

Towards revision of conventional theory of turbulence in stratified flows

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Frontiers in Oceanic, Atmospheric and Cryospheric Boundary Layers

University of California, Santa Barbara, 21-25.05. 2018



CONTENT

Current paradigm of the theory of turbulence basically follows Kolmogorov (**K-1941**). However, Kolmogorov focused on the shear-generated turbulence in neutrally stratified flows

Extension of his paradigm to stably-stratified and boyancy-generated turbulence was done by his followers without proof

New vision / paradigm of turbulence accounts for its SELF-CONTROL IN STABLE STRATIFICATION AND SELF-ORGANISATION IN UNSTABLE STRATIFICATION

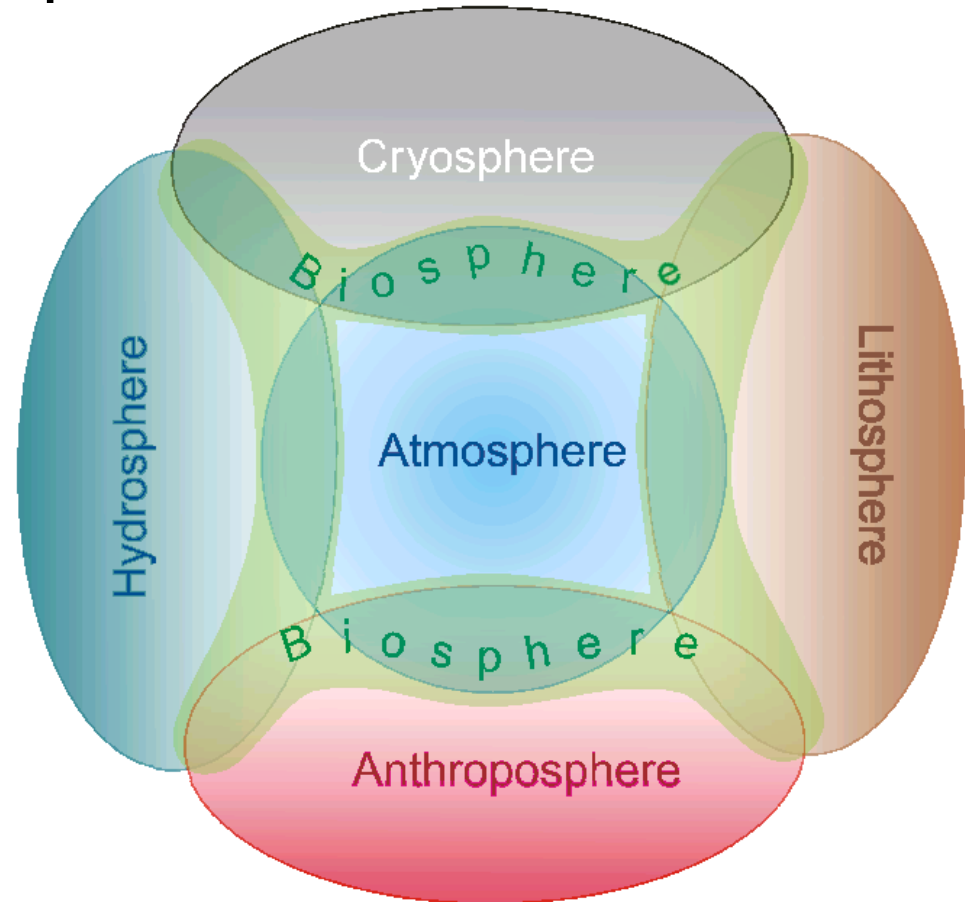
Turbulence and planetary boundary layers (PBLs)

Turbulence is strong in **PBLs** and **weak** beyond (in the free atmosphere and ocean thermocline), coexists with mean flow, and makes our planet **comfortable for life**

PBLs (green lenses)

couple geospheres into Earth system

host 99% of biosphere and the anthroposphere



Turbulence in climate-bio/technosphere system

Vitally important:

- **Performs vertical transports** across the atmosphere, and hydrosphere
- **Couples geospheres into climate-biosphere-technosphere system**

Ever present in harmony with mean flow:

- Doesn't degenerate due to self-control **even in super-critical stability**
- Doesn't dominate due to its self-organization **even in calm-weather turbulent convection**



