



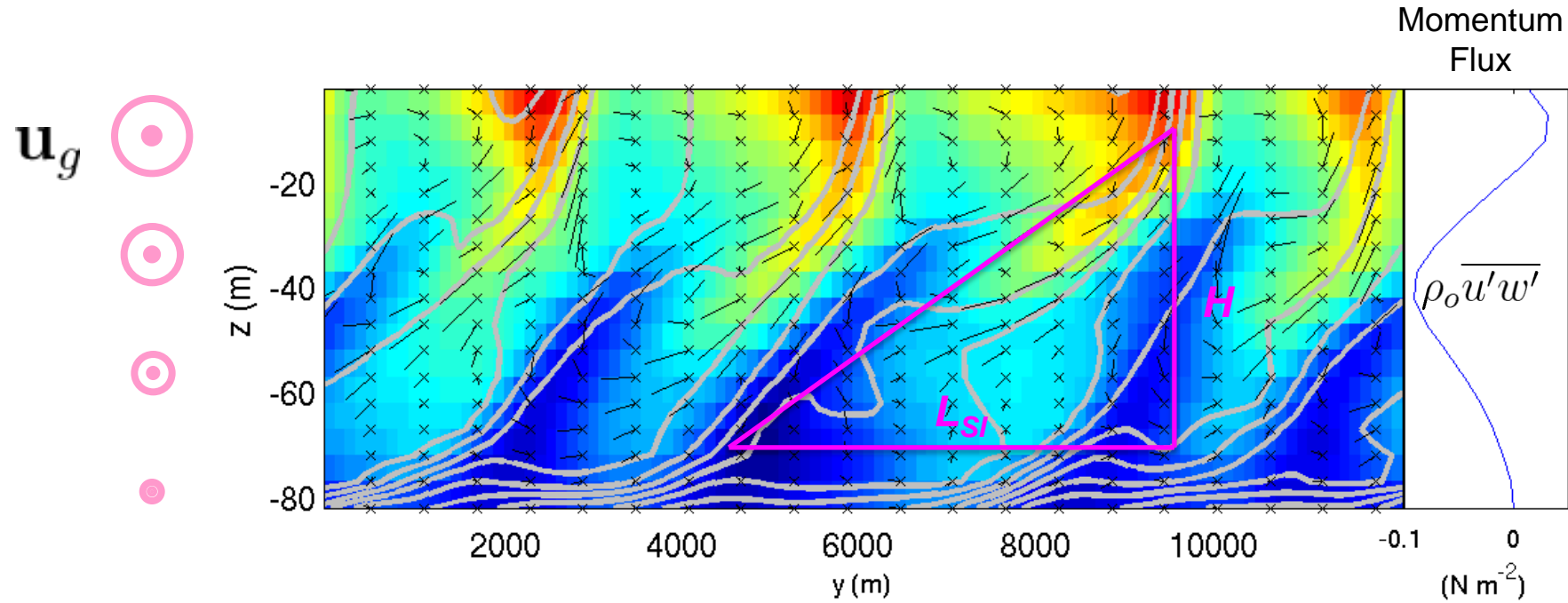
**Symmetric Instability (SI)-Turbulence: A  
Unique Form of Boundary Layer Turbulence--  
The Observational Evidence**

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# Symmetric instability (SI)



- SI is an overturning instability that forms in baroclinic currents that derives its KE from the thermal wind shear.

- Characterized by coherent structures

- Slantwise overturning cells with flow along isopycnals

- Frontlets with jets

- The width of the overturning cells, frontlets, and jets scale as:

- $L_{SI} \sim (\text{depth of the boundary layer}) / (\text{isopycnal slope in the boundary layer})$

$$\frac{\partial \mathbf{u}_g}{\partial z} = \frac{1}{f} \hat{\mathbf{k}} \times \nabla_h b$$

*THERMAL WIND SHEAR*

$$b = -\frac{g\rho}{\rho_0}$$



# SI turbulence

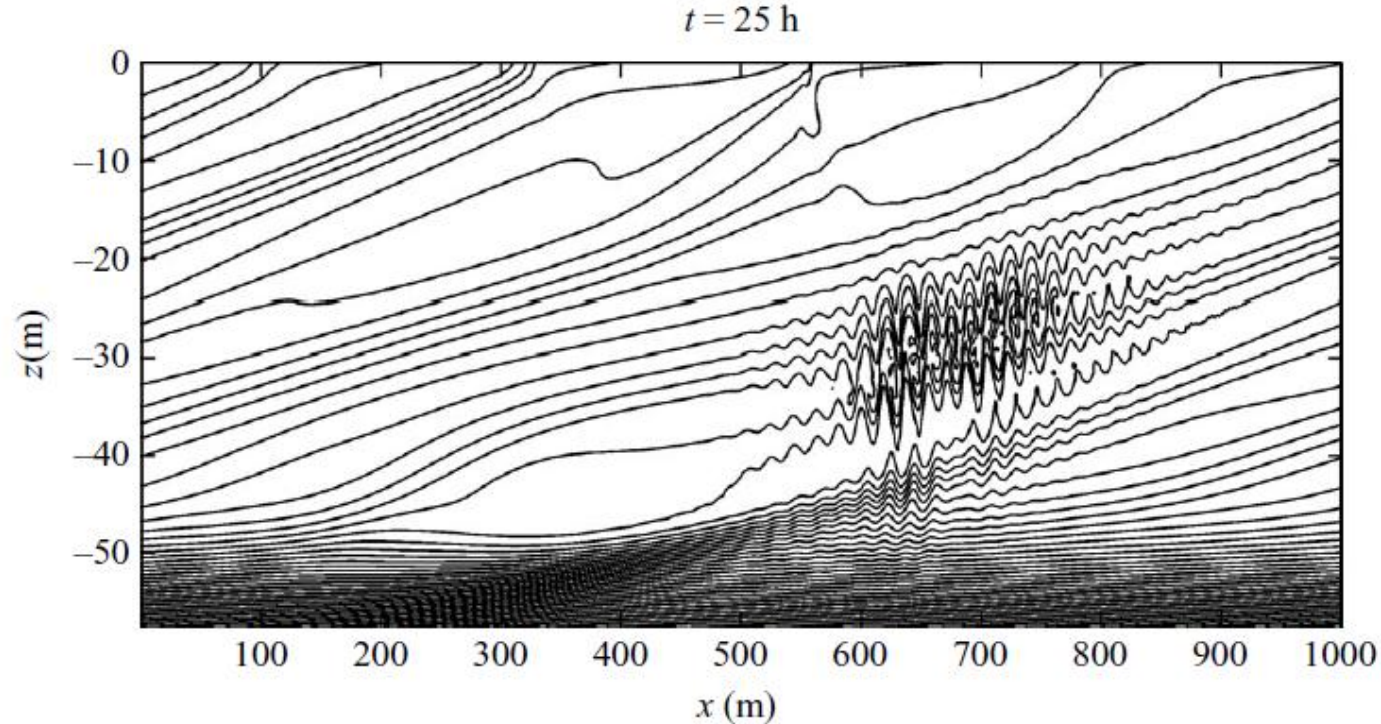


Figure from Taylor and Ferrari, *JFM*, 2009

- The SI overturning cells are themselves unstable to secondary instabilities that enhance boundary layer turbulence.
- Collectively, the coherent structures and small scale turbulence is termed **SI turbulence**.
- It is unique in that it **forms in stable stratification** and derives its energy from the ocean circulation rather than from winds, surface waves, or buoyancy loss and **represents a sink of KE for the circulation**.

# Outline

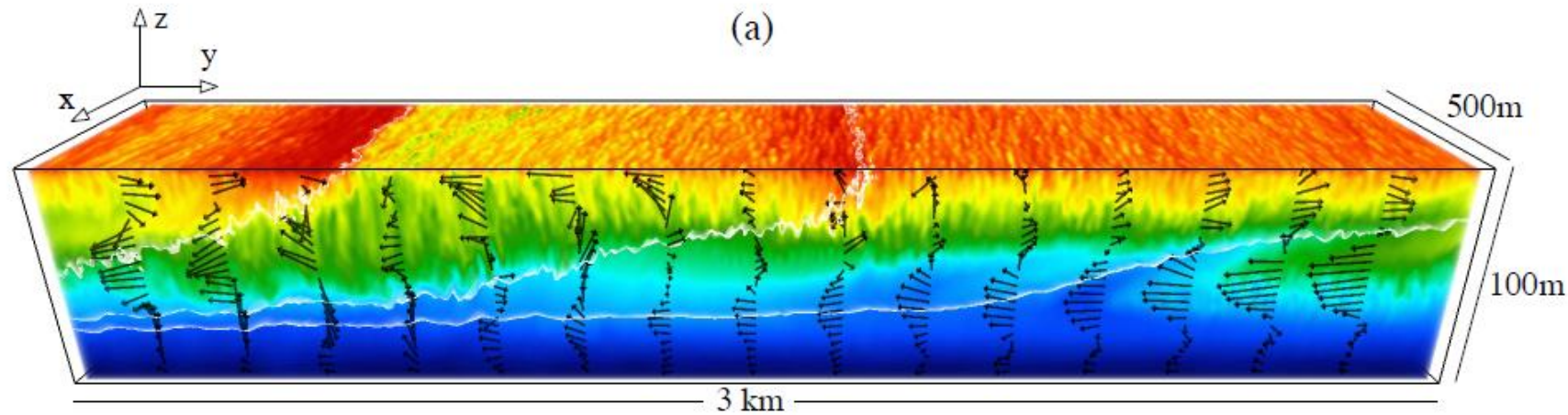
- Identifying SI

- Instability criterion
- Energetics
- Conductive forcing

- Observational evidence of SI

- Persistent upper ocean fronts--Kuroshio and Gulf Stream
- Ephemeral fronts (E. North Atlantic, OSMOSIS)
- Bottom boundary layers associated with deep waters in the Southern Ocean

- Thoughts on parameterizing SI turbulence



# Potential vorticity and flow stability

Ertel potential vorticity

$$q = \omega_a \cdot \nabla b = (f + \zeta)N^2 + \omega_h \cdot \nabla_h b$$

Absolute vorticity

$$\omega_a = f\hat{k} + \nabla \times \mathbf{u}$$

- The sign of the PV determines whether the fluid is unstable to overturning instabilities (gravitational, inertial/centrifugal, symmetric):

$$fq < 0 \quad \text{FLOW UNSTABLE}$$

	Gravitational Instability	Symmetric Instability	Centrifugal Instability
Instability Criterion	$N^2 < 0$	$Rib = \frac{N^2 f^2}{ \nabla_h b ^2} < \frac{1}{(1 + \zeta/f)}$ $N^2 > 0 \quad f(f + \zeta) > 0$	$f(f + \zeta) < 0$
KE Source	Buoyancy flux $\overline{w'b'}$	Geostrophic Shear Production (GSP) $-\overline{\mathbf{u}'w'} \cdot \frac{\partial \bar{\mathbf{u}}_g}{\partial z}$	Lateral Shear Production (LSP) $-\overline{\mathbf{u}'v'_s} \cdot \frac{\partial \bar{\mathbf{u}}_g}{\partial s}$

# State of marginal stability

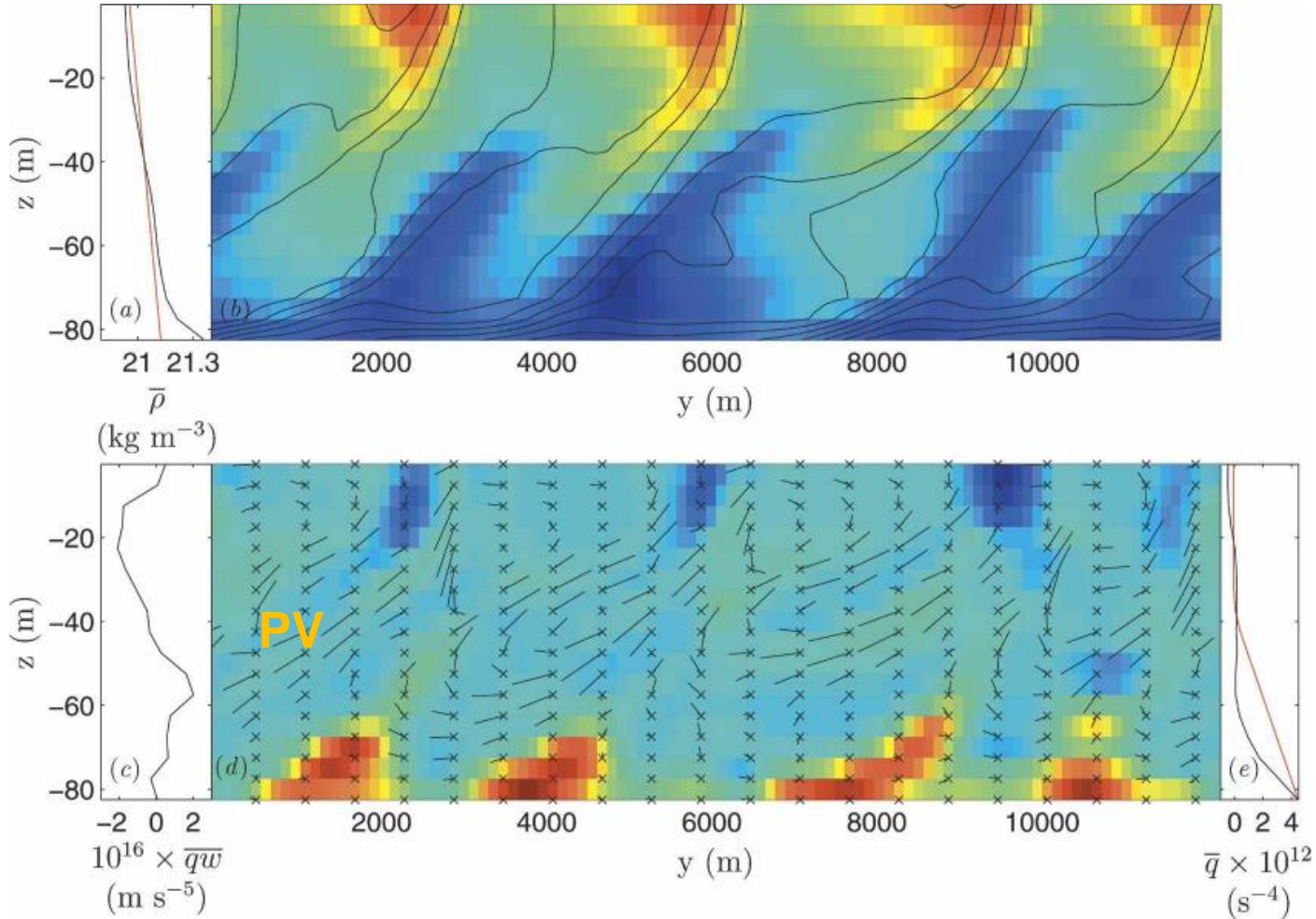


Fig. from Thomas (2005)

