Higher Spin Gravity from 2d CFT

Tom Hartman
Institute for Advanced Study

Bits, Branes, Black Holes @ KITP
March 29, 2012

Simplified Holography

A goal

- Find a holographic duality simple enough to solve, but complicated enough to look like gravity in d>2.

This talk

- Simple bulk: 3d higher spin gravity
- Simple boundary: 2d CFT with $\mathcal{W}_N$ symmetry

Motivation

What makes holography tick?

- How does the radial direction emerge from renormalization?

  in free field theory: Douglas, Mazzucato, Razamat '10

- What is a black hole? (Coarse-graining of microstates, information retrieval, etc.)

- How general is holography? (de Sitter, cosmology, etc.)

Higher Spin Gravity

Gravity plus large (or infinite) number of massless fields,

$$A_{\mu_1 \cdots \mu_s}$$

with spins

$$s = 0, 1, 2, 3, 4, \cdots ,$$

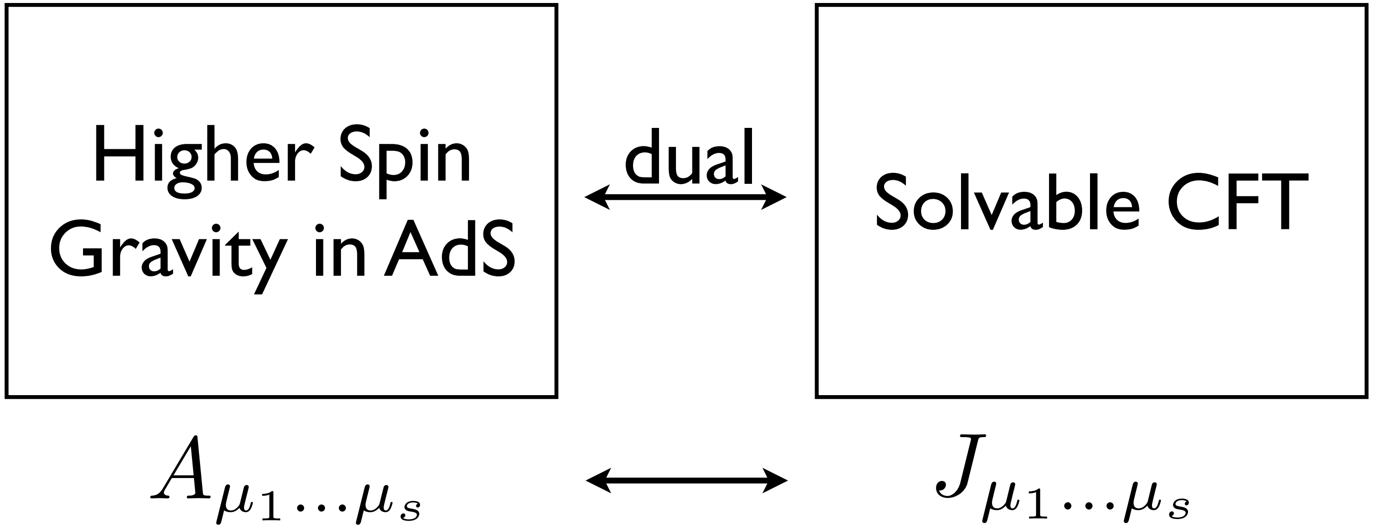
Spin-2 = graviton. Massless higher spin fields mean very large gauge symmetry extending diffeomorphism invariance.

Consistent interacting theory exists for $\Lambda \neq 0$

Fradkin and Vasiliev, 1987
Vasiliev, 1990

Toy model for string theory in the stringy limit

Higher Spin Holography



Higher Spin Dualities

d=4 gravity

- Vasiliev gravity in AdS_4 ⟷ $O(N)$ CFT_3
- Similar construction in 4d de Sitter

Fronsdal '79
Witten
Sundborg
Mikhailov
Sezgin & Sundell
Klebanov & Polyakov
Giombi & Yin
etc...

d=3 gravity

- Vasiliev gravity in AdS_3 ⟷ W_N CFT_2
  - Gravity side is simpler than 4d
  - Interacting, solvable CFT duals

Campoleoni et al
Henneaux & Ray
Gaberdiel & Gopakumar
Gaberdiel & TH
etc.

Outline

- ✔ Introduction
- Minimal Model CFTs
- 3d Higher Spin Gravity
- The duality
- Black holes

Minimal Model CFTs

BPZ Virasoro Minimal Models

Belavin, Polyakov, Zamolodchikov '84

- Central charge $c < 1$
- Ising model, etc
- Rational: finite number of primaries

W_N Minimal Models

- $c < N - 1$; allows for large N, classical limit in the bulk
- Extended conformal symmetry: W_N instead of Virasoro
- Rational and exactly solvable

Extended Conformal Symmetry

3d CFT

- Maldacena and Zhiboedov 2011: A single conserved current with spin >2 implies an infinite set of conserved higher spin currents, and the <JJJ...> correlators are those of a free theory.