TURBULENCE



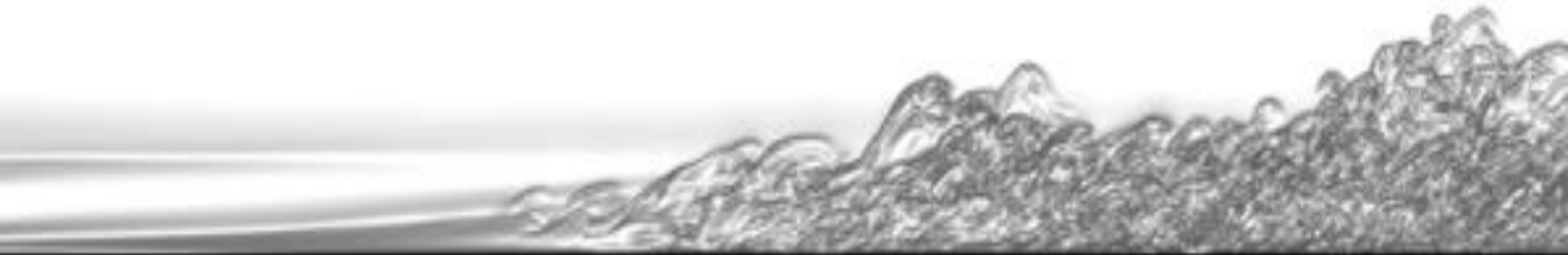
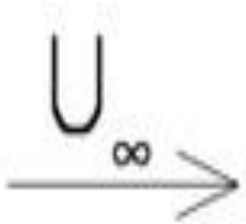
Leonardo da Vinci: **Mult-scale** chaotic eddies

Richardson (1920): **Direct energy cascade**

Kolmogorov (1941): **Quantified the cascade**



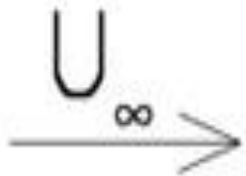
“Engineering” TURBULENCE in neutral boundary-layer flow over flat plate



Common vision (**paradigm**): dynamic instability causes energy cascade from large to small eddies towards dissipation: **chaos out of order** (Kolmogorov, 1941)

Conventional vision: Kolmogorov-1941

- Two types of motion: **regular mean flow** + **chaotic turbulence**
- Instability of sheared flow → large eddies break down to produce smaller 3-D eddies → **direct (forward) cascade** of Turbulent Kinetic Energy (TKE) → towards viscous dissipation at smallest scales → the **dissipation rate** defined as $TKE / t_T \sim (TKE)^{3/2}/z$
- Turbulence **energetics** fully defined by the **TKE budget Equation**
- Turbulent **fluxes** = **gradients** multiplied by **exchange coefficients**



**Shear-generated turbulence
in neutrally-stratified flows**



Followers of Kolmogorov adopted his paradigm to stratified flows word for word (missing Turbulent Potential Energy and many other principal features)

Thanks to realistic results **for weakly / moderately stratified flows** the paradigm still underlies calculation of turbulence in geophysical, astrophysical and engineering applications, e.g., in atmospheric surface layer (since Monin & Obukhov, 1954)

Modern experiments and DNS reveal

- **non-gradient fluxes in strongly stable and unstable lows**
- **inverse energy cascade (anarchy) in convective turbulence**

The time is ripe to revise old paradigm, and to develop new theory of stratified turbulence



Breakdown of laminar flow

Very narrow spectrum: **Pre-TURBULENCE?**



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Convective turbulence in the atmosphere



**Buoyant plumes do not break down
in contrast to shear-generated eddies!**



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Turbulence in economy, policy, life ... and art



Vincent van Gogh *The Starry Night*, June 1889, The Museum of Modern Art, New York

The word **TURBULENCE** is used in very wide sense
with no exact definition even in physics!