- SI modifies the background flow, driving the system to the state of marginal stability:

$$\bar{q} = (f + \bar{\zeta})\bar{N}^2 - \frac{|\overline{\nabla_h b}|^2}{f} = 0$$

- For SI the state of marginal stability is stratified with stratification

$$\bar{N}^2 = \frac{|\overline{\nabla_h b}|^2}{f^2} \left(1 + \frac{\bar{\zeta}}{f}\right)^{-1}$$

and Richardson number

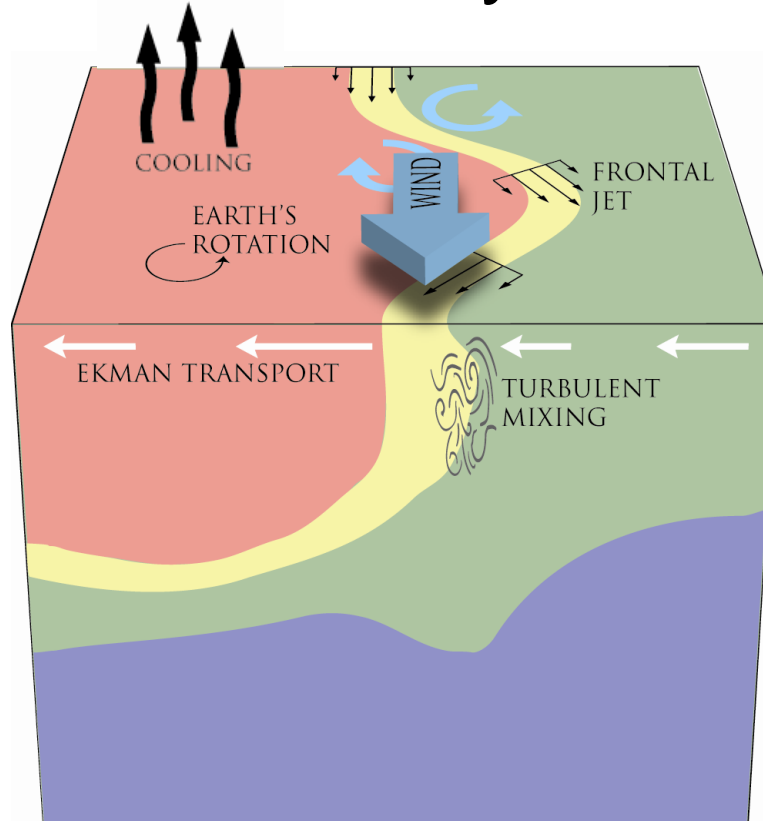
$$Ri_b = \frac{1}{(1 + \bar{\zeta}/f)}$$

- When the PV=0, SI will extinguish itself without destabilizing forcing.

# PV reduction by atmospheric forcing

BUOYANCY  
LOSS

$$B_o > 0$$



• Flux form of PV equation

$$\frac{\partial q}{\partial t} = -\nabla \cdot \mathbf{J} \qquad \frac{Db}{Dt} = \mathcal{D}$$

$$\mathbf{J} = \mathbf{u}q + \nabla b \times \mathbf{F} - \mathcal{D}\omega_a$$

Diabatic PV flux

$$J_z^D = -(f + \zeta)\mathcal{D}$$

Frictional PV flux

$$J_z^F = \nabla_h b \times \mathbf{F}$$

• The PV in the boundary layer is reduced when

• There is buoyancy loss  $B_o > 0$

• The wind is *down-front*, i.e. it has a component *along* the frontal jet,

*EKMAN BUOYANCY FLUX*

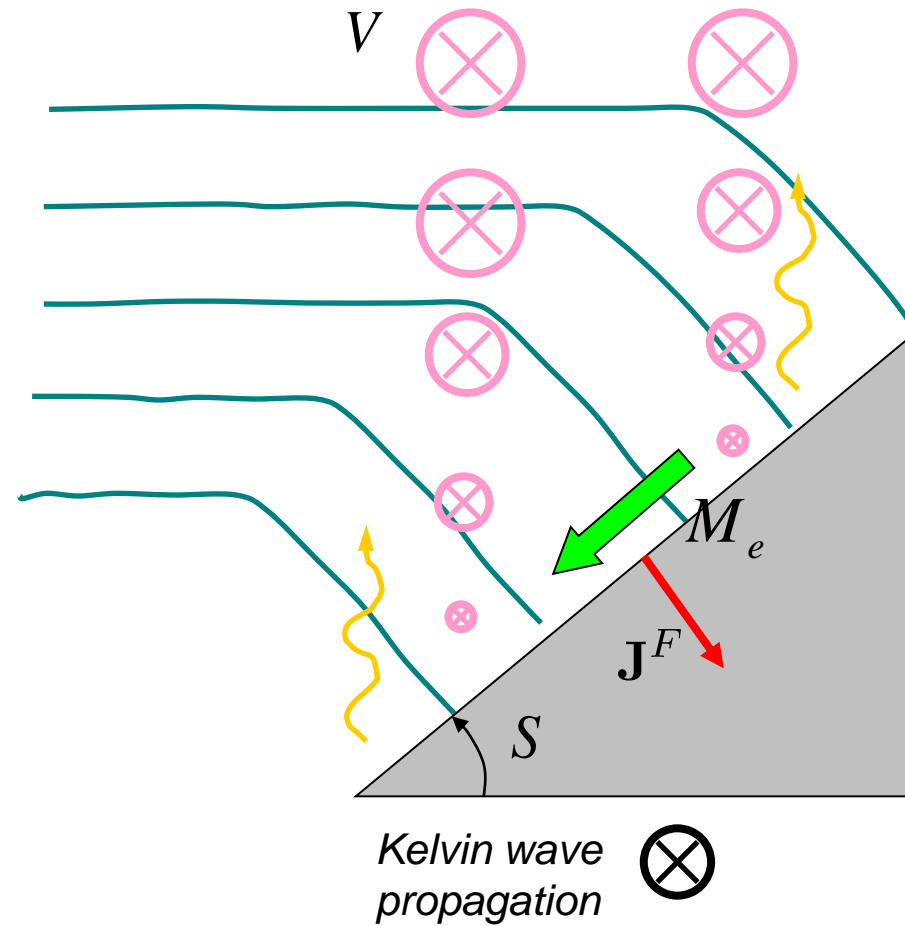
$$EBF \equiv \mathbf{M}_e \cdot \nabla_h b > 0$$

$$\mathbf{M}_e = \frac{\boldsymbol{\tau}^w \times \hat{k}}{\rho_o f}$$

Ekman transport



# PV reduction in the bottom boundary layer



Northern Hemisphere  
example

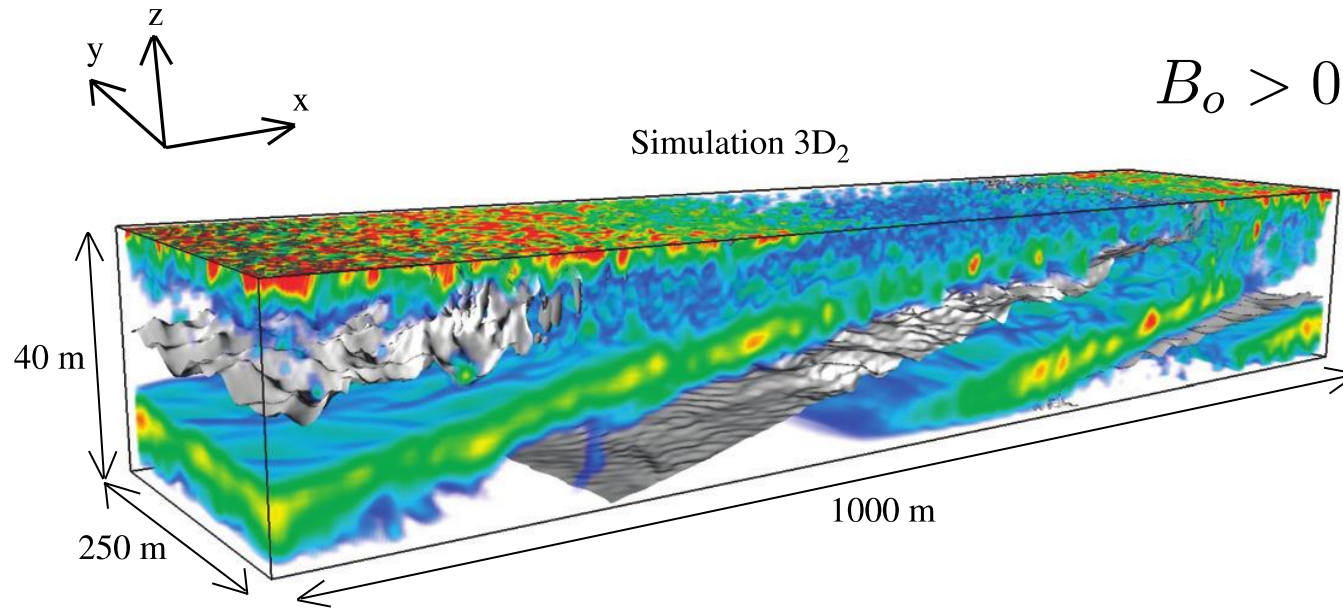
Frictional PV flux

$$\mathbf{J}^F = \nabla b \times \mathbf{F}$$

- The frictional deceleration of a current that flows in the direction of Kelvin wave propagation results in a downslope Ekman transport that advects lighter water under dense, lowering the stratification and PV.

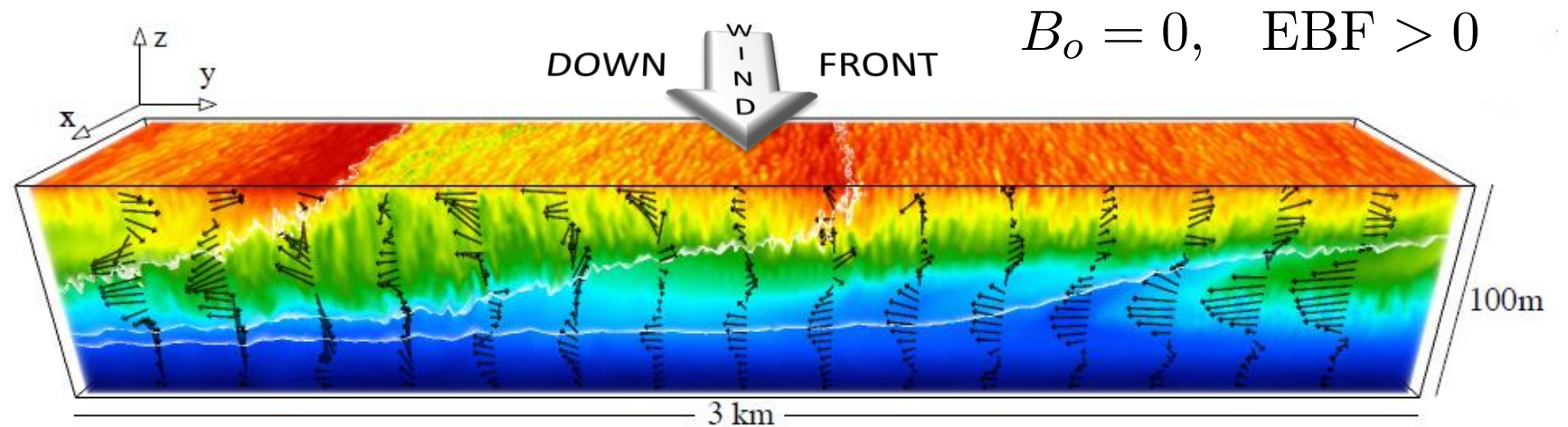


# Large eddy simulations of *forced* SI



$$B_o > 0, \quad \text{EBF} = 0$$

Taylor and Ferrari (2010)

