HILBERT SPACE NETWORKS AND UNITARY MODELS FOR BLACK HOLE EVOLUTION

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Refs: SBG 1108.2015, 1201.1037;
SBG and Y. Shi, work in progress

The black hole information "paradox," or unitarity crisis -

(most clearly illustrated in ultraplanckian scattering -- colloquium!)

A fundamental conflict:

Local quantum field theory (LQFT)

Spacetime

Analogous to stability crisis for classical atom?

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(most clearly illustrated in ultraplanckian scattering -- colloquium!)

Local quantum field theory (LQFT)

A fundamental conflict:

Analogous to stability crisis for classical atom?

- not sharply defined in Q. gravity
- perturbative breakdowns LQFT
- Growing range w/ E

etc.

Locality: - true to excellent approximation

- tightly interwoven w/consistency (causality)

Don't abandon completely, but generalize appropriately?

LQFT: $[\mathcal{O}(x), \mathcal{O}(y)] = 0 \quad , \quad (x-y)^2 > 0$

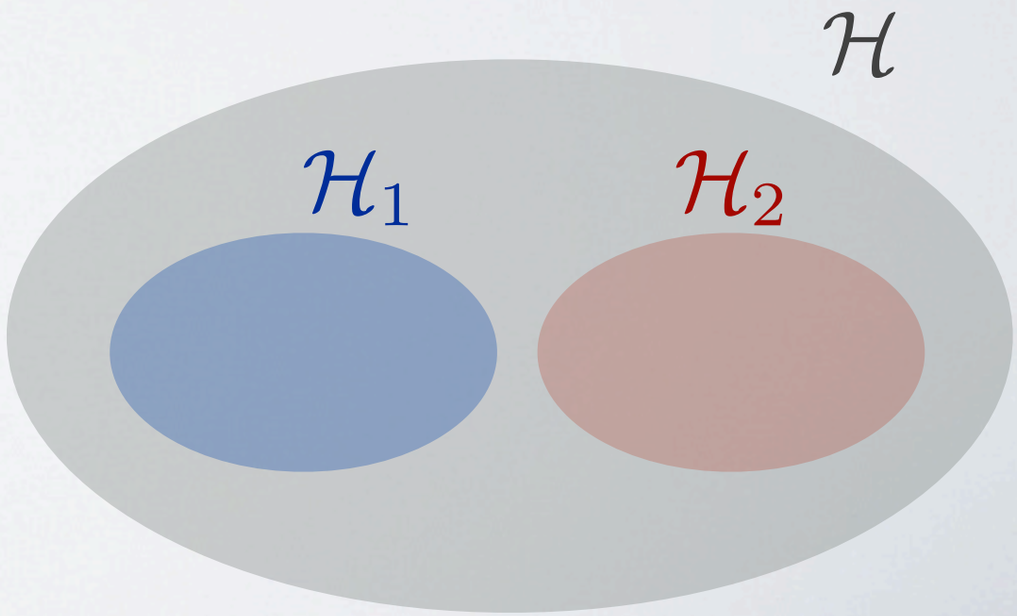
Generalize: QM $\Rightarrow$ Hilbert space

(but perhaps not local ops, fields, etc.)

Locality? $\mathcal{H} \supset \mathcal{H}_1 \times \mathcal{H}_2$

"different regions"

(coarser notion)



traditional view

Spacetime, fields $\simeq$ Hilbert space

(If AdS/CFT gives fine-grained bulk physics,
seemingly would realize such a story.)

(*If* - another seminar)

A basic message:

~~Local quantum fields in spacetime~~

Then what is physics?

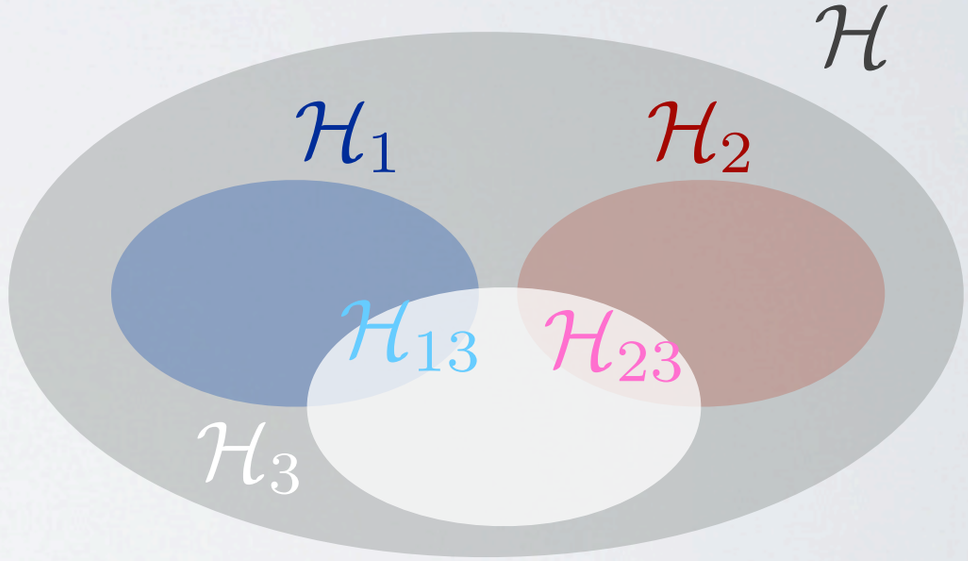
Approach: Hilbert space network with appropriate structure; recover LQFT in an approximation

(in practice, finite, but large, dimension?)

- all Hilbert spaces of given finite dimension are the same

~ Spacetime localization from nesting/overlapping tensor factor structure

(some common ideas w/ algebraic QFT; also Banks "HST" -- though important differences)



Evolution?

Unitary maps U w/ appropriate properties

(e.g. think of Schrodinger picture)

Study this perspective in context of BH evolution:

$$\mathcal{H} = \mathcal{H}_{BH} \otimes \mathcal{H}_{ext}$$

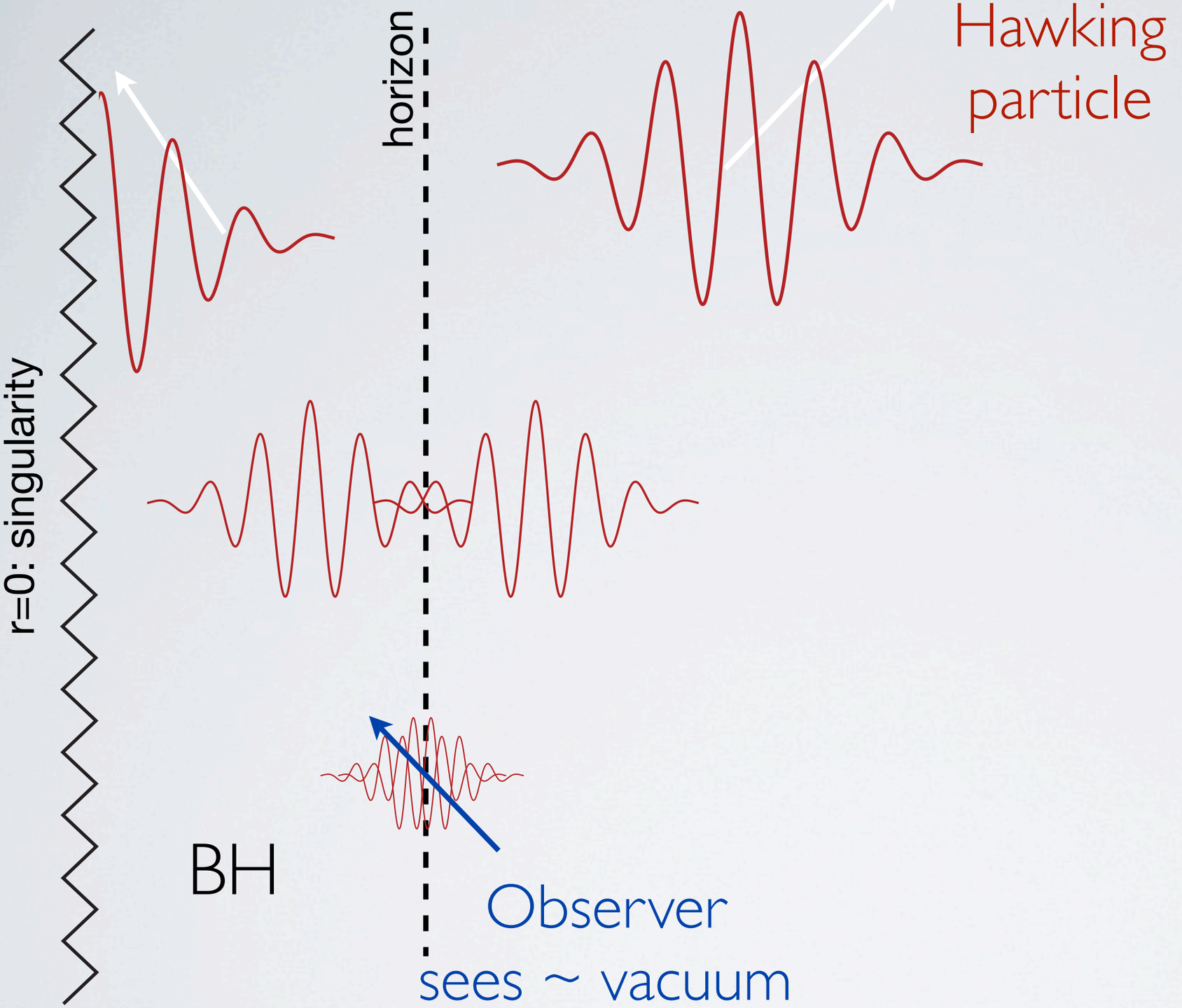
Seek evolution that:

1) Matches known/expected facts

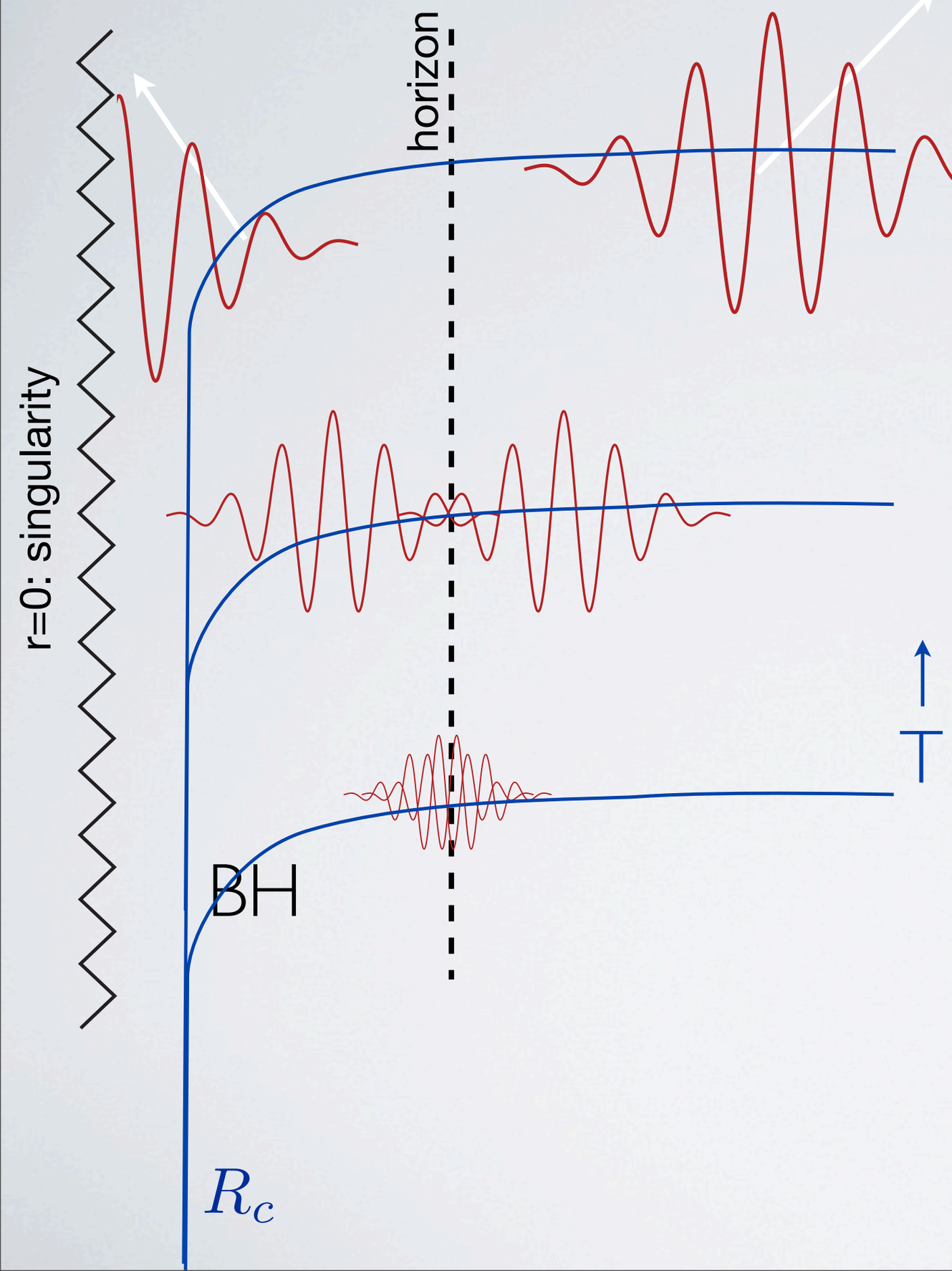
2) Is unitary

But first: Hawking (LQFT) evolution (then modify ...)

Review:


Outgoing
Hawking
particle
horizon
r=0: singularity
BH
Observer
sees ~ vacuum

Review:



Slicing → description of Hilbert space of LQFT states

Different possible slicings:

$X^+,\ X^-$: null, Kruskal

$$X^+X^-|_{r=0} = R_0^2$$

$$X^+(X^- + e^{-2T}X^+) = R_c^2$$

$a) R_c = 0$: Schwarzschild

→ $b) R_0 > R_c > 0$: "Nice"

$c) R_c > R_0$: "Natural"

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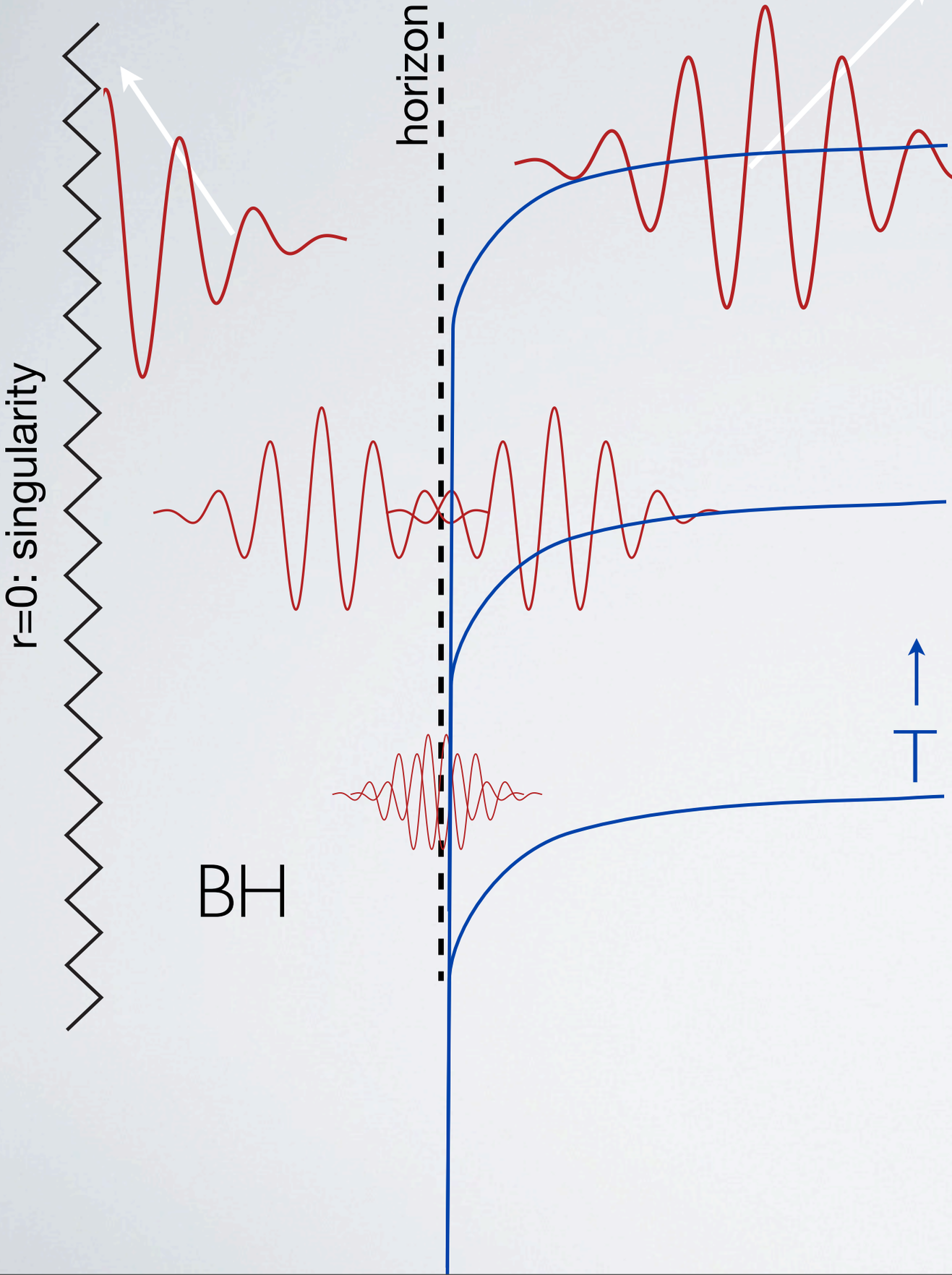

