Accelerated Expansion and AdS/CFT

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with JB Hartle, SW Hawking

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Can AdS/CFT be applied to cosmology?

- Time-dependent Lorentzian AdS/CFT

  $\rightarrow$ "cosmological" backgrounds

Can AdS/CFT be applied to cosmology?

- Time-dependent Lorentzian AdS/CFT

  $\rightarrow$ "cosmological" backgrounds

- Analytic continuation from Euclidean AdS

  $\rightarrow$ wave function of perturbations

Wave function of the universe:

- Euclidean AdS/CFT
  [Horowitz & Maldacena '04]

$$\exp(-I^R_{ADS}[h,\chi]/\hbar) = Z_{QFT}[h,\chi]$$

- No-boundary State
  [Hartle & Hawking '83]

$$\Psi[h,\chi] = \exp(-I_{dS}[h,\phi]/\hbar)$$

This talk: *connect both ideas.*

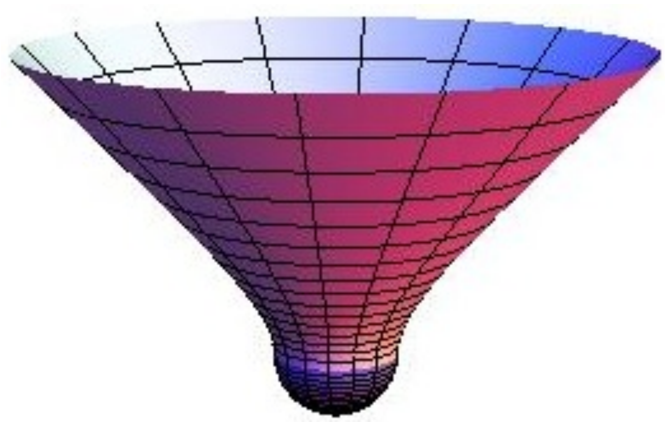
Motivation

- precise formulation of no-boundary state
- singularity resolution
- theoretical foundations of inflation
- probabilities in eternal inflation

No-Boundary State

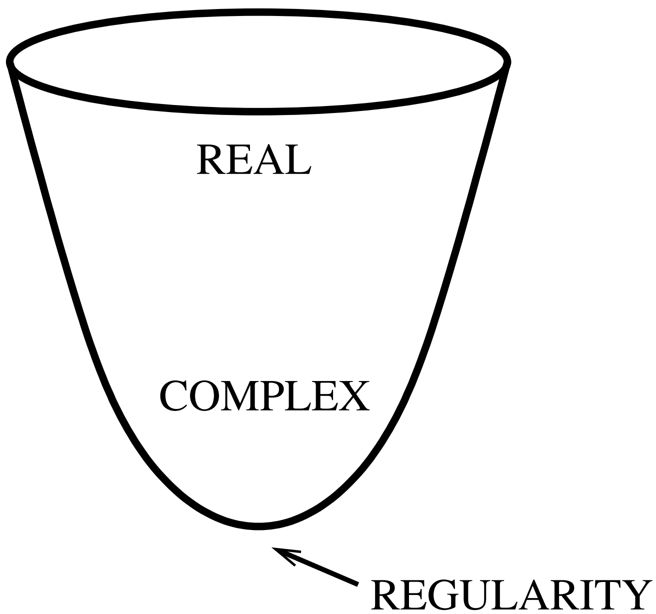
$$\Psi[^3g,\chi] = \int_C \delta g \delta\phi \exp(-I_{dS}[g,\phi]/\hbar)$$

"The amplitude of configurations $(^3g,\chi)$ on a three-surface Σ is given by the integral over all regular metrics g and matter fields ϕ that match $(^3g,\chi)$ on their only boundary." [Hartle & Hawking '83]



