pi_{D1} \epsilon = \Pi_{D5} \epsilon = \Pi_P \epsilon = 0$$

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

Compatible projectors:

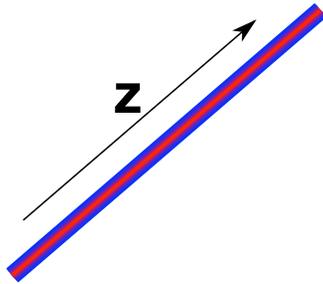
$$[\Gamma^{0z6789} \sigma_1, \Gamma^{0z} \sigma_1] = [\Gamma^{0z6789} \sigma_1, \Gamma^{0z}] = [\Gamma^{0z} \sigma_1, \Gamma^{0z}] = 0$$

D1-D5-P configuration  $\Rightarrow \frac{1}{8}$  BPS objects; 4 supersymmetries

# The Original Supertube

Mateos and Townsend, hep-th/0103030

Start with **D0 + F1** (compatible) charges



$$\Pi_{D0} = \frac{1}{2} (\mathbb{1} + \Gamma^0 i \sigma_2)$$

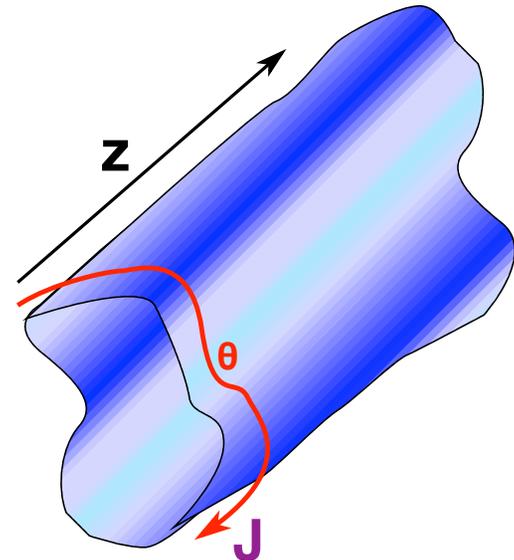
$$\Pi_{F1} = \frac{1}{2} (\mathbb{1} + \Gamma^{0z} \sigma_3)$$

$$\Rightarrow \frac{1}{2} \times \frac{1}{2} \times 32 = 8 \text{ supersymmetries}$$

## The supertube transition:

- ◆ Start with D2 brane along  $(z, \theta)$  where  $\theta$  defines a closed curve  
 $\Rightarrow$  **d2 dipole charge + arbitrary shape**
- ◆ Dissolve D0 and F1 charge in the D2 brane
- ◆ Spin up with angular momentum, **J**,  
in  $\theta$  direction

$$\begin{pmatrix} D0 (0z) \\ F1 (z) \end{pmatrix} \rightarrow \begin{pmatrix} d2 (0\theta) \\ J (\theta) \end{pmatrix}$$



# The supersymmetry projectors

D2 brane projector:  $\Pi_{D2} = \frac{1}{2} (\mathbb{1} + \Gamma^{0z\theta} \sigma_1)$

**Dissolve D0 + F1** charge and spin it up. Projector becomes:

$$\hat{\Pi}_{D2} = \frac{1}{2} (\mathbb{1} + \underbrace{v_1 \Gamma^{0z\theta} \sigma_1}_{\mathbf{d2}} + \underbrace{v_2 \Gamma^0 i \sigma_2}_{\mathbf{D0}} + \underbrace{v_3 \Gamma^{0z} \sigma_3}_{\mathbf{F1}} + \underbrace{v_4 \Gamma^{0\theta}}_{\mathbf{J}})$$

$\mathbf{v}_i \leftrightarrow$  brane charges

## Constraints

Projector :  $\sum v_i^2 = 1$

Supersymmetry  $\Rightarrow \mathbf{Q}_{F1} \mathbf{Q}_{D0} = \mathbf{J} \mathbf{q}_{d2} \Rightarrow v_1 v_4 = v_2 v_3$

$$\hat{\Pi}_{D2} = \frac{1}{2} (\mathbb{1} + \cos \alpha \cos \beta \Gamma^{0z\theta} \sigma_1 + \sin \alpha \cos \beta \Gamma^0 i \sigma_2 + \cos \alpha \sin \beta \Gamma^{0z} \sigma_3 + \sin \alpha \sin \beta \Gamma^{0\theta})$$

**Preserves** same supersymmetries as **D0 + F1** charges:

$$\hat{\Pi}_{D2} = a \Pi_{D0} + b \Pi_{F1} \Leftrightarrow \beta = \frac{\pi}{2} - \alpha$$

Supertube projector, depending upon 1 parameter,  $\alpha$ :

$$\hat{\Pi}_{D2} = \frac{1}{2} (\mathbb{1} + \cos^2 \alpha \Gamma^{0z} \sigma_3 + \sin^2 \alpha \Gamma^0 i \sigma_2 + \sin \alpha \cos \alpha (\Gamma^{0z\theta} \sigma_1 + \Gamma^{0\theta}))$$

