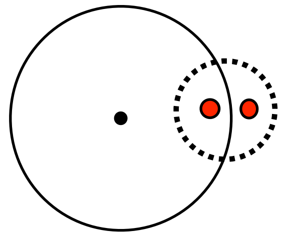
The black hole information paradox

Santa Barbara 2012

1. The information paradox

Hawking argument can be made rigorous

Inequality

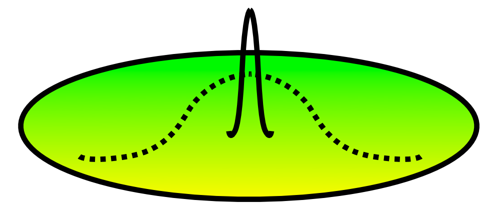


(arXiv: 0909.1038)

2. Constructing microstates ....

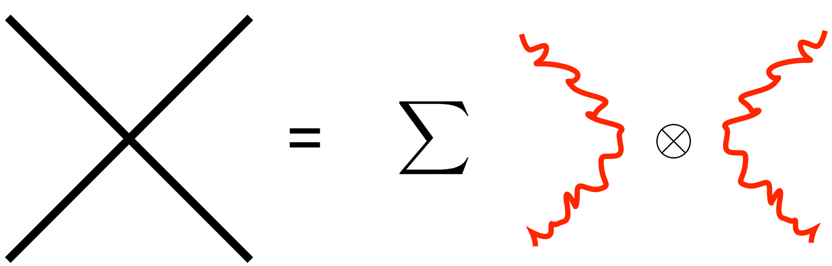
Find no regular horizons (fuzzballs)
('hair' in string theory)

Collapsing shell: wavefunction spreads over large phase space of solutions

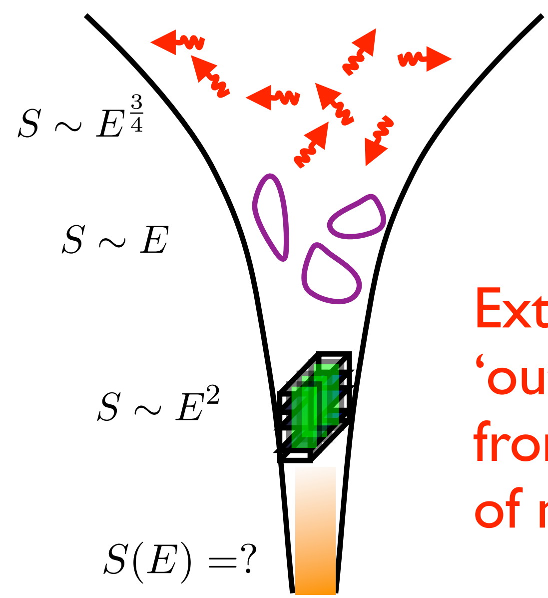


3. The infall problem

For generic microstates, is there a sense in which infalling observers see traditional black hole physics in some approximation ?



4. Cosmology

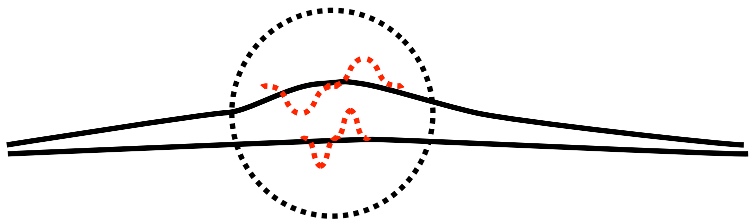


Extra
'outwards 'push'
from level density
of microstates ?

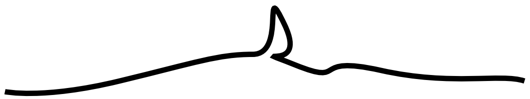
What is the information paradox ?

(i) The full theory of nature includes quantum gravity

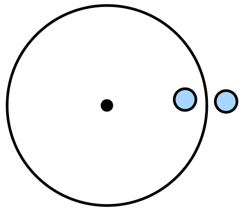
(ii) There is a limit in which we get semiclassical physics, where quantum gravity effects are small



(iii) This semiclassical limit breaks down when curvatures are planck scale



(iv) Hawking 'theorem': There must be a second mode of breakdown which does not involve planck scale curvature.



If there is no such second mode, then black hole evaporation will lead to information loss / remnants

Question for any theory of gravity:

Is there a second mode of breakdown for the semiclassical approximation ?

If yes, what is it ?

What are the conditions under which it happens ?
(It must happen in the good slicing of the black hole, but not in a good slice through our room)

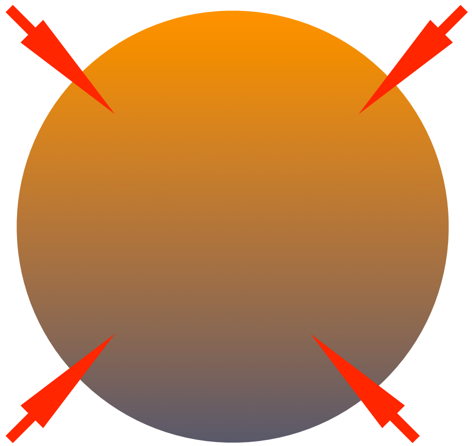
Different theories of gravity behave differently ...

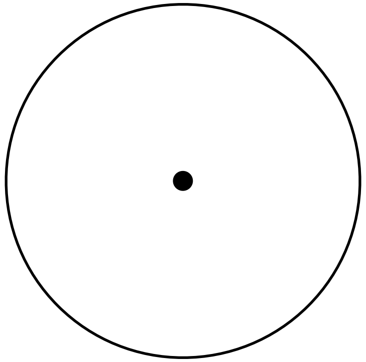
Canonically quantized gravity: information loss/remnants

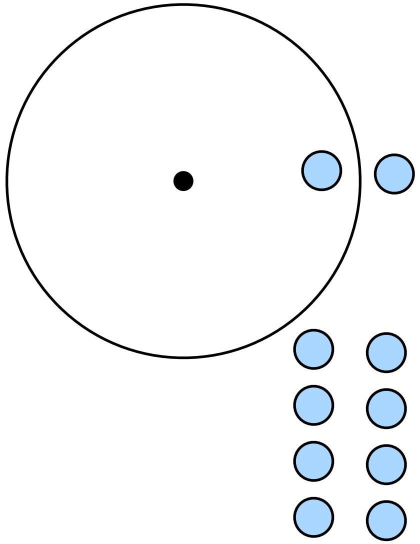
Loop quantum gravity: slow leaking remnants ...

String theory ... Information in Hawking radiation ...

The information problem







Ψ_M

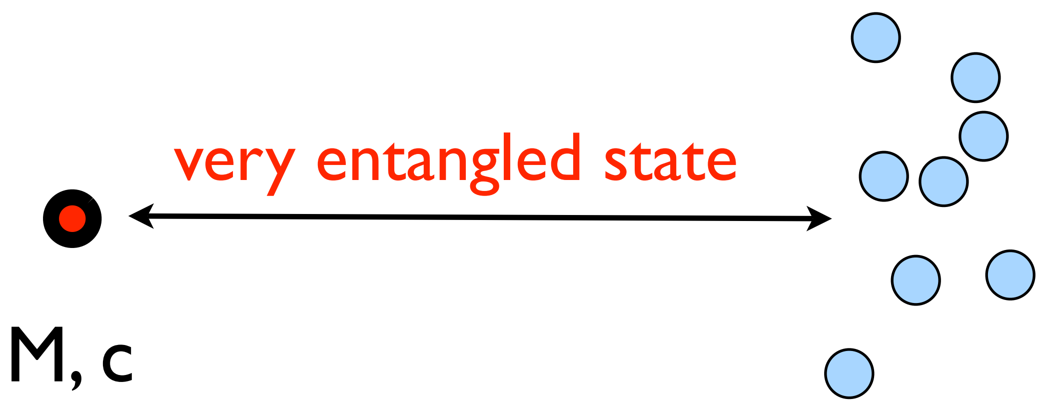
$\otimes \quad |0\rangle_1|0\rangle_{1'} + |1\rangle_1|1\rangle_{1'}$

$\otimes \quad |0\rangle_2|0\rangle_{2'} + |1\rangle_2|1\rangle_{2'}$

$\cdots$

$\otimes \quad |0\rangle_n|0\rangle_{n'} + |1\rangle_n|1\rangle_{n'}$

Possible endpoints



Remnant

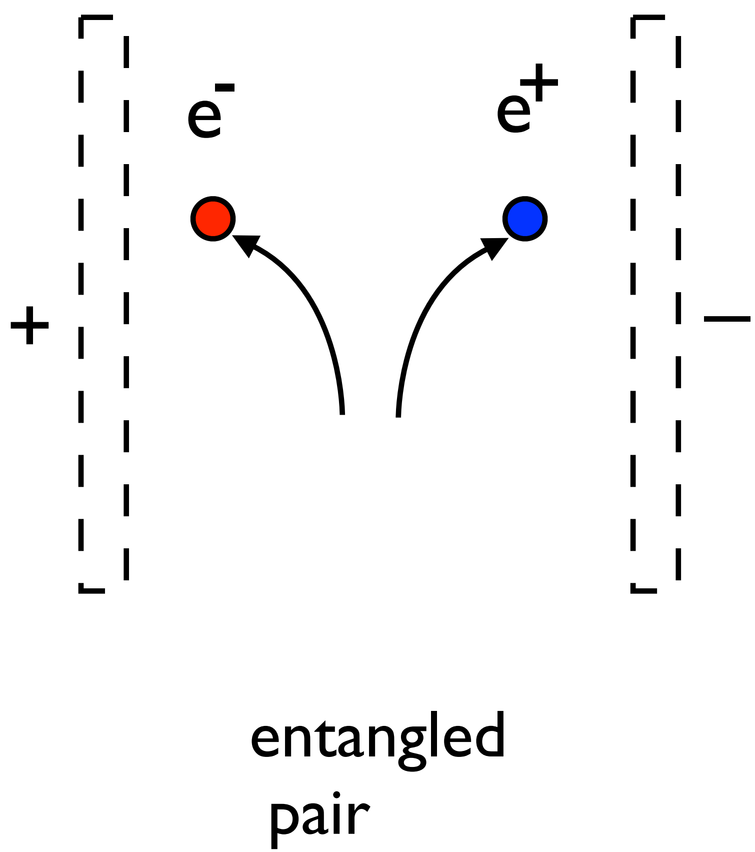
Planck mass, planck sized objects with unbounded degeneracy

Radiation can only be defined by density matrix

No wavefunction can be written for the radiation

State of radiation is ‘mixed’ in a fundamental way

Let us first look at the Schwinger process ...



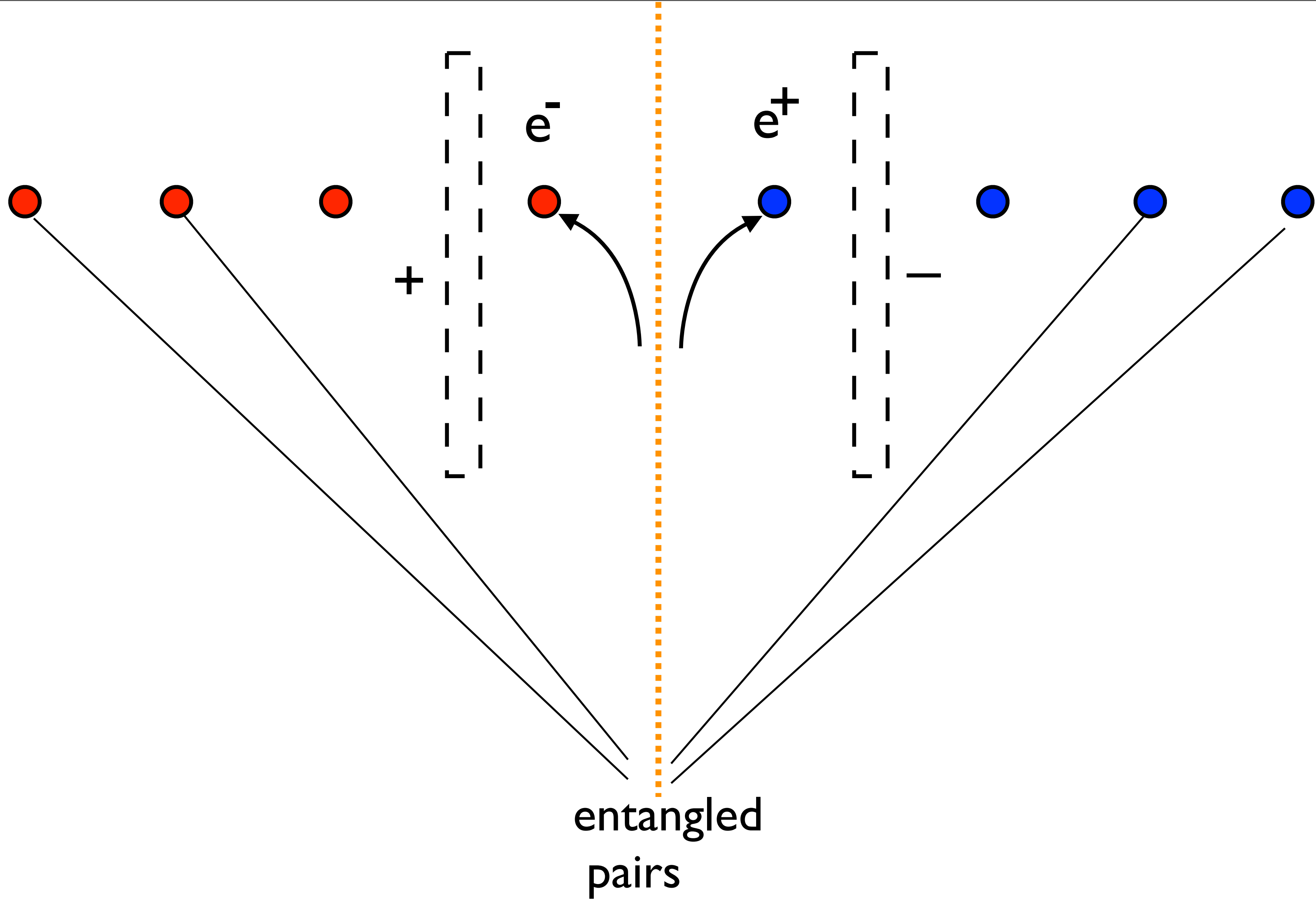
Schwinger pair production

State of created quanta is entangled

$$\uparrow\downarrow - \downarrow\uparrow$$

Entanglement entropy

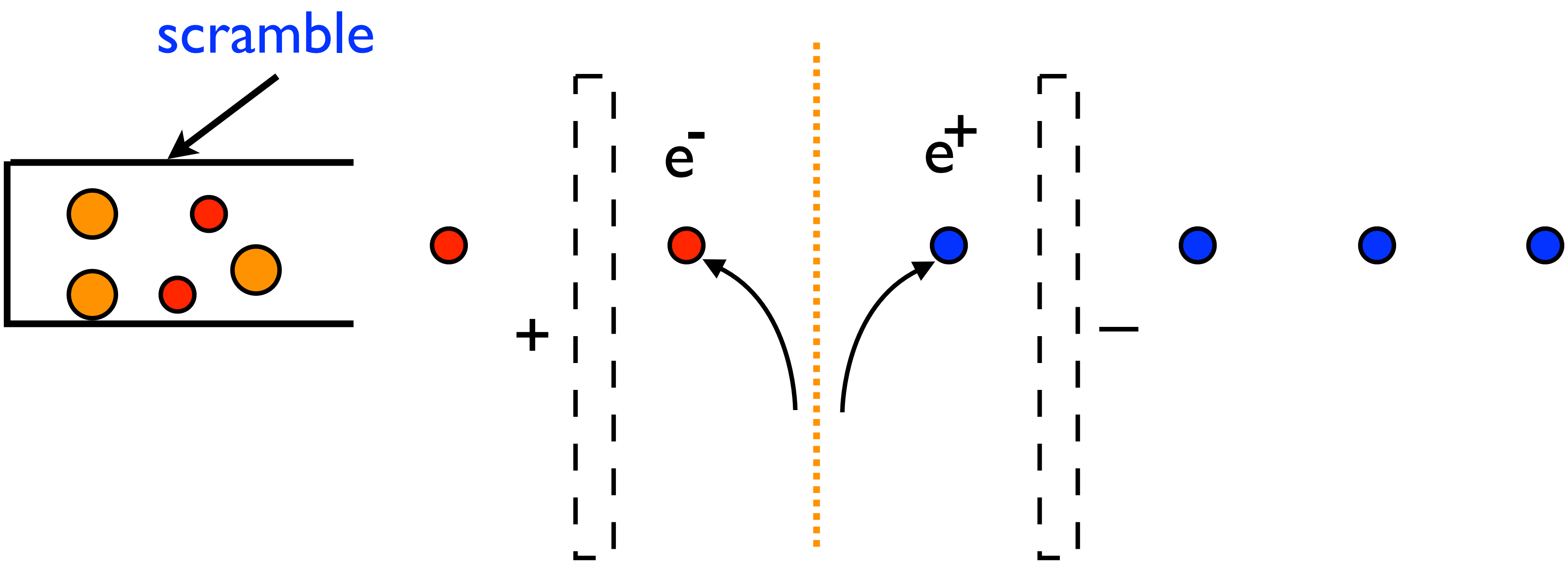
$$S_{ent} = \ln 2$$



After N steps, the leading order computation gives

$$S_{ent} \ = \ N \ln 2$$

Can we change something so that S_{ent} becomes close to ***zero*** ?



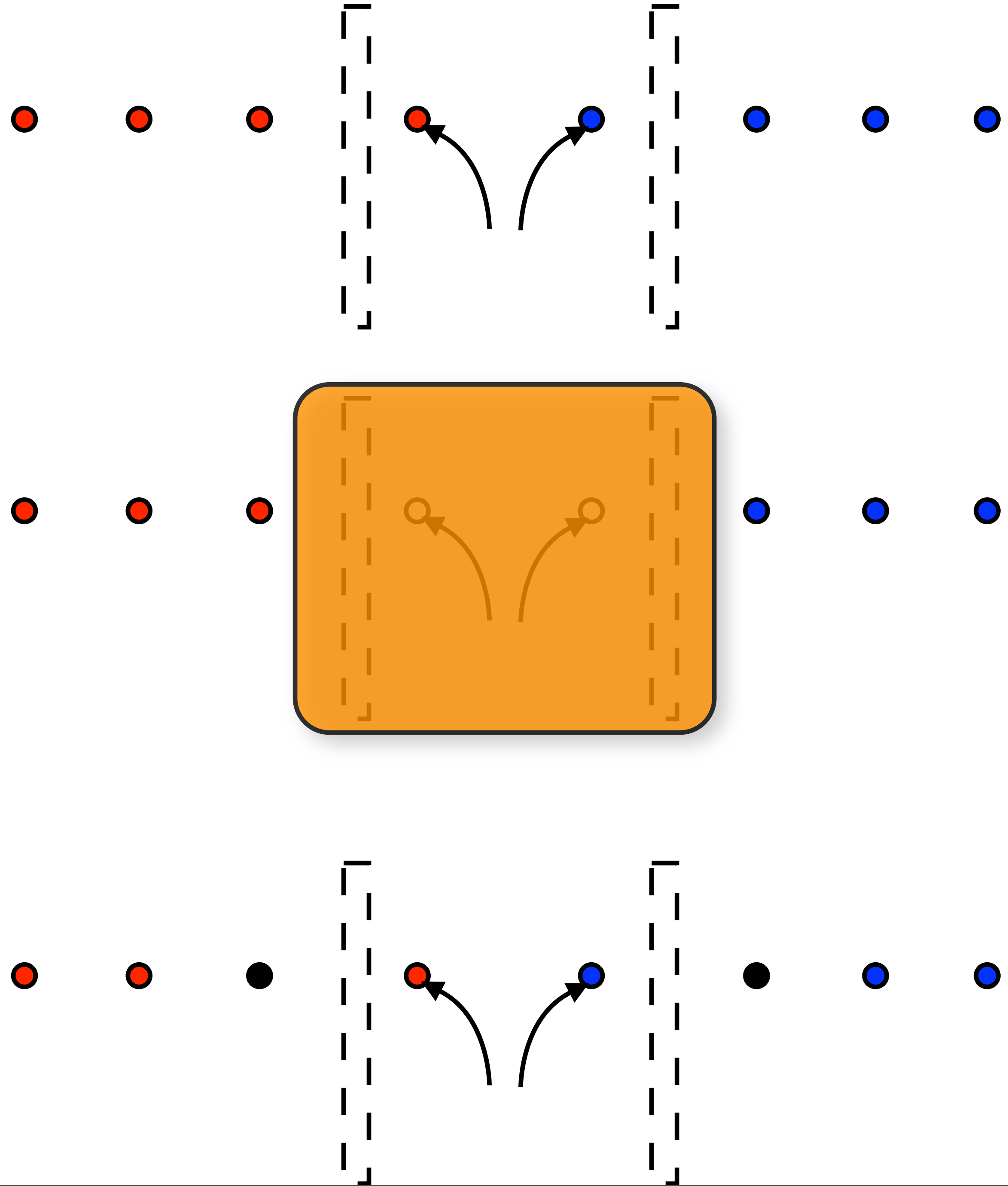
Scrambling the quanta that have already been created does not change the entanglement at all ....

$$\Psi = \sum_i C_i \, \psi_i \otimes \chi_i$$

$$\psi_i \;\rightarrow\; U_{ij}\psi_j$$

$$S_{ent} \;=\; \sum_i |C_i|^2$$ remains unchanged

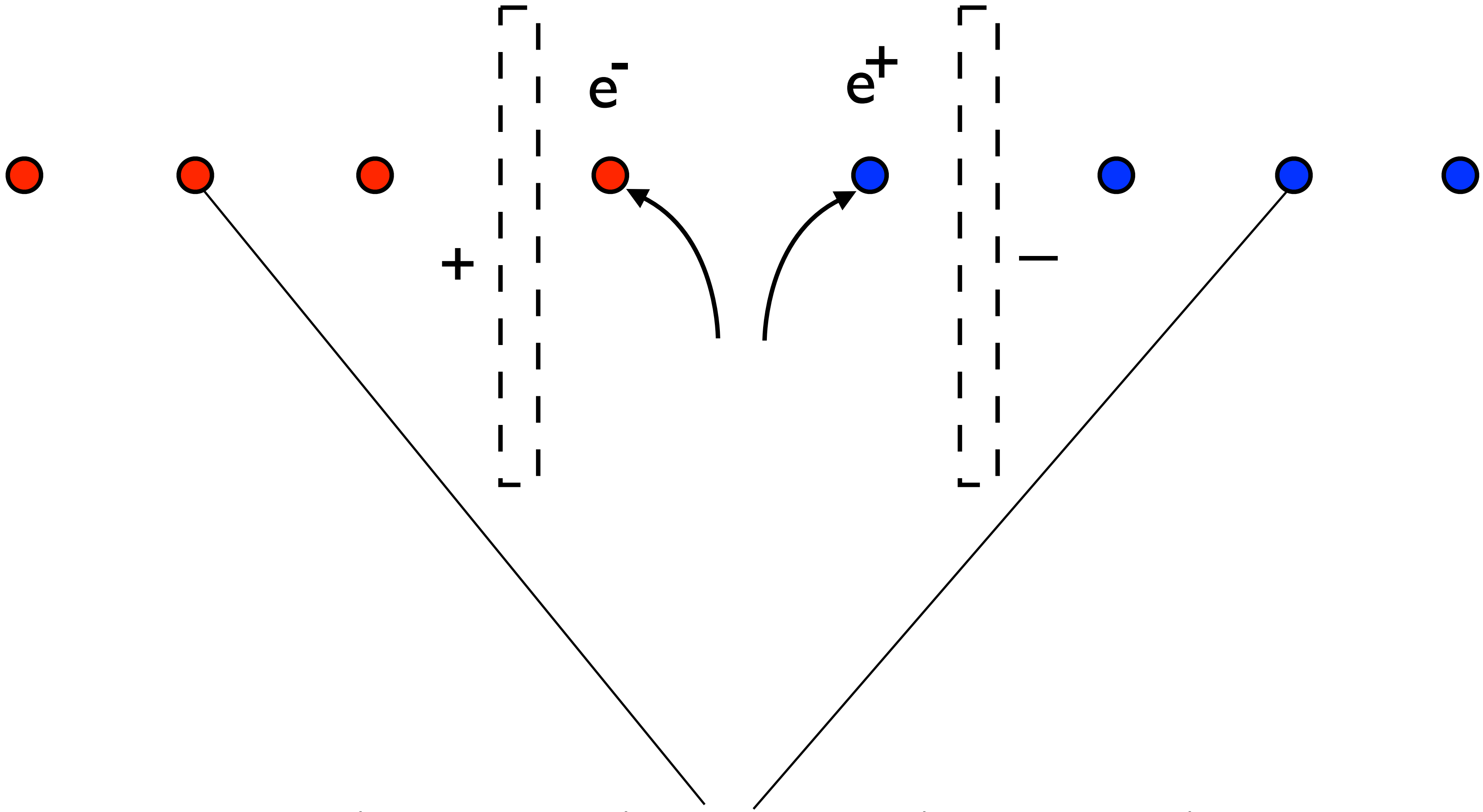
Large corrections, occurring very infrequently, don't help either ...



System can tunnel into another state with an exponentially small probability

Produce one pair that may not be entangled ...

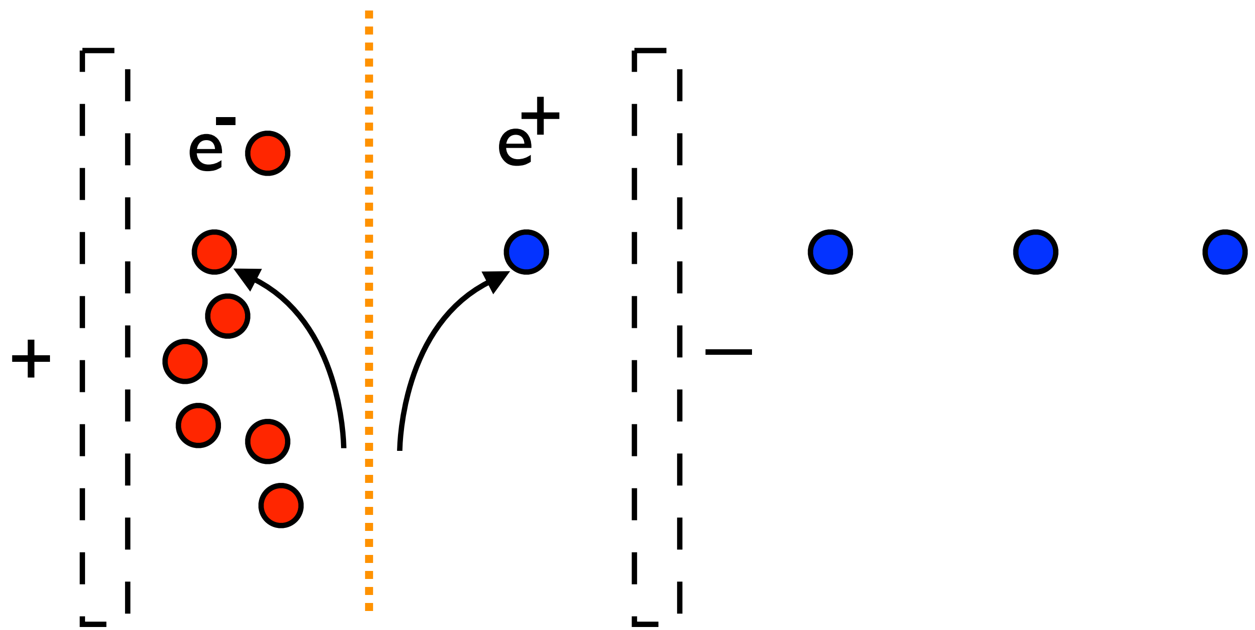
Small correction at each step, large number of pairs N ...



$$\frac{1}{\sqrt{2}}\Big(\uparrow\downarrow - \downarrow\uparrow\Big) + \epsilon_k \frac{1}{\sqrt{2}}\Big(\uparrow\downarrow + \downarrow\uparrow\Big)$$

Its not completely obvious, but it can be shown that this does not help either ...

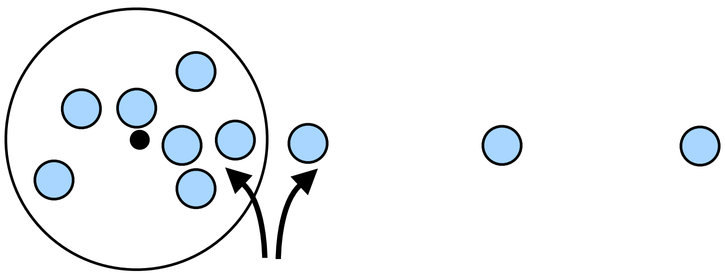
We ***would*** get a significant modification if the earlier created quanta did not move away ....



