Bits, Branes, Black Holes
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Vasiliev theory as a string theory

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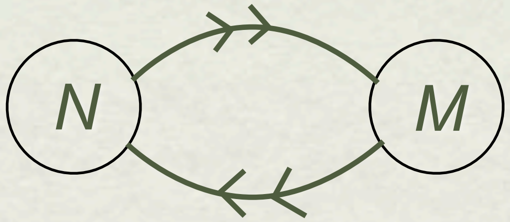
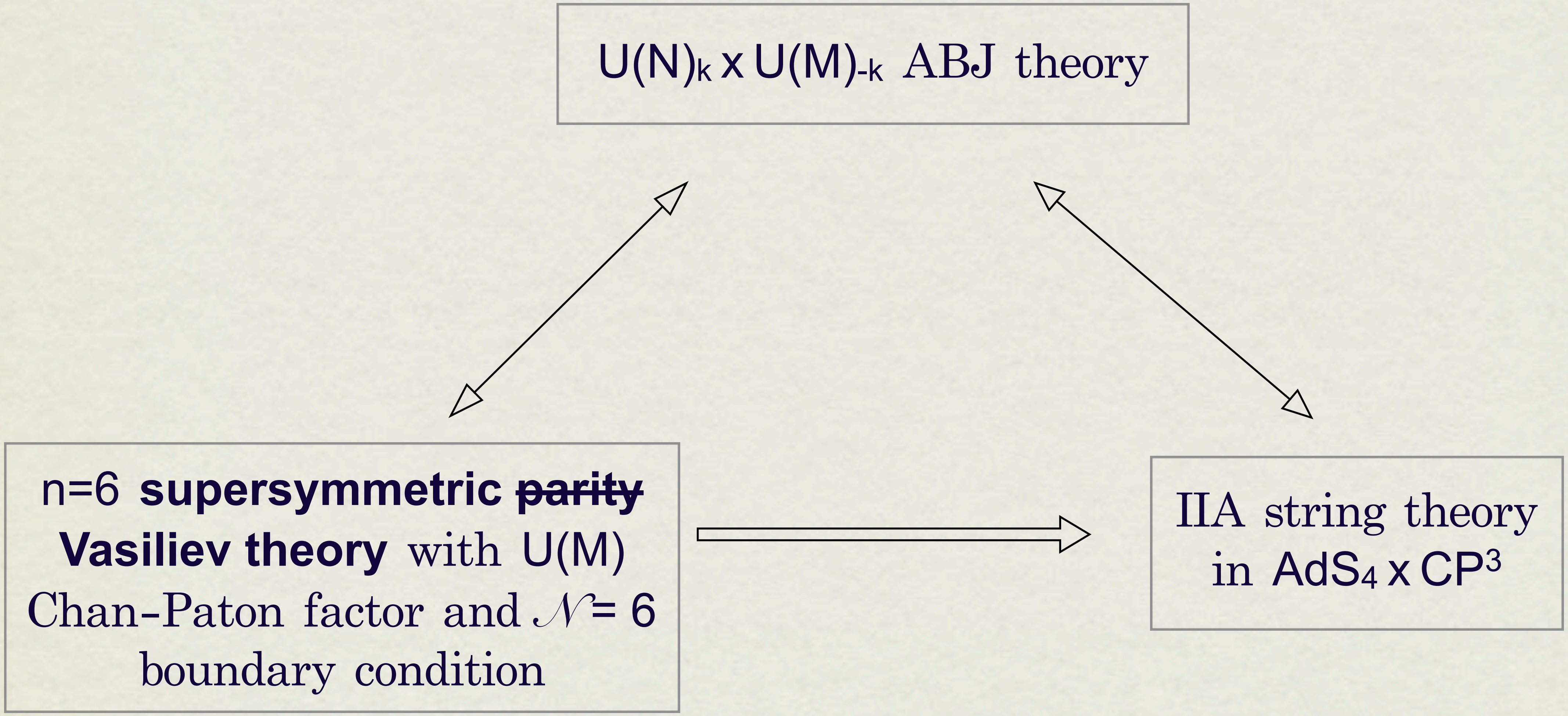
based on work with Chi-Ming Chang, Simone Giombi, Shiraz Minwalla, Shiroman Prakash, Tarun Sharma, Sandip Trivedi, Spenta Wadia, ...