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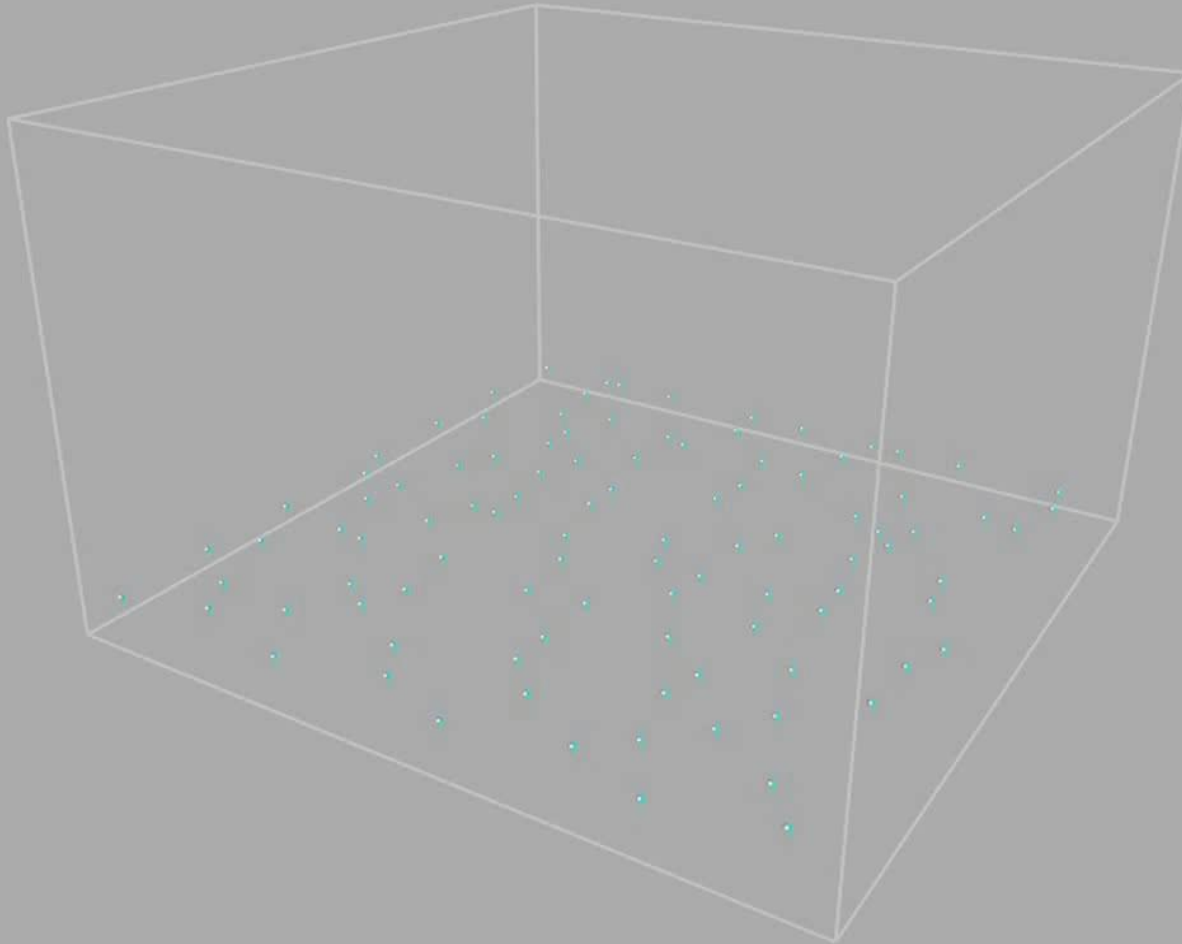
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CONVECTIVE TURBULENCE

in Procrustean bad of conventional theory



Buoyant plumes
do not break
down but merge
to built larger
plumes → inverse
energy cascade
to self-organised
cells or rolls →
order out of chaos
= **ANARCHY**
TURBULENCE