## Important Identities:

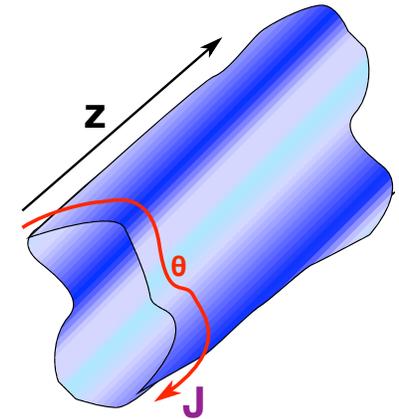
$$\hat{\Pi}_{D2} = \frac{1}{2} (\mathbb{1} + \cos^2 \alpha \Gamma^{0z} \sigma_3 + \sin^2 \alpha \Gamma^0 i \sigma_2 + \sin \alpha \cos \alpha (\Gamma^{0z\theta} \sigma_1 + \Gamma^{0\theta}))$$

$$(i) \quad \hat{\Pi}_{D2} = \frac{1}{2} (\mathbb{1} + \mathbb{P}), \quad \mathbb{P} \equiv \Gamma^{0z} (\cos \alpha \sigma_3 + \sin \alpha \Gamma^{z\theta}) (\cos \alpha \mathbb{1} + \sin \alpha \Gamma^{\theta} i \sigma_2)$$

$$\Rightarrow \mathbb{P}^2 = \mathbb{1}, \quad \text{Tr}(\mathbb{P}) = 0$$

$\Rightarrow$  Preserves 16 supersymmetries **locally**

Supersymmetries depend upon direction,  $\theta$ .



$$(ii) \quad \hat{\Pi}_{D2} = \cos \alpha (\cos \alpha \mathbb{1} - \sin \alpha \Gamma^{z\theta} \sigma_3) \Pi_{F1} + \sin \alpha (\sin \alpha \mathbb{1} + \cos \alpha \Gamma^{z\theta} \sigma_3) \Pi_{D0}$$

$$\Pi_{F1} = \frac{1}{2} (\mathbb{1} + \Gamma^{0z} \sigma_3) \quad \Pi_{D0} = \frac{1}{2} (\mathbb{1} + \Gamma^0 i \sigma_2)$$

$\Rightarrow$  Preserves 8 supersymmetries **globally**, independent of the direction,  $\theta$ .

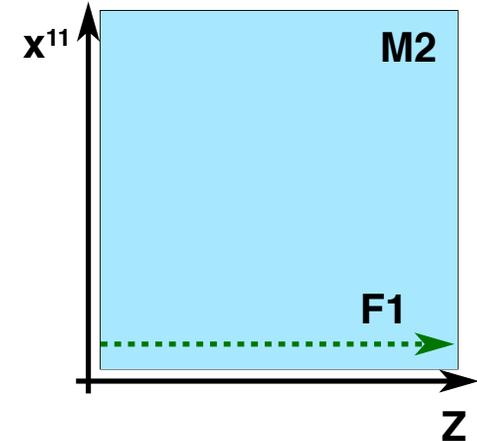
Preserves the same supersymmetries as the original **D0 + F1** configuration

# Simple Picture: Tilting and Boosting an M2 Brane

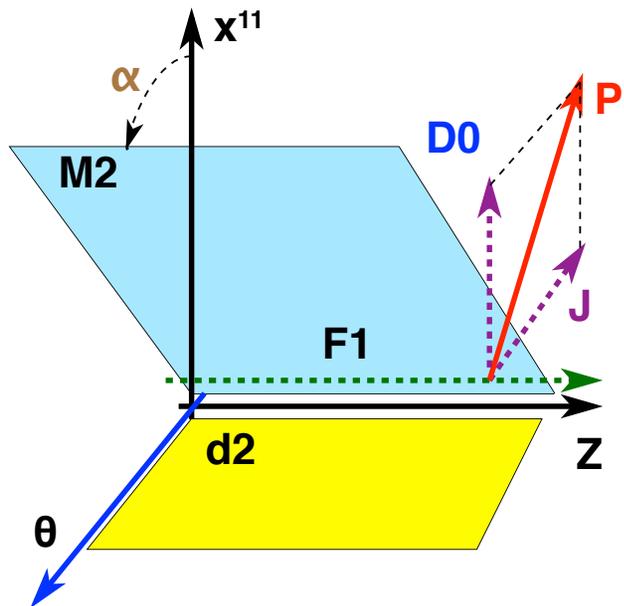


Actually M2 brane

16 supersymmetries



Tilt induces **d2** charge and **Boost** induces **D0, J**:

