Observing the Ocean in the Lagrangian Frame

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Observations (quantitative) <- instruments <- problems (qualitative)

Advection

Eddy Diffusion

$$\frac{\partial}{\partial t} C = \frac{\partial}{\partial y}\left(\kappa \frac{\partial}{\partial y} C\right)$$

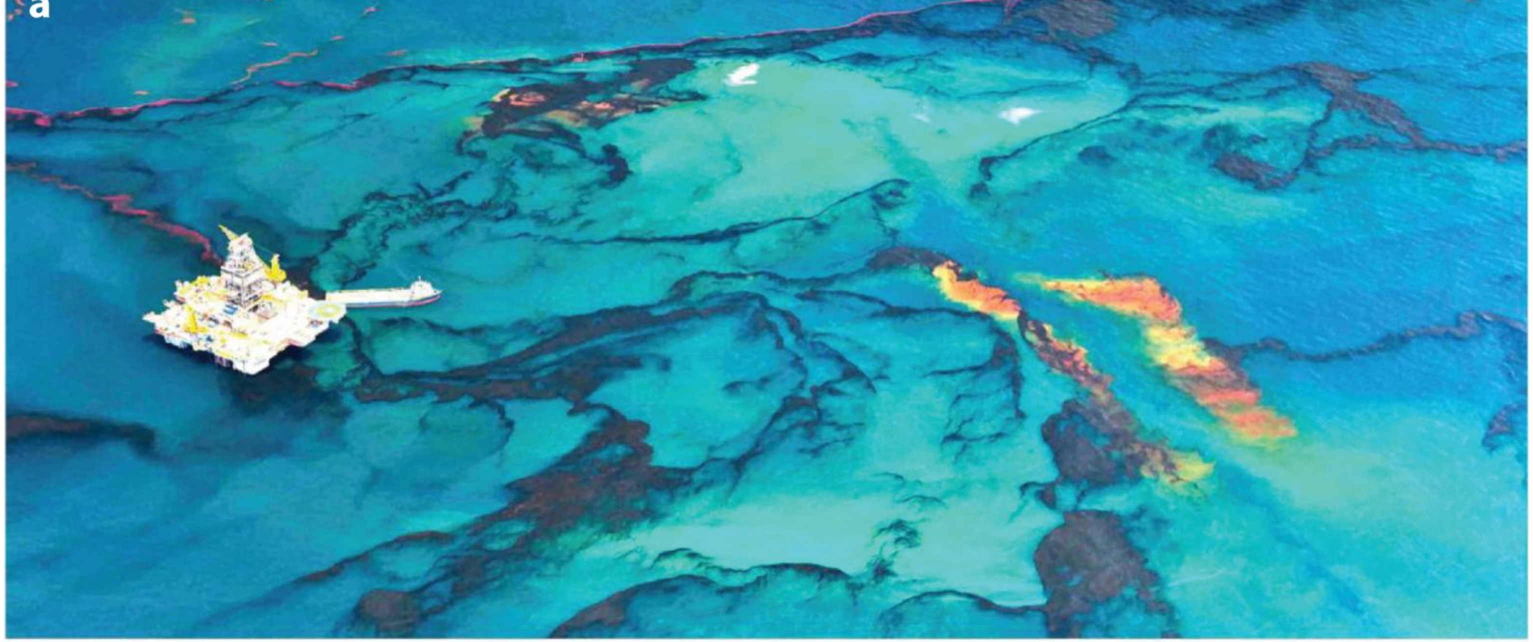
$$\kappa \propto y^{4/3}$$

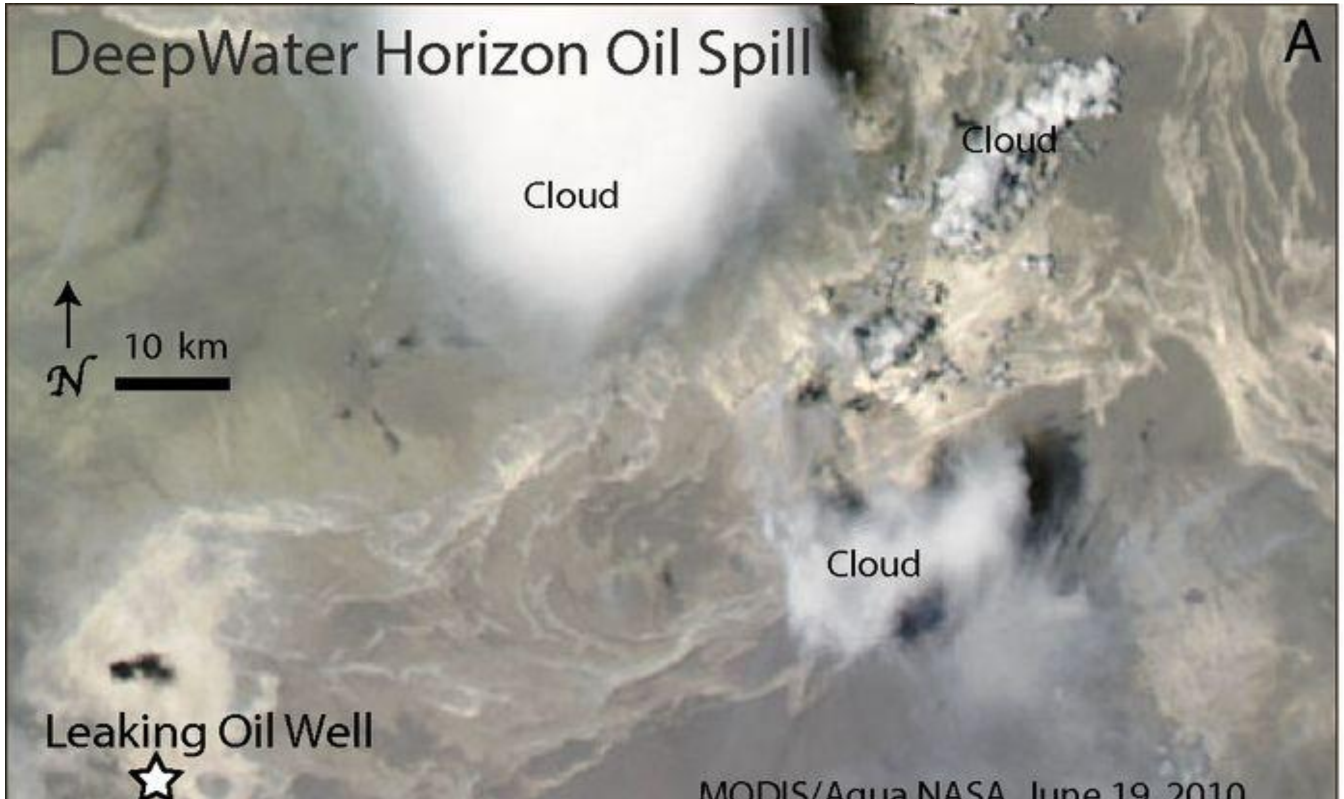
From observations of smoke plume spreading

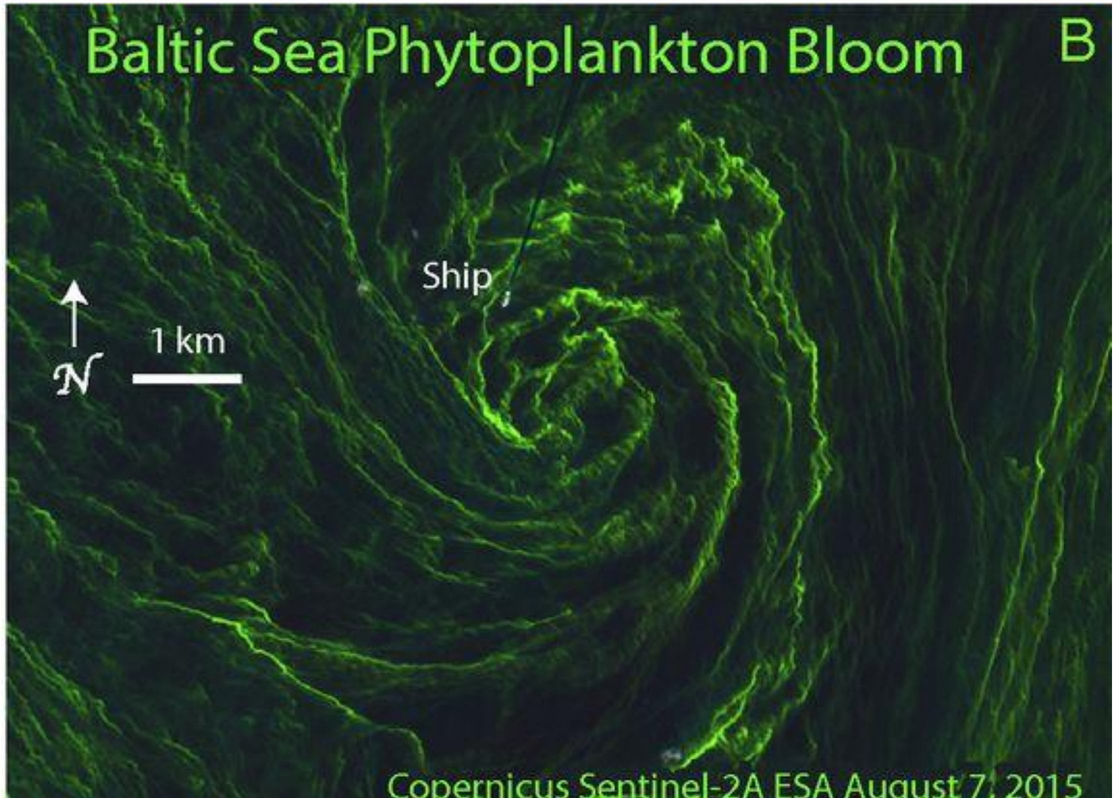
Taylor, G. I. (1921): Diffusion by continuous moments. Proc. London Math. Soc., (20), 196-211.
Richardson, L.F. (1926): Atmospheric diffusion on a distance-neighbour graph. Proc. Royal Soc. London A (110), 709-737.
Richardson,L. F., and H. Stommel, Note on eddy Diffusion in the sea, J. Meteo. Rol., 5, 238-240, 1948.

Tracer distribution at the Ocean Surface

Lumpkin et al 2016,
D'Asaro et al 2018







Eddy Diffusion – early observations related to turbulence/mixing theory
Richardson-Obukhov law

$$\kappa \propto y^{4/3}$$

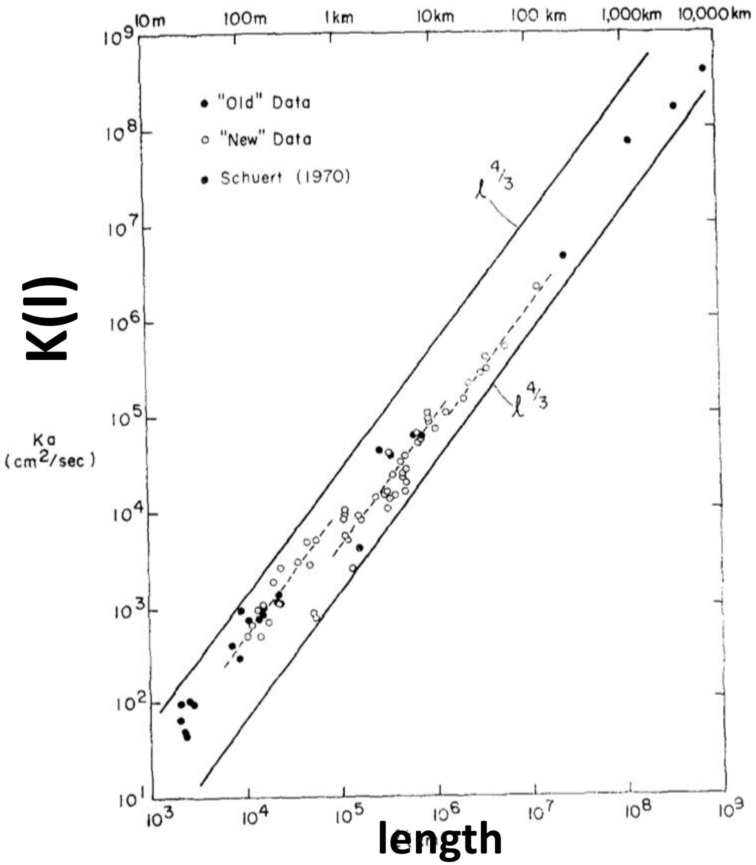


Fig. 4. Apparant diffusivity versus scale of diffusion (old and new data): fit of the 4/3 power law locally.