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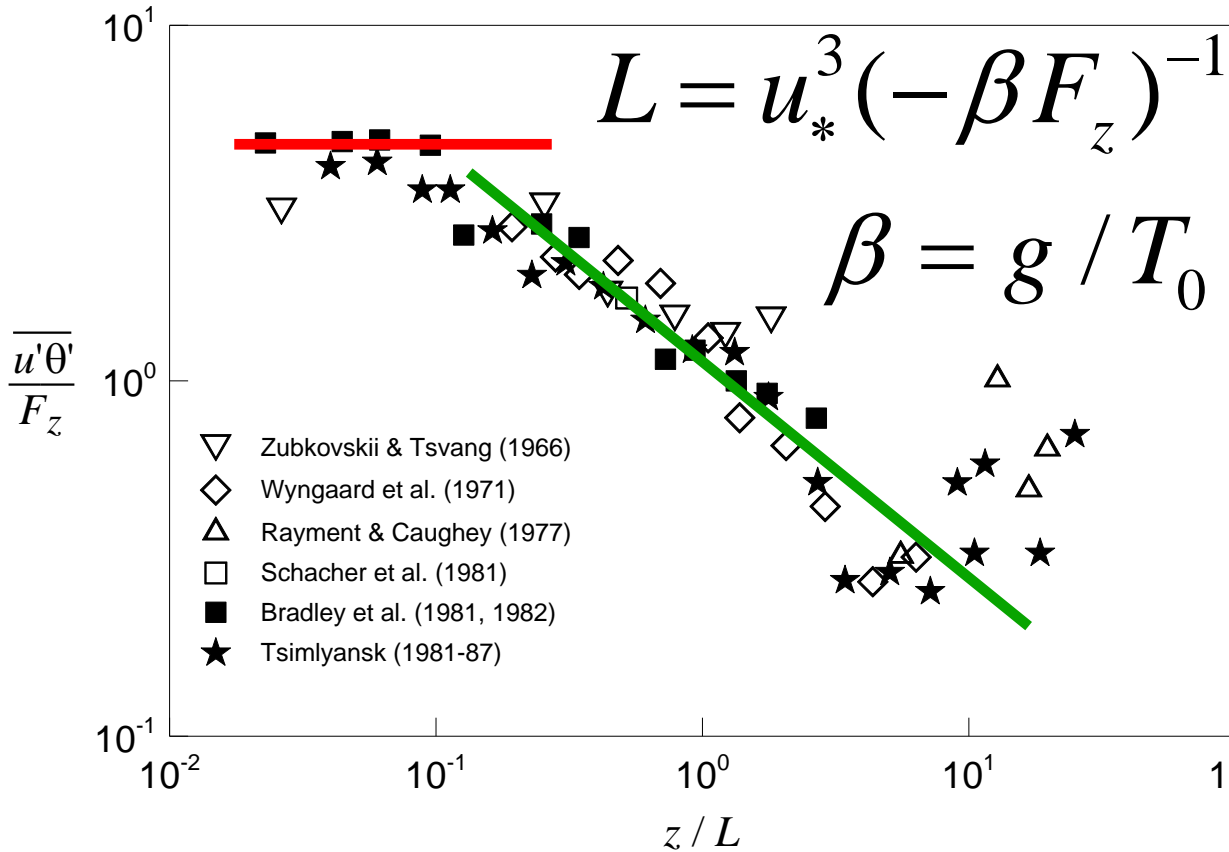


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Ratio horizontal/vertical heat flux F_x/F_z vs z/L