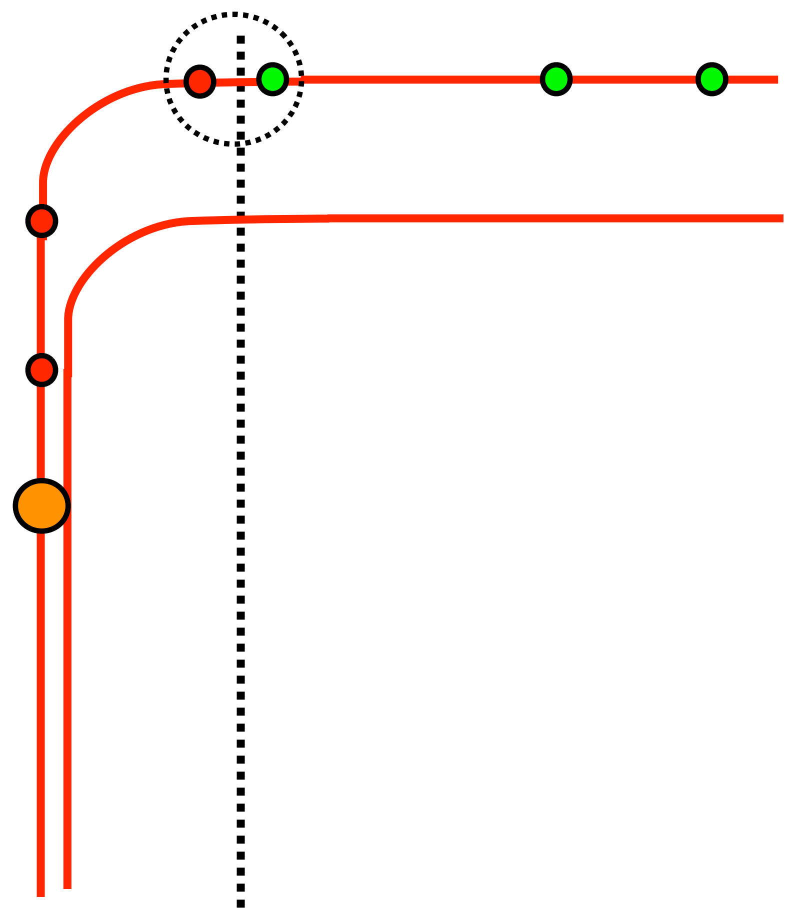
$$\frac{1}{\sqrt{2}}\Big(\uparrow\downarrow-\downarrow\uparrow\Big) \;\rightarrow\; \frac{1}{\sqrt{2}}\Big(\uparrow\downarrow-\downarrow\uparrow\Big)+\alpha_k\frac{1}{\sqrt{2}}\Big(\uparrow\downarrow+\downarrow\uparrow\Big)$$

with $\alpha_k = O(1)$

In any normal warm body, e.g. a star, we can have radiation, leading to entanglement ...

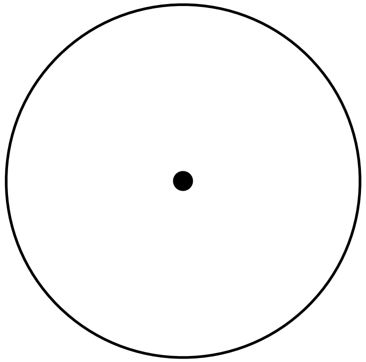


State of new pair will be corrected to order unity by interaction with earlier created quanta ...



But in the black hole we have a horizon, and then the older quanta get flushed away ...

Structure of the black hole



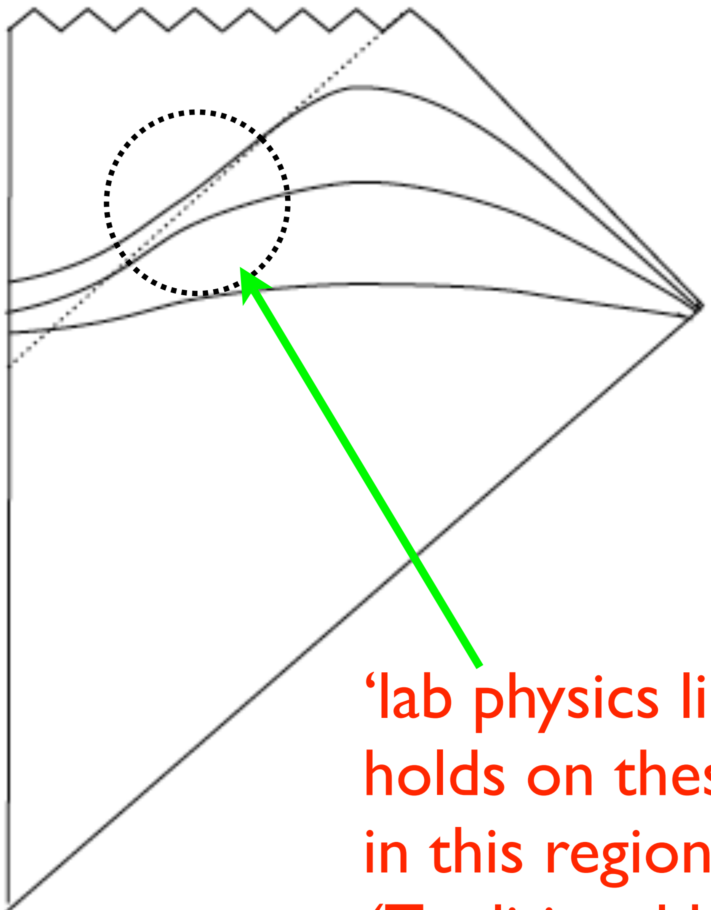
The black hole is described by the Schwarzschild metric

$$ds^2 = -(1-\frac{2M}{r})dt^2 + \frac{dr^2}{(1-\frac{2M}{r})} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

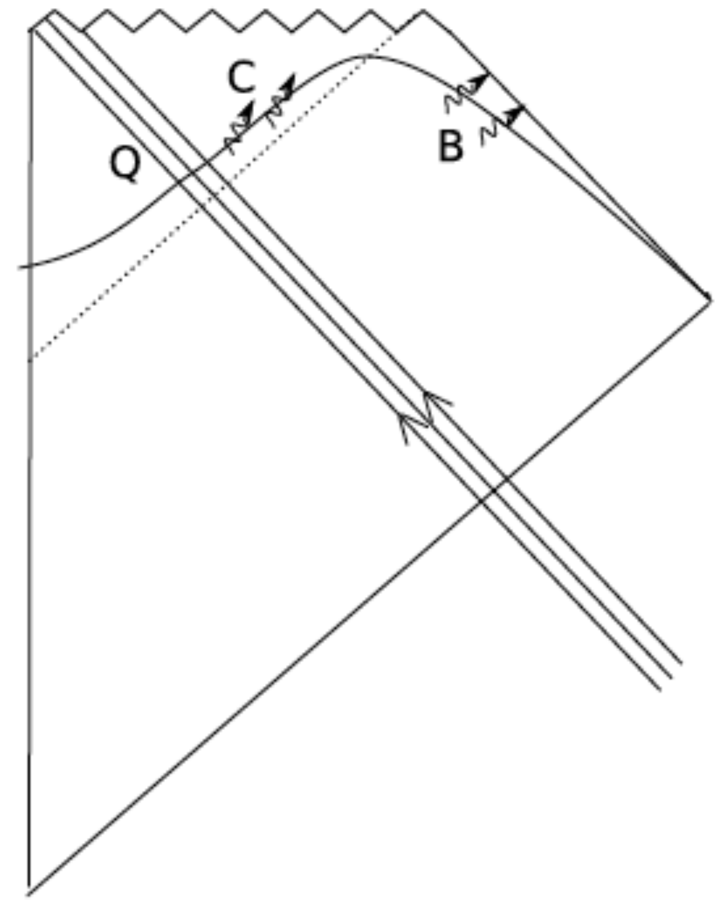
Crucial point about the black hole:

For $r > 2M$ the surface $t = constant$ is spacelike

For $r < 2M$ the surface $r = constant$ is spacelike

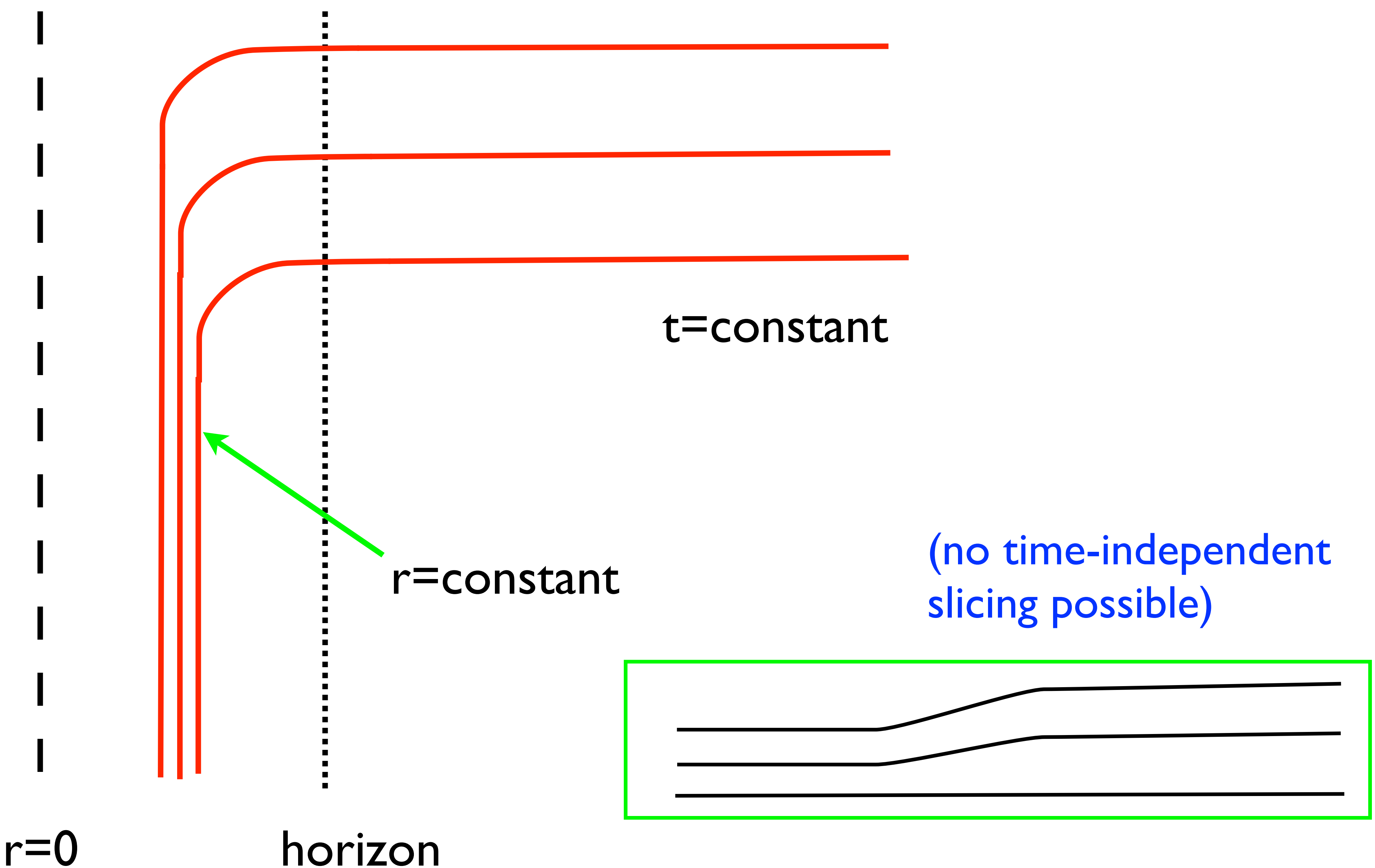


‘lab physics limit’
holds on these slices
in this region
(Traditional horizon)

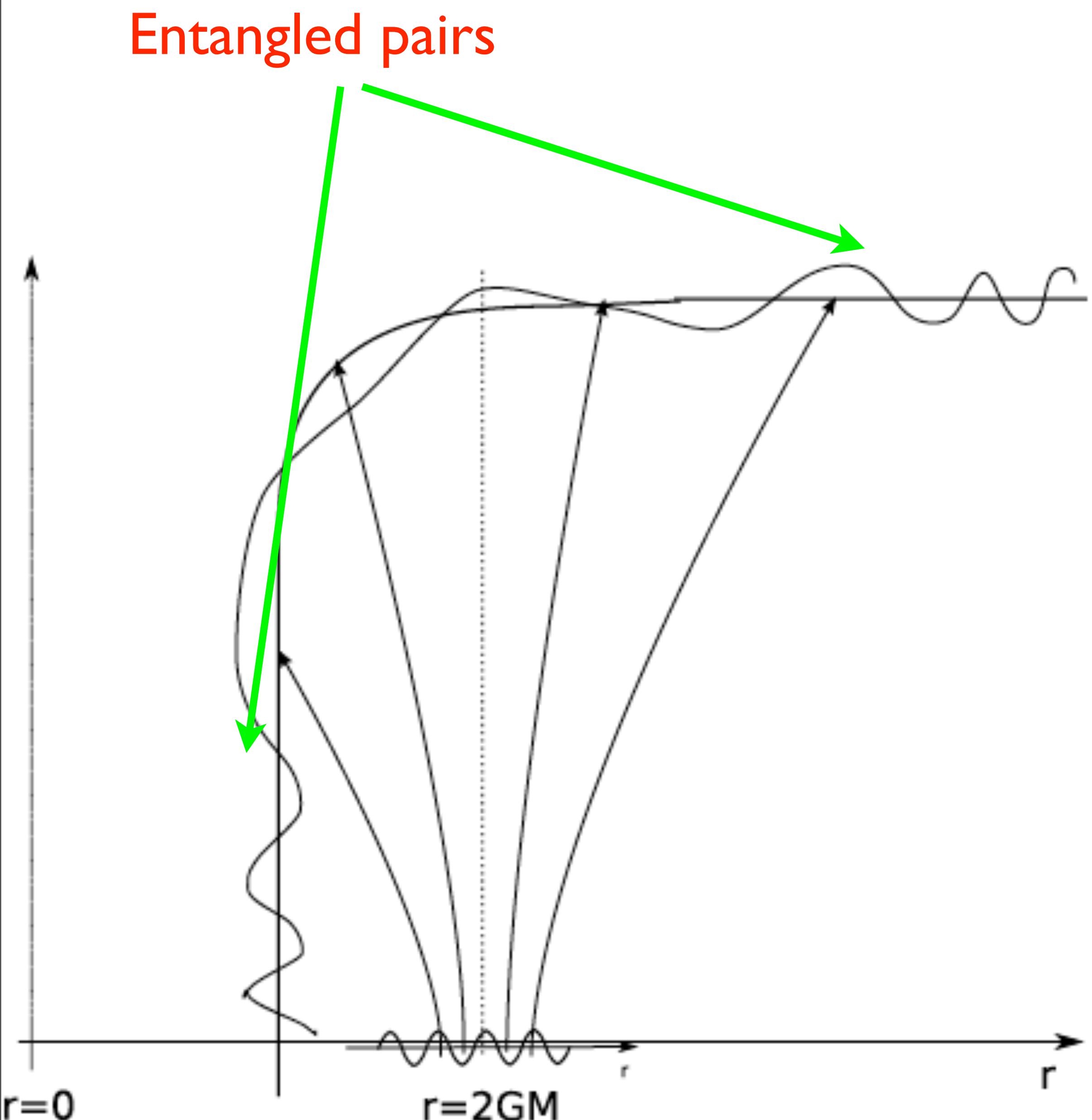


The infalling matter, and the created pairs, are all at low energy on the slice

The spacelike slices in a schematic picture



The Hawking process

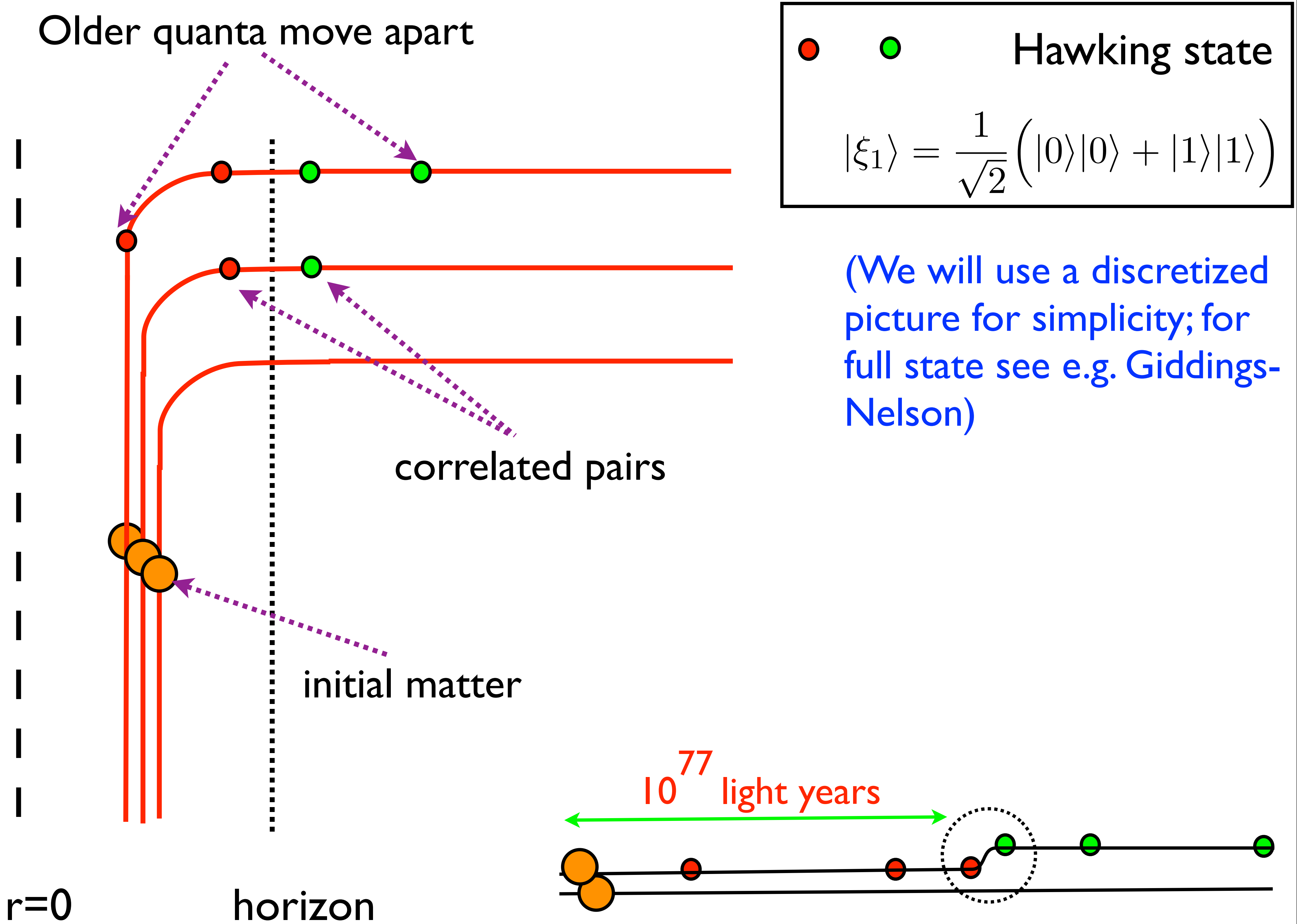


Follow the wavemode from say 1 fm to 1 Km

At 1 fm the mode must be in the vacuum state, else there would be a high energy density at the horizon
(would violate ‘traditional horizon’ assumption)

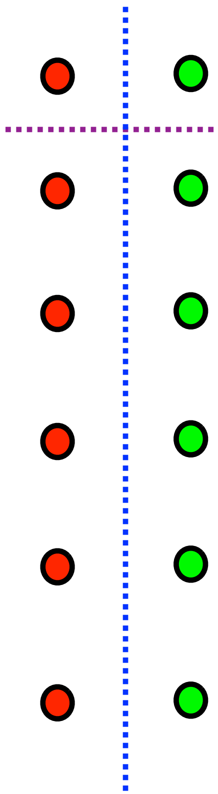
At 1 Km we have particle pairs, with wavefunction the Hawking entangled state

(Transplanckian physics not needed; bypassed by uniqueness of vacuum assumption)



(We will use a discretized picture for simplicity; for full state see e.g. Giddings-Nelson)

Hawking's argument



$$|\xi_1\rangle = \frac{1}{\sqrt{2}}\Big(|0\rangle|0\rangle + |1\rangle|1\rangle\Big)$$

$$S_{N+1} = S_N + \ln 2$$

S_N: Entanglement entropy after N pairs have been created

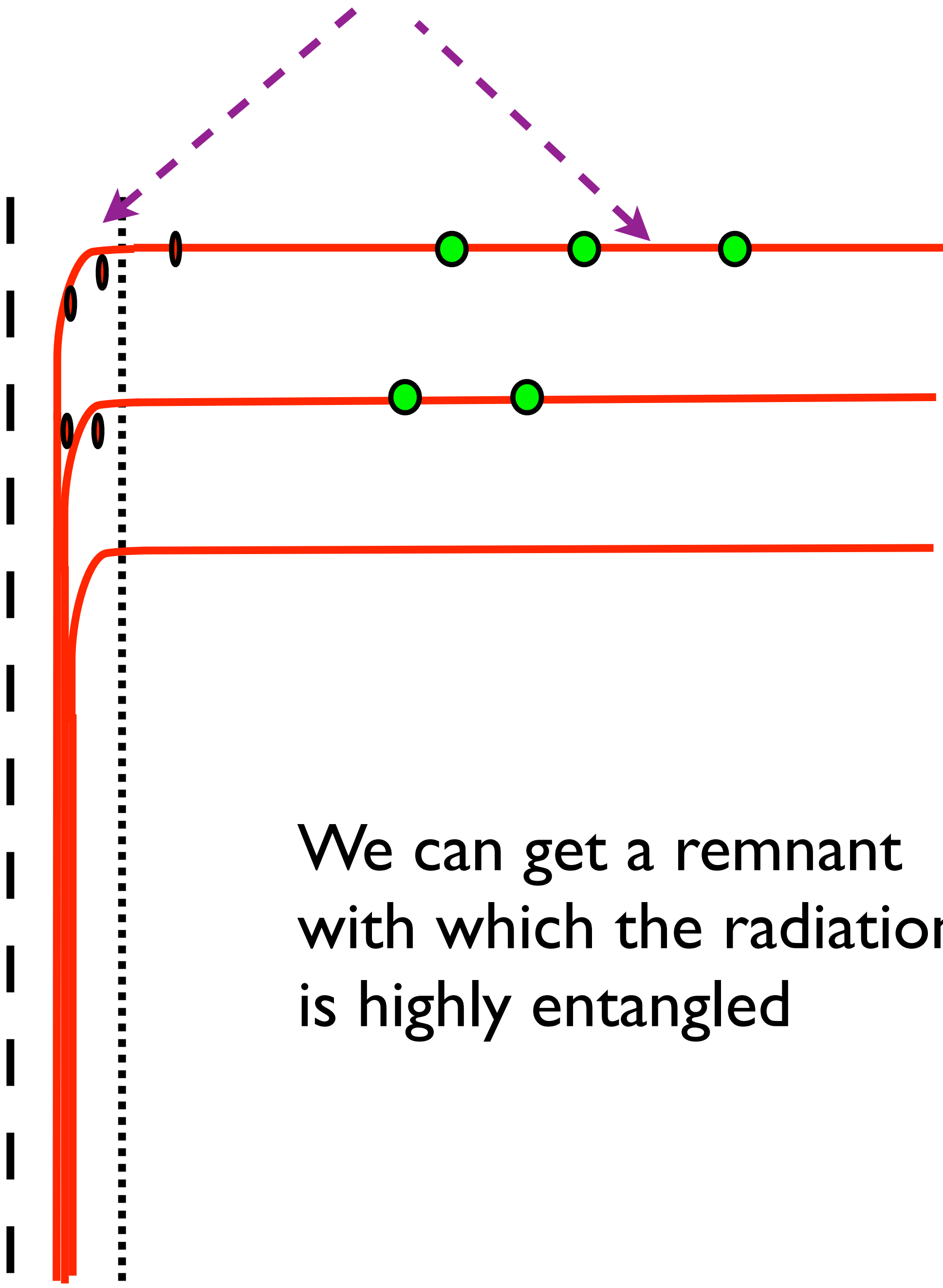
$$|\xi_1\rangle = \frac{1}{\sqrt{2}}\Big(|0\rangle|0\rangle + |1\rangle|1\rangle\Big)$$

$$|\xi_1\rangle = \frac{1}{\sqrt{2}}\Big(|0\rangle|0\rangle + |1\rangle|1\rangle\Big)$$

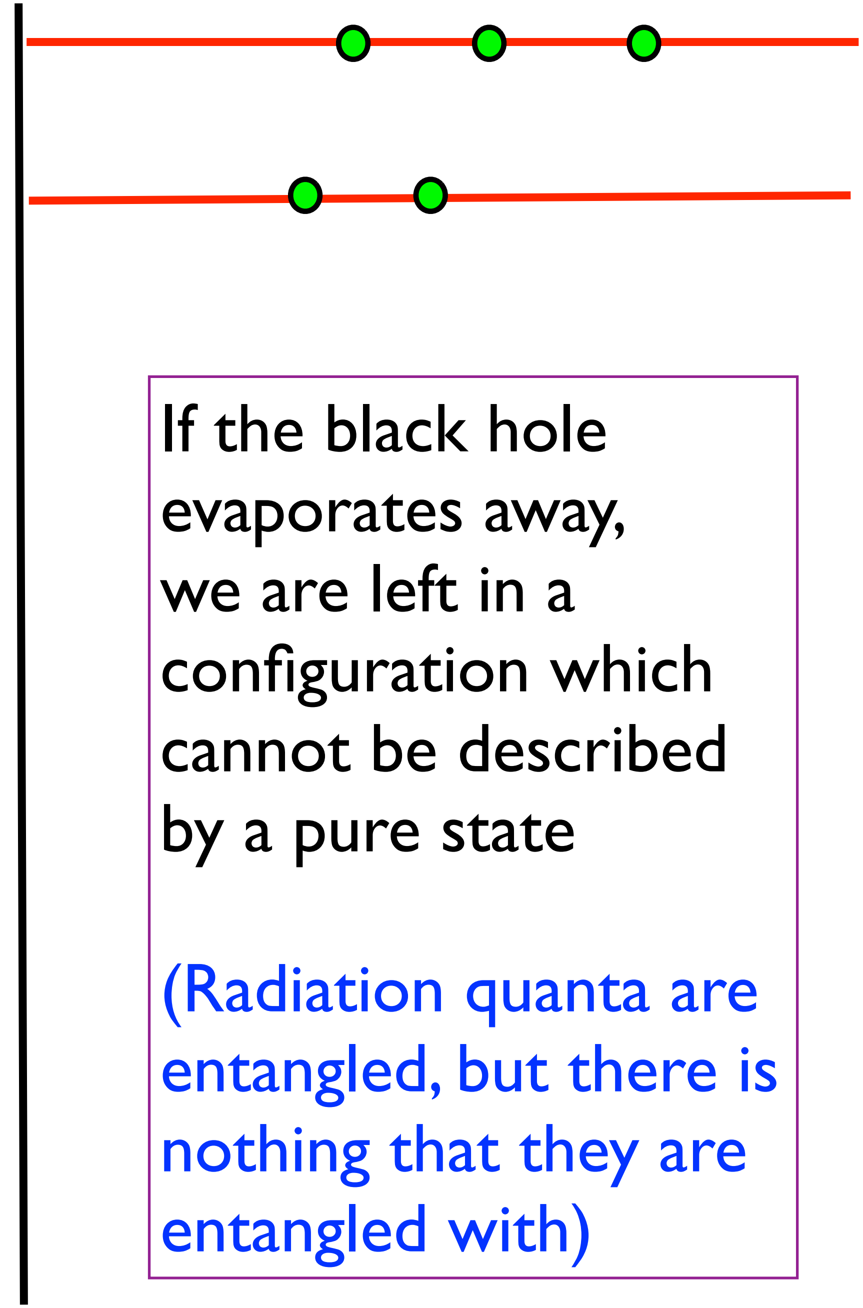
The radiation state (green quanta) are highly entangled with the infalling members of the Hawking pairs (red quanta)

$$S = N_{total} \ln 2$$

Entangled state



We can get a remnant with which the radiation is highly entangled

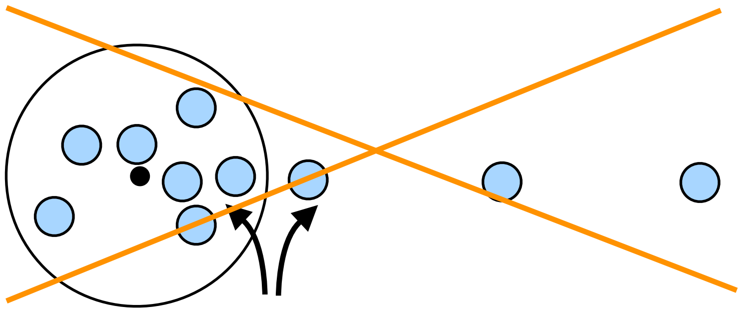


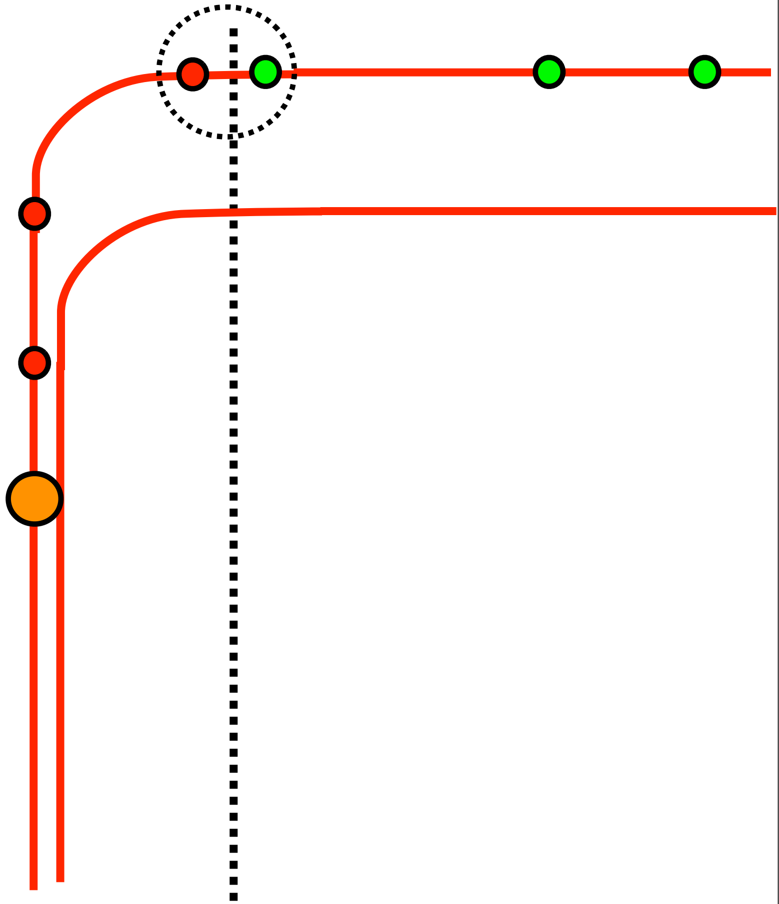
If the black hole evaporates away, we are left in a configuration which cannot be described by a pure state

(Radiation quanta are entangled, but there is nothing that they are entangled with)

Corrections ?