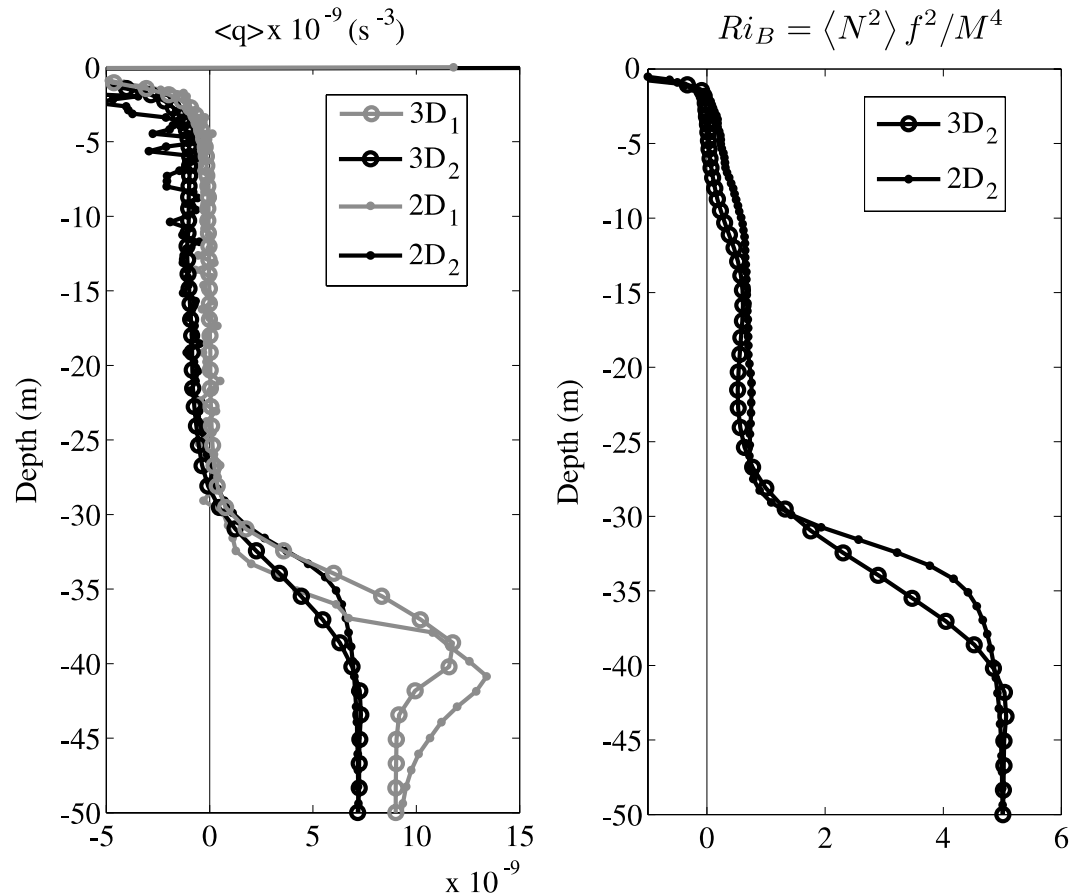


$$B_o = 0, \quad \text{EBF} > 0$$

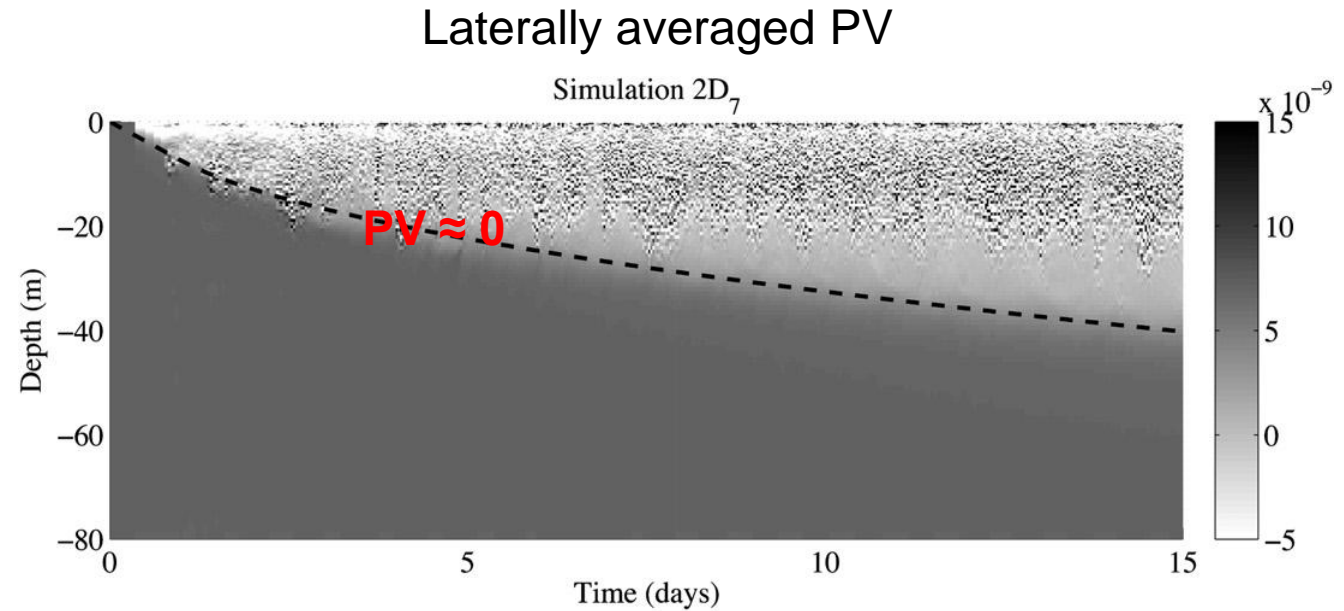
Thomas and Taylor (2010)

# Deepening of a stratified boundary layer with near-zero PV

$$B_o > 0, \quad \text{EBF} = 0$$



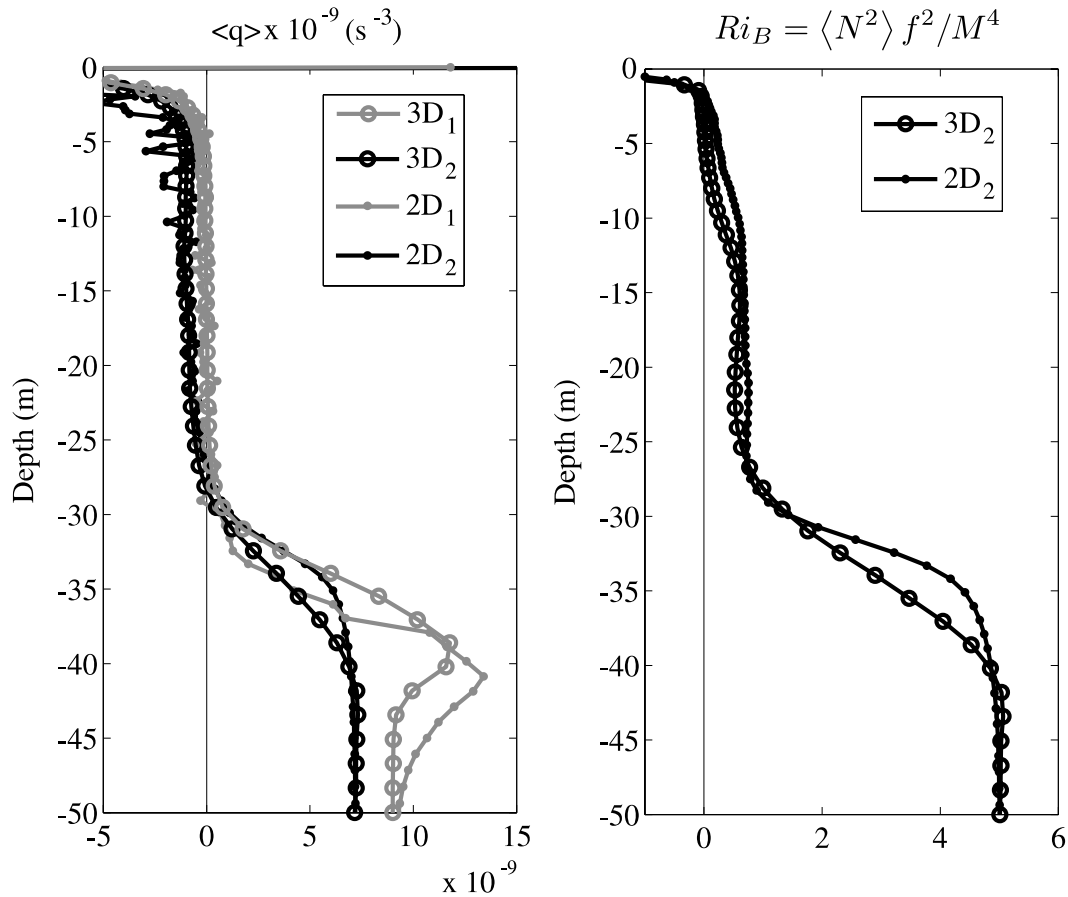
Profiles of the PV and  $Ri_b$  at 10 days



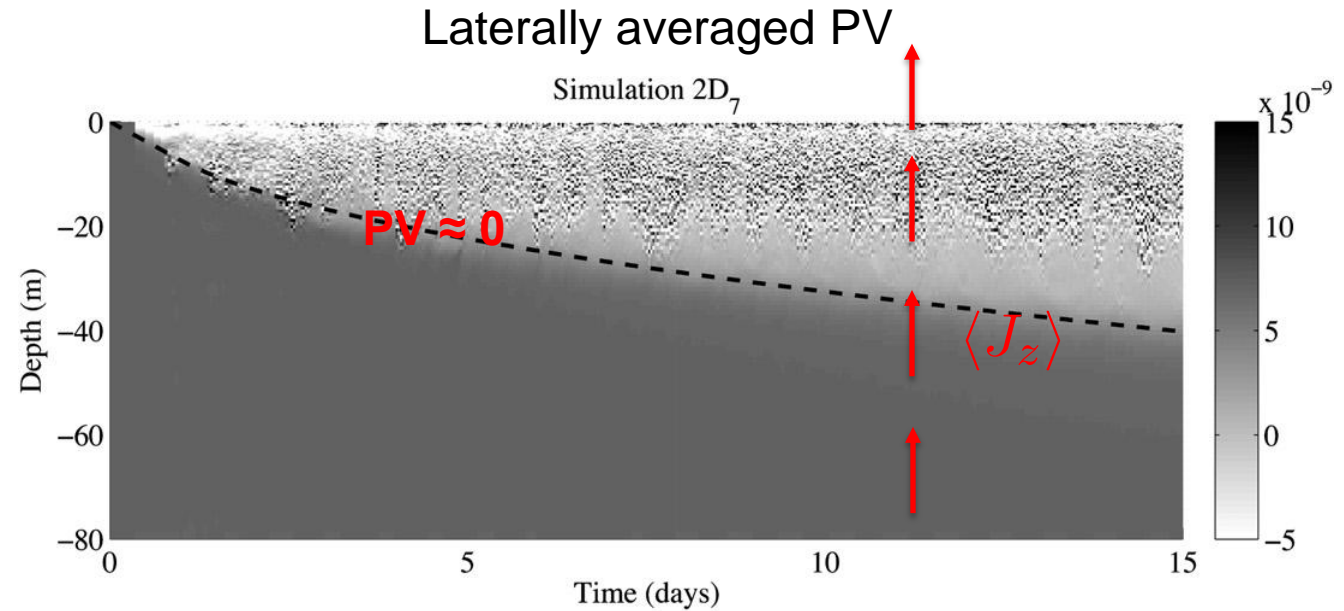
- Buoyancy loss drives a deepening boundary layer with a
  - near-surface layer with negative PV and unstable stratification
  - deeper stratified layer with near zero PV

# Deepening of a stratified boundary layer with near-zero PV

$$B_o > 0, \quad \text{EBF} = 0$$



Profiles of the PV and  $Ri_b$  at 10 days



- A layer with uniform, near-zero PV, that deepens in time requires a non-zero vertically-uniform PV flux:

$$\frac{\partial}{\partial t} \langle q \rangle = - \frac{\partial}{\partial z} \langle J_z \rangle \approx 0$$

In PV=0 layer

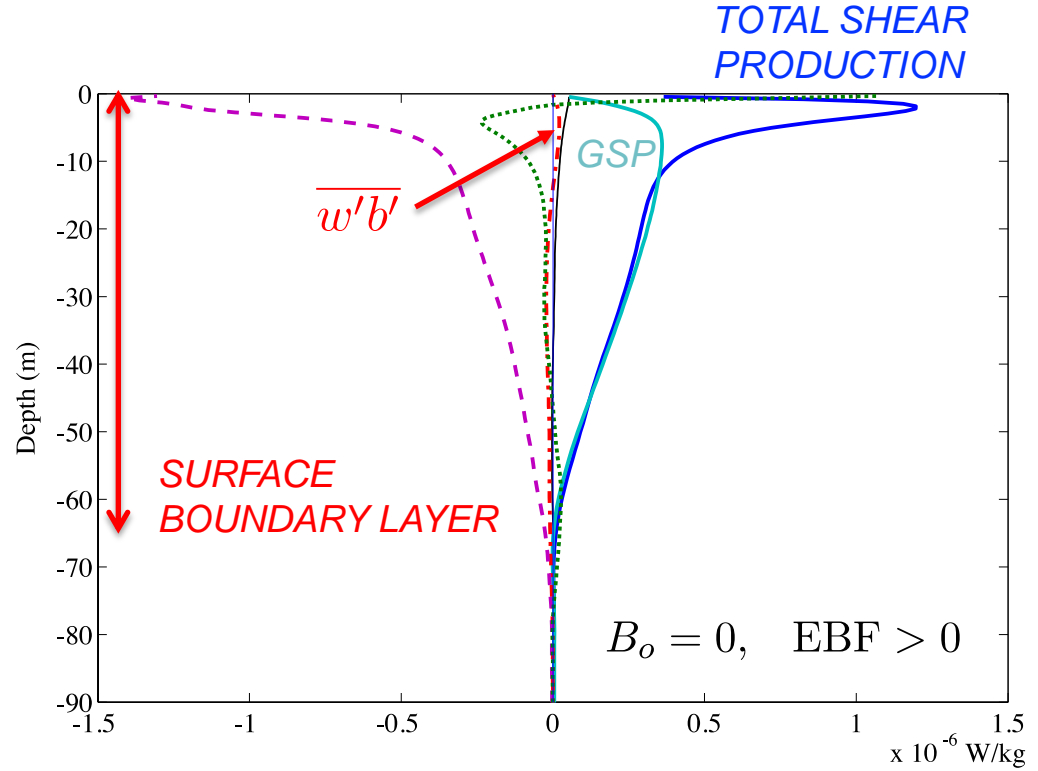
$$\langle J_z \rangle \neq 0$$



# Energetics of forced SI and the vertical structure of boundary layer

- The surface boundary layer is split into two parts distinguished by different sources of TKE.