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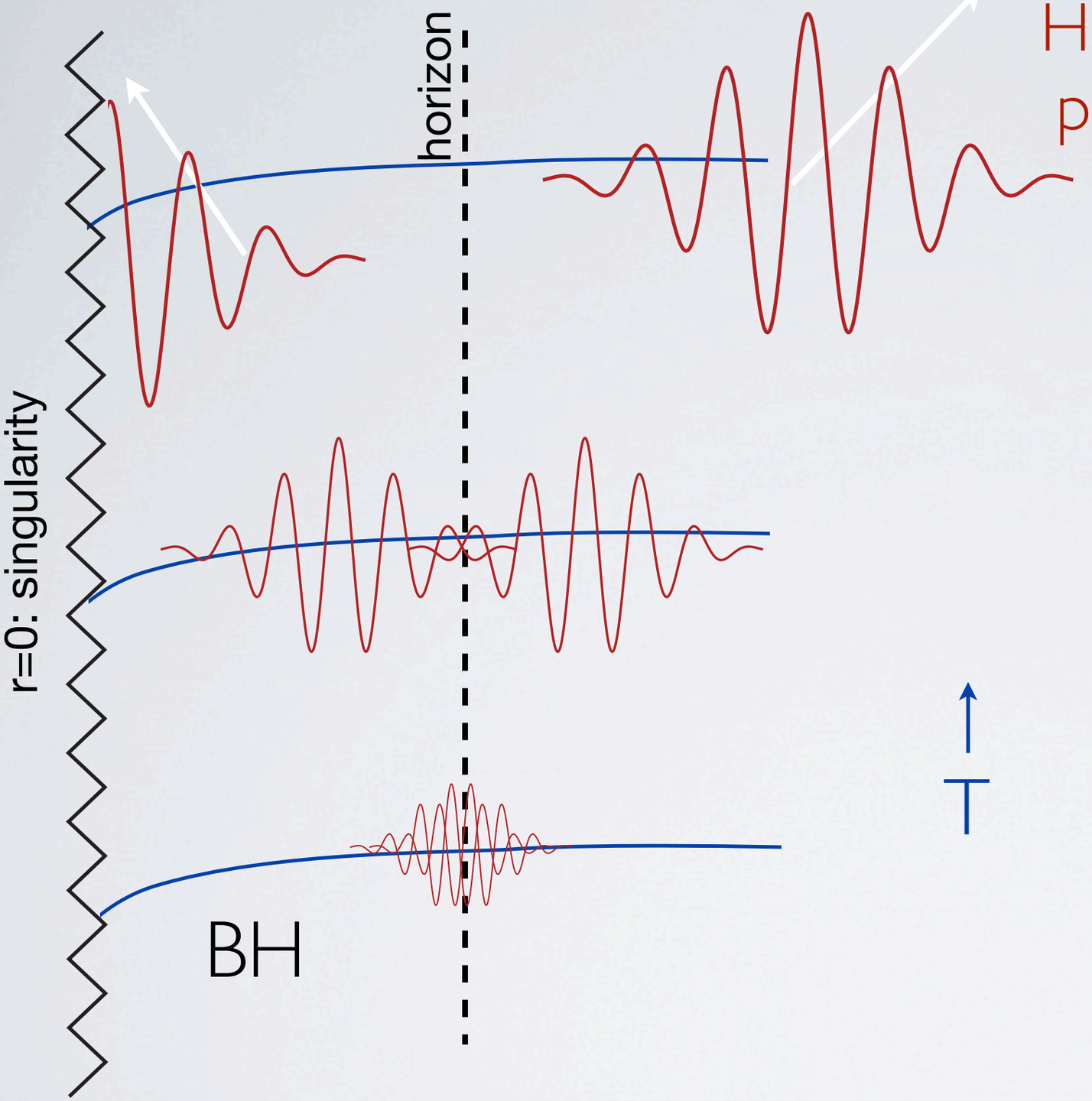

Outgoing Hawking particle

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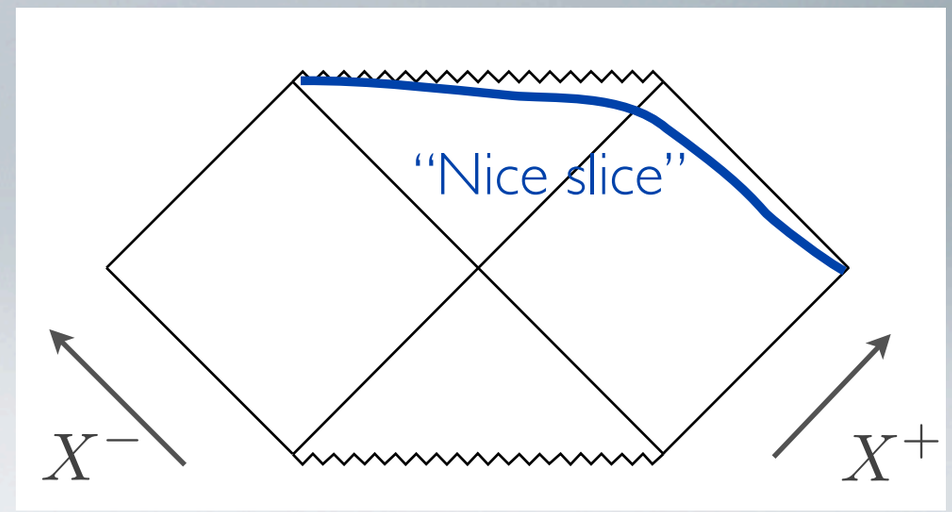
Fields and evolution:

- e.g. free field

- expand in spherical harmonics $\Phi_J = \frac{u_J(R,T)}{r^{D/2-1}} Y_J(\Omega)$

