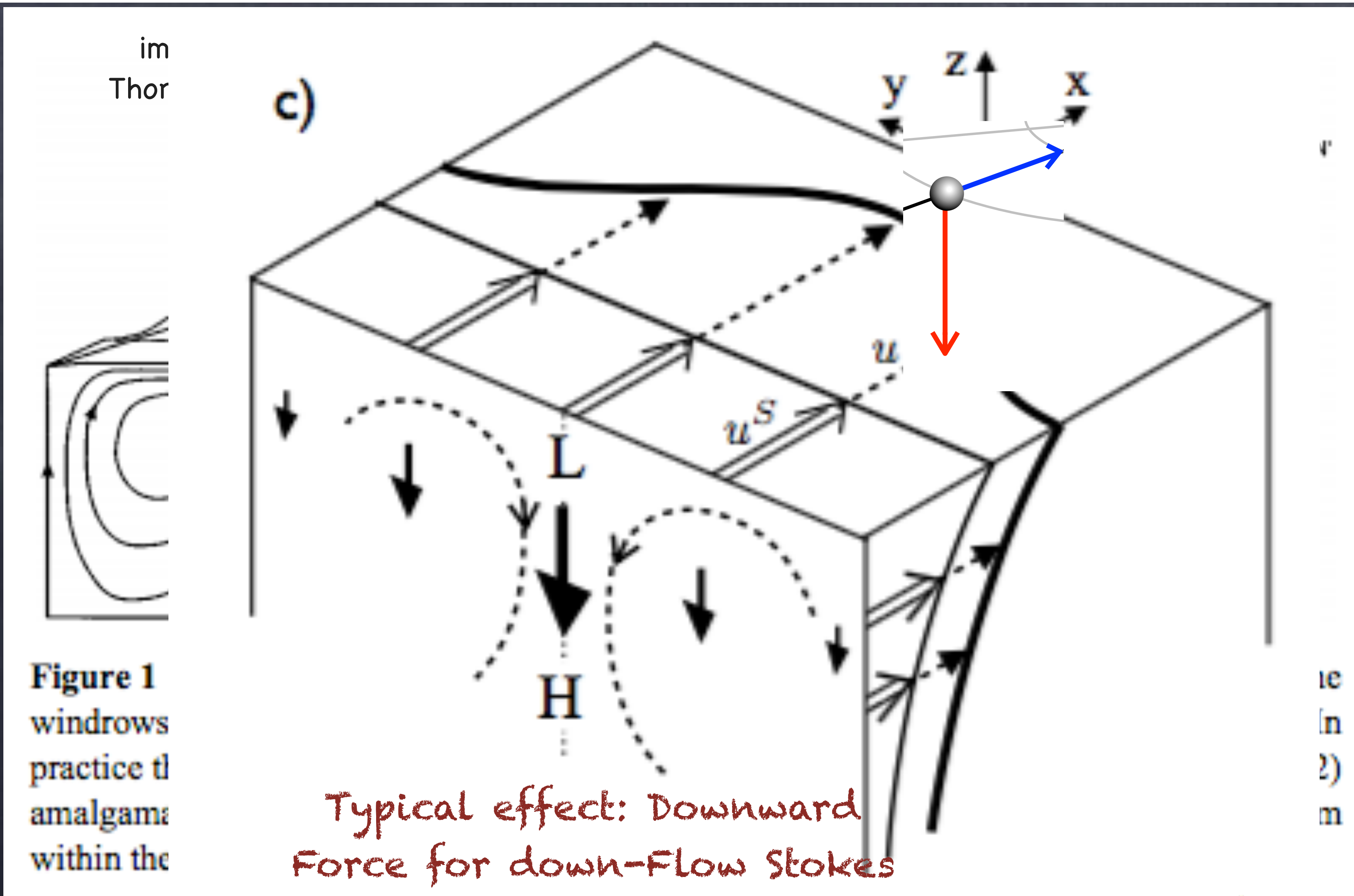


$$M_{Ro} \equiv \max(1, Ro)$$

$$Re = \frac{UL}{\nu}$$

$$Ro = \frac{U}{fL}$$

$$Ri = \frac{N^2}{(U_{,z})^2}$$

$$\alpha = H/L$$

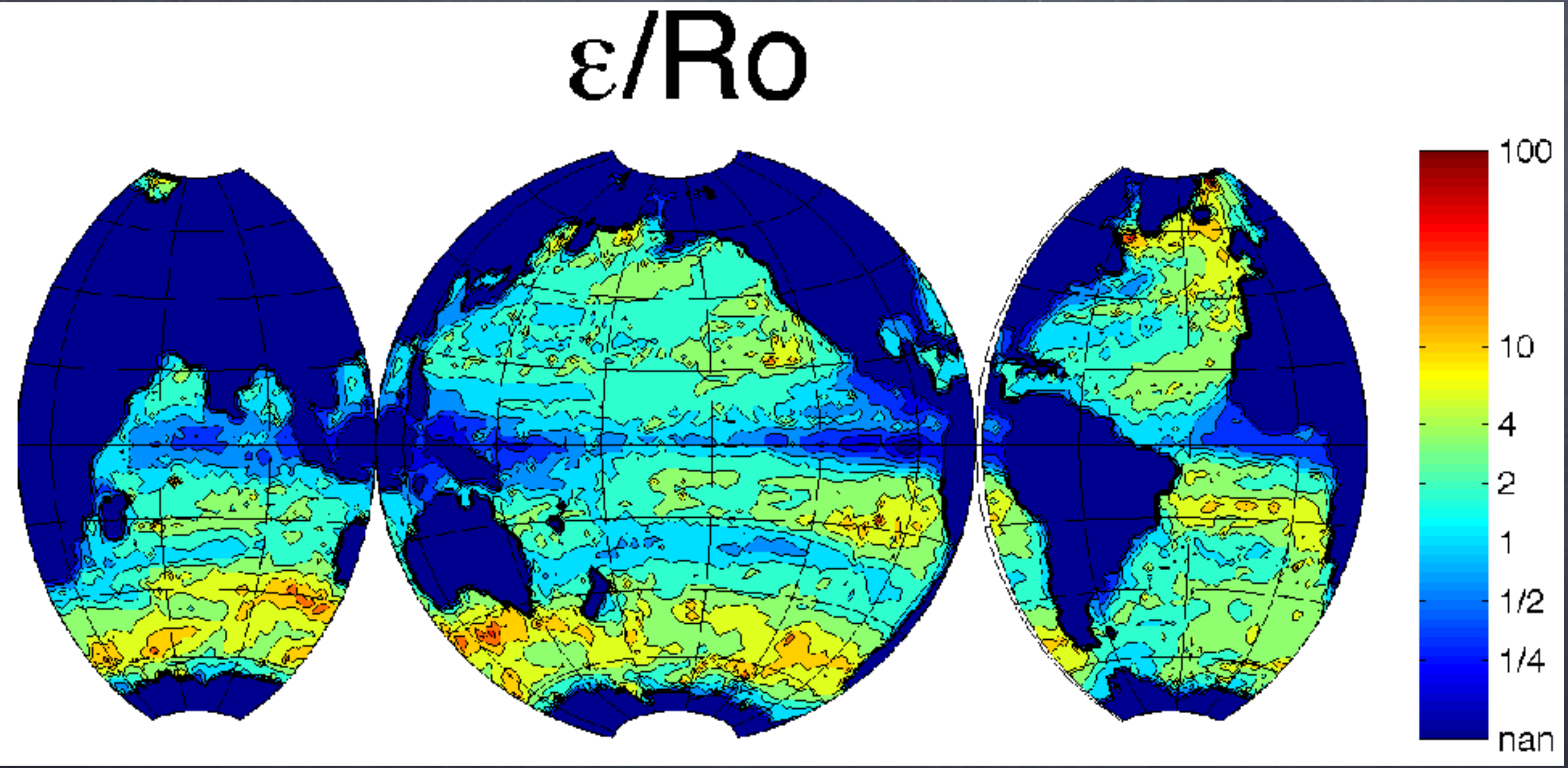
$$\varepsilon = \frac{V^s H}{f L H_s}$$

"wavy hydrostatic" if $\epsilon \gg 1$

$$\frac{\alpha^2}{Ri}\left[w_{,t} + v_j^L w_{,j} + \frac{M_{Ro}}{RoRi} w w_{,z}\right] = \boxed{-\pi_{,z} + b - \varepsilon v_j^L v_{j,z}^s} + \frac{\alpha^2}{ReRi} w_{,jj}$$

N. Suzuki and BFK. Understanding Stokes forces in the wave-averaged equations. Journal of Geophysical Research-Oceans, 121:1-18, 2016.

Do Stokes force directly affect larger scales?