Simulation forced by down-front winds



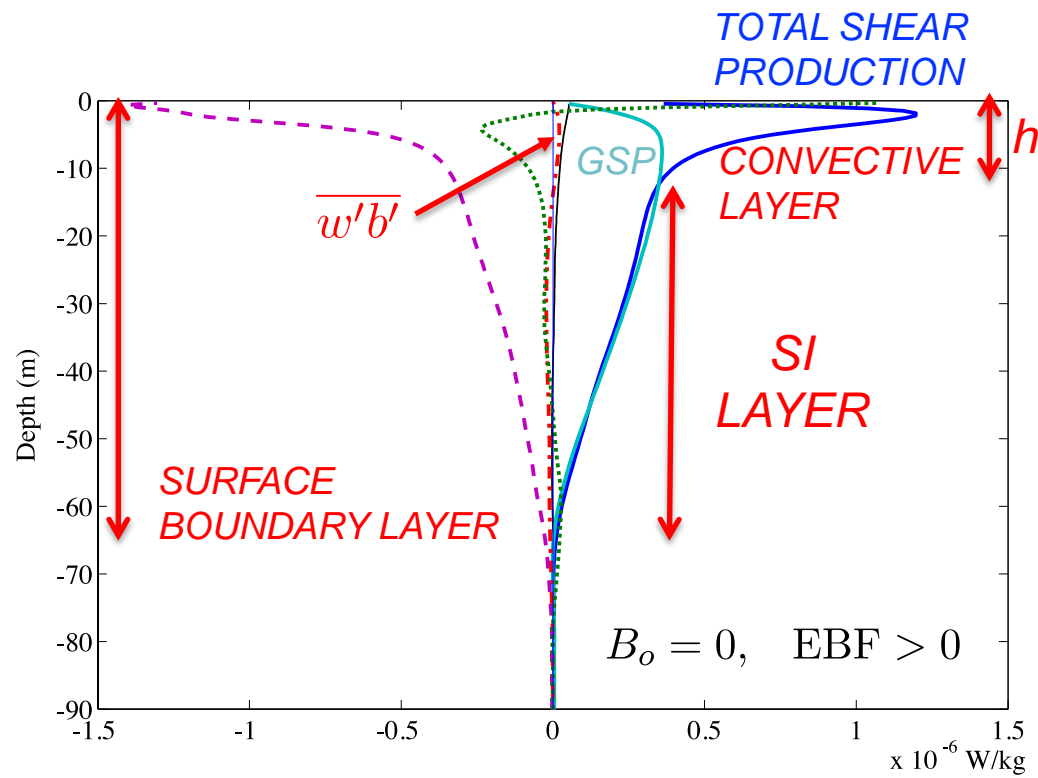
- A near-surface layer of thickness  $h$  that Taylor and Ferrari (2010) termed the *convective layer*
  - Sources of TKE: ageostrophic shear production, the buoyancy flux, etc.
  - Unstably or weakly stratified,  $Ri < 1$
- A *SI layer* for  $-H < z < -h$ 
  - Source of TKE: geostrophic shear production.
  - Stratified with  $Ri \sim 1$

$$\begin{array}{cccc}
 \underbrace{-\left\langle \frac{\partial k}{\partial t} \right\rangle}_{\text{red dashed}} & \underbrace{-\langle u'w' \rangle \frac{S^2}{f}}_{\text{blue solid}} & \underbrace{-\epsilon_{res} - \epsilon_{sgs}}_{\text{purple dashed}} & \underbrace{\langle b'w' \rangle}_{\text{red dotted}} \\
 \underbrace{-\langle \mathbf{u}'w' \rangle \cdot \frac{\partial}{\partial z} \langle \mathbf{u} + \mathbf{u}_g \rangle}_{\text{blue solid}} & & \underbrace{\frac{\partial}{\partial z} \left\langle -w'k' - \frac{1}{\rho_0} w'p' + (\nu + \nu_{sgs}) \frac{\partial k}{\partial z} \right\rangle}_{\text{green dotted}} & 
 \end{array}$$

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 \end{array}$$

# The convective layer depth and the criterion for *forced* SI

$$1 - \frac{h}{H} = \frac{1}{c} \left[ \frac{w_*^3}{|\Delta u_g|^3} + \left( \frac{u_*^2}{|\Delta u_g|^2} \right) \cos \theta \right]^{-2/3} \left[ \frac{h}{H} \right]^{4/3}$$

$$w_* = (B_o H)^{1/3}$$

$$u_* = \sqrt{|\tau_w|/\rho_o}$$

$$\Delta u_g = |\partial \mathbf{u}_g / \partial z| H$$

$$c \simeq 14$$

Taylor and Ferrari (2010); Thomas et al. (2013)

- Limits for  $h/H$  for no surface buoyancy flux,  $B_o=0$

Classical Mixed Layer

$$\frac{u_*}{\Delta u_g} \gg 1 \rightarrow \frac{h}{H} \sim 1$$

- No SI layer
- Turbulence in SBL gets its energy from the wind

SI Turbulence Dominates

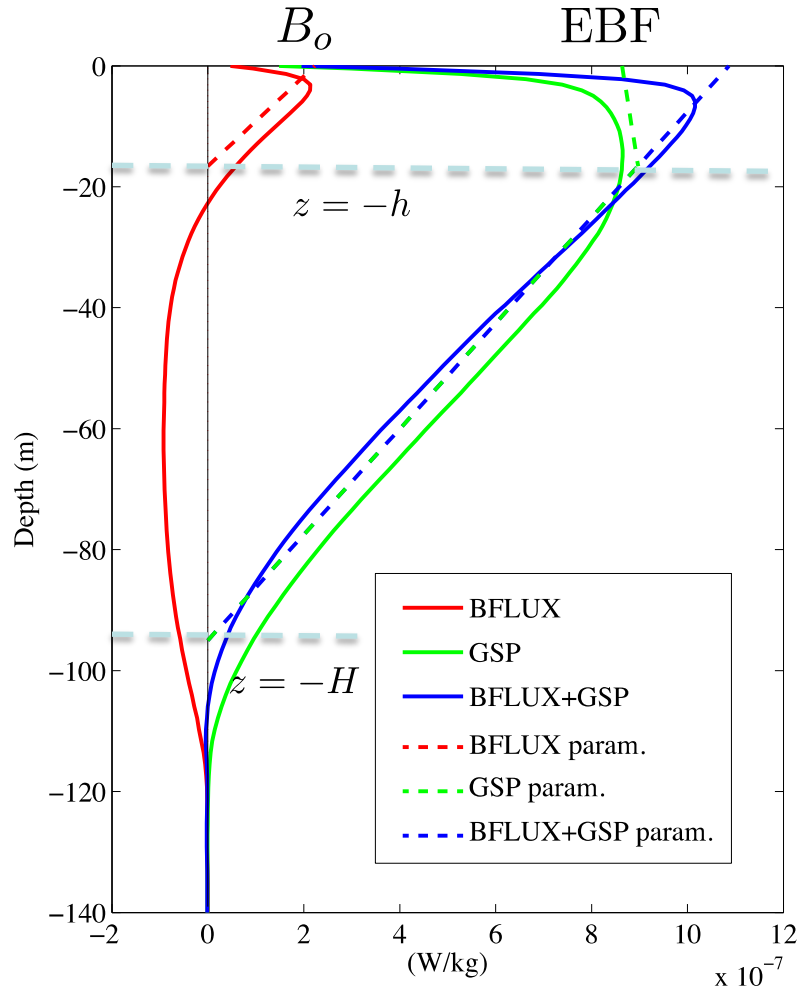
$$\frac{u_*}{\Delta u_g} \ll 1 \rightarrow \frac{h}{H} \ll 1$$

- SI layer fills the SBL
- Turbulence in SBL gets its energy from the geostrophic flow.

→ SI plays an important role in the dynamics of the SBL when  $f_q < 0$  **AND**  $\frac{u_*}{\Delta u_g} \ll 1$



# Parameterization for the energetics of forced SI



- Total SI TKE production:

$$\text{GSP} + \overline{w'b'} \approx (\text{EBF} + B_o) \left( \frac{z + H}{H} \right)$$

- Convection in convective layer:

$$\overline{w'b'} \approx \begin{cases} B_o(z + h)/h & z > -h \\ 0 & z < -h \end{cases}$$

- Rate of KE extraction from geostrophic flow:

GSP

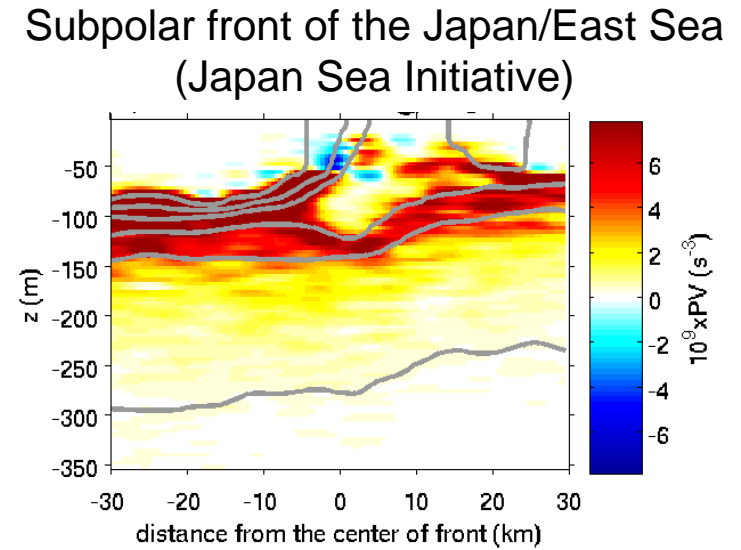
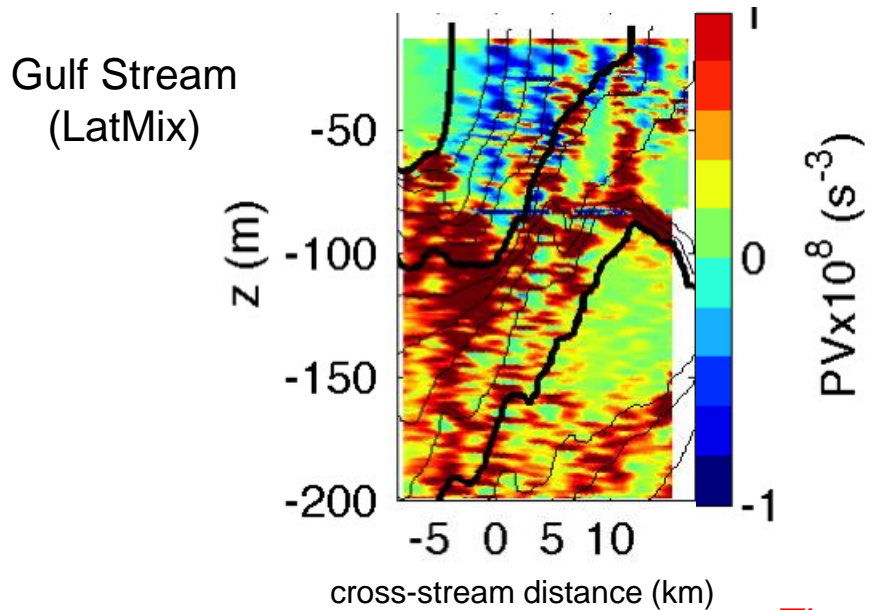
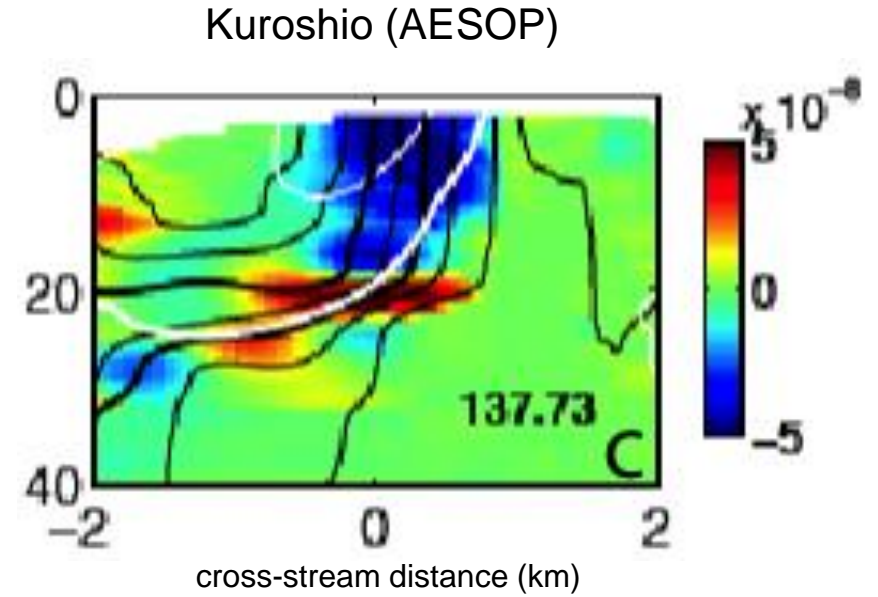
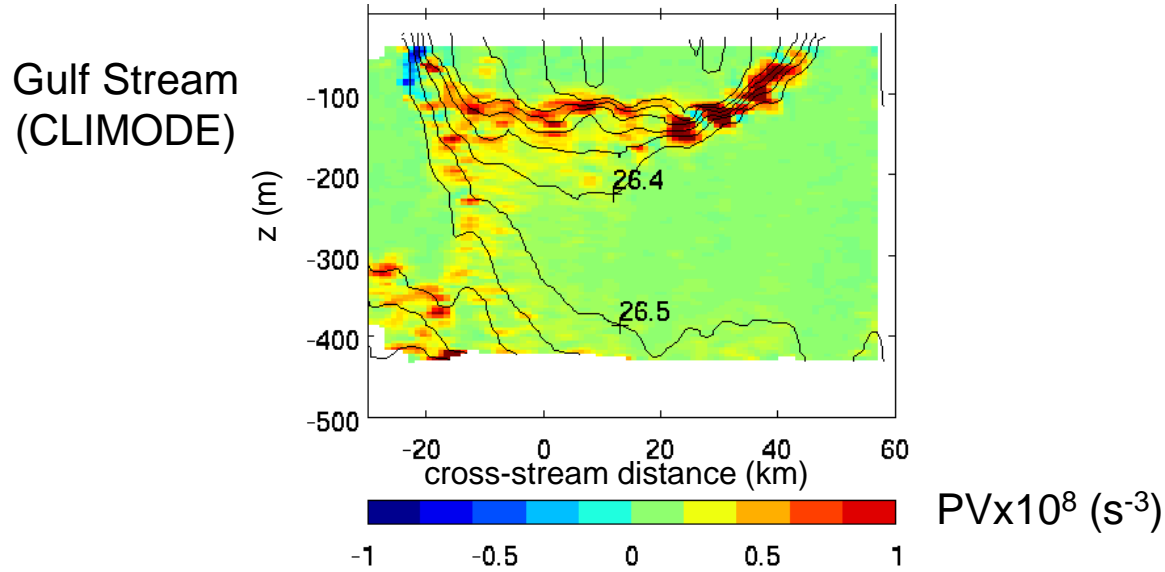
$$\approx \begin{cases} (\text{EBF} + B_o) \left( \frac{z + H}{H} \right) - B_o \left( \frac{z + h}{h} \right) & z \geq -h \\ (\text{EBF} + B_o) \left( \frac{z + H}{H} \right) & -H < z < -h \\ 0 & z \leq -H \end{cases}$$