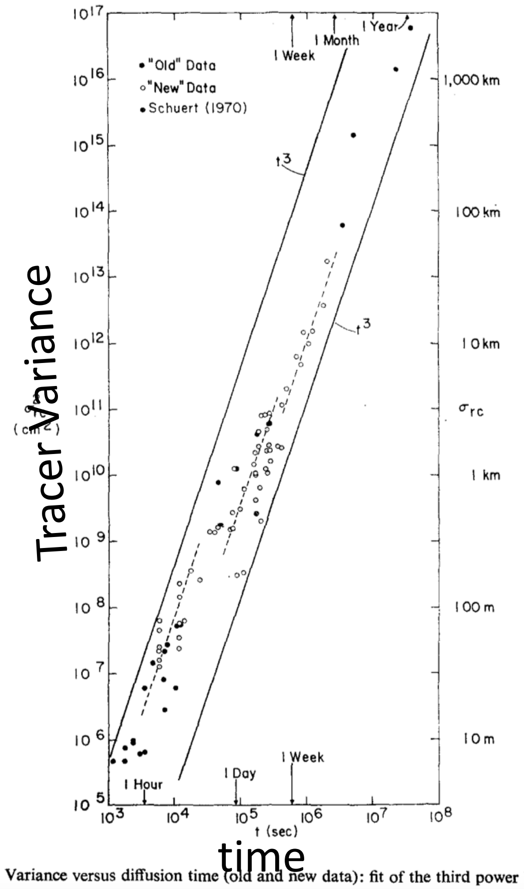


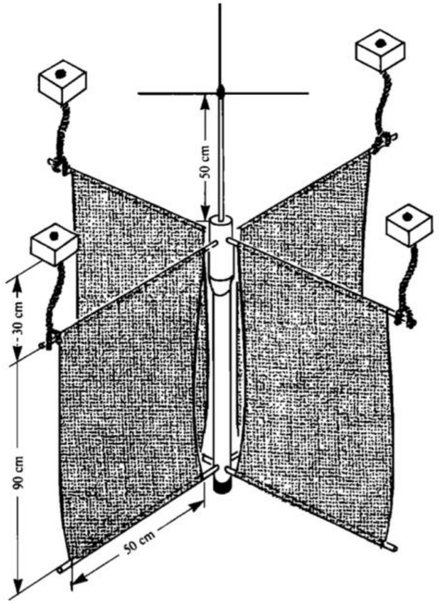
Fig. 3. Variance versus diffusion time (old and new data): fit of the third power law locally.

$$\sigma(t) \propto t^3$$

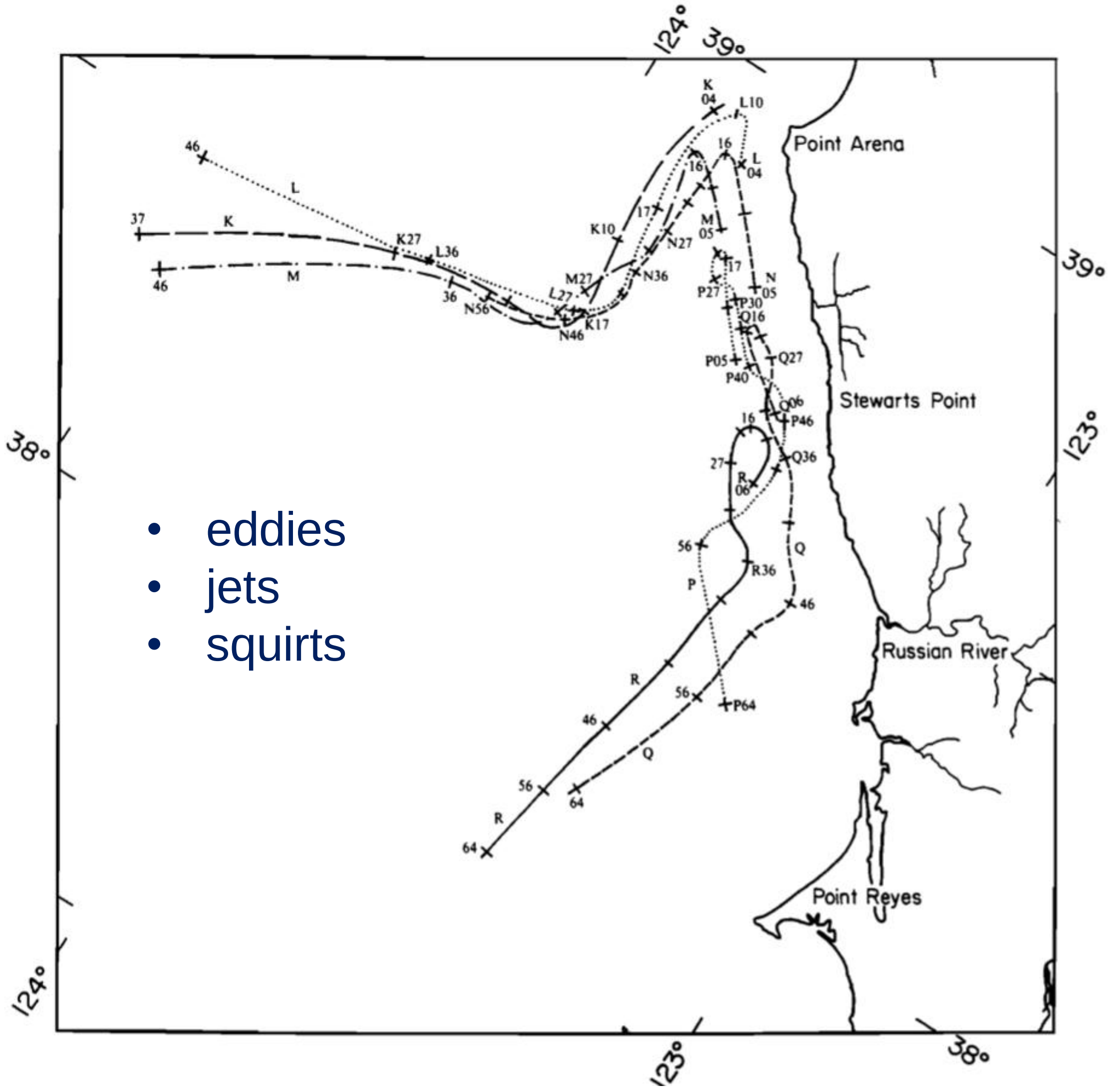
Diffusivity estimates using instantaneous dye releases.
Okubo 1971: Oceanic Diffusion Diagrams

History of modern observations

Coastal Ocean Dynamics Experiment (CODE) desire to observe small scale structure of ocean



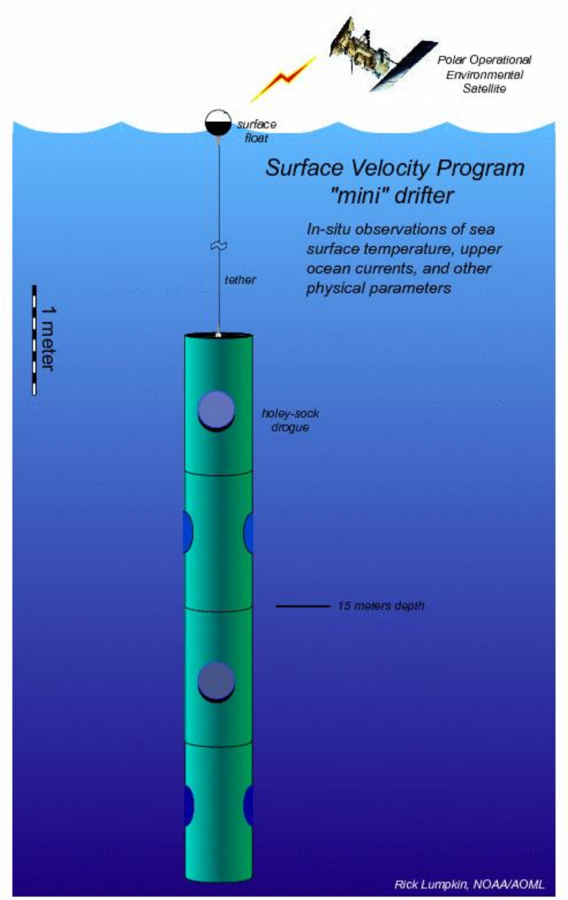
CODE drifter – drogue depth 1 m; position every few hours (Davis, 1985)



- eddies
- jets
- squirts

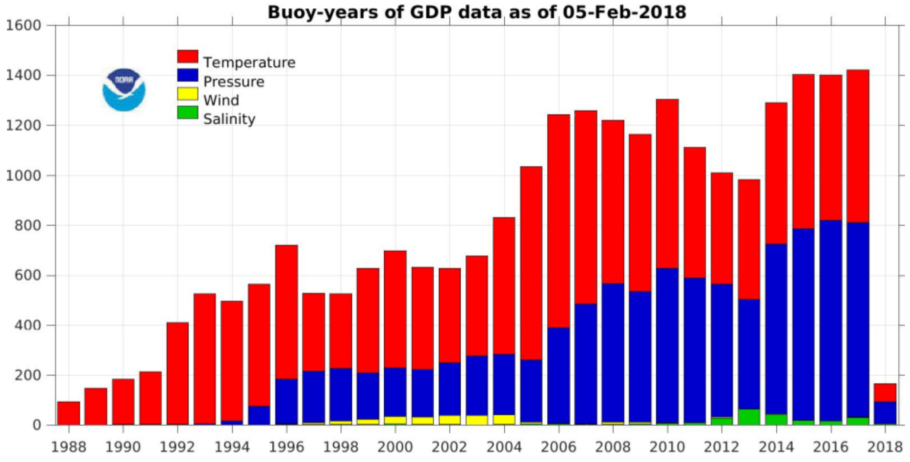
History of modern observations

Global Drifter Program



SVP drifter - drogue depth 15 m; position every few hours (Niiler et al. 1987)