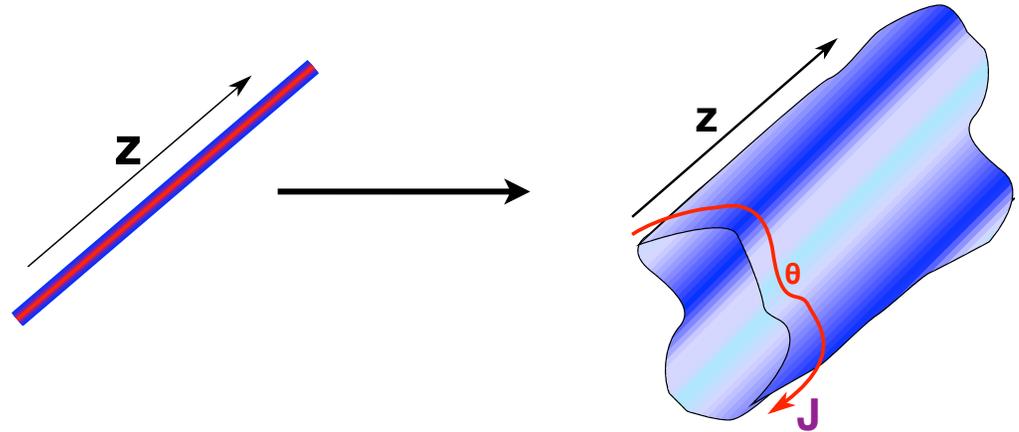


- All charges fixed by  $Q_{M2}$  and tilt angle
- Still has 16 supersymmetries
- Projector induced by tilt and boost

$$\hat{\Pi}_{D2} = \frac{1}{2} \left( \mathbb{1} + \cos^2 \alpha \Gamma^{0z} \sigma_3 + \sin^2 \alpha \Gamma^0 i \sigma_2 + \sin \alpha \cos \alpha \left( \Gamma^{0z\theta} \sigma_1 + \Gamma^{0\theta} \right) \right)$$

## Key points

$$\begin{pmatrix} D0 \\ F1(z) \end{pmatrix} \rightarrow \begin{pmatrix} d2(z\theta) \\ J(\theta) \end{pmatrix}$$



- ◆ Carry *two electric charges* and one *dipole charge* + angular momentum, **J**
- ◆ Configuration can be given an **arbitrary shape** parametrized by  **$\theta$**
- ◆ Locally  $\frac{1}{2}$  BPS objects: **16 supersymmetries (locally primitive)** depending on  **$\theta$**  direction
- ◆ Globally  $\frac{1}{4}$  BPS objects: **8 supersymmetries** same supersymmetries as original charge configuration

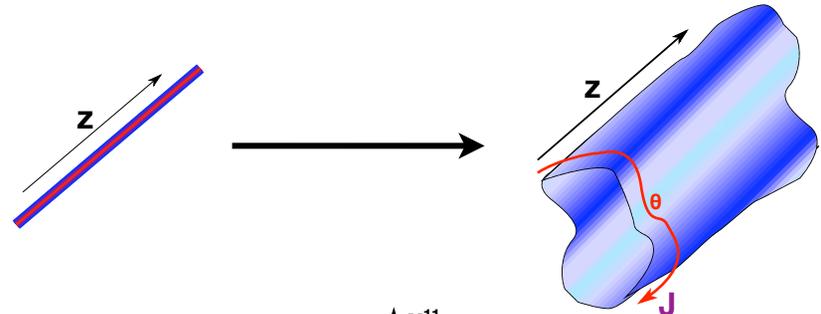
# Generic duality frames

One can dualize the supertube construction to arbitrary duality frames.

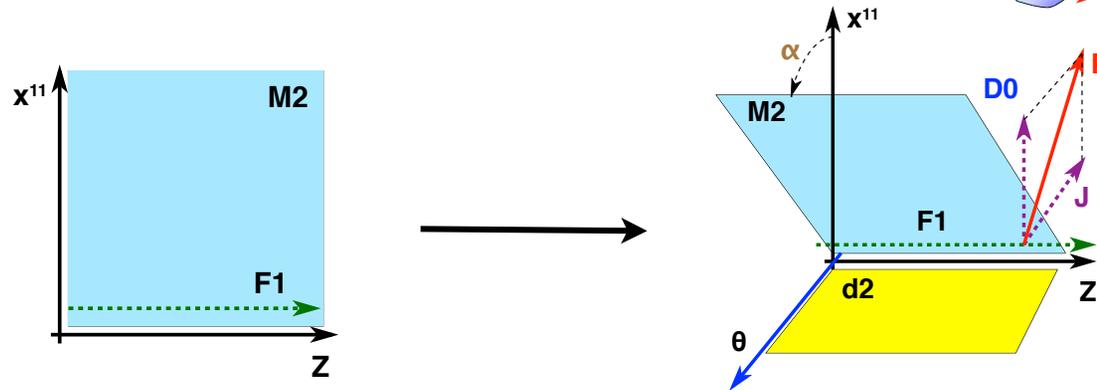
$$\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} \rightarrow \begin{pmatrix} d_X(\theta) \\ J(\theta) \end{pmatrix} \quad \text{Configuration extends and rotates along } \theta$$

The supertube transition extends the configuration in an extra direction

⇒ The locus of the supertube often gains a dimension ( $\theta$ )



But not always ...