**EXTRA SOURCE OF BL TKE DISTINCT FROM WINDS, BUOYANCY LOSS, WAVES**

# Identifying characteristics of a boundary layer with SI turbulence

1. PV of the opposite sign of  $f$ , or near zero, in a flow with thermal wind shear.
2. Stable stratification.
3. Absolute vorticity the same sign as  $f$ .
4. Forcing that reduces the PV.
5. Convective depth is a small fraction of the boundary layer depth.
6. Strength of turbulence exceeds that expected from the atmospheric forcing.

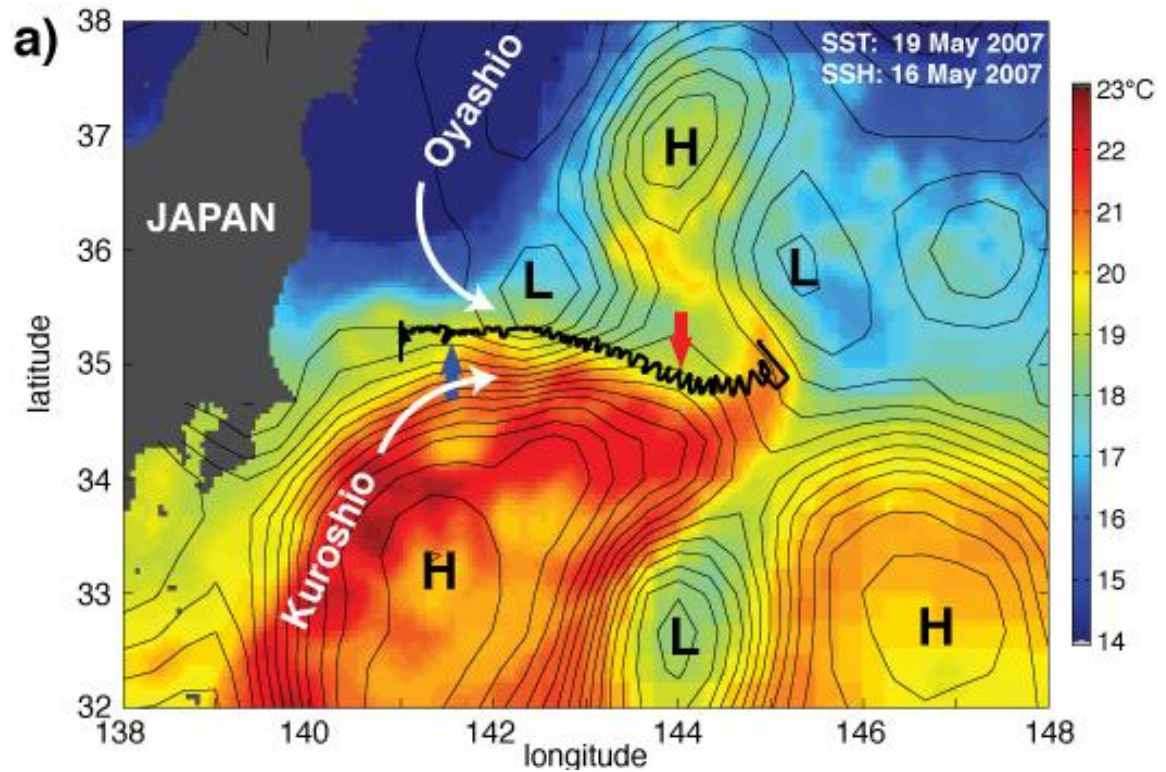
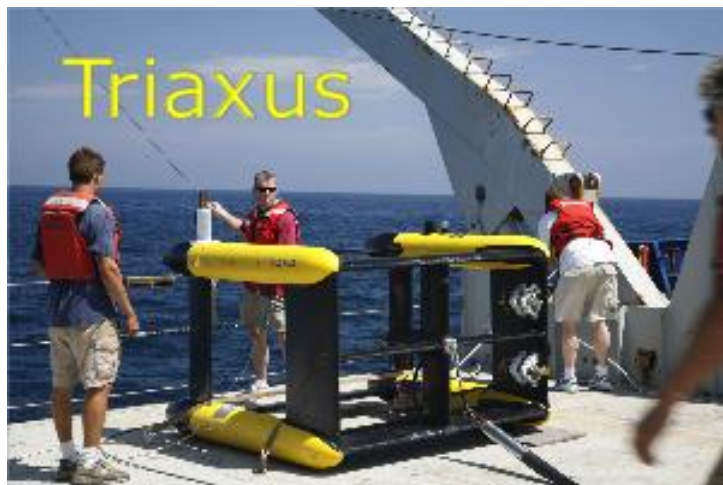
# Symmetrically unstable currents in persistent fronts



Thomas and Lee (2005), D'Asaro et al (2011), Thomas et al (2013), Thomas et al (2016).



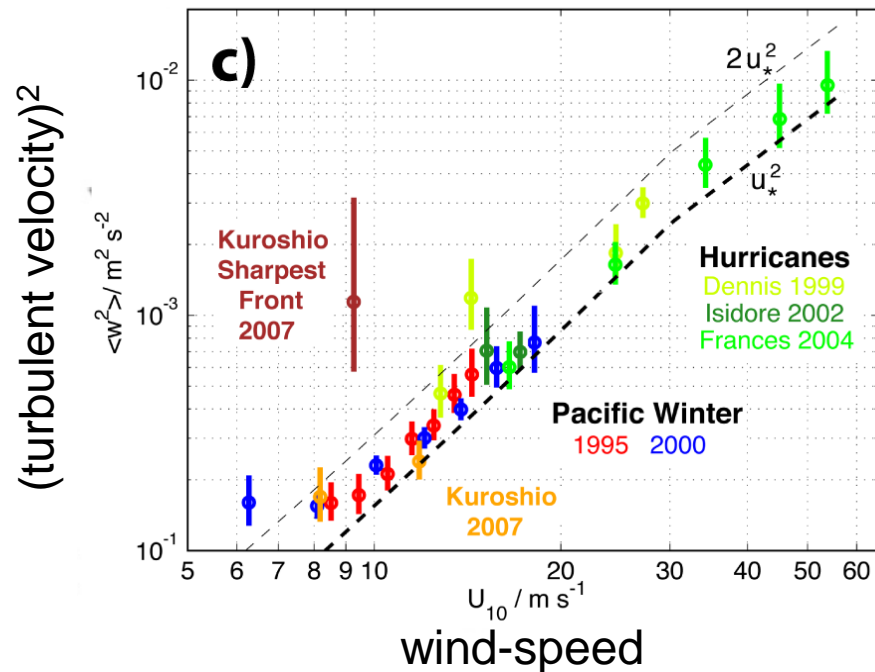
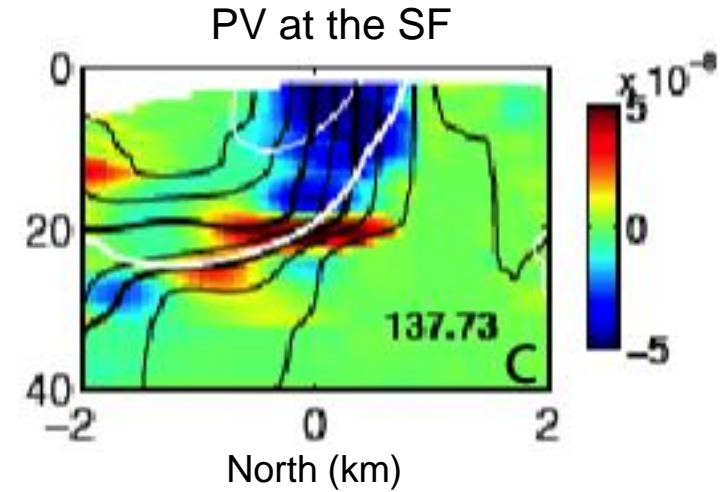
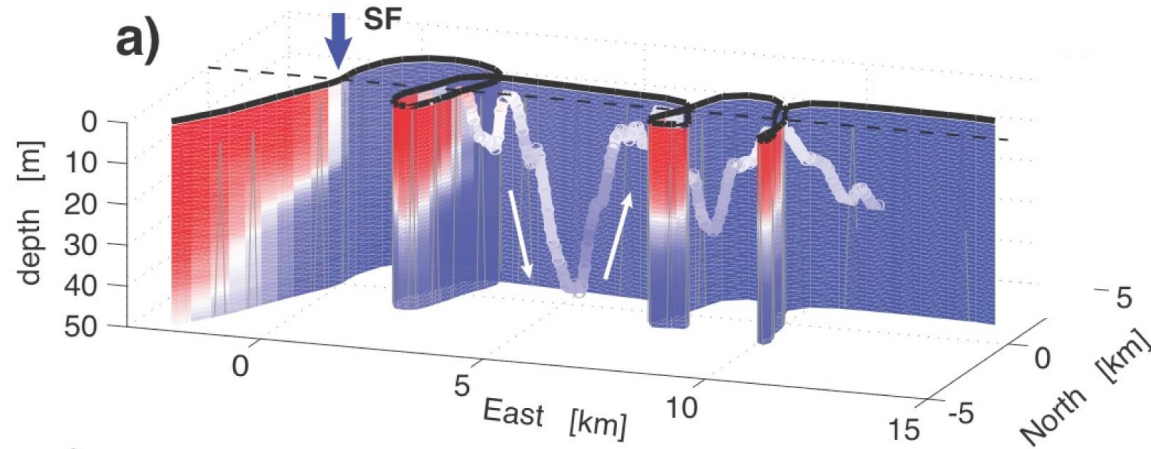
# Direct measurements of turbulence in the Kuroshio



- Eric D'Asaro and Craig Lee (APL/UW) deployed one of their mixed layer floats in the Kuroshio.
- The float was followed by a ship that towed a Triaxis to measure cross-sections of density and velocity at the front.