Saddle Point Limit

$$\Psi[^3g, \chi] \approx \exp\{[-I_{dS}(^3g, \chi)]/\hbar\}$$



$$I_{dS}(^3g, \chi) = I_R(^3g, \chi) - iS(^3g, \chi)$$

WKB Interpretation

$$\Psi[^3g,\chi] \approx \exp\{[-I_R(^3g,\chi) + iS(^3g,\chi)]/\hbar\}$$

Lorentzian space-time evolution emerges if

$$|\nabla_A I_R| \ll |\nabla_A S|$$

The predicted classical histories are

$$p_A = \nabla_A S$$

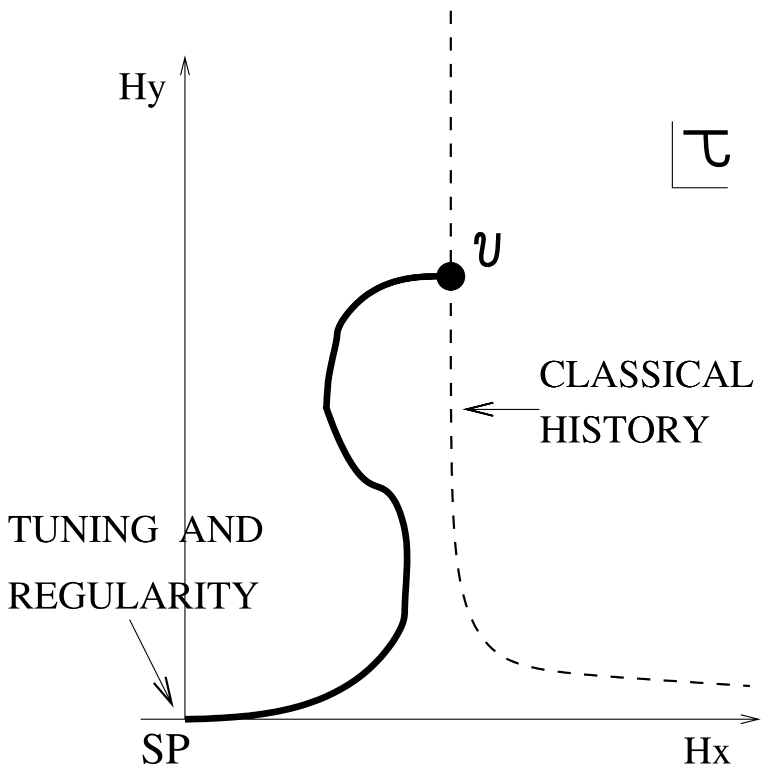
and have conserved probabilities

$$P_{history} \propto \exp[-2I_R/\hbar]$$

No-boundary state $\rightarrow$ prior on multiverse

Complex Saddle points

$$ds^2 = d\tau^2 + g_{ij}(\tau, x)dx^i dx^j, \quad \phi(\tau, x)$$



Tuning at SP: $\phi(0) = \phi_0 e^{i\gamma}$

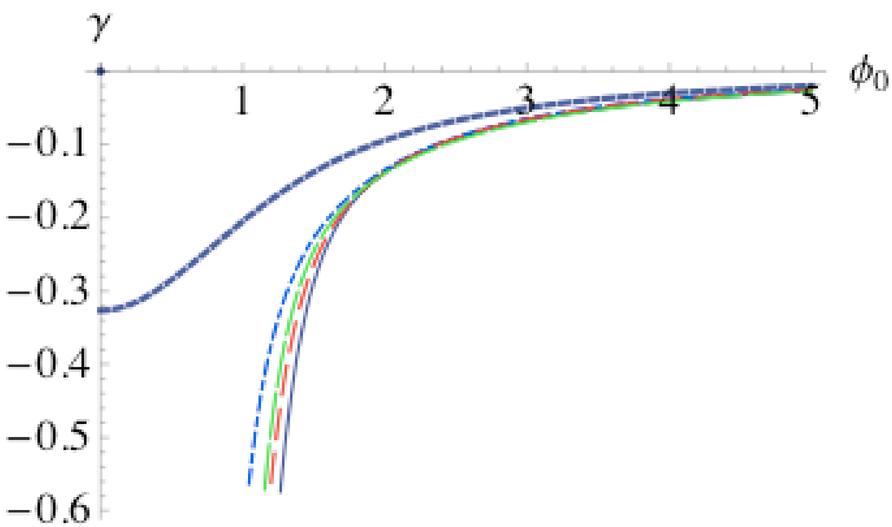
Example

Homogeneous/isotropic ensemble: $\Psi[b, \chi]$

$$ds^2 = d\tau^2 + a^2(\tau)d\Omega_3, \quad \phi(\tau)$$

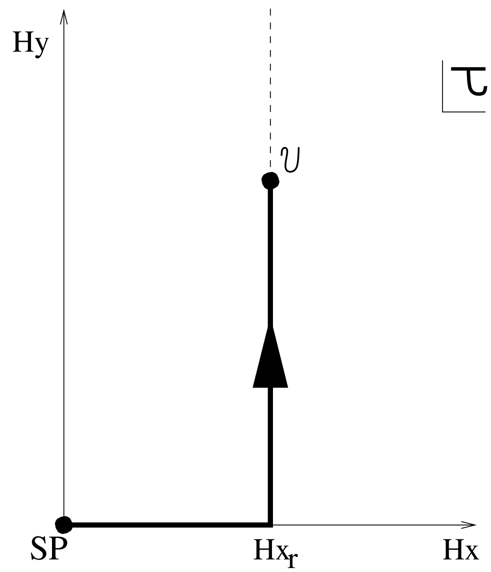
$$V(\phi) = \Lambda + \tfrac{1}{2}m^2\phi^2$$

Classical evolution requires tuning:



$\rightarrow$ multiverse of FLRW backgrounds

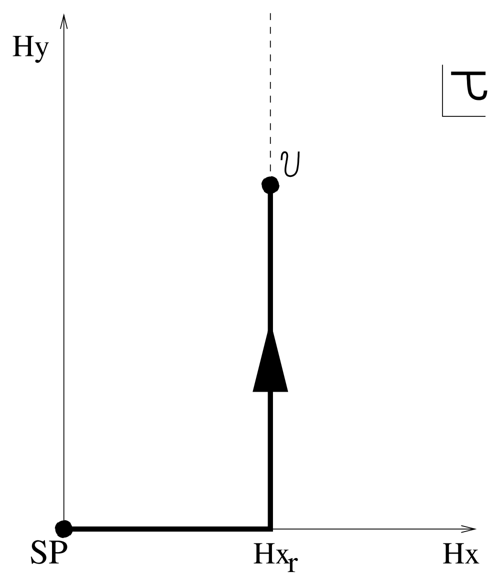
Probability measure



$$I(\upsilon) = \frac{3\pi}{2}\int_C d\tau a[a^2(H^2 + 2V(\phi)) - 1]$$

$$I_R(\chi) \approx -\frac{\pi}{4V(\phi_0)}$$

Probability measure



$$I(\upsilon) = \frac{3\pi}{2} \int_C d\tau a[a^2(H^2 + 2V(\phi)) - 1]$$

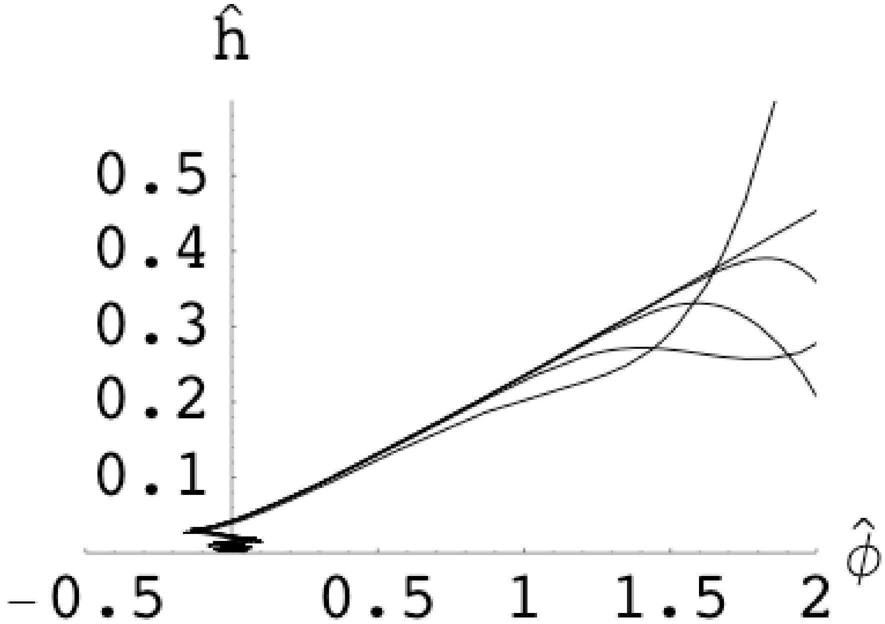
$$I_R(\chi) \approx -\frac{\pi}{4V(\phi_0)}$$

Including perturbations:

$$I_R(\delta\zeta_n) \approx +(\epsilon/H^2) n^3 (\delta\zeta_n)^2$$

Inflation

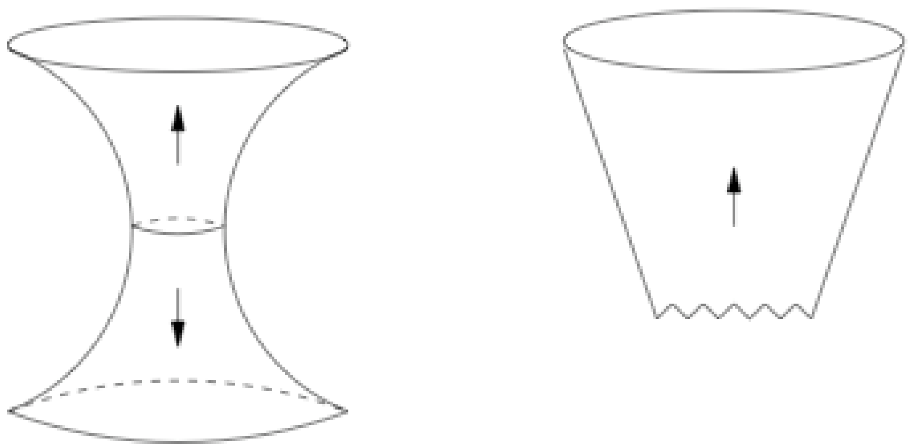
$$p_A = \nabla_A S$$



$$\hat{h} \approx m\hat{\phi}$$

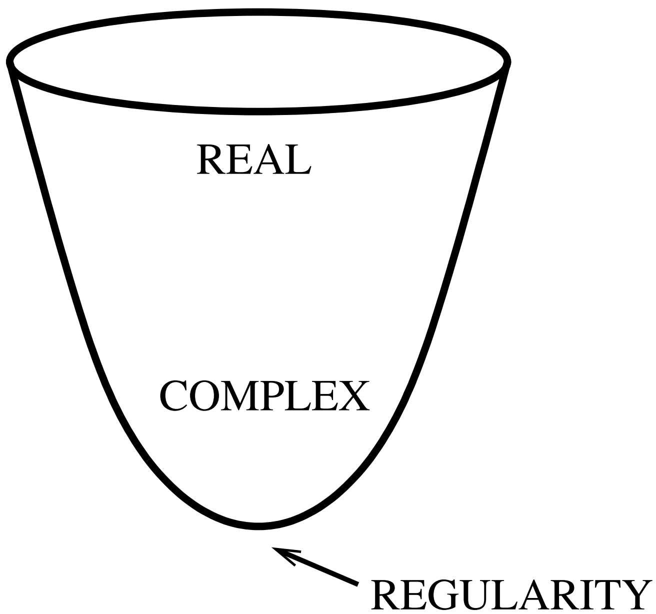
No-boundary state predicts inflation

Was there a Beginning?



large ϕ_0 small ϕ_0

Was there a Beginning?