(A)



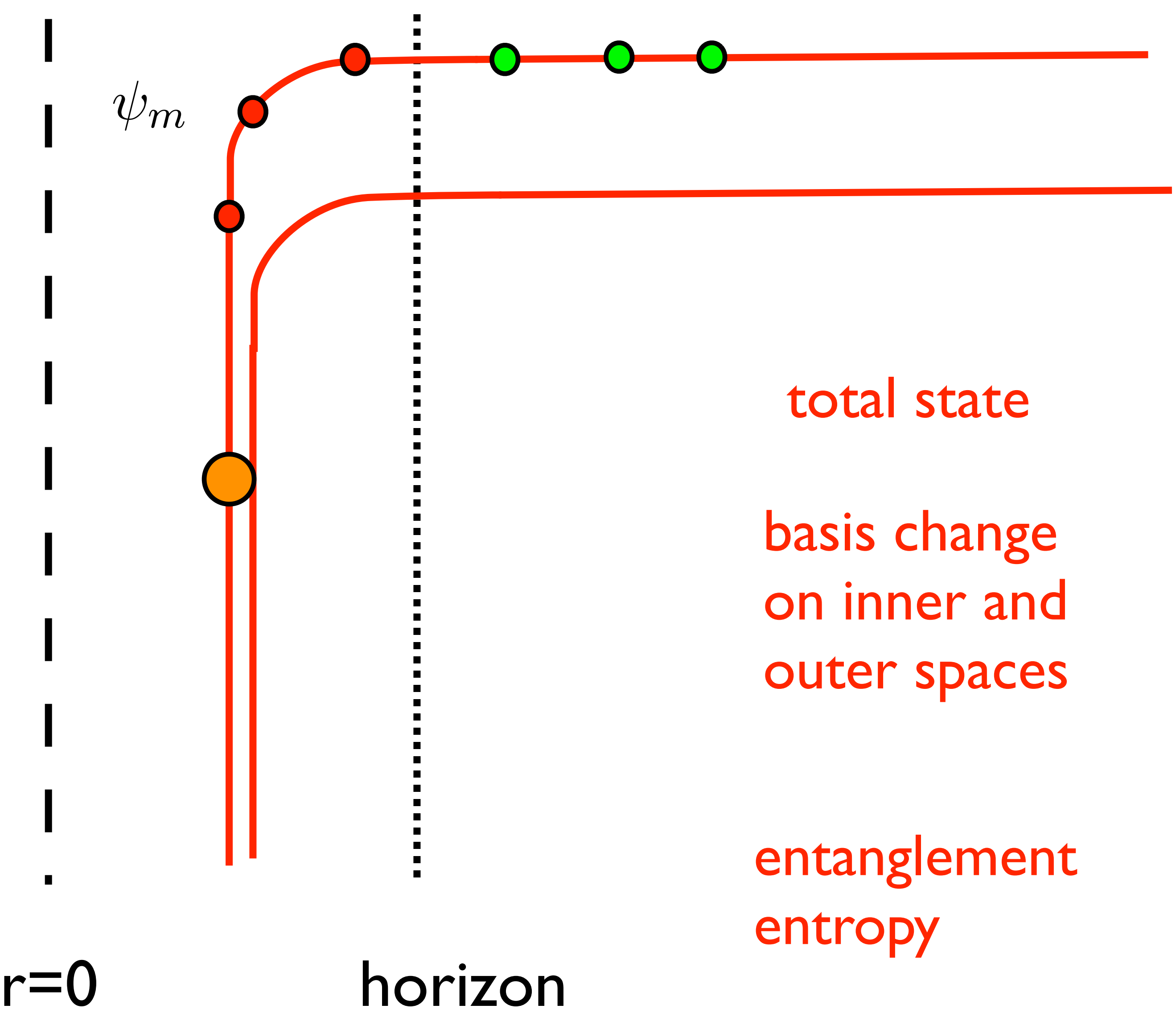


Older quanta get flushed away ...

At step N :

(basis for initial matter and inside quanta)

χ_n (basis for outside quanta)



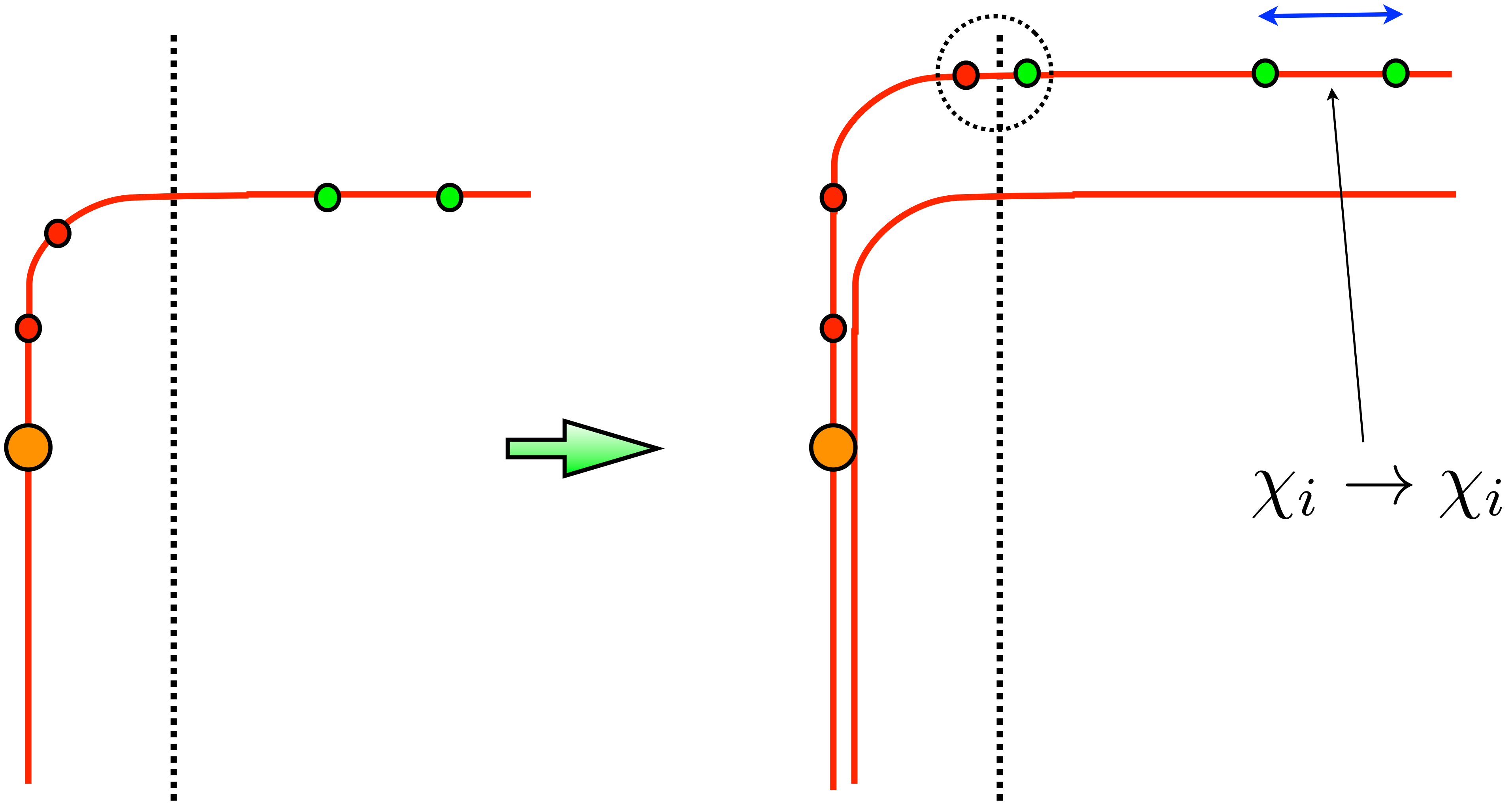
total state $$|\Psi\rangle = \sum C_{mn}\psi_m\chi_n$$

basis change on inner and outer spaces $$|\Psi\rangle = \sum_i C_i\psi_i\chi_i$$

entanglement entropy $$S_N = -\sum_i |C_i|^2 \ln |C_i|^2$$

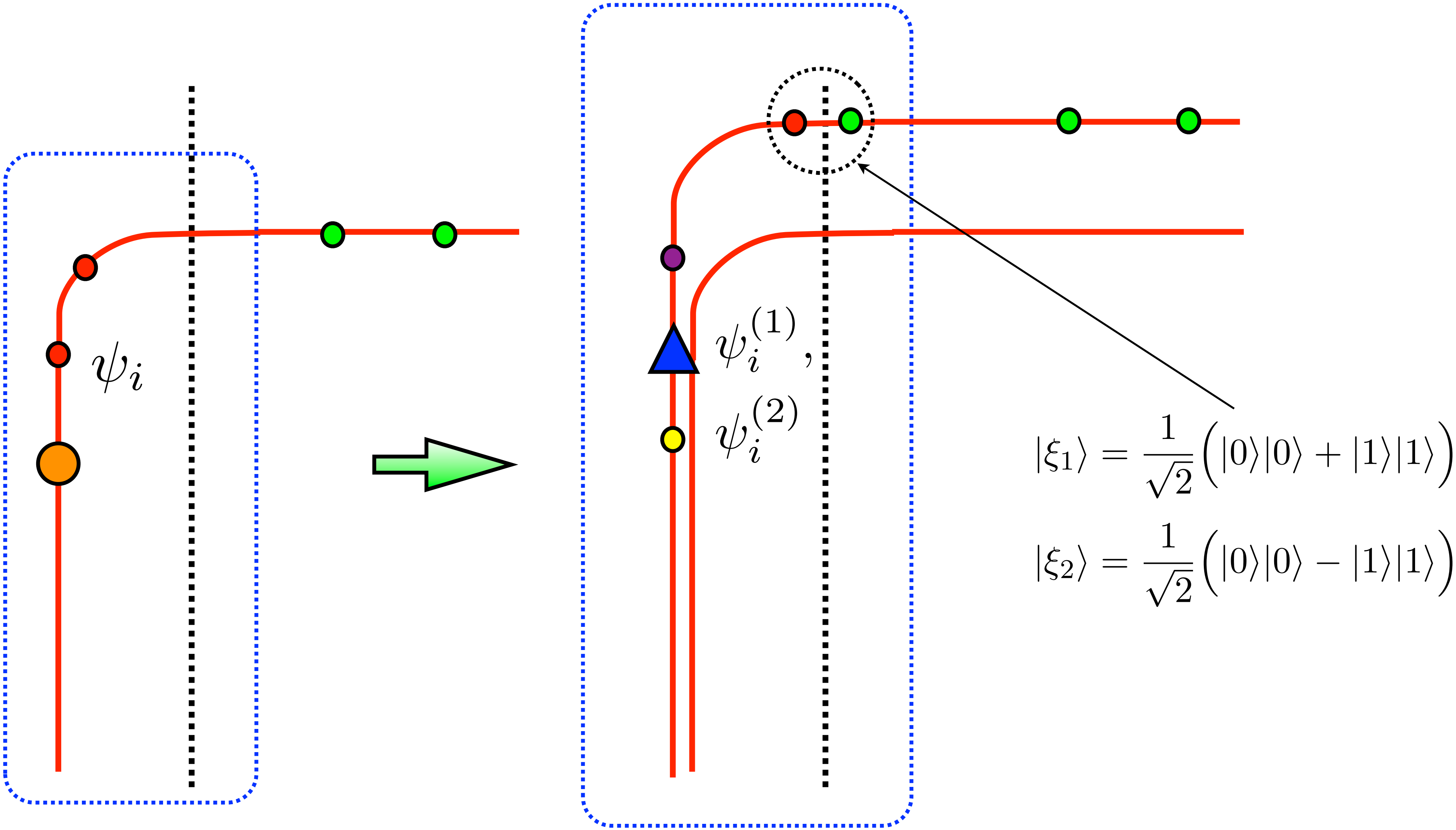
Rules for evolution from step N to step N+1:

(a) Quanta that have already left at earlier steps are not modified



(This also happens for burning paper)

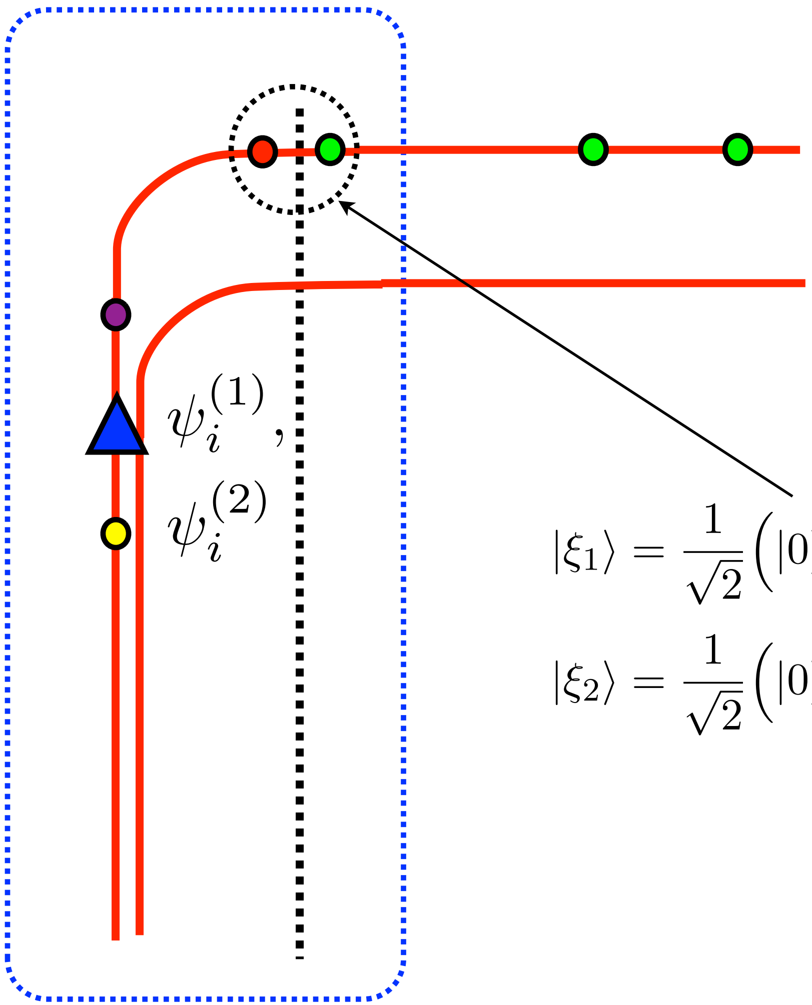
(b) The quanta in the hole from earlier steps, and the initial matter, can mix up arbitrarily to a new state



$$\psi_i \to \psi_i^{(1)}\xi^{(1)} + \psi_i^{(2)}\xi^{(2)}$$

(Unitary evolution)

Evolution of the state from timestep N to N+1:



$$|\xi_1\rangle = \frac{1}{\sqrt{2}}\Big(|0\rangle|0\rangle + |1\rangle|1\rangle\Big)$$

$$|\xi_2\rangle = \frac{1}{\sqrt{2}}\Big(|0\rangle|0\rangle - |1\rangle|1\rangle\Big)$$

$$|\Psi\rangle = \sum_i C_i \psi_i \chi_i$$

$$\to \sum_i C_i [\psi_i^{(1)} \xi^{(1)} + \psi_i^{(2)} \xi^{(2)}]$$

$$\equiv \xi^{(1)} \Lambda^{(1)} + \xi^{(2)} \Lambda^{(2)}$$

$$\Lambda^{(1)} = \sum_i C_i \psi_i^{(1)} \chi_i$$

$$\Lambda^{(2)} = \sum_i C_i \psi_i^{(2)} \chi_i$$

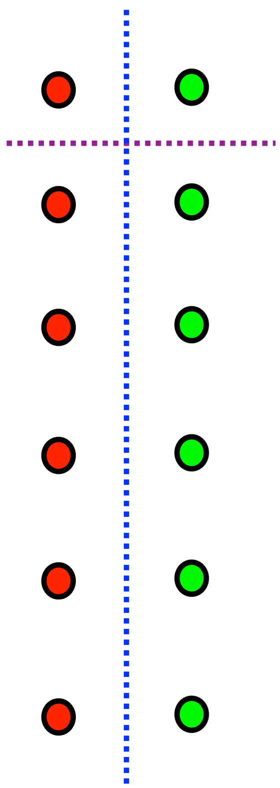
$$||\Lambda^{(1)}||^2 + ||\Lambda^{(2)}||^2 = 1$$

Horizon is a ‘normal place’ : $||\Lambda^{(2)}|| < \epsilon, \quad \epsilon \ll 1$

Theorem: Small corrections to Hawking's leading order computation do NOT remove the entanglement

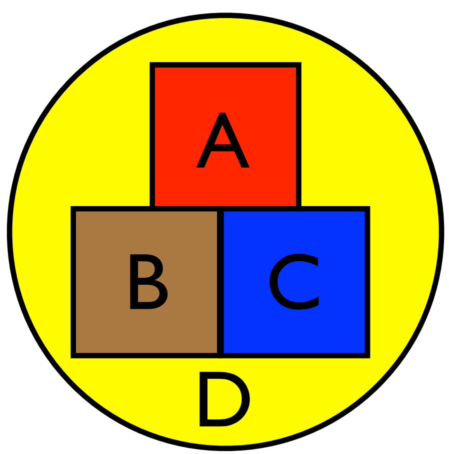
$$\frac{\delta S_{ent}}{S_{ent}} < 2\epsilon$$

(SDM arXiv: 09091038)



Bound does not depend on the number of pairs N

Basic tool : Strong Subadditivity (Lieb + Ruskai '73)



$$S(A) = -Tr[\rho_A \ln \rho_A] \quad \text{etc.}$$

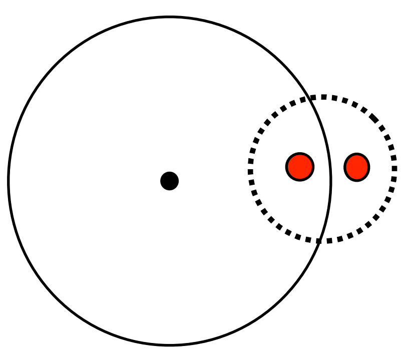
$$S(A+B) + S(B+C) \geq S(A) + S(C)$$

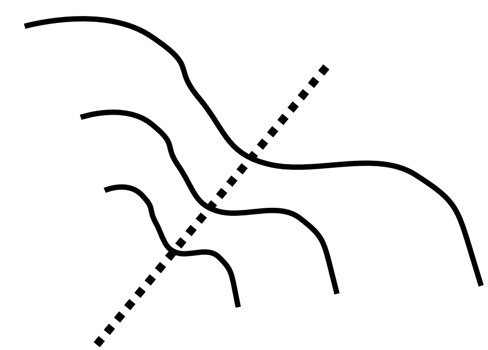
The Hawking argument → ‘Theorem’

Can be made as rigorous as we want ...

(a) We either have a ‘Traditional horizon’, or ‘hair’

Traditional horizon



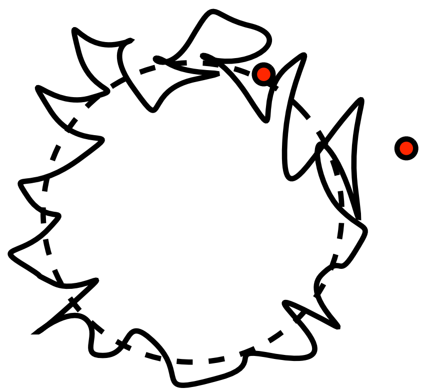


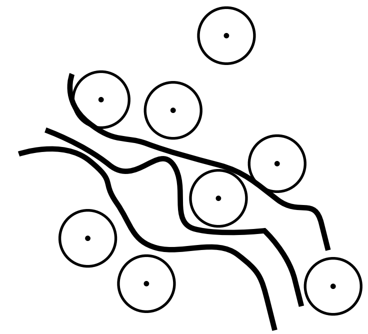
There is a good slicing at the horizon in which a neighbourhood of this horizon is low energy physics just like the one in this room ....

Then the stretching of vacuum modes will create an entangled pair at each time step ...

S_{ent} keeps growing ...

(b) 'Hair' : Anything else ...





If we do not have the standard vacuum pair production at the horizon, then there is no Hawking argument ...
(This would remove the paradox)

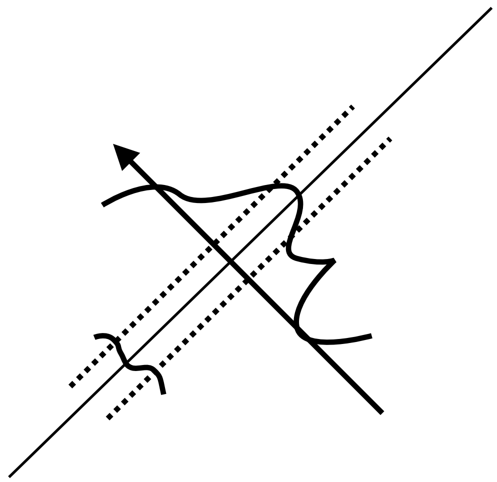
Note that the corrections to low energy evolution have to be order unity, not 'small' ....

What do we have to do to resolve the paradox ?

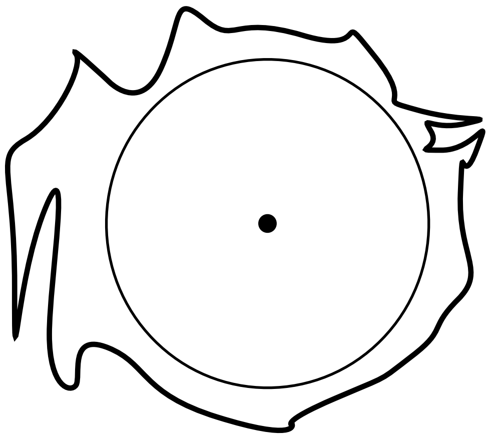
In one sense very little, in another sense, a lot ...

Little, because all we have to show is that there is an effect that will obstruct normal physics at the horizon ...

But this has been very hard, since people tried but could not find hair ... 'no hair theorem' ...

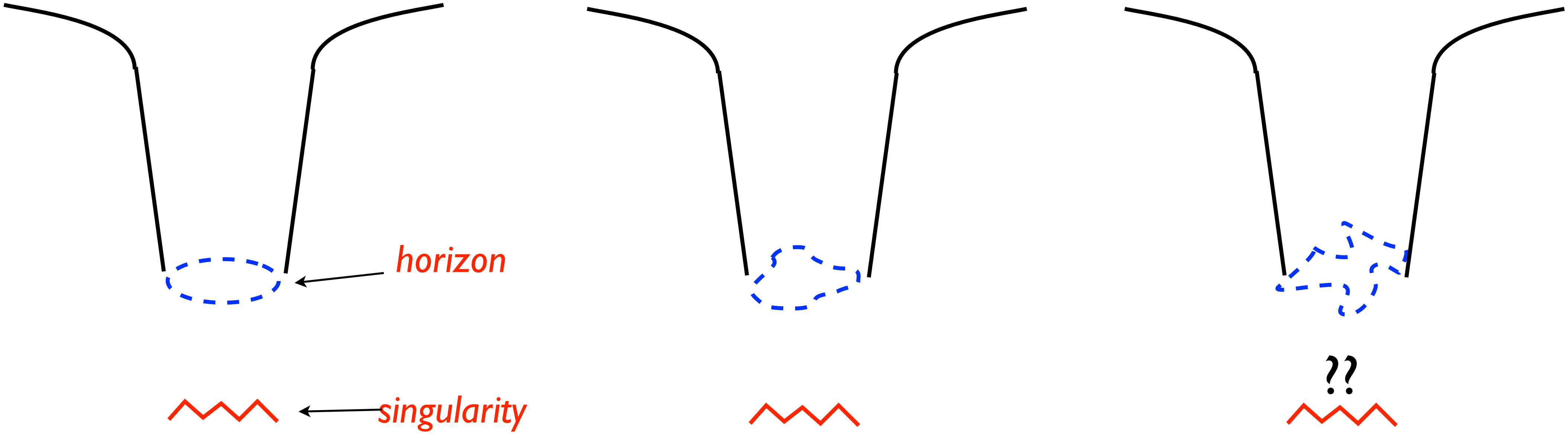


't hooft: infalling quanta create large shifts in outgoing rays ...



Susskind: strings at the horizon ... corrupt normal evolution ...

The basic difficulty can be seen from the earliest computations ...

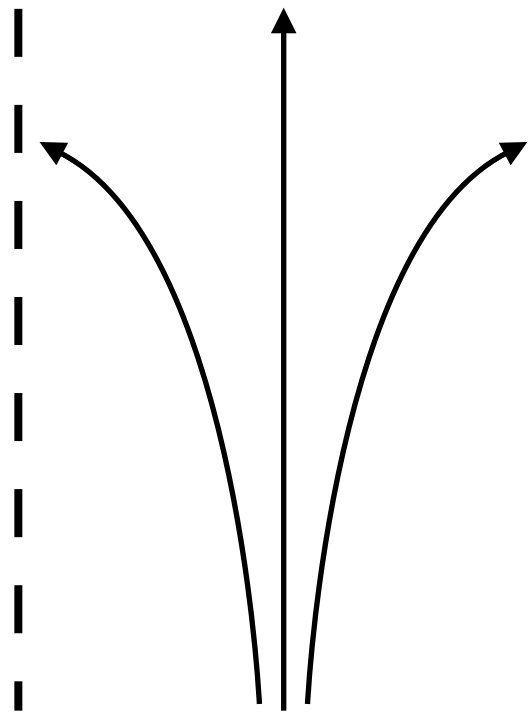


People wrote down the wave equation for scalars, gauge fields, gravitons ...
Looked for solutions with L=1,2, 3,...