- simplify presentation: 2D BH

$$ds^2 = -\frac{dX^+ dX^-}{M - X^+ X^-}$$

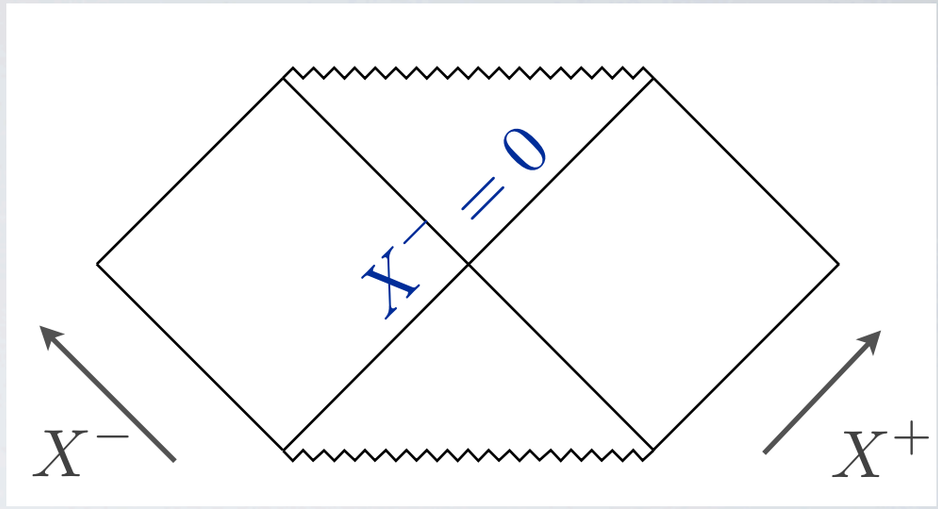


Use X^- : spatial coord

$$\phi(X^-) = \int d\omega \left(a_\omega e^{-i\omega X^-} + \text{h.c.} \right) + LM$$

Product structure?

$$X^- = \begin{cases} -e^{-x^-} & \text{out} \\ e^{-\hat{x}^-} & \text{in} \end{cases}$$

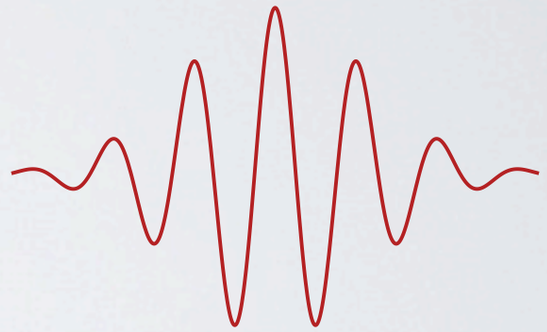


$$v(\omega) = e^{-i\omega x^-} \qquad \hat{v}(\omega) = e^{-i\omega \hat{x}^-}$$

choose complete basis of localized modes

v_{jn} $\hat{v}_{jn}$

$\omega \simeq j\epsilon$ $x^- \simeq 2\pi n/\epsilon$



$$\phi = \sum_{jn} b_{jn} v_{jn} + \hat{b}_{jn} \hat{v}_{jn} + \text{h.c.}$$

"Hawking state:"

(SBG and Nelson, 1992)

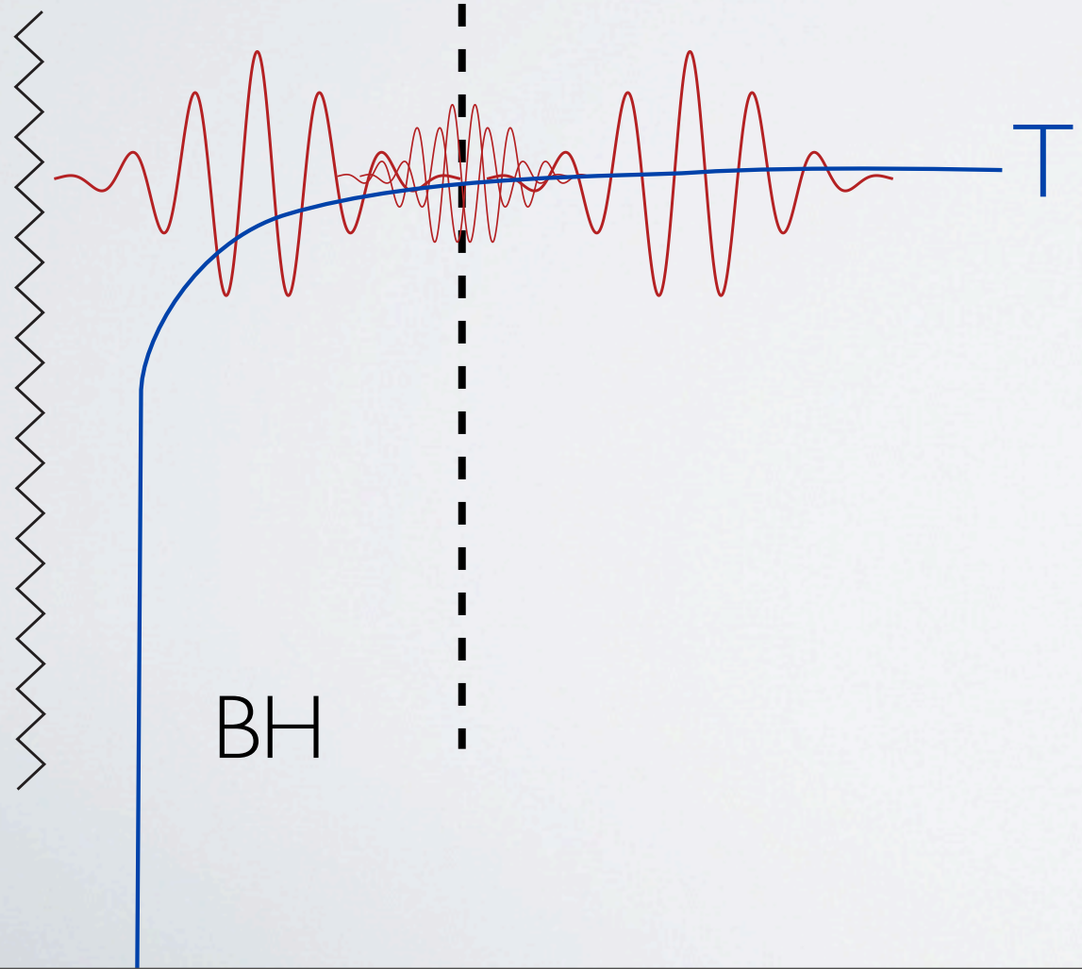
$$|0\rangle_X = \prod_{jn} S_{jn} |\hat{0}\rangle |0\rangle$$

$$2D: \ \beta = \frac{1}{2\pi}$$

$a_\omega = 0$

$$\exp\left\{ tanh^{-1}[e^{-\beta\omega_j/2}] \left(\hat{b}^\dagger_{jn} b^\dagger_{jn} - \text{h.c.} \right) \right\}$$

$$= \frac{1}{Z} \sum_{\{n_{jn}\}} e^{-\beta H/2} |\widehat{\{n_{jn}\}}\rangle |\{n_{jn}\}\rangle$$



For given T:

$$n \to \infty \ \leftrightarrow \to \text{horizon}$$

Regularize:

$$n < N(T) \ \sim \ \lambda > L$$

Shorter modes "look like" vacuum

"Hawking state" (cont'd):

So rewrite:

$$|0\rangle_X = \prod_{jn} S_{jn}|\hat{0}\rangle|0\rangle = |0\rangle_N \prod_j \prod_n^{N-1} \left(S|\hat{0}\rangle|0\rangle\right)_{jn}$$

$$\in \mathcal{H}_{BH} \otimes \mathcal{H}_{ext}$$

As discussed

- $|0\rangle_N$ can either go with BH, or "ancillary"
- Hilbert spaces effectively finite dim. (if finite time)
- This is a simple example of LQFT evolution. Generalizations: D>2, dynamical gravity, etc. (e.g. ADM framework) -- see 1201.1037

Evolution: $U = 1$

$$|0\rangle_X = |0\rangle_N \prod_j \prod_n^{N-1} \left(S|\hat{0}\rangle|0\rangle\right)_{jn}$$

$$= \frac{|0\rangle_N}{Z} \sum_{\{n_{jn}\}, n<N} e^{-\beta H/2} |\widehat{\{n_{jn}\}}\rangle |\{n_{jn}\}\rangle$$

more generally: U_{LQFT}

(also, can include infalling matter)

and: $N \to N+1$

Note: "generalized" unitary transform: dimensions of $\mathcal{H}_{BH}$, $\mathcal{H}_{ext}$ change