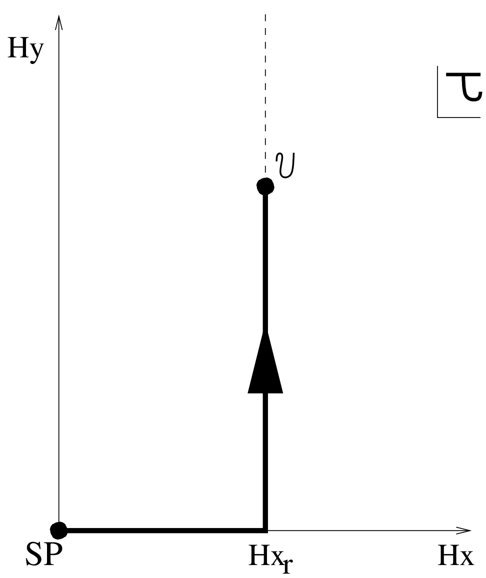


Saddle points everywhere regular

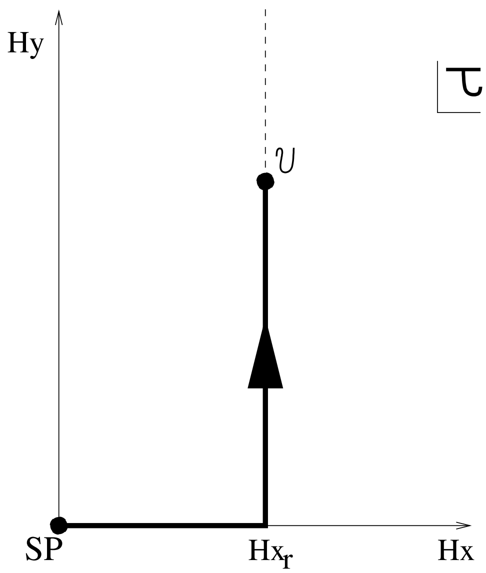
$\rightarrow$ singularities in classical extrapolation
no obstacle to asymptotic predictions

No-Boundary State:
ADS form

Complex Saddle points



Complex Saddle points

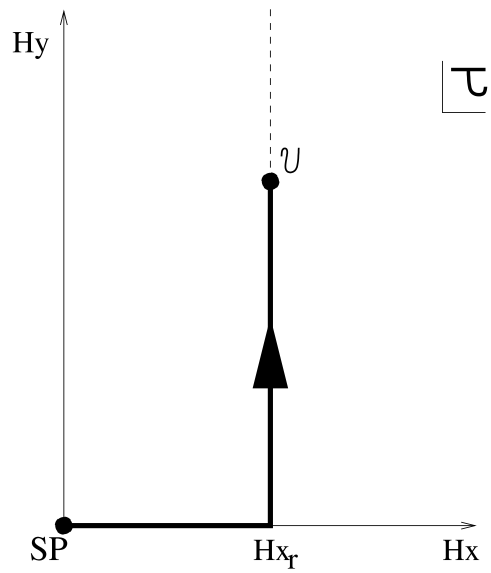


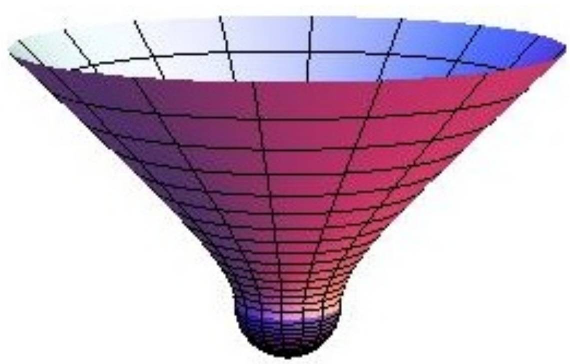
e.g. $a(\tau) = \frac{1}{H}\sin(H\tau), \qquad \phi(\tau) = 0$