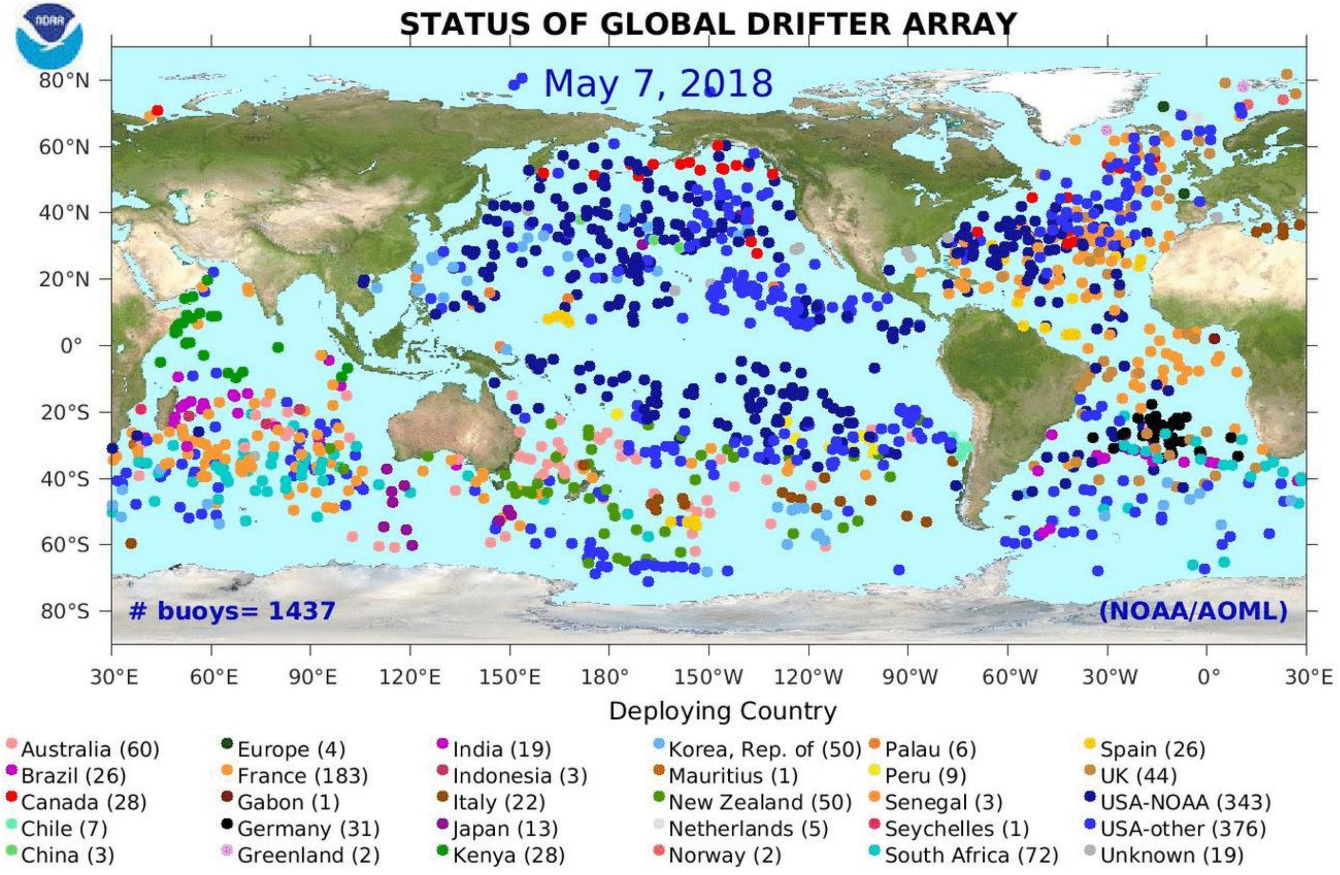
Niiler and Lumpkin



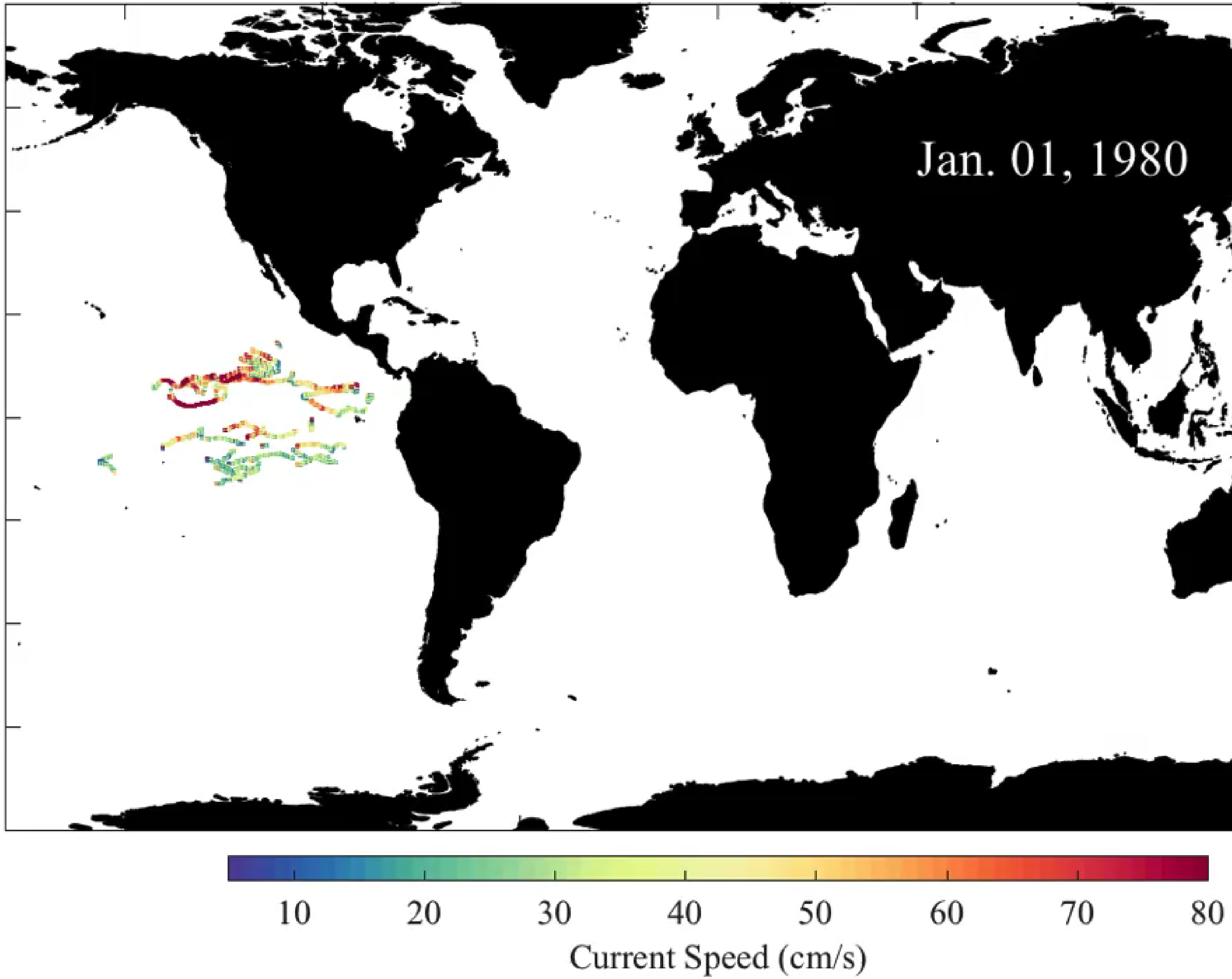
- Position recorded every few hours with accuracy of O(100 m).
- Platform for global observations of SST and atmospheric pressure.
- Studies often ”bin” data.

History of modern observations

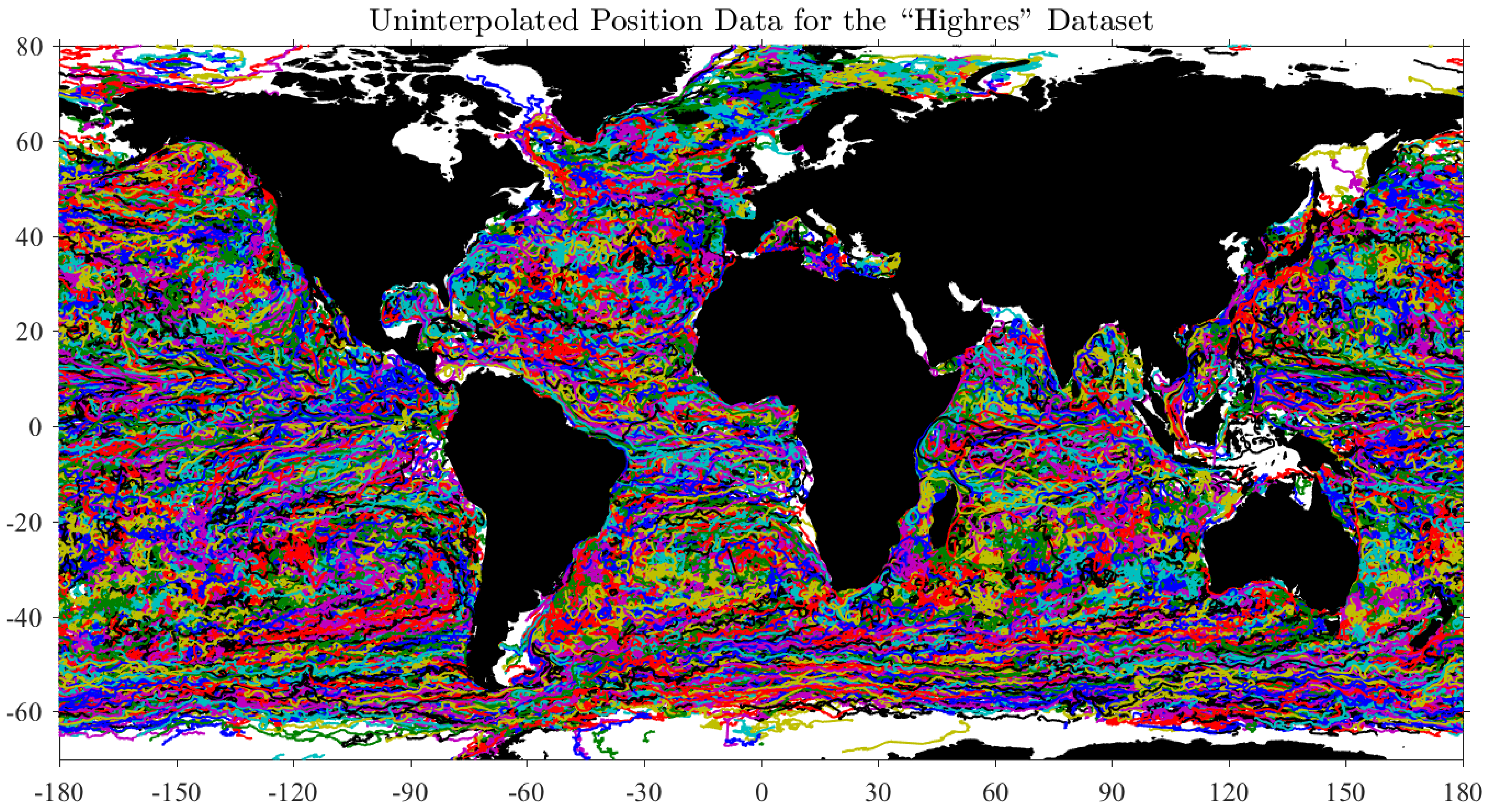
Global Drifter Program



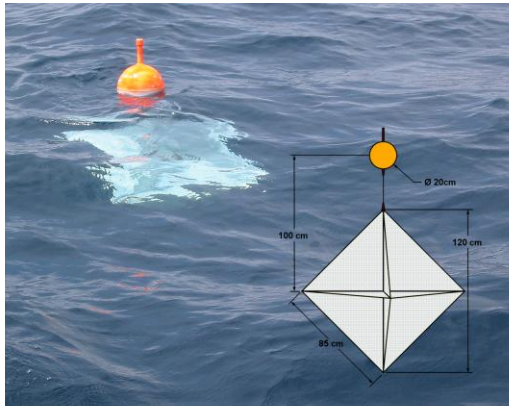
<u>GDP Science goals</u>: global surface currents, global EKE distribution, global SST, atmospheric pressure for assimilation into weather models