“ABJ Triality”





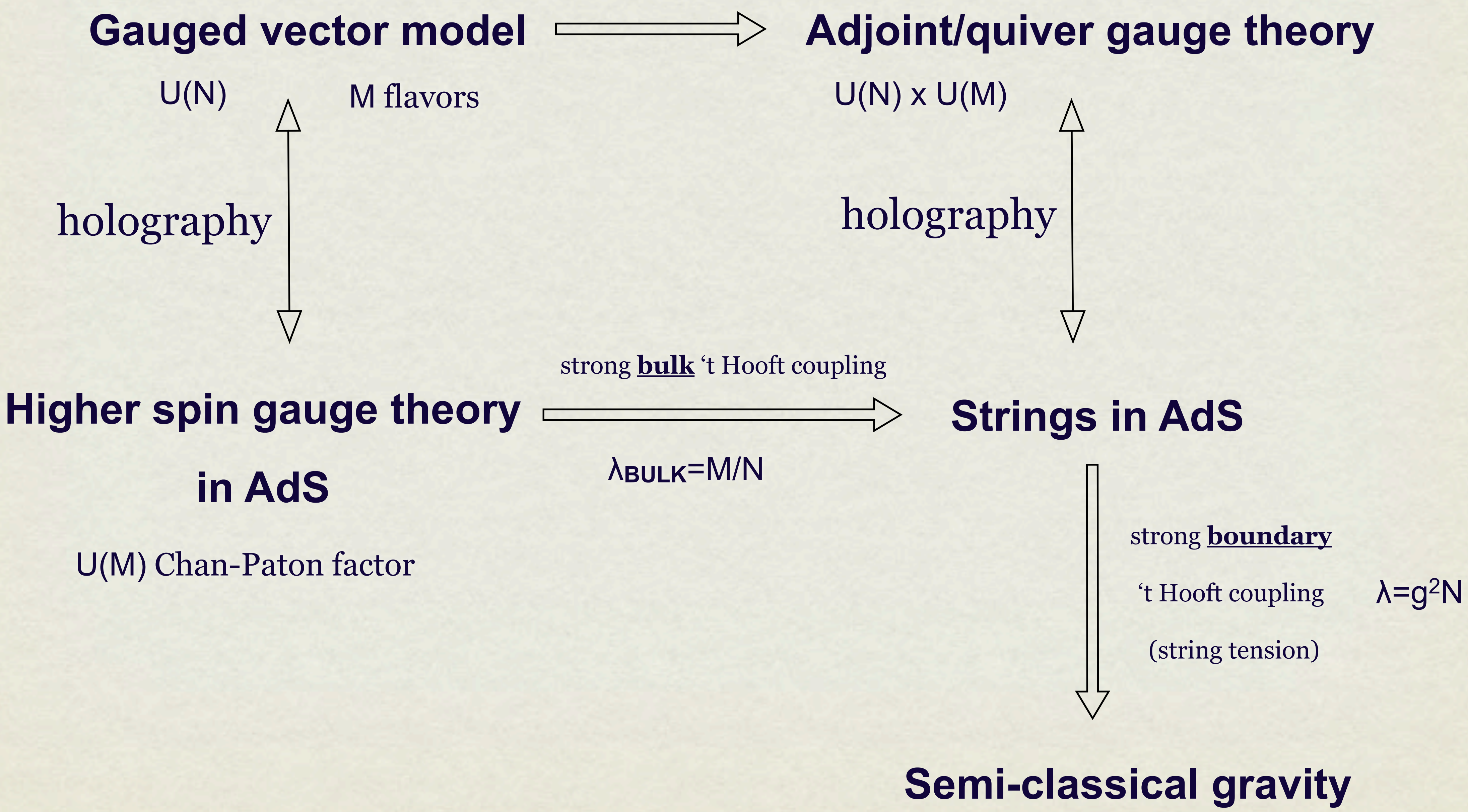
$U(N)_k \times U(M)_{-k}$ ABJ theory

n=6 **supersymmetric** ~~**parity**~~ **Vasiliev theory** with U(M) Chan-Paton factor and $\mathcal{N}= 6$ boundary condition

IIA string theory in $AdS_4 \times CP^3$

bound states of higher spin particles ⇢ strings

The role of higher spin gauge theory in AdS/CFT



Gauged vector model

Adjoint/quiver gauge theory

U(N) M flavors

U(N) x U(M)

holography

holography

strong **bulk** ‘t Hooft coupling

Higher spin gauge theory in AdS

Strings in AdS

λ_{BULK}=M/N

U(M) Chan-Paton factor

strong **boundary** ‘t Hooft coupling (string tension)

$\lambda=g^2N$

Semi-classical gravity

Pure higher spin gauge theory in AdS

M.A. Vasiliev (90s): complete nonlinear gauge invariant classical equations of motion for pure higher spin gauge theory in AdS_4.

