Local observables in AdS

work with Hamilton, Lifschytz, Lowe, Roy, Sarkar

recent work by Heemskerk, Marolf, Polchinski

KITP

4/10/12

Motivation

Many puzzles and properties of quantum gravity involve local measurements.

BH unitarity, CMB fluctuations

Take holography as a fundamental principle, with AdS/CFT as a concrete realization. Can we understand the emergence of (approximately) local bulk physics from the CFT?

=> We're here.

One approach, followed since the earliest days of AdS/CFT: write local field operators in the bulk in terms of the CFT.

Horowitz,...

free field:

$$\phi(x,z) = \int dx' K(x,z|x')\mathcal{O}(x')$$

interacting field:

$$\phi(x,z) = \sum_i \int dx' K_i(x,z|x')\mathcal{O}_i(x')$$

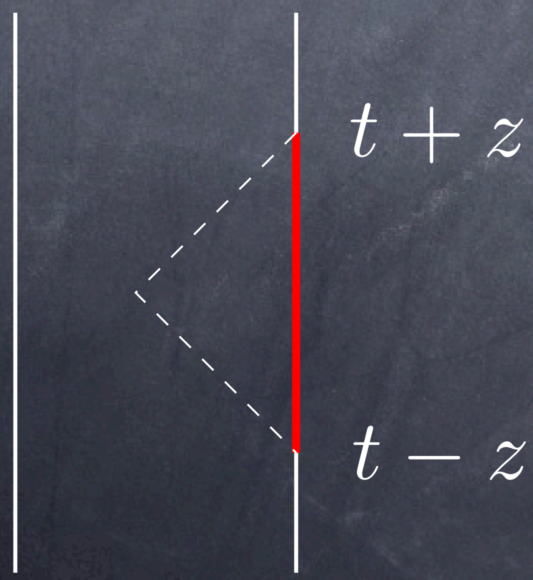
Free massless scalar in AdS_2

$$ds^2 = \frac{R^2}{z^2}\left(-dt^2 + dz^2\right)$$

wave eqn $\quad \Box\phi = 0$

bdy cond $\quad \phi(t,z) \sim z\mathcal{O}(t) \quad$ as $z \to 0$

Solution: $\quad \phi(t,z) = \frac{1}{2}\int_{t-z}^{t+z} dt'\,\mathcal{O}(t')$



support at spacelike separation

Free scalar in AdS_{d+1}

Generalizes pretty easily, provided you work on the complexified boundary.

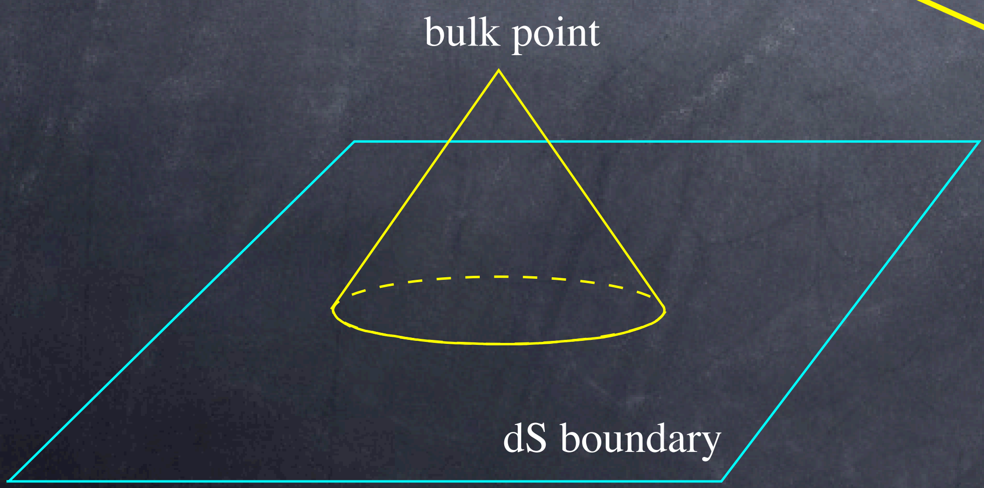
(complexifying seems necessary)

Veronika + Mukund?

For a massless scalar

$$\phi(t, x, z) \sim \int_{t'^2 + y'^2 < z^2} dt' \, dy' \, \mathcal{O}(t + t', x + iy')$$

smear over a ball of radius z



For a massive scalar

bulk - bdy distance

$$\phi(t,x,z) \sim \int_{t'^2+y'^2<z^2} dt'\,dy'\,(\sigma z')^{\Delta-d}\,\mathcal{O}(t+t',x+iy')$$

Basically fixed by symmetries.