If they had found such solutions, then one would expect that the entropy comes from horizon fluctuations, and there would be no information problem

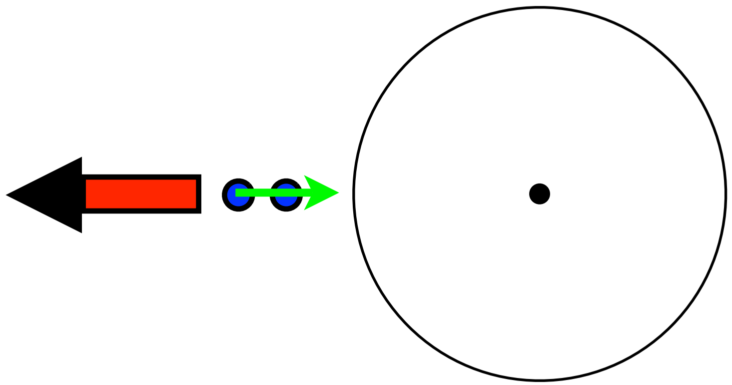
But no hair was found ... "no hair theorem"

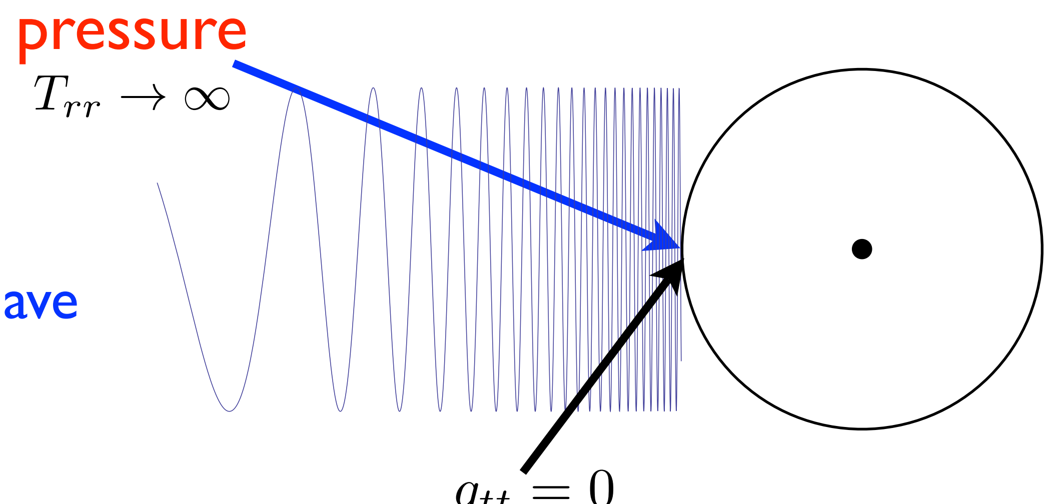
Why is it hard to find hair?



Horizon is an unstable place ...

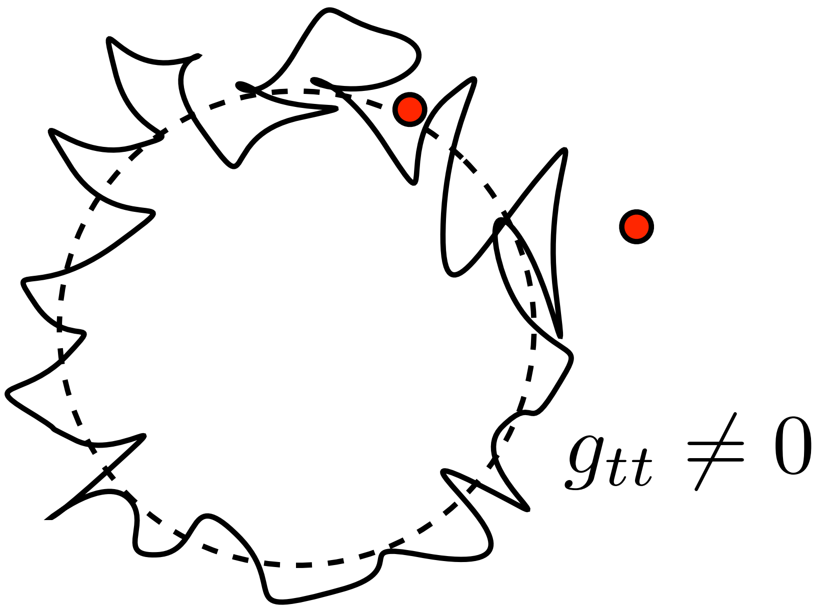
Large relative momentum needed to keep the rocket stationary





Field modes have divergent stress-energy

But there might still be non-perturbative "hair" ?



Backreaction from the distortion is self consistent ... no horizon forms

Hole radiates like 'paper'

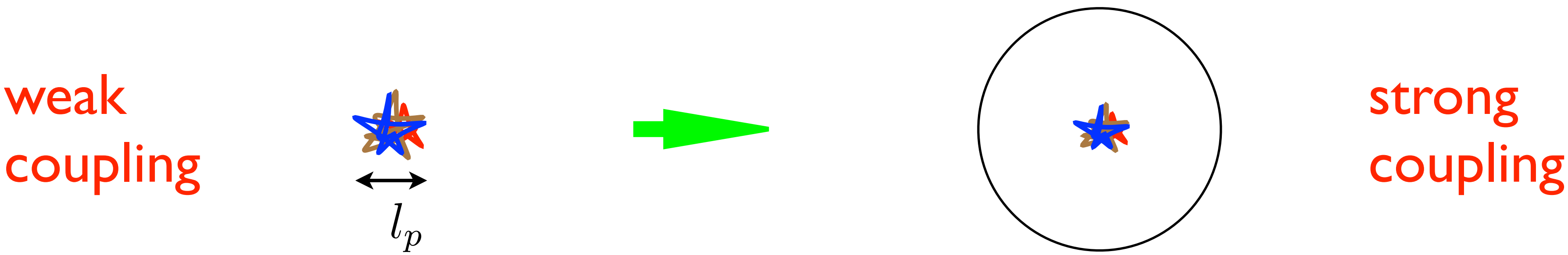
How do we find such solutions ?

String theory gives a new expansion parameter, the 'complexity of the microstate' ...

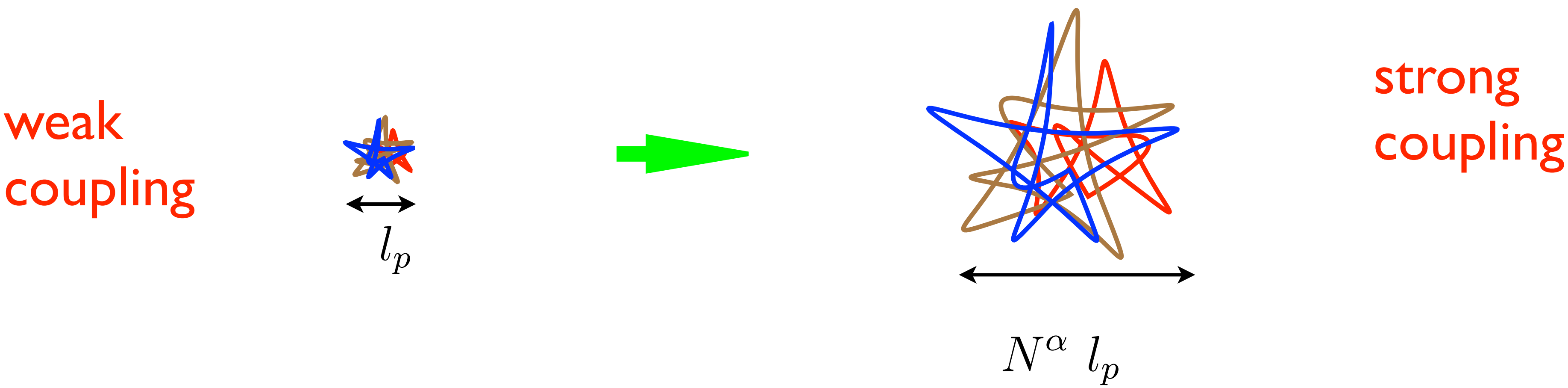
Resolving the puzzle

(Avery, Balasubramanian, Bena, Chowdhury, de Boer, Gimon, Giusto, Keski-Vakkuri, Levi, Lunin, Maldacena, Maoz, Park, Peet, Potvin, Ross, Ruef, Saxena, Simon, Skenderis, Srivastava, Taylor, Turton, Warner ...)

The traditional expectation ...



But it seems in string theory the opposite happens ...



$$R \sim [\frac{g^2 \alpha'^3 \sqrt{n_1 n_5 n_p}}{RV}]^{\frac{1}{3}} \sim R_s$$

(SDM 97)

The 'no-hair' theorem tells us that the black hole metric is unique:

$$ds^2 = -(1-\frac{2M}{r})dt^2 + \frac{dr^2}{(1-\frac{2M}{r})} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

But how did we get this metric ?

We take an ansatz where the metric coefficients had no dependence on angular variables or on the compact directions

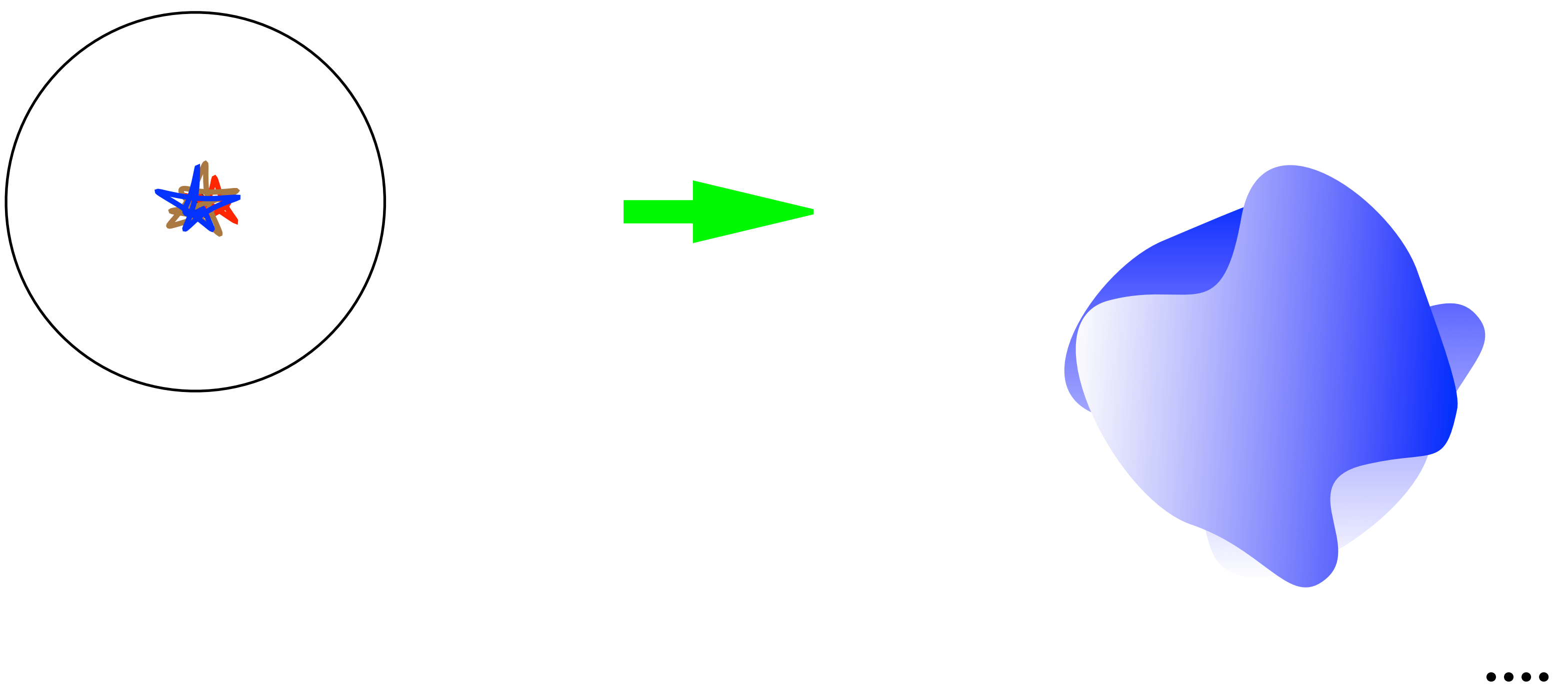
$$ds^2 = -f(r)dt^2 + g(r)dr^2 + r^2 d\Omega_2^2 + dz_i dz_i$$

The solution we get is singular, however, at the origin, so we cannot be sure it is a solution of the full quantum gravity theory

Now let us look for solutions that have no spherical symmetry and the compact directions are also not trivially tensored

Then there are a large number of regular solutions - no horizon and no singularity - with the same M, Q, J as the black hole

→ ‘Fuzzballs’



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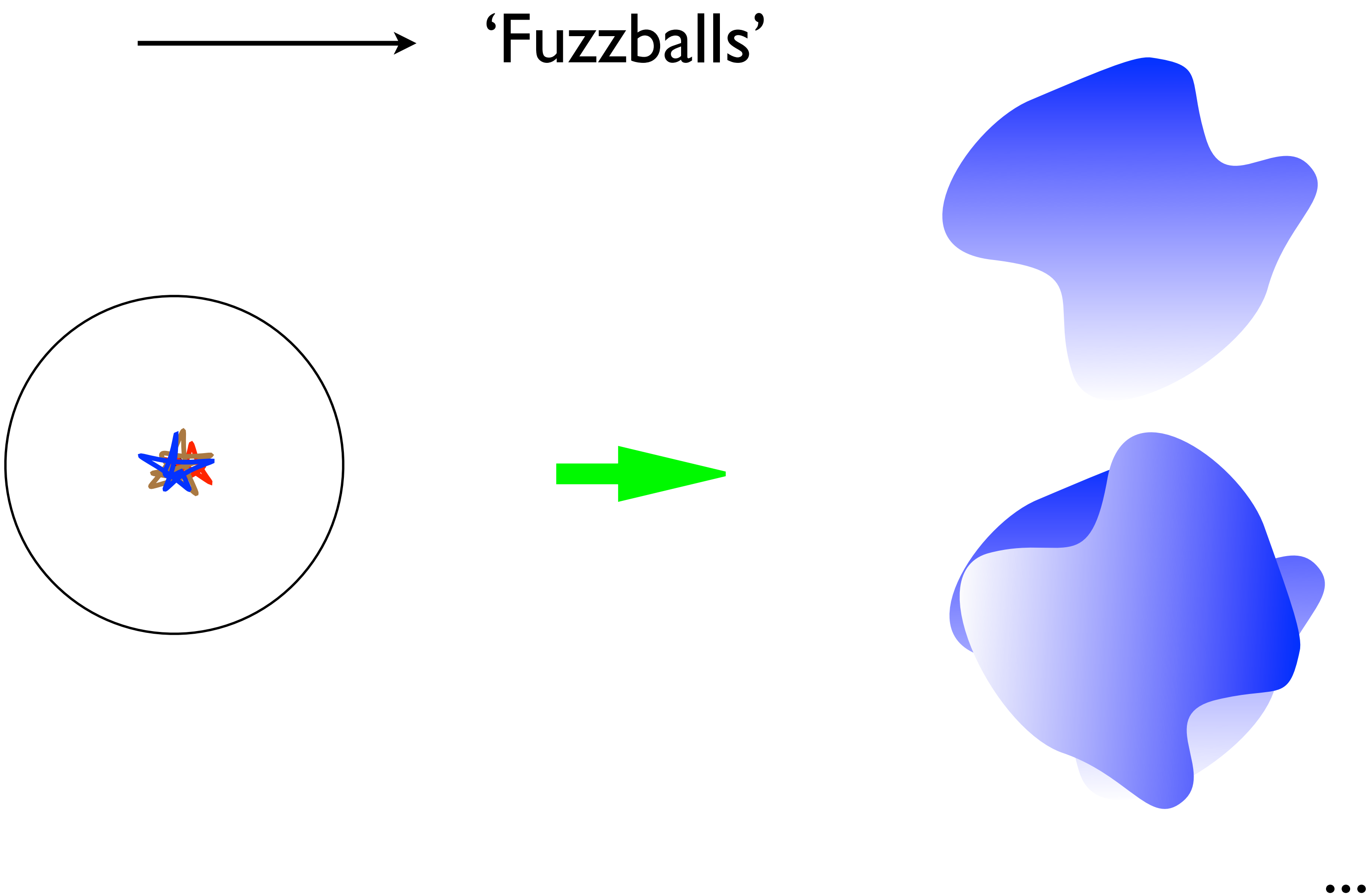
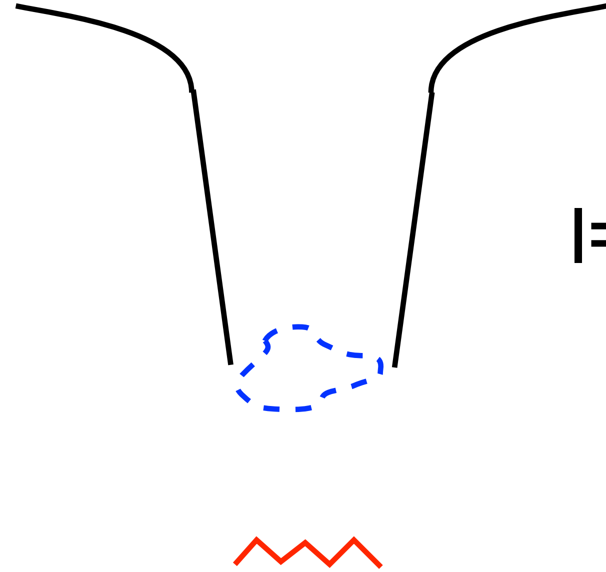
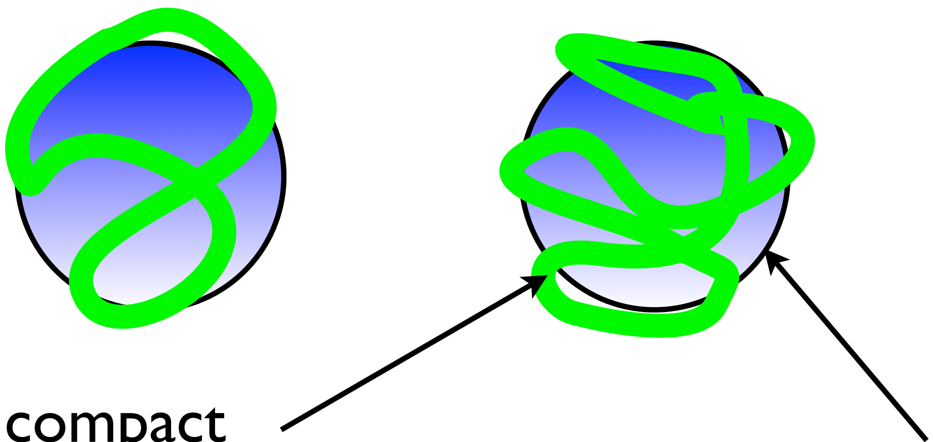


Hair in string theory ....



Nature of the hair:



small compact
direction circle

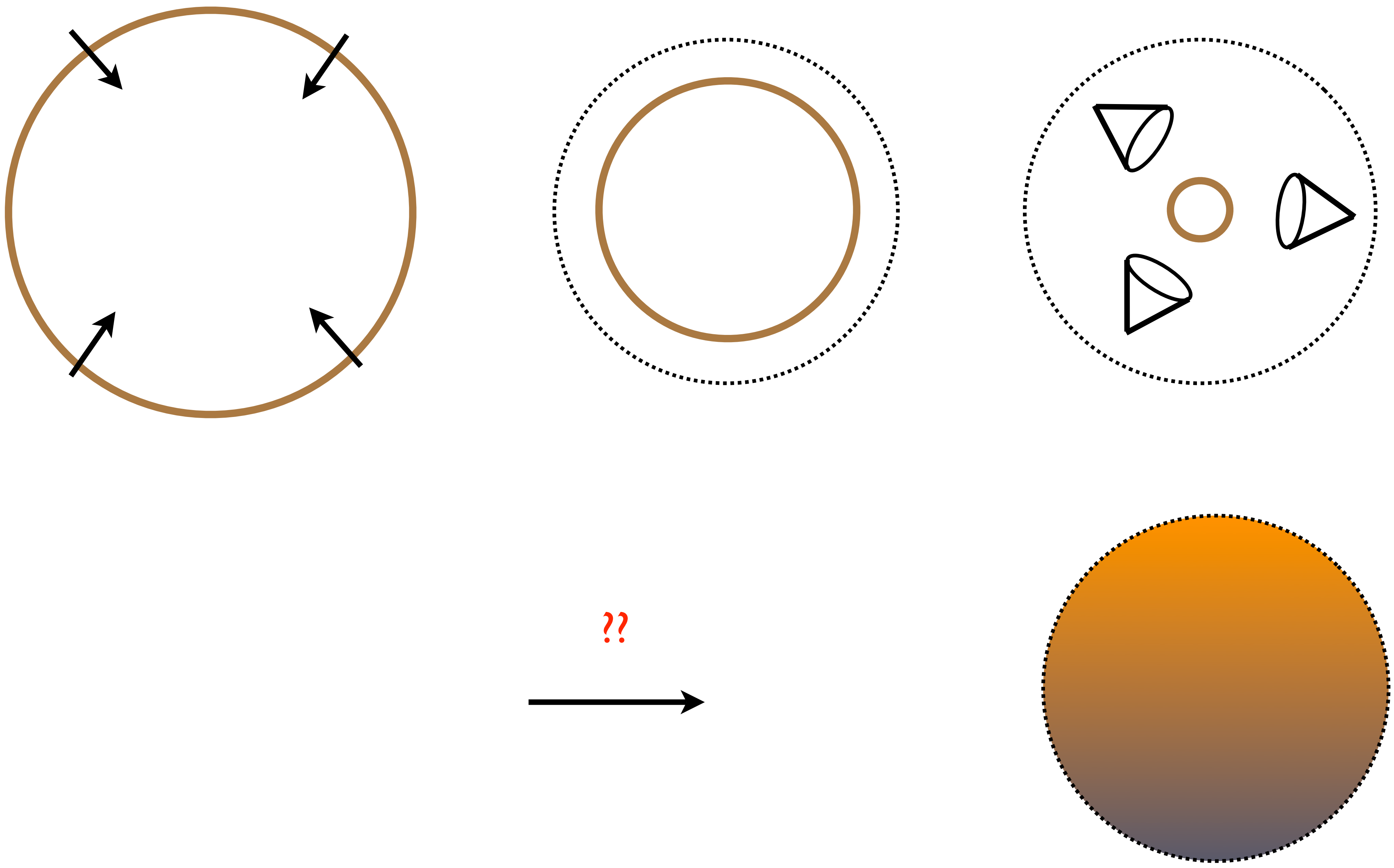
Angular sphere of
noncompact directions

Compact directions make
locally nontrivial fibrations over
the noncompact directions

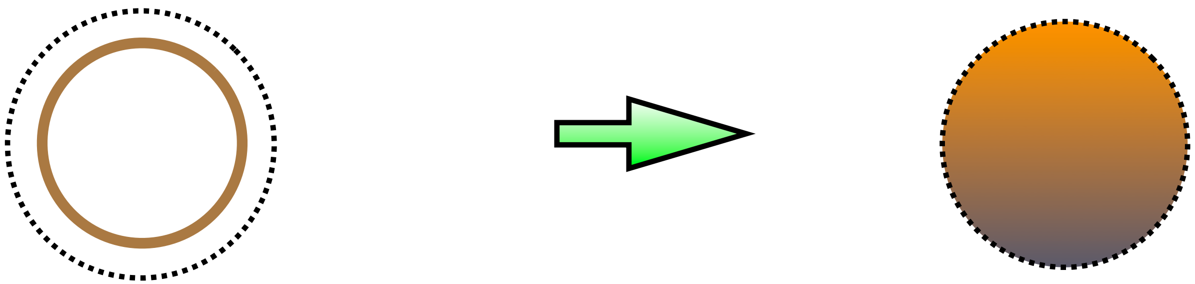
Thus the hair are fundamentally a nonperturbative construct involving the extra dimensions ...

How does semiclassical
intuition go wrong ?

How does a collapsing shell become fuzzballs ?



Consider the amplitude for the shell to tunnel to a fuzzball state



$$S_{tunnel} \sim \frac{1}{G}\int R d^4x \sim \frac{1}{G}\frac{1}{(GM)^2}(GM)^4 \sim GM^2$$

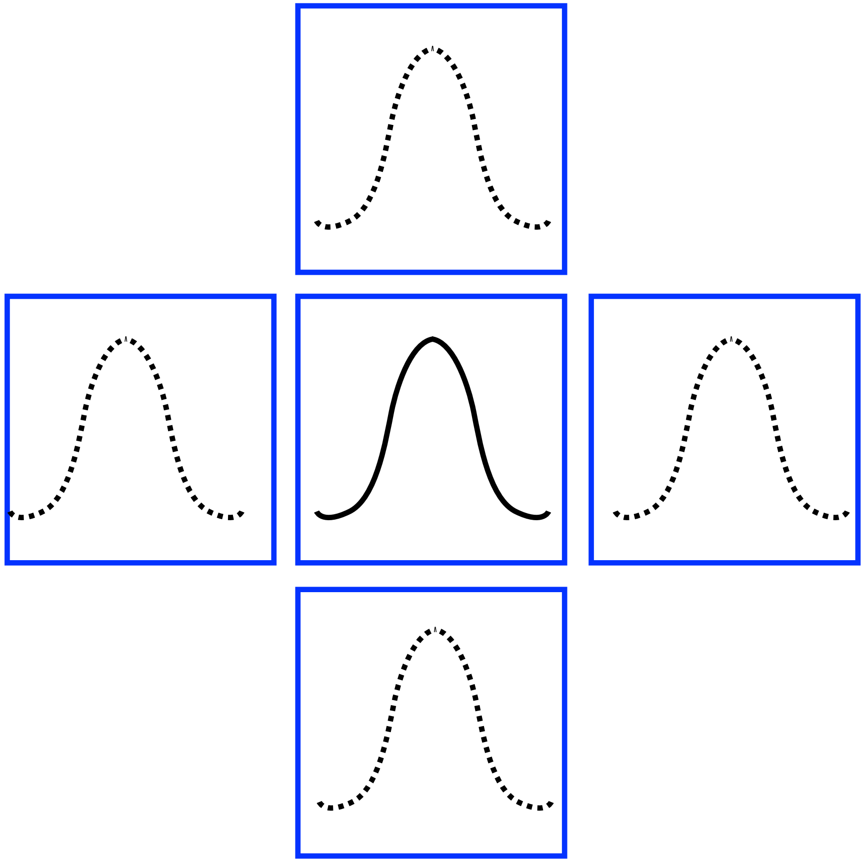
$$\mathcal{A} \sim e^{-S_{tunnel}}$$

Amplitude to tunnel is very small

$$\mathcal{N} \sim e^{S_{bek}} \sim e^{GM^2}$$

But the number of states that one can tunnel to is very large !

Toy model: Small amplitude to tunnel to a neighboring well, but there are a correspondingly large number of adjacent wells

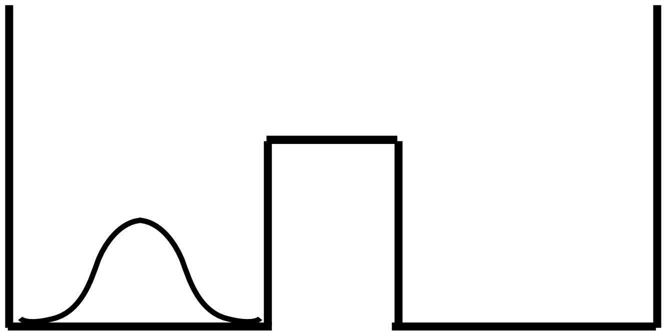


In a time of order unity, the wavefunction in the central well becomes a linear combination of states in all wells (SDM 07)

How long does this tunneling process take ?

If it takes longer than Hawking evaporation time then it does not help ...