horizontal part: $ds^2 = d\tau^2 + \frac{1}{H^2}\sin^2(H\tau)d\Omega_3^2$

vertical part: $ds^2 = -dy^2 + \frac{1}{H^2}\cosh^2(Hy)d\Omega_3^2$

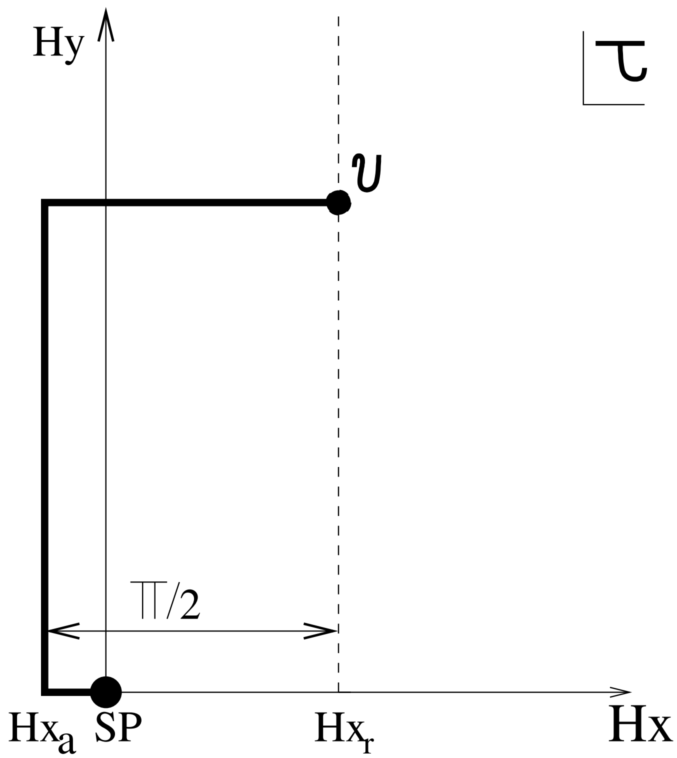
Complex Saddle points



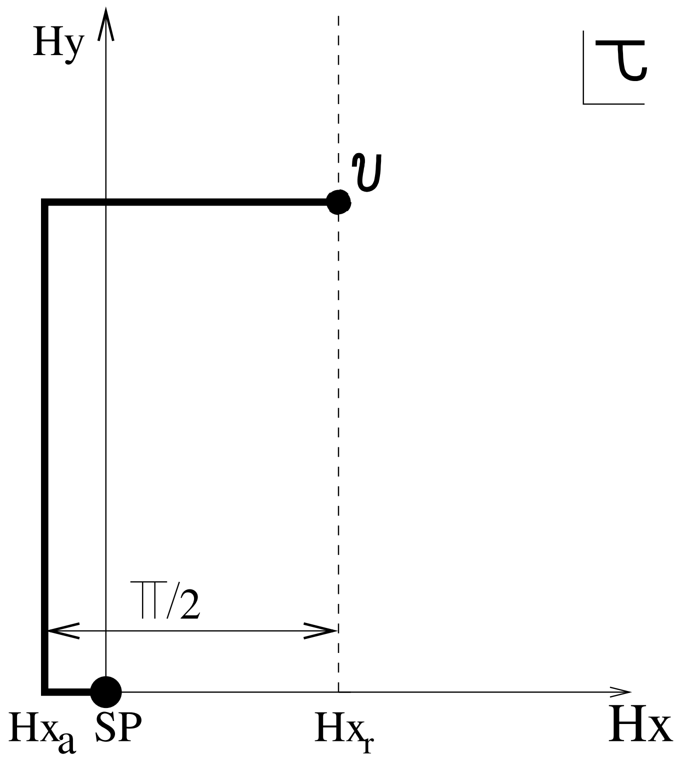


Representations

Different representation of same solution:



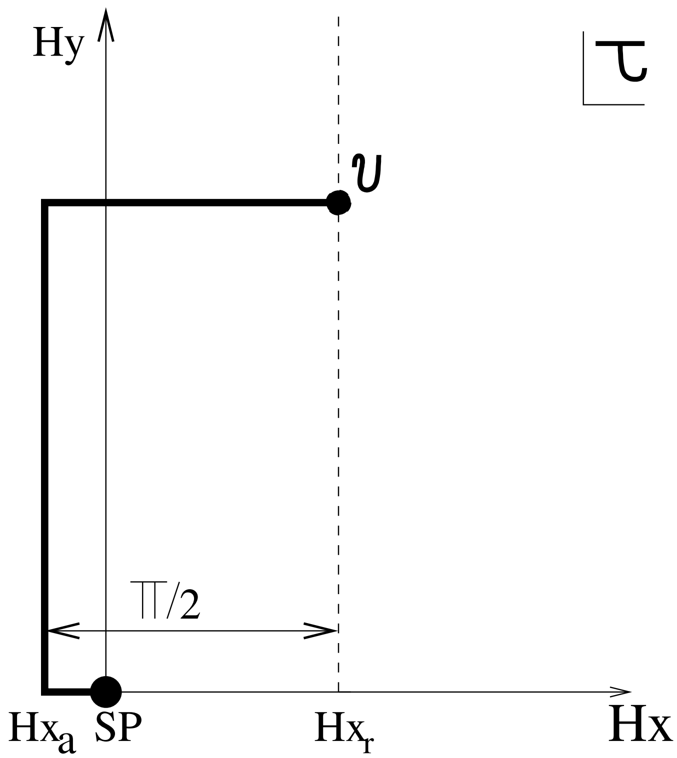
Representations



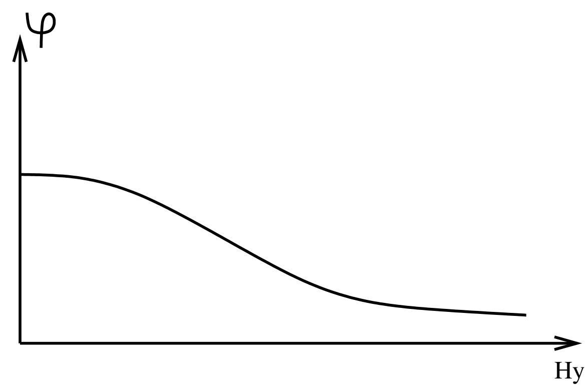
vertical part: Euclidean ADS

$$ds^2 = -dy^2 - \frac{1}{H^2}\sinh^2(Hy)d\Omega_3^2$$

Representations



With matter: Euclidean ADS domain wall



- V acts as effective $-V$ in AdS regime because the signature of the complex saddle point metric varies in the τ-plane,

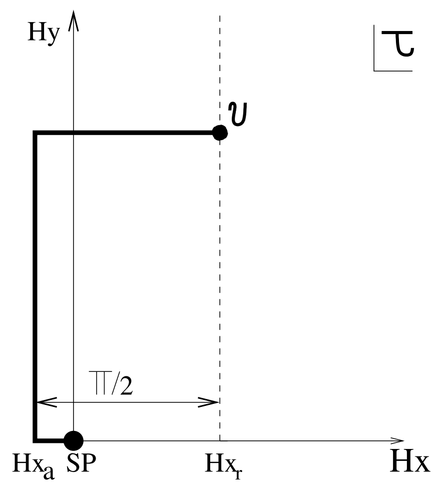
$$V_{eff} = -V$$

- Somewhat reminiscent of Domain Wall/Cosmology correspondence in SUGRA.
  [Cvetic; Skenderis,Townsend, Van Proeyen]

- Realized here at the level of the wave function of the universe which involves a given complexified theory.