For a massive scalar

bulk - bdy distance

$$\phi(t,x,z) \sim \int_{t'^2+y'^2<z^2} dt'\,dy'\,(\sigma z')^{\Delta-d}\,\mathcal{O}(t+t',x+iy')$$

dimension

Δ

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$d-\Delta$

dimension

Δ

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For a massive scalar

bulk - bdy distance

$$\phi(t,x,z) \sim \int_{t'^2+y'^2<z^2} dt'dy'\,(\sigma z')^{\Delta-d}\,\mathcal{O}(t+t',x+iy')$$

$-d$ $\quad$ $d-\Delta$ $\quad$ dimension Δ

Basically fixed by symmetries.

Gauge fields

Set $A_z = 0$ and

$$zA_\mu \sim \int_{t'^2+y'^2=z^2} dt'dy'\, j_\mu(t+t', x+iy')$$

shell of radius z

In AdS3 a chiral current $j_- = j_-(x^-)$ is dual to a Chern-Simons gauge field with

$$A_+ = A_z = 0$$

$$A_-(x^+, x^-, z) = j_-(x^-)$$

Metric

Set $h_{zz} = h_{z\mu} = 0$ and

$$z^2 h_{\mu\nu} \sim \int_{t'^2 + y'^2 < z^2} dt' dy' \, T_{\mu\nu}(t + t', x + iy')$$

In AdS3 $h_{\mu\nu} = T_{\mu\nu}$ so there's a Virasoro algebra

$$i\left[h_{--}(x^+, x^-, z), h_{--}(x'^+, x'^-, z')\right] = \frac{c}{24\pi} \delta'''(x^- - x'^-)$$

Interactions?

For scalar fields there are two approaches.

1. Solve bulk e.o.m. perturbatively, e.g.

$$\nabla\phi = \lambda\phi^2 \quad \Rightarrow \phi = \phi^{(0)} + \phi^{(1)} + \cdots$$

$$\nabla\phi^{(0)} = 0$$

$$\nabla\phi^{(1)} = \lambda\left(\phi^{(0)}\right)^2$$

$$\vdots$$

Heemskerk - Marolf - Polchinski

2. Impose bulk micro-causality   Dowker

$\langle \int K \mathcal{O}_1 \, \mathcal{O}_2 \rangle$ causal by construction

$\langle \int K \mathcal{O}_1 \, \mathcal{O}_2 \, \mathcal{O}_3 \rangle$ violates causality

In the 1/N expansion you can add higher-dimension multi-trace operators, with coefficients chosen to restore bulk causality. No bulk e.o.m. required!

For scalar fields the two approaches seem to be equivalent.

Rindler coordinates on AdS_3

$$ds^2 = -(r^2-1)dt^2 + (r^2-1)^{-1}dr^2 + r^2 dx^2$$

$$\phi(t,x,r) = \int d\omega dk\, a_{\omega k} e^{-i\omega t} e^{ikx} f_{\omega k}(r)$$

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$$\downarrow$$

$$r^{-\Delta}\mathcal{O} \qquad \text{as } r \to \infty$$

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$\downarrow$ $\qquad$ $\downarrow$

$r^{-\Delta}\mathcal{O}$ $\qquad$ as $r \to \infty$ $\qquad$ $r^{-\Delta}$

So $a_{\omega k} = \int e^{i\omega t} e^{-ikx} \mathcal{O}(t,x)$ Plug back in and formally $\phi = \int K\mathcal{O}$ with $K = \text{F.T.}\left(f_{\omega k}\right)$