Minimal bosonic theory: spin *s=0,2,4,6,....*

Non-minimal bosonic theory: spin *s=0,1,2,3,.....*

Supersymmetric & non-Abelian extensions

Parity ⇒ only two theories: A-type and B-type

~~Parity~~ ⇒ infinite parameter family of theories (yet still highly constrained by higher spin symmetry)

Not unrelated to the zoo of parity violating Chern-Simons-matter CFTs in 3d.

Vasiliev's system

Twister variables: Y, Z (carry spinor indices), noncommutative $*$ product

Master fields: $\mathbb{A}(x|Y,Z) = W_\mu \, dx^\mu + S_A \, dz^A + S_{\dot{A}} \, d\bar{z}^{\dot{A}}, \ \mathbb{B}(x|Y,Z)$

Equation of motion: $d\mathbb{A} + \mathbb{A} * \mathbb{A} = f_*(\mathbb{B} * K) dz^2 + \bar{f}_*(\mathbb{B} * \bar{K}) d\bar{z}^2.$

$K, \bar{K}$ - Kleinian operator

$f(X) = X \exp(i\,\theta(X)) = e^{i\theta_0} X + b_3 X^3 + b_5 X^5 + \ldots$ up to field redefinition

Parity $\Rightarrow$ $f(X) = X$ (A-type theory) or $f(X) = i\,X$ (B-type theory)

A little more details

Spacetime coordinates x^μ, twister variables: $Y=(y^A,\bar{y}^{\dot{A}})$, $Z=(z^A,\bar{z}^{\dot{A}})$ $(A, \dot{A}=1,2)$, noncommutative $*$ product

$$f(y,z)*g(y,z)=f(y,z)\exp(\epsilon^{AB}(\overleftarrow{\partial}_{y_A}+\overleftarrow{\partial}_{z_A})(\overrightarrow{\partial}_{y_B}-\overrightarrow{\partial}_{z_B}))\,g(y,z)$$

Master fields: $\mathbb{A}(x|Y,Z) = W_\mu dx^\mu + S_A dz^A + S_{\dot{A}} d\bar{z}^{\dot{A}}$, $\mathbb{B}(x|Y,Z)$, both contain symmetric traceless tensor fields of all spins along with infinitely many auxiliary fields.

Equation of motion: $d\mathbb{A}+\mathbb{A}*\mathbb{A} = f_*(\mathbb{B}*K)dz^2 + f_*(\mathbb{B}*\overline{K})d\bar{z}^2$. Implies a Bianchi identity of the form $d\mathbb{B}+\mathbb{A}*\mathbb{B}-\mathbb{B}*\pi(\mathbb{A}) = 0$.

$K = \delta(y)*\delta(z) = \exp(z^A y_A)$, $K = \exp(\bar{z}^{\dot{A}}\bar{y}_{\dot{A}})$ - Kleinian operators, obey $K*K=\overline{K}*\overline{K}=1$.

$f(X) = X\exp(i\,\theta(X)) = e^{i\theta_0}X + b_3X^3 + b_5X^5 + \ldots$ up to field redefinition

Parity $\Rightarrow$ $f(X) = X$ (A-type theory) or $f(X) = i\,X$ (B-type theory)

And a little more …

Equations of motion in W,S,B :

$$d_xW+W*W=0,$$
$$d_zW+d_xS+W*S+S*W=0,$$
$$d_zS+S*S=B*(e^{i\theta_0}Kdz^2+e^{-i\theta_0}\overline{K}d\bar{z}^2),$$
$$d_xB+W*B-B*\pi(W)=0,$$
$$d_zB+S*B-B*\pi(S)=0.$$

Gauge symmetry: $\delta W=d_x\varepsilon+[W,\varepsilon]_*$, $\delta S=d_z\varepsilon+[S,\varepsilon]_*$, $\delta B=-\varepsilon*B+B*\pi(\varepsilon)$, $\varepsilon=\varepsilon(x|Y,Z)$. When the master fields are expressed in terms of the physical degrees of freedom after partial gauge fixing, Vasiliev's gauge symmetry reduces to (nonlinear) higher spin gauge symmetry.