2d CFT

- There is no such theorem. Interacting theories with a finite set of higher spin currents can be constructed explicitly.
- Definition of W-algebra: An extension of the Virasoro algebra by higher spin currents.

- Example: $\mathcal{W}_3$
  - Extended chiral algebra: spin-2: $T(z)$ spin-3: $W(z)$

Zamolodchikov '85

- Example: $\mathcal{W}_N$
  - Higher spin currents of all spins s=2,...,N

W_N Minimal Models

W_N specifies the symmetries. The simplest CFTs with this symmetry are the "minimal models":

$$\frac{SU(N)_k \times SU(N)_1}{SU(N)_{k+1}} \qquad c < N - 1$$

- N=2 gives the c < 1 Virasoro minimal models -- Ising, etc.
- For $k = \infty$ this is the singlet sector of N-1 free bosons

These are solvable CFTs, ie we can compute:

- Dimensions and charges of primary operators
- Partition function
- All correlation functions
- W-algebra commutation relations

... at least in principle

Outline

- ✔ Introduction
- ✔ Extended conformal symmetry in 2d
- 3d Higher Spin Gravity
- The duality
- Black hole entropy

3d Gravity/Chern-Simons

AdS_3 gravity is related to Chern-Simons gauge theory

Achucarro, Townsend '86
Witten '88

$$g_{\mu\nu} \to \omega, e \to A_\pm = \omega \pm e$$

$$S_{Einstein} = \frac{k}{4\pi} \int \mathrm{tr} \left(A_- dA_- + \frac{2}{3} A_-^3 \right) - \frac{k}{4\pi} \int \mathrm{tr} \left(A_+ dA_+ + \frac{2}{3} A_+^3 \right)$$

$$A_\pm \in SL(2, R) \qquad\qquad k = \frac{\ell_{AdS}}{4 G_N}$$

Comments:

- This sector of the theory is topological. Any extra matter is not.
- These theories are inequivalent. For our purposes, the CS action is just a rewriting of the Einstein action in convenient variables, and should not be thought of as a gauge theory.

Higher Spin Gravity

Enlarge the gauge group

$$sl(2,R) \to g$$

Finite N

$$g = sl(N)\ , \quad \text{spins} = 2, 3, \ldots, N$$

Infinite N (all spins >1)

$$g = sl(\infty)$$

$$\boxed{g = hs(\lambda)} \leftarrow \text{non-integer } sl(\lambda)$$

Fradkin, Vasiliev '80s
Blencowe '88
etc.

Higher Spin Gravity

Comments

- We will discuss the one-parameter family of higher spin theories based on the infinite Lie algebra $hs(\lambda)$

- Translation back to metric-like variables is known implicitly, but complicated

- In d > 3, action is unknown, and truncation to any finite number of higher spins is impossible

Outline

- ✓ Introduction
- ✓ Extended conformal symmetry in 2d
- ✓ 3d Higher Spin Gravity
- The duality
- Black hole entropy

The Conjecture

A duality between

Gaberdiel, Gopakumar '10

- The 3d higher spin gravity theory based on $hs(\lambda)$ plus two additional scalar matter fields $\phi_\pm$ with masses

$$M^2 = -1 + \lambda^2$$

- The 2d W_N minimal model CFT at level k, with large N

with the tunable 't Hooft-like coupling

$$\lambda = \lim_{N,k\to\infty} \frac{N}{N+k} \quad , \quad 0 < \lambda < 1$$

$$c \approx N(1-\lambda^2)$$

Evidence

Symmetries

Spectrum of Primaries

Correlation functions

Black holes

Bulk Symmetries

Brown & Henneaux computed asymptotic symmetries in ordinary AdS_3 gravity:

$$sl(2) \quad \longrightarrow \quad \text{Virasoro}$$

In higher spin gravity, there is a similar enhancement to a W-algebra:

$$hs(\lambda) \quad \longrightarrow \quad \mathcal{W}_\infty(\lambda)$$

Campoleoni et al
Henneaux & Ray
Gaberdiel & TH

$\mathcal{W}_\infty(\lambda)$ is a nonlinear algebra with an infinite number of conserved currents.