Cartoon: timestep ~R

(See also Mathur)

$$|0\rangle_N \to |0\rangle_{N+1} \left(|\hat{0}0\rangle + |\hat{1}1\rangle\right)_{n=N}$$

"qubit model"

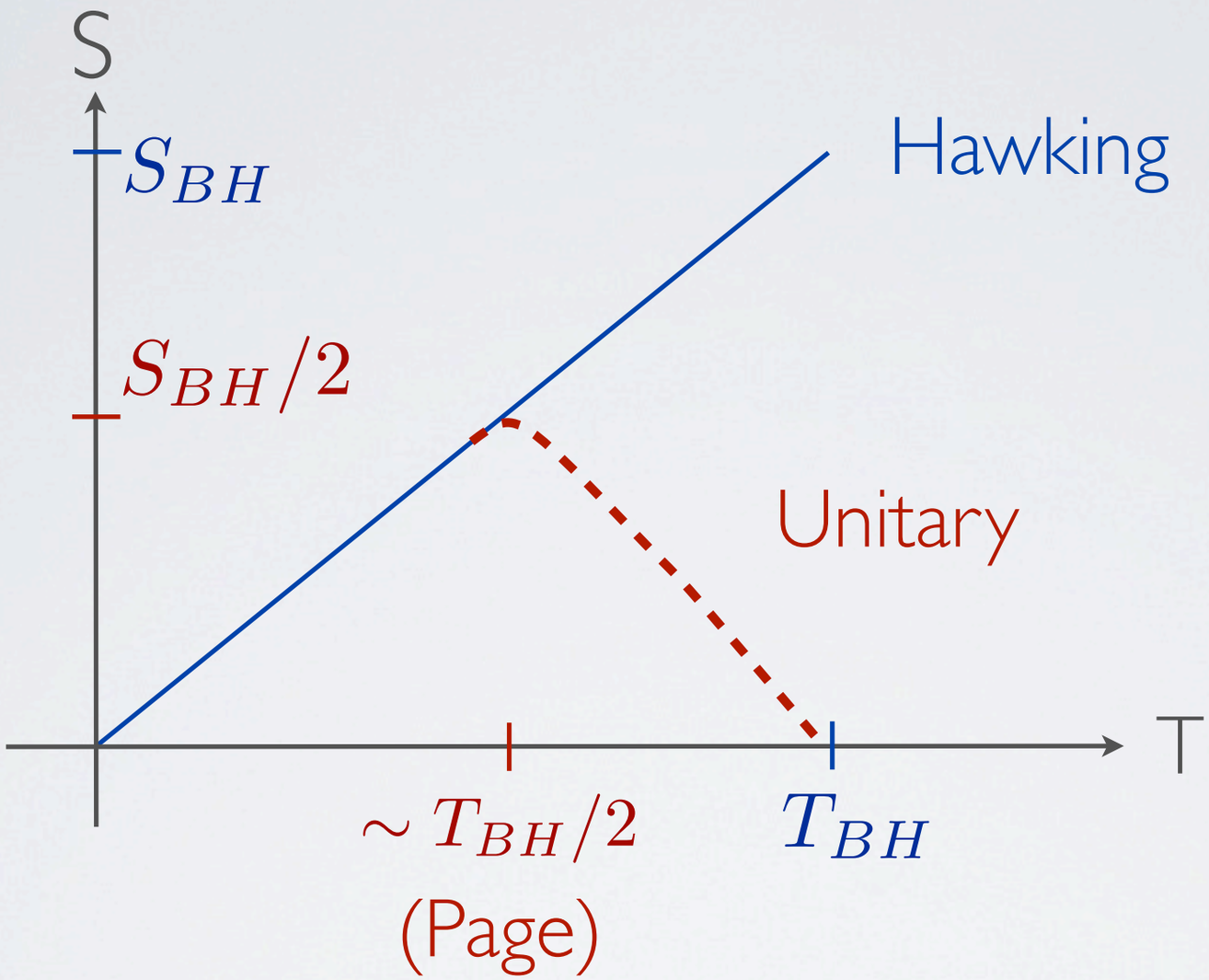
pairs produced

$(\omega_j \simeq 1/\beta)$

Nonunitarity

$$\rho_{ext} = \mathrm{Tr}_{BH}\left(|0\rangle_X\langle 0|\right) \propto \sum_{\{n_{jn}\},n<N} e^{-\beta H}|\{n_{jn}\}\rangle\langle\{n_{jn}\}|$$

$$S_{ext} = -\mathrm{Tr}\rho_{ext}\ln\rho_{ext}$$



- elsewhere (hep-th/0703116, 0911.3395, etc.) I've argued for breakdown of systematic LQFT/nice slice approximation

- but how do we describe more plausible, unitary, evolution?

Use general Hilbert space framework:

E.g. expect:

- modify $\mathcal{H}_{BH}$: $\neq \mathcal{H}_{BH,LQFT}$
- modify U : $\neq U_{LQFT}$
- mods to $\mathcal{H}_{ext}$ small

(but close?)

Goal:

Seek models of, and
constraints on, unitary evolution

(explore and parameterize tension between
LQFT and unitary evolution)

(~effective quantum theory; more?)

Physical constraints on evolution:

A) $S \to 0$

B) Innocuous to infalling observers (?)

(preserve "pain-free" horizon)

C) $dS/dt \sim 1/R$ (?)

D) Near-Hawking; ~thermodynamic (?)

E) Correspondence limit: LQFT + GR

F) Energy conservation

G) Complete, consistent

...

Basic guideline: be as conservative as possible!

What is "as close as possible to LQFT"?

Constraining/characterizing information transfer

Shi and SBG, in preparation

(all information is quantum information)

General form:

$$U : \psi \in \mathcal{H}_{BH} \times \mathcal{H}_{ext} \rightarrow \mathcal{H}'_{BH} \times \mathcal{H}'_{ext}$$

C), D): expect $dim \approx \exp S_{BH}$

~LQFT

dims change $\Delta S \sim (t' - t)/R$

U: 1-1, preserves inner product

(on subspace -- image of map - "isometry")

$\dim \mathcal{H}_{BH}$ decreases; explore other constraints

E.g. "minimal" info transfer between subsystems:

"least" excitation of $\mathcal{H}_{ext}$ for given info out of $\mathcal{H}_{BH}$

(technical condition: saturates subadditivity)

"bit transfer" (more generally, subspace transfer)

e.g.

$$|\hat{0}\rangle|\hat{a}\rangle|a\rangle \to |\hat{a}\rangle|0\rangle|a\rangle$$
$$|\hat{1}\rangle|\hat{a}\rangle|a\rangle \to |\hat{a}\rangle|1\rangle|a\rangle$$

mod unitaries on $\mathcal{H}_{BH}, \mathcal{H}_{ext}$

non-minimal:

$$|\hat{0}\rangle \to |\hat{0}\rangle|0\rangle$$
$$|\hat{1}\rangle \to |\hat{1}\rangle|1\rangle$$

Explore examples
(e.g. qubit models)

Not most conservative?

