Progress on dS/dS & FRW Holography

Based on works with Alishahiha, Dong, Horn, Karch, Matsuura Polchinski, Tong, Torroba (2004 - 2012)

0. brief intro/review of framework

1. Unitarity bounds & t-dependent (dual) QFT

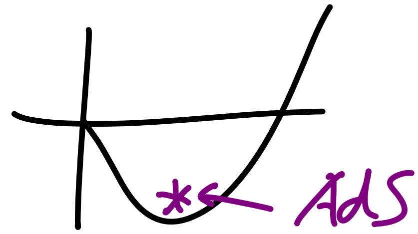
   Dong Horn ES Torroba '12

2. · UV structure of dS/dS & dS symmetries

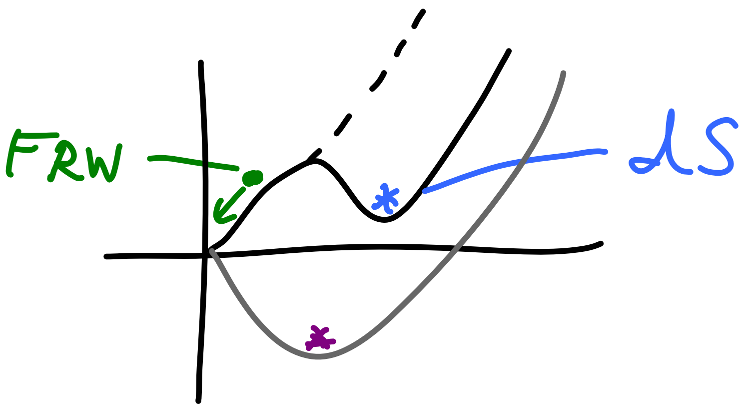
   · a simpler construction(?)

3. additional comments & questions

Start by asking what happens to AdS/CFT



when we uplift to cosmology

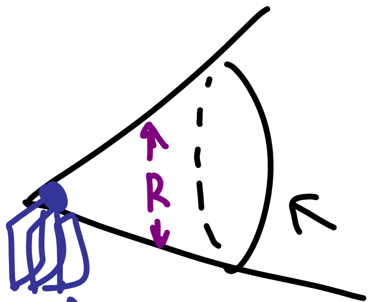


in concrete examples.

Goal : make holographic cosmology a precision science ... with understanding of error bars along the way.

AdS/CFT brane construction

$$\left(\frac{dR}{dr}\right)^2 = \frac{1}{R^2} \to R = r \quad \text{(cone)}$$



$\mathcal{R} > 0$ Einstein space

color branes

Gravitational Redshift $g_{00} \sim 1 - \frac{R^4}{r^4}$

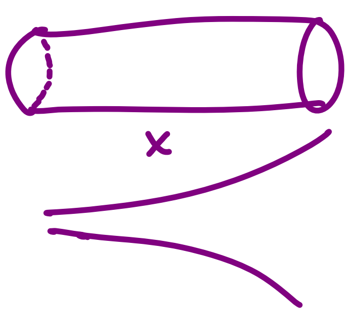
= Low-energy region

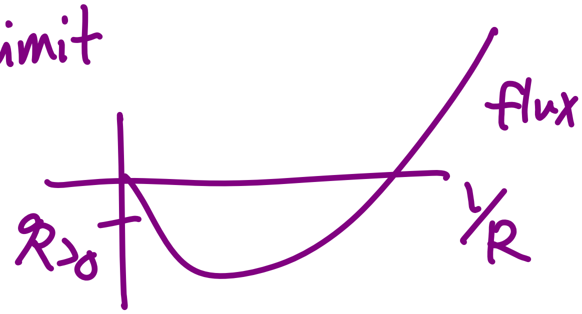
Our Strategy: Look for this in cosmo uplift

dS, FRW

$$E = \sqrt{-g_{00}}\, E_{pr} \ll E_{pr}$$

→ Effective field theory (EFT) dual, complete QFT in strict near-horizon limit



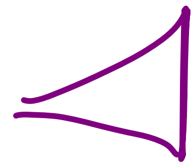


Note: CFT does not live on the boundary.