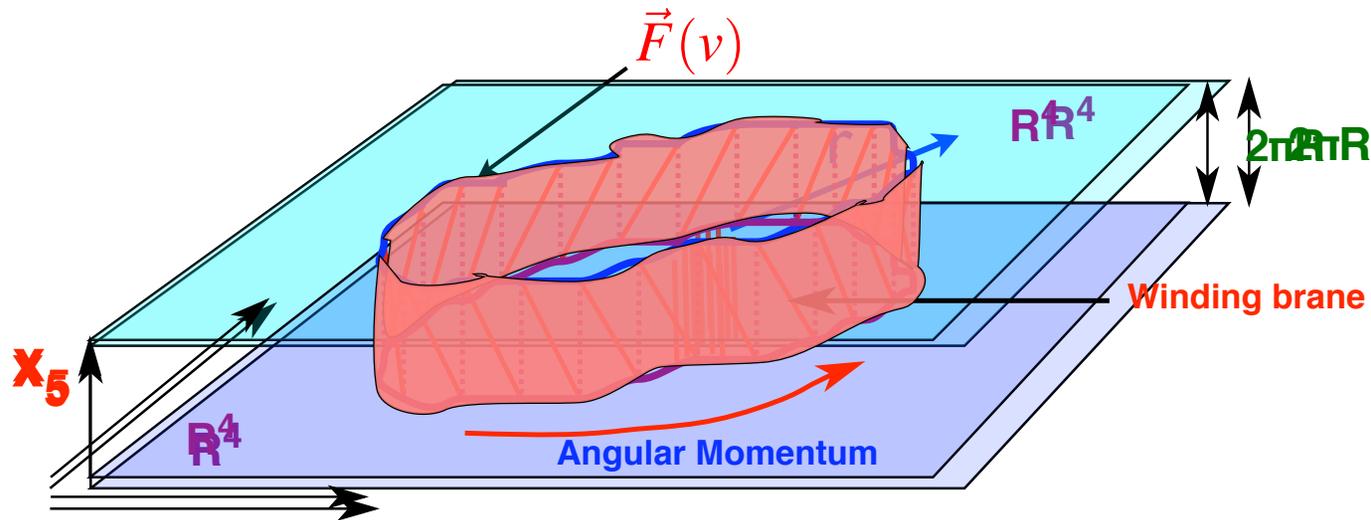
**The exception:** When one charge is a **momentum charge**.

⇒ Supertube transition is achieved by **tilting** and **boosting**

⇒ Supertube transition does not add to the dimension of the configuration

# An Important Example: the D1-D5 supertube in six dimensions

D1-D5 branes + KKM charge + angular momentum



$$\begin{pmatrix} D1(z) \\ D5(z\theta 6789) \end{pmatrix} \downarrow \begin{pmatrix} KKM(z\theta 6789) \\ J(\theta) \end{pmatrix}$$

$$ds_6^2 = (H_1 H_5)^{-\frac{1}{2}} \left[ - (dt - \vec{A} \cdot d\vec{y})^2 + (dz + \vec{B} \cdot d\vec{y})^2 \right] + (H_1 H_5)^{\frac{1}{2}} d\vec{y} \cdot d\vec{y}$$

Original charge distribution  $H_1 = 1 + \frac{Q_1}{r^2}$   $H_5 = 1 + \frac{Q_5}{r^2}$

⇒ Metric is singular near D-branes

Supertube transition: Freely choosable profile,  $\vec{F}(v)$

$$H_1 = 1 + \frac{Q_1}{L} \int_0^L \frac{dv}{|\vec{x} - \vec{F}(v)|^2} \quad H_5 = 1 + \frac{Q_1}{L} \int_0^L \frac{\left| \frac{d}{dv} \vec{F}(v) \right|^2 dv}{|\vec{x} - \vec{F}(v)|^2}$$

The softening to a line singularity makes this solution completely smooth:

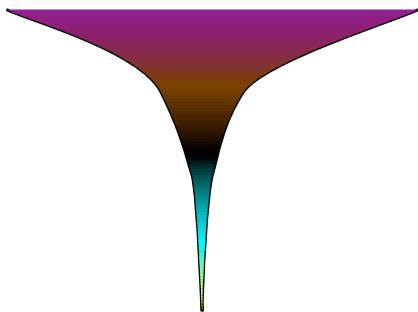
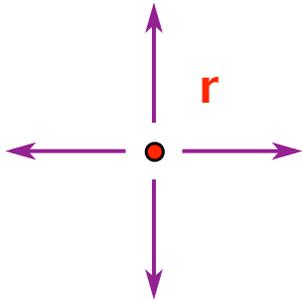
*A microstate geometry ...*

# Regularity

Maldacena, Maoz, hep-th/0012025; Lunin, Mathur, hep-th/0202072; Lunin, Maldacena, Maoz hep-th/0212210