"wavy hydrostatic"

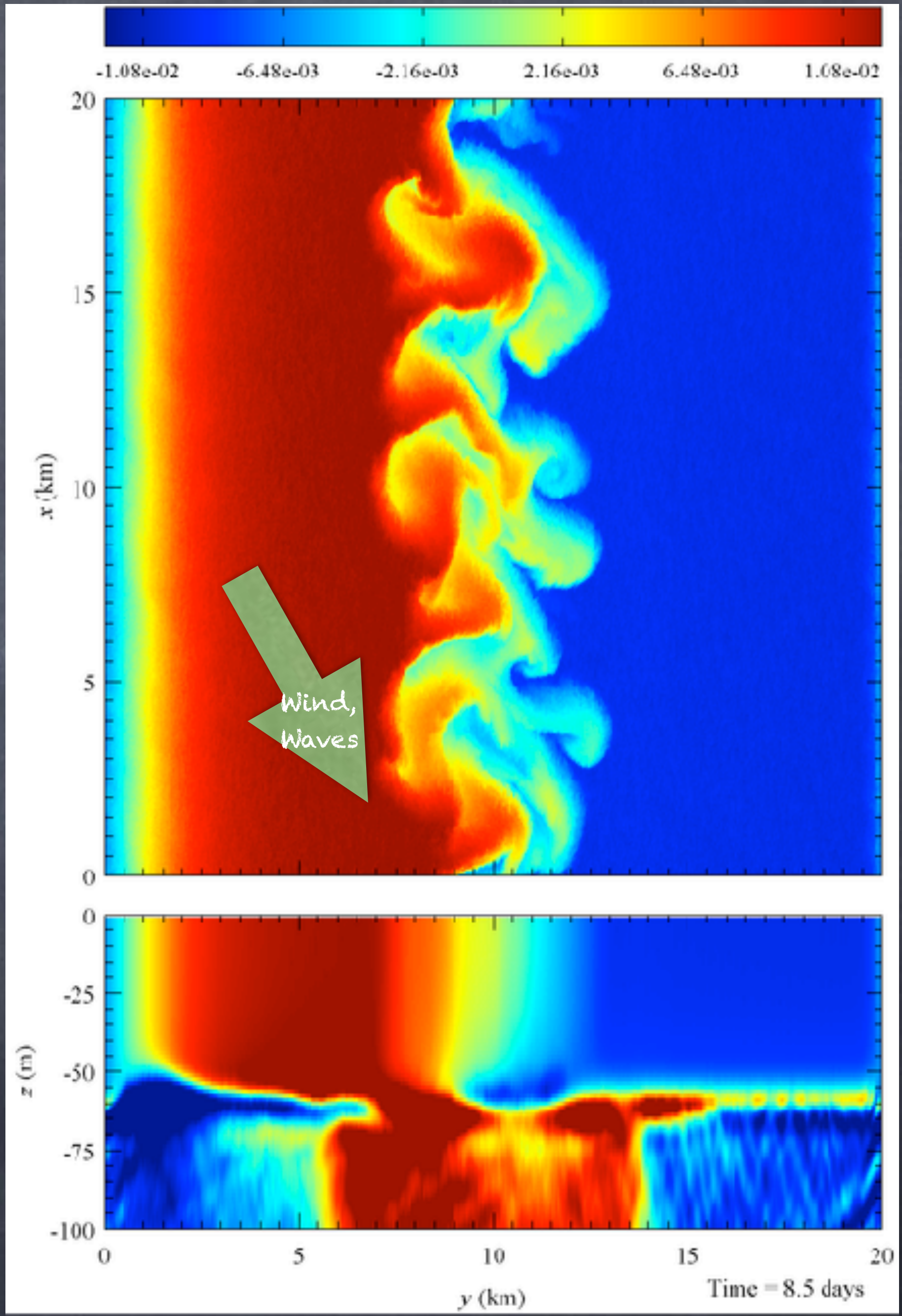
$$\epsilon \gg 1$$

$$\varepsilon = \frac{V^s H}{f L H_s} \qquad Ro = \frac{U}{fL}$$

J. C. McWilliams and BFK. Oceanic wave-balanced surface fronts and filaments. Journal of Fluid Mechanics, 730:464-490, 2013.

Do Stokes forces affect (sub)Meso-Scales?

Movie: P. Hamlington



LES of Langmuir turbulence with a submesoscale temperature front

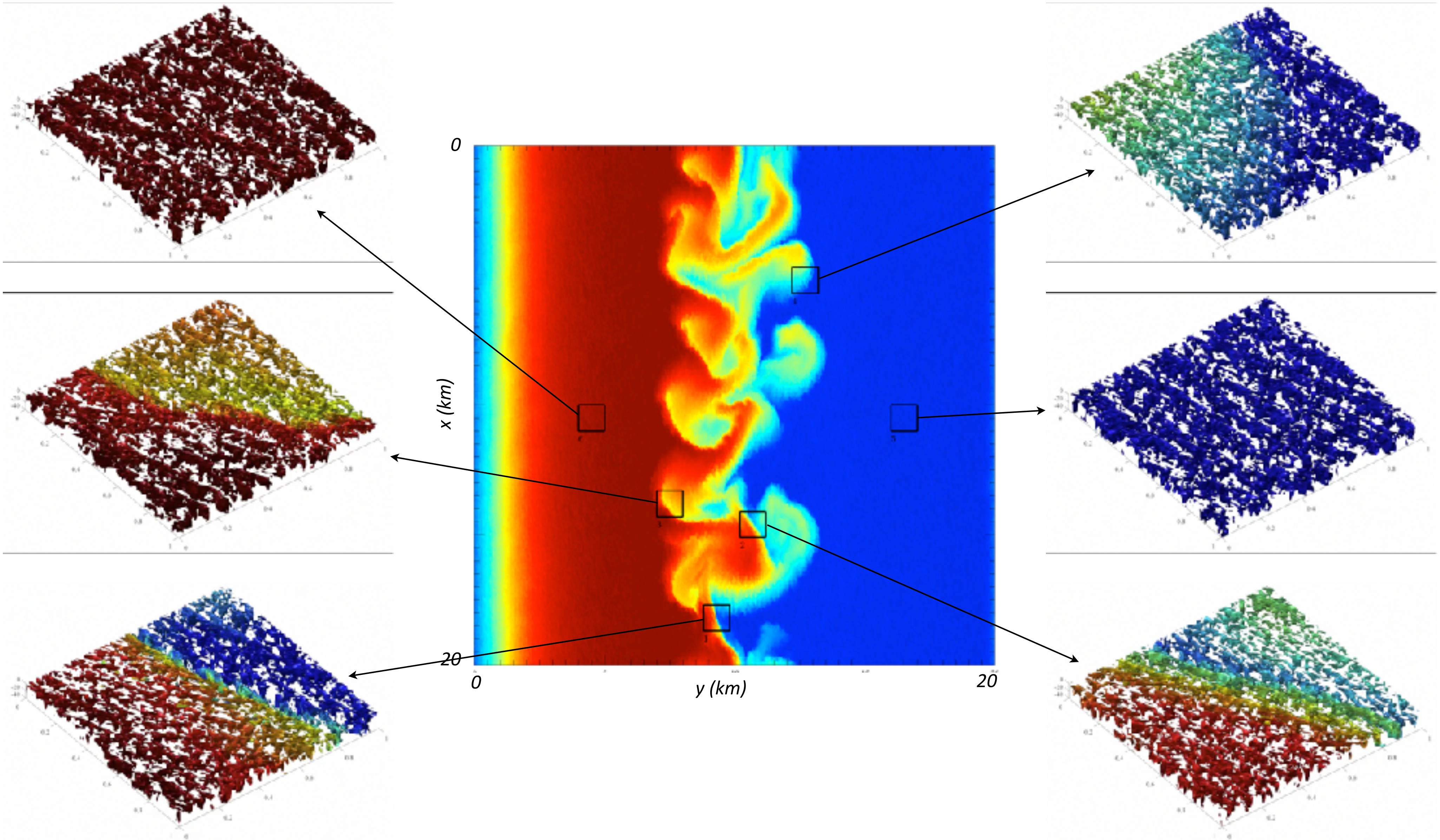
Use NCAR LES model to solve Wave-Averaged Eqtns.

2 Versions: 1 With Waves & Winds
1 With only Winds

Computational parameters:
Domain size: 20km x 20km x -160m
Grid points: 4096 x 4096 x 128
Resolution: 5m x 5m x -1.25m

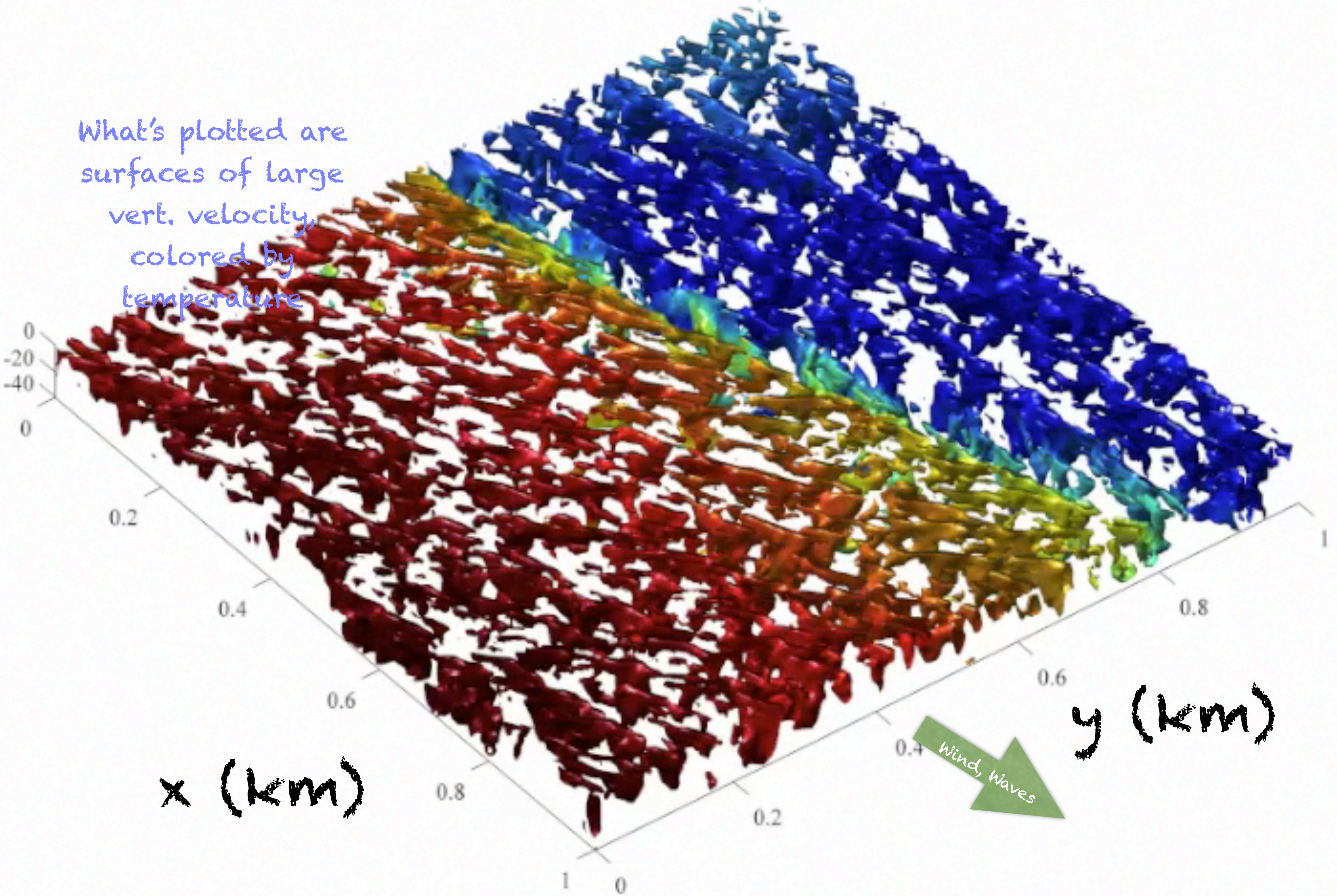
P. E. Hamlington, L. P. Van Roekel, BFK, K. Julien, and G. P. Chini. Langmuir-submesoscale interactions: Descriptive analysis of multiscale frontal spin-down simulations. Journal of Physical Oceanography, 44(9): 2249-2272, September 2014.