Tunneling in the double well:



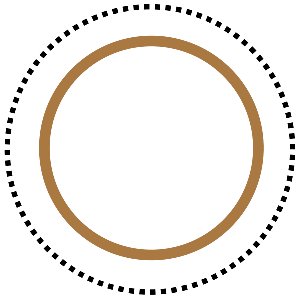
$$\psi = e^{-iE_S t}\psi_S \; + \; e^{-iE_A t}\psi_A$$

The wavefunction tunnels to the other well in a time

$$\Delta t = \frac{\pi}{\Delta E}$$

where $\Delta E = E_A - E_S$

For the collapsing shell ...



$$|\psi\rangle = \sum_k c_k |E_k\rangle$$

$$E \sim \frac{P^2}{2M}$$

$$\Delta P \gg \frac{1}{R}$$

$$\Delta E \sim \frac{P \Delta P}{M} \gg \frac{(\Delta P)^2}{M} \gg \frac{1}{MR^2}$$

$$t_{dephase} \sim \frac{1}{\Delta E} \ll MR^2$$

$$t_{evap} \sim MR^2$$

$$t_{dephase} \ll t_{evap}$$

Thus the collapsing shell turns into a linear combination of fuzzball states in a time short compared to Hawking evaporation time

The matrix model

If we define our gravity as the dual to a CFT:

(a) Remnant ?

(b) Slow leaking remnant ?

(c) No black hole ?

(d) No good local gravity theory ?

(e) Information in Hawking radiation ?

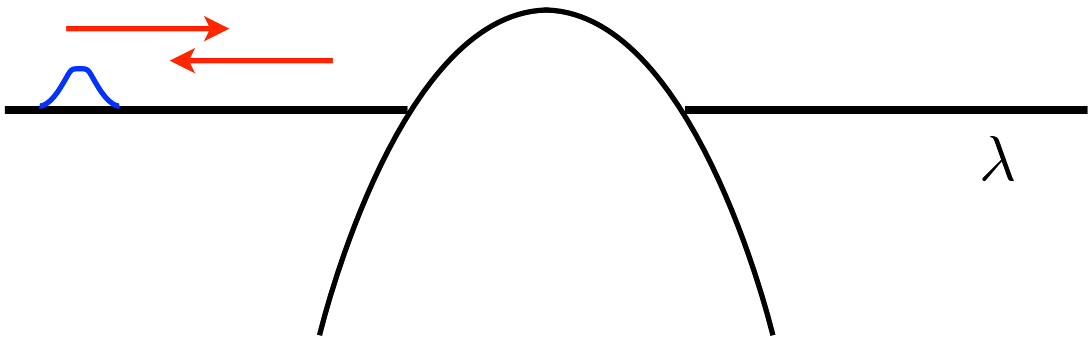
If there are no fuzzballs, then we cannot get (e) ...

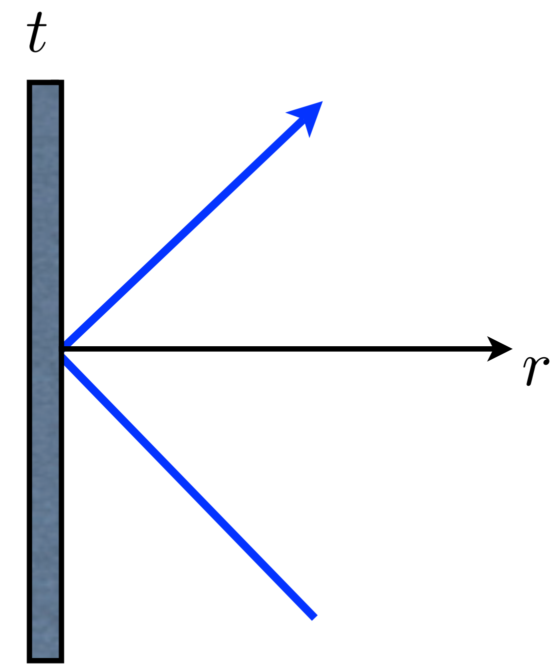
$c = 1$ Matrix model: A toy model of AdS/CFT

$M \; : \; N \times N$ matrix

Eigenvalues form
a 1-d fermi sea

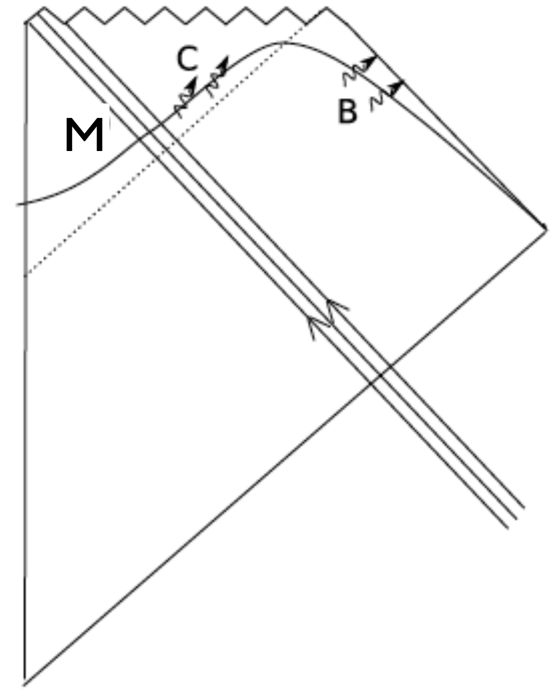
$$L = Tr\left(\frac{1}{2}\dot{M}(t)^2 - V(M)\right)$$





Scattering of low energy pulses from potential wall agrees with the scattering of pulses in 1+1 dim dilaton gravity + scalar

1+1 dilaton gravity has black holes ...



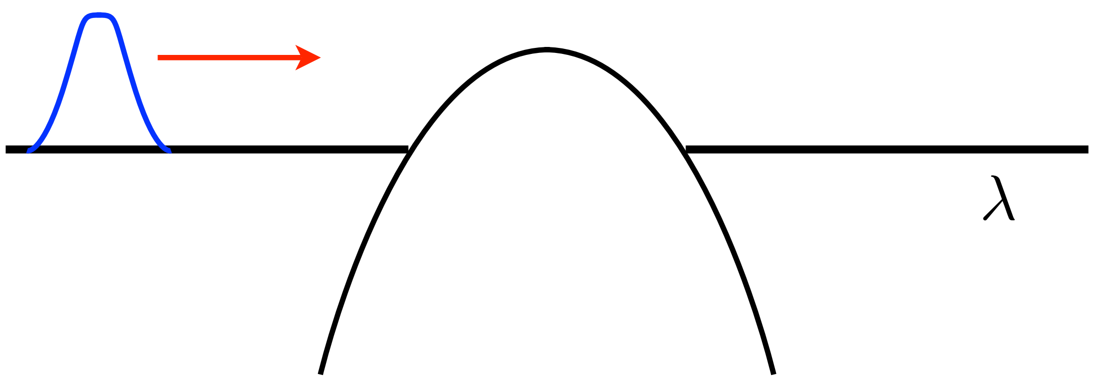
The matrix model is unitary,
so information cannot be lost ...

So we should find that evaporation
in 1+1 dim dilaton gravity is unitary ?

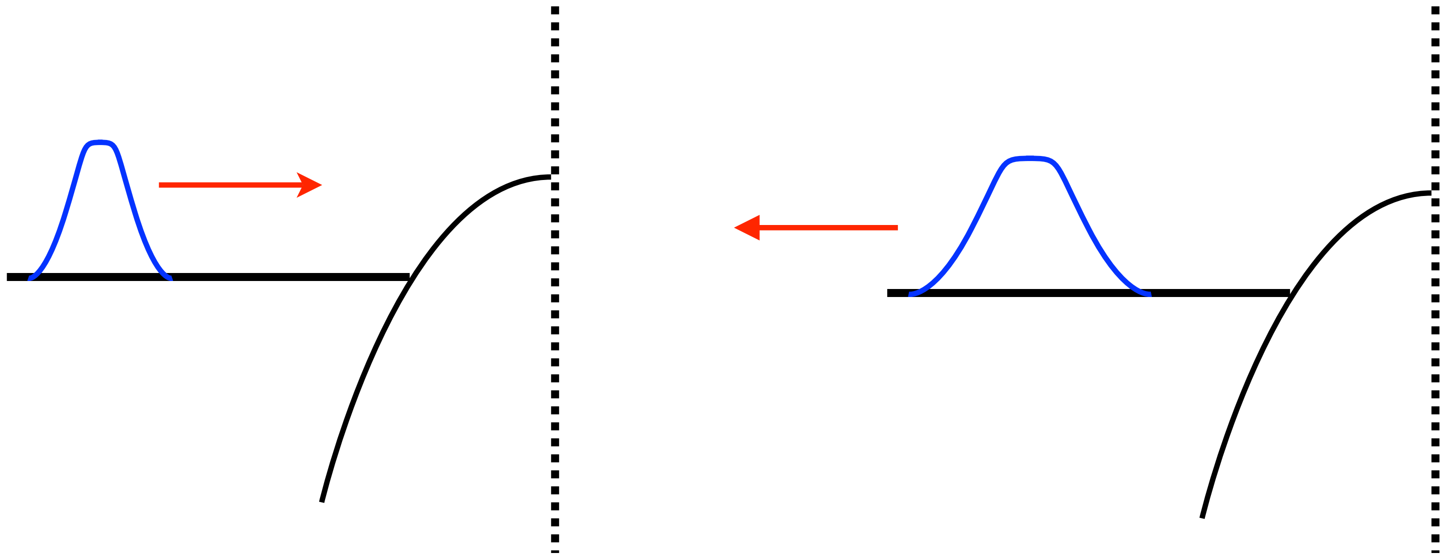
NO !! Low energy scattering agrees between matrix model dilaton gravity

But if we try to make a black hole in the matrix model ....

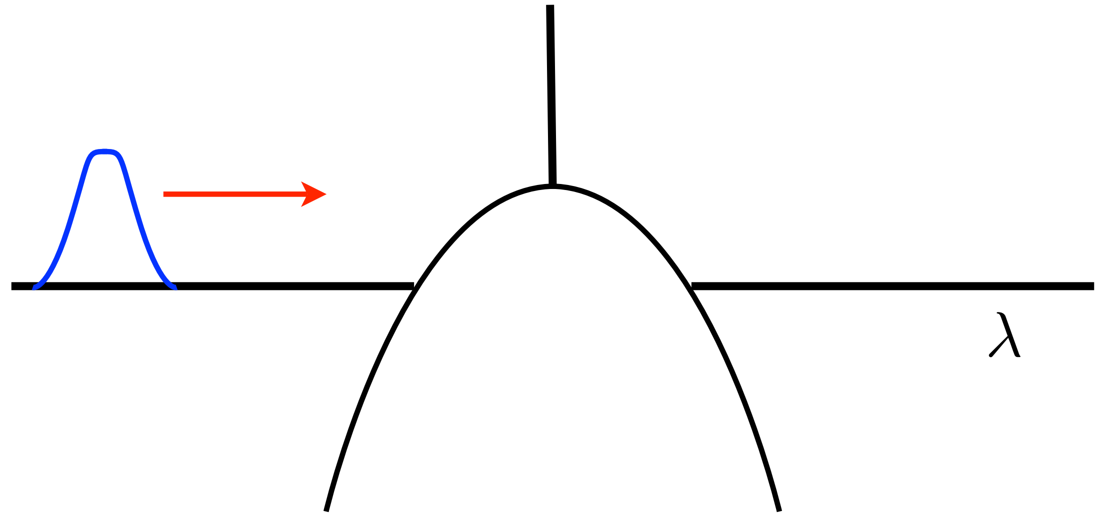
(a) Pulse with enough energy to make black hole spill over and goes to a second asymptotic infinity : ‘information loss’



(b) If we orbifold the two sides (e.g. susy theories), the pulse returns in crossing time: no black hole formation



(c) If we try to modify potential, gravity theory cannot in general be derived from a local lagrangian



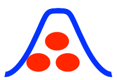
What we do not get is a black hole, persisting for a time greater than crossing time by a factor $(M/m_{pl})^{\alpha}$, information coming out in Hawking radiation

No fuzzballs ⟶ cannot get info in Hawking radiation

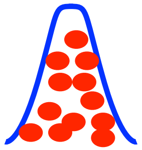
No remnants and no information loss

Theory takes only way out: no black hole formation ...

How do we avoid black hole formation in the matrix model ?



A pulse made of n quanta has small deviations from exact dilaton gravity, because of interaction between the quanta

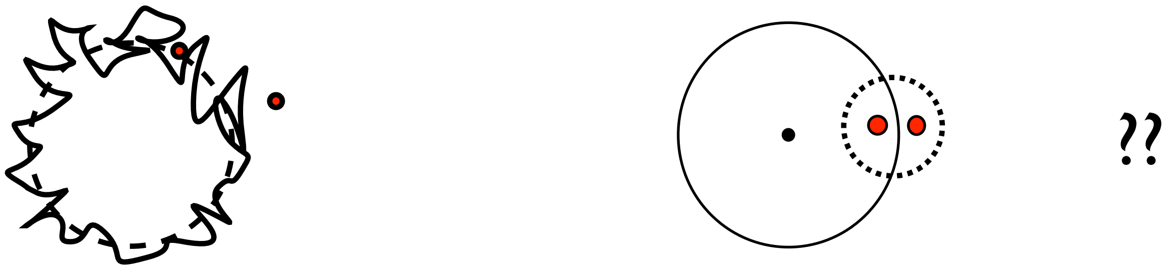


When the pulse becomes big enough to make a black hole, these interactions create order unity deviation from dilaton gravity

Large number of mutual interactions stop semiclassical collapse at horizon scale

Common questions about fuzzballs

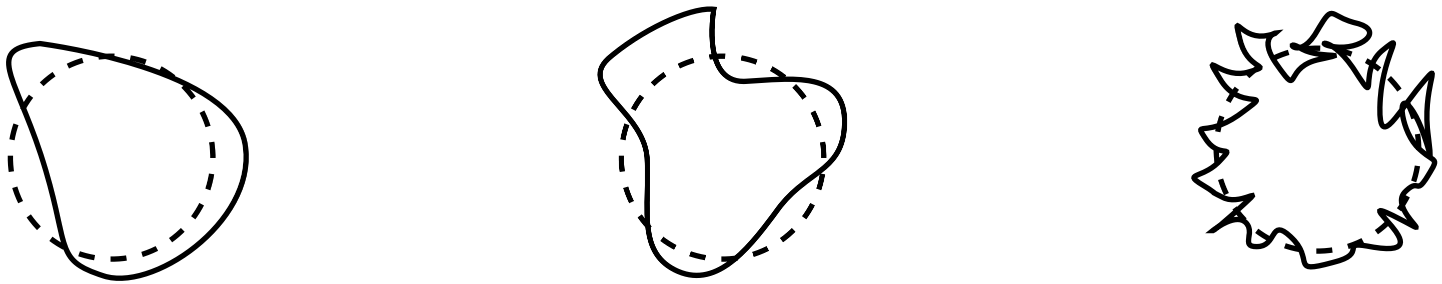
(A) How many fuzzballs have you found ?

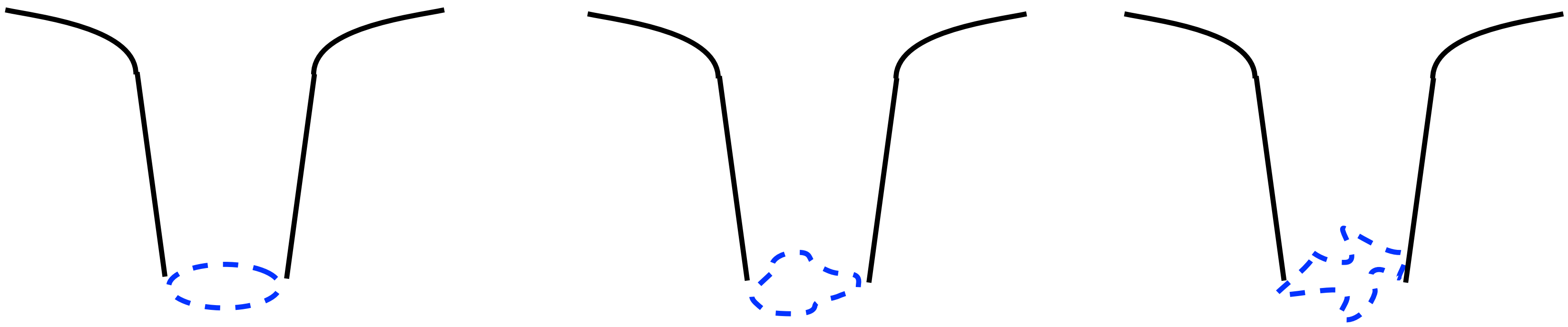


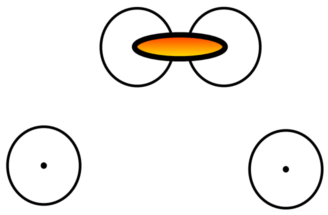
fuzzballs   black hole with traditional horizon

But if even one traditional black hole state exists, then we can choose it and get information loss/remnants ...

(B) How well can you describe generic fuzzballs ?







When KK monopoles are close,
D2 brane pairs will be excited
between them ... and so on ...

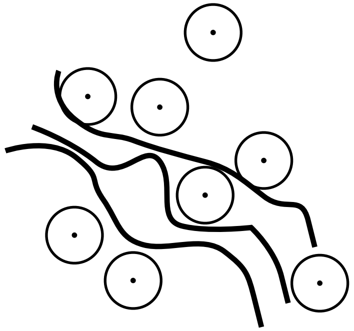
Why does this matter ?

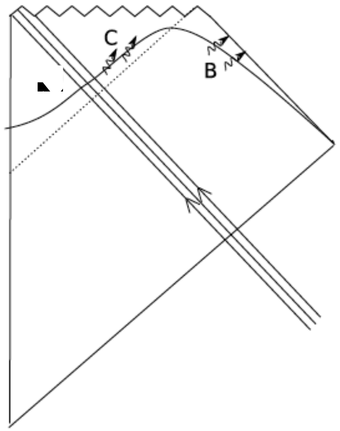
Question: can it be that the more messy generic states become 'close' to the vacuum for all practical purposes?

Then we have not learnt much by making the simple fuzzballs ...

But there is only one practical purpose relevant to the information puzzle: Does the evolution of low energy modes at the horizon differ by order unity from the traditional vacuum ?

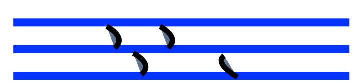
As we take the limit to more generic states, the evolution of these modes does NOT go towards evolution in the vacuum ...



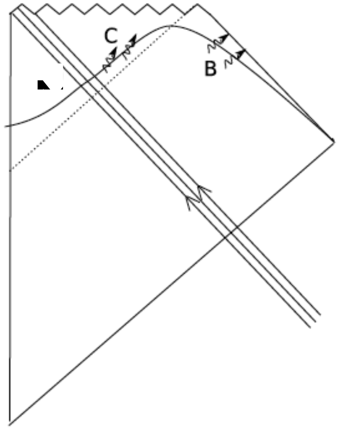


Information paradox
needs structure at
the horizon, but we dont
quite know how to get it

Fuzzballs provide this structure.
But generic fuzzballs are
complicated, so we can
describe physics only in dual CFT



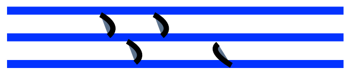
In the dual CFT it is hard
to see locality, etc, so we
dont quite know the
solution to the information
problem



Information paradox needs structure at the horizon

Fuzzballs provide this structure. Hawking evolution in generic fuzzballs shows no sign of being vacuum evolution

Can use picture to understand Complementarity, issues in Cosmology, etc ...



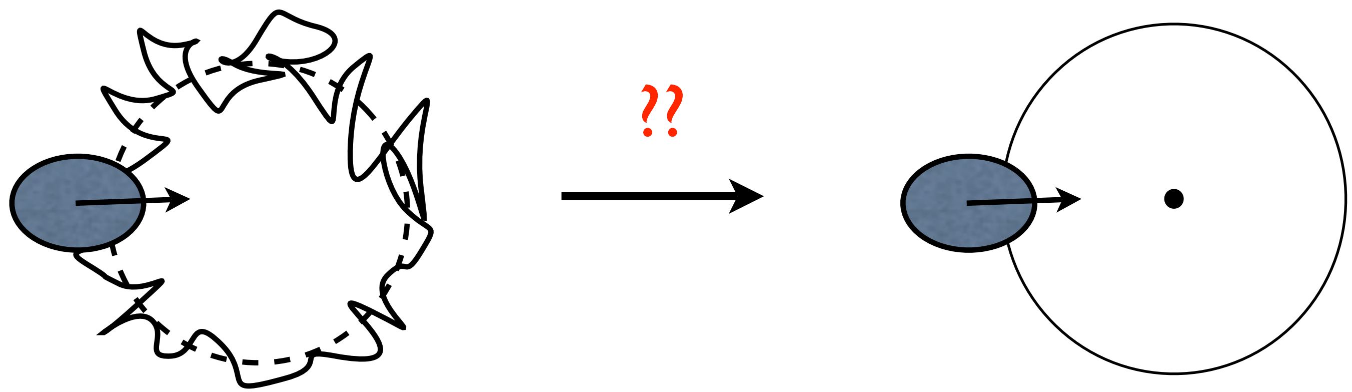
AdS/CFT duality is correct, but is not useful for understanding whether there is structure at the horizon

The infall problem

The infall problem

What happens to an object (E >> kT) that falls into the black hole?

What does an infalling observer ‘feel’ ?



Low energy radiation modes are corrected by order unity, no information loss in process of creation

Is it possible that the dynamics of high energy infalling objects can approximated by the traditional black hole geometry in some way?

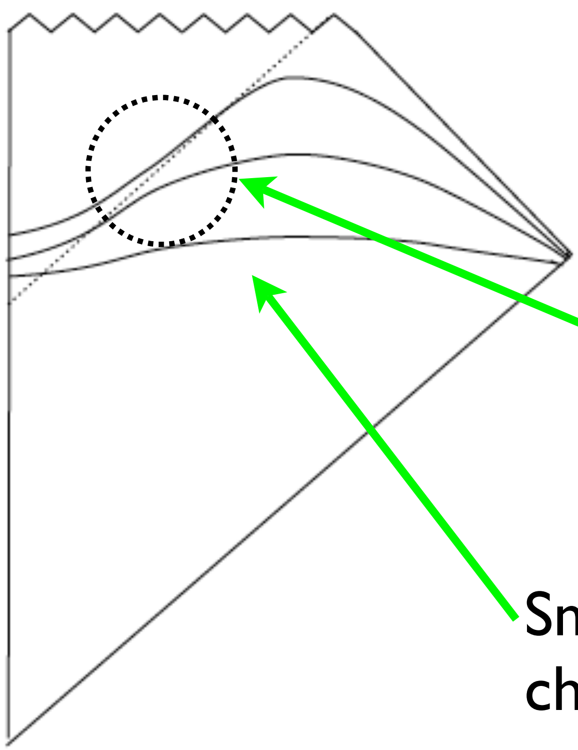
‘t Hooft, Susskind: Complementarity :

Infalling observer get destroyed by Hawking radiation at horizon (so that information never falls into the hole)

BUT

In a dual description he continues to fall inside (so we take into account that the horizon is not a ‘special place’

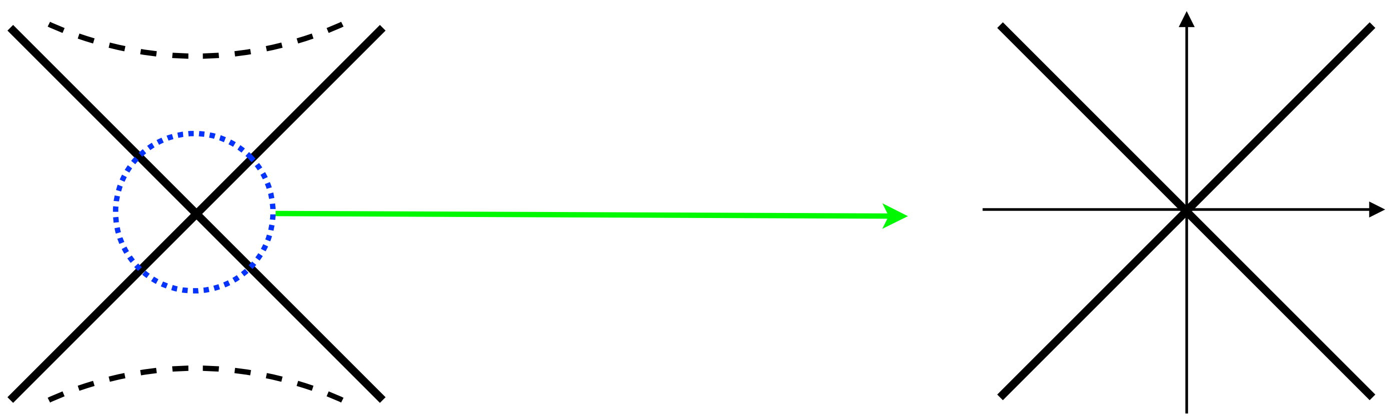
With the traditional picture of the black hole, people could see no way to make complementarity work ...



Infalling observer feels no Hawking radiation

Smooth evolution, no reason to change description at a regular point

Now we know that black hole microstates are fuzzballs. let us see if we can do any better ...