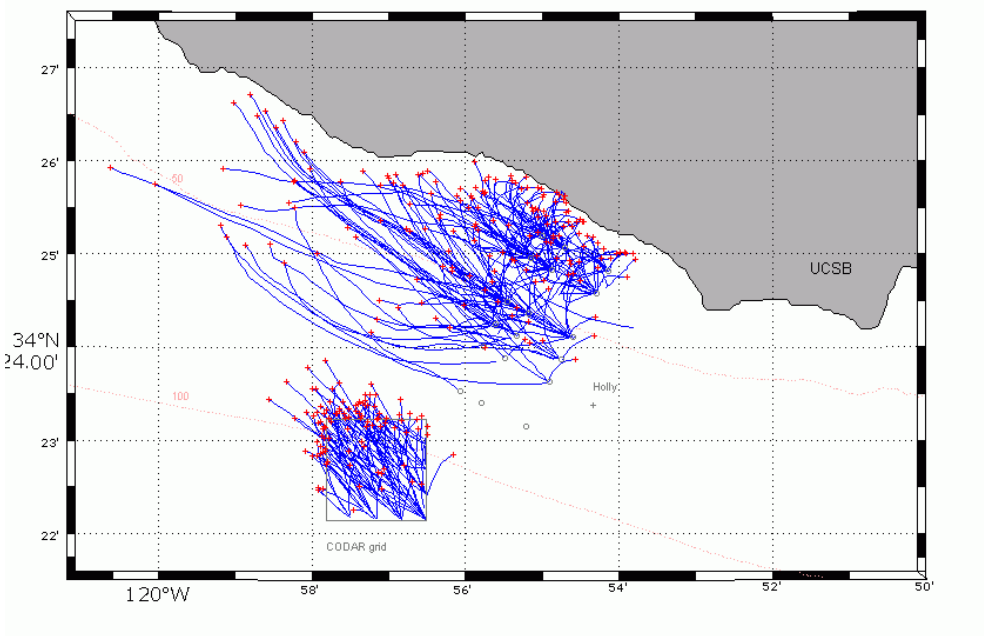
Jonathan Lilly - http://www.jmlilly.net/


Uninterpolated Position Data for the "Highres" Dataset
80
60
40
20
0
-20
-40
-60
-180
-150
-120
-90
-60
-30
0
30
60
90
120
150
180

GPS drifters for high resolution coastal observations



Microstar drifter – drogue depth 1 m; position every 10 mins with O(m) accuracy. Recoverable!! (Ohlmann et al 2005).

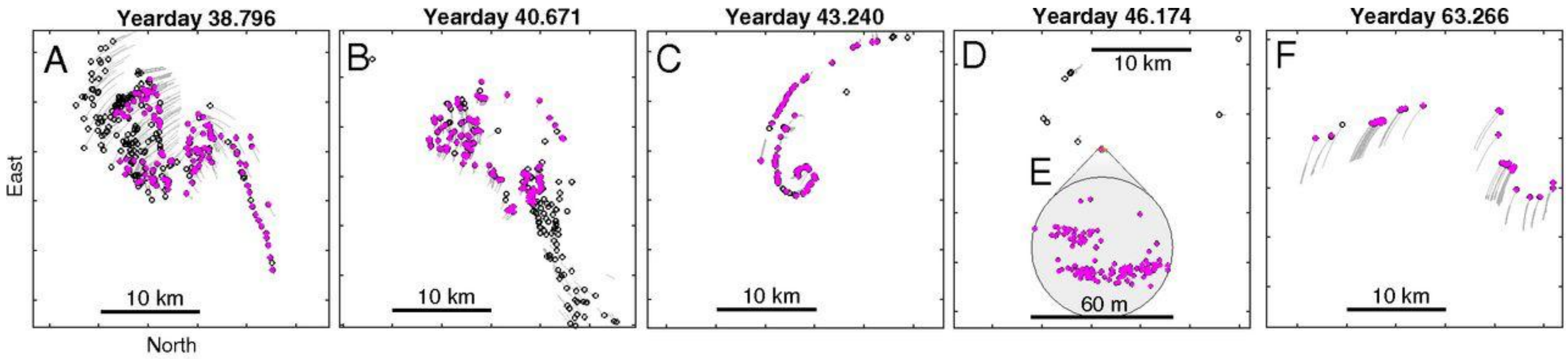


300 velocity observations within a 2 x 2 km grid collected over 5 days with 16 drifters (Ohlmann et al. 2007).

Microstar demonstrated the power of deploying clusters. Drifters no longer expendable.

Smaller more economical (biodegradable) drifters for deploying large numbers

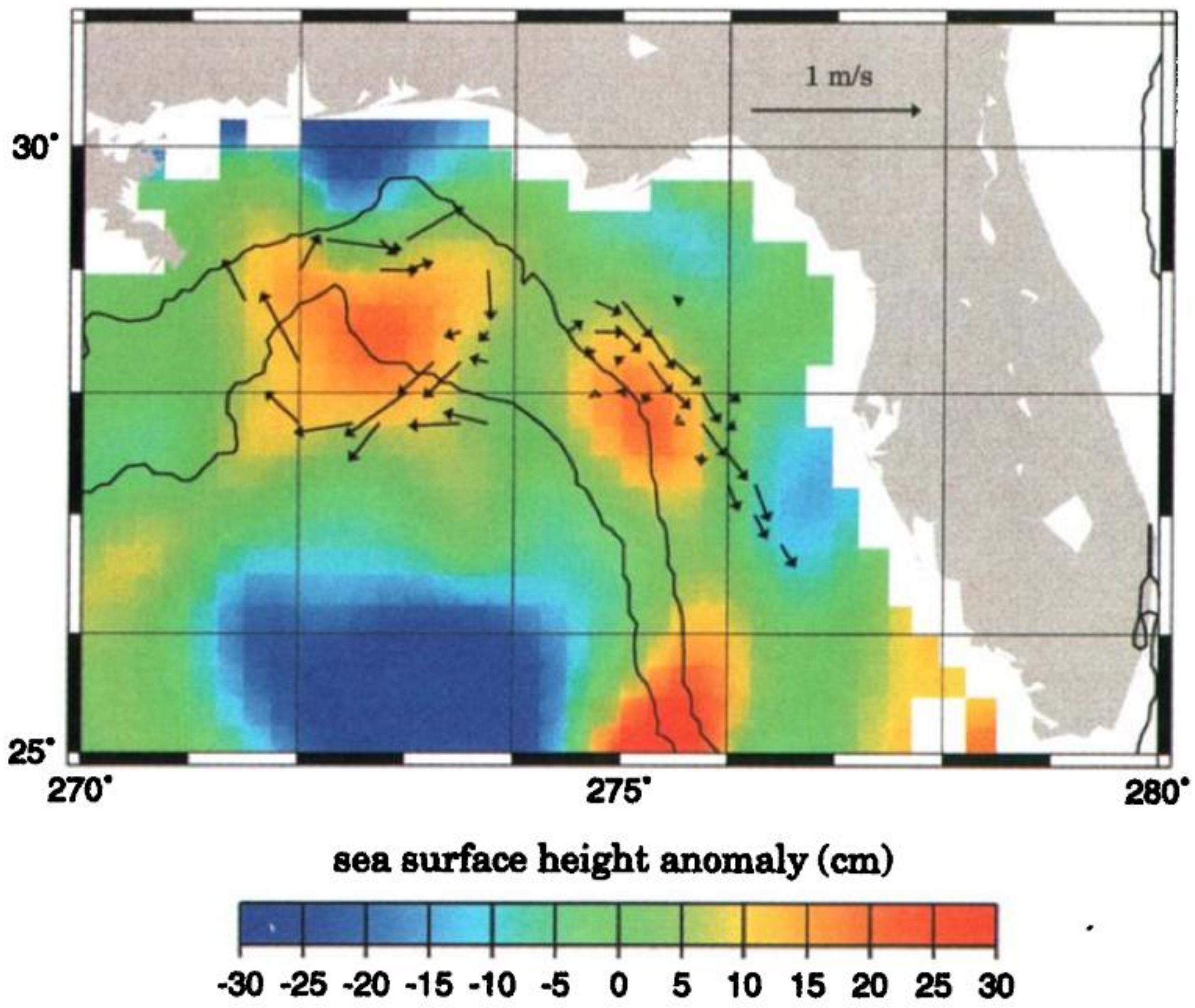
Carthe drifter – drogue depth 0.5 m; GPS position every few minutes (Novelli et al. 2017).



Carthe drifter – allowed 200 drifters to be deployed at once over a 20 x 20 km region. "More than half of an array of ~200 surface drifters covering ~20 × 20 km^2 converged into a 60 × 60 m region within a week (D'Asaro et al. 2018).

What velocity field does the drifter measure?