I A) e.g. qubit "q=0, I" in:

$$\rightarrow \underbrace{|\hat{a}\rangle|\hat{q}\rangle}|\psi\rangle$$

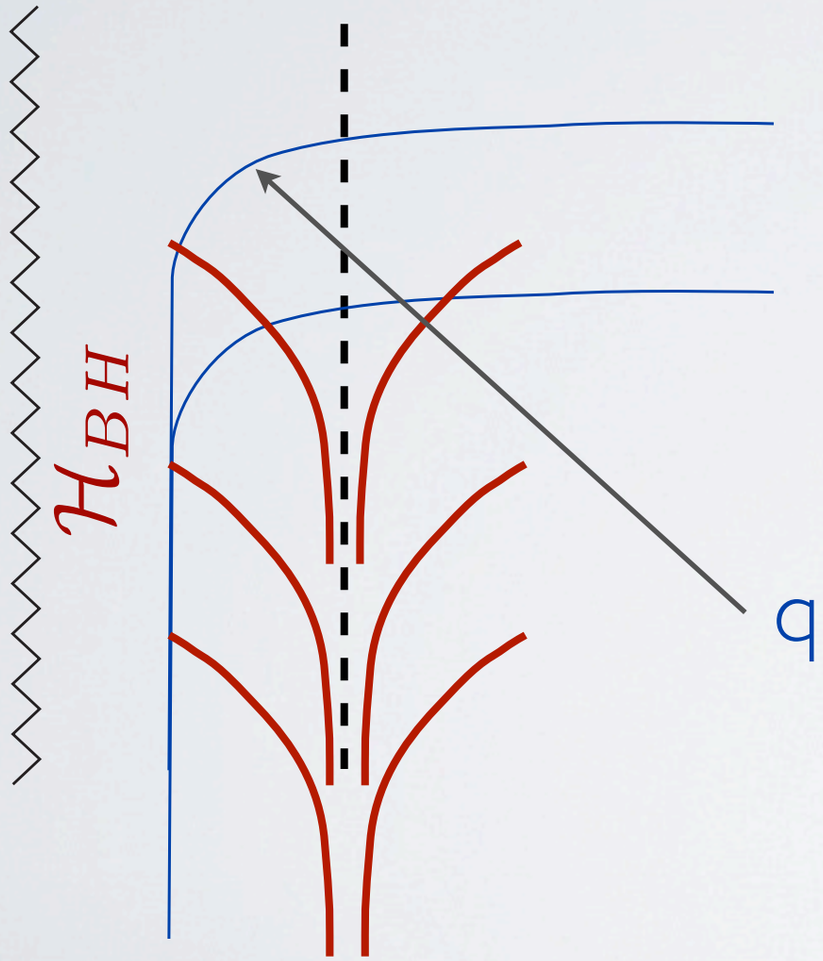
$\hat{U}\left(|\hat{a}\rangle|\hat{q}\rangle\right)$ $\quad\quad$ $\Delta t \sim R$

"Fast scrambling" (or, w/ $|\hat{0}0\rangle + |\hat{1}1\rangle$)

then: $|\hat{q}'\hat{a}'\rangle|\psi\rangle \rightarrow |\hat{a}'\rangle|q'\rangle|\psi\rangle$

(separate scrambling/transfer)

Recap:



(these are representative of more general finite dim models)

Not most conservative?

Big departure from LQFT evolution

(↔complementarity?)

Indeed, Hayden/Preskill:

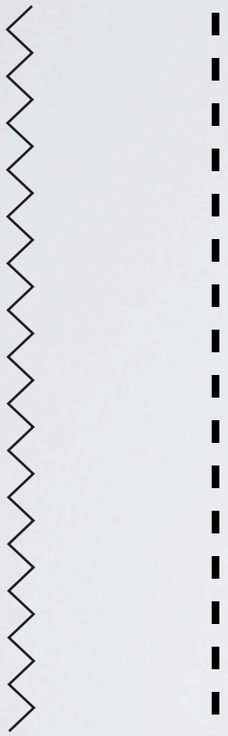
After sufficient evolution, a BH behaves as an information mirror on the scrambling time!

One classification:

	$T_{scramble}$	$T_{transfer}$
Susskind	$R \ln R$	$R \ln R$
Page		$< \mathcal{O}(RS)$
HR, nat. slice	$\sim R$?	∞
HR, nice slice	∞	∞

Not most conservative? (cont'd)

1B) Massive remnant; fuzzball



Not most conservative? (cont'd)

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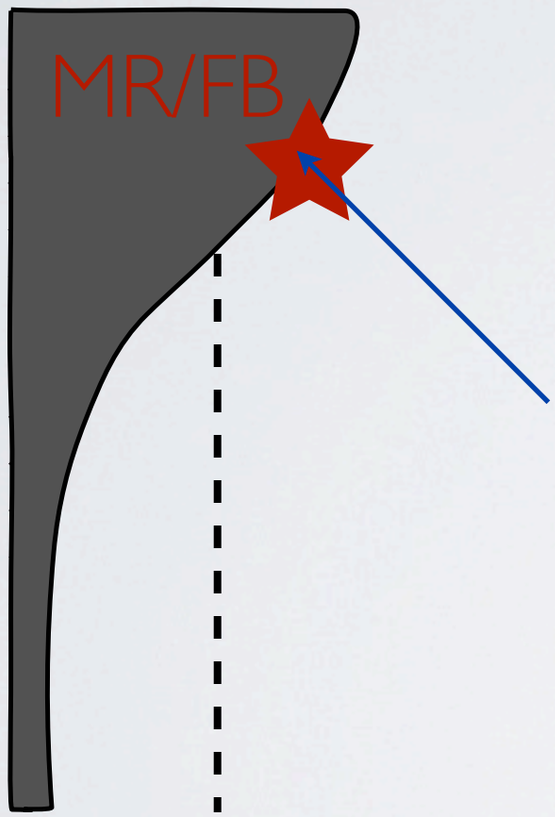

expect:

big mods. to $\mathcal{H}_{BH}$

rapidly varying microstructure outside horizon

rapid, more limited(?) scrambling

(~ "neutron star")

Unitary models: "more conservative"

2)

$$|\hat{0}\hat{0}\hat{a}\rangle|a\rangle \rightarrow \hat{U}|\hat{a}\rangle \frac{(|\hat{0}0\rangle + |\hat{1}1\rangle)}{\sqrt{2}} U|a\rangle$$

$$|\hat{0}\hat{1}\hat{a}\rangle|a\rangle \rightarrow \hat{U}|\hat{a}\rangle|\hat{0}1\rangle U|a\rangle$$

$$|\hat{1}\hat{0}\hat{a}\rangle|a\rangle \rightarrow \hat{U}|\hat{a}\rangle|\hat{1}0\rangle U|a\rangle$$

$$|\hat{1}\hat{1}\hat{a}\rangle|a\rangle \rightarrow \hat{U}|\hat{a}\rangle \frac{(|\hat{0}0\rangle - |\hat{1}1\rangle)}{\sqrt{2}} U|a\rangle$$

e.g. LQFT

e.g. "leftmost" qubits

... representative of more general (e.g. multimode) models:

(and e.g. extra unitaries)

"Hawking-like" "minimal" mods to evolution

- info imprinted in typical Hawking modes
- $\langle T^0_r \rangle = \langle T^0_r \rangle_{Hawking}$

3) $$|\hat{q}_1\hat{q}_2\hat{a}\rangle|a\rangle \rightarrow \hat{U}|\hat{a}\rangle \underbrace{\frac{(|\hat{0}\rangle|0\rangle + |\hat{1}\rangle|1\rangle)}{\sqrt{2}}}_{\text{Usual Hawking particles}} \underbrace{|\hat{0}'\hat{0}''\rangle|q_1'q_2''\rangle}_{\text{Not typically occupied}} U|a\rangle$$

(again, modulo unitaries)

... representative of more general (e.g. multimode) models:

- more generic

- yield extra energy flux

(But: $dE/dI > 1/R$?)

Comments:

1) "Minimal" info. transfer leads to restricted evolution

(how close to this are we?)

2) Our physical constraints lead to more restrictions

(incl. rate constraints)

(Further exploration: SBG and Y. Shi, in prep)

3) Nonlocal

4) What about B) Innocuous to infalling observer(?) ?