Diverse types of interaction: Stronger Langmuir (small) Turbulence

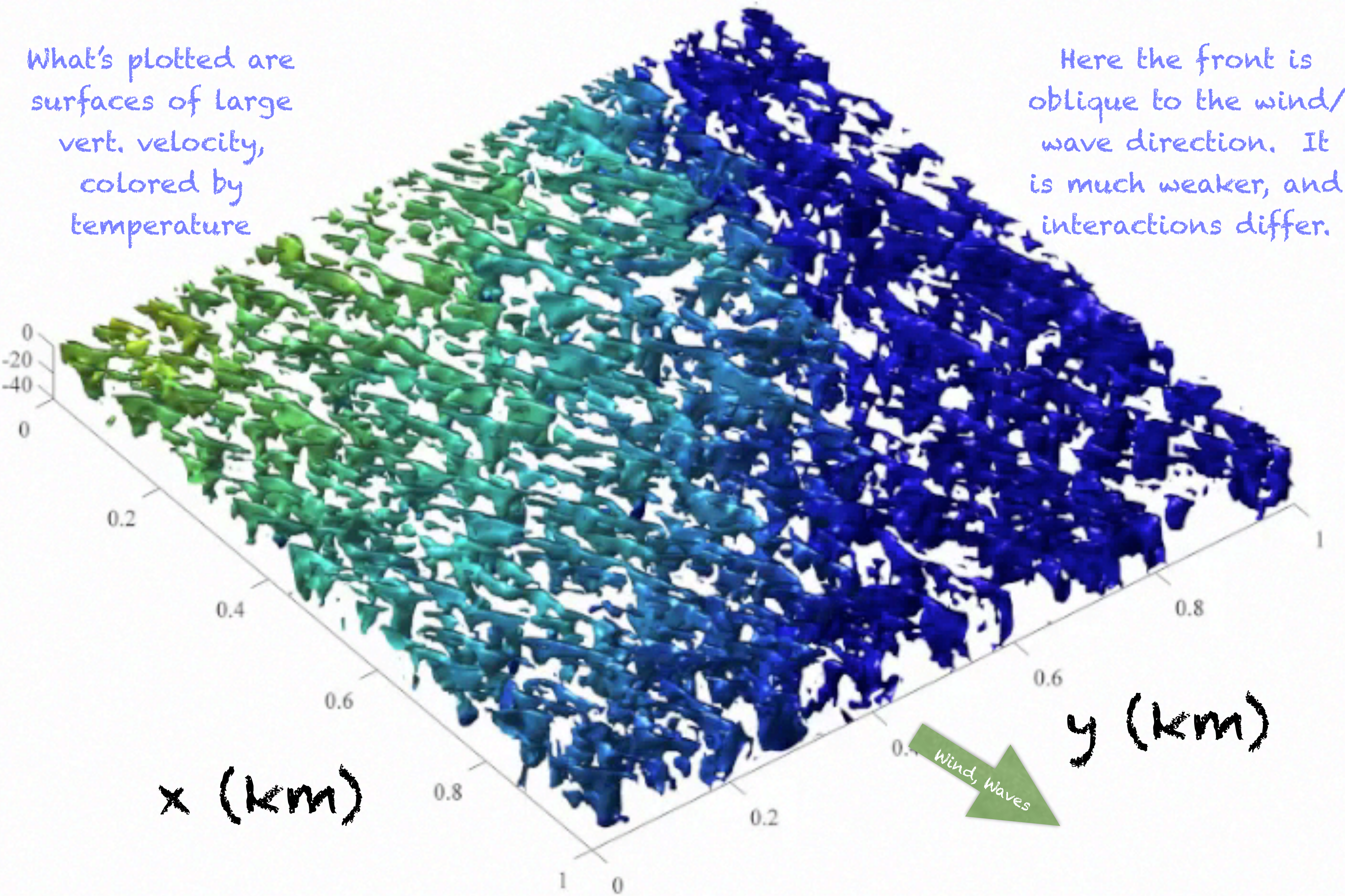


P. E. Hamlington, L. P. Van Roekel, BFK, K. Julien, and G. P. Chini. Langmuir-submesoscale interactions: Descriptive analysis of multiscale frontal spin-down simulations. Journal of Physical Oceanography, 44(9): 2249-2272, September 2014.

Zoom: Submeso-Langmuir Interaction!



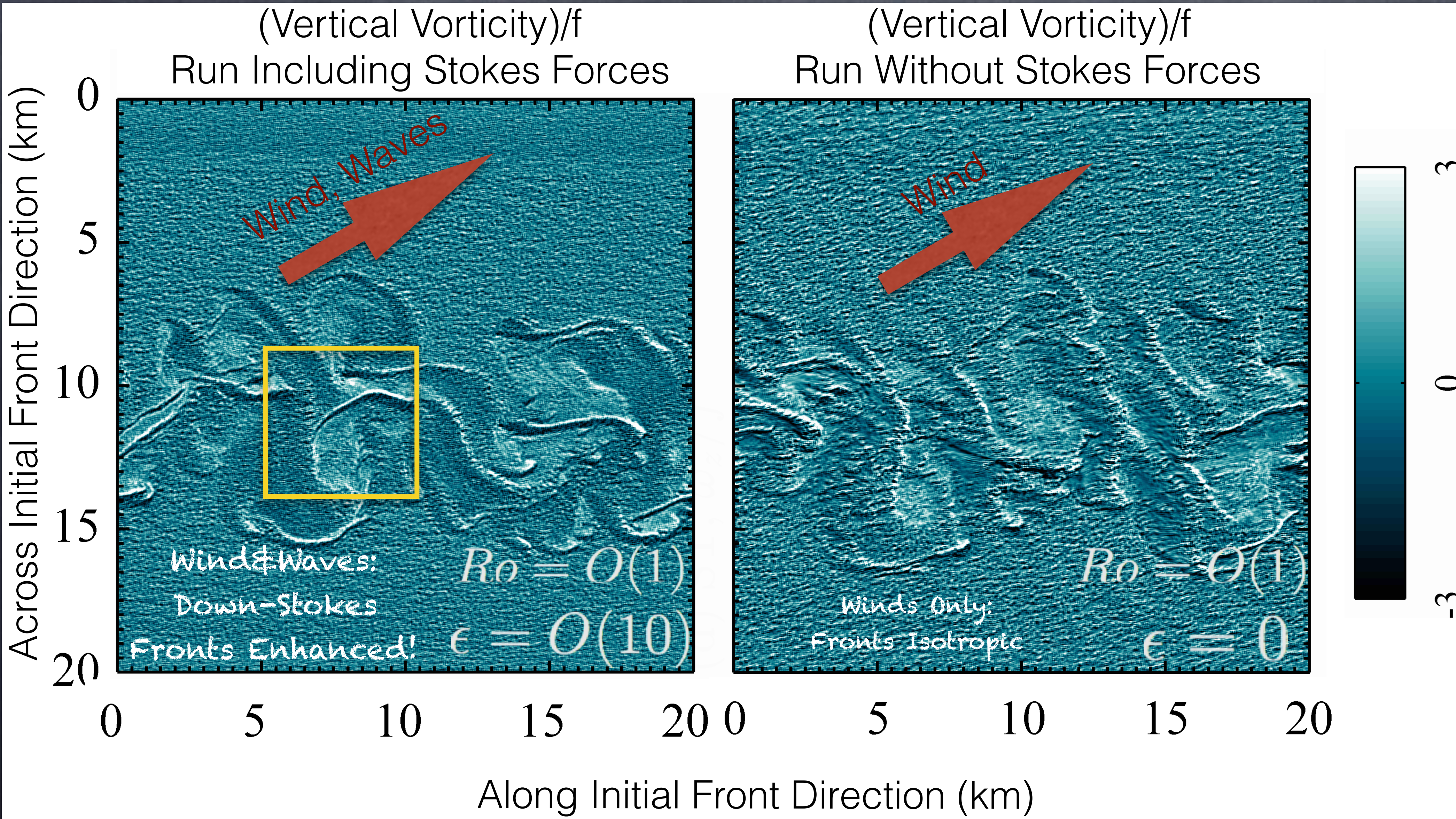
P. E. Hamlington, L. P. Van Roekel, BFK, K. Julien, and G. P. Chini. Langmuir-submesoscale interactions: Descriptive analysis of multiscale frontal spin-down simulations. JPO, 44(9):2249-2272, 2014.