Sea level anomaly from satellite and coincident drifter observations



Geostrophic velocity obtained from SSH + **MORE!**

Figure from Ohlmann et al. 2001

Deep Ocean Measurements

Swallow Floats

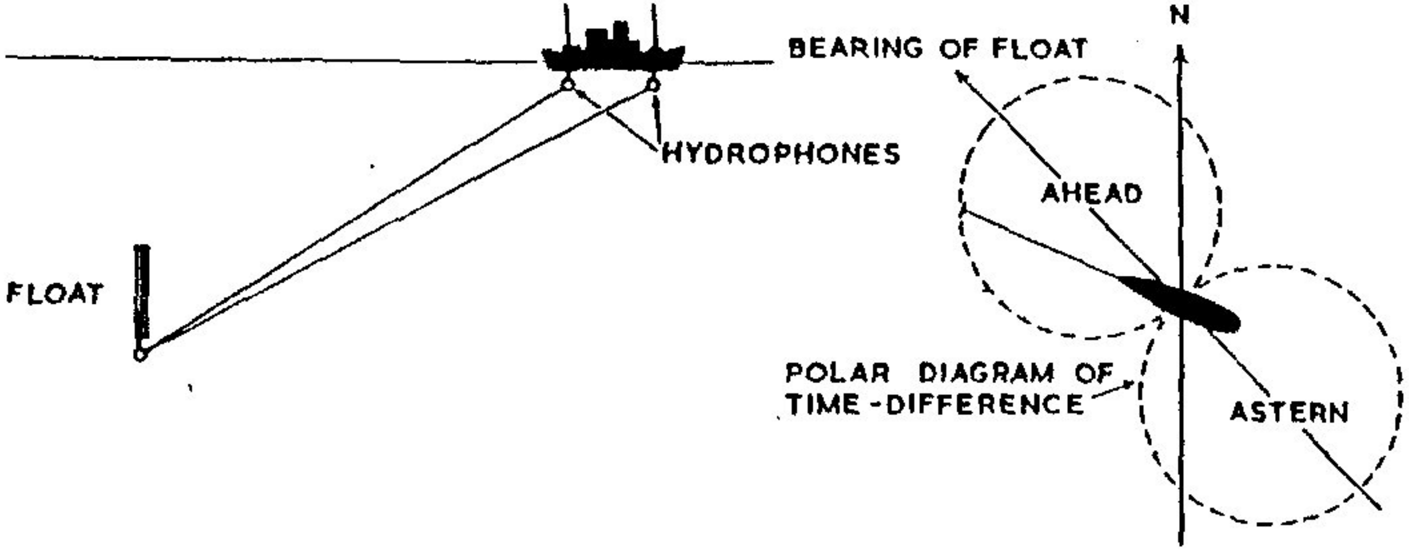
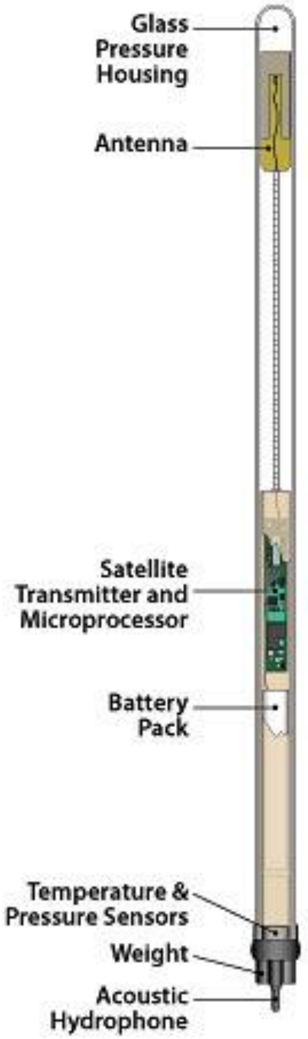
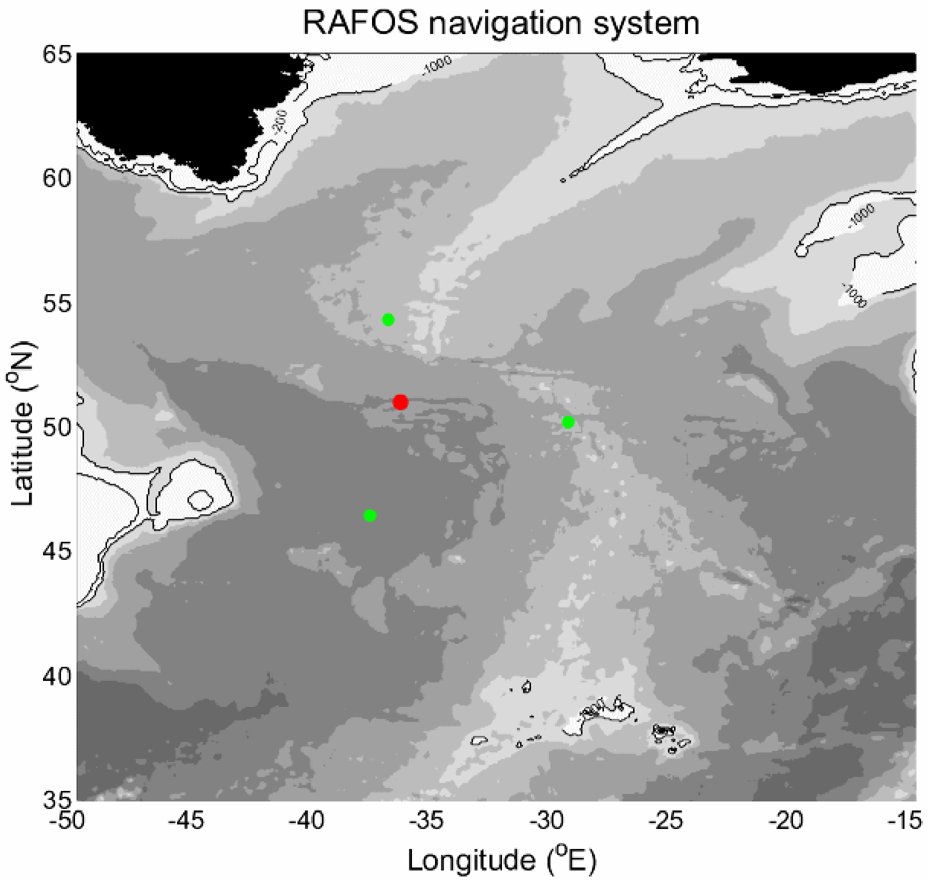


Fig. 3. Method of locating float.