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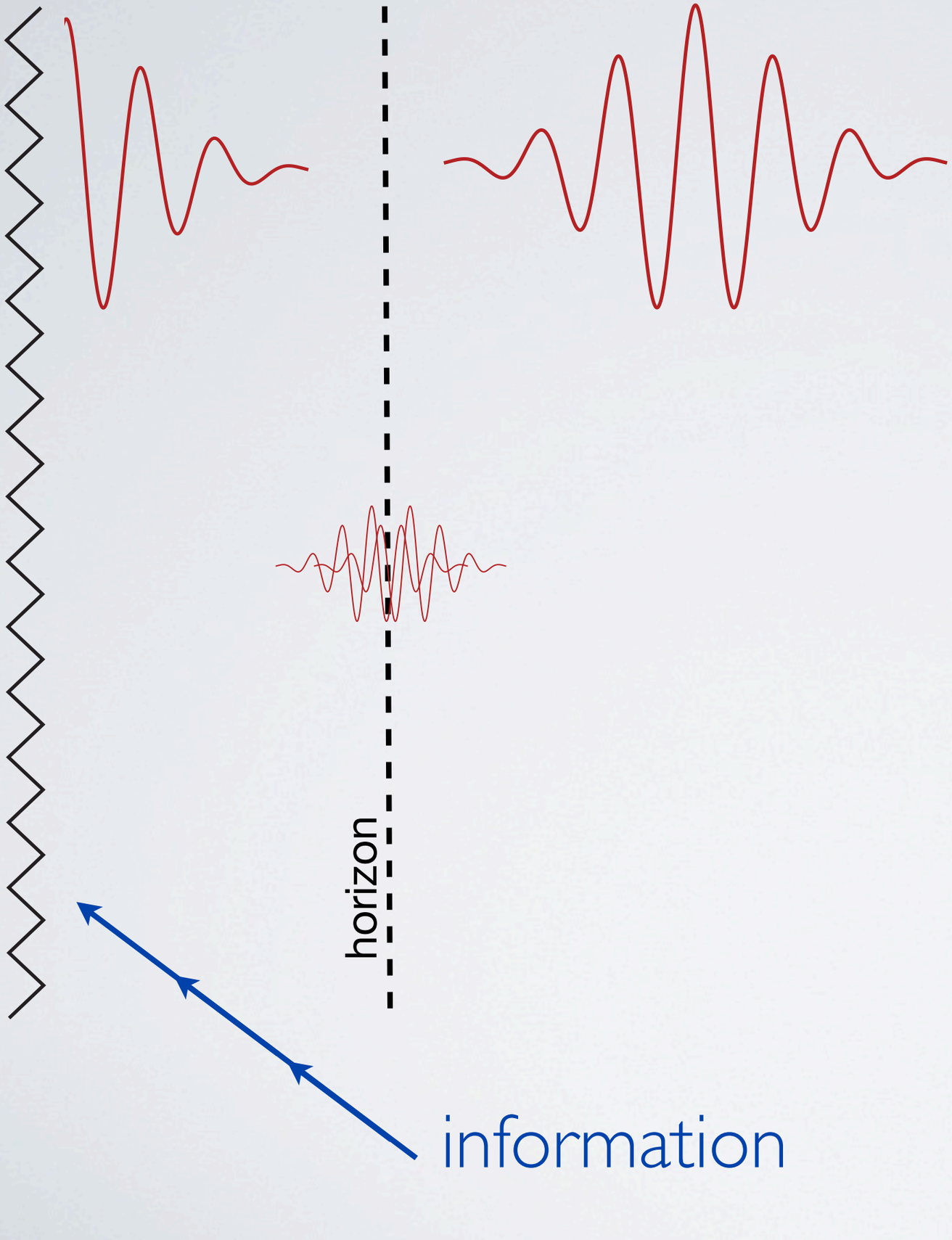

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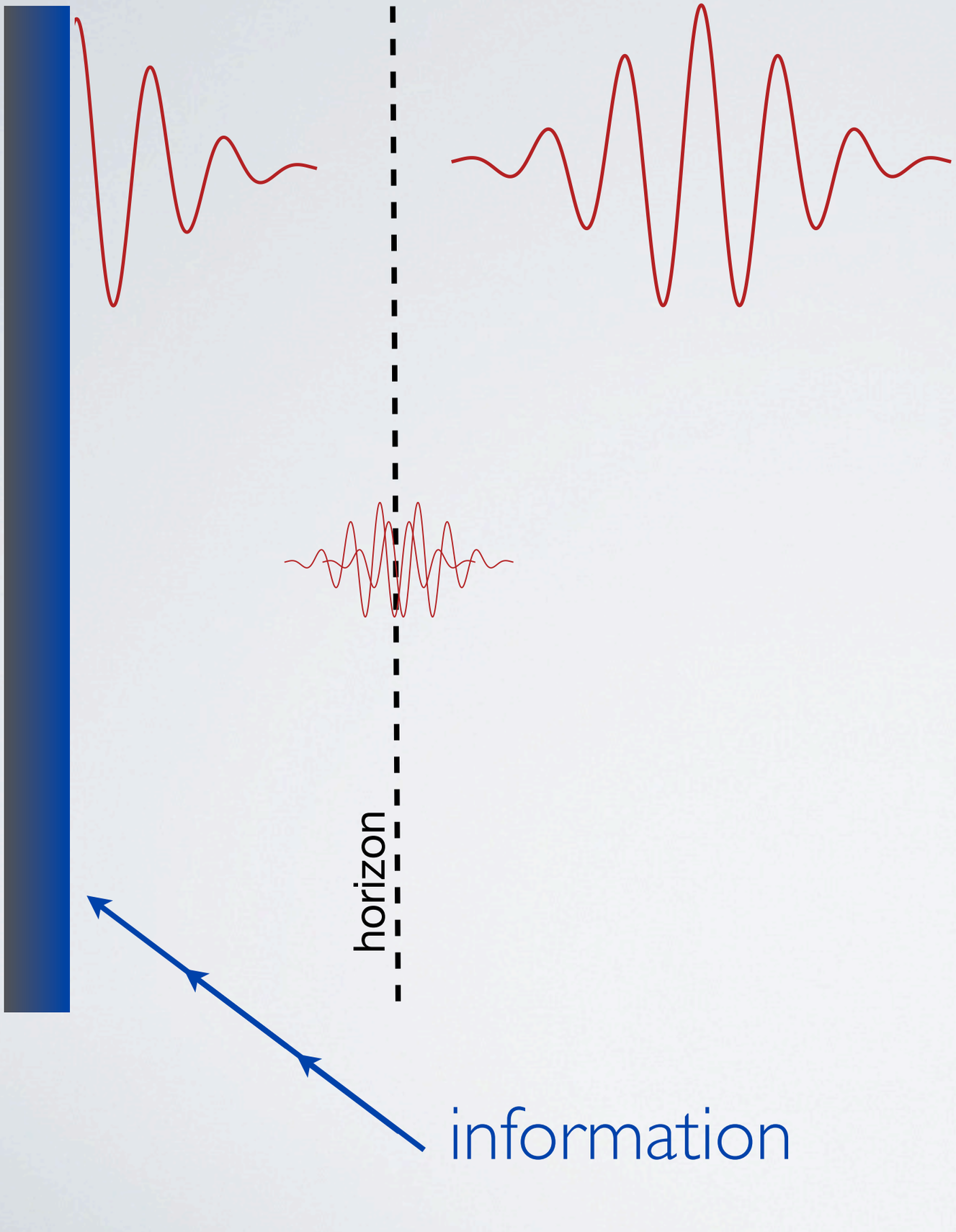

