Submesoscale Frontogenesis, Arrest, and Decay: With and Without Surface Waves

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The oceanic submeoscale is a bunch of lines on the sea surface; i.e., surfactants gathered into horizontal convergences above downwelling jets
Spirals on the Sea

Sun-glint showing ~ 5 km "spirals on the sea" not far off the Mediterranean coast of Africa photographed from space (Scully-Power, 1986)

Lines are at density filaments (mimima) or density fronts (steps).
lines are horizontal density gradients, along-line jets, and overturning secondary circulations

they arise from frontogenesis

the energy source is available potential energy, $w_b > 0$, while the processes of its release can be of several types

occasionally the lines roll up or tear apart into coherent vortices

$Ro, Fr, Ri \sim 1$
Frontogenesis induced by an ambient horizontal strain field
(a la Bergeron, Hoskins, ...)

Density Front

$\mathbf{v}(x,z)$

down-front flow

frontal axis

secondary circulation $(u, w)$

isopycnals: $\frac{R_o}{Fr^2} <b> + b'$

deformation flow $(u_d, v_d)$

$0$

heavy $b' < 0$

cyclonic

light $b' > 0$

anti-cyclonic

$x \rightarrow$

$z$

$y$
filaments have stronger convergence and frontogenesis per unit density gradient and strain
Convergence vs. Strain?

As an “external” velocity, both fields imply exponential growth for any passive \( |\nabla c| \sim |\nabla \rho| \).

As a local secondary circulation, convergence implies super-exponential growth for \( |\nabla \rho| \).

Oceanic mesoscale and atmospheric synoptic-scale have stronger strain than convergence because \( Ro \ll 1 \).

Oceanic submesoscale fronts have both comparable strain and convergence because \( Ro \gtrsim 1 \).
Turbulent Thermal Wind Balance (TTW): combined effects of a geostrophic shear and vertical momentum mixing

\[ - \partial_z [\nu_v \partial_z u] - f v = - \partial_x \int \limits^z b' \, dz \]
\[ - \partial_z [\nu_v \partial_z v] + f u = - \partial_y \int \limits^z b' \, dz \]
\[ \partial_x u + \partial_y v + \partial_z w = 0 \]

e.g., for \( S = \partial_z v_g(z) > 0 \) in the surface boundary layer \((z > -h)\), TTW \( \Rightarrow \) the ageostrophic flow \( u_a(z) \) is left-rearward relative to \( v_g(z) \).

\( \Rightarrow \) in 2D \( u_a(x,z) \) is a frontogenetic cross-frontal secondary circulation in the same sense as in strain-induced frontogenesis.

TTW is only an approximation to the balanced diagnostic model; e.g., it is not aware of stratification or ageostrophic advection.
Linear TTW without Wind or Waves

Front

Dense Filament

буяность

buoyancy field

BL depth

$z = -h(x)$

convergence on dense side

convergence in center

“TTW”

ageostrophic streamfunction for secondary circulation

$(u,w) (x,z)$
Diagnosing Secondary Circulation and Frontogenetic Tendency (SCFT)

After the circulation dynamics has created submesoscale coherent structures with only “slow” advective and boundary layer evolution rates, diagnose the instantaneous 3D secondary circulation $u_a$ and frontogenetic tendencies $(T^b, T^u)$ from $b(x,z)$ and the turbulent forcing and mixing ($\nu_v$, $\kappa_v(x,y)$, $\tau$, $Q$) by assuming, as the only dynamical approximation to the hydrostatic, incompressible Primitive Equations with vertical mixing and wave-averaged Stokes vortex forces, the neglect of the ageostrophic time tendency in the horizontal momentum equations.

This is a kind of maximally inclusive Balance Equations that excludes, e.g., inertia-gravity waves; i.e., a kind of generalized Sawyer-Eliassen model, but not based on the usual $\delta \ll \zeta = \partial_x v - \partial_y u$ approximation. Solved iteratively.

It does not preclude evolution in $b$ and $u_g$, hence in $u_a$.

Its PDE solutions can be expected to fail to exist at some finite $Ro > 1$. The true evolution is presumed to stay close to this balanced state, as in realistic ROMS simulations.

Today SCFT is applied to 2D $b(x,z)$ fronts and dense filaments.