Pure D1-D5 source

$$H_i \sim \frac{Q_i}{r^2}$$

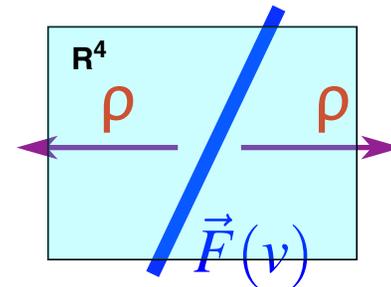


Singular Geometry

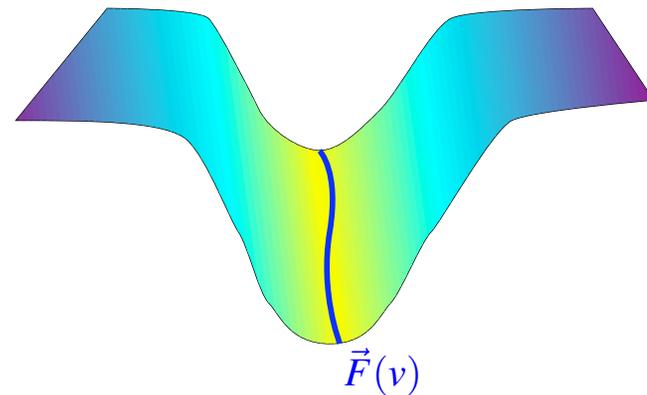


Smearred D1-D5 source + KKM charge

$$H_i \sim \frac{\tilde{Q}_i}{\rho}$$



Transverse to tube: Metric is spatial  $R^4$



A Completely regular six-dimensional geometry: Regular for all supertube profiles

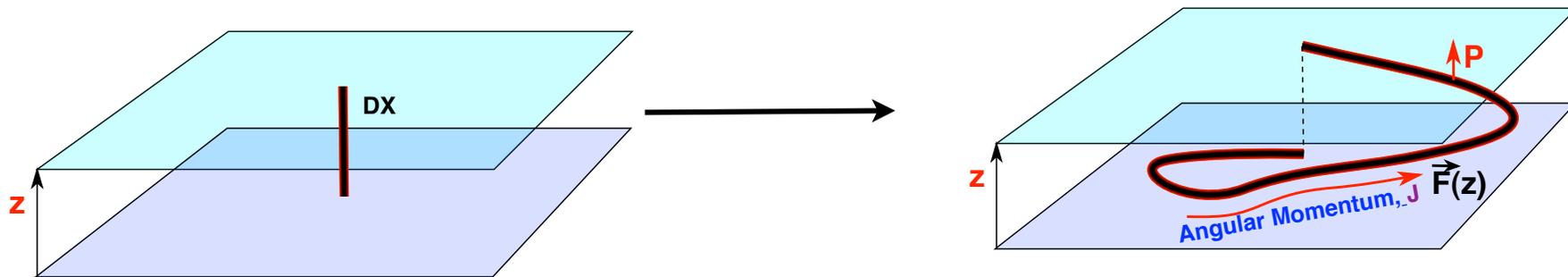
# Double Bubbling the D1-D5-P system

Bena, de Boer, Shigemori and Warner, I107.2650

Split the momentum between the D1 and D5  $P = P^{(1)} + P^{(2)}$

First puff-up  $\Leftrightarrow$  first dipole charge: Tilt and boost D1 and D5 branes in parallel

$$\begin{pmatrix} D1(z) \\ P^{(1)}(z) \end{pmatrix} \longrightarrow \begin{pmatrix} d1(\theta) \\ J^{(1)}(\theta) \end{pmatrix} \quad \begin{pmatrix} D5(z) \\ P^{(2)}(z) \end{pmatrix} \longrightarrow \begin{pmatrix} d5(\theta) \\ J^{(2)}(\theta) \end{pmatrix}$$



Two puffed up projectors:  $X=1,5$

$$\hat{\Pi}_{DX} = \cos \alpha (\cos \alpha \mathbb{1} + \sin \alpha \Gamma^{z\theta} \sigma_1) \Pi_{DX} + \sin \alpha (\sin \alpha \mathbb{1} - \cos \alpha \Gamma^{z\theta} \sigma_1) \Pi_P$$