Higher spin fields

AdS$_4$ vacuum: $W=W_0(x|Y)$, $S=0$, $B=0$.

The equation $dW_0=W_0 * W_0$ is solved by $W_0 = (\omega_0)_{AB}y^A y^B + (\omega_0)_{\dot{A}\dot{B}}\bar{y}^{\dot{A}}\bar{y}^{\dot{B}} + (e_0)_{A\dot{B}}y^A\bar{y}^{\dot{B}}$, where ω_0 and e_0 are the spin connection and vierbein of AdS$_4$. The bilinears of y's generate $so(3,2)$ under the $*$ product.

Linearized fields: $[\ \Omega(x|Y) \equiv W|_{Z=0}\ ]$

$$\begin{tikzcd}
\Omega^{(2s-2,0)} \arrow[d] & \cdots \arrow[l] & \Omega^{(s,s-2)} \arrow[l] & \Omega^{(s-1,s-1)} \arrow[l] \arrow[r] \arrow[d, phantom, "\cup"] & \Omega^{(s-2,s)} \arrow[r] & \cdots \arrow[r] & \Omega^{(0,2s-2)} \arrow[d] \\
B^{(2s,0)} \arrow[d] & & & \text{spin-}s\text{ fields} & & & B^{(0,2s)} \arrow[d] \\
B^{(2s+1,1)} \arrow[d] & & & \text{spin-}s\text{ Weyl tensor} \arrow[ulll] \arrow[urrr] & & & B^{(1,2s+1)} \arrow[d] \\
\cdots & & & & & & \cdots
\end{tikzcd}$$

The HS/VM Duality

Vasiliev's system describes classical interacting higher spin gauge fields in AdS_4. The interactions are highly constrained (and almost uniquely fixed) by higher spin symmetry, which makes it plausible that the quantum theory is renormalizable and possibly finite, despite the higher derivative (and seemingly nonlocal) interactions. If Vasiliev's theory is a consistent quantum theory of gravity, then it should have a three-dimensional CFT dual.

Conjecture (Klebanov-Polyakov, Sezgin-Sundell '02): Vasiliev's minimal bosonic theory in AdS_4 is holographically dual to the free or critical O(N) vector model.

	A-type	B-type
Δ=1	free O(N) boson	critical O(N) fermion (Gross-Neveu)
Δ=2	critical O(N) boson	free O(N) fermion

A few basic comments on HS/VM duality

1. They are among the simplest class of examples of AdS/CFT correspondence.

2. Explicit realization of holographic dual of free CFTs (with large N factorization), and their deformations (interacting but typically exactly solvable at infinite N).

3. Bulk interactions involve arbitrarily high order derivative coupling and are generally non-local.

4. Full classical equations of motion known explicitly (in contrast to string theory in AdS where relatively little is known about the full closed string field theory). But the classical action is not explicitly known and appears to be very complicated (if one tries to recover it order by order in fields).

5. In principle tree level correlators can be calculated straightforwardly using Vasiliev's equations. Going to loop order, prescriptions for regularization and gauge fixing/ghosts remain to be understood. Modulo this technicality (presumingly), both sides of the duality are computable order by order in 1/N.

6. AdS boundary conditions are important, and often break higher spin symmetry (and other symmetries).

Evidence for the duality

Giombi-Yin '09,'10: three-point functions of higher spin currents computed from Vasiliev theory at tree level match exactly with those of free and critical O(N) vector model.

Maldacena-Zhiboedov '11,'12:

exactly conserved higher spin current ⇒ CFT is free.

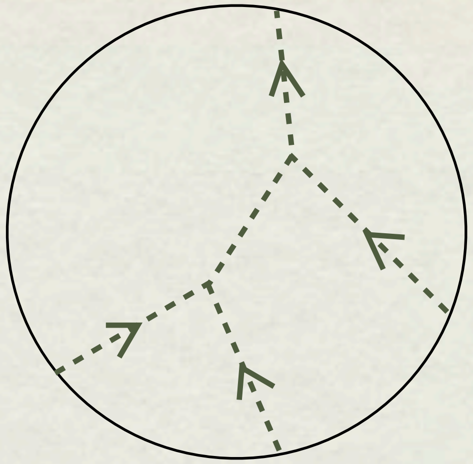
"Approximate" HS symmetry ⇒ three-point functions constrained.