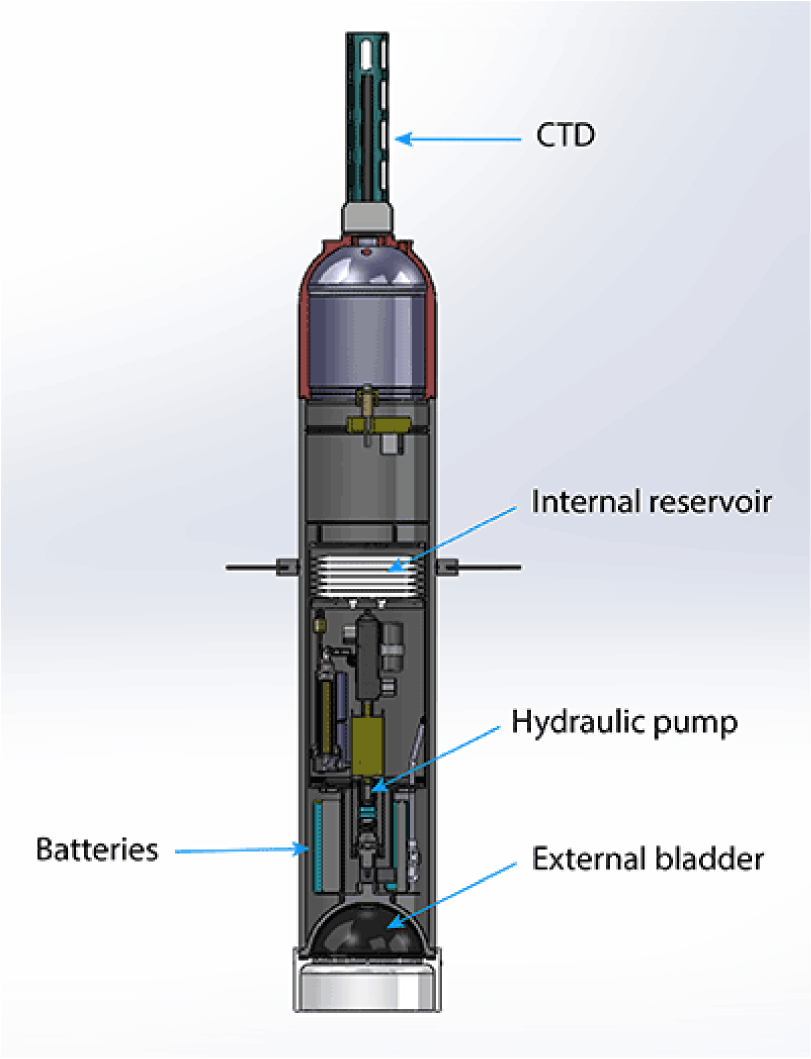
Deep Ocean Measurements

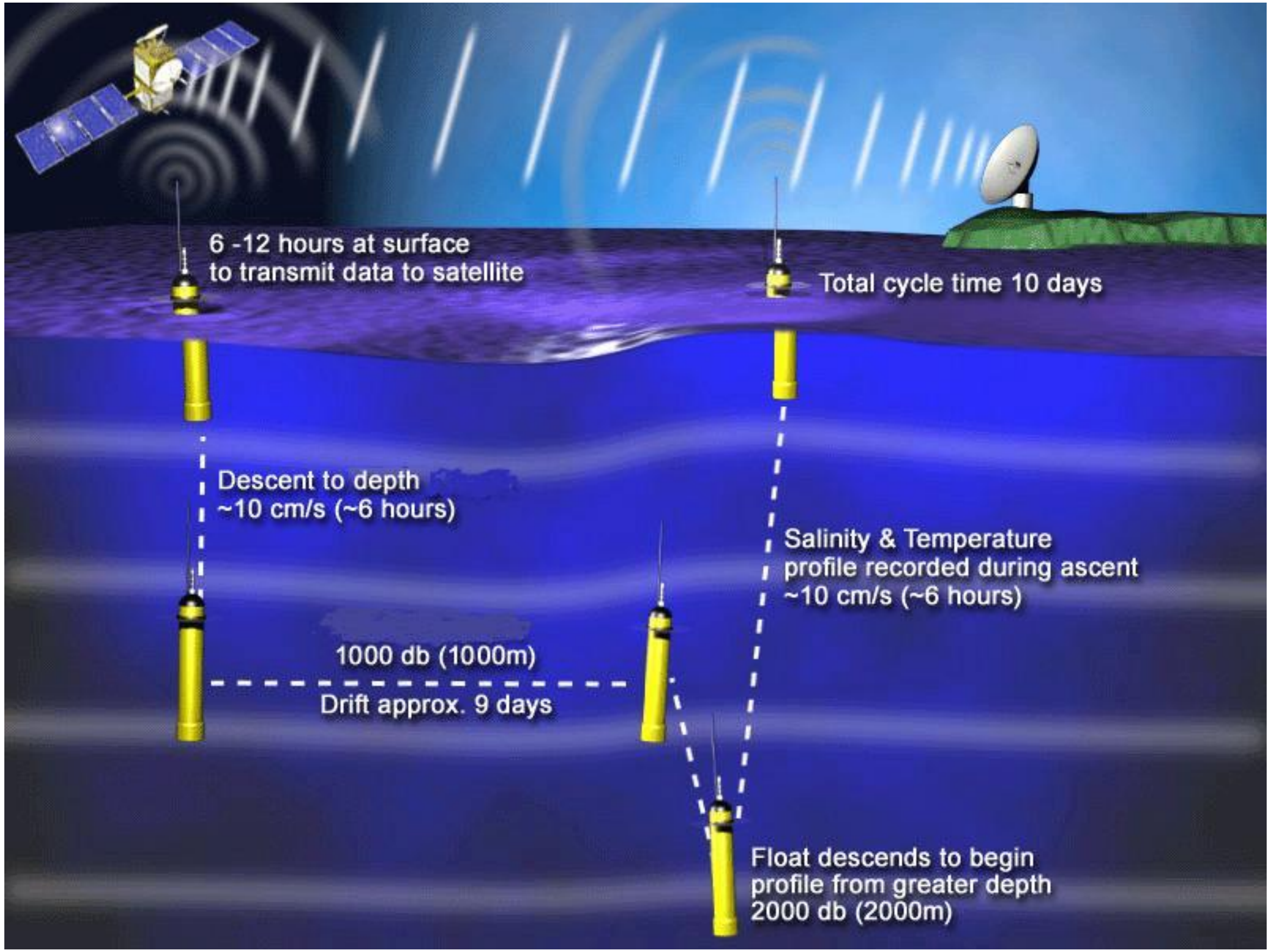


RAFOS Floats

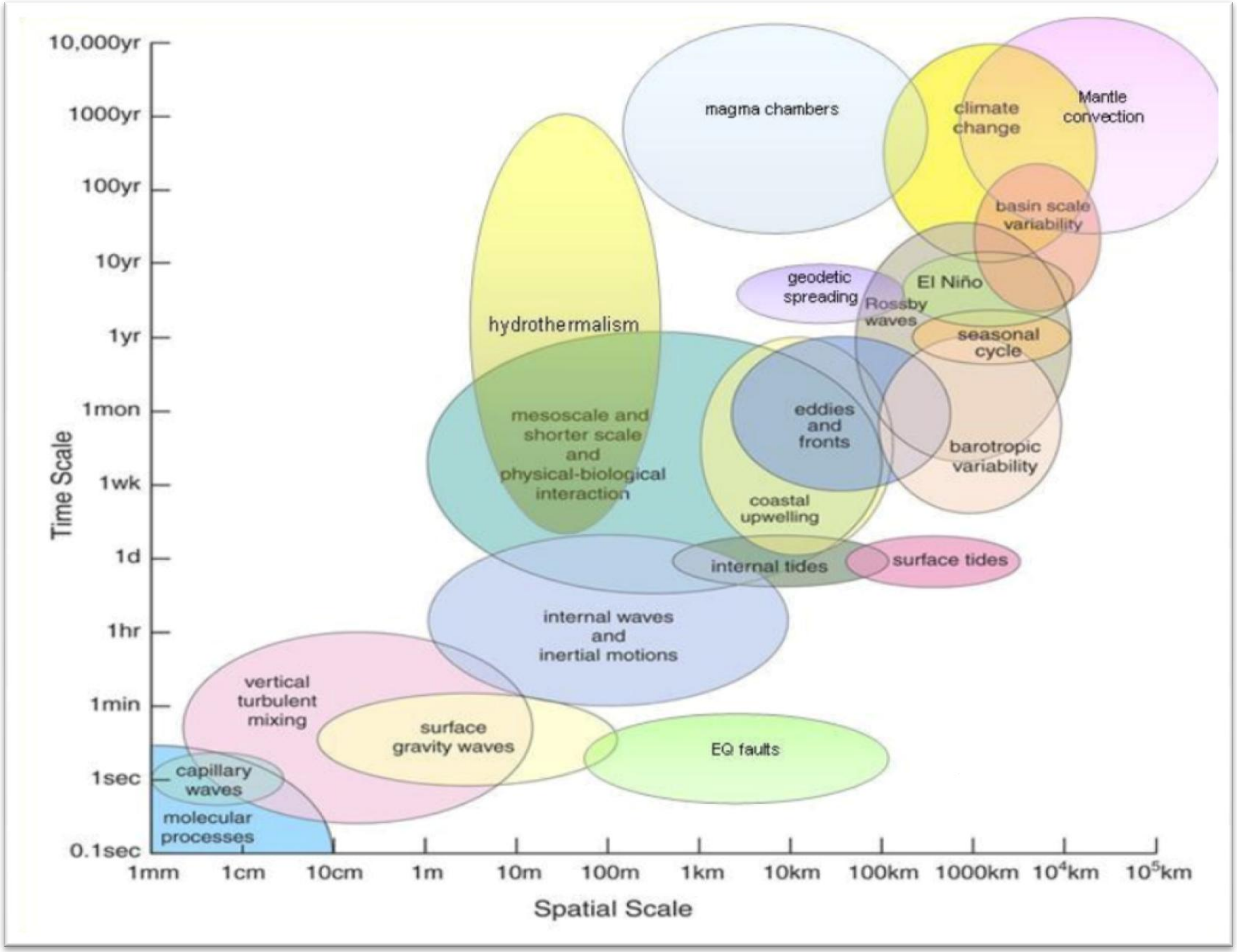


Argo floats


CTD
Internal reservoir
Hydraulic pump
Batteries
External bladder

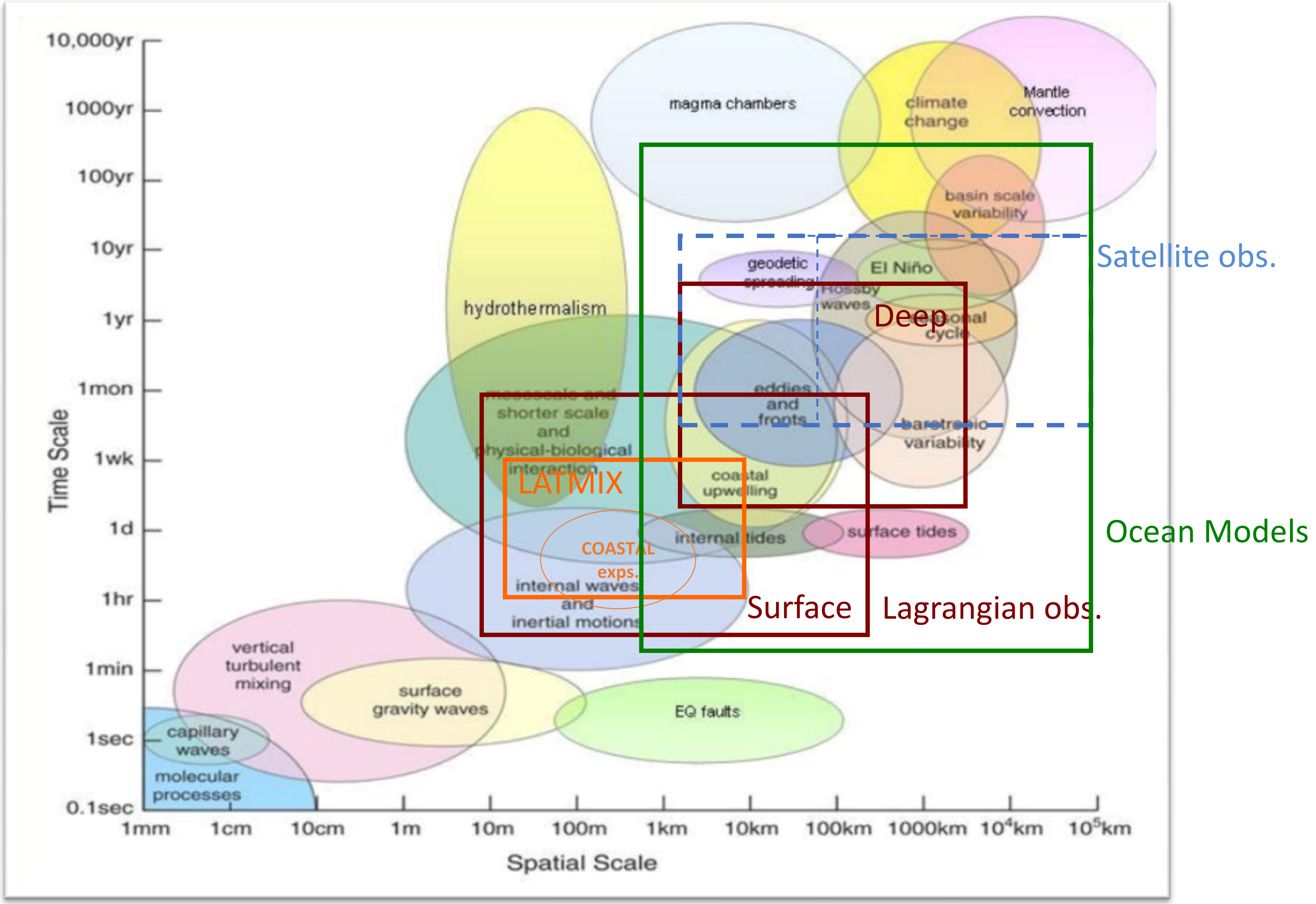

6 -12 hours at surface
to transmit data to satellite
Total cycle time 10 days
Descent to depth
~10 cm/s (~6 hours)
Salinity & Temperature
profile recorded during ascent
~10 cm/s (~6 hours)
1000 db (1000m)
Drift approx. 9 days
Float descends to begin
profile from greater depth
2000 db (2000m)

Processes



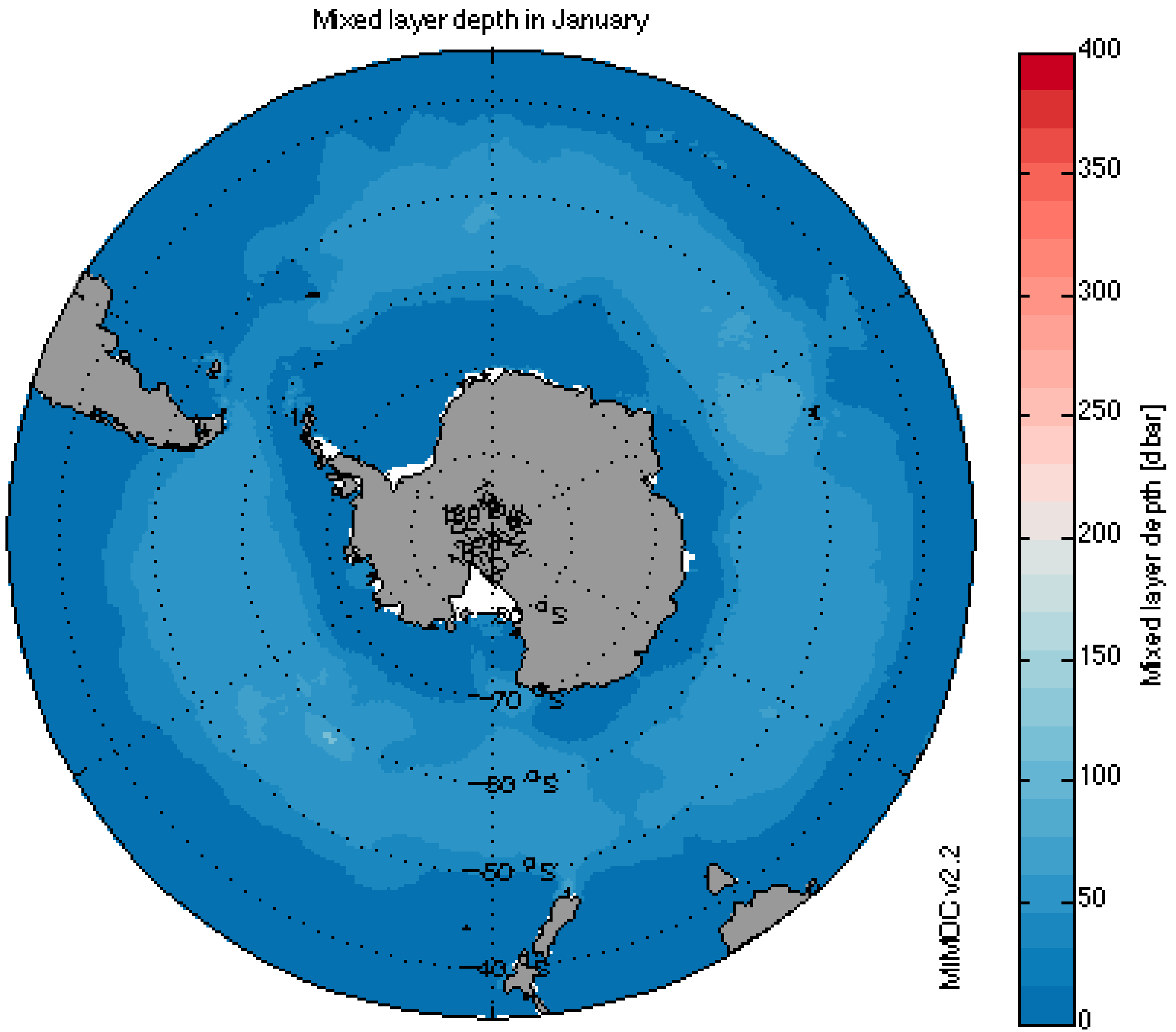
Stommel Diagram, Lampitt et al 2009 (adopted from D. Chelton)