## Supersymmetries

Locally/profile independent of  $\theta$

Globally/profile arbitrary,  $\vec{F}(\theta)$

D1 or D5

16

8

D1 and D5 combined

8

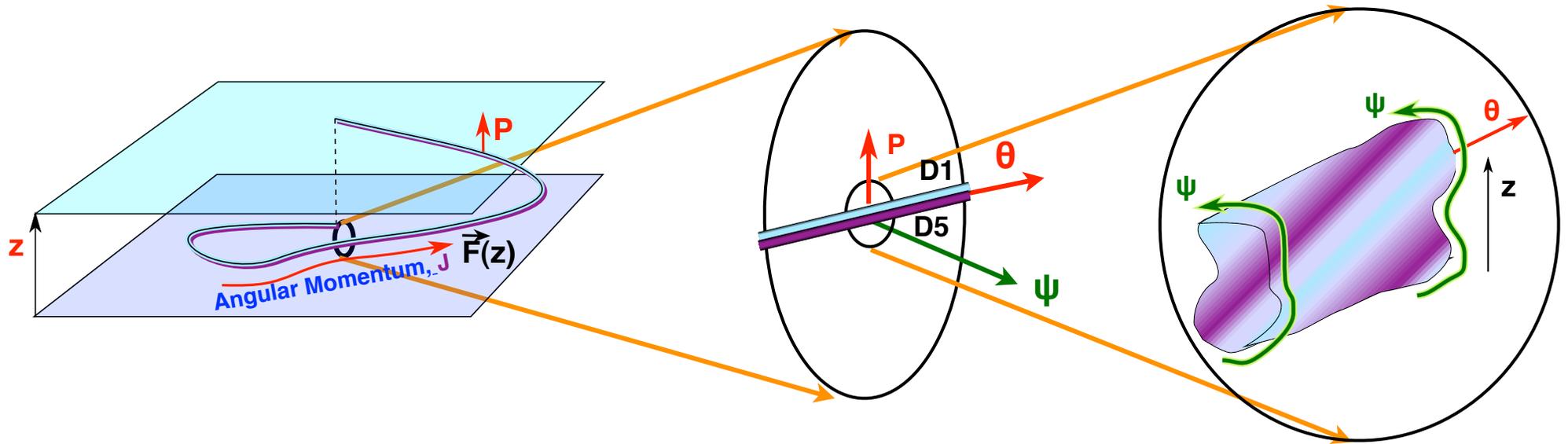
4

*Remains a co-dimension 4 object in this transition ..*

## Second puff-up $\Leftrightarrow$ second dipole charge: The Superstratum

So far:  $\hat{\Pi}_{DX} = \cos \alpha (\cos \alpha \mathbb{1} + \sin \alpha \Gamma^{z\theta} \sigma_1) \Pi_{DX} + \sin \alpha (\sin \alpha \mathbb{1} - \cos \alpha \Gamma^{z\theta} \sigma_1) \Pi_P$

Locally a tilted **D1-D5** system with some profile in  $\theta$ : (*not smeared*)



Puff up again into **D1-D5** + *KKM* ( $\psi$ ),  $J$  ( $\psi$ ):

$$\hat{\Pi} = \cos \beta (\cos \beta \mathbb{1} + \sin \beta \Gamma^{\hat{z}\psi} \sigma_1) \hat{\Pi}_{D1} + \sin \beta (\sin \beta \mathbb{1} - \cos \beta \Gamma^{\hat{z}\psi} \sigma_1) \hat{\Pi}_{D5}$$

$$\Gamma^{\hat{z}} \equiv \cos \alpha \Gamma^z - \sin \alpha \Gamma^\theta$$

Two parameters,  $(\alpha, \beta) \Leftrightarrow$  two independent dipole charges

Profile now depends upon two variables,  $(\theta, \psi)$  while sweeping out  $z$   
 $\Rightarrow$  a *co-dimension-3 object*

## The Double Bubbled 3-Charge, 2-Dipole Charge Superstratum

$$\begin{aligned}\hat{\Pi} &= \cos \beta \left( \cos \beta \mathbb{1} + \sin \beta \Gamma^{\hat{z}\psi} \sigma_1 \right) \hat{\Pi}_{D1} + \sin \beta \left( \sin \beta \mathbb{1} - \cos \beta \Gamma^{\hat{z}\psi} \sigma_1 \right) \hat{\Pi}_{D5} \\ &= A_1 \Pi_{D1} + A_2 \Pi_{D5} + A_3 \Pi_P\end{aligned}$$

- Three electric charges  $\mathbf{Q}_{D1}, \mathbf{Q}_{D5}, \mathbf{Q}_P$
- Two independent dipole charges parametrized by  $(\alpha, \beta)$
- The supertube transitions allow independent profiles in  $(\theta, \psi)$

### Supersymmetries

Profile independent of  $(\theta, \psi)$

**16**

Profile depends on  $\theta$  but not  $\psi$  (or on  $\psi$  but not  $\theta$ )

**8**

Arbitrary profile as a function of  $(\theta, \psi)$

**4**

- Generic *superstratum* is *locally primitive* (**16 supersymmetries**)....  
but globally has same supersymmetries as the original D1-D5-P system
- The *superstratum* has a two-dimensional set of shape modes and has co-dimension 3 in six (ten) dimensions ....
- The *superstratum* should be a completely smooth BPS solution:  
*Microstate geometries that depends upon functions of two variables.*

**Can you construct it as a smooth supergravity solution?**

## BPS Solutions in Six Dimensions

Six-dimensions: IIB compactified on  $T^4$  = other four dimensions of the D5  
Study minimal, ( $N=1$ ) supergravity coupled to one anti-self-dual tensor multiplet.

Bosonic field content  $(g_{\mu\nu}, B^+_{\mu\nu}) + (B^-_{\mu\nu}, \phi) \Leftrightarrow (g_{\mu\nu}, B_{\mu\nu}, \phi)$

Trivial dimensional reduction to five dimensions yields  $N=2$  supergravity coupled to two vector multiplets

Bosonic field content (five dimensions)  $g_{\mu\nu}, A^K_\mu, X^a \quad K=1,2,3; a=1,2$

*This six-dimensional theory is precisely the one that underpins the study of three-charge black holes in five dimensions ...*

BPS equations derived and studied in great detail in:

**Gutowski, Martelli and Reall hep-th/0306235**

**Cariglia, Mac Conamhna hep-th/0402055**

Huge surprise: **Bena, Giusto, Shigemori, Warner arXiv:1110.2781**

Once the base geometry is fixed, *the BPS equations are linear.*

- ◆ Much easier to solve
- ◆ Superposition solutions  $\Rightarrow$  phase space structure much simpler
- ◆ Highly relevant to  $AdS_3 \times S^3$  holography