Girardello-Porrati-Zaffaroni '98, Hartman-Rastelli '06, Giombi-Yin '10: In A-type Vasiliev theory, higher spin symmetry broken by $\Delta=2$ boundary condition. Duality with free O(N) theory for $\Delta=1$ boundary condition implies the duality with critical O(N) theory for $\Delta=2$ boundary condition, to all order in perturbation theory (1/N).

What remains to be shown: Vasiliev's system can be quantized in a manner in which higher spin symmetry is not anomalous nor broken by boundary conditions.

Explicit computation of 4-point function and loop corrections (should be absent for $\Delta=1$ b.c.) in Vasiliev theory yet to be performed.

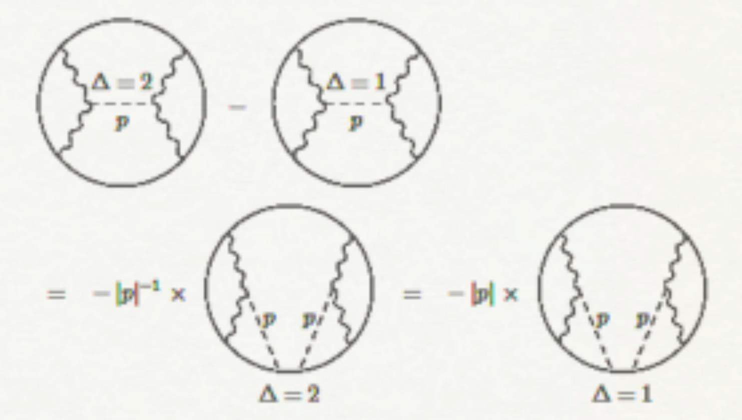
How to do bulk computations



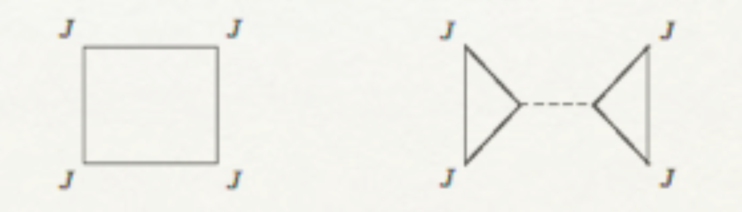
Tree level *n*-point correlators: treat *n-1* boundary currents as sources, solve bulk equation of the schematic form $D\varphi = \varphi * \varphi$ to (*n-1*)-th order and extract *n*-point function from boundary value of $\varphi^{(n-1)}$.

"*W=0* gauge": In Vasiliev theory, *W* is a flat connection in spacetime. Can perform a formal gauge transformation (violates AdS boundary condition condition) to eliminate all explicit dependence on spacetime coordinates (all data encoded in twistor variables). Easy to solve equation of motion perturbatively in this "gauge", then transform back to physical gauge.

Subtleties/difficulties: the *W=0* gauge appears singular for many purposes (4-point function, one-loop), potential gauge ambiguity in extracting final answer (obstacle in obtaining parity odd contributions?)



Changing boundary conditions (on fields of spin 0,1/2,1): Bulk propagators modified, in ways that precisely account for deformations of the boundary theory.



Generalizations

Gaiotto-Yin '07: Chern-Simons-matter theories provide a large class of 3d CFTs ⇔ various supersymmetric or non-supersymmetric string theories in AdS_4.

In some examples, there is a semi-classical gravity limit (Aharony-Bergman-Jafferis-Maldacena '08). In many other examples, the dual must always involve higher spin fields (Minwalla-Narayan-Sharma-Umesh-Yin '11).

Giombi-Minwalla-Prakash-Trivedi-Wadia-Yin, Aharony-Gur-Ari-Yacoby '11: Chern-Simons vector models have approximately conserved higher spin currents at large N.

Conjecture: Chern-Simons vector models are dual to ~~parity~~ Vasiliev theories in AdS_4.