(McWilliams, 2017)
SCFT Diagnostic Analysis:
Vertical Velocity in a Dense Filament

Linear TTW without wind

Nonlinear TTW with wind

\[ \theta_w = \frac{\pi}{4} \ (NE) \]
Simplified Theory of Strong Frontogenesis

Strain Induced Frontogenesis (Semigeostrophy) Scaling

Without loss of generality we assume the anisotropic submesoscale features are aligned in the y direction. Apply semigeostrophic scaling:

- Cross-front: $x \sim l$, $u \sim \epsilon V$
- Along-front: $y \sim L$, $v \sim V$
- Mixed-layer depth: $z \sim h_{ml}$, $w \sim u \frac{h_{ml}}{l}$
- Front depth: $T \sim \frac{l}{\epsilon V}$
- Along-front geostrophic balance: $\Phi \sim fVl$, $b \sim \frac{fVl}{h_{ml}}$

Strain Induced Frontogenesis Scaling

\[ Ro = \frac{V}{fl} \sim O(1) \]  \[ \epsilon = \frac{l}{L} \ll 1 \]

(Hoskins and Bretherton, 1972)
motivated by the rapid frontogenetic time scales and comparable cross-front and along-front velocity magnitudes we use:

$$x \sim l, \quad y \sim L, \quad z \sim h_{ml},$$

$$v \sim V, \quad u \sim RoV, \quad w \sim u \frac{h_{ml}}{l},$$

$$T \sim \frac{l}{RoV}, \quad \Phi \sim fVl, \quad b \sim \frac{fVl}{h_{ml}},$$

$$Ro = \frac{V}{fl} \sim O(1)$$

$$\epsilon = \frac{l}{L} \ll 1$$
Frontogenetic Tendencies and Rates

buoyancy frontal tendency
\[ \frac{D}{Dt} \frac{1}{2} |\nabla_h b|^2 = Q \cdot \nabla_h b \rightarrow -\delta |\nabla_h b|^2 \]

velocity frontal tendency
\[ \frac{D}{Dt} \frac{1}{2} |\nabla_h u_h|^2 = F_u \rightarrow -\delta |\nabla_h u_h|^2 \]

frontogenetic tendency rates
\[ T_b = \frac{F_b}{|\nabla_h b|^2}, \ T_u = \frac{F_u}{|\nabla_h u|^2} \]

\[ T_b \approx T_u \approx -\delta \]
\[ \delta = u_x + v_y \]
What about the divergence evolution?

\[
\frac{D}{Dt} \frac{1}{2} |\nabla_h b|^2 \approx -\delta |\nabla_h b|^2 + \ldots \quad \mathcal{F}_b
\]

\[
\frac{D}{Dt} \frac{1}{2} |\nabla_h u_h|^2 \approx -\delta |\nabla_h u_h|^2 + \ldots \quad \mathcal{F}_u
\]

\[
\frac{D}{Dt} \delta \approx -\delta^2 + f\zeta_{ag} + V_{\text{mix}} + V_{\text{adv}} + H_{\text{diff}}
\]

\[
\frac{D}{Dt} \zeta = -\delta \zeta + f\delta + V_{\text{mix}} + V_{\text{adv}} + H_{\text{diff}}
\]

after removing geostrophic balance

TTW balances
strong frontogenesis in a Lagrangian frame with $Ro \gg 1$:

$$\dot{\delta} = -\delta^2, \quad \dot{\zeta} = -\delta \zeta.$$ 

with $\delta < 0$, $\zeta > 0$ at the center of the front (i.e., convergent and cyclonic).

$$\Rightarrow \quad \delta(t) \approx -\frac{1}{t_s - t}, \quad \zeta(t) \propto +\frac{1}{t_s - t} \quad \text{as } t \to t_s^-.$$ 

a finite time singularity with $-\delta$ as its growth rate. Also,

$$|\nabla b|^2 = -\delta |\nabla b|^2 \Rightarrow |\nabla b|(t) \propto +\frac{1}{(t_s - t)^{1/2}}.$$
snapshot of surface divergence in a realistic ROMS simulation for the Gulf of Mexico

tests of asymptotic theory: composite evolution over 656 detected frontogenesis events in this simulation

frontogenesis slows at late time in model due to instability & diffusive arrest

(Barkan et al., 2018)
Large-Eddy Simulation: Filament Frontogenesis, Arrest, & Decay
boundary-layer turbulence generated by surface cooling