But $f_{\omega k}$ grows exponentially as $k \to \pm\infty$

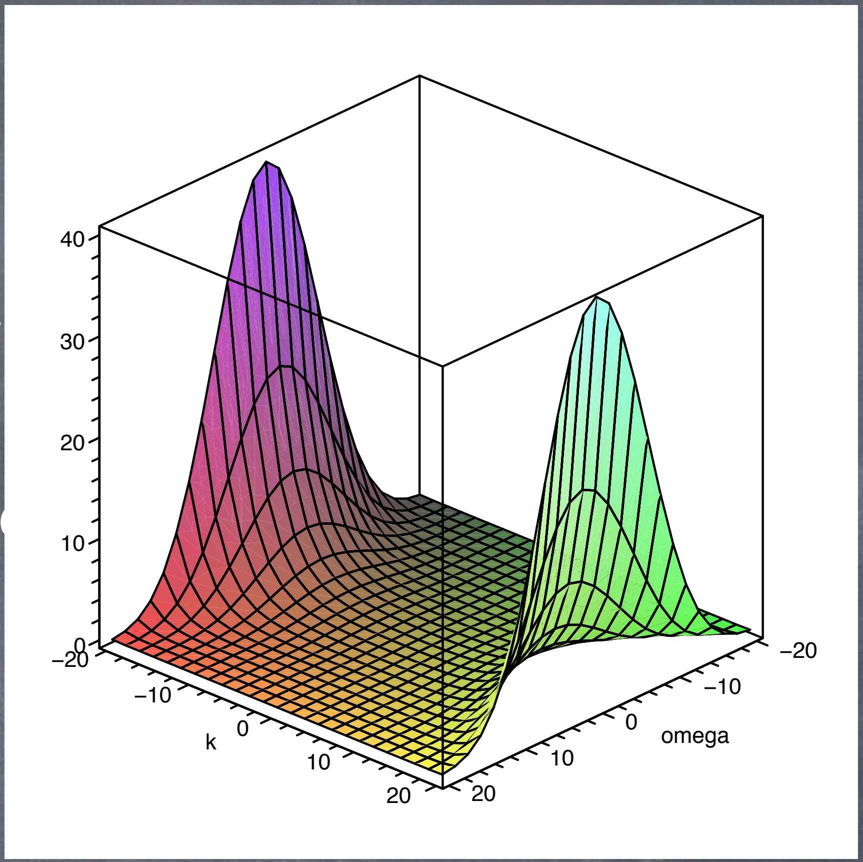
Rindler coordinates on AdS_3

$$ds^2 = -(r^2 - 1)dt^2 + (r^2 - 1$$

$$\phi(t, x, r) = \int d\omega dk \, a_{\omega k} e^{-i\omega t}$$

$\downarrow$

$r^{-\Delta}\mathcal{O}$ as $r \to \infty$



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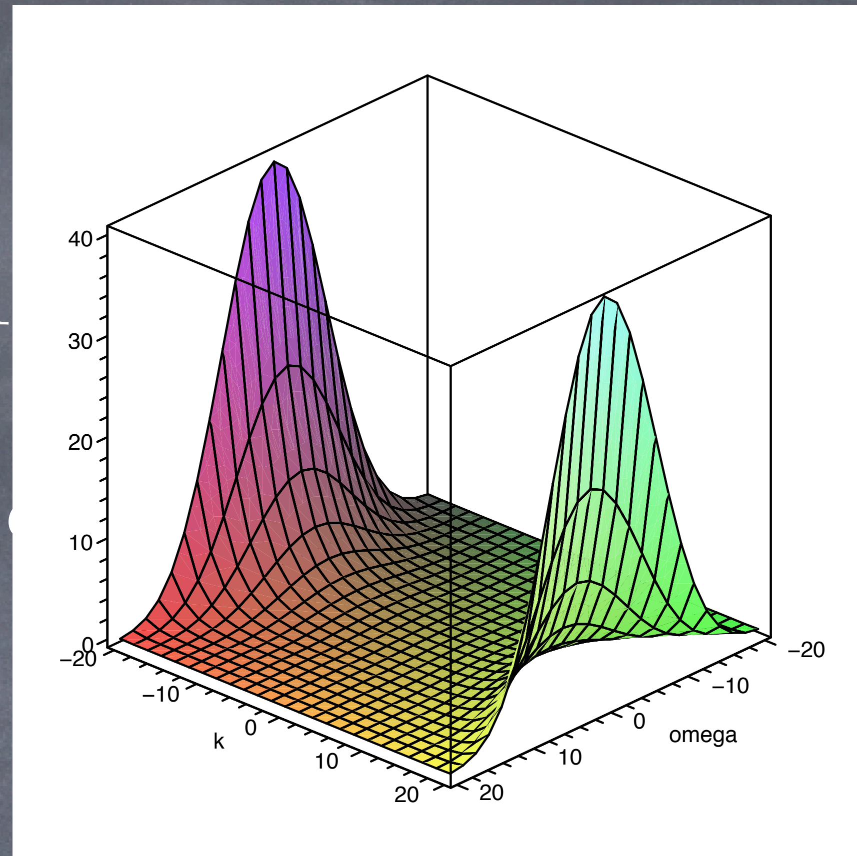
Rindler coordinates on AdS$_3$



$$ds^2 = -(r^2 - 1)dt^2 + (r^2 - 1$$

$$\phi(t, x, r) = \int d\omega dk \, a_{\omega k} e^{-i\omega t}$$

$\downarrow$

$r^{-\Delta}\mathcal{O}$ as $r \to \infty$

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Interpret as continuation to complex x.