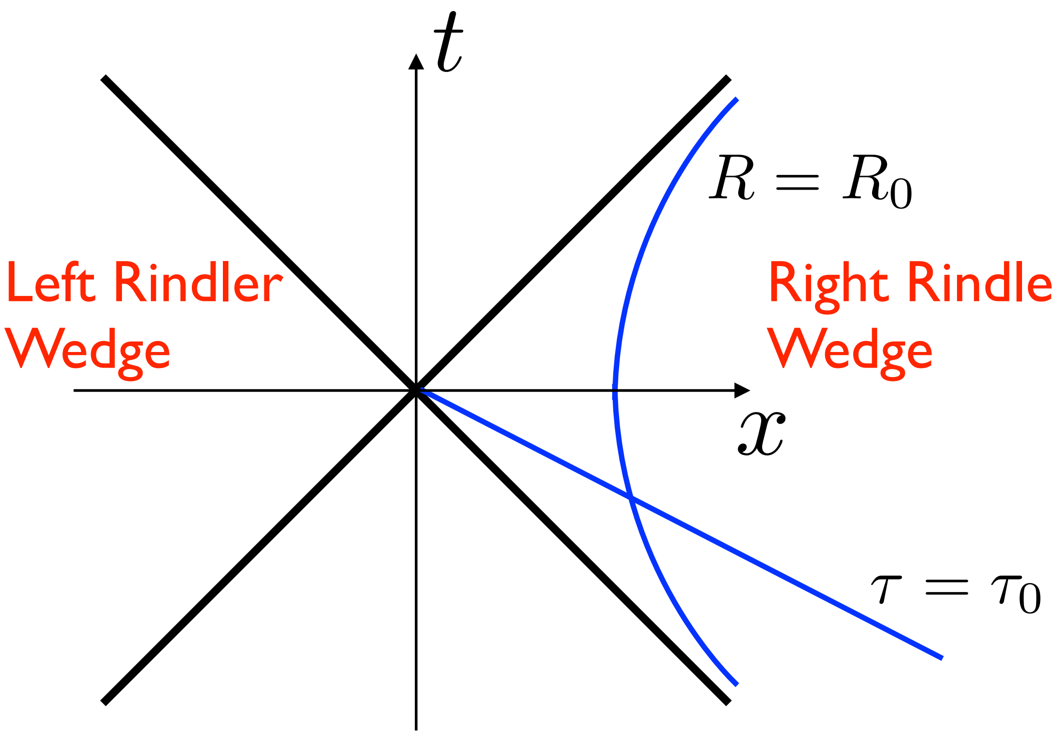


Central part of eternal black hole diagram looks like a piece of Minkowski spacetime, Horizons look like Rindler horizons

So complementarity looks as strange as asking that we get destroyed at a Rindler horizon, and in a dual description we continue past the horizon

Rindler space: Accelerated observers see a thermal bath

Minkowski spacetime



$$t = R \sinh \tau$$

$$x = R \cosh \tau$$

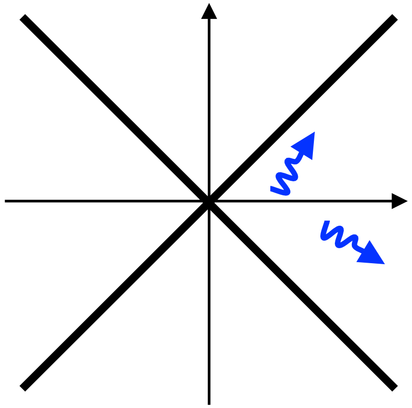
An observer moving along $R = R_0$ sees a temperature

$$T = \frac{1}{2\pi R_0}$$

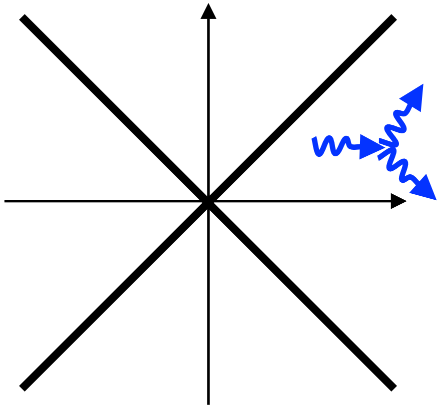
The Minkowski vacuum can be written an an entangled sum of Rindler states

$$|0\rangle_M = \sum_k e^{-\frac{E_k}{2\pi}} \, |E_k\rangle_L \; {}_R\langle E_k|$$

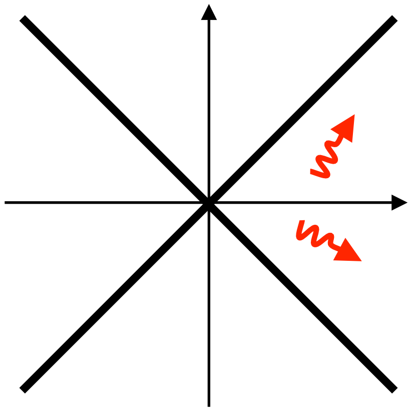
An observation



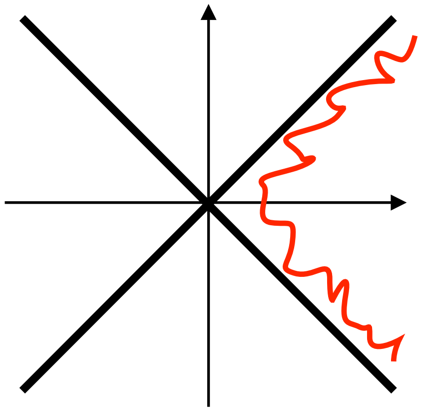
If there is a scalar field ϕ, then the Rindler states will have a bath of scalar quanta



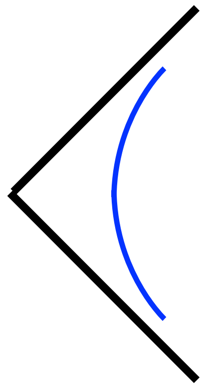
If ϕ has a ϕ^3 interaction, then this bath of scalar quanta will be interacting



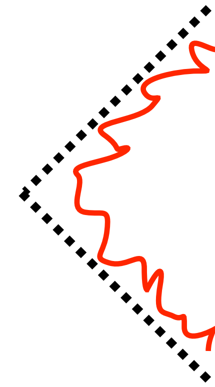
The graviton is a field that is always present, so we will have a bath of (interacting) gravitons



Thus expect fully nonlinear quantum gravity near Rindler horizon



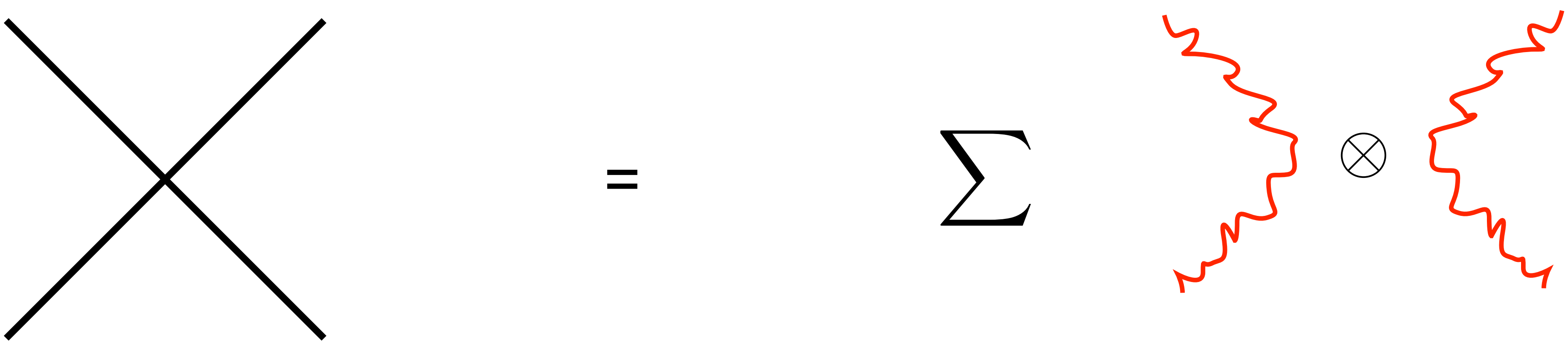
Rindler coordinates 'end' at the boundary of the wedge



Thus it is logical to expect that the gravity solution for Rindler states should also 'end'

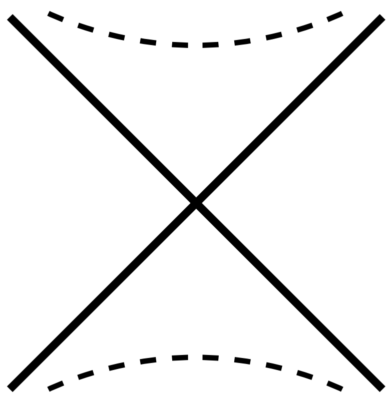
But this is exactly what fuzzball microstates do !

Thus we expect :

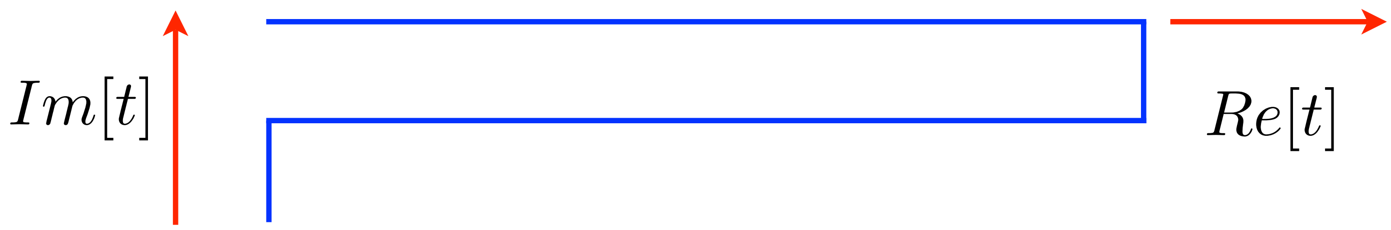


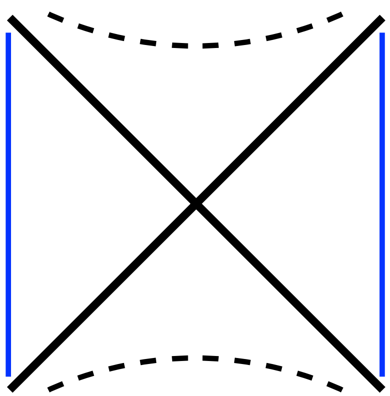
$$|0\rangle_M = \sum_k e^{-\frac{E_k}{2\pi}} \, |E_k\rangle_L \, {}_R\langle E_k|$$

Black Holes :

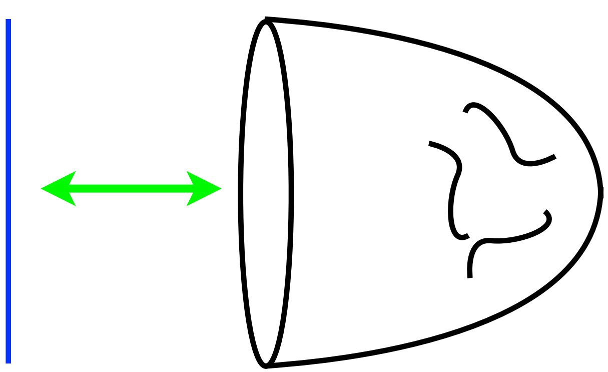


Israel (1976): The two sides of the eternal black hole are the two entangled copies of a thermal system in thermo-field-dynamics

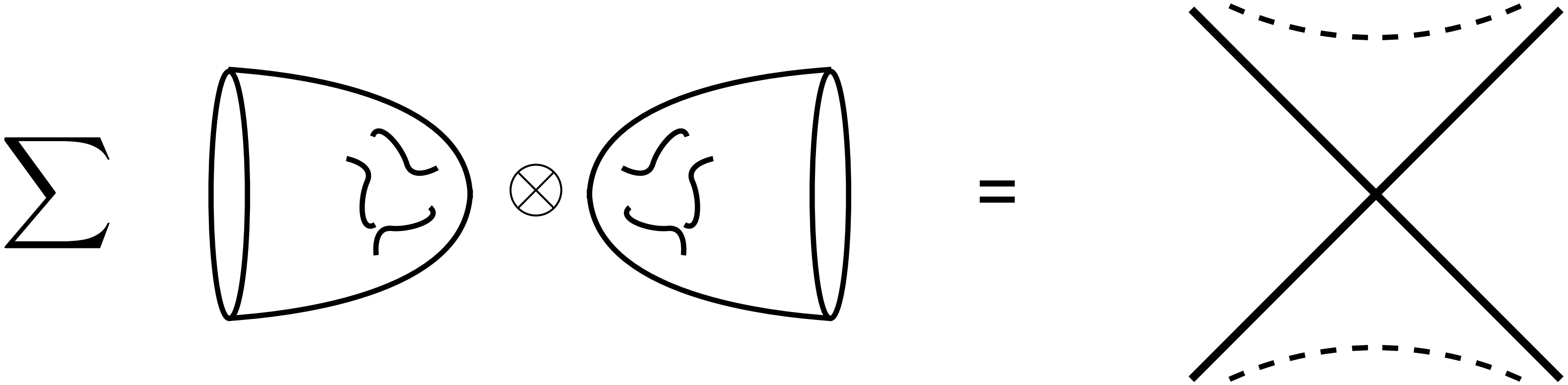




Maldacena (2001): This implies that the dual to the eternal black hole is two entangled copies of a CFT

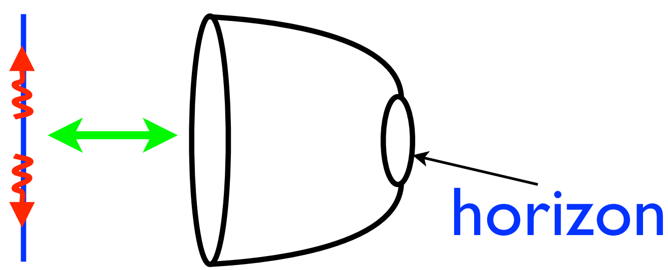


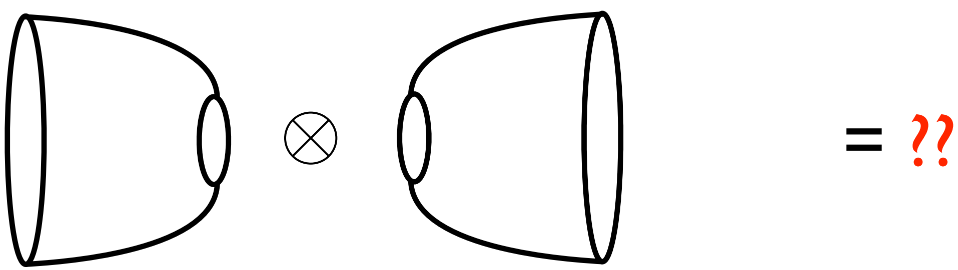
Van Raamsdonk (2009): CFT states are dual to gravity solutions ... so we should be able to write an entangled sum of CFT states as an entangled sum of gravity states ...



(Van Raamsdonk 2009)

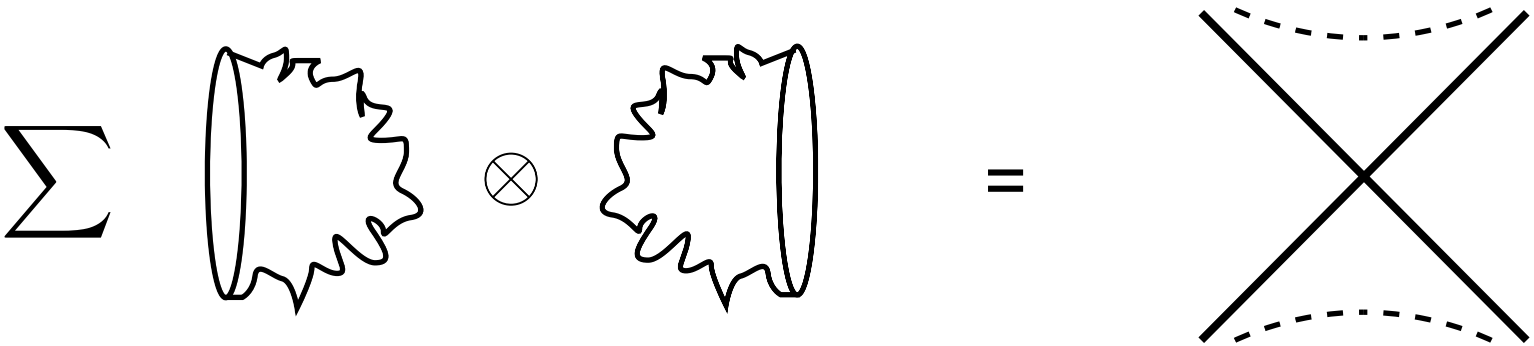
But what do we do with CFT states which are dual to black holes with a horizon ?





But the lesson from fuzzballs is that there are no microstates with horizons !! Thus there is only one 'class' of microstates, they just vary in their complexity

Thus we can expect that summing over fuzzball microstates will generate the eternal black hole spacetime



(SDM + Plumberg 2011)

The fuzzball microstates do not have horizons, but the eternal black hole spacetime does ...

Is it reasonable to expect that sums over (disconnected) gravitational solutions can be a different (connected) gravitational solution ?

Something like this happens in 2-d Euclidean CFT ...

(a) Low energy dynamics $(E \sim kT)$

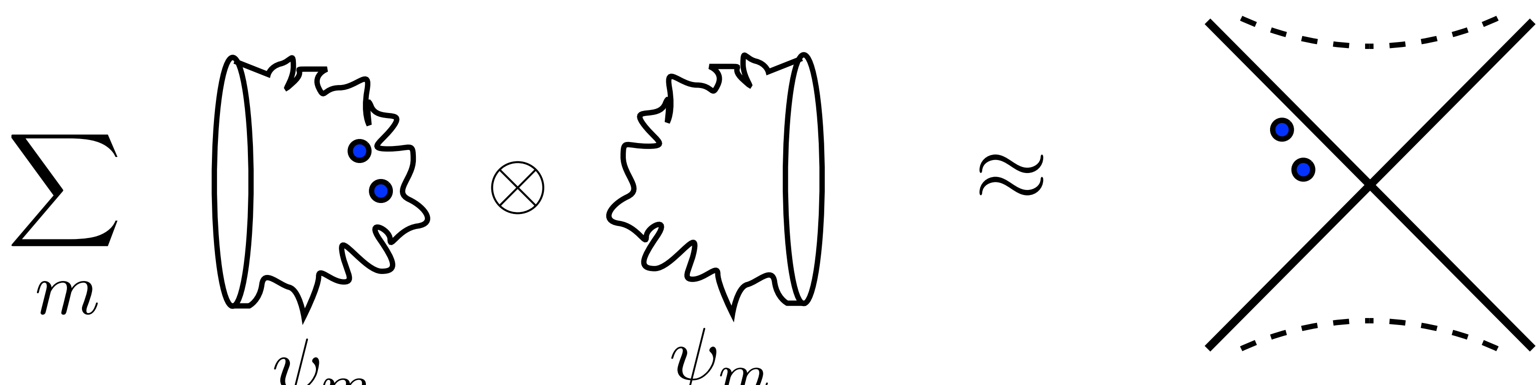
No horizon, radiation from ergoregions, so radiation like that from any warm body

no information loss since radiation depends on choice of microstate ψ_k

(b) Correlators in high energy infalling frame $(E \gg kT)$

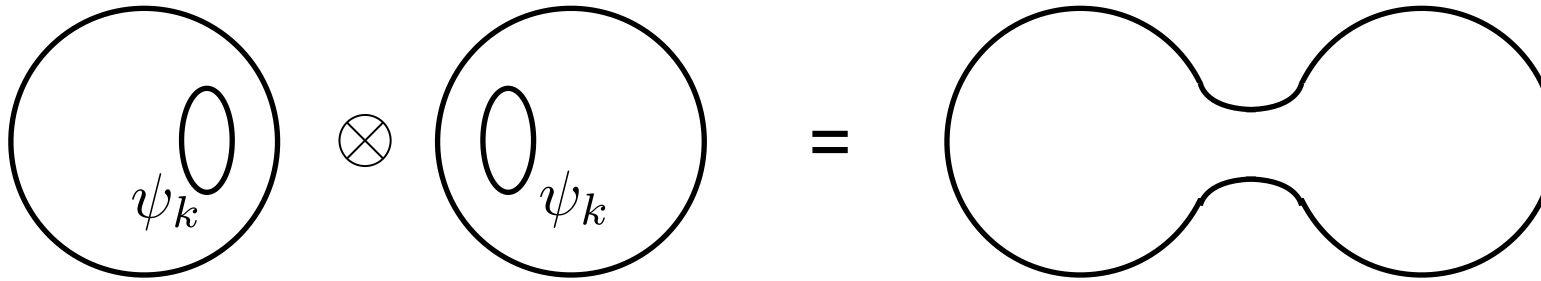
$$\langle \psi_k | \hat{O}_1 \hat{O}_2 | \psi_k \rangle \approx \sum_m e^{-\beta E_m} \langle \psi_m | \hat{O}_1 \hat{O}_2 | \psi_m \rangle$$

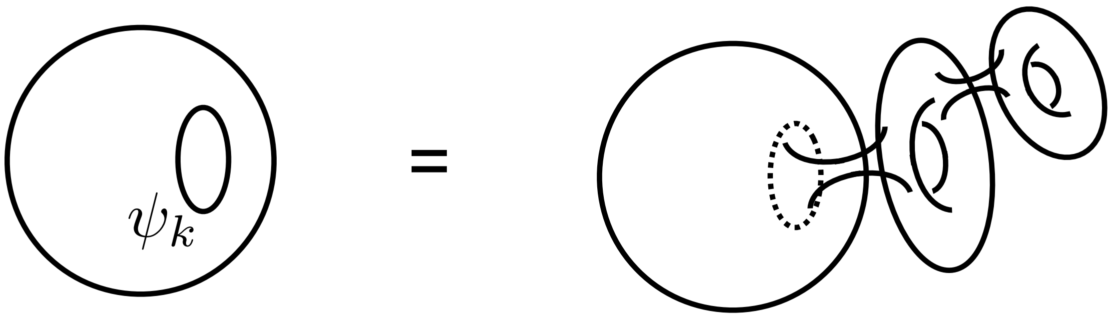
for generic states ψ_k



‘Sewing’ process in CFT

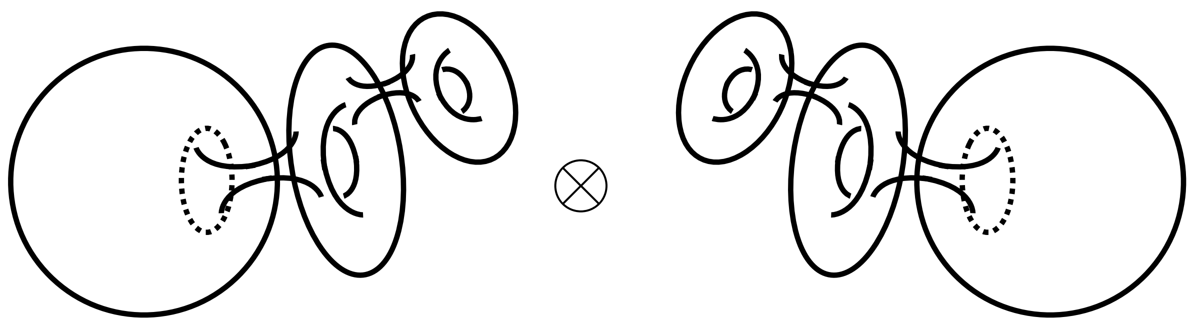
$$\sum_k e^{-\tau h_k - \bar{\tau}\bar{h}_k}$$



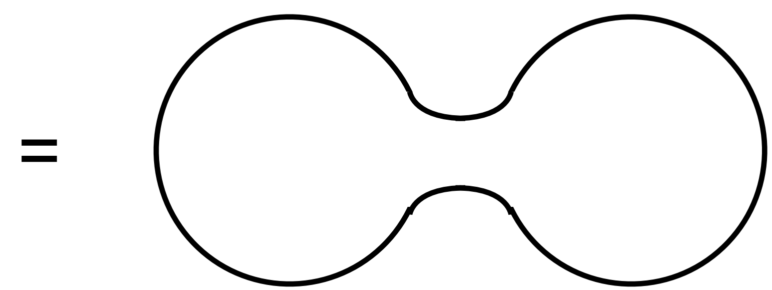


Structure of state if continued inside

$$\sum_k e^{-\tau h_k - \bar{\tau}\bar{h}_k}$$



Sum over a set of messy entangled geometries equals a smooth geometry



Minkowski vacuum

$$|0\rangle_M = C\sum_i e^{-\frac{E_i}{2}}|E_i\rangle_L|E_i\rangle_R, \qquad C = \left(\sum_i e^{-E_i}\right)^{-\frac{1}{2}}$$

Expectation value of an operator in the right wedge

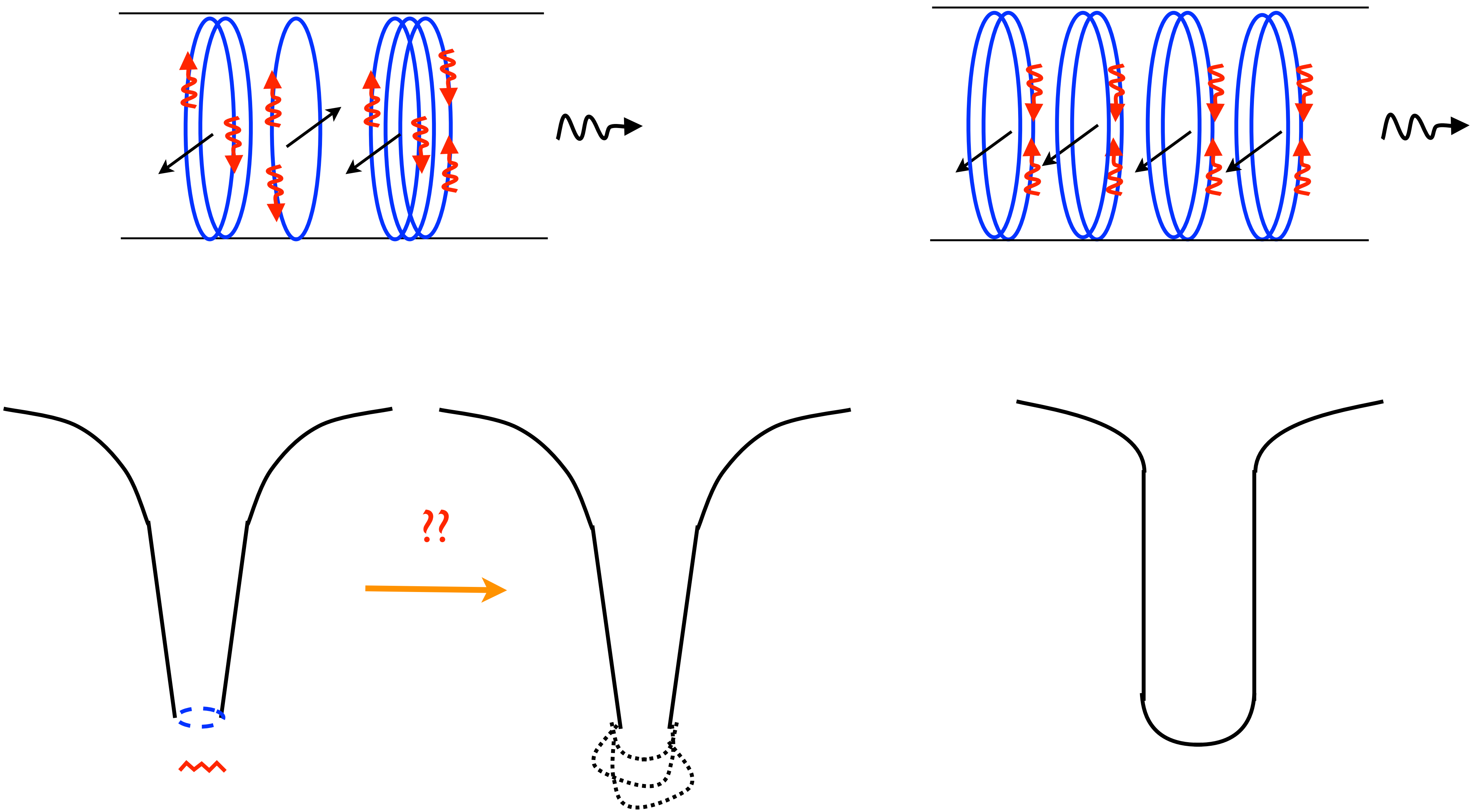
$$\begin{aligned} {}_M\langle 0|\hat{O}_R|0\rangle_M &= C^2\sum_{i,j} e^{-\frac{E_i}{2}}e^{-\frac{E_j}{2}}\,{}_L\langle E_i|E_j\rangle_L\,{}_R\langle E_i|\hat{O}_R|E_j\rangle_R \\ &= C^2\sum_i e^{-E_i}\,{}_R\langle E_i|\hat{O}_R|E_i\rangle_R \end{aligned}$$

Thus for suitable (high impact) operators, the expectation value in a single (generic) fuzzball equals the expectation value in the naive extended geometry with horizons

$${}_R\langle E_k|\hat{O}_R|E_k\rangle_R \approx \frac{1}{\sum_l e^{-E_l}}\sum_i e^{-E_i}\,{}_R\langle E_i|\hat{O}_R|E_i\rangle_R = {}_M\langle 0|\hat{O}_R|0\rangle_M$$

Nonextremal states and Hawking radiation

Nonextremal states: D1D5 + nonextremal energy



$$
\begin{aligned}
\mathrm{d}s^2 &= -\frac{f}{\sqrt{\tilde{H}_1\tilde{H}_5}}(\mathrm{d}t^2-\mathrm{d}y^2)+\frac{M}{\sqrt{\tilde{H}_1\tilde{H}_5}}(s_p\mathrm{d}y-c_p\mathrm{d}t)^2 \\
&+\sqrt{\tilde{H}_1\tilde{H}_5}\left(\frac{r^2\mathrm{d}r^2}{(r^2+a_1^2)(r^2+a_2^2)-Mr^2}+\mathrm{d}\theta^2\right) \\
&+\left(\sqrt{\tilde{H}_1\tilde{H}_5}-(a_2^2-a_1^2)\frac{(\tilde{H}_1+\tilde{H}_5-f)\cos^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}\right)\cos^2\theta\mathrm{d}\psi^2 \\
&+\left(\sqrt{\tilde{H}_1\tilde{H}_5}+(a_2^2-a_1^2)\frac{(\tilde{H}_1+\tilde{H}_5-f)\sin^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}\right)\sin^2\theta\mathrm{d}\phi^2 \\
&+\frac{M}{\sqrt{\tilde{H}_1\tilde{H}_5}}(a_1\cos^2\theta\mathrm{d}\psi+a_2\sin^2\theta\mathrm{d}\phi)^2 \\
&+\frac{2M\cos^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}[(a_1c_1c_5c_p-a_2s_1s_5s_p)\mathrm{d}t+(a_2s_1s_5c_p-a_1c_1c_5s_p)\mathrm{d}y]\mathrm{d}\psi \\
&+\frac{2M\sin^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}[(a_2c_1c_5c_p-a_1s_1s_5s_p)\mathrm{d}t+(a_1s_1s_5c_p-a_2c_1c_5s_p)\mathrm{d}y]\mathrm{d}\phi \\
&+\sqrt{\frac{\tilde{H}_1}{\tilde{H}_5}}\sum_{i=1}^{4}\mathrm{d}z_i^2
\end{aligned}
$$

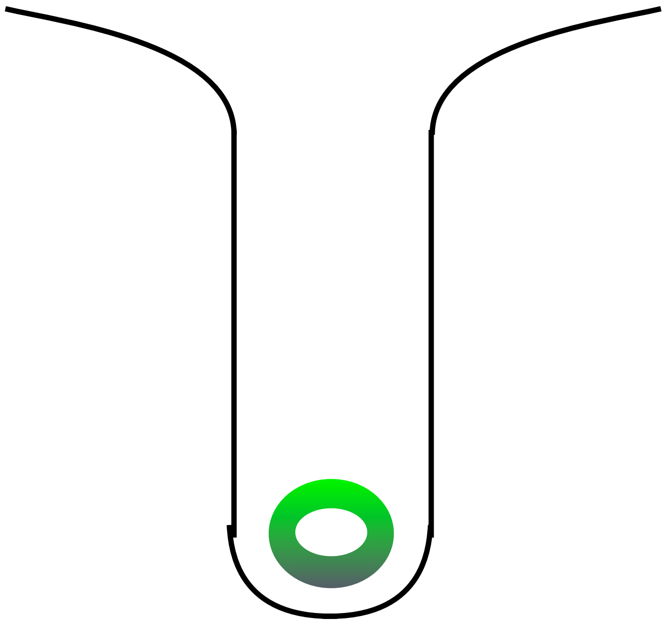
$$
Q_1 = \frac{g\alpha'^3}{V}n_1 \qquad Q_5 = g\alpha' n_5 \qquad Q_p = \frac{g^2\alpha'^4}{VR^2}n_p
$$