RS/warped compactifications

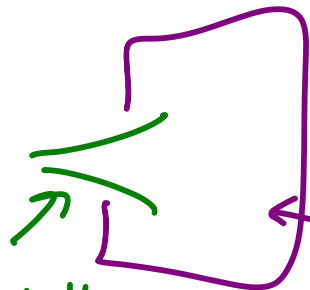
$$ds^2 \cong \frac{r^2}{R^2}\,dx^2 + \frac{R^2}{r^2}\,dr^2 + \text{internal}$$

$r < r_{uv}$



r_{uv}

or



H. Verlinde

Giddings/Kachru/Polchinski

compactification X

warped throat

$$\cong \underbrace{CFT}_{D-1} + \underbrace{GR_{D-1}} + \cdots$$

$$E < \Lambda_c = \frac{r_{uv}}{R^2} \Bigg| \; M_p^2 \sim \frac{r_{uv}^2}{R^4} N^2 + \frac{Vol(X)}{g_s^2}$$

$\rightarrow$ complete QFT in limit

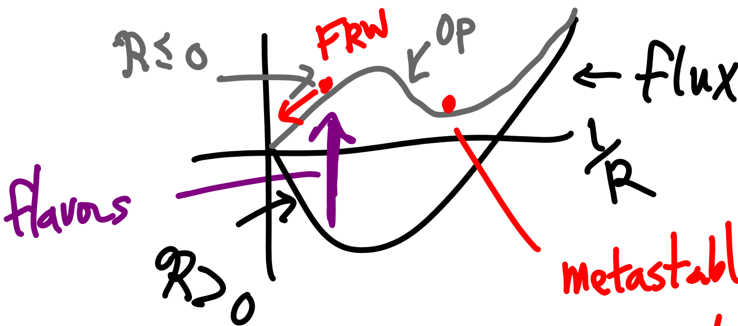
$$r_{uv} \rightarrow \infty \quad \text{with} \quad \frac{r_{uv}^2}{R^4} N^2 > \frac{Vol(X)}{g_s^2}$$

(Can happen in t-dependent way.)

Turns out to be a good analogy for

$dS \rightarrow FRW$

Uplifting AdS/CFT: Brane Constructions

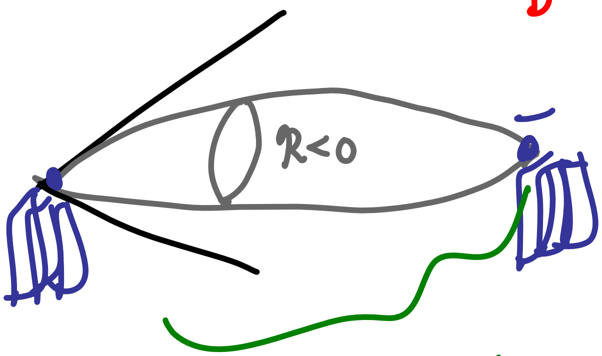


(+ other ingredients to fully stabilize)

metastable dS

$$ds^2_{dS_D} = \sin^2\frac{w}{L}\, \underbrace{ds^2_{dS_{D-1}}}_{} + dw^2$$

2 redshifted regions



2 tips ⟹

$$\left(\frac{dR}{dr}\right)^2 = \frac{-1}{R^2} + \frac{\text{const}}{R^{n>2}}$$

⟹ 2 EFTs

Micro ⟷ Macro

2 EFTs

para-metric entropy counts

- $dS_D \cong 2$ $EFTs_{D-1}$ w/ GR_{D-1}
  → we'll specify UV behavior shortly.
- FRW also has 2 redshifted* regions, and $\cdot\, GR_{d-1}$ decouples at late times
  - $S \to \infty$
  - t-dependent (power law) couplings
    ↪ matter content via unitarity