MO-1954 based on K-1941, is valid in shear-generated turbulence layer

$z_0 < z < 0.1L$ but fails in the anarchy turbulence layer

$0.1L > z > 10L$

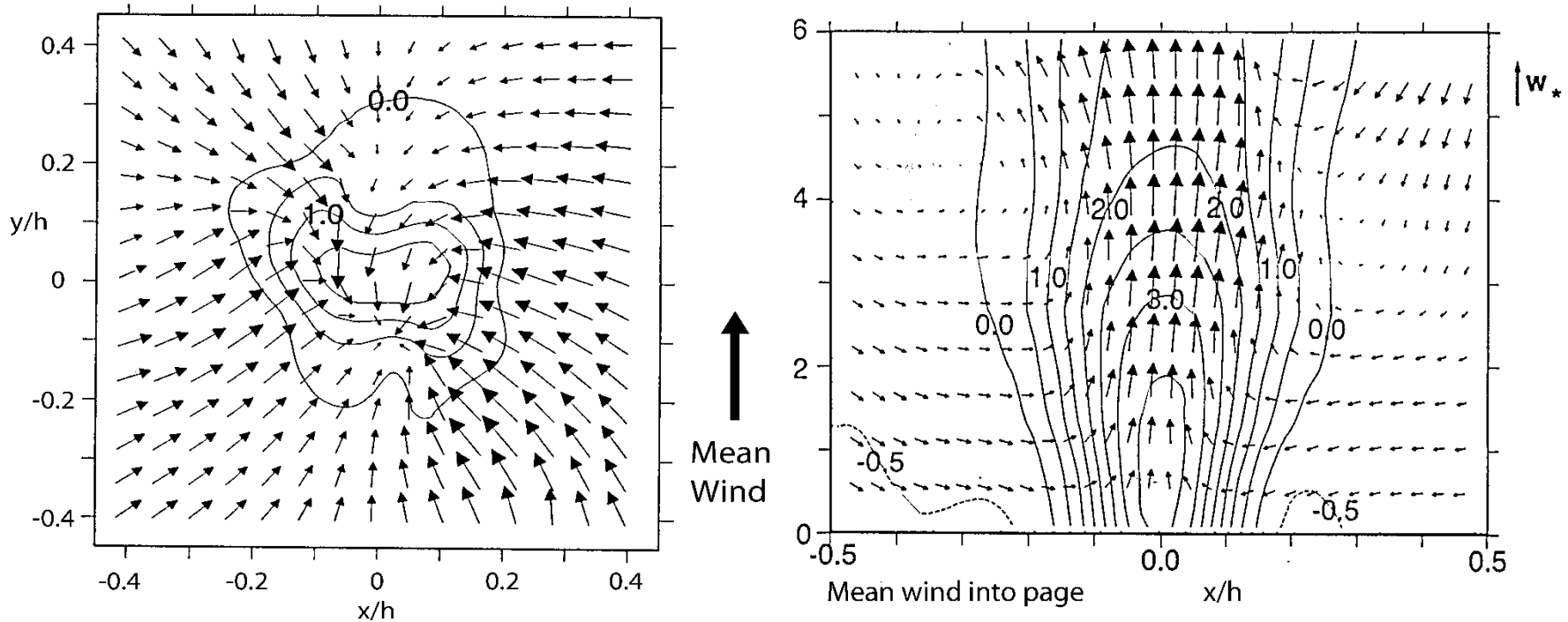
MO = direct cascade = generation of horizontal TKE by