P. E. Hamlington, L. P. Van Roekel, BFK, K. Julien, and G. P. Chini. Langmuir-submesoscale interactions: Descriptive analysis of multiscale frontal spin-down simulations. JPO, 44(9):2249-2272, 2014.

Are Fronts and Filaments different with Stokes shear force?

$$\frac{\alpha^2}{Ri}\left[w_{,t}+v_j^L w_{,j}+\frac{M_{Ro}}{RoRi}ww_{,z}\right]=-\pi_{,z}+b-\varepsilon v_j^L v_{j,z}^s+\frac{\alpha^2}{ReRi}w_{,jj}$$

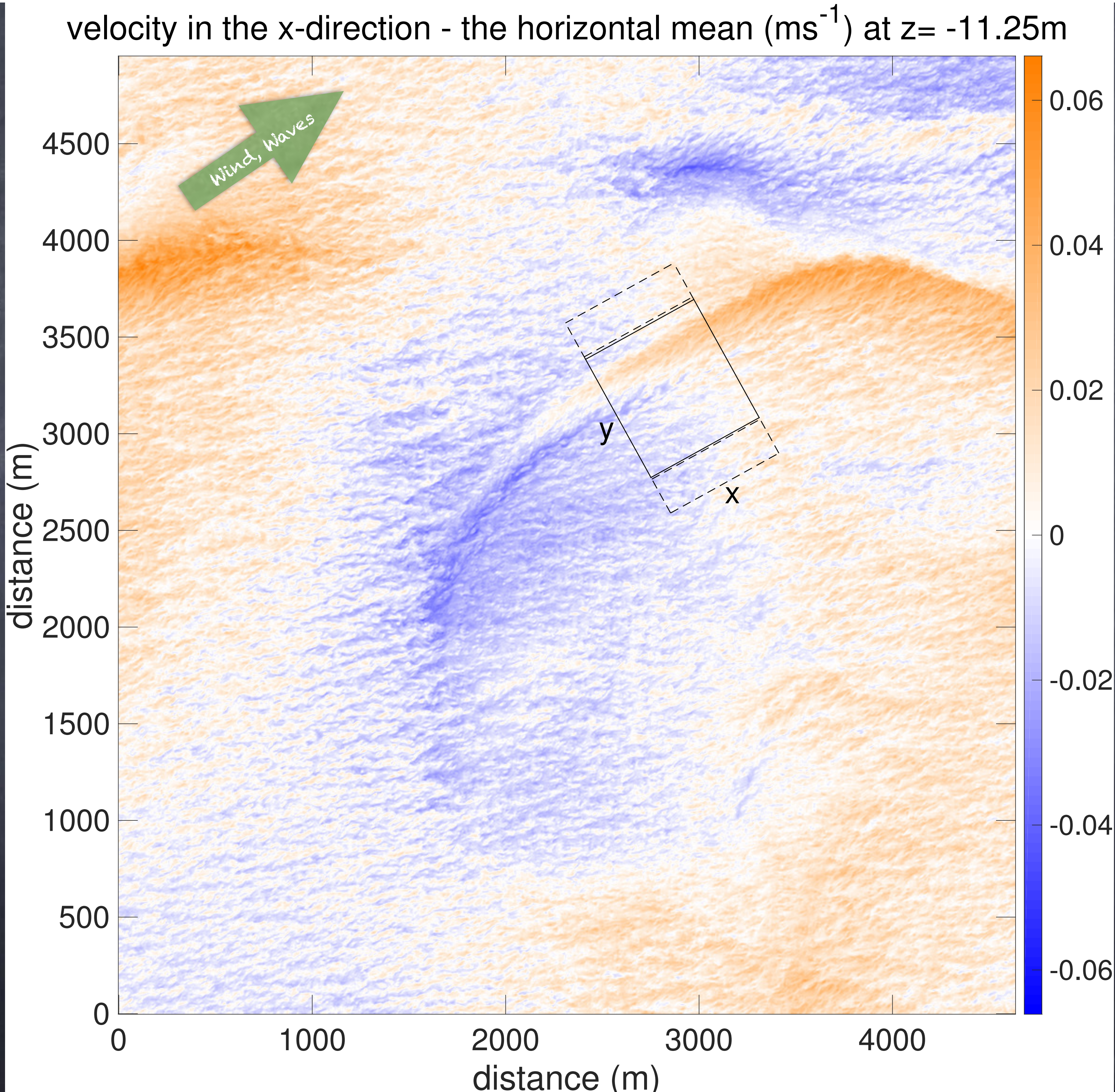


N. Suzuki, BFK, P. E. Hamlington, and L. P. Van Roekel. Surface waves affect frontogenesis. Journal of Geophysical Research-Oceans, 121:1-28, 2016.

N. Suzuki and BFK. Understanding Stokes forces in the wave-averaged equations. Journal of Geophysical Research-Oceans, 121:1-18, 2016.

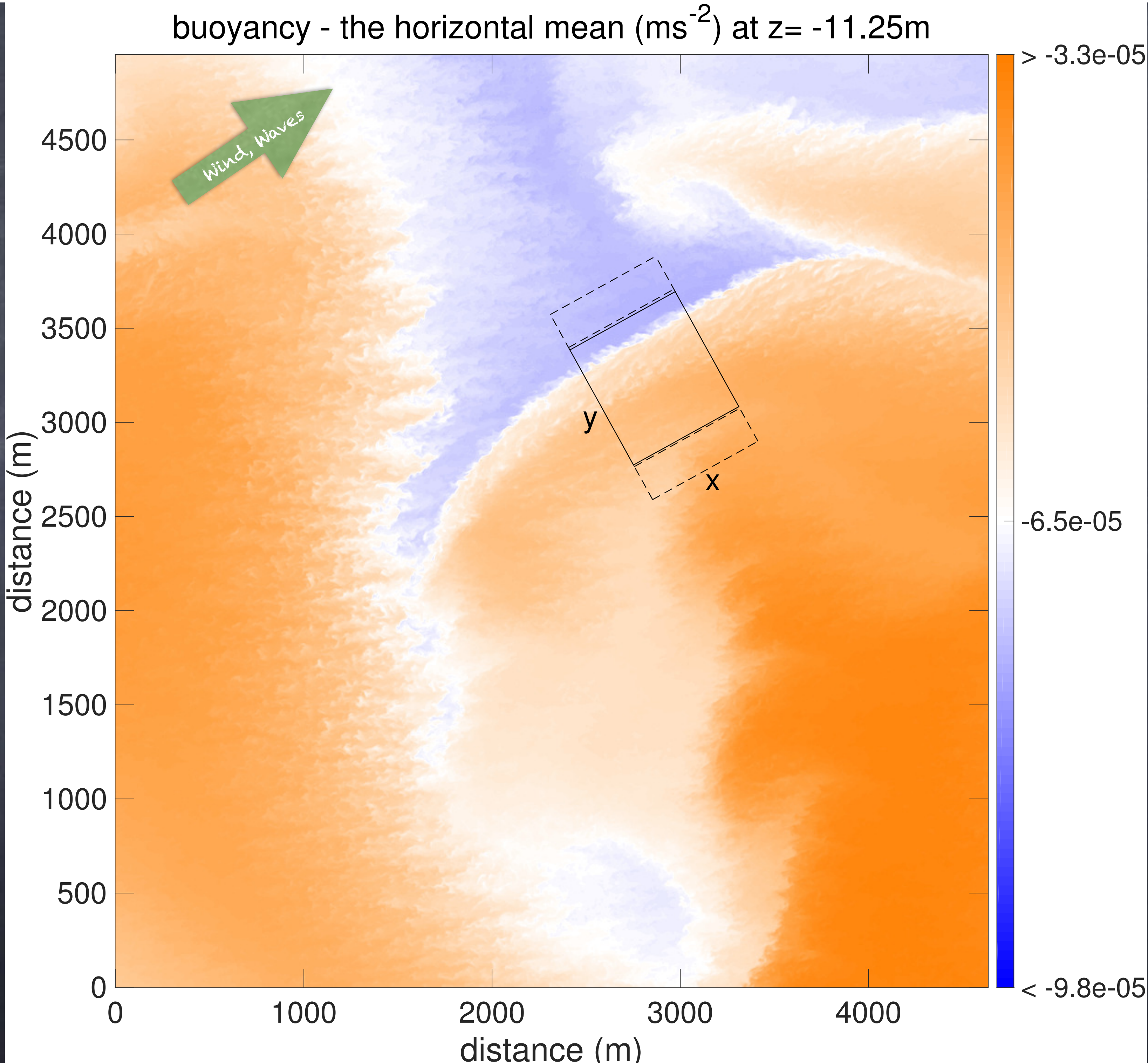
J. C. McWilliams and BFK. Oceanic wave-balanced surface fronts and filaments. Journal of Fluid Mechanics, 730:464-490, 2013.