## The Elements of BPS Solutions

The metric:

$$ds^2 = 2 (Z_1 Z_2)^{-1/2} (dv + \beta) (du + \omega - 2 Z_3 (dv + \beta)) - (Z_1 Z_2)^{1/2} ds_4^2$$

The 3-form flux and its dual have an electric and magnetic decomposition:

$$G = d \left[ -\frac{1}{2} Z_1^{-1} (du + \omega) \wedge (dv + \beta) \right] + \hat{G}_1,$$

$$e^{2\sqrt{2}\phi} *_6 G = d \left[ -\frac{1}{2} Z_2^{-1} (du + \omega) \wedge (dv + \beta) \right] + \hat{G}_2$$

where

$$\hat{G}_1 \equiv \frac{1}{2} *_4 (DZ_2 + (\partial_v \beta) Z_2) + (dv + \beta) \wedge \Theta_1$$

$$\hat{G}_2 \equiv \frac{1}{2} *_4 (DZ_1 + (\partial_v \beta) Z_1) + (dv + \beta) \wedge \Theta_2$$

Dilaton:  $e^{2\sqrt{2}\phi} \equiv \frac{Z_1}{Z_2}$

**Building blocks:** Base metric,  $ds_4^2$ ; (fibration) vector field,  $\beta$ ;

Potential functions  $Z_1, Z_2, Z_3$ ; Magnetic 2-forms  $\Theta_1, \Theta_2$ ;

Angular momentum vector,  $\omega$ .

Everything (including  $ds_4^2$ ) is *independent* of  $u$  but can depend on  $v$  and the coordinates,  $\vec{y}$ , on the four-dimensional base.

## The geometric elements

The base geometry,  $ds_4^2$ , is required to be “almost hyper-Kähler”

Anti-self-dual 2-forms,  $J^{(A)}$ , on four-dimensional base, satisfying:

$$J^{(A)m}{}_p J^{(B)p}{}_n = \epsilon^{ABC} J^{(C)m}{}_n - \delta^{AB} \delta_n^m$$
$$\tilde{d}J^{(A)} = \partial_v(\beta \wedge J^{(A)})$$

where  $\tilde{d}$  is the exterior derivative on the base,  $ds_4^2$ , (i.e. in  $\vec{y}$  alone)

The vector field,  $\beta$ , that defines the v-fibration is required to satisfy:

$$D\beta = *_4 D\beta \quad D \equiv \tilde{d} - \beta \wedge \partial_v$$

*Obvious solution:  $ds_4^2$  and  $\beta$  are v-independent and  $ds_4^2$  is hyper-Kähler.*

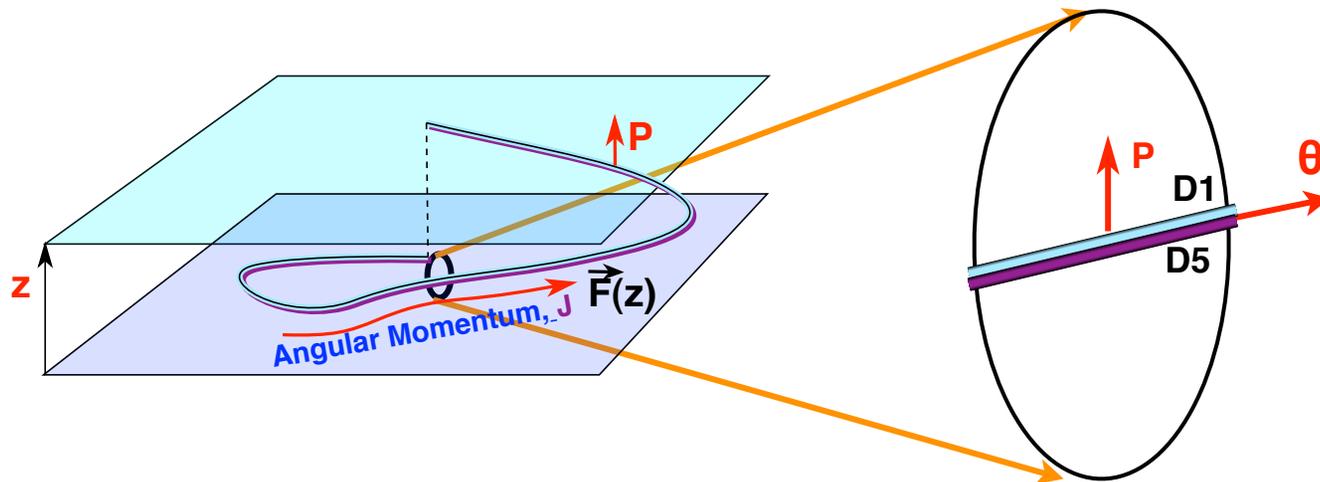
**N.B.** These are the only non-linear parts of the system and either define, or are defined entirely in terms of, the base geometry

The equations (as functions of  $v$  and  $\vec{y}$ ) for the potential functions  $Z_1, Z_2, Z_3$ , the magnetic 2-forms  $\Theta_1, \Theta_2$  and the angular momentum vector,  $\omega$ , **are linear!**

# First step to the superstratum: Superthreads

Double bubbling is a constructive prescription.