* from flavors

We can be more precise:

$$ds^2_{dS_D} = \sin^2\frac{w}{L}\, ds^2_{dS_{D-1\equiv d}} + dw^2$$

$$Z_{\text{bulk}} = \int D\hat\Phi \underbrace{\int [D\Phi]\Big|_{\substack{w<\frac{\pi}{2}L \\ \Phi(\frac{\pi}{2}L)=\hat\Phi}} e^{iS_<}}_{Z^{(1)}_{QFT}[\hat\Phi]} \underbrace{\int [D\Phi]\Big|_{\substack{w>\frac{\pi}{2}L \\ \Phi(\frac{\pi}{2}L)=\hat\Phi}} e^{iS_>}}_{Z^{(2)}_{QFT}[\hat\Phi]}$$

e.g. at Gaussian level,

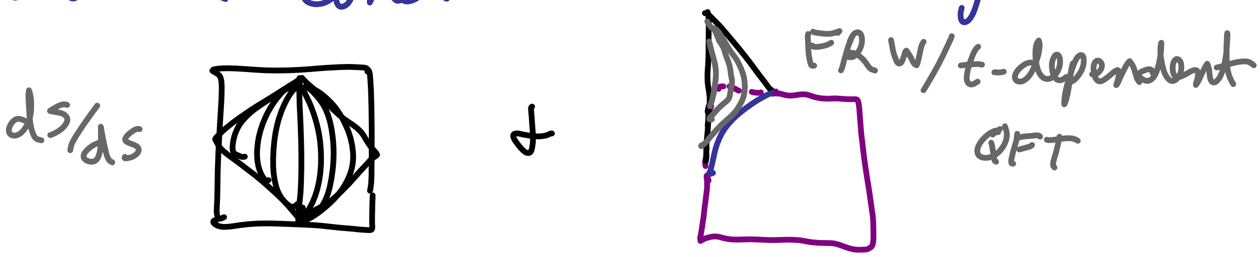
$$\langle \mathcal{O}\mathcal{O}\rangle^{(1)}_\ell = \frac{\delta^2 Z_1}{\delta\hat\Phi_1^2} = \frac{\Gamma\left[\frac{1}{2}(1+d-\hat\Delta+\ell)\right]\Gamma\left[\frac{1}{2}(1+\hat\Delta+\ell)\right]}{\Gamma\left[\frac{1}{2}(d-\hat\Delta+\ell)\right]\Gamma\left[\frac{\hat\Delta+\ell}{2}\right]}$$

- $\hat\Delta = \frac{d}{2} + \sqrt{\left(\frac{d}{2}\right)^2 - M^2L^2}$ (but $\langle \mathcal{O}\mathcal{O}\rangle_\ell$ real)

  cf Bousso Maloney Strominger
- Δ flows to $\frac{d}{2} + \frac{1}{2}$ in UV
- encodes (max. symmetric) shape of dS_D warp factor

Comparative holography:

Recall in AdS/CFT that p-brane construction lands on Poincaré slicing. In dS + FRW, above brane constructions → slicings



- inside a causal region
- a spatial direction (↔ scale) emerges.

⇒ • # of degrees of freedom real, >0 and unitarity more transparent

• Symmetries (such as they are) less manifest

As in AdS → Global, this may connect to other slicings (dS/CFT, FRW/CFT)

* Must make sense of $\int D\hat{h}_{\mu\nu}\, D\hat{g}$ in all cases.

Anninos/Hartman/Strominger
Harlow/Shenker/Stanford/Susskind '12

We would like to extract the essential features of the dual theories, given the concrete brane constructions

→ <u>Plan</u>:

(1) Flavor content of FRW duals, unitarity, & time-dependent QFT couplings

(2) Comments on Structure of dS_D duals

Magnetic flavors & uplifting

e.g. IIB (p,q) 7Bs wrapping $\Sigma_3 \subset S^5$

Tension $\propto \Lambda \times \frac{1}{R^2}\frac{1}{g_s^2}$

Competes with curvature

$S^1 \to S^{5 \text{ or } 3}$
$\downarrow$
$\mathbb{CP}^{2 \text{ or } 1}$

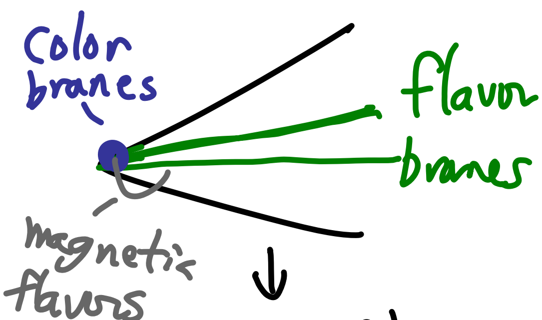
on $\mathbb{CP}^1$: 24 7Bs $\to \mathcal{R} = 0$

$*$ $\mathbb{CP}^2$: 36 7Bs $\to \mathcal{R} = 0$

Banks
Douglas
Seiberg,
Aharony
Maldacena
Fayyazuddin
Joe P ES

$$\Delta n \equiv n - n_{\mathcal{R}=0}$$

$\Delta n < 0$	$\Delta n \geq 0$
AdS	Cosmo



$\downarrow$

Static soln

no static solution
but $\exists$ simple,
t-dep't solutions

cf Kleban + Redi

$\Delta n \gtrless 0$ distinction in dual QFT?

Unitarity bounds & t-dependent QFT.

generic
(also FRW dual)

Given well-defined QFT at some scale, unitarity bounds (e.g. $\Delta_{scalar} > \frac{d-2}{2}$) help constrain IR physics.

e.g. $\mathcal{N}=1$ SQCD N_c colors & N_f flavors $Q, \tilde{Q}$

- IF SCFT: $\Delta_{chiral} = \frac{3}{2}|R|$

  unitarity $\Rightarrow \Delta \geq \frac{d-2}{2} = 1$

- $\Delta_{(Q\tilde{Q})} = 3\frac{(N_f - N_c)}{N_f} \geq 1 \Rightarrow N_f \geq \frac{3}{2} N_c$

Seiberg, ...

In D3 – (p,q)7 system,

$\Delta n > 0 \Rightarrow$ no static solution.

☆ In the simplest case (with parallel 7-branes $\Leftrightarrow$ $\mathcal{N}=2$ susy) this follows from unitarity.

Seiberg-Witten curve $\quad y^2 = x^3 - \overset{\sim u^r}{f(u)}\, x - \overset{\sim u^s}{g(u)}$

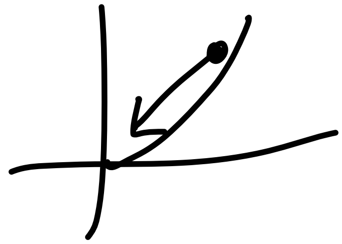
$(s < \frac{3}{2} r)$ $\boxed{2\Delta_y = 3\Delta_x = s\Delta_u}$

$$\frac{d\lambda_{SW}}{du} = \frac{dx}{y} \qquad a \sim \int \lambda_{SW} \sim \frac{u\,dx}{y}$$

BPS masses dim 1 $\Rightarrow$ $\boxed{\Delta_u = 1 + \Delta_y - \Delta_x}$

$$\boxed{\Rightarrow \Delta_u = \frac{12}{12 - N_f}, \quad N_f > 12 \text{ would violate unitarity}}$$

But there do exist t-dependent solutions



with the required properties (redshift, N_{dof}, $M_p \to \infty$) for a dual t-dependent EFT $\Rightarrow$ QFT.