(Jejalla, Madden, Ross Titchener '05)

$$
\tilde{H}_i = f + M\sinh^2\delta_i, \quad f = r^2 + a_1^2\sin^2\theta + a_2^2\cos^2\theta
$$

$$
Q_1 = M\sinh\delta_1\cosh\delta_1, \quad Q_5 = M\sinh\delta_5\cosh\delta_5, \quad Q_p = M\sinh\delta_p\cosh\delta_p
$$

Structure of the geometry



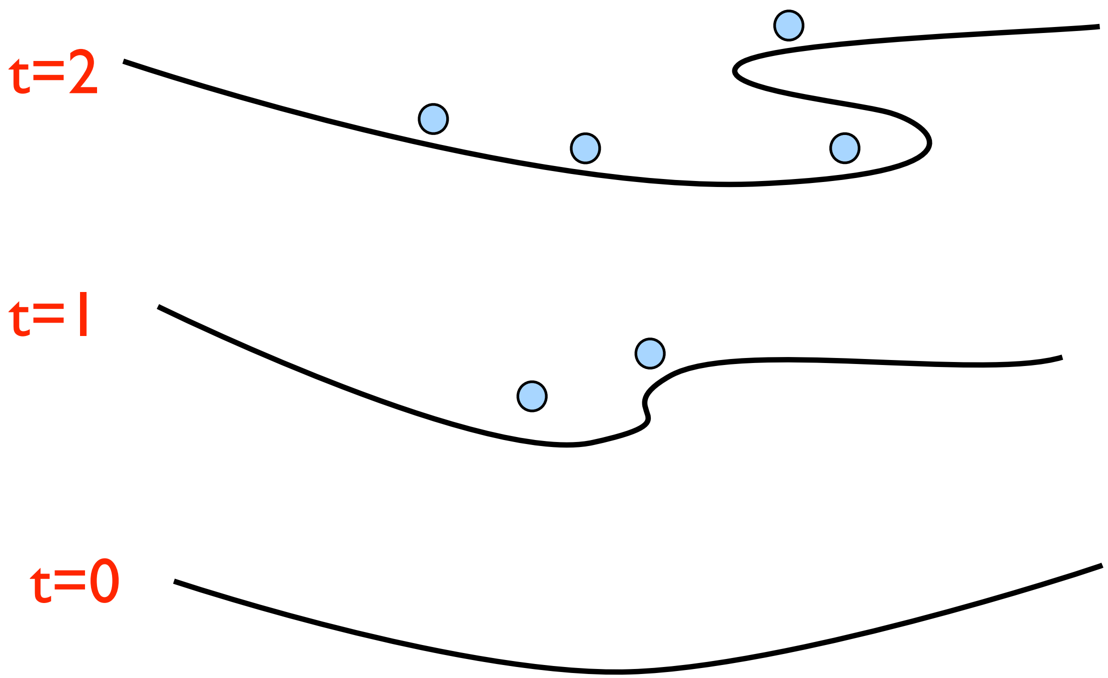
There is no horizon

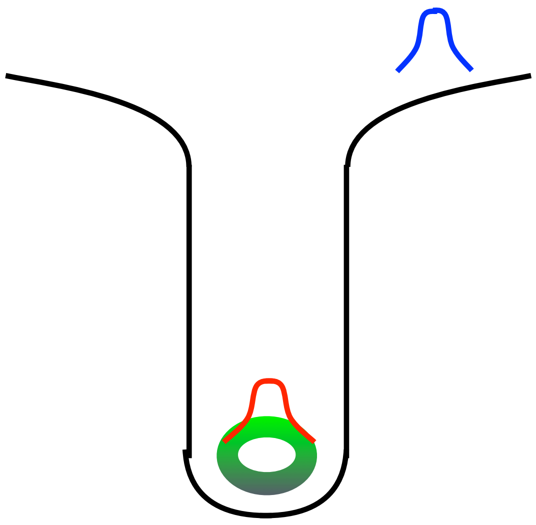
But there is an ERGOREGION

The geometry does not depend on t, so $\partial/\partial t$ is a Killing vector

But this Killing vector is not timelike everywhere:
It is SPACELIKE in the ergoregion

So even though the metric is independent of t, any foliation with spacelike hypersurfaces will be TIME-DEPENDENT



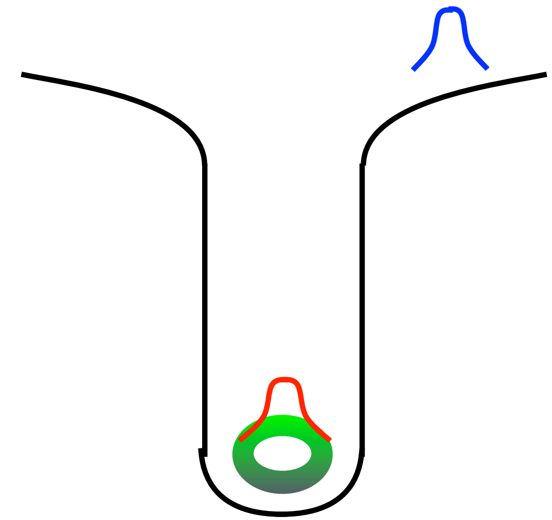


Since the geometry of the slice keeps changing, the vacuum of the initial slice is not the vacuum on later slices, and we have particle pair creation near the ergoregion

One member of the pair falls into the eregoregion, and tends to 'cancel' the frame dragging causing the ergoregion

The other member escapes to infinity as radiation

Radiation from the ergoregion



Scalar field $\Box\Psi = 0$

$$\Psi = \psi(x)e^{-i\omega t}$$

$$\omega \simeq \omega_R = \frac{1}{R}\left(-l - m_\psi m + m_\phi n - |-\lambda - m_\psi n + m_\phi m| - 2(N+1)\right)$$

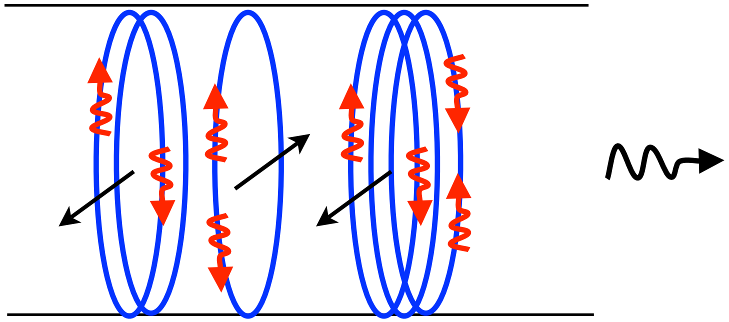
$$\omega_I = \frac{1}{R}\left(\frac{2\pi}{[l!]^2}\left[(\omega^2 - \frac{\lambda^2}{R^2})\frac{Q_1 Q_5}{4R^2}\right]^{l+1} {}^{l+1+N}C_{l+1} \;\; {}^{l+1+N+|\zeta|}C_{l+1}\right)$$

$$\zeta \equiv -\lambda - m_\psi n + m_\phi m$$

$$N \geq 0$$

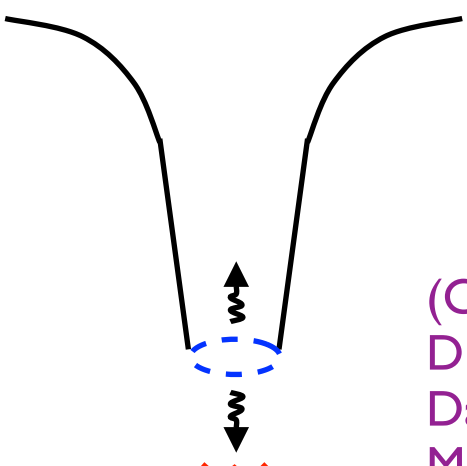
(Cardoso, Dias, Jordan, Hovdebo, Myers, '06)

Is this 'Hawking radiation' for this particular microstate ?

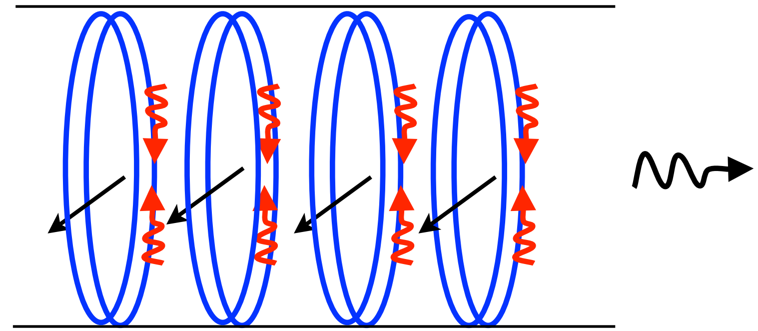


$$\Gamma = V \rho_L \rho_R$$

Setting ρ_L, ρ_R
as bose and fermi thermal
distributions gives the
Hawking radiation spectrum
from the black hole



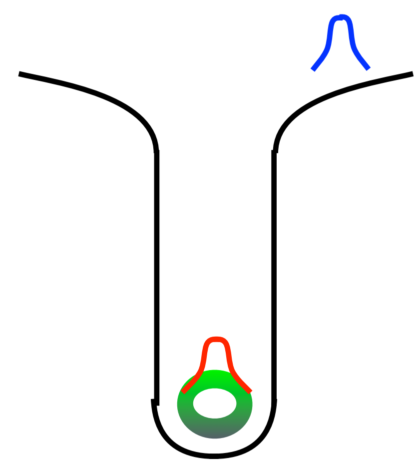
(Callan-Maldacena 96,
Dhar-Mandal-Wadia 96,
Das-Mathur 96,
Maldacena-Strominger 96)



$$\Gamma_{microstate} = V \bar{\rho}_L \bar{\rho}_R$$

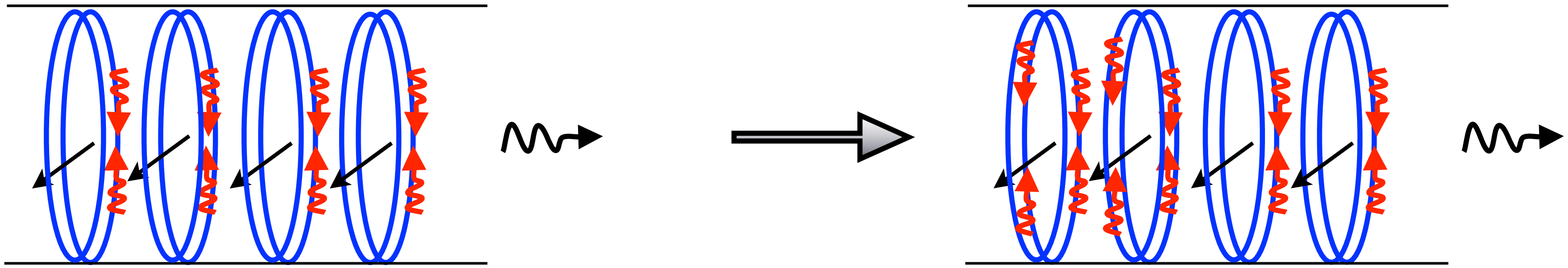
We find

$$\boxed{\Gamma_{microstate} = \Gamma_{gravity}}$$



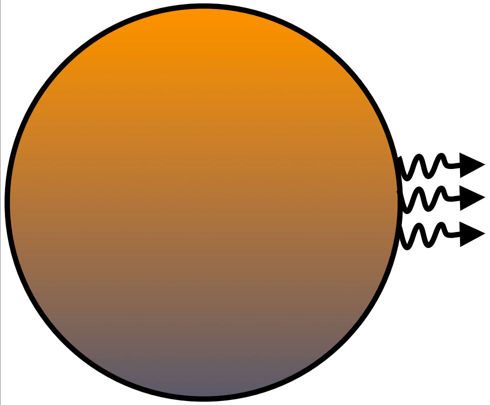
Thus we can explicitly see the interior geometry for this microstate and unitary Hawking radiation carrying out information from the interior of the microstate

We can similarly find ergoregions for less symmetrical microstates :

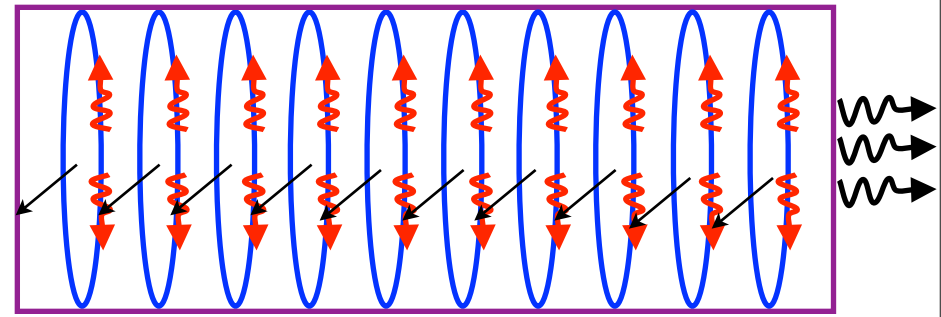


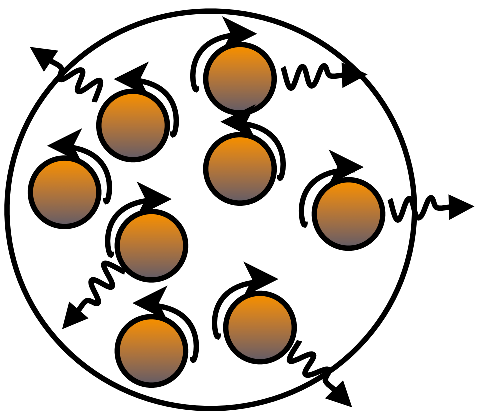
(Chowdhury+SDM 07, 08)

Special and generic states in gravity: conjecture

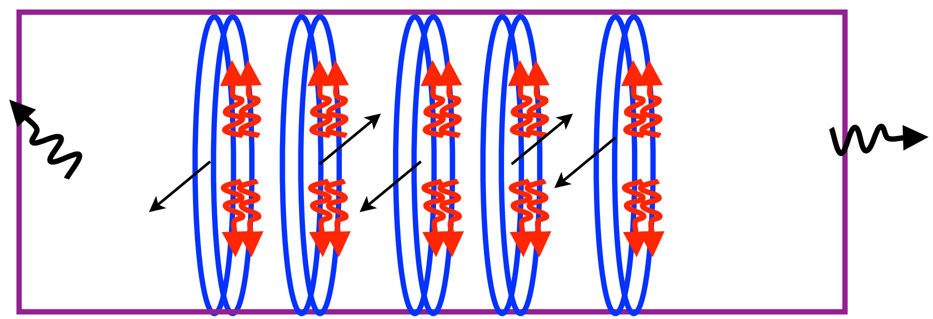


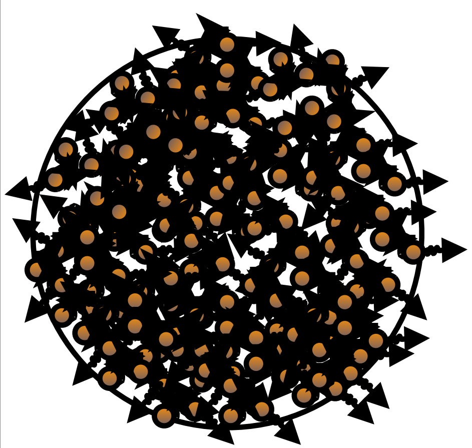
Classical geometry, axial symmetry, standard ergoregion, enhanced emission



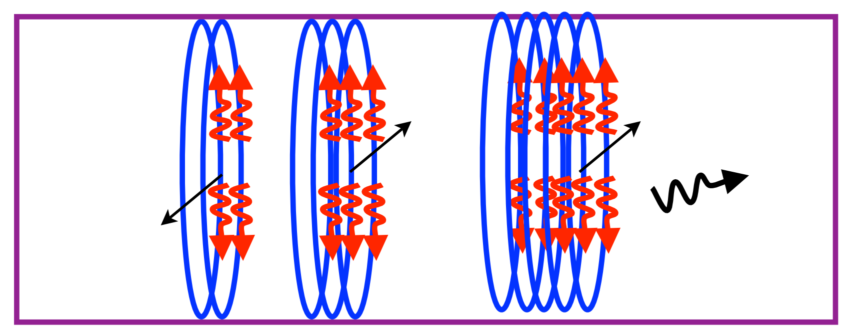


'Star cluster'. Different stars have ergoregions with different orientations, so there is no axial symmetry in the emission



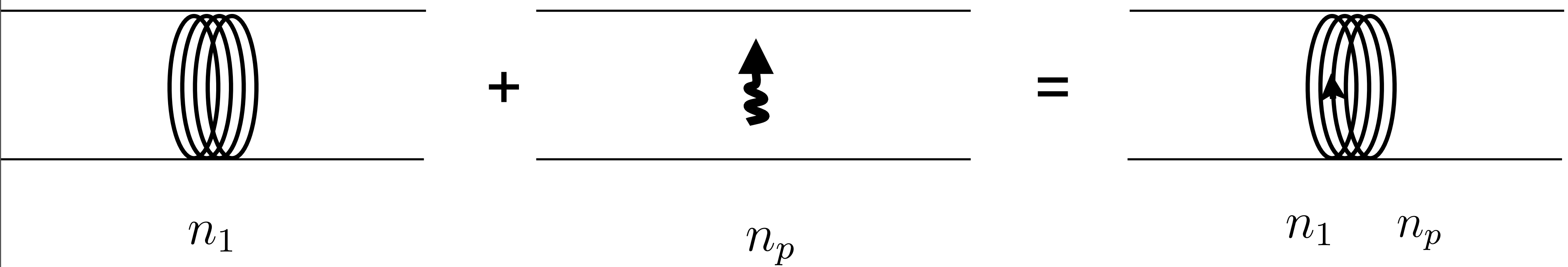


A generic state is very quantum, with very 'shallow' ergoregions, and quanta leak out slowly as Hawking radiation



Constructing 2-charge extremal fuzzballs

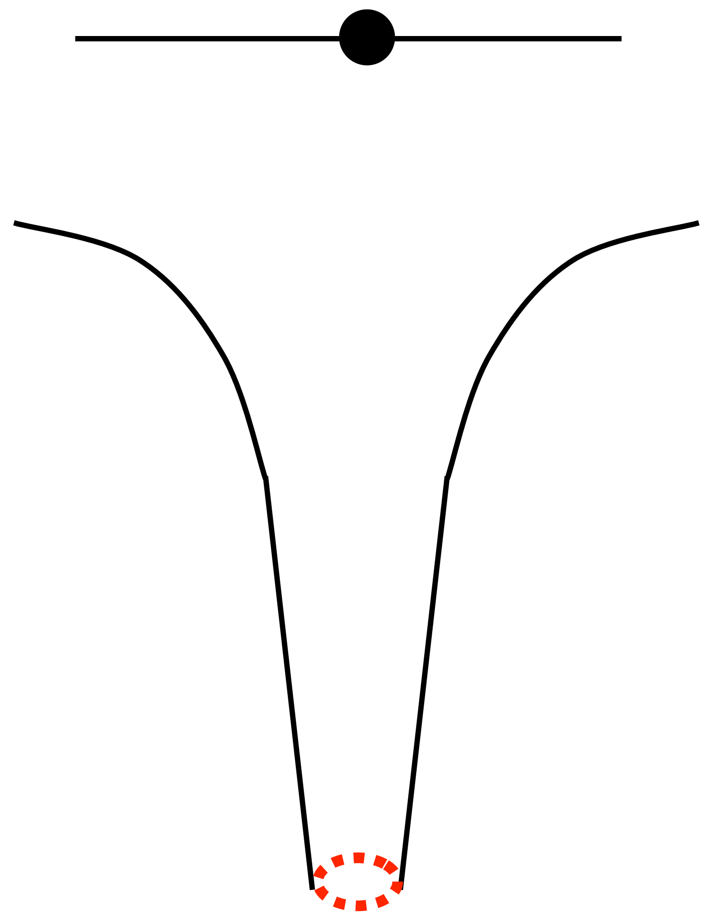
Making black holes in string theory



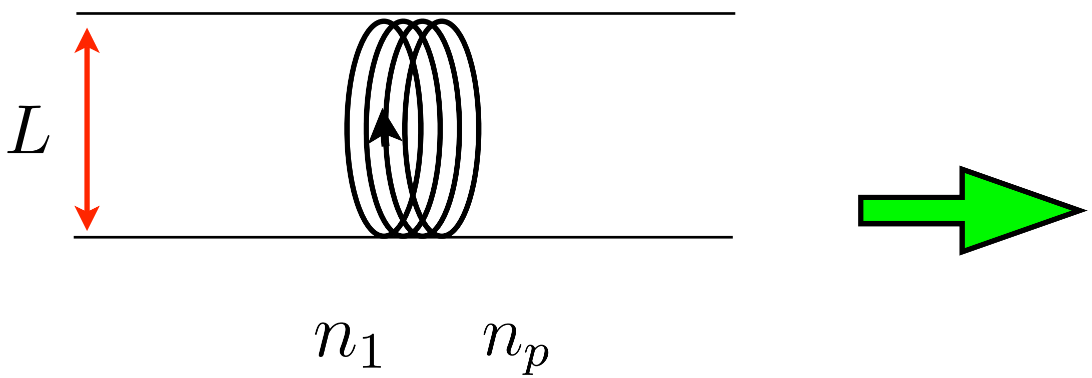
For $K3 \times S^1$ compactification, geometry gives a Bekenstein - Wald entropy

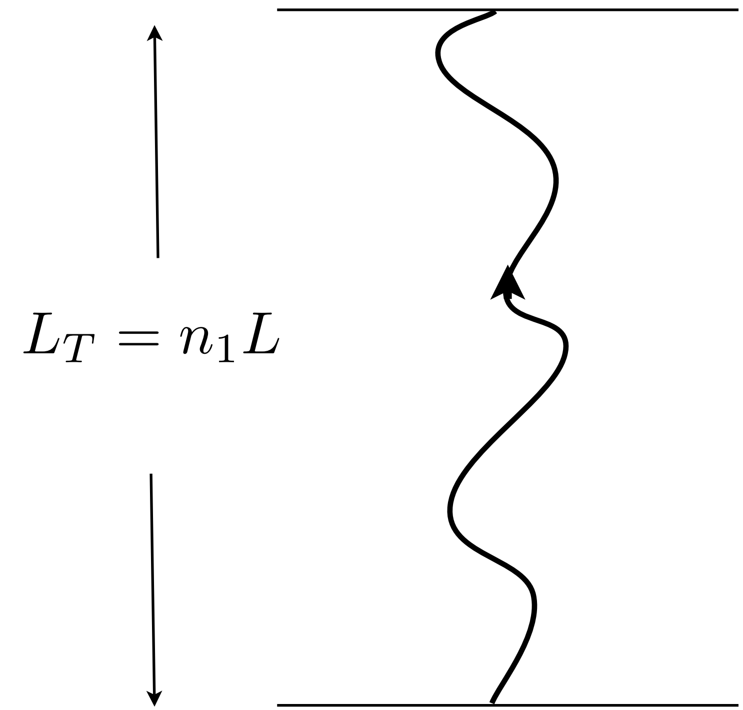
$$S_{bek} = \frac{A}{2G} = 4\pi\sqrt{n_1 n_p}$$

(Dabholkar '04)



Can we get this entropy by a microscopic count of states ?





Open up string to its 'covering space'
We have transverse vibrations carrying momentum up the string

$$S_{micro} = 4\pi\sqrt{n_1 n_p}$$

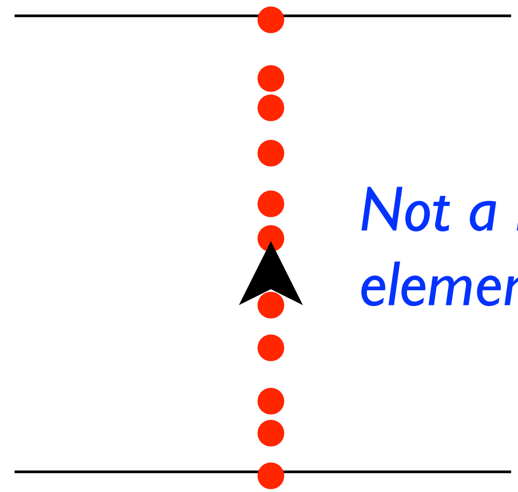
$$S_{bek} = \frac{A}{2G} = 4\pi\sqrt{n_1 n_p} = S_{micro}$$

(Susskind '93, Sen '94)

A key point

The elementary string (NS1) does not have any LONGITUDINAL vibration modes

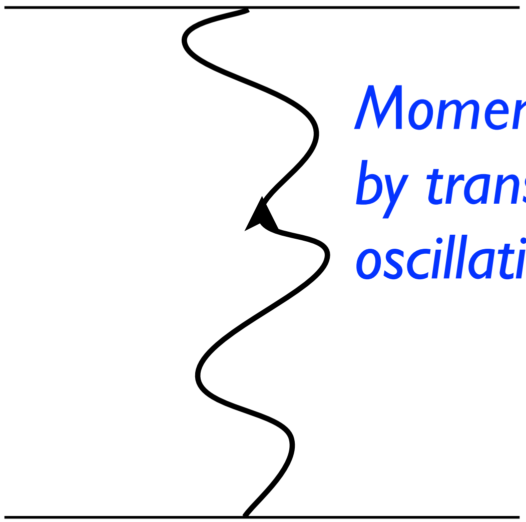
This is because it is not made up of 'more elementary particles'



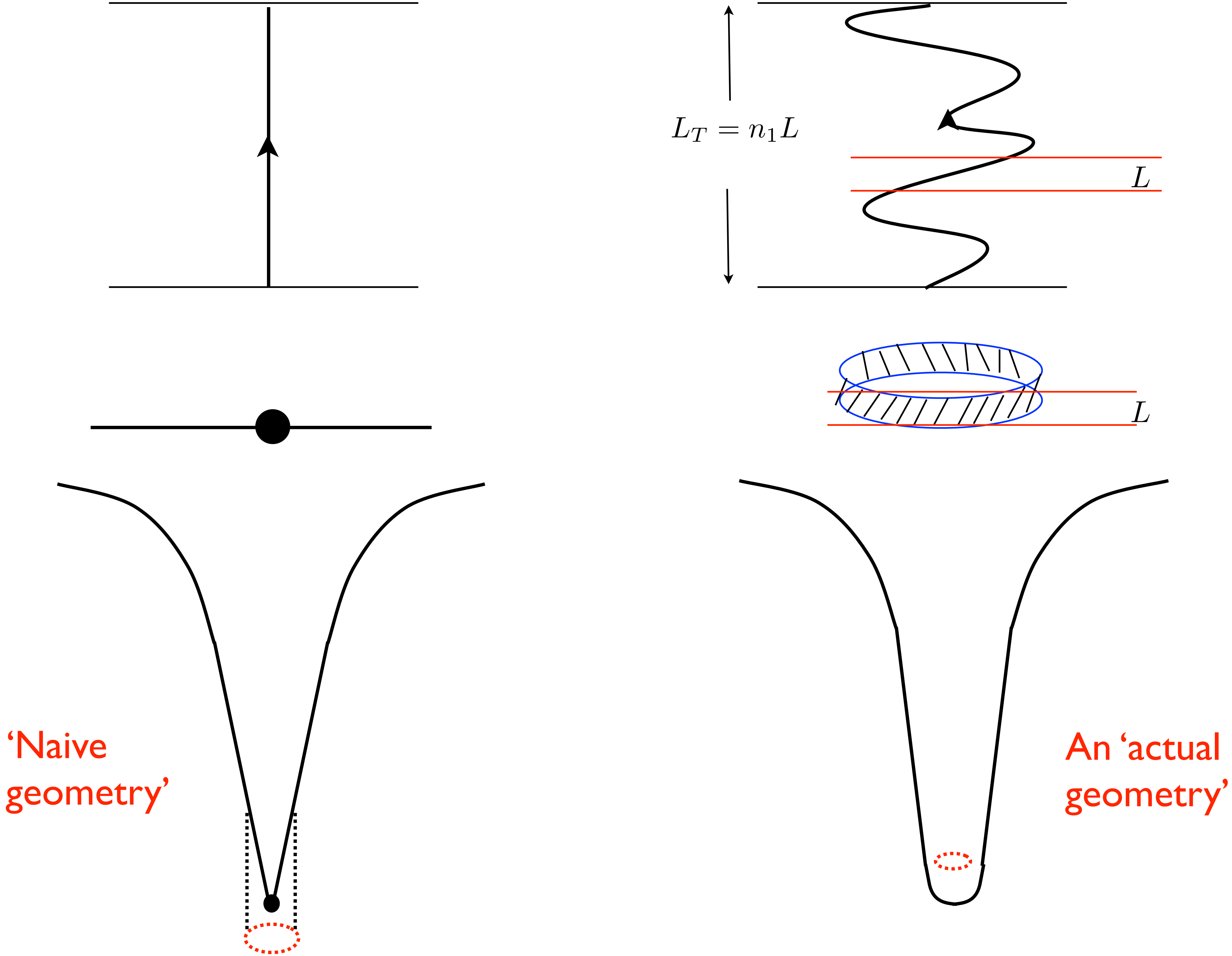
Not a mode for the elementary string

Thus only transverse oscillations are permitted

This causes the string to spread over a nonzero transverse area



Momentum is carried by transverse oscillations


$L_T = n_1 L$
L
L
'Naive geometry'
An 'actual geometry'