Chern-Simons vector models

U(N) or SU(N) Chern-Simons theories coupled to massless scalars (AGY '11) or massless fermions (GMPTWY '11) in the fundamental representation are exactly conformal vector models. 't Hooft limit: N large, λ= N/k finite. Exactly solvable at infinite N. Conservation of higher spin currents broken by 1/N effects:

$$\partial^{\mu} J^{(s)}_{\mu\ldots} = f(\lambda) \sum \partial^{n_1} J^{(s_1)} \partial^{n_2} J^{(s_2)} + g(\lambda) \sum \partial^{n_1} J^{(s_1)} \partial^{n_2} J^{(s_2)} \partial^{n_3} J^{(s_3)}$$

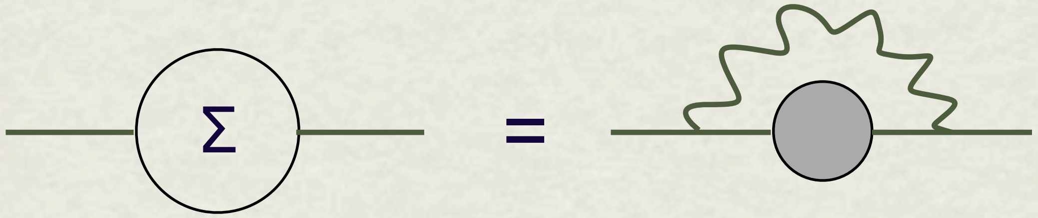
where the spin-s current $J^{(s)}_{\mu\ldots}$ is normalized so that $\langle J^{(s)} J^{(s)} \rangle \sim N$.

Easy generalization to more than one flavor and to the supersymmetric case.

With the same matter fields and Chern-Simons level, $\mathcal{N}$= 0,1,2,3 CS vector models differ merely by double trace and triple trace deformations. With suitable matter content, $\mathcal{N}$= 4 (Gaiotto-Witten) or $\mathcal{N}$= 6 (ABJ) CS vector models are obtained by simply gauging a flavor group with CS coupling.

Exact solution at infinite N

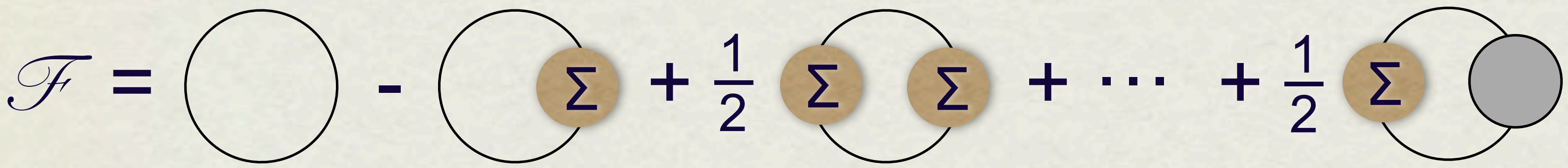
Schwinger-Dyson equations (CS in lightcone gauge)

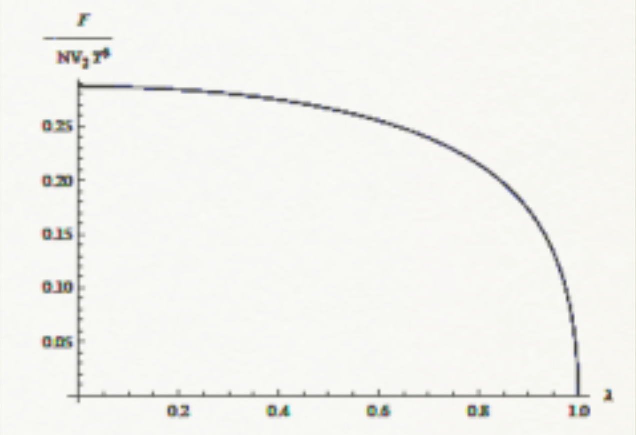


⇒ can solve exact planar matter self energy

Free energy (at finite temperature)

e.g. CS-fermion VM





Correlation functions

The higher spin dual

~~Parity~~ Vasiliev theory determine by the function

$$f(X) = X \exp(i\,\theta(X)) = e^{i\theta_0} X + b_3 X^3 + b_5 X^5 + \ldots$$

Recall equation of motion: $d\mathbb{A} + \mathbb{A} * \mathbb{A} = f_*(\mathbb{B} * K)dz^2 + f_*(\mathbb{B} * \overline{K})d\bar{z}^2$.

Coefficient of X^n determines $n+2$ and higher order coupling. E.g. $e^{i\theta_0}$ controls tree level three-point function, b_3 controls five-point function, etc.