V



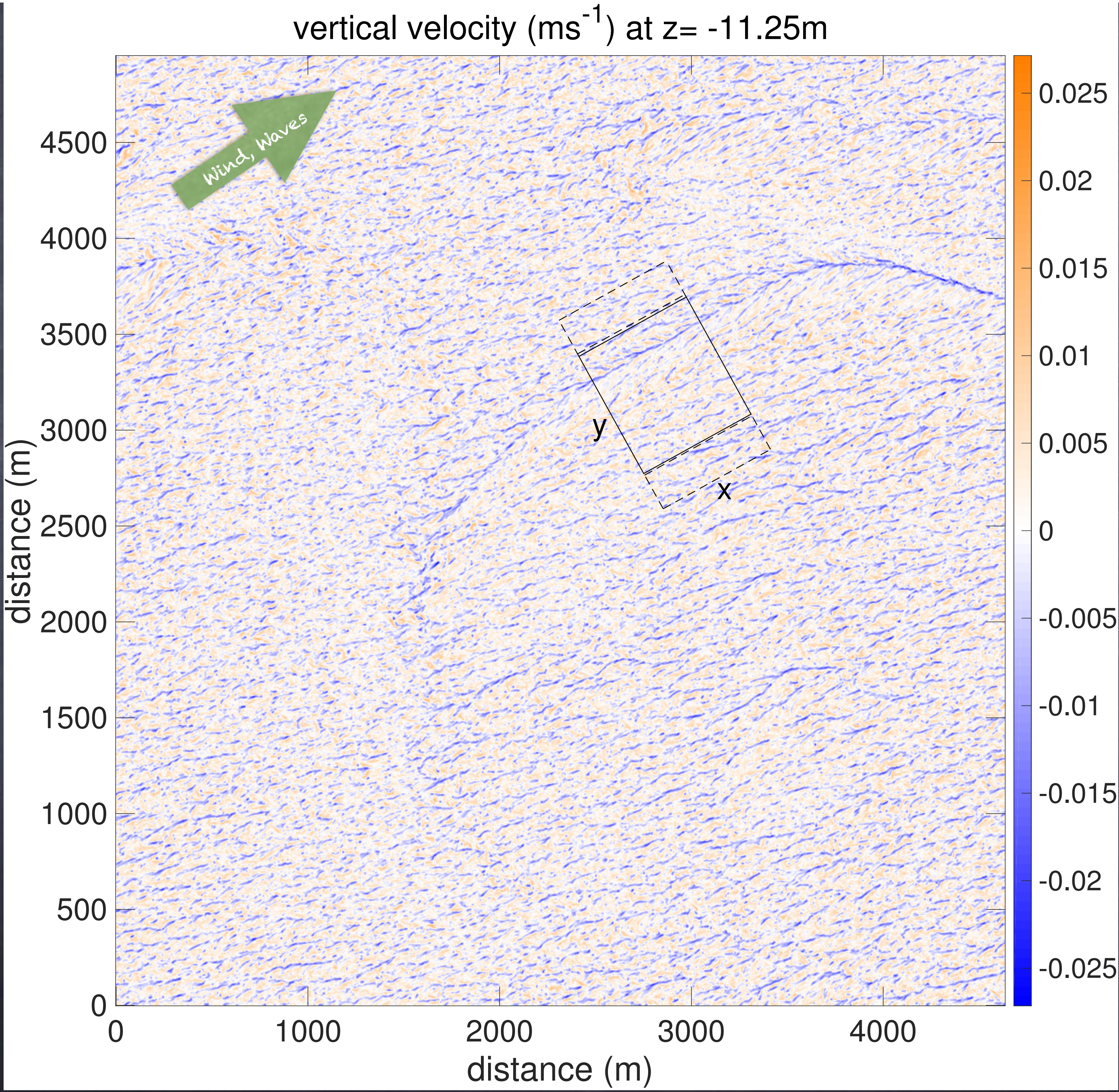
N. Suzuki, B. Fox-Kemper, P. E. Hamlington, and L. P. Van Roekel. Surface waves affect frontogenesis. Journal of Geophysical Research-Oceans, 121:1-28, May 2016.

T



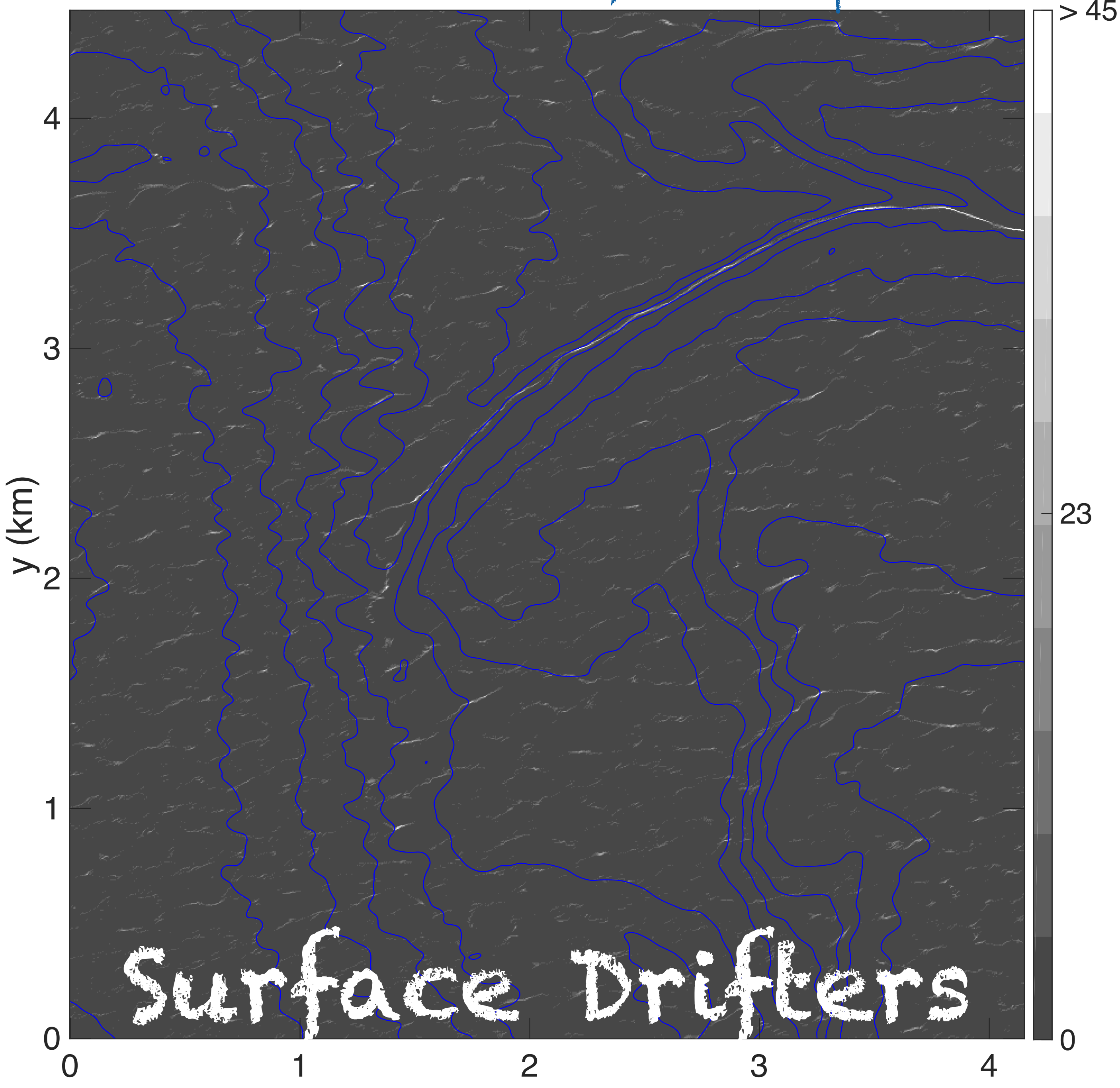
N. Suzuki, B. Fox-Kemper, P. E. Hamlington, and L. P. Van Roekel. Surface waves affect frontogenesis. Journal of Geophysical Research-Oceans, 121:1-28, May 2016.