breaking plumes: $u' \sim w' \sim (\beta F_z z)^{-1/3} \rightarrow F_x/F_z = \text{constant}$

Anarchy (Z, 2013): $u' \sim \tau/(\beta F_z z)^{-1/3} \rightarrow F_x/F_z \sim (z/L)^{-2/3}$

Data: Kader & Yaglom (1990)

Self organization of convective TURBULENCE



Airborne measurements in **atmosphere** (Williams, Hacker, 1992). Arrows show **self-organised wind pattern**. Solid lines show deviations of potential temperature θ from its mean value $\langle\theta\rangle$; the lines $\theta - \langle\theta\rangle = 0$ mark side walls of large buoyant plume.



Enhanced heat/mass transfer in free convection

Large-scale self-organised structures



Convective winds towards the plume base

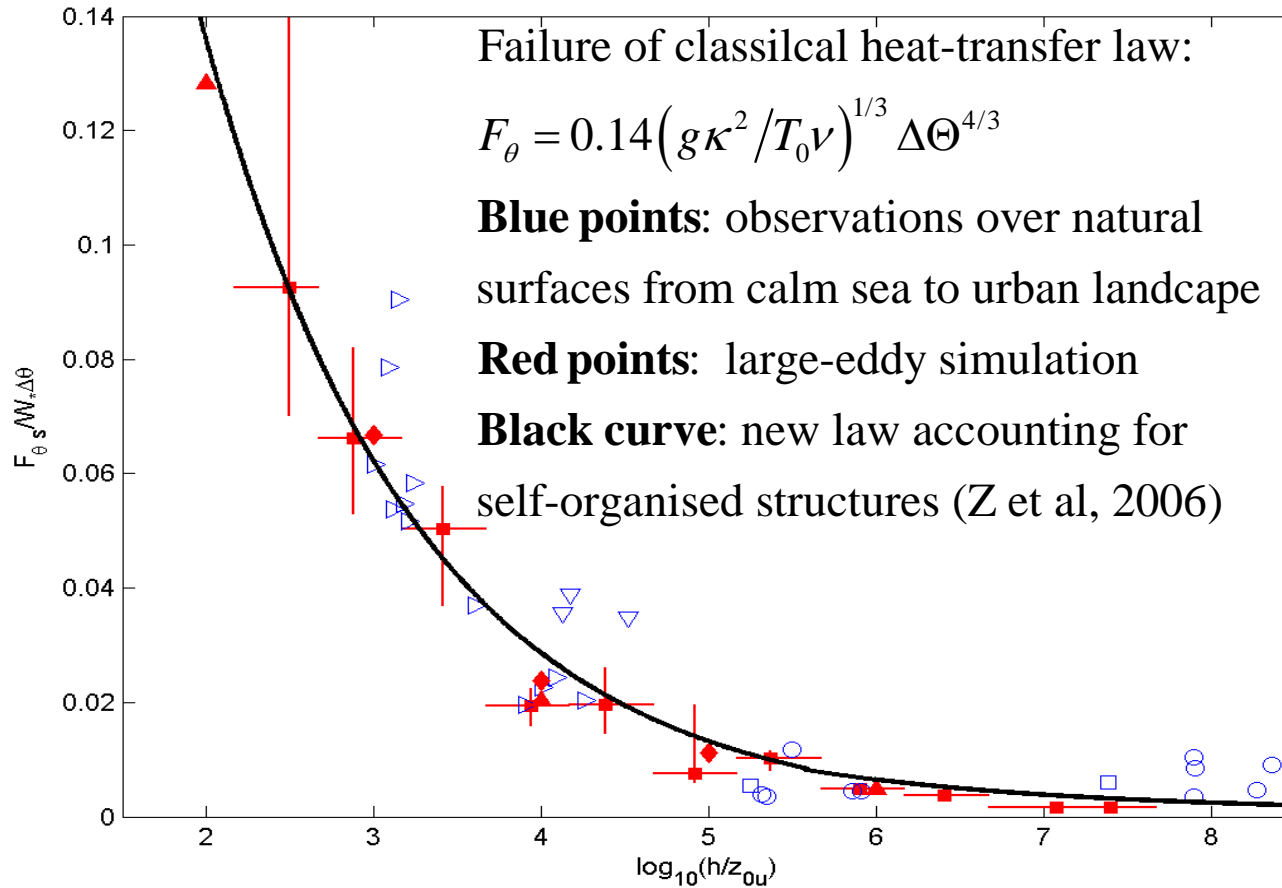


Internal boundary layer → mechanical turbulence
(**overlooked in conventional theories**)



Strongly enhanced heat/mass transfer

Heat-transfer in calm-weather convection



Heat transfer is much stronger than traditional theory predicts.

It depends on PBL height (h) and roughness length (z_{0u}), both missed in traditional theory

New law $F_{\theta} = f(h / z_{0u}) (gh / T_0)^{1/2} \Delta \Theta^{3/2}$ properly calculates the heat flux over rough surfaces **whereas traditional theory underestimates this flux to an order of magnitude and more**

Remarks on turbulence in unstable stratification

Traditional theory:

does not distinguish shear-generated ***mechanical*** turbulence *and* buoyancy-generated ***anarchy*** turbulence characterised by **inverse energy cascade from smaller to larger plumes towards self-organised structures** → **order out of chaos** (Prigogine-1982)

is erroneous as applied to horizontal mixing, fluxes and diffusion

overlooks large-scale self-organised motions, and strongly **underestimates** surface fluxes, especially in calm weather

In modelling practice, these failures are corrected but only empirically, without physical explanation



TURBULENCE IN STABLE STRATIFICATION



Sub-critically stable **well-mixed** PBL differs from **super-critically** stable **but yet turbulent** free flow aloft
(Altay, Russia, 28.08.2010, photo SZ)



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Stably stratified turbulence: *strong-mixing* in PBL and *weak-conductivity* aloft ($Ri > Ri_c$)



Shallow PBL is seen due to water haze (Bergen). Old theory did not distinguish turbulence in stable PBL and supercritically stable free flow. The problem is solved by the EFB closure theory (Z et al., 2007-2013).



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PBL height and air pollution



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Self-control of turbulence in stable stratification via counter-gradient heat flux missed in K-1941, MO-1954

F_θ -budget reveals downgradient
and countergradient terms
comprising the **factual** heat flux

$$F_\theta = C_1 t_T \beta \langle \theta^2 \rangle - C_2 t_T E_z \frac{\partial \Theta}{\partial z}$$

Key feedback assuring self-control (Z et al., 2007, 2013):

Increasing temperature gradient $\partial \Theta / \partial z$ immediately **enhances**