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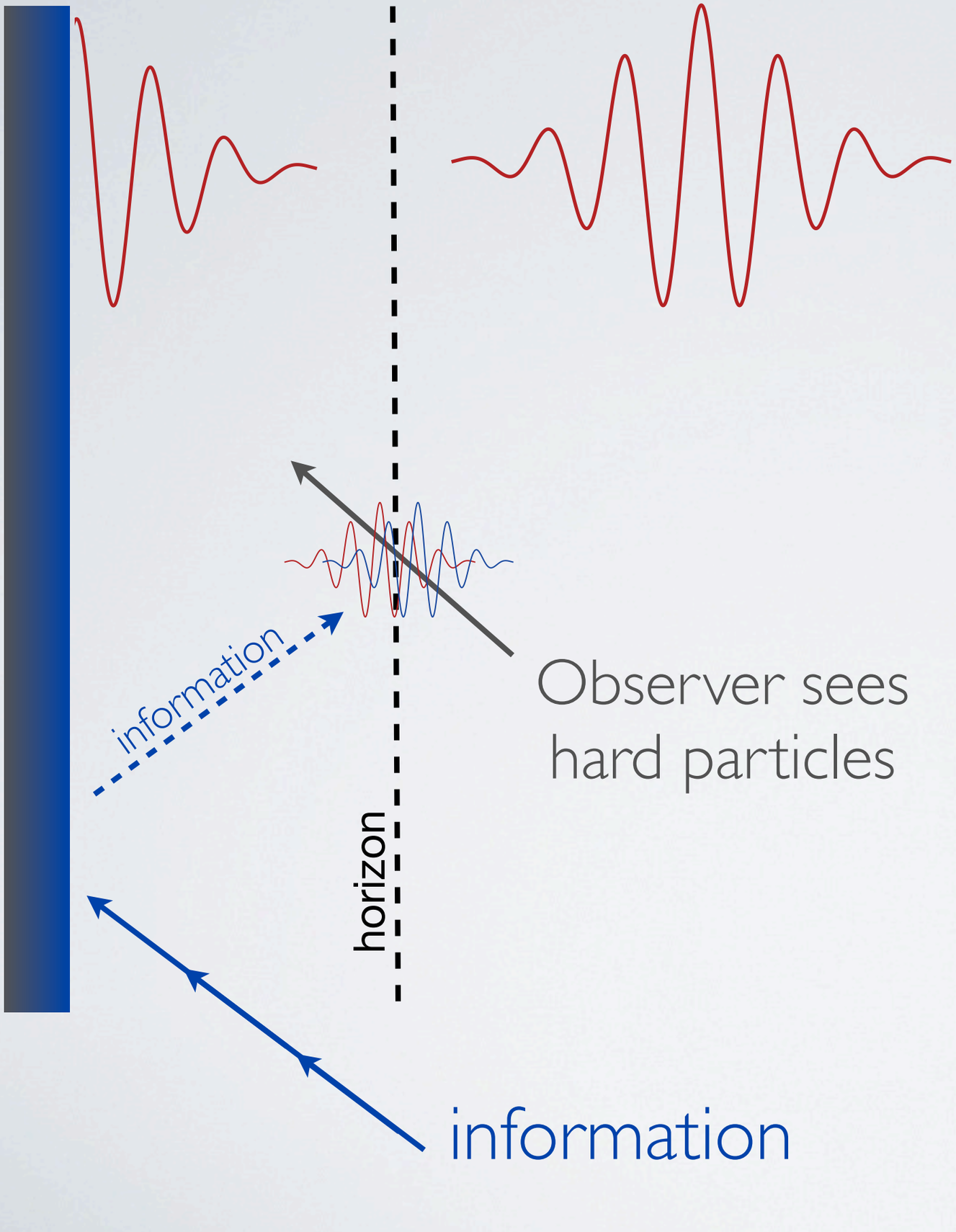

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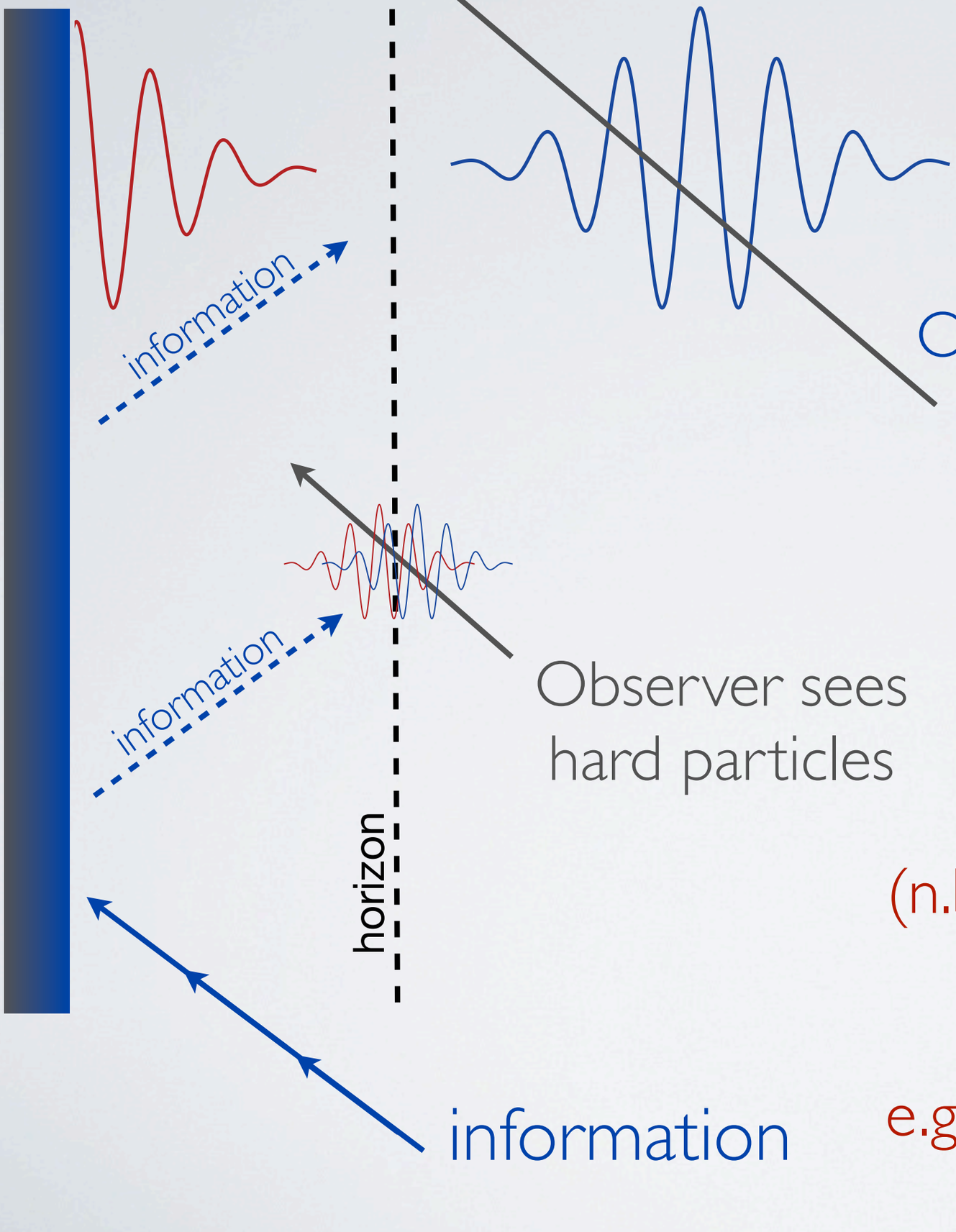

Observer sees ~1 extra quantum
of energy ~1/R per time R
(~innocuous)

(n.b.: idea is: geometry not strictly correct picture of physics - spacetime=approx.! ;
e.g. nice slice descript. not good at long times)

Comments, cont'd:

5) Models #2, 3: "nearly" LQFT evolution

6) Underlying mechanism/mechanics?

Centrally important question

Ref. report:

For example, do known string theory models lead to these rules ? In the absence of a concrete realization in some proposed theory of quantum gravity ... reject

(perhaps an AdS/CFT miracle ...)

6) Underlying mechanism/mechanics?

Let's return to the history of the atom and QM:

Correct Physics (atomic stability) —Bohr→ Guess ad-hoc rules → Development of QM

So:

1) we might try to describe/parametrize the Correct Physics (presumably, unitary evolution)

2) such an approach may yield important clues (otherwise in short supply) towards the underlying mechanics!

Indeed, once headed the right direction, there were two apparently different paths to QM (matrix mechanics, wave mechanics)!

At present: various constraints and clues.

Constraints and Clues:

- If QM, need to fit within Hilbert-space framework

- Locality/spacetime from Hilbert tensor structure

$\mathcal{H}_1 \otimes \mathcal{H}_2$, etc.

- Unitary evolution - significant info-theoretic constraints

- "Nearly" LQFT

Innocuous to infalling observer(?); ~Hawking; ~thermodynamic(?)

Correspondence limit: LQFT + GR

Constraints and Clues (cont'd):

- Needed generalization to/clues from Q. Cosmo

Sufficiently general formulation of QM?

spacetimes, histories not input "Universal QM" 0711.0757

Question of correspondence limit

- Properties of S-matrix

Analyticity; Symmetries; HE behavior; macrocausality; ...

SBG & Srednicki, 0711.5012

SBG & Porto, 0908.0004

Erice lectures, 1105.2036

another talk...

Reasons to be optimistic!