W

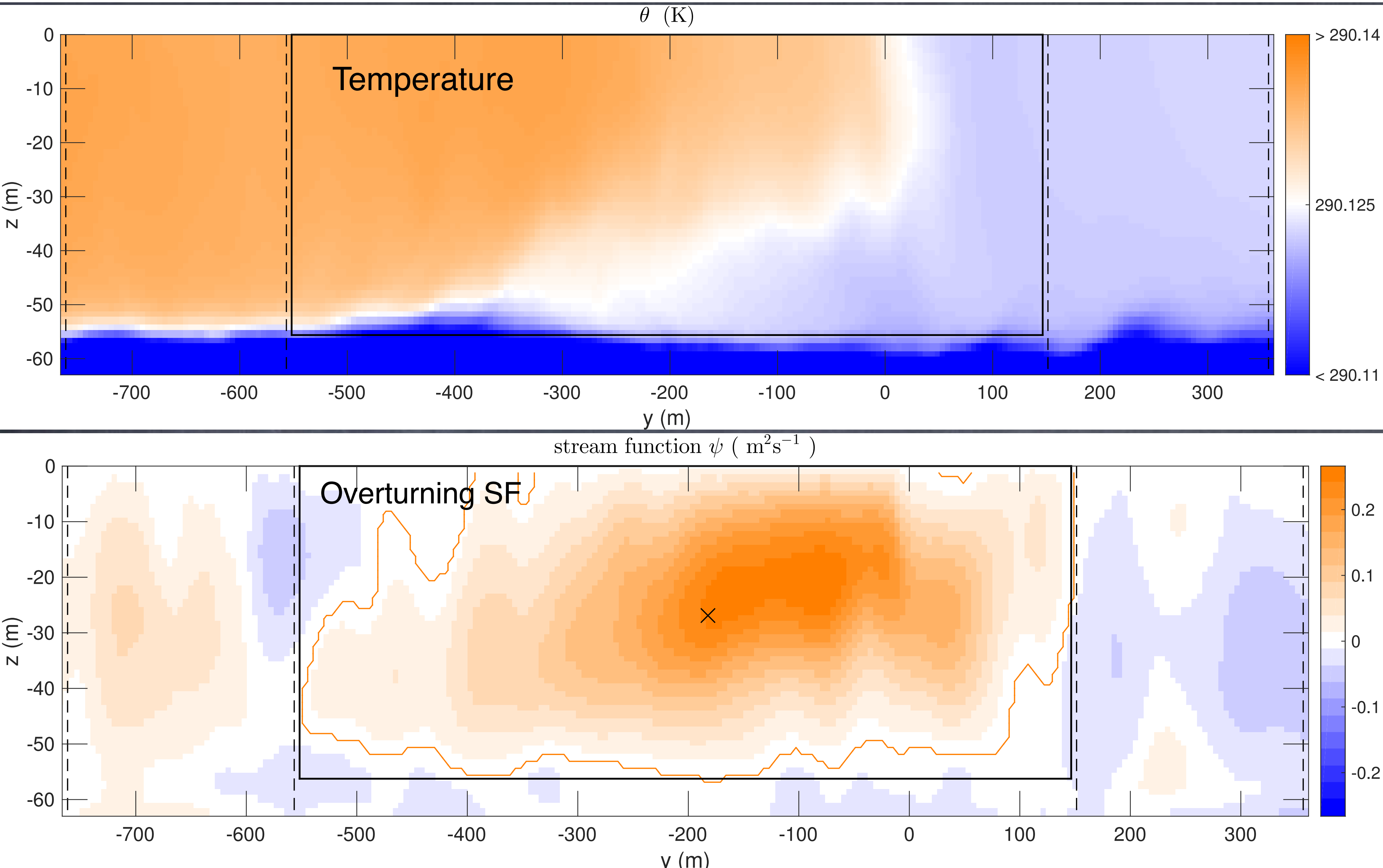


N. Suzuki, B. Fox-Kemper, P. E. Hamlington, and L. P. Van Roekel. Surface waves affect frontogenesis. Journal of Geophysical Research-Oceans, 121:1-28, May 2016.

Initially every surface node has 1 drifter, so there are 851796 drifters in the picture



After 80 Min



N. Suzuki, BFK, P. E. Hamlington, and L. P. Van Roekel. Surface waves affect frontogenesis. Journal of Geophysical Research-Oceans, 121:1-28, 2016.

Do (wavy hydrostatic) Stokes Forces Matter?

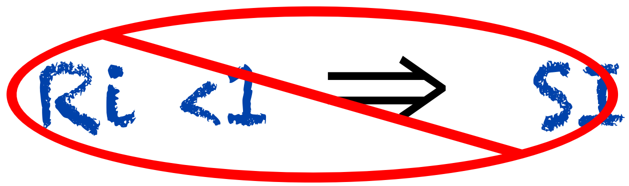
Yes! At Leading Order (in LES)

Table 3. Integrated Budget for Overturning Vorticity[a]

Responsible Force	Relative Value
Relative Tendency of Overturning Circulation along the Cell Boundary	
Net tendency	$11 \pm 8\%$
Sources	
Buoyancy anomaly	100%
Stokes shear force anomaly	$44 \pm 4\%$
Interaction with v^H	$44 \pm 8\%$
Frontal anomaly in pressure gradient	
	$6 \pm 9\%$
Nonlinear interaction with v^B:	$2 \pm 1\%$
Sinks	
Frontal turbulence anomaly	
(mostly, imbalance in wavy Ekman relation)	$-82 \pm 11\%$
Coriolis on along-front jet	$-66 \pm 2\%$
Lagrangian advection of (v^ψ, w^ψ)	$-36 \pm 7\%$

N. Suzuki and BFK. Understanding Stokes forces in the wave-averaged equations. Journal of Geophysical Research-Oceans, 121:1-18, April 2016.

N. Suzuki, BFK, P. E. Hamlington, and L. P. Van Roekel. Surface waves affect frontogenesis. Journal of Geophysical Research-Oceans, 121:1-28, May 2016.

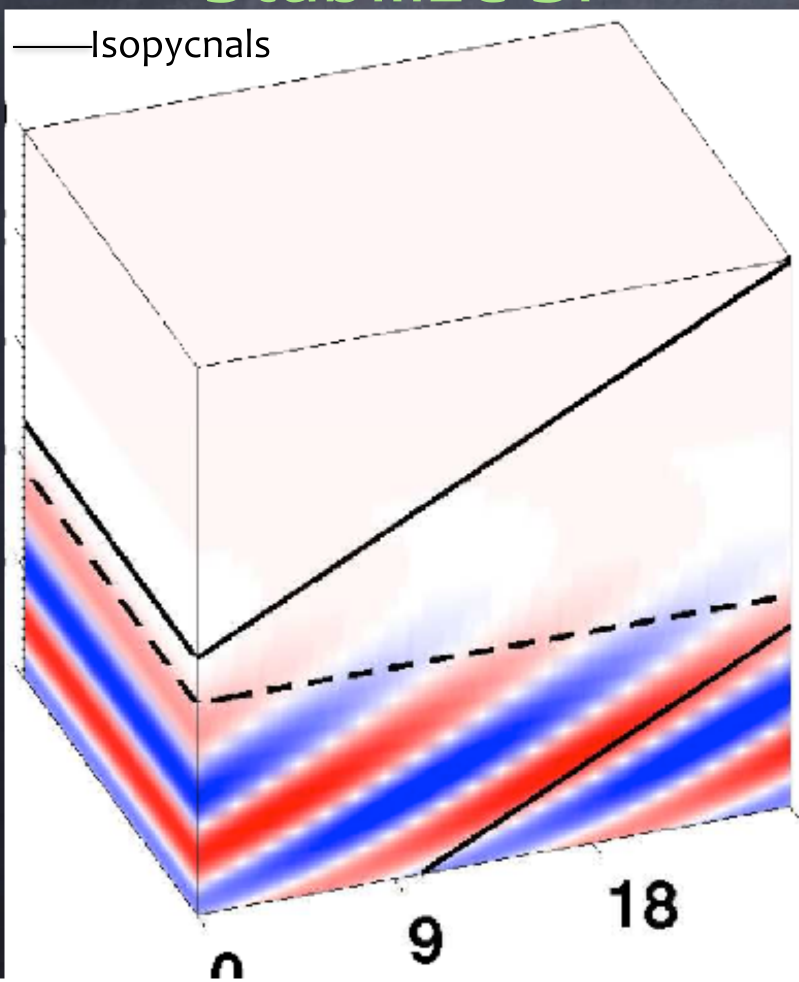


Wavy Submesoscale Instability Different: Symmetric Instability

★ $fQ<0 \Rightarrow SI$

Ri = 0.5

Stokes Forces Stabilize SI

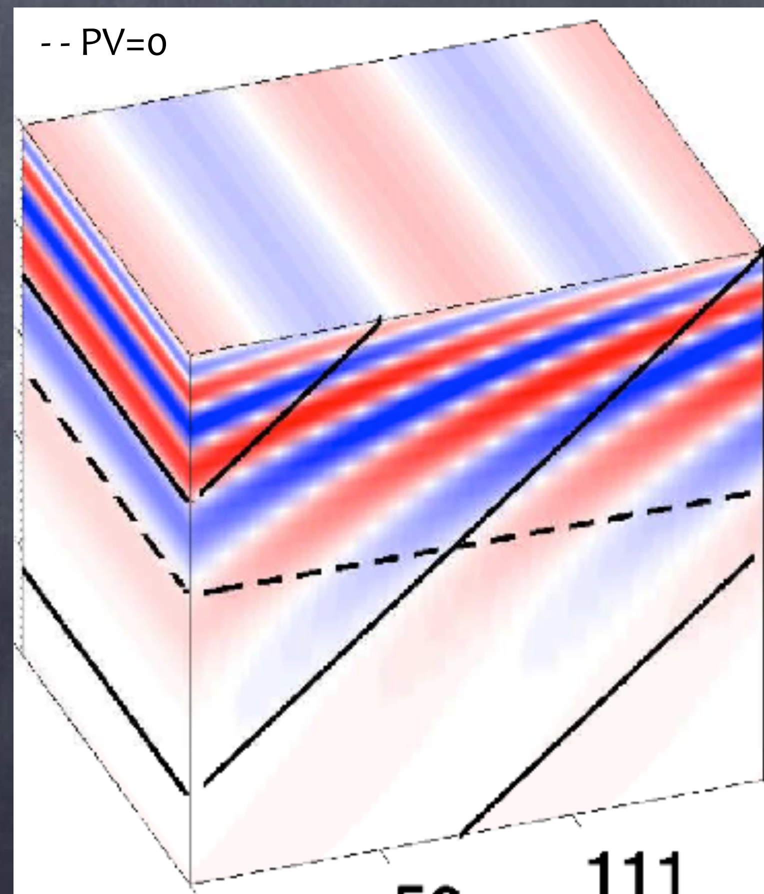


Cross front velocity for the fastest growing mode

S. Haney, BFK, K. Julien, and A. Webb. Symmetric and geostrophic instabilities in the wave-forced ocean mixed layer. JPO 45:3033-3056, 2015.

Ri = 2

Stokes Forces Destabilize SI



Conclusions

- The Stokes vortex or Stokes shear force simplifies the wave-mean interactions, simples enough that simulations are becoming common
- Stokes forces, as treated here, can be included in hydrostatic models like GCMs (wavy hydrostatic)
- Stokes forces affect Langmuir turbulence, but also (sub)mesoscale fronts (more energy, anisotropy) and submesoscale instabilities. Need to assess climate & environmental impact!