Saddle Point Action

$$I_{dS}(b,\chi) = I_v + I_h$$



- Contribution from vertical part:

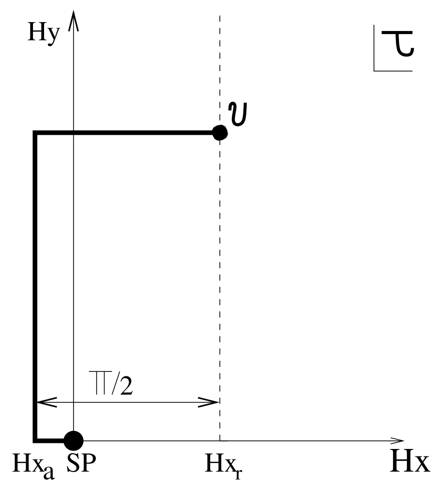
$$I_v = \int_v I_{dS}[g,\phi] = -I^R_{AdS}(^3\tilde{g},\tilde{\chi}) + S_{ct}(a,\tilde{\chi})$$

where I^R_{AdS} is finite when $y \to \infty$.

$$\phi = \alpha e^{-\lambda_- y} + \beta e^{-\lambda_+ y}$$

Saddle Point Action

$$I_{dS}(b, \chi) = I_v + I_h$$



- Contribution from horizontal part:

$$I_h = \int_h I[g, \phi] = -S_{ct}(a, \tilde{\chi}) + iS_{ct}(b, \chi)$$

and no finite contribution.

Asymptotic Structure

Expanded in small $u \equiv e^{i\tau} = e^{-y+ix}$,

$$g_{ij}(u,\Omega) = \frac{-1}{4u^2}[h_{ij}(\Omega) + h^{(2)}_{ij}(\Omega)u^2 + h^{(-)}_{ij}(\Omega)u^{\lambda_-} + h^{(3)}_{ij}(\Omega)u^3 + \cdots]$$

$$\phi(u,\Omega) = u^{\lambda_-}(\alpha(\Omega) + \alpha_1(\Omega)u + \cdots) + u^{\lambda_+}(\beta(\Omega) + \beta_1(\Omega)u + \cdots)$$

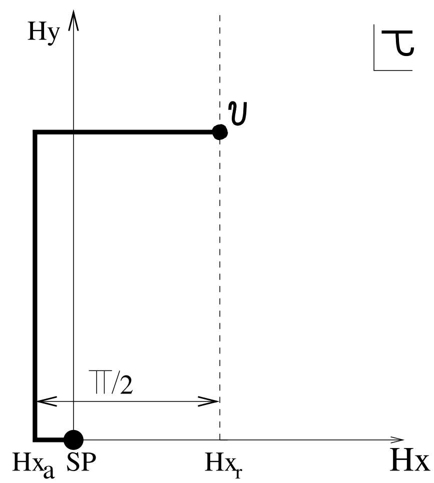
with $\lambda_\pm \equiv \frac{3}{2}[1 \pm \sqrt{1-(2m/3)^2}]$

and arbitrary 'boundary values' (h_{ij}, α).

$$I_h = \int_h I[g,\phi] = -S_{ct}(a,{}^3\tilde{g},\tilde{\chi}) + iS_{ct}(b,{}^3\tilde{g},\chi)$$

A universal AdS/dS connection follows directly from an asymptotic analysis

Saddle Point Action



$$I_{dS}(^3g, \chi) = -I^R_{AdS}(^3\tilde{g}, \tilde{\chi}) + iS_{ct}(^3g, \chi)$$