Processes

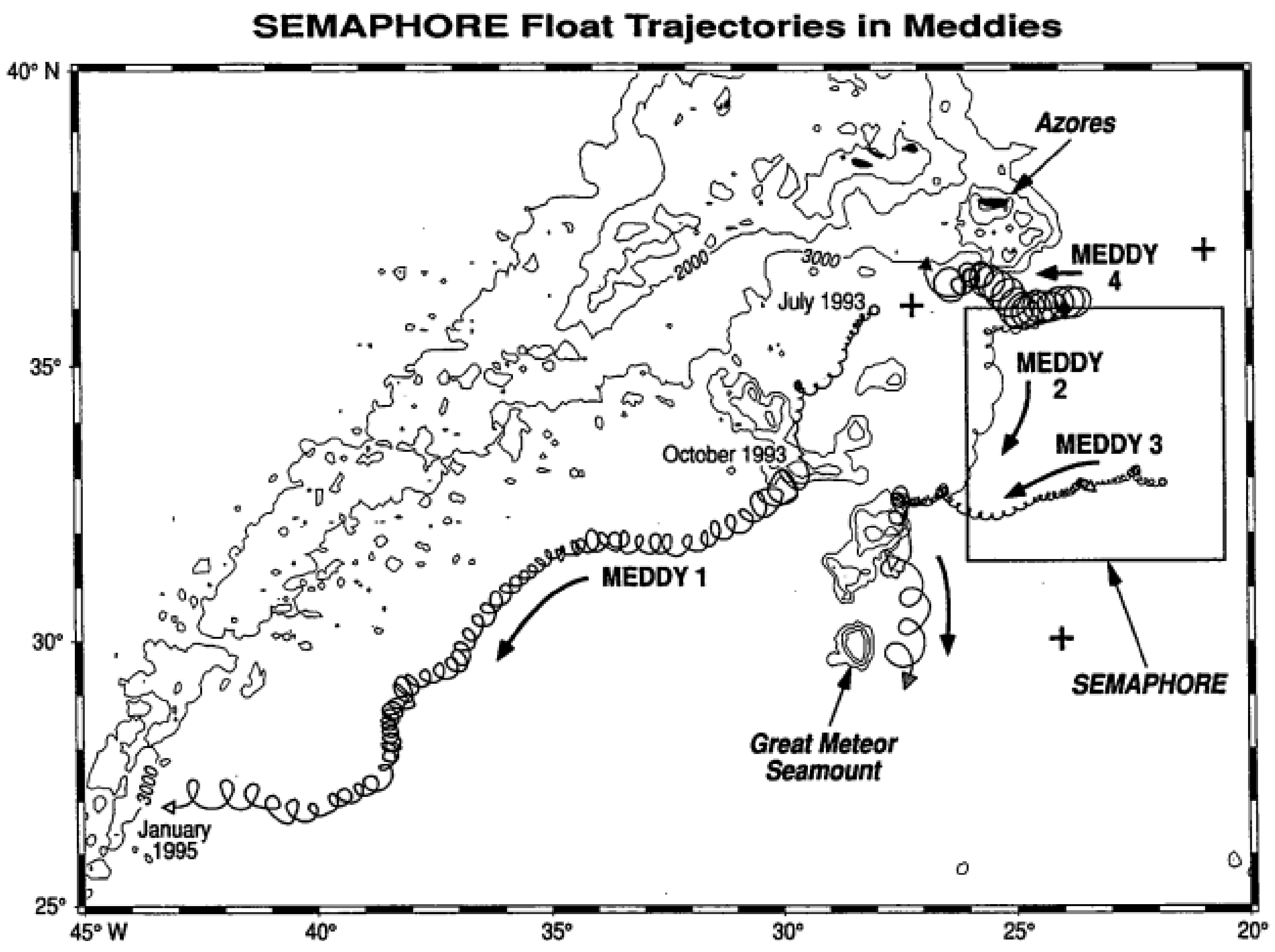


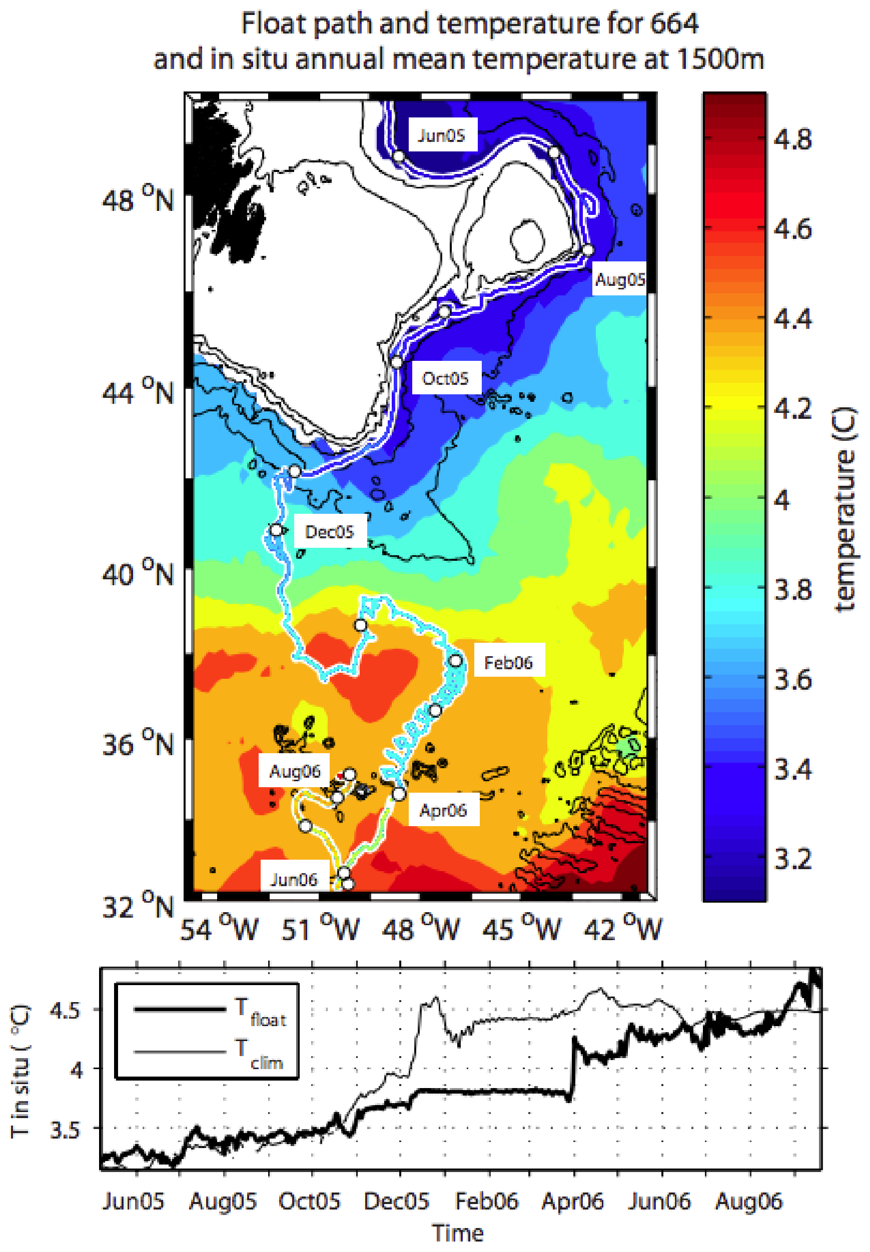
10,000yr
1000yr
100yr
10yr
1yr
1mon
1wk
1d
1hr
1min
1sec
0.1sec
Time Scale
1mm
1cm
10cm
1m
10m
100m
1km
10km
100km
1000km
10^4km
10^5km
Spatial Scale
magma chambers
climate change
Mantle convection
basin scale variability
geodetic spreading
El Niño
Rossby waves
seasonal cycle
hydrothermalism
mesoscale and shorter scale and physical-biological interaction
eddies and fronts
barotropic variability
coastal upwelling
internal tides
surface tides
internal waves and inertial motions
vertical turbulent mixing
surface gravity waves
EQ faults
capillary waves
molecular processes
Satellite obs.
Deep
LATMIX
COASTAL exps.
Surface
Lagrangian obs.
Ocean Models

Mixed layer depth seasonality in the Southern Ocean.