Wave-Averaged Equations following Lane et al. (07), McWilliams & F-K (13)

$$\underbrace{v_j^L}_{\text{Lagrangian}} = \underbrace{v_j}_{\text{Eulerian}} + \underbrace{v_j^S}_{\text{Stokes}}$$

Boundary conditions,

plus:

(Lagrangian) geostrophic

$$Ro\left[v_{i,t} + v_j^L v_{i,j}\right] + \frac{M_{Ro}}{Ri} w v_{i,z} + \boxed{\epsilon_{izj} v_j^L = -M_{Ro}\pi_{,i}} + \frac{Ro}{Re} v_{i,jj}$$

$$\frac{\alpha^2}{Ri}\left[w_{,t} + v_j^L w_{,j} + \frac{M_{Ro}}{RoRi} w w_{,z}\right] = \boxed{-\pi_{,z} + b} - \varepsilon v_j^L v_{j,z}^s + \frac{\alpha^2}{ReRi} w_{,jj}$$

hydrostatic

$$b_t + v_j^L b_{,j} + \frac{M_{Ro}}{RoRi} w b_z = \frac{1}{Pe} b_{,jj}$$

$$v_{j,j} + \frac{M_{Ro}}{RoRi} w_z = 0$$

$$\boxed{\varepsilon = \frac{V^s H}{f L H_s}}$$

$$Re = \frac{UL}{\nu} \quad Ro = \frac{U}{fL} \quad Ri = \frac{N^2}{(U_{,z})^2} \quad \alpha = H/L \quad M_{Ro} \equiv \max(1, Ro)$$

J. C. McWilliams and BFK. Oceanic wave-balanced surface fronts and filaments. Journal of Fluid Mechanics, 730:464-490, 2013.

N. Suzuki and BFK. Understanding Stokes forces in the wave-averaged equations. Journal of Geophysical Research-Oceans, 121:1-18, 2016.

3 Wave Effects, 1: Lagrangian Advection:

Particles, tracers, momentum flow with Lagrangian, not Eulerian flow

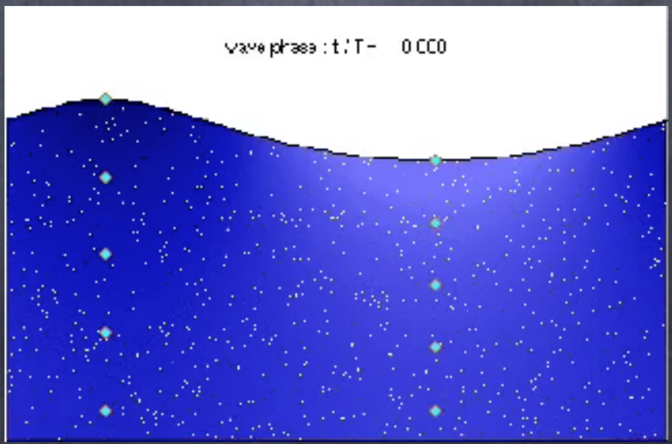
$$Ro\left[v_{i,t} + v_j^L v_{i,j}\right] + \frac{M_{Ro}}{Ri} w v_{i,z} + \epsilon_{izj} v_j^L = -M_{Ro}\pi_{,i} + \frac{Ro}{Re} v_{i,jj}$$

$$\frac{\alpha^2}{Ri}\left[w_{,t} + v_j^L w_{,j} + \frac{M_{Ro}}{RoRi} w w_{,z}\right] = -\pi_{,z} + b - \varepsilon v_j^L v_{j,z}^s + \frac{\alpha^2}{ReRi} w_{,jj}$$

$$b_t + v_j^L b_{,j} + \frac{M_{Ro}}{RoRi} w b_z = \frac{1}{Pe} b_{,jj}$$

Adding a Stokes advection term converts total to Lagrangian advection

$$\underbrace{v_j^L}_{\text{Lagrangian}} = \underbrace{v_j}_{\text{Eulerian}} + \underbrace{v_j^S}_{\text{Stokes}}$$



N. Suzuki and BFK. Understanding Stokes forces in the wave-averaged equations. Journal of Geophysical Research-Oceans, 121:1-18, 2016.

3 Wave Effects, 2: Lagrangian Coriolis:

Particles, tracers, momentum flow with Lagrangian, not Eulerian flow—Experience Coriolis force during this motion

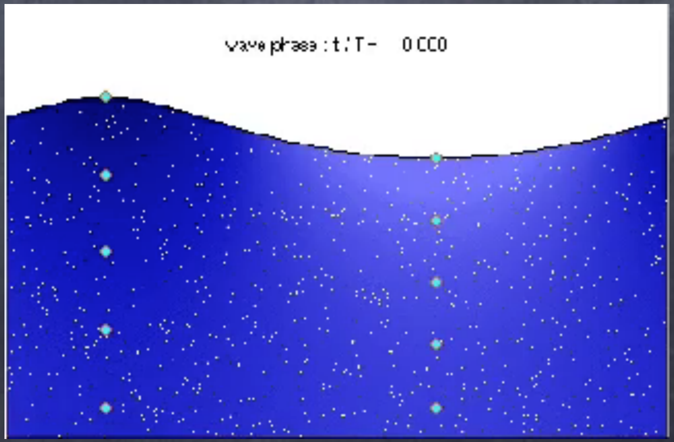
$$Ro\left[v_{i,t} + v_j^L v_{i,j}\right] + \frac{M_{Ro}}{Ri} w v_{i,z} + \boxed{\epsilon_{izj} v_j^L} = -M_{Ro}\pi_{,i} + \frac{Ro}{Re} v_{i,jj}$$

$$\frac{\alpha^2}{Ri}\left[w_{,t} + v_j^L w_{,j} + \frac{M_{Ro}}{RoRi} w w_{,z}\right] = -\pi_{,z} + b - \varepsilon v_j^L v_{j,z}^s + \frac{\alpha^2}{ReRi} w_{,jj}$$

$$b_t + v_j^L b_{,j} + \frac{M_{Ro}}{RoRi} w b_z = \frac{1}{Pe} b_{,jj}$$

Adding a Stokes Coriolis term converts total to Lagrangian

$$\underbrace{v_j^L}_{\text{Lagrangian}} = \underbrace{v_j}_{\text{Eulerian}} + \underbrace{v_j^S}_{\text{Stokes}}$$



N. Suzuki and BFK. Understanding Stokes forces in the wave-averaged equations. Journal of Geophysical Research-Oceans, 121:1-18, 2016.

Imperfect Frontogenesis: Governing equations with Eddy Viscosity and Diffusivity

x momentum equation (horizontal cross front)

$$\frac{Du}{Dt} - fv = \alpha u - \frac{1}{\rho_0}\frac{\partial p}{\partial x} + \boxed{\nu \nabla^2 u}$$

(Viscosity)

y momentum equation (horizontal along front)

$$\frac{Dv}{Dt} + fu - -\alpha v + \boxed{\nu \nabla^2 v}$$

z momentum equation (vertical)

$$0 = b - \frac{1}{\rho_0}\frac{\partial p}{\partial z}$$

Thermodynamic equation

$$\frac{Db}{Dt} = \boxed{\kappa \nabla^2 b}$$ (Diffusivity)

Conservation of mass

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

- u - Cross front velocity
- v - Along front velocity
- w - Vertical velocity
- p - Pressure
- b - Buoyancy
- ρ_0 - Density
- f - Coriolis
- α - Strain field
- $\frac{D}{Dt}$ - Lagrangian derivative
- κ - Diffusivity
- ν - viscosity

A. Bodnef, BFK, Luke Van Roekel. Effects of point source PV on frontogenesis, in preparation.

Perturbation analysis

Assuming eddy viscosity and diffusivity are small corrections to the conventional strain induced frontogenesis equations.

$$U = \bar{U} + u = \varepsilon^0(\bar{U} + u^0) + \varepsilon^1 u^1 + O(\varepsilon^2)$$
$$V = \bar{V} + v = \varepsilon^0(\bar{V} + v^0) + \varepsilon^1 v^1 + O(\varepsilon^2)$$
$$W = w = \varepsilon^0 w^0 + \varepsilon^1 w^1 + O(\varepsilon^2)$$

Note, zeroth order contains both geostrophic and ageostrophic flows—but only *inviscid* flows

$$\varepsilon = \frac{1}{Ek} = \frac{\nu}{fL^2}$$

Eddy viscosity
(Ek – Ekman number)

$$\varepsilon = \frac{1}{Pe} = \frac{\kappa}{fL^2}$$

Eddy diffusivity
(Pe - Peclet-ish number)

Zeroth order – known solution.

First order – addition due to forcing terms

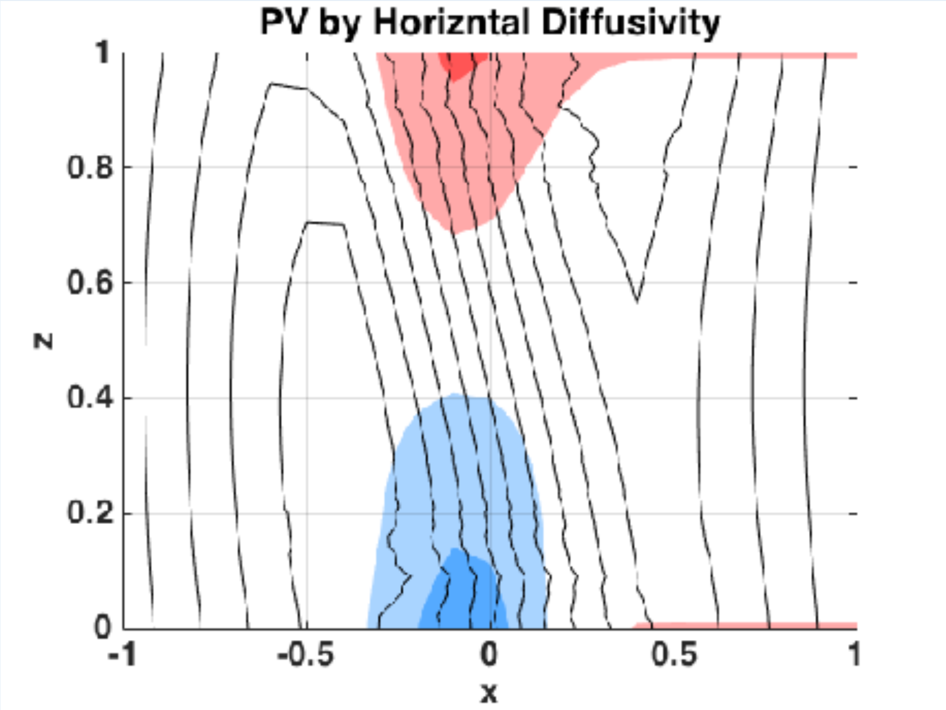
At zeroth order, an inviscid, zero-PV solution is found/used (e.g., Shakespeare & Taylor, 2013)

At first order, viscous and diffusive effects are found, and a single PV is injected.