(1) Total (negative) fluxes of heat F_θ and buoyancy $F_b = \beta F_\theta$

(2) Hence mean squared temperature

$$\langle \theta^2 \rangle = -C_3 t_T F_\theta \partial \Theta / \partial z$$

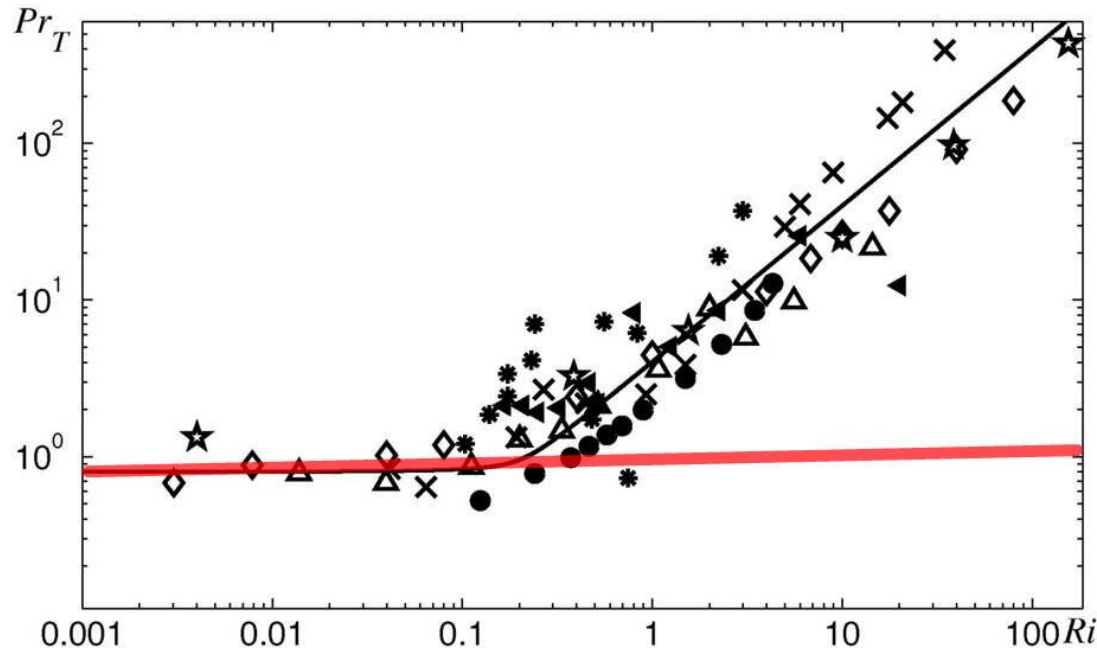
(3) And thus the counter-gradient **positive** share of heat flux $C_1 t_T \langle \theta^2 \rangle$

This compensates for enhancing total (negative) heat flux and prevents collapse of turbulence in super-critical stratification

Prandtl no. Pr_T vs. Richardson no. Ri

K-1941, MO-1954 ignore self-control of heat flux, F_θ , and suggest similarity of viscosity and conductivity: $Pr_T = K_M/K_H = constant$

This entails erroneous turbulence cut off at $Ri > Ri_c = 0.25$



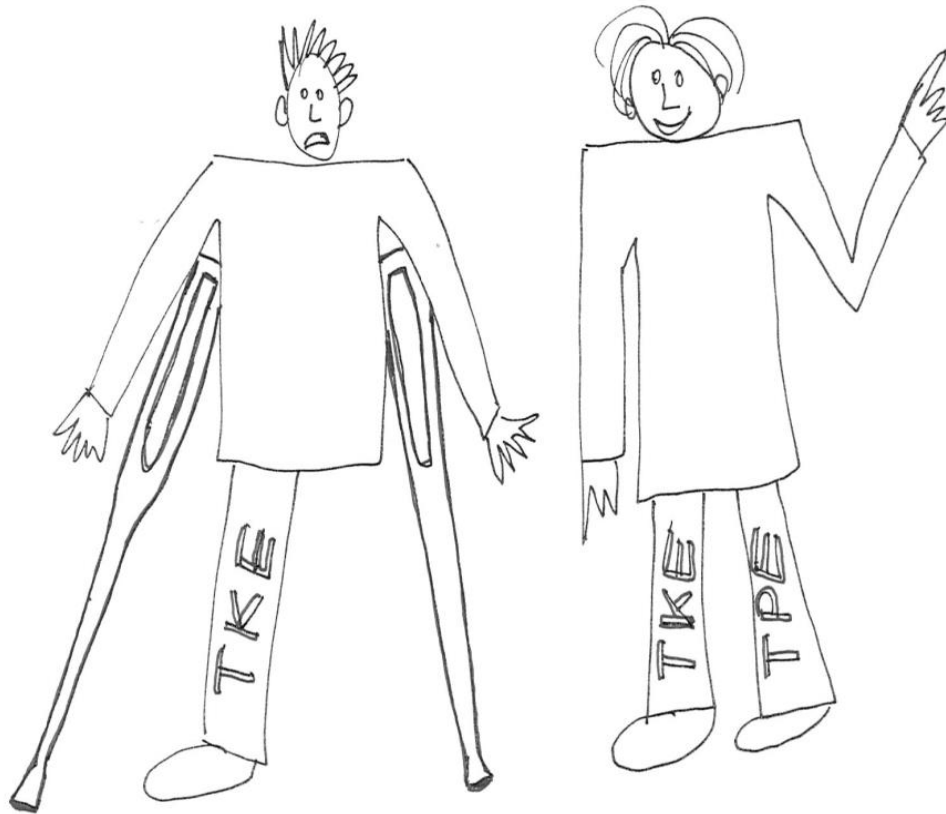
Black line: Pr_T after the EFB turbulence closure (Z et al., 2007-2017)