Matching Symmetries

The symmetry in the CFT at finite N is

$$\mathcal{W}_N$$

So the duality requires

$$W_\infty[\lambda] = \lim_{N,k\to\infty} W_N \quad \text{with} \quad \lambda = \frac{N}{N+k}$$

This is unproven but appears to be true

Gaberdiel, TH
Gaberdiel, Gopakumar, TH, Raju

- (Evidence = properties of degenerate representations)
- The higher spin algebra $hs(\lambda)$ is hiding inside $\mathcal{W}_N$ in the 't Hooft limit

Spectrum

Bulk

Gauge fields + two complex scalars, with

$$h_\pm = \frac{1}{2}\left(\frac{3}{2} \pm \sqrt{\frac{9}{4} + m^2\ell^2}\right) = \frac{1 \pm \lambda}{2}$$

CFT

States in the coset theory are labeled by two representations of SU(N). Examples:

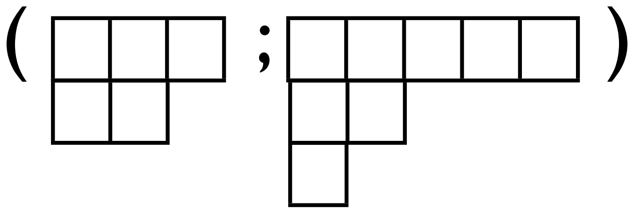
$$\frac{SU(N)_k \times SU(N)_1}{SU(N)_{k+1}}$$

$(0 ; \square)$ $h = \dfrac{1-\lambda}{2}$ dual to first scalar

$(\square ; 0)$ $h = \dfrac{1+\lambda}{2}$ dual to other scalar

Spectrum II

More complicated reps



$\leftrightarrow$ multiparticle state of bulk scalars

A problem with light states

CFT states of the form $(\Lambda;\Lambda)$ are very light

$$h \sim \frac{\lambda^2}{N}$$

but have (so far) no clear bulk interpretation. cf: S. Shenker's talk.

Spectrum & Correlators

Possible Escapes:

- Drop one of the scalars; resulting theory is consistent perturbatively but not modular invariant Chang & Yin
- Lots of classical solutions in the bulk, which account for these additional states Castro, Gopakumar, Gutperle, Raeymaekers

Quick Summary

- Excluding light states, bulk spectrum and CFT spectrum match exactly as $N \to \infty$

Gaberdiel & Gopakumar
Gaberdiel, Gopakumar, TH, Raju

- The `t Hooft limit is nice: ie, correlators factorize.

- Certain correlators have been computed in Vasiliev theory in the bulk, and matched to the CFT

Chang & Yin
Ammon, Kraus, & Perlmutter

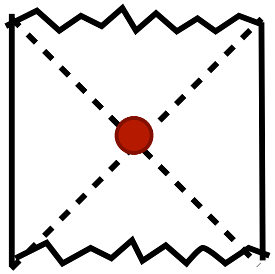
Outline

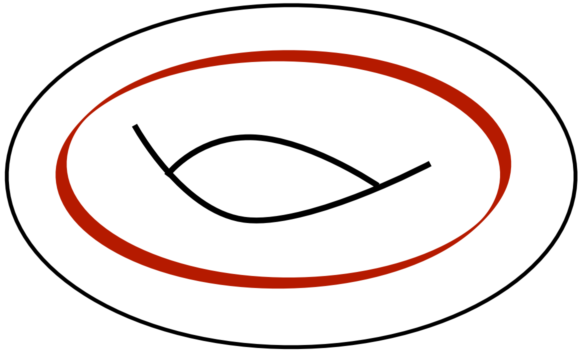
- ✓ Introduction
- ✓ Extended conformal symmetry in 2d
- ✓ 3d Higher Spin Gravity
- ✓ The duality
- Black holes

Black Holes in Higher Spin Gravity

What is a black hole?

- Lorentzian: Classical solution with a horizon
- Euclidean: smooth, solid torus





However,

- The metric and gauge fields mix under higher spin gauge transformations

- Thus Ricci and causal structure are *not* gauge invariant.

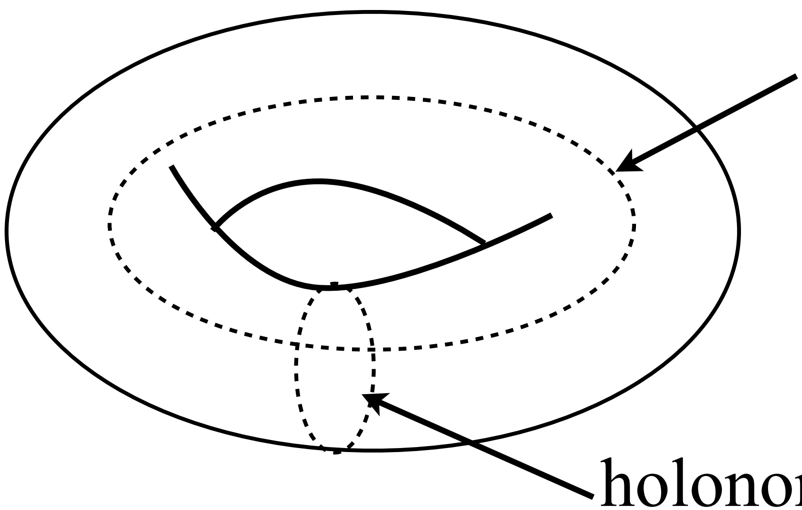
  - What is "smooth"? What is a "horizon"?