A. Bodnef, BFK, Luke Van Roekel. Effects of point source PV on frontogenesis, in preparation.

Localized potential vorticity -> Green's fct. analysis

- Studies (e.g. Thomas 2005) show frontal PV is localized due to friction.
- Zeroth-order perturbation solution is inviscid, zero-PV.
- Most of the turbulence and mixing (i.e., PV source) is at the front.
- Thus the potential vorticity can be approximated as a **point source/sink** at the front.



Solid lines represent buoyancy,
red- positive PV, blue- negative PV (Thomas 05)

A. Bodnef, BFK, Luke Van Roekel. Effects of point source PV on frontogenesis, in preparation.

First order solution—
Amenable to Green's Fct. treatment due to point PV sources

PV equation nontrivial at first order:

Viscous forcing Diffusion

$$\frac{D}{Dt_0}Q^1 = (\nabla \times \mathbf{D}_u^1) \cdot \nabla b^0 + \omega_a^0 \cdot (\nabla D_b^1)$$

First order PV using zeroth order terms

$$Q^1 = \frac{1}{f^2}\frac{\partial^2 \phi^0}{\partial z^2}\boxed{\frac{\partial^2 \phi^1}{\partial x^2}} - \frac{2}{f^2}\frac{\partial^2 \phi^0}{\partial x \partial z}\boxed{\frac{\partial^2 \phi^1}{\partial x \partial z}} + \left(1 + \frac{1}{f^2}\frac{\partial^2 \phi^0}{\partial x^2}\right)\boxed{\frac{\partial^2 \phi^1}{\partial z^2}}$$

↑

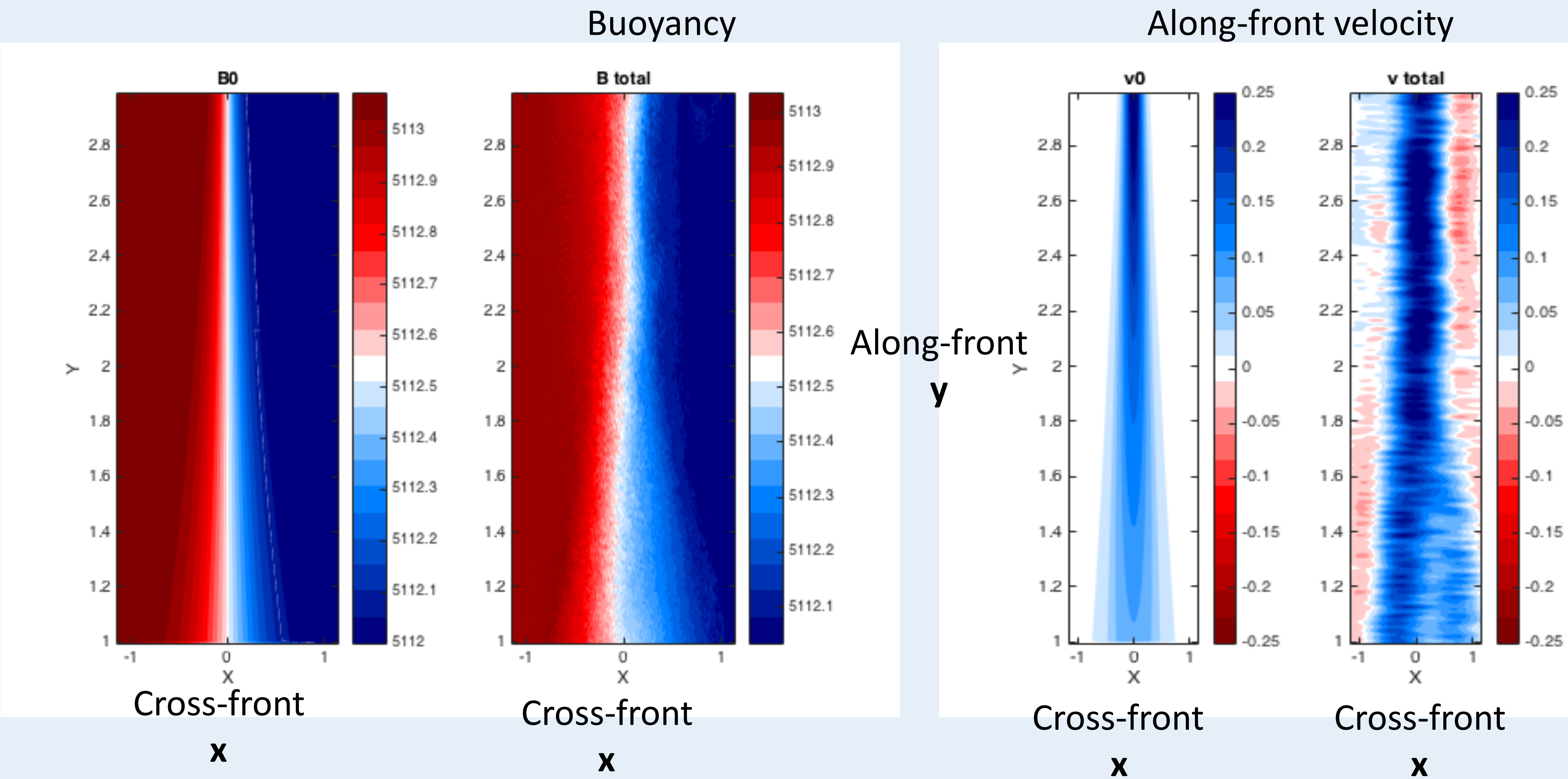
$\delta(x - x_{front})$

Second order linear PDE in first-order pressure correction.

Analytically invertible if PV is a point source (delta fct.)

A. Bodnef, BFK, Luke Van Roekel. Effects of point source PV on frontogenesis, in preparation.

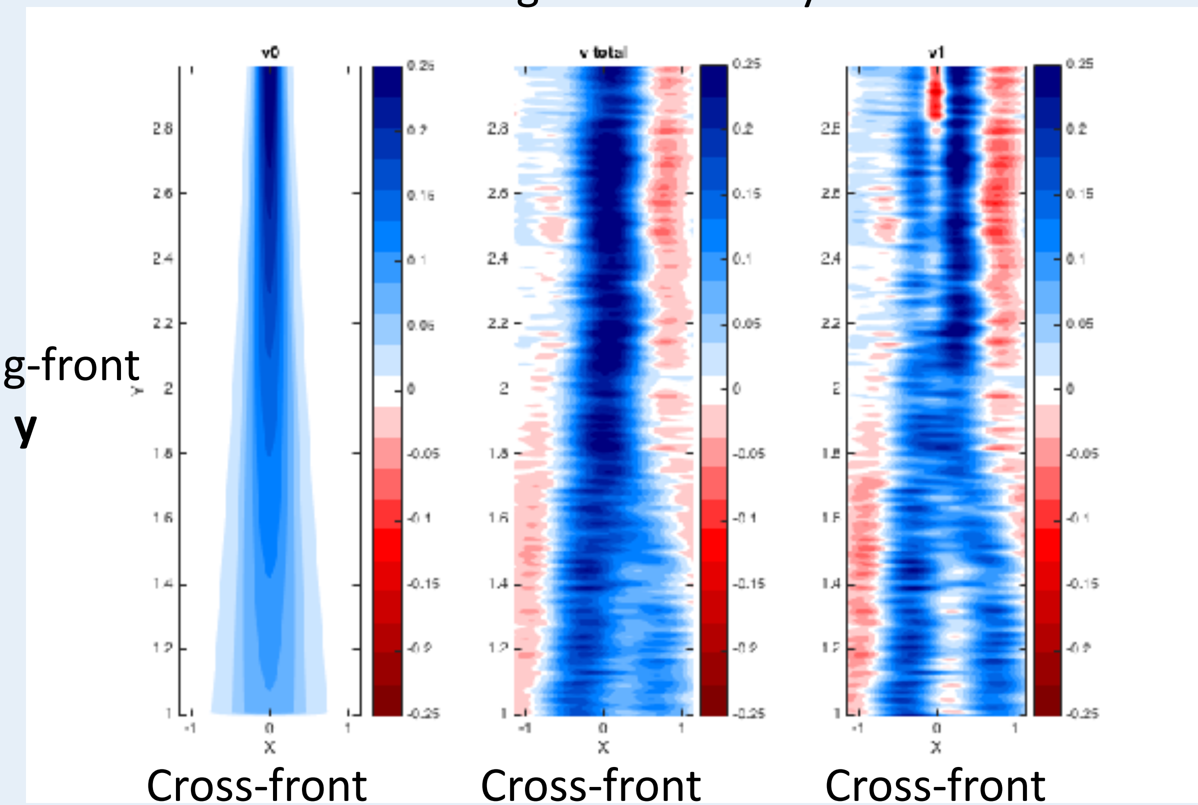
Perturbation analysis guides analysis of frontal Large Eddy Simulations



A. Bodnef, BFK, Luke Van Roekel. Effects of point source PV on frontogenesis, in preparation.

First order solution

- Perturbation parameter ε for both viscosity and diffusivity is O(0.1)
- The largest contributor is horizontal diffusivity.
- Perturbation theory appears to be valid.
- We can find first order velocities.



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