The first puff-up: Give the **D1-D5-P** system an arbitrary profile,  $\vec{F}(v)$ , in six dimensions adding **d1-d5 dipoles** + angular momentum, **J**.



Solved in  $R^4$  base. **Bena, Giusto, Shigemori, Warner** arXiv:1110.2781

Completely new **three-charge, two-dipole charge** BPS solution with arbitrary profile.

Superthreads have highly **non-trivial and non-linear** interactions:

**Layered linear system**: **Linear solutions**  $\rightarrow$  **Non-linear sources for next linear layer**

- ◆ **Magnetic field** and **electric fields** source each other
- ◆ **Magnetic field** + **electric fields** source **angular momenta**

Multi-superthreads have a very rich structure ...

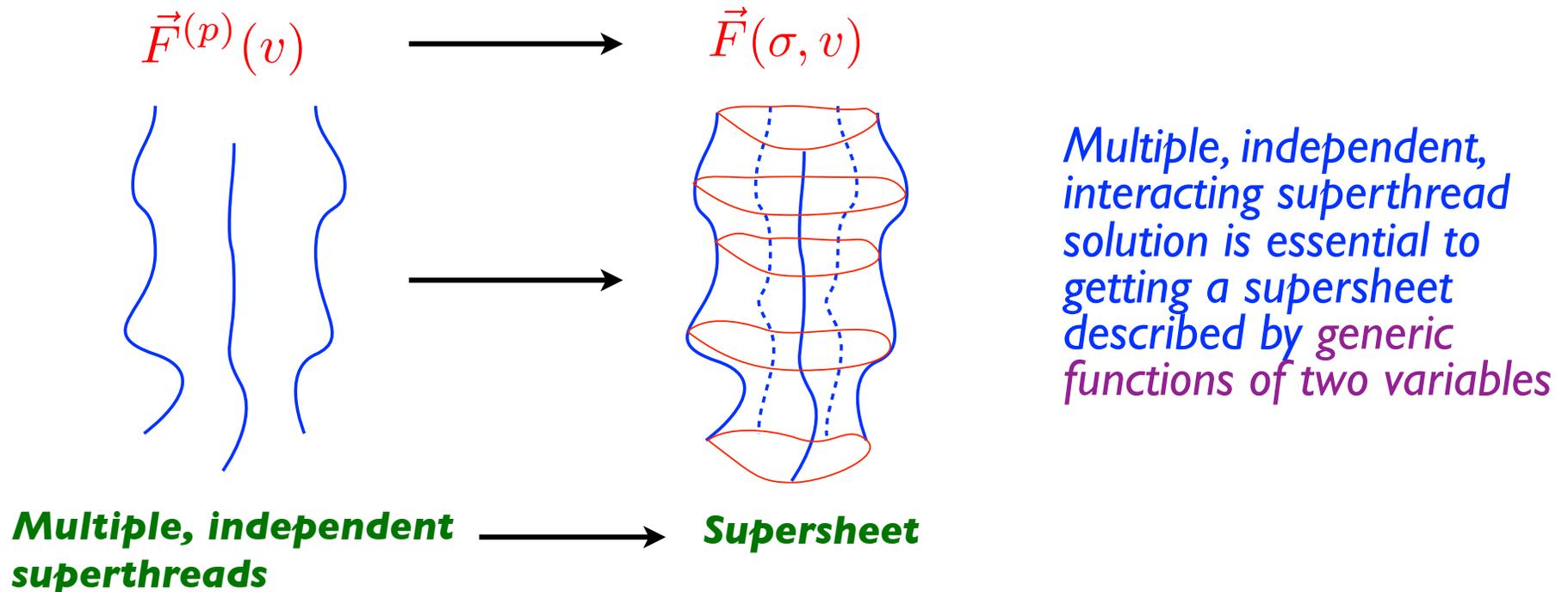
# The Next Step Multi-Superthreads and Supersheets

Niehoff, Vasilakis and Warner arXiv:1203.1348

Multiple, *independent, interacting* superthreads:

New *three-charge, two-dipole charge* BPS solutions that depends upon multiple, independent functions,  $\vec{F}^{(p)}(v)$ , of one variable

One can take the *three-charge* multiple superthread solution and *smear* into a supersheet that depends upon functions of two variables.



**Supersheet:** New *three-charge, two-dipole* charge BPS solutions that depends upon functions of two variables. *Details:* See poster by Niehoff and Vasilakis

## Conclusions

The classification of microstate geometries has made remarkable progress

There is a very important class of new BPS solutions called *superstrata*

- Solitonic bound-state with three charges, two independent dipole charges
- 4 supersymmetries in general; 16 supersymmetries locally.
- Arbitrary shapes as functions of *two variables*
- *Smooth* in D1-D5-P duality frame

⇒ New, extremely rich families of microstate geometries

Route to construction: Six-dimensional  $N=1$  supergravity

- BPS Equations are *linear* six dimensions
  - ◆ Much easier to solve; superposition ⇒ phase space structure
- Many new multicomponent BPS solutions; *Superthreads and supersheets*
- *Superstratum* requires *supersheets* + KKM.

Solve  $D\beta = *_4 D\beta$ ,  $D \equiv \tilde{d} - \beta \wedge \partial_v$  in general?