Black Holes in Higher Spin Gravity

- The Chern-Simons description is useful to define black holes in a gauge-invariant way
- Gauge invariant data = holonomies of the Chern-Simons gauge field $A \in hs(\lambda)$

$$\mathrm{Pexp} \oint A$$

"Black hole" = flat connection on a torus, with vanishing holonomy around one cycle



non-trivial holonomy encodes mass, ang. mom., and higher spin charges

holonomy = 0

Phase structure?

BTZ black hole dominates the free energy at asymptotically high temperatures,

$$T \gg c$$

where the CFT obeys the Cardy formula. (cf. 4d)

Otherwise the phase structure is completely unknown, in both bulk and CFT.

- Is there a Hawking-Page transition at $T \gtrsim 1$?
- Or perhaps a Shenker-Yin-like transition at Planck temperatures?
- Do other saddles dominate at other temperatures?

Although exact CFT partition function is known, it is too complicated to solve this problem by brute force.

Instead I will discuss charged black holes at very high temperature.

Charged Black Holes

The black hole in $hs(\lambda)$-gravity carrying spin-3 charge has been constructed explicitly.

Gutperle & Kraus '11
Ammon, Gutperle, Kraus, & Perlmutter
Castro, Hijano, Lepage-Jutier, & Maloney
Kraus & Perlmutter

Parameters

$$L_0, \overline{L}_0 = \text{mass} \pm \text{ang. mom.}$$

$$Q_3, \overline{Q}_3 = \text{spin-3 charges}$$

Potentials

$$\tau, \overline{\tau} = \text{inverse Hawking temperature}$$

$$\mu, \overline{\mu} = \text{spin-3 potential}$$

Smoothness

- The zero-holonomy condition relates charges to potentials,

$$L_0 = L_0(\tau, \mu; \lambda) \qquad Q_3 = Q_3(\tau, \mu; \lambda)$$

Aside: RG Flows

Ammon, Gutperle, Kraus, & Perlmutter

Turning on a higher spin potential is like deforming the CFT by an irrelevant operator,

$$S_{cft} \to S_{cft} + \mu \int d^2 z W(z)$$

Therefore these black holes violate the original Brown-Henneaux boundary conditions.

However:

- It *is* asymptotically AdS in a different higher-spin gauge
- A different set of higher spin fields are identified as "metric" in the UV
- Thus these black holes describe an RG flow between two CFTs

Black Hole Entropy

Horizon area / Wald entropy is not higher-spin-gauge invariant. But Wald entropy was designed to integrate the first law of thermodynamics, so we might as well just do this directly:

$$L_0 \sim \partial_\tau \log Z$$

$$Q_3 \sim \partial_\mu \log Z$$

$$\Rightarrow Z = \mathrm{Tr}\, e^{2\pi i(\tau L_0 + \mu Q_3)}$$

Smoothness (zero-holonomy) condition gives the charges. Integrating the thermodynamic relations gives the free energy,

Thermodynamics

Bulk partition function

$$\log Z = \frac{i\pi c}{12\tau}\left[1 - \frac{4}{3}\frac{\mu^2}{\tau^4} + \frac{400}{27}\frac{\lambda^2-7}{\lambda^2-4}\frac{\mu^4}{\tau^8} - \frac{1600}{27}\frac{5\lambda^4-85\lambda^2+377}{(\lambda^2-4)^2}\frac{\mu^6}{\tau^{12}} + \cdots\right]$$

Kraus & Perlmutter

$$\text{entropy} = (1 - \tau\partial_\tau - \mu\partial_\mu)\log Z$$

- this should equal the on-shell action of the Vasiliev theory, but this computation has not been done.
- first term is the Cardy formula Strominger '97

CFT partition function

Gaberdiel, TH, Jin

- Compute $Z = \mathrm{Tr}\, e^{2\pi i(\tau L_0 + \mu Q_3)}$ in a CFT with $\mathcal{W}_\infty(\lambda)$
- At high temperature, result agrees exactly with the bulk
- In what regime does this formula apply to minimal models?

Conclusions

Summary

- 3d higher spin gravity = 2d CFT with W_N symmetry

The Future

- Can we use these models to tackle difficult issues in holography and quantum gravity, like RG, de Sitter space, information paradox, etc.?

Jevicki and Jin
Douglas, Mazzucato, & Razamat
etc.