with

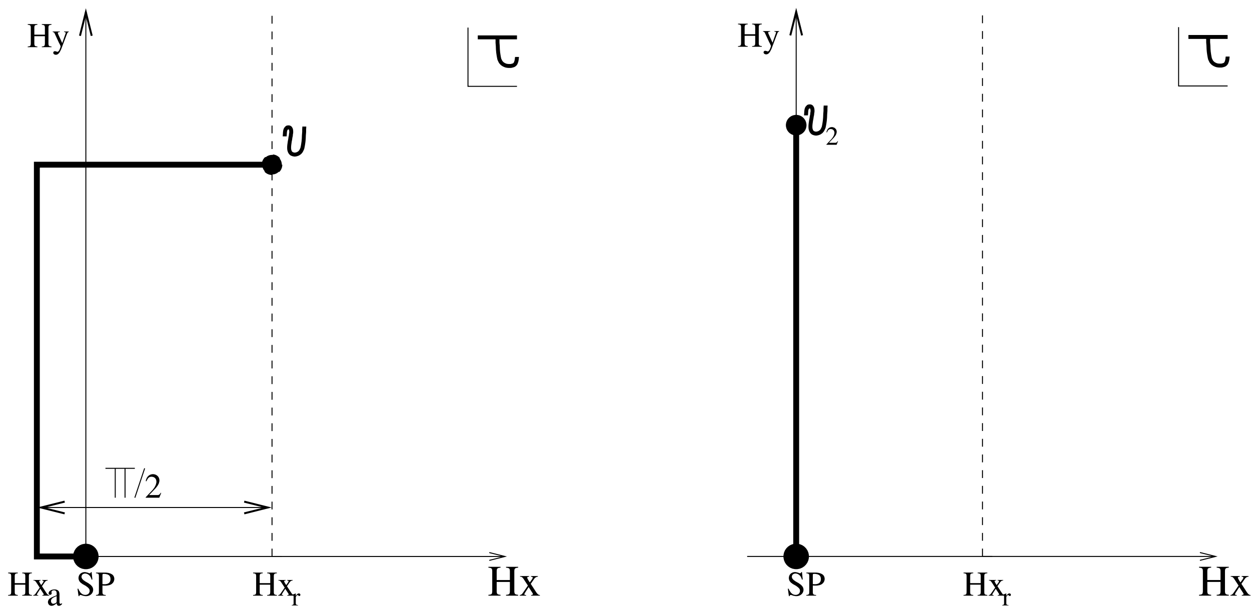
$$S_{ct}(^3g, \phi) = a_0 \int \sqrt{^3g} + a_1 \int \sqrt{^3g} R^{(3)} + \cdots$$

No-Boundary State:

$$\Psi[b, \tilde{h}, \chi] = \exp\{[+I^R_{AdS}(\tilde{h}, \tilde{\chi}) - iS_{ct}(b, \tilde{h}, \chi)]/\hbar\}$$

Two Sets of Saddle points

$$\phi(v) = \phi(v_2) = \chi, \qquad \tilde{g}_{ij}(v) = \tilde{g}_{ij}(v_2)$$



COMPLEX ϕ_0 REAL ϕ_0

Accelerated Expansion and AdS/CFT

Holographic Cosmology

- No-boundary State:

$$\Psi[b, \tilde{h}, \chi] = \exp\{[+I^R_{AdS}(\tilde{h}, \tilde{\chi}) + iS_{ct}(b, \tilde{h}, \chi)]/\hbar\}$$

- Euclidean AdS/CFT:

$$\exp(-I^R_{AdS}[\tilde{h}, \tilde{\chi}]/\hbar) = Z_{QFT}[\tilde{h}, \tilde{\chi}]$$

$$\Psi[b, \tilde{h}, \chi] = \frac{1}{Z_{QFT}[\tilde{h}, \tilde{\chi}, \epsilon]} \exp\{[iS_{ct}(b, \tilde{h}, \chi)]/\hbar\}$$

with complex source $\tilde{\chi}$ and UV cutoff $\epsilon \sim \frac{l}{b}$

Cosmology with AdS Gravity

Using AdS/CFT we evaluate the holographic no-boundary state,

$$\exp(-I^R_{ADS}[\tilde{h},\tilde{\chi}]/\hbar) = Z_{QFT}[\tilde{h},\tilde{\chi}]$$

There are two sets of saddle points

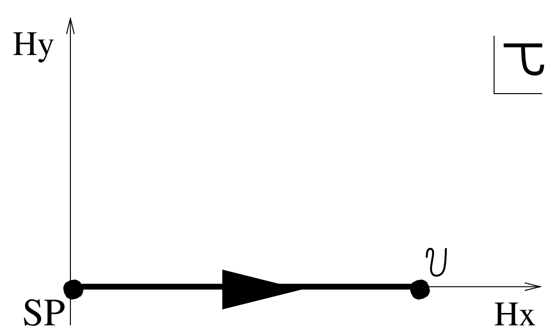
Cosmology with AdS Gravity

Using AdS/CFT we can evaluate the holographic no-boundary state,

$$\exp(-I^R_{ADS}[\tilde{h},\tilde{\chi}]/\hbar) = Z_{QFT}[\tilde{h},\tilde{\chi}]$$

There are two sets of saddle points

A: Real $\tilde{\chi}$ $\rightarrow$ real Euclidean AdS domain walls



$$\Psi[b,\tilde{h},\chi] \approx \exp\{[+I^R_{AdS}(\tilde{h},\tilde{\chi}) - S_{ct}(b,\tilde{h},\chi)]/\hbar\}$$

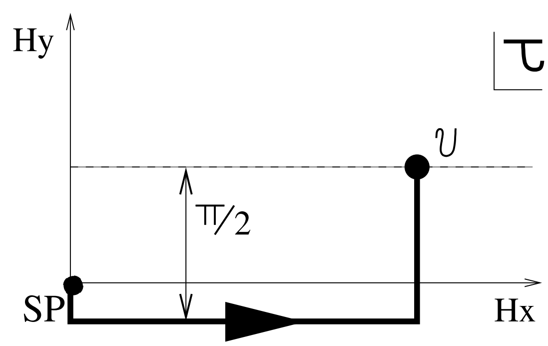
Cosmology with AdS Gravity

Using AdS/CFT we can evaluate the holographic no-boundary state,

$$\exp(-I^R_{ADS}[\tilde{h}, \tilde{\chi}]/\hbar) = Z_{QFT}[\tilde{h}, \tilde{\chi}]$$

There are two sets of saddle points

B: Complex $\tilde{\chi} \to$ complex Euclidean domain walls



$$\Psi[b, \tilde{h}, \chi] \approx \exp\{[+I^R_{AdS}(\tilde{h}, \tilde{\chi}) - iS_{ct}(b, \tilde{h}, \chi)]/\hbar\}$$