→ More general question: how do t-dependent couplings affect unitarity bounds on IR behavior $\leftrightarrow$ field content

Consider

$$\int \Delta \mathcal{L} = \int dt\, d^{d-1}\vec{x}\; g(t,\vec{x})\, \mathcal{O}$$

$$g = g_0 t^{\alpha} \quad \text{or} \quad g_0 \left(t^2 - \vec{x}^{\,2}\right)^{\frac{\alpha}{2}}$$

as $t \to \infty$

- α can change whether $\Delta\mathcal{L}$ dominates at late times (IR)
- If $\Delta\mathcal{L}$ marginal in IR under $x^\mu \to \lambda x^\mu$, then

$$[\mathcal{O}] = d + \alpha$$

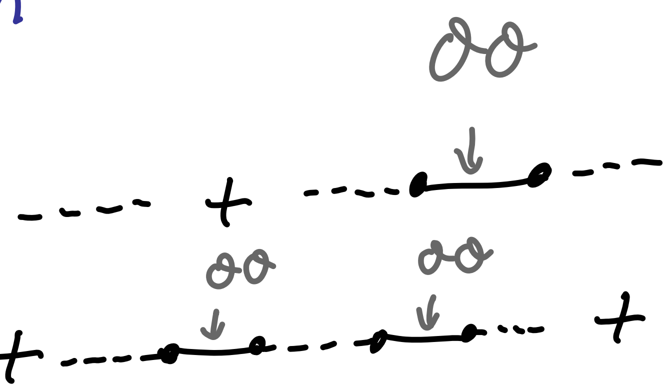
$\to$ Expect α can shift relevance condition & unitarity bounds

We can analyze this explicitly in large-N double trace flows.

$$\Delta S = \int \tfrac{1}{2}\phi\, m^2 \phi + g(t,\vec{x})\, \mathcal{O}\, \phi$$

$$\simeq \int \frac{g^2}{2m^2}\, \mathcal{O}\mathcal{O}$$

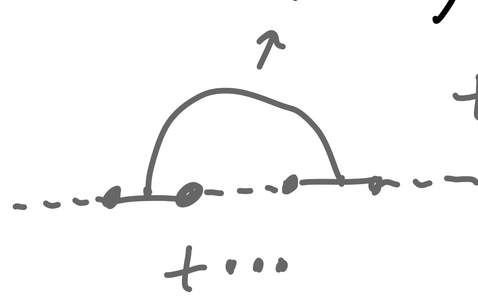
$$\langle \phi\, \phi \rangle =$$



$+ \cdots$

Effectively Gaussian

$+ \mathcal{O}\left(\frac{1}{N}\right)$



+ non-adiabatic effects

$+ \cdots$

Static Limit:

For analysis in holographic case see Andrade, Faulkner, Marolf ...

$$\Delta_\pm = \frac{d}{2} \pm \nu$$

$$\langle \mathcal{O}_\pm(p)\, \mathcal{O}_\pm(-p) \rangle = -i\, C_{\pm\nu} (p^2 - i\varepsilon)^{\pm\nu}$$

$$S^{(+)}_{CFT} + \int \frac{g^2}{4m^2} \underbrace{\mathcal{O}_+ \mathcal{O}_+}_{\Delta = d + 2\nu}$$

irrelevant

$$\langle \phi(p)\, \phi(-q) \rangle = \frac{-i\, \delta(p-q)}{m^2 - g^2 C_\nu (p^2)^\nu}$$

$$C_\nu = \boxed{2^{-2\nu} \pi^{\frac{d}{2}} \frac{\Gamma(-\nu)}{\Gamma(\frac{d}{2}+\nu)}}$$

$\rightarrow \langle \mathcal{O}_- \mathcal{O}_- \rangle$

as $p^2 \rightarrow \infty$

UV $\qquad \mathcal{O}_-$

nonunitary for $\nu > 1$

$$\Lambda_g^{\nu} \quad \frac{1}{g^{\frac{1}{\nu-1}}}$$

ok as cut off QFT

IR $\qquad \mathcal{O}_+$ unitary

$\boxed{\text{t-dependent case}}$

- $\int \lambda_0 t^{2\gamma} \mathcal{O}_+^2$ is <u>relevant</u>

(dominates 2 pt fns at large Δx)

when $[\lambda_0] = 2(\gamma - \nu) > 0$

- Unitarity maintained, including $\nu > 1$

- Can UV complete, e.g. SUSY models

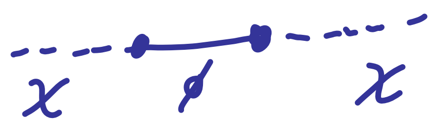
(effects of $\dot\lambda$ lost for $\Delta t < \lambda/\dot\lambda$)