Tree level three-point function

$$\langle J\,J\,J\rangle = \cos^2\theta_0 \langle J\,J\,J\rangle_{FB} + \sin^2\theta_0 \langle J\,J\,J\rangle_{FF} + \sin\theta_0 \cos\theta_0 \langle J\,J\,J\rangle_{odd}$$

↑ ↑ ↑

free boson free fermion parity odd

This structure was also found by perturbative two-loop computation in Chern-Simons-fermion theory (GMPTWY) and shown to follow generally from weakly broken higher spin symmetry by Maldacena-Zhiboedov.

The higher spin dual to $\mathcal{N} = 6$ ABJ vector model, and more

n-extended supersymmetric ~~parity~~ Vasiliev theory: introduce Grassmannian auxiliary variables $\psi_1, \ldots \psi_n$ in the master fields. They obey Clifford algebra $\{\psi_i, \psi_j\} = \delta_{ij}$. Equation of motion modified to

$$d\mathbb{A} + \mathbb{A} * \mathbb{A} = f_*(\mathbb{B} * K) dz^2 + f_*(\mathbb{B} * \overline{K}\Gamma) d\bar{z}^2.$$

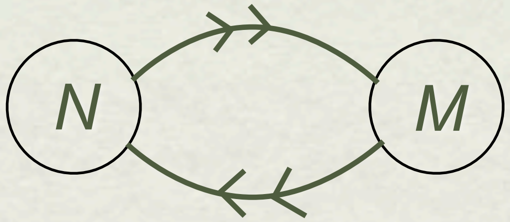
The $\mathcal{N} = 0,1,2,3,4,6$ CS vector models differ merely by double trace and triple trace deformations, and gauging a flavor symmetry with Chern-Simons coupling. These correspond to, in the holographic dual, simply changes of boundary condition on the bulk fields.

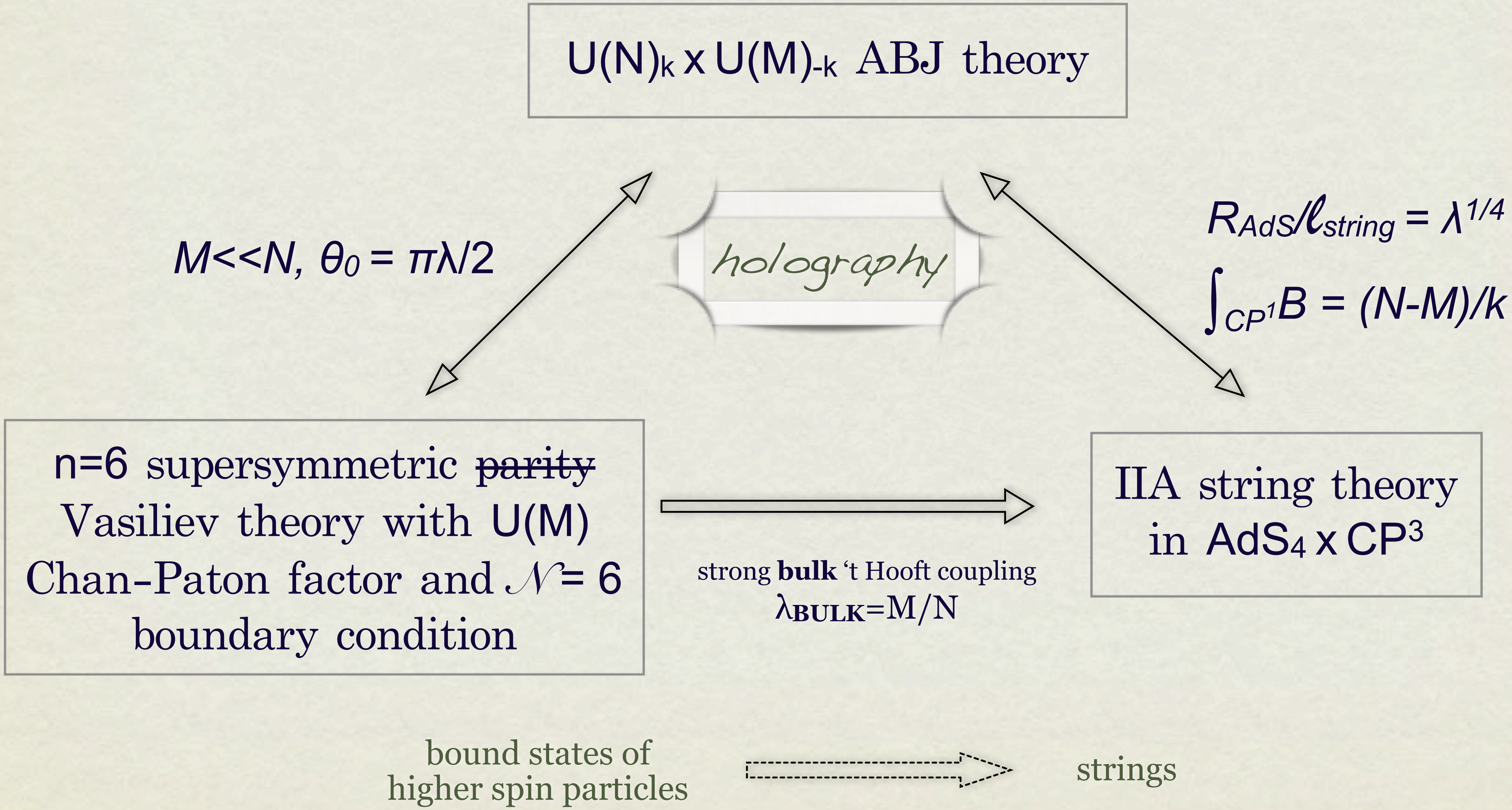
Note: A global symmetry of the boundary CFT ↔ an asymptotic symmetry δ_ε of the bulk theory. The symmetry is broken if $\delta_\varepsilon \Phi$ does not respect boundary conditions on the bulk field Φ. In particular, all higher spin symmetries and some of the supersymmetries are broken in the parity violating Vasiliev theory in this manner.