Making the geometry

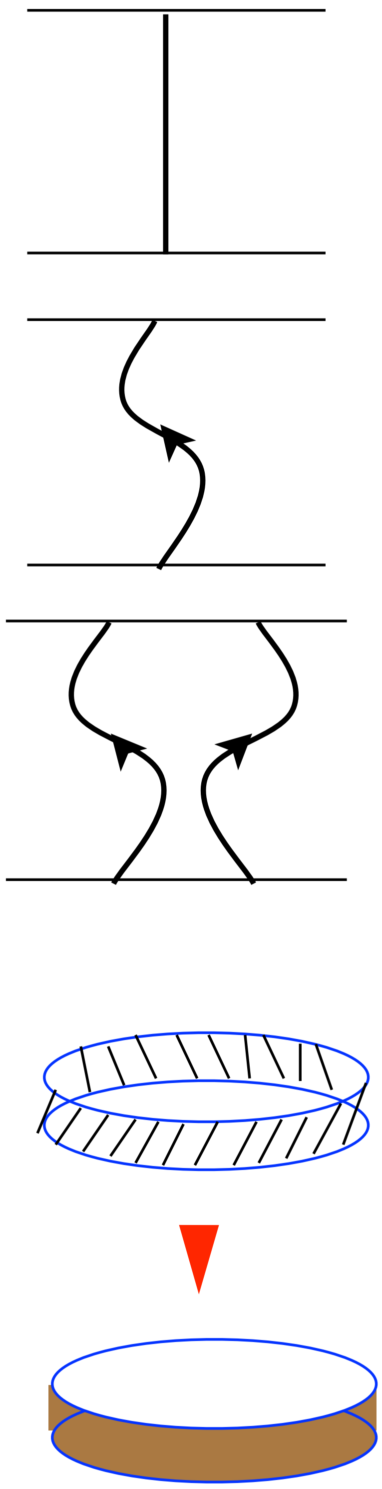
We know the metric of one straight strand of string

We know the metric of a string carrying a wave -- 'Vachaspati transform'

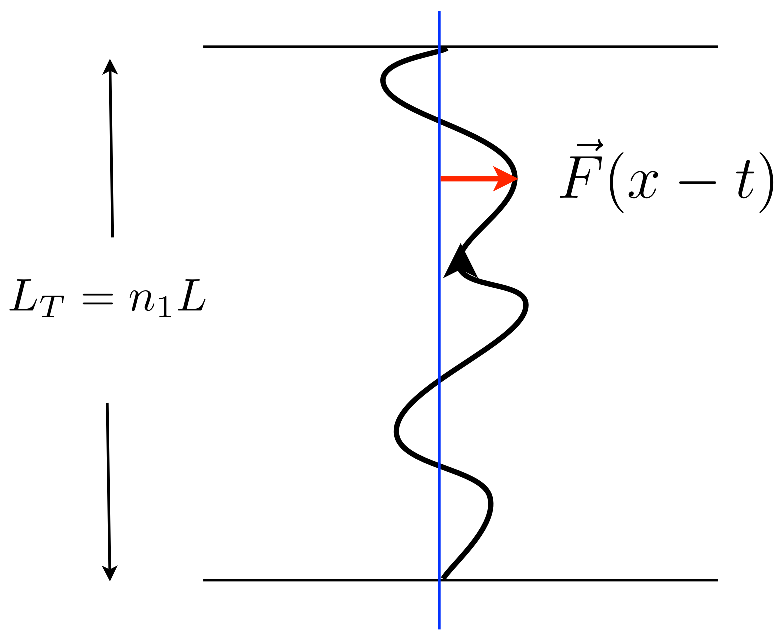
We get the metric for many strands by superposing harmonic functions from each strand

(Dabholkar, Gauntlett, Harvey, Waldram '95, Callan, Maldacena, Peet '95)

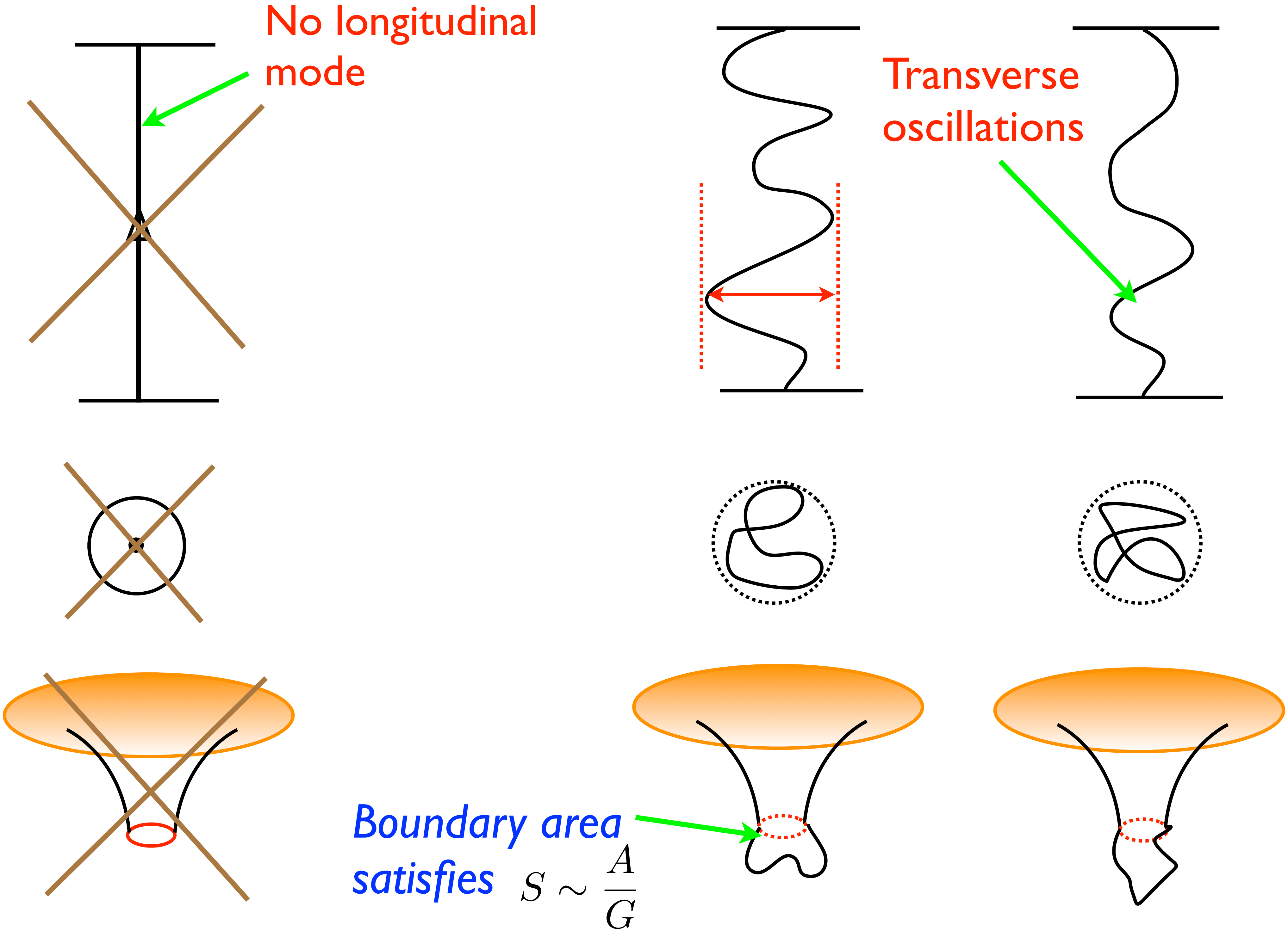
In our present case, we have a large number of strands, so we 'smear over them to make a continuous 'strip' (Lunin+SDM '01)



$$
\begin{aligned}
ds^2_{string} &= H[-dudv + Kdv^2 + 2A_i dx_i dv] + \sum_{i=1}^{4} dx_i dx_i + \sum_{a=1}^{4} dz_a dz_a \\
B_{uv} &= \frac{1}{2}[H-1], \qquad B_{vi} = HA_i \\
e^{2\phi} &= H
\end{aligned}
$$



$$
\begin{aligned}
H^{-1} &= 1 + \frac{Q_1}{L_T}\int_0^{L_T} \frac{dv}{|\vec{x}-\vec{F}(v)|^2} \\
K &= \frac{Q_1}{L_T}\int_0^{L_T} \frac{dv(\dot{F}(v))^2}{|\vec{x}-\vec{F}(v)|^2} \\
A_i &= -\frac{Q_1}{L_T}\int_0^{L_T} \frac{dv\dot{F}_i(v)}{|\vec{x}-\vec{F}(v)|^2}
\end{aligned}
$$


No longitudinal mode
Transverse oscillations
Boundary area satisfies $S \sim \frac{A}{G}$

We can map the NS1 - P bound state to a D5 - D1 bound state

$$NS1\,P\,(IIB) \stackrel{S}{\to} D1\,P\,(IIB)$$

$$\stackrel{T_{T^4_{1234}}}{\to} D5\,P\,(IIB)$$

$$\stackrel{S}{\to} NS5\,P\,(IIB)$$

$$\stackrel{T_{S^1}}{\to} NS5\,NS1\,(IIA)$$

$$\stackrel{T_{T^4_1}}{\to} NS5\,NS1\,(IIB)$$

$$\stackrel{S}{\to} D5\,D1\,(IIB)$$

Geometry for D1-D5

$$ds^2 = \sqrt{\frac{H}{1+K}}[-(dt - A_i dx^i)^2 + (dy + B_i dx^i)^2] + \sqrt{\frac{1+K}{H}} dx_i dx_i + \sqrt{H(1+K)} dz_a dz_a$$

$$H^{-1} = 1 + \frac{Q}{L_T} \int_0^{L_T} \frac{dv}{|\vec{x} - \vec{F}(v)|^2}$$

$$K = \frac{Q}{L_T} \int_0^{L_T} \frac{dv (\dot{F}(v))^2}{|\vec{x} - \vec{F}(v)|^2}$$

$$A_i = -\frac{Q}{L_T} \int_0^{L_T} \frac{dv \ \dot{F}_i(v)}{|\vec{x} - \vec{F}(v)|^2}$$

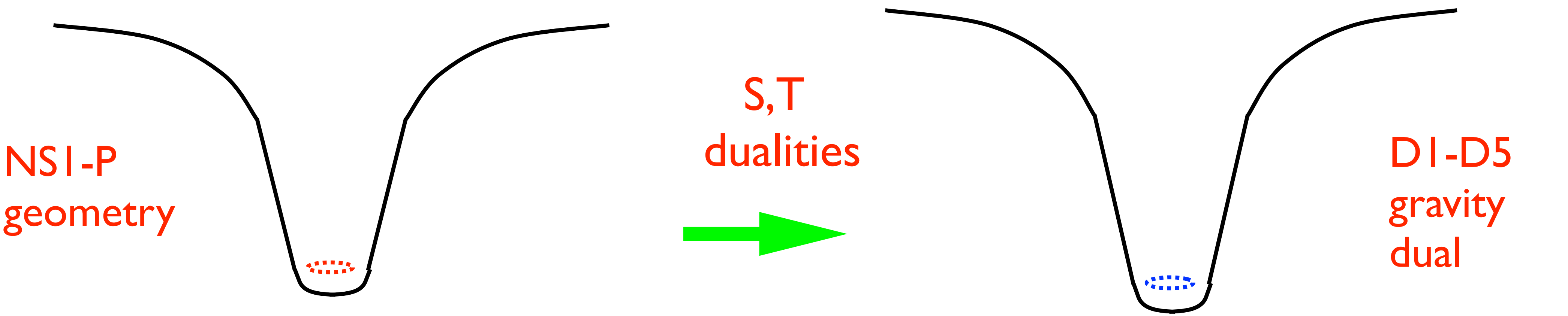
$$dB = - *_4 dA$$

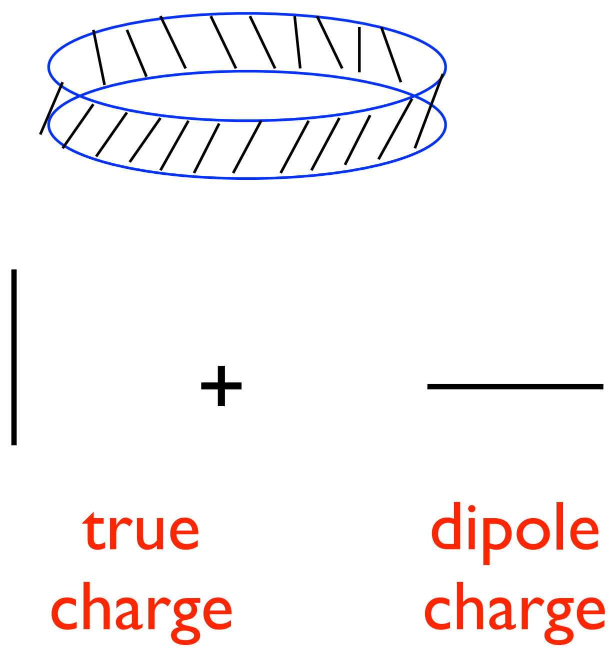
(Lunin+SDM '01,

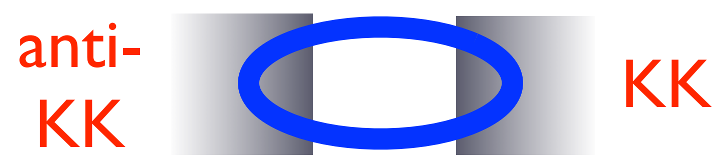
Lunin+Maldacena+Maoz 02

Taylor 05, Skenderis+Taylor 06)

Dipole charges





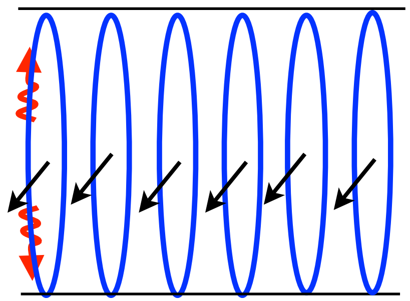


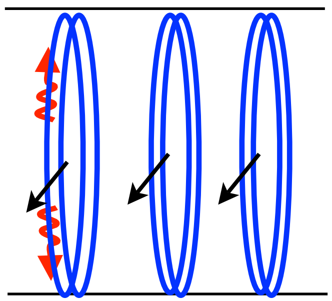
True charges are D1, D5

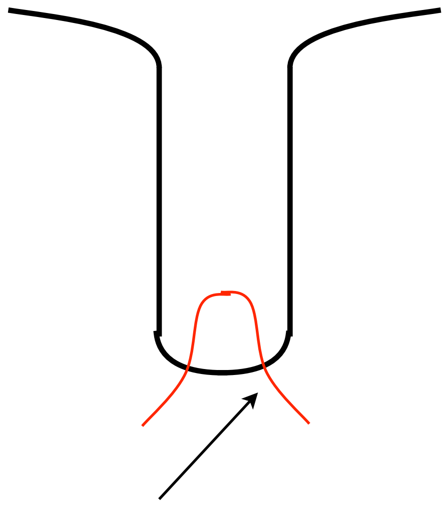
Dipole charge is KK monopole

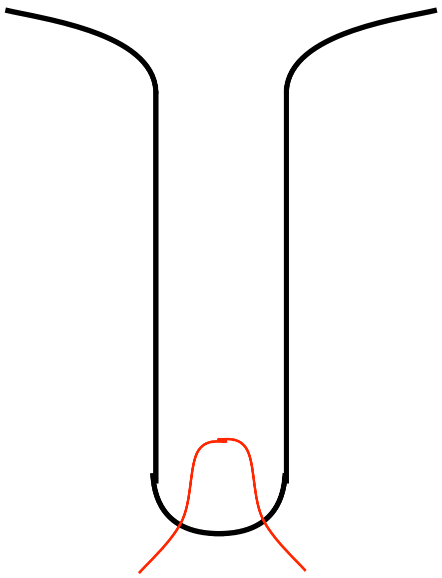
Energy gaps exactly agree between the CFT and the gravity solution...

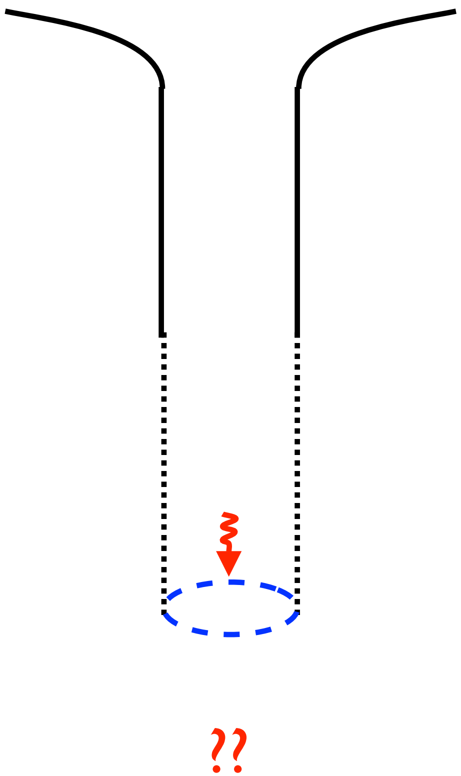
(so we must have fuzzballs ...)





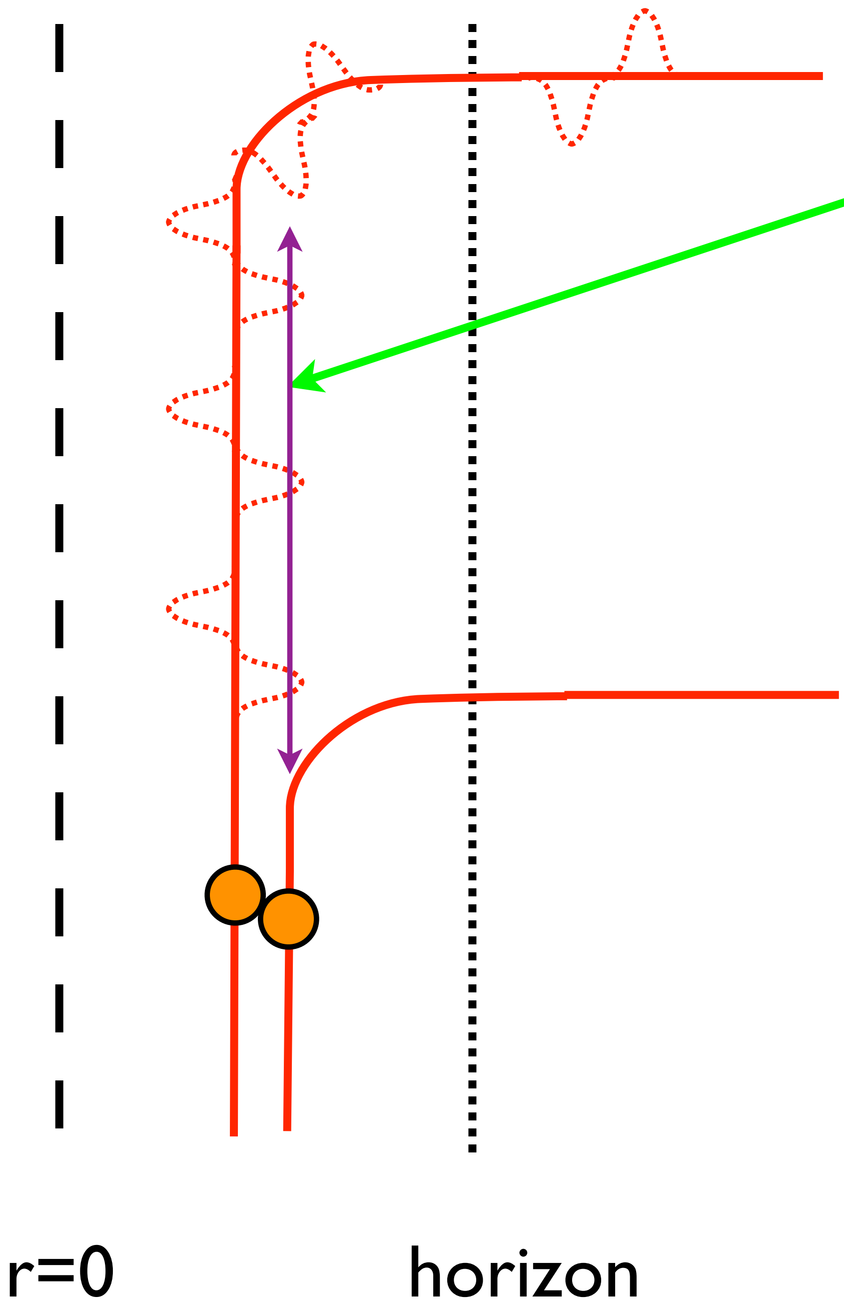






Nonlocality

‘Over-stretching’ of slices



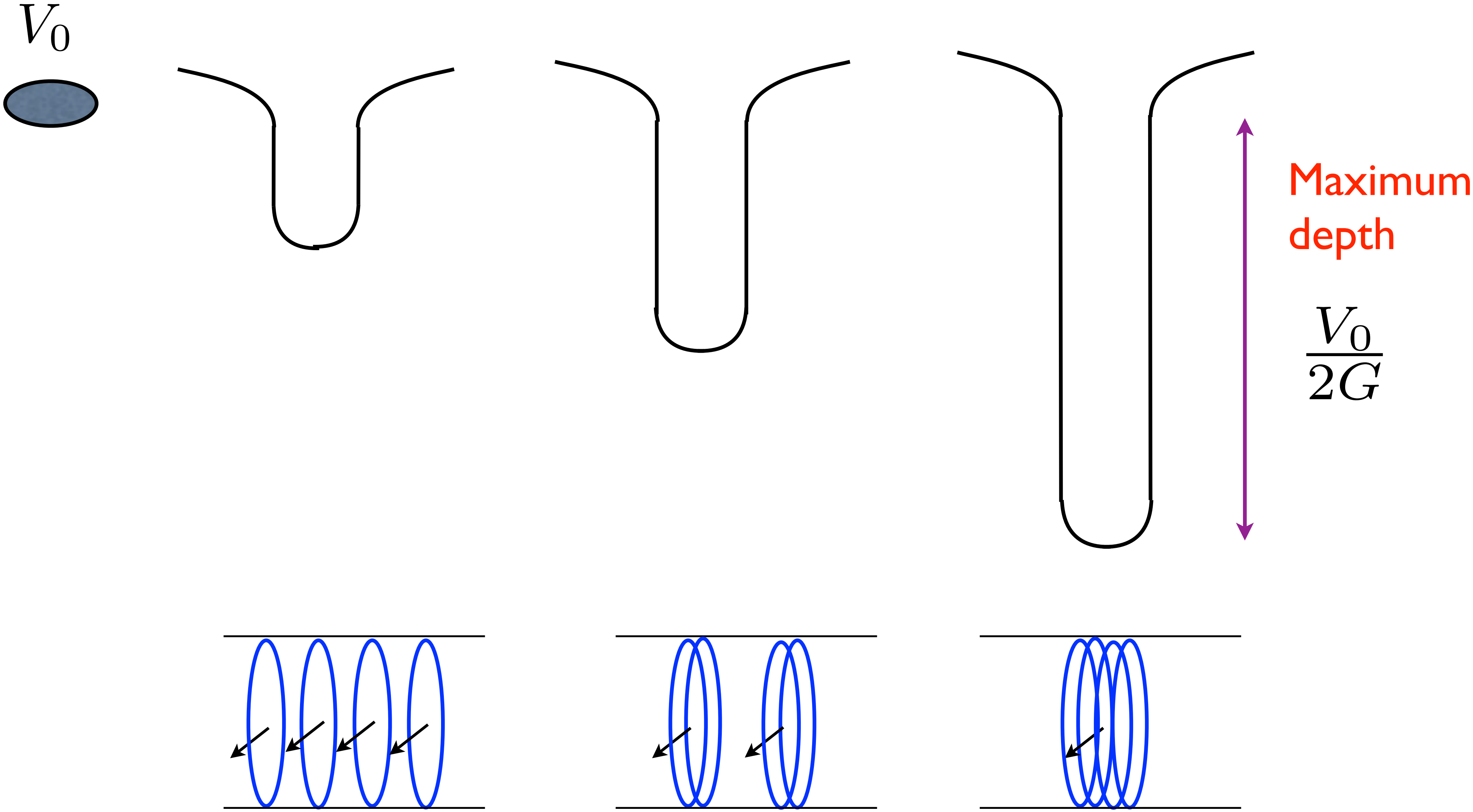
$$R_s \times S_{bek} \sim \frac{R_s^3}{G} \sim \frac{V}{G}$$

(a very long length)

Suppose that new nonlocal effects arose when a slice was stretched too much ...

Then the information problem would be solved: lab physics is local because stretching is small, but black hole slices have nonlocal physics after they stretch enough to hold half the Hawking radiation ...

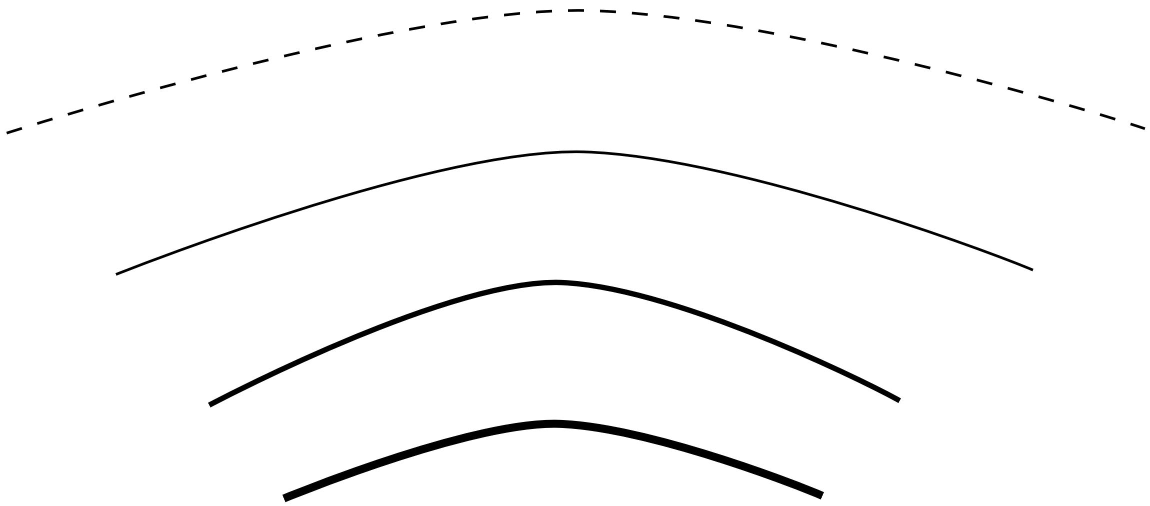
(SDM gr-qc/0007011, Giddings hepth/0911.3395)



D1D5 extremal states: it appears the space can be 'stretched' only upto a maximal depth, which is of the correct order ...

(SDM hepth/0205192)

But what happens if we apply this to the Universe ?



Spacelike slices are not just an abstract manifold, but have a ‘thickness’

If we ‘stretch too much’, semiclassical physics breaks down, nonlocal effects start

(SDM hepth/0305204)

But our Universe started with the size of a marble and ‘stretched’ to 3000 Mega-parsecs ... and we seem to have normal physics today ...