$$\Delta S = \int \tfrac{1}{2}\phi\, m^2 \phi + \underbrace{g(t,\vec{x})}_{\equiv\ \tilde{\phi}}\, \phi\, \mathcal{O}_+$$

$$\langle \tilde{\phi}(p)\, \tilde{\phi}(-q) \rangle \xrightarrow[\mathrm{IR}]{} i\,\delta(p-q)\, \frac{(p^2 - i\varepsilon)^{-\nu}}{c_\nu}$$

* Not norm of a state

$$\Rightarrow \langle \phi(x)\, \phi(x') \rangle = \frac{-1}{c_\nu c_{-\nu}\, g(x) g(x') \left[(x-x')^2\right]^{\Delta_-}} \; *$$

Despite the Δ_- here, forward scattering*  is unitary

$$\mathrm{Im}\, A(\chi \to \chi) \propto -\sin \pi\nu / c_\nu > 0$$

(Intriligator/Grinstein: $\mathrm{Im} A_f \propto C_{\mathcal{O}} \left(\Delta - \left(\frac{d-2}{2}\right)\right)$ in CFTs)

Scales in t-dependent case:

$\Rightarrow$ UV complete...

- $\lambda(t)$ dynamical field
- SUSY gauge theory e.g. in which $\phi, \mathcal{O}$ composite below Λ_λ

t-dependence of λ doesn't matter

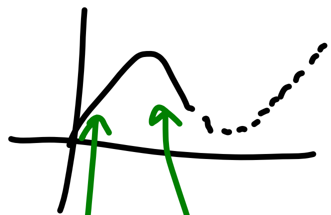
$\left.\begin{array}{l}\lambda \to \text{strong} \\ \text{would get} \\ \text{ghosts at}\end{array}\right\} \to \Lambda_\lambda$

... or cut off

$$\boxed{\frac{\dot{\lambda}}{\lambda} \sim \frac{\gamma}{t}}$$

$$\boxed{\int \lambda_0 t^{2\gamma} \mathcal{O}_+^2 \ \text{relevant \& unitary}}$$

In dS_D, we expect a similar effect:



heavy flavor branes — alone non-unitary (static)

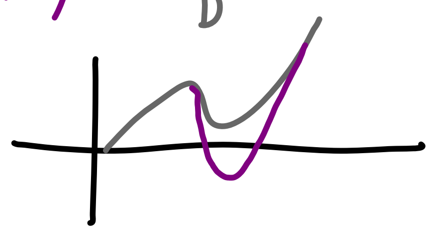
O-planes → correct the metric on Coulomb branch → restore unitarity

(in progress on QFT side of explicit uplifted dS_3 construction)

Further Comments on dS_D duals:

- <u>Non-perturbative instabilities</u>

Our explicit $(A)dS_D$ vacua are all metastable



In $AdS_D \Rightarrow$ no dual CFT at $\lambda \gg 1$, but long-lived with cutoff (e.g. warped compactification). Similar to

$AdS \times S^5/\mathbb{Z}_k$ / Quiver $U(N)^k$ QFT's
(SUSY with free action)

Non-perturbative instability to develop VEVs.

Kachru ES, ... Horowitz/Orgera/Polchinski, Kachru/Simic/Trivedi

• Maximal symmetry & RG

$$ds^2_D = \sin(h)^2 \frac{w}{L}\, ds^2_{dS_{D-1}} + dw^2$$

In AdS_D, Moduli fixing is dual to $\beta_{\{\lambda\}} = 0$.

- In dS_D, we also fix the moduli (more strongly, since no allowed tachyons).
- Reduces at low energy ($\sin w \to \sinh w \to w$) to CFT on dS_{D-1} but dimensions evolve IR $\to$ UV. Maximal symmetry $\Leftrightarrow$ $\sin^2 \frac{w}{L}$ warp factor.