Red line: Pr_T predicted by conventional theory (e.g. MO-1954)

Data Atmospheric: Kondo et al., 1978, Bertin et al., 1997;
Laboratory: Rehmann, Koseff, 2004, Ohya, 2001, Strang, Fernando, 2001; **DNS:** Stretch et al., 2001; **LES:** Esau, 2009

Remarks on turbulence in stable stratification

From hypothetical turbulent exchange coefficients and energetics limited to TKE



to flux-budget equations yielding down-gradient **and non-gradient** transports, **TKE + TPE** energy budget, **self-control of heat flux**, surviving of **turbulence in supercritical stratification**, and many other “heretical” features of turbulence ...

We have finally got rid of misleading analogy with molecular transports



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Prior paradigms originated from doubtful analogy with chaotic molecular motions

1. Boussinesq (1877)

The concept of **turbulent exchange coefficients**: viscosity, K_M , heat-conductivity, K_H , and diffusivity, K_D , analogous to molecular coefficients but much larger and to be determined empirically

2. Prandtl (1930)

$K_{M,H,D} = l dU/dz$ are products of **mixing length** l and **turbulent velocity** u_T analogous to molecular **pathway** and **velocity**. In boundary layers: $l \sim z$.

3. Kolmogorov (1941-1942)

TKE cascade towards dissipation = $(\text{TKE})^{3/2}/l$

TKE budget equation to determine $u_T = (\text{TKE})^{1/2}$,

but yet downgradient fluxes ($K_{M,H,D}$ defined after Prandtl)

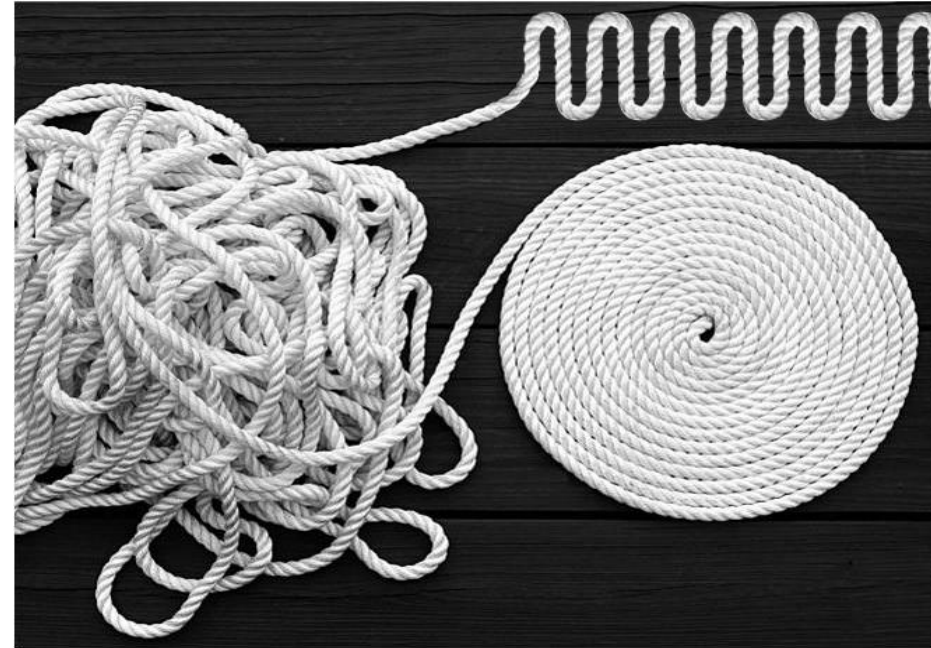


Conclusion: revised vision of turbulence

Conventional PARADIGM:

K-1941 NEUTRAL stratification

- (1) regular mean flow
- (2) chaotic turbulence → only the forward energy cascade from larger to smaller eddies towards viscous dissipation
- (3) down-gradient transport



Revised PARADIGM: **ANY** stratification

- (1) mean flow
- (2) usual turbulence with forward cascade towards dissipation
- (3) both down-gradient and **non-gradient** turbulent transports
- (4) **anarchy turbulence** with **inverse energy cascade** towards
- (5) **large-scale organised structures** (secondary circulations)



ALEXANDER OBUKHOV,

physicist, mathematician,
key founder of theory of
stratified turbulence

**was born 100 year
ago: in May 1918**

The Obukhov length

$$L = u_*^3 (-\beta F_z)^{-1}$$

is forever inherent part of
the theory of turbulence



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