[Chang-Minwalla-Sharma-Yin, coming soon]

The duality makes a nontrivial prediction on two and three point function coefficients that do not follow from known symmetries!

A Triality





$U(N)_k \times U(M)_{-k}$ ABJ theory

$M << N$, $\theta_0 = \pi\lambda/2$

holography

$R_{AdS}/\ell_{string} = \lambda^{1/4}$

$\int_{CP^1} B = (N-M)/k$

n=6 supersymmetric ~~parity~~ Vasiliev theory with U(M) Chan-Paton factor and $\mathcal{N} = 6$ boundary condition

strong **bulk** 't Hooft coupling

$\lambda_{\mathrm{BULK}} = M/N$

IIA string theory in $AdS_4 \times CP^3$

bound states of
higher spin particles

strings

Vasiliev theory from string (field) theory?

(Supersymmetric) ~~parity~~ Vasiliev equations can be put in the form

$$\left.\begin{array}{l} d\mathbb{A} + \mathbb{A} * \mathbb{A} = e^{i\theta}\,\mathcal{B}\, dz^2 + e^{-i\theta}\,\overline{\mathcal{B}}\, d\bar{z}^2 \\ \partial_{\bar{z}}\mathcal{B} + [\mathbb{A}_{\bar{z}}, \mathcal{B}]_* = 0 \\ \partial_{z}\overline{\mathcal{B}} + [\mathbb{A}_{z}, \overline{\mathcal{B}}]_* = 0 \end{array}\right] \quad \Rrightarrow \quad Q_0\mathbb{V} + \mathbb{V} * \mathbb{V} = 0$$

$$\mathcal{B} = \mathcal{B} * K\overline{K}\Gamma \qquad \Rrightarrow \qquad (b_0 - \bar{b}_0)\,\mathbb{V} = 0$$

In the “ABJ triality”, θ is identified with the B-field flux of type IIA string theory on $AdS_4 \times CP^3$, suggesting that the RHS of Vasiliev’s equation come from worldsheet instantons (wrapping CP^1) !

How to derive Vasiliev’s system from tensionless limit of the string field theory remains to be seen. A topological open+closed string field theory from D6 brane wrapped on $AdS_4 \times RP^3$ (Jafferis-Gaiotto setup) in the zero radius limit?

Challenges

Quantize Vasiliev system in AdS_4: formulate the path integral with an action, gauge fixing, ghosts, …in ways that preserve higher spin symmetry, and prove the duality with vector models.

Derive (supersymmetric) Vasiliev system from string (field) theory in AdS in small radius/tensionless limit.

Beyond pure higher spin gauge theory: coupling to (more) matter fields in the bulk, higher dual of adjoint type gauge theories…

A rich $\mathrm{AdS}_3/\mathrm{CFT}_2$ story (that would be another talk.) Other dimensions… 5D vector models?