D'Asaro et al (2011)

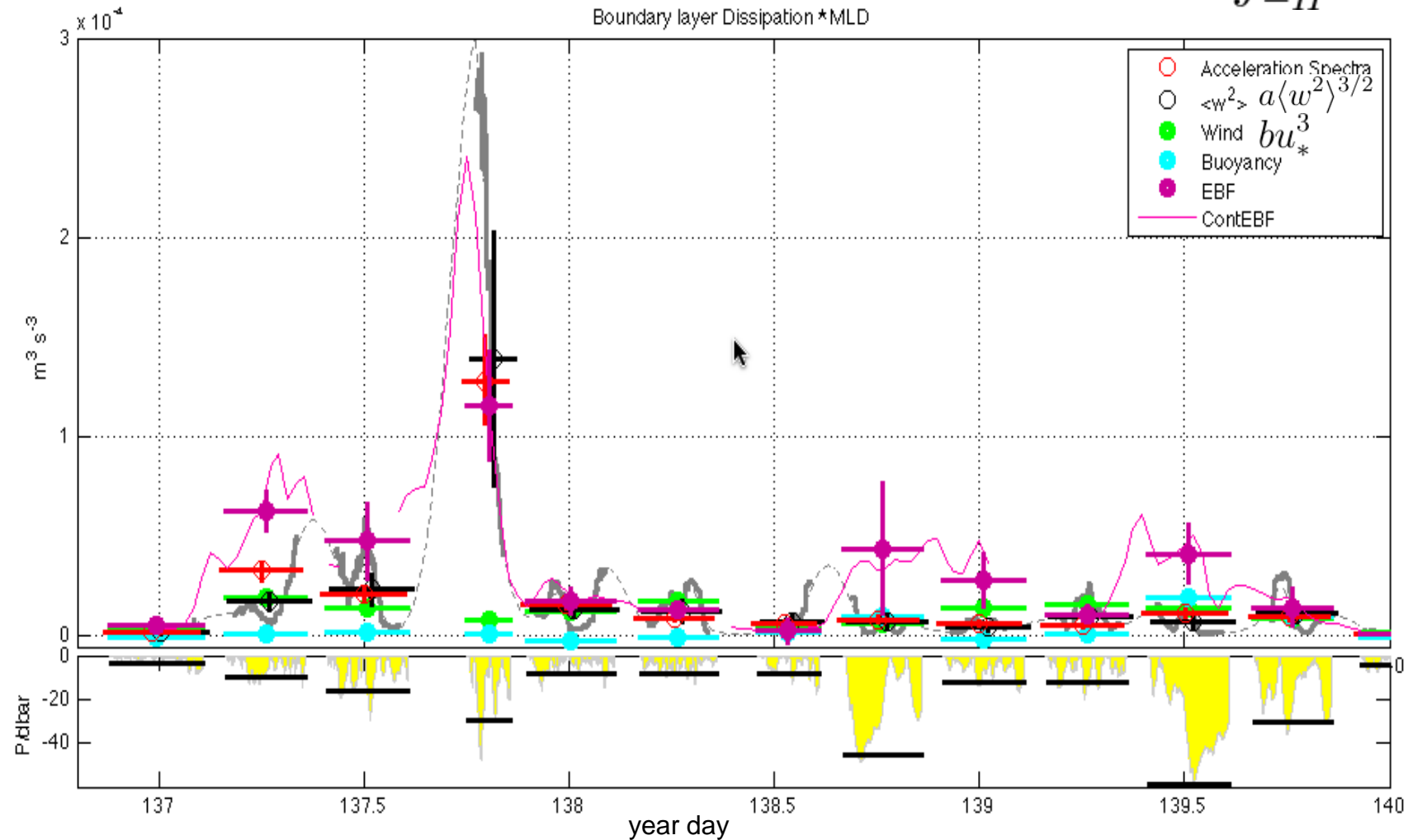
# Enhanced turbulence at a symmetrically unstable front



- At the SF the PV  $<0$  and  $\Delta u_g \sim 0.5 \text{ m/s} \gg u_* \sim 0.01 \text{ m/s}$   $\rightarrow$  the conditions for forced SI are satisfied.
- The KE of turbulence in the mixed layer scales with the square of the wind-speed, *except* at the sharpest front (SF).
- The excess KE must come from the circulation at the front and SI must play a key role in the transfer of energy from the balanced flow to turbulence.

D'Asaro et al (2011)

# Estimates of net dissipation $\int_{-H}^0 \epsilon dz$



- At the SF the net dissipation peaks with the EBF, consistent with the parameterization for the GSP.



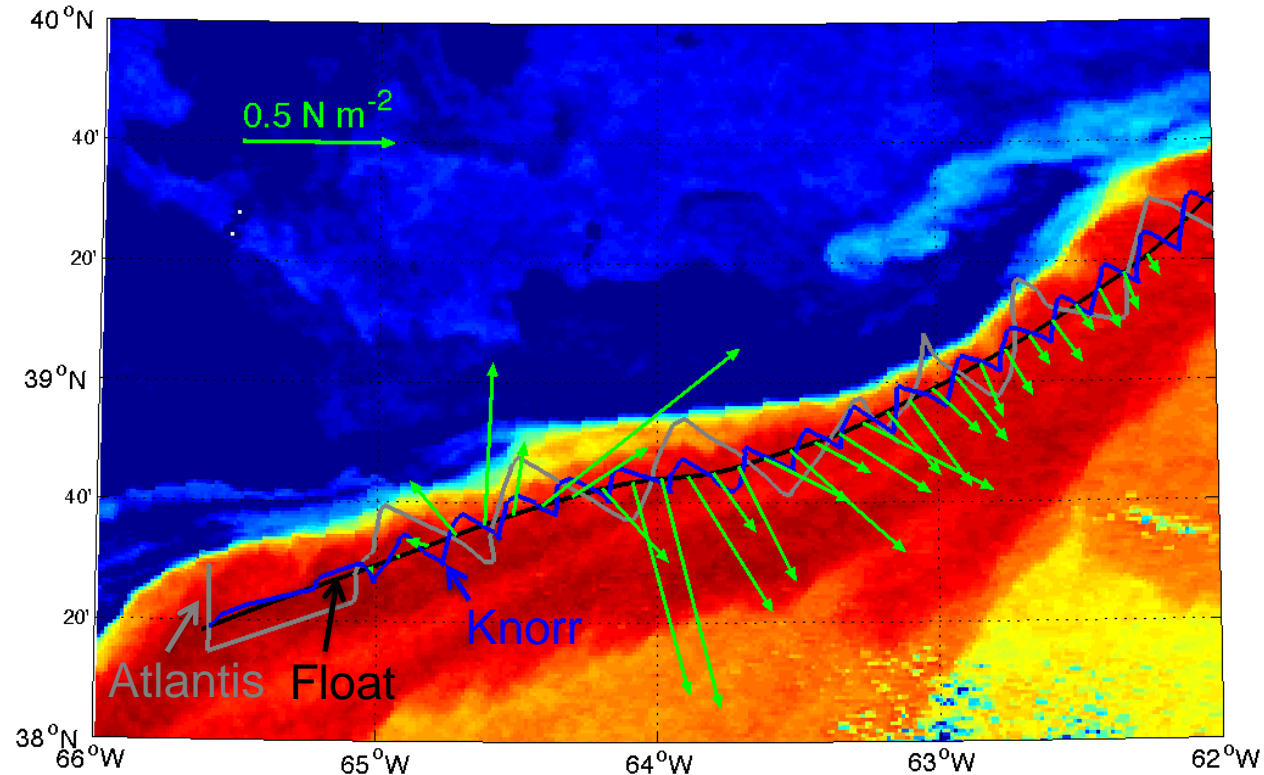
# LatMix 2012 field campaign: SI-drift II

*D'Asaro:*  
Lagrangian  
float

*Lee:* Triaxus

*Klymak:*  
Moving Vessel  
Profiler

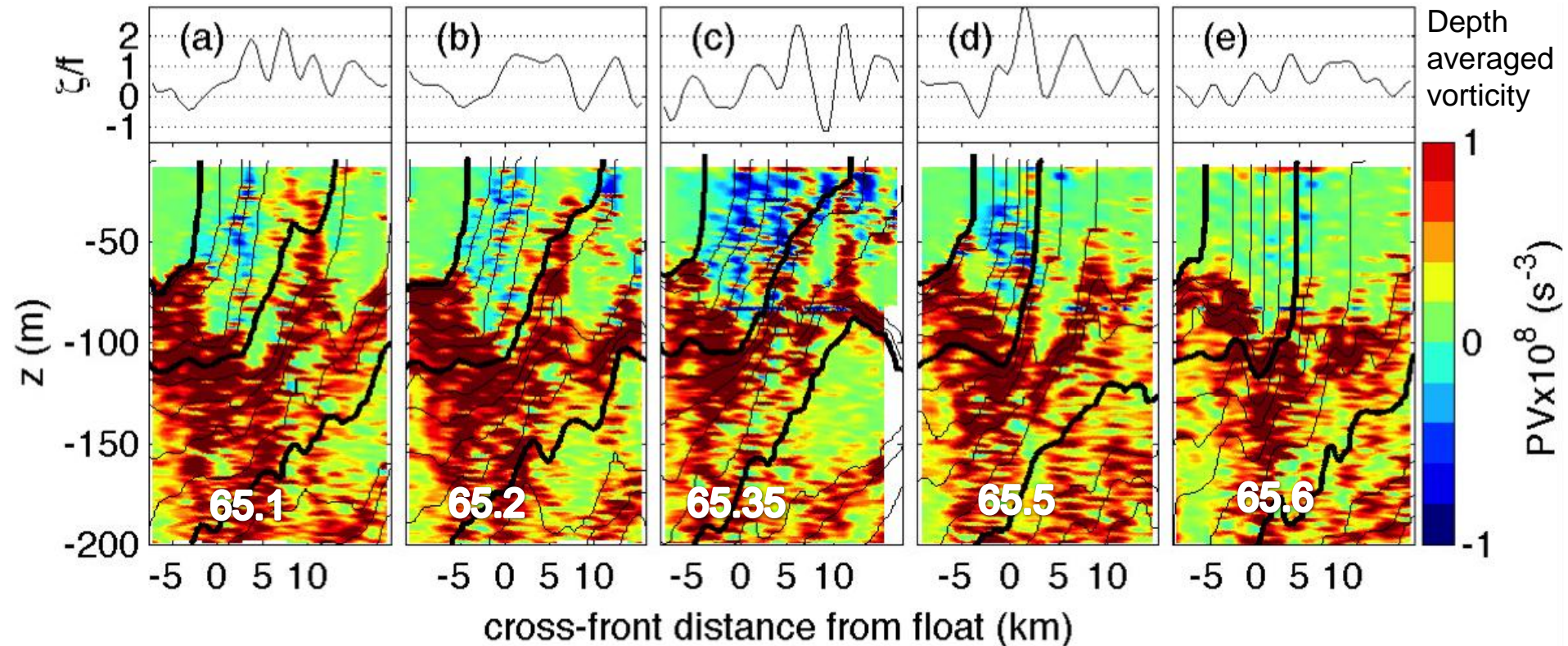
*Sundermeyer:*  
Dye injection  
and tracking



March 5-8

- Mixed layer float deployed in Gulf Stream, followed by two research vessels making cross-front sections.
- Storm passed during the drift, max winds had a significant down-front component → SI.
- Winds strong, rapidly varying, and rotated clockwise with time → inertial oscillations.
- Fluorescein dye was injected near the float and its dispersal was tracked.

# Stratification, vorticity, and potential vorticity

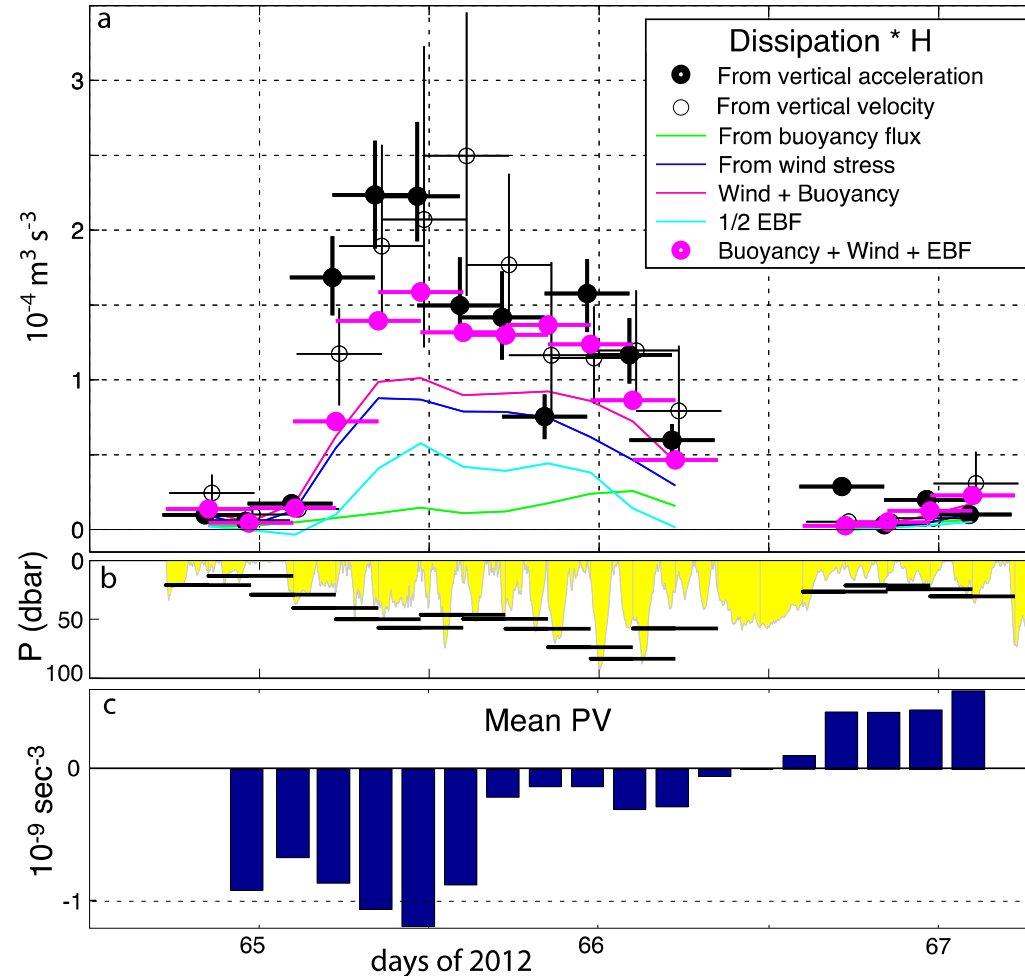


- The vertical vorticity was **positive** while the Ertel potential vorticity was **negative**.
- The stratification in the boundary layer was **stable**.
- Convective layer less than 30 m thick, i.e. thinner than the boundary layer depth.