Saddle points correspond to inflationary universes

Remarks

$$Z_{QFT}[\tilde{h}, \tilde{\chi}] = \langle \exp \int d^3x \sqrt{\tilde{h}} \tilde{\chi} \mathcal{O} \rangle$$

- The dependence of Z on the external sources provides a cosmological measure on the space of configurations $(b, \tilde{h}, \chi)$.
- AdS/CFT implements no-boundary condition of regularity in saddle point limit
- Scale factor evolution arises as inverse RG flow
- Physical interpretation of counterterms in AdS

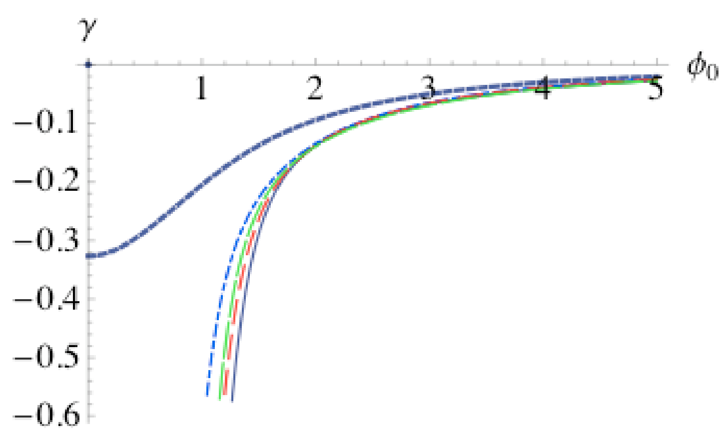
What models?

What models?

$$Z_{QFT}[\tilde{h}, \tilde{\chi}] = \langle \exp \int d^3x \sqrt{\tilde{h}} \tilde{\chi} \mathcal{O} \rangle$$

- Lorentzian stability criteria too stringent
- Unitary physics in given classical background
- Ensemble of inflationary backgrounds

$$V_{AdS}(\phi) = -\Lambda - \frac{1}{2} m^2 \phi^2$$



Classical evolution constrains V

Singularity Resolution
(for Gary)

Singularity Resolution (for Gary)

$$\Psi[b,\tilde{h},\chi] = \frac{1}{Z_{QFT}[\tilde{h},\tilde{\chi},\epsilon]} \exp\{[iS_{ct}(b,\tilde{h},\chi)]/\hbar\}$$

Given an asymptotic structure one can probe the deep interior by taking the RG flow all the way down to the IR.

Singularity described by IR fixed point.

Conclusion

The no-boundary proposal and Euclidean AdS/CFT are intimately connected.

- A wave function defined in terms of a gravitational theory with a negative cosmological constant Λ can predict expanding universes with an 'effective' positive cosmological constant $-\Lambda$.

- The Euclidean AdS/CFT dual provides a more precise, 'holographic' formulation of the semiclassical no-boundary state.

Applications:

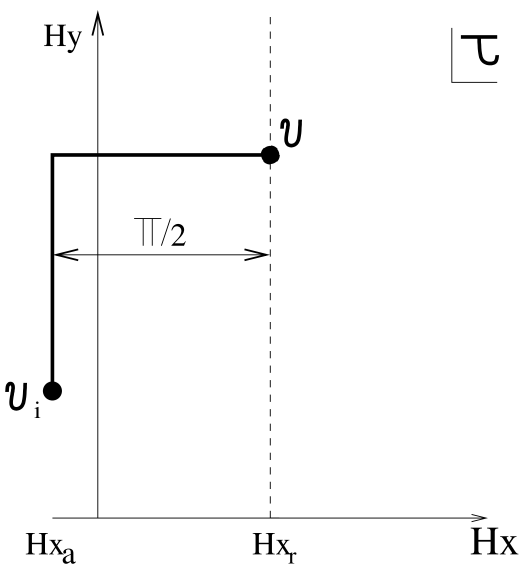
- Holographic calculation of CMB correlators.

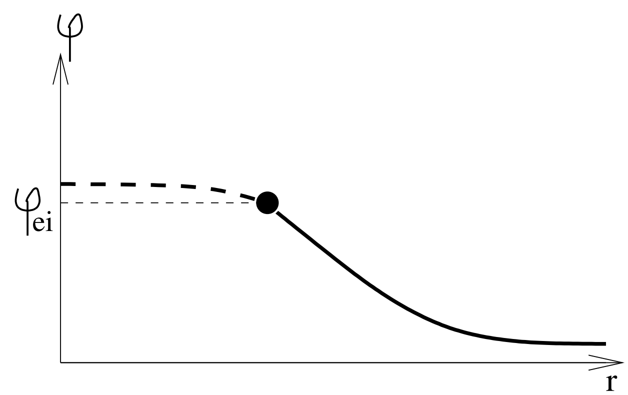
- Get a better handle on eternal inflation?

Euclidean Eternal Inflation

[Hartle, Hawking & TH, in progress]

Proposal: replace the inner region of eternal inflation by a dual CFT on the threshold surface at υ_i:





- IR CFT with a deformation set by threshold ϕ_{EI}. (similar to [Maldacena '10])

- $< \mathcal{O} >$ on inner boundary replaces regularity condition at origin.