Recall

$$ds^2_{dS_d} = \sin^2\frac{w}{L}\, ds^2_{dS_{D-1}} + dw^2$$

$$Z_{bulk} = \int D\hat{\Phi} \underbrace{\int [D\Phi]\Big|_{\substack{w<\frac{\pi}{2}L \\ \Phi(\frac{\pi}{2}L)=\hat{\Phi}}} e^{iS_<}}_{Z^{(1)}_{QFT}[\hat{\Phi}]} \underbrace{\int [D\Phi]\Big|_{\substack{w>\frac{\pi}{2}L \\ \Phi(\frac{\pi}{2}L) = \hat{\Phi}}} e^{iS_>}}_{Z^{(2)}_{QFT}[\hat{\Phi}]}$$

e.g. at Gaussian level,

$$\langle \mathcal{O}\mathcal{O}\rangle^{(1)}_\ell = \frac{\delta^2 Z_1}{\delta \hat{\Phi}_1^2} = \frac{\Gamma\left[\frac{1}{2}(1+d-\hat{\Delta}+\ell)\right]\Gamma\left[\frac{1}{2}(1+\hat{\Delta}+\ell)\right]}{\Gamma\left[\frac{1}{2}(d-\hat{\Delta}+\ell)\right]\Gamma\left[\frac{\hat{\Delta}+\ell}{2}\right]}$$

- $\hat{\Delta} = \frac{d}{2} + \sqrt{\left(\frac{d}{2}\right)^2 - M^2L^2}$ (but $\langle \mathcal{O}\mathcal{O}\rangle_\ell$ real)

  cf Bousso Maloney Strominger
- Δ flows to $\frac{d}{2} + \frac{1}{2}$ in UV
- encodes (max. symmetric) shape of dS_D warp factor

A more general construction?

We can reproduce $Z[\hat{\Phi}]$, e.g.

$$\langle \mathcal{O}\mathcal{O}\rangle^{(1)}_\ell = \frac{\delta^2 Z_1}{\delta \hat{\Phi}_1^2} = \frac{\Gamma[\frac{1}{2}(1+d-\hat{\Delta}+\ell)]\,\Gamma[\frac{1}{2}(1+\hat{\Delta}+\ell)]}{\Gamma[\frac{1}{2}(d-\hat{\Delta}+\ell)]\,\Gamma[\frac{\hat{\Delta}+\ell}{2}]}$$

by any

$$S_{CFT} + \sum_\ell \hat{K}(\ell)\,\mathcal{O}(\ell)\mathcal{O}(\ell) + \cdots$$

$$+ \sum_{\{\ell\}} \lambda_{\{\ell\}}\,\mathcal{O}\cdots\mathcal{O}$$

no relevant operators

e.g. deWolfe Giryavets Kachru Taylor

Multiple trace ops can be internally non-local...

$$\hat{K} + \frac{1}{\langle\mathcal{O}\mathcal{O}\rangle_{CFT}} = \frac{1}{\langle\mathcal{O}\mathcal{O}\rangle^{(1)}}$$; similarly solve for λ's

By construction, produces dS_D results at semiclassical level... potential problems at $1/N$, & UV completing $\int D\hat{h}\, D\hat{\Phi}$

Summary so far

- Can uplift AdS/CFT to cosmology
  - 2 EFT's + GR_{D-1}

    Concrete brane constructions

    decouples at late times
  - magnetic flavor content surviving at low energies,
    $$N_f > N_f^{CFT}$$
    consistent with basic Unitarity checks.
  - Can go beyond low energy description, e.g. capturing the $dS_D \to dS_{D-1}$ warp factor

Other directions

- flux vacua: trade flux for branes to expose dual d.o.f.

  cf $\mathcal{N}=4$ SYM Coulomb branch

  e.g. KKLT: reproduces $S \sim N^6$

  and $\frac{c_3}{c_3'} \sim \frac{N'}{N}$

- spherical slicing of static patch: (Anninos, Hartnoll, Hofman '11)

  build up from $AdS_2 \times S^2 \times CY$, another opportunity for concrete brane construction.