→ The flow satisfied the necessary conditions for forced SI.

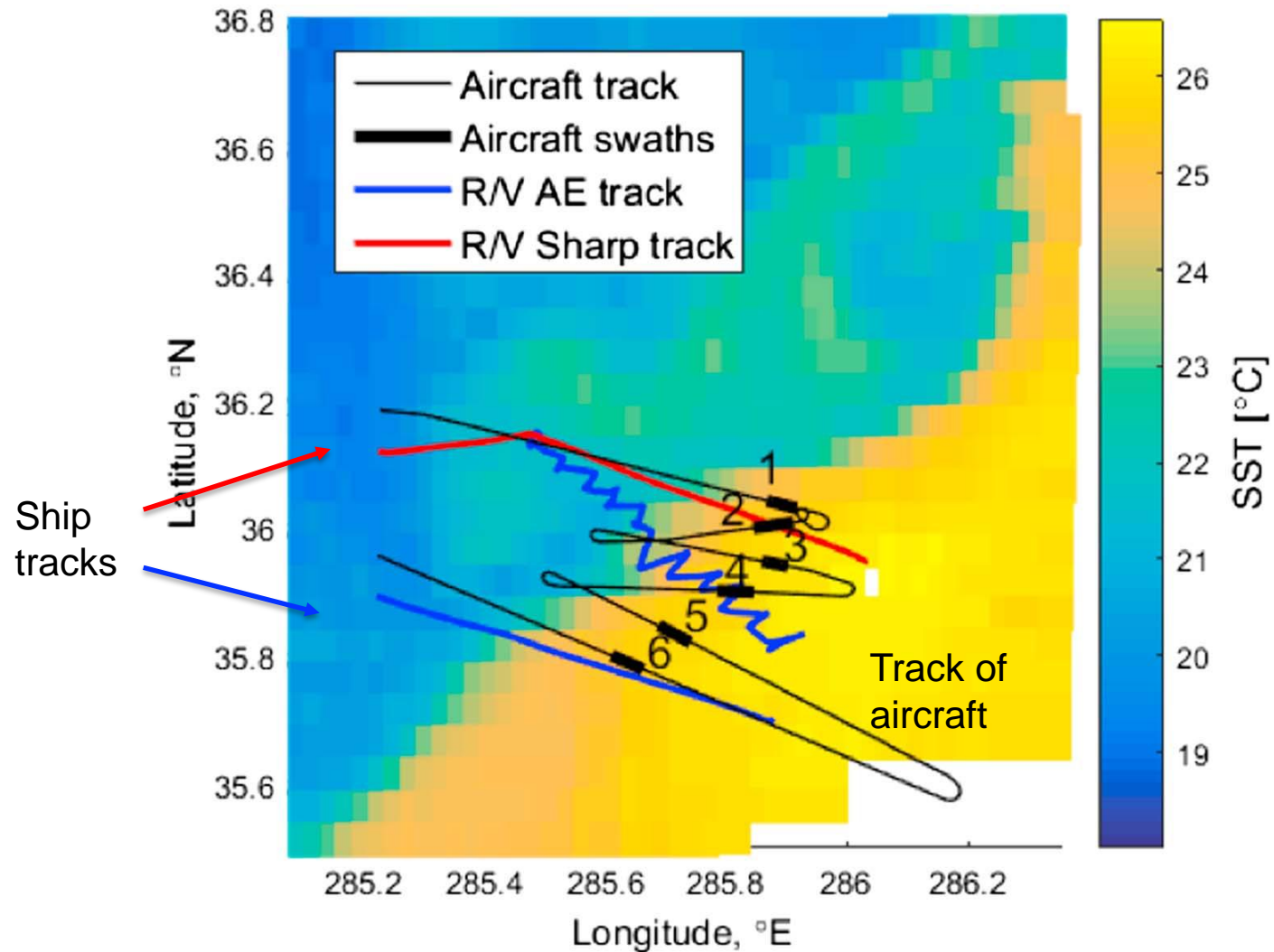
# Excess turbulent dissipation in the boundary layer

PV averaged  
across front  
and upper 60 m



- Strong turbulence in a boundary layer with negative PV and stable stratification.
- Depth integrated dissipation exceeds estimates for TKE production by atmospheric forcing → **front is a source of TKE.**

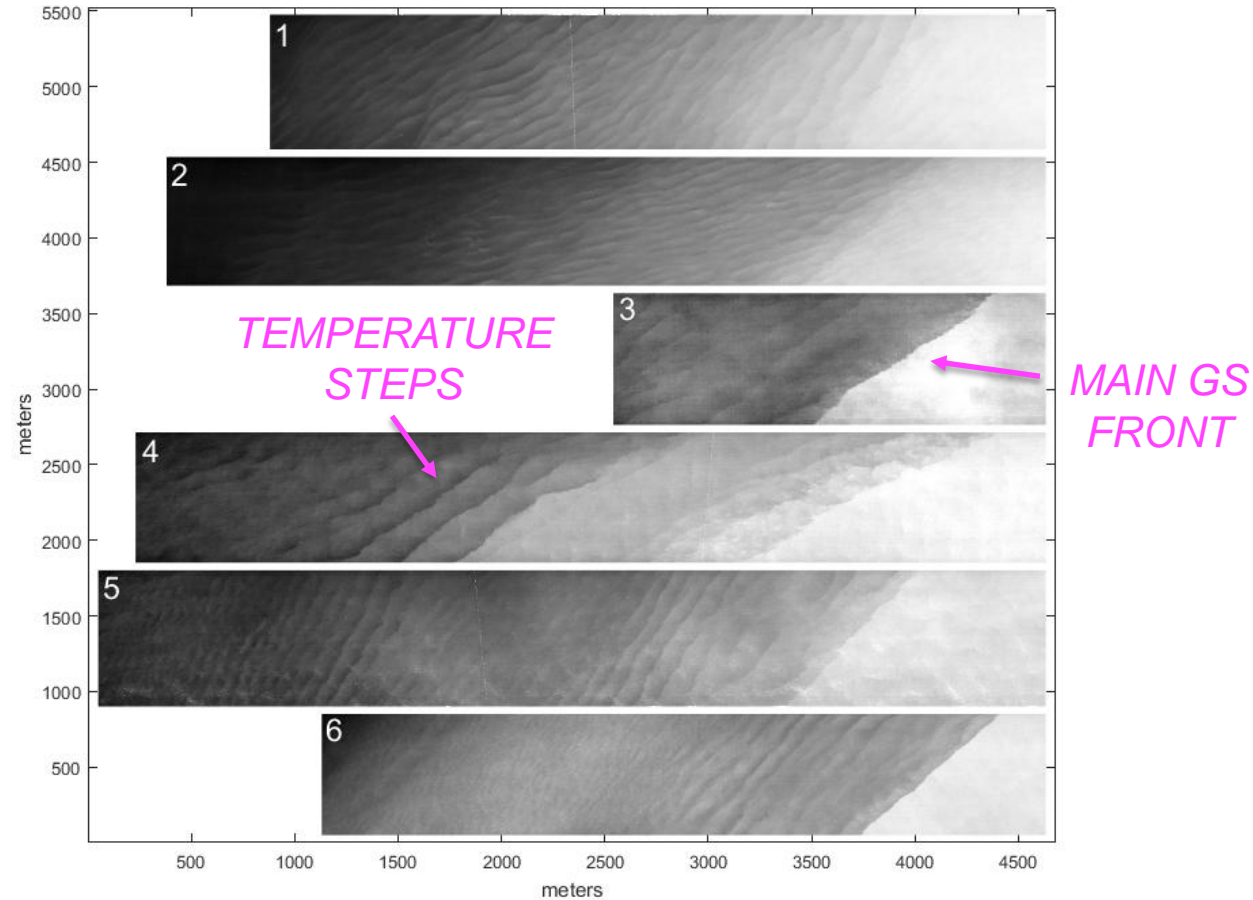
# Aerial observations of SI in the Gulf Stream



- The aim of the CASPER field campaign in Oct 2015 was to study air-sea interactions using a combination of airborne SST imagery and in situ hydrography and velocity measurements in the proximity of the Gulf Stream.

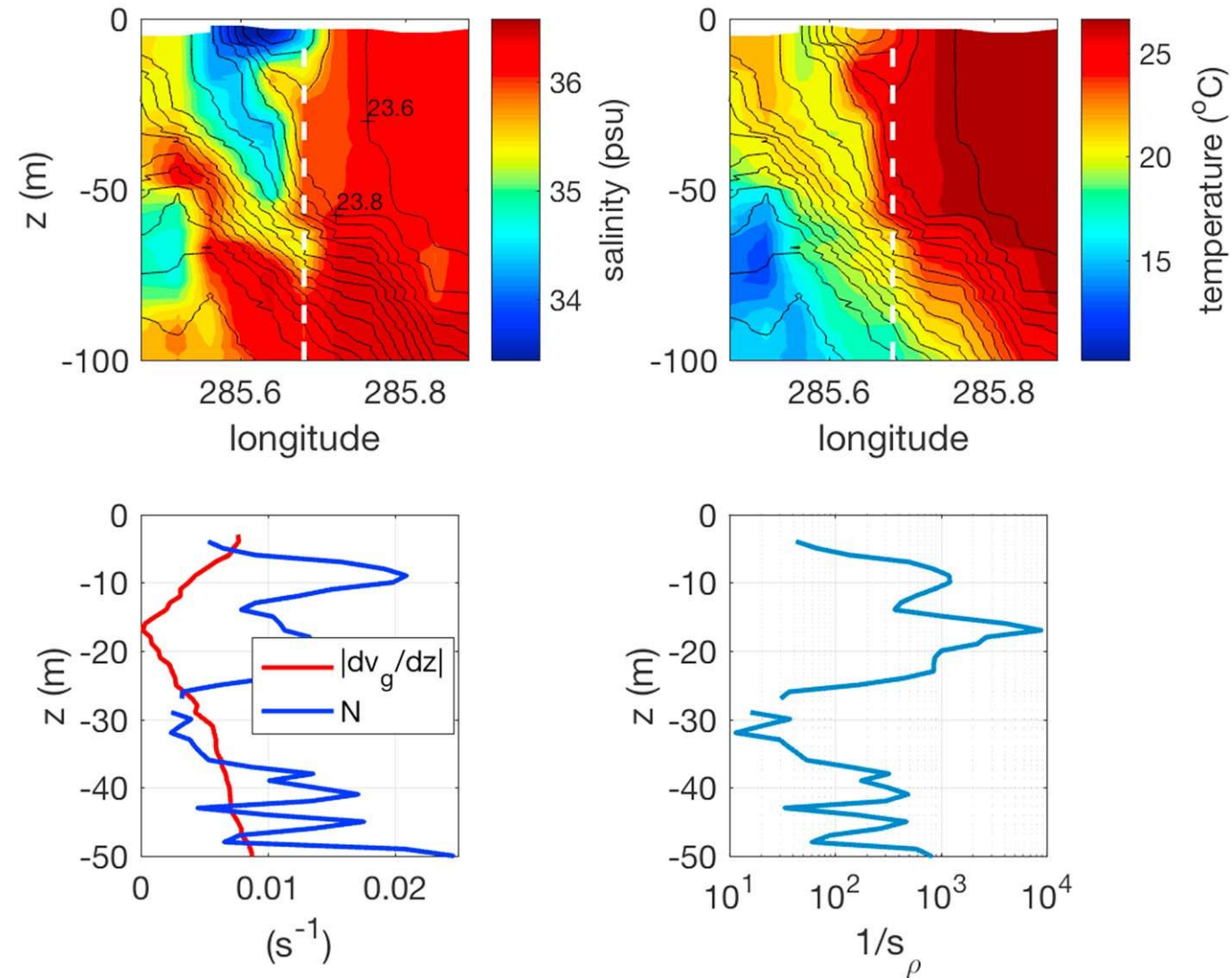


# Coherent structures in Gulf Stream airborne SST imagery



- Step-like structures are visible in SST on the cold side of the front.
- The steps tend to be aligned with the axis of the main front.
- Width of the steps ~100 m.
- Surface waves propagate at an oblique angle to steps—not Langmuir cells