fully turbulent i.c. at $t=6$ hrs., peak frontal strength and arrest

along-front averaged fields

(Sullivan & McWilliams, JFM, 2017)
Large-Eddy Simulation of Frontogenesis, Arrest, and Decay:

time series of y-averaged peak vorticity, $\zeta_p$, and turbulent KE

![Graph showing time series of y-averaged peak vorticity, $\zeta_p$, and turbulent KE vs. time (t) in hours.](image)
Large-Eddy Simulation of Frontogenesis, Arrest, & Decay:
near-surface $u$ indicating arrest by horizontal shear instability of the sharp front

note the small arrest scale $\sim 100 \text{ m} \sim h_{bl}$

arrest by horizontal Reynolds stress, $\langle u'v' \rangle < 0$
alongfront wavenumber spectrum at time of peak arrest
Different LES cases: E and N winds and free convection:
TTW frontogenesis, arrest, and decay always happen, but differently in detail.
Surface Gravity Wave-Averaged Effects on Currents (WEC)

\[
\begin{align*}
\partial_t u + \ldots &= u_{st} \times (f\hat{z} + \nabla \times u) + F^w \\
\partial_t C + \ldots &= -u_{st} \cdot \nabla C + Q^w \\
\nabla \cdot u &= \nabla \cdot u_{st} = 0.
\end{align*}
\]

- \(\mathbf{u}\) ... 3D wave-averaged velocity
- \(u_{st}\) ... 3D Stokes drift
- \(C\) ... wave-averaged material tracer (including buoyancy)
- \(F^w\) ... non-conservative wave-related force (often Radiation Stress divergence, but also added mixing)
- \(Q^w\) ... non-conservative wave-related effects on \(C\), mostly mixing

Now proceed to do similar SCFT diagnoses, ROMS simulations, and LES with WEC.
Filament in Wind-Wave Equilibrium: **Secondary Circulation**

\[ \theta_{\text{wind}} = \theta_{\text{wave}} = \frac{\pi}{4} \]

**with WEC**

**without WEC**

fully nonlinear SCFT with TTW and WEC and Ekman currents

\[ w \] here is stronger than with either TTW or WEC alone through constructive alignment with Ekman current

(McWilliams, 2018)
Filament in Wind-Wave Equilibrium: **Frontogenetic Tendency**

$\theta_{\text{wind}} = \theta_{\text{wave}} = \frac{\pi}{4}$

$T^u [10^{-13} \text{ s}^{-3}]$

$\mathbf{T}^u$ here is stronger than with either TTW or WEC alone through constructive alignments with Ekman current and ageostrophic advection.
Filament Frontogenesis in LES with Surface Waves
Why is TTW frontogenesis made so much weaker with along-front waves?

Because the wave forces induce a shallow eastward geostrophic flow in the filament center that tears of the top of the front by eastward advection:

\[ \partial_t u = \cdots + \int_{-\infty}^{z} \partial_z v^{st} \zeta \, dz' > 0 \]

with \( v^{st}, \zeta > 0 \). This has the effect of creating two east-side fronts with competing secondary circulations, hence "detuning" the frontogenesis.
alongfront-averaged vertical velocity after frontal rragmentation

\[ \langle w \rangle(x, z) \]
$w(x,y)$ at $z = -5$ m after frontal fragmentation: inhomogeneous Langmuir turbulence
frontogenesis (FG) is a key process for surface submesoscale currents

in realistic simulations TTW-induced convergence is more common than mesoscale strain as a cause of FG

“balanced” diagnoses from $b$ and $\nu_y$ (or background strain rate) $\rightarrow$ Secondary Circulation and Frontogenetic Tendencies (SCFT)

surface convergence $\delta$ associated with the frontal ageostrophic secondary circulation is the cause of strong FG ($Ro > 1$) over a time of a few hrs.

frontal arrest is often caused by lateral shear instability

frontal arrest set the lower size limit for the submesoscale regime, here with a width $\sim h_{BL}$

frontal decay extends over several days with slowly weakening currents

surface waves' Stokes drift has an important influence on strong fronts

we don't yet have a phenomenological overview of submesoscale $\leftrightarrow$ surface wave $\leftrightarrow$ boundary-layer interactions, but vertical momentum mixing is key to FG