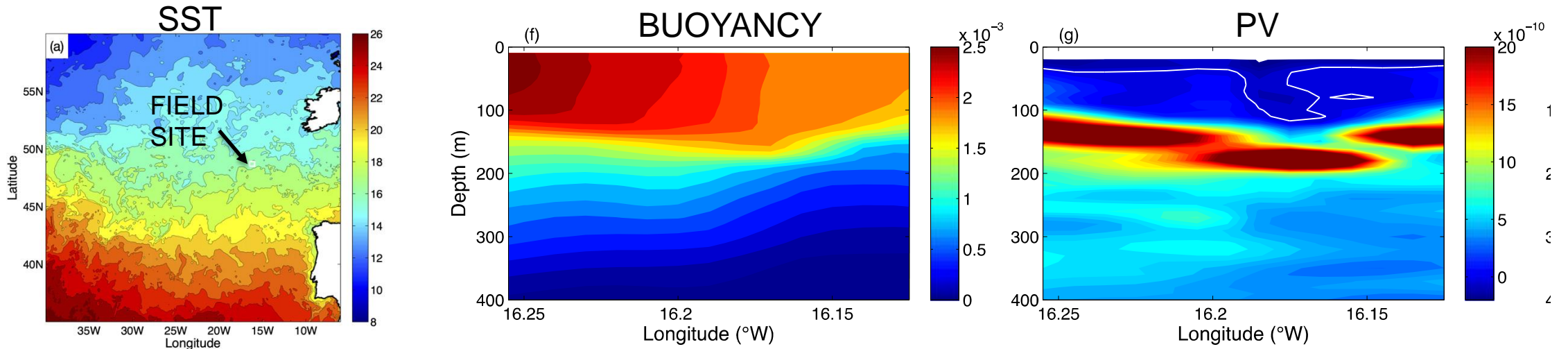
# SI unstable flow in the BL near where steps were observed



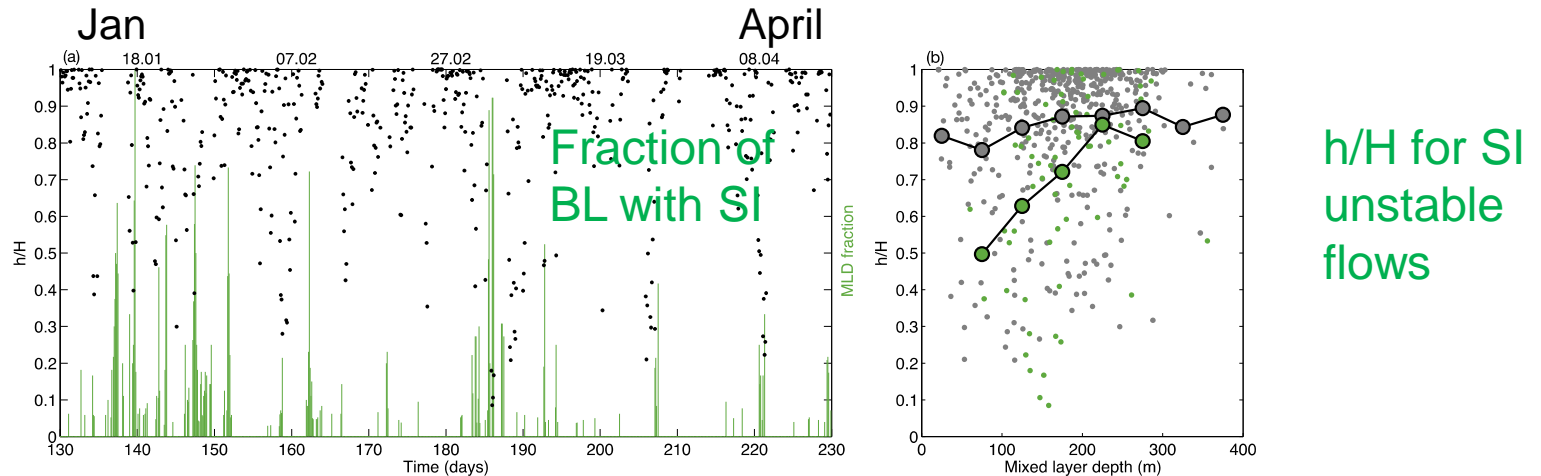
- Near the temperature steps  $Ri_b$  is less than one over a thin BL  $\sim 5$  m thick.
- The width of SI cells  $H_{BL}/(\text{isopycnal slope}) \sim 200$  m is similar to the observed width of the temperature steps.

Savelyev et al GRL (2018)

# Symmetrically unstable ephemeral fronts in the NE Atlantic



- Porcupine Abyssal Plain
  - Lateral buoyancy gradients 10-100 times weaker than those in WBCs.
  - Ephemeral fronts
  - Moderate EKE



• Glider based hydrographic observations made as part of the OSMOSIS project reveal that during the late fall and winter when the BL was deepening, flows in the BL were often symmetrically unstable.

• During these periods  $0.5 < h/H < 0.8$

Thompson et al (2016)

# Secondary circulation driven by SI momentum fluxes and shear dispersion

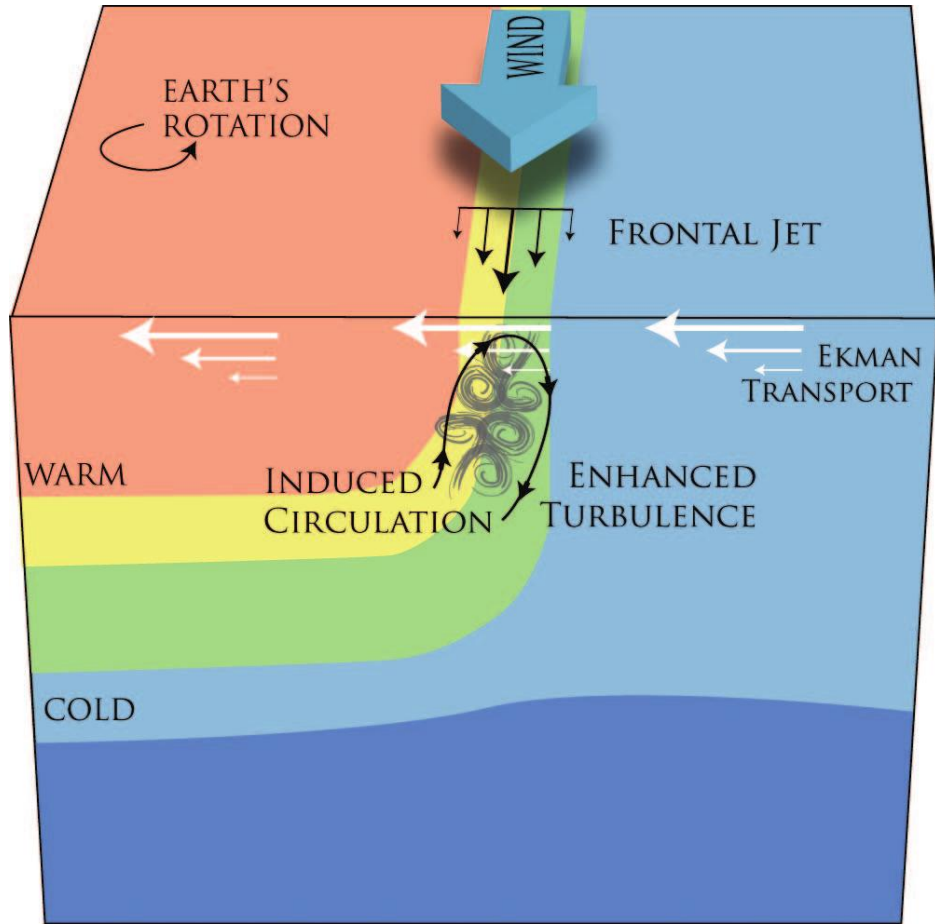


Fig. from D'Asaro et al (2011)

- Vertical momentum fluxes associated with SI turbulence drive a cross-front secondary circulation following a turbulent Ekman balance

$$0 = -\frac{\partial}{\partial z} \overline{u'w'}_{SI} + f\bar{v}_{SI} \quad \text{Taylor and Ferrari (2010)}$$

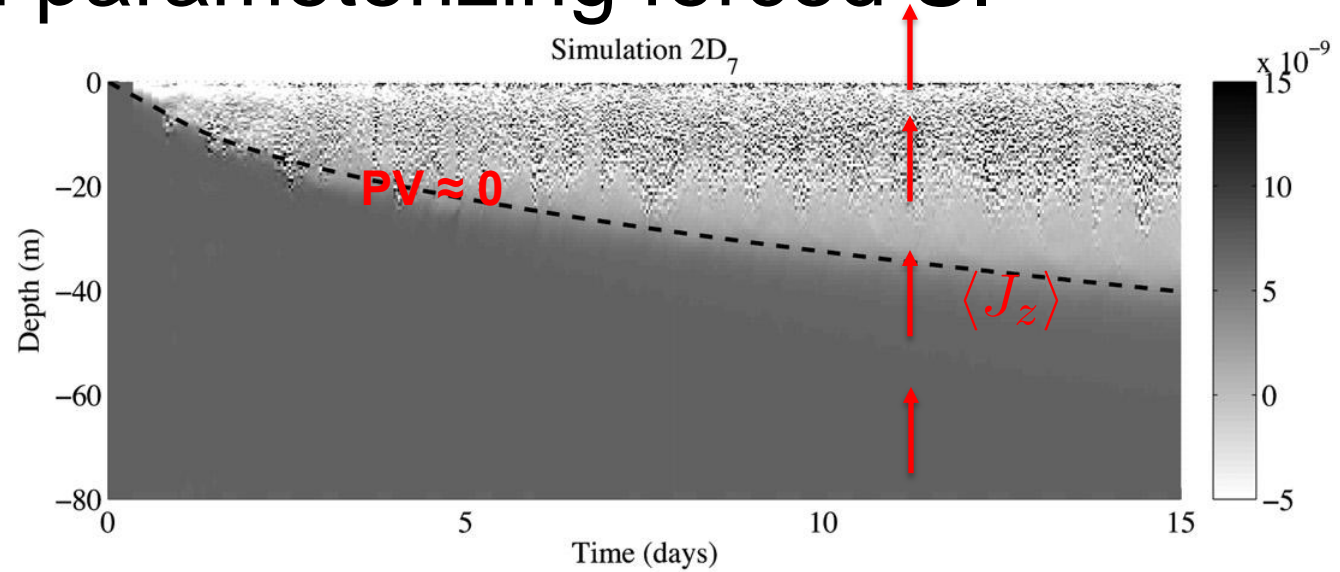
- Follows the same force balance governing the secondary circulation associated with Turbulent Thermal Wind (TTW) dynamics (e.g. Sullivan and McWilliams 2018).
  - SI-turbulence can drive TTW secondary circulations
- The combination of the shearing motion of the TTW secondary circulation and small-scale turbulence can result in the lateral dispersal of tracers through shear dispersion (Crowe and Taylor 2018).



# Considerations when parameterizing forced SI

- Goals---a parameterization for forced SI should
1. deepen the BL while keeping the PV near zero;
  2. result in a vertically-uniform, non-zero PV flux

Can a parameterization using down-gradient turbulent fluxes meet these goals?



## FRONTAL ZONE

$$f \frac{\partial}{\partial z} \langle u_g \rangle = -\frac{\partial}{\partial y} \langle b \rangle = \text{const}$$

$$\langle \zeta \rangle = 0 \quad \langle q \rangle = 0$$

## DOWN-GRADIENT FLUX FORMULATION

$$F_x = \frac{\partial}{\partial z} \nu \frac{\partial}{\partial z} \langle u \rangle$$

$$Pr = \frac{\nu}{\kappa}$$

$$D = \frac{\partial}{\partial z} \kappa \frac{\partial}{\partial z} \langle b \rangle$$

## PV FLUX

$$\langle J_z \rangle = -\frac{\partial \langle b \rangle}{\partial y} F_x - f \mathcal{D}$$

$$= f \frac{\partial \langle u_g \rangle}{\partial z} \left( \frac{\partial}{\partial z} \nu \frac{\partial \langle u \rangle}{\partial z} \right) - f \frac{\partial}{\partial z} \kappa \frac{\partial \langle b \rangle}{\partial z}$$

If  $\langle u \rangle = \langle u_g \rangle$  and  $\langle q \rangle = 0 \rightarrow \langle J_z \rangle = f \frac{\partial \langle b \rangle}{\partial z} \frac{\partial \kappa}{\partial z} (Pr - 1)$

- A down-gradient formulation for turbulent fluxes with a Prandtl number (Pr) of one is incapable of parameterizing forced SI.
- Parameterizations with  $Pr \neq 1$  (e.g. Bachman et al 2017) or non-local transports should be used.

# Conclusions

- *SI-turbulence* is unique in that it derives its energy from the geostrophic flow rather than the atmospheric forcing and therefore is a sink of KE for the circulation. SI-turbulence:
  - is characterized by coherent structures and small-scale turbulence in a stratified BL;
  - forms in baroclinic currents with PV the opposite sign of  $f$ , stable stratification, and centrifugally stable flow;
  - is driven by buoyancy loss, down-front winds, or the frictional spin-down of currents flowing in the direction of Kelvin waves;
  - dominates the BL turbulence only when the convective layer depth is a small fraction of the BL depth;
  - extracts KE from geostrophic flows at a rate that scales with the Ekman buoyancy flux and surface buoyancy loss;
  - has been observed in strong and weak fronts in the SBL and in the BBL of a deep boundary current;
  - contributes to closing the KE budget of the circulation, mixing tracers laterally, and setting the PV that enters the ocean interior.