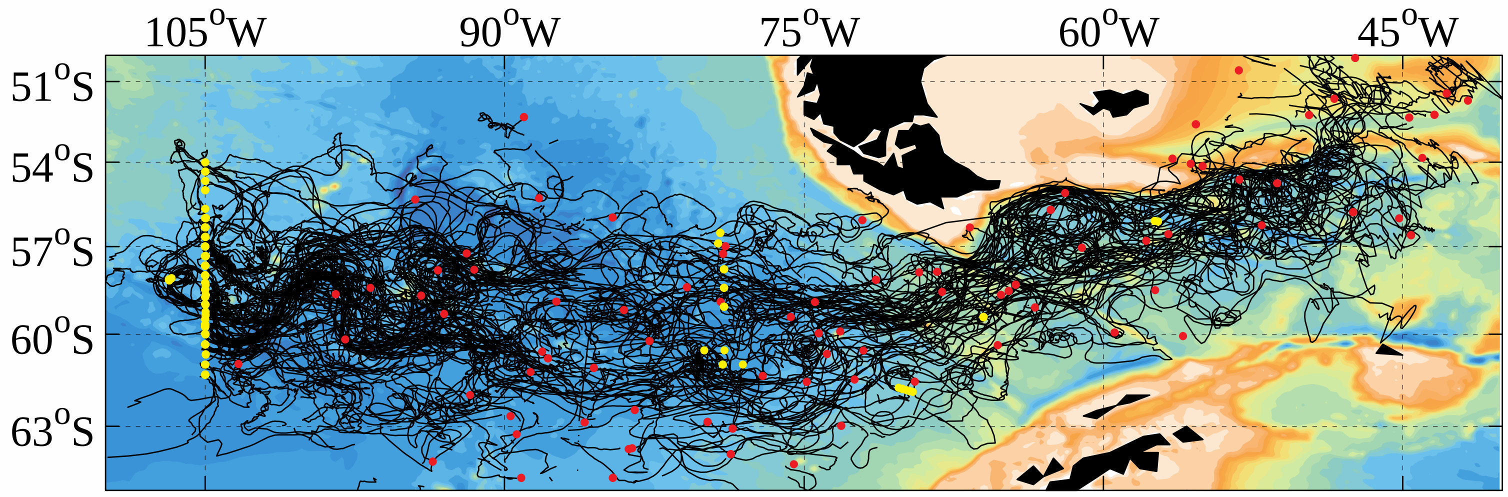


Deep ocean eddies/lenses

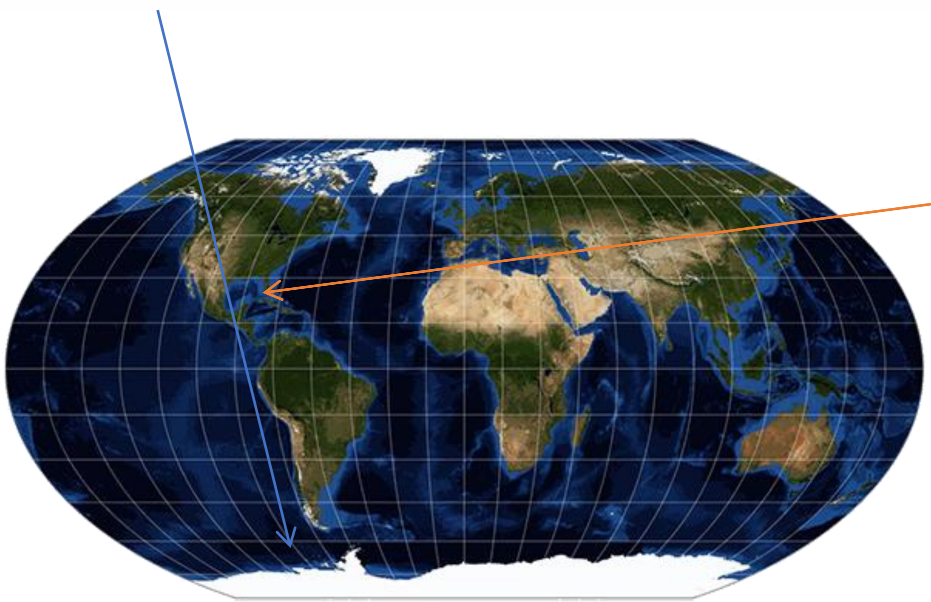




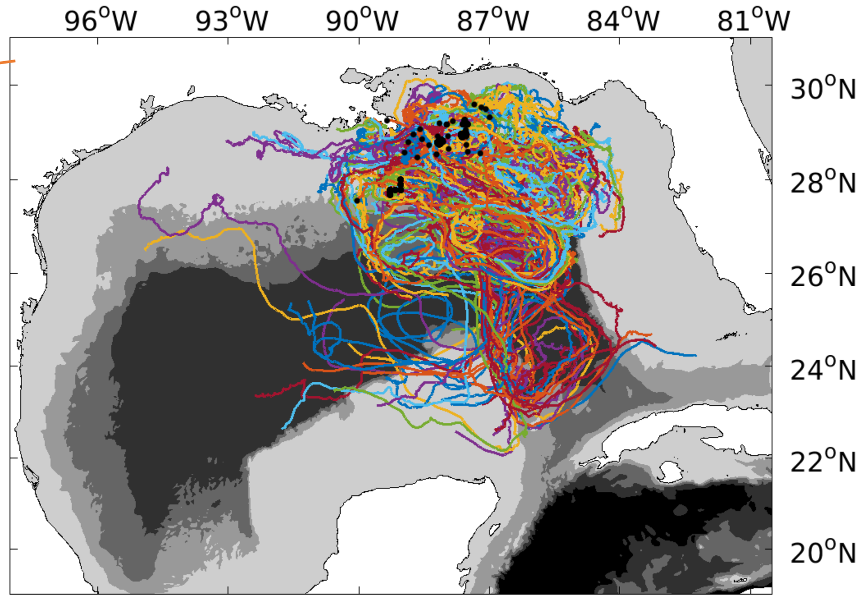
2 Recent Experiments



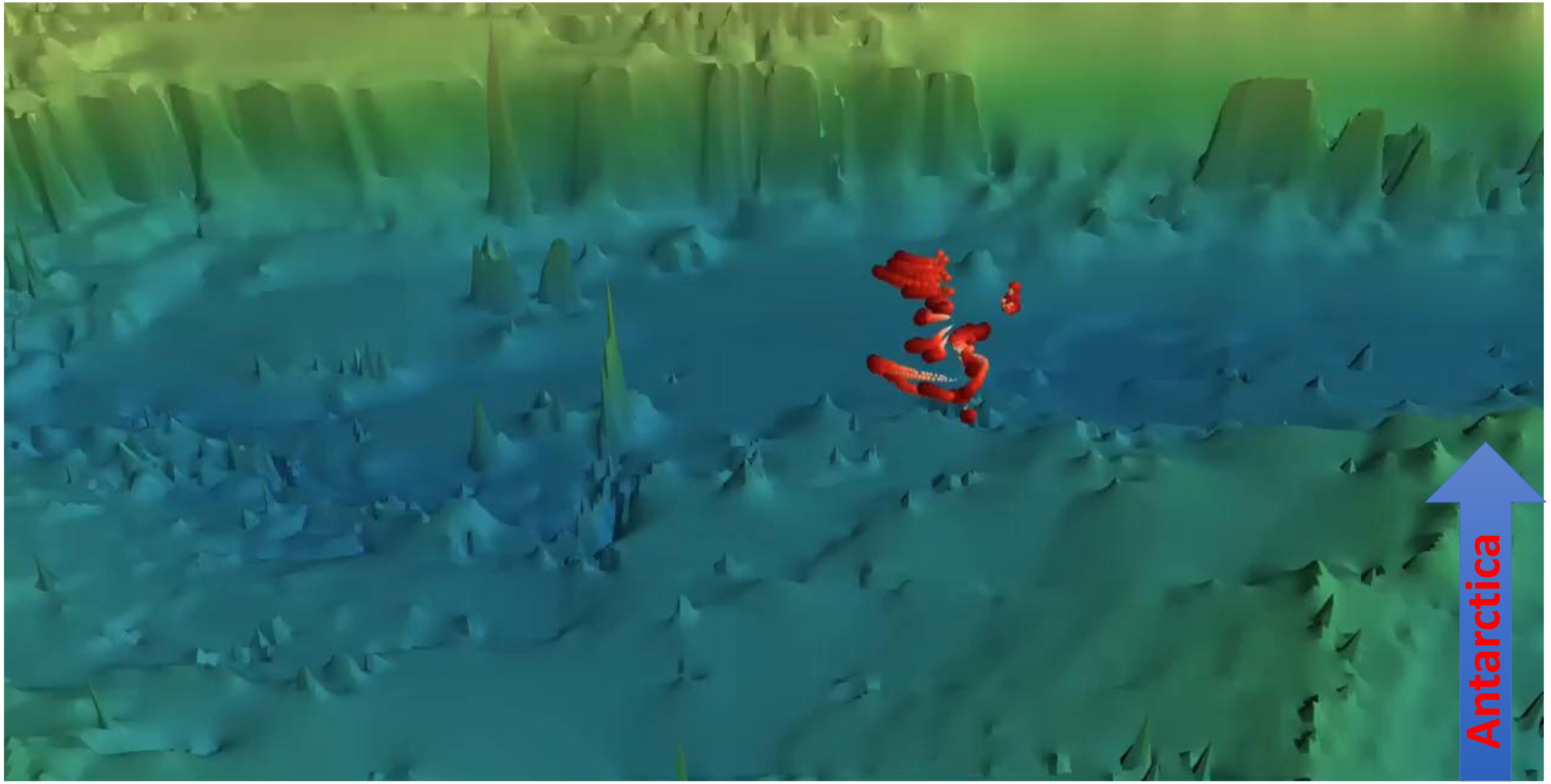
DIMES
Mid-depth (750-2500m) Southern Ocean
~Daily sampling



GLAD
Surface
Gulf of Mexico
~ 15 mins



Diapycnal and Isopycnal Mixing Experiment in the Southern Ocean (DIMES)



Statistical View –
Can some general statements be made?

Lagrangian measurements can sample the spatial structure of features in regions of zero mean Eulerian flow.

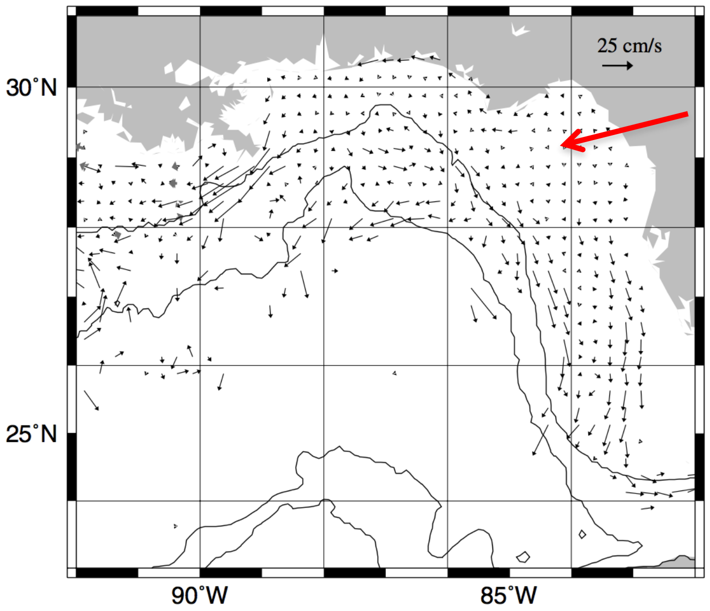


Fig. 11. Mean surface currents (as in Fig. 9) for the northeastern gulf.

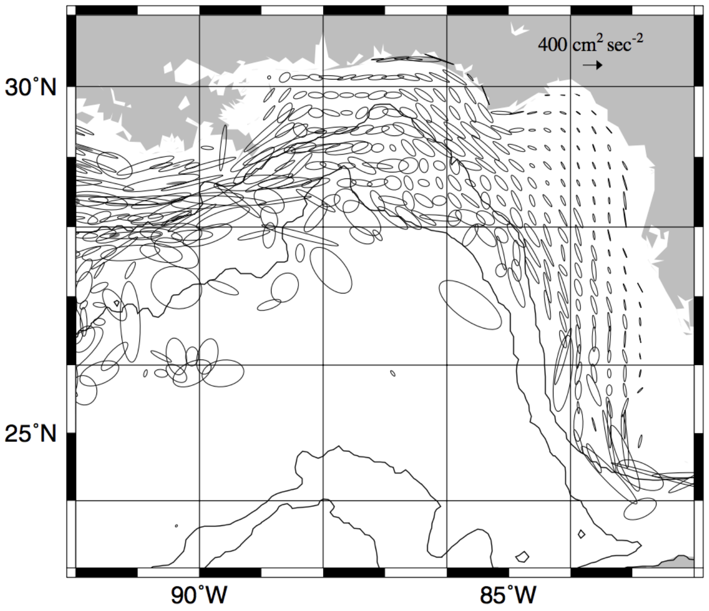
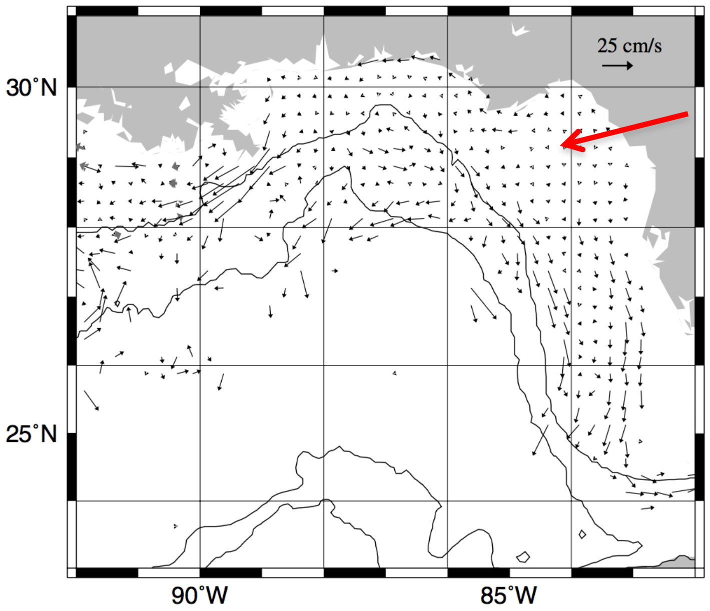


Fig. 12. Velocity variance ellipses (as in Fig. 10) for the northeastern gulf.

Eulerian mean velocities are near zero in location indicated by red arrow

Figures from Ohlmann and Niiler, 2005

Lagrangian measurements can sample the spatial structure of features in regions of zero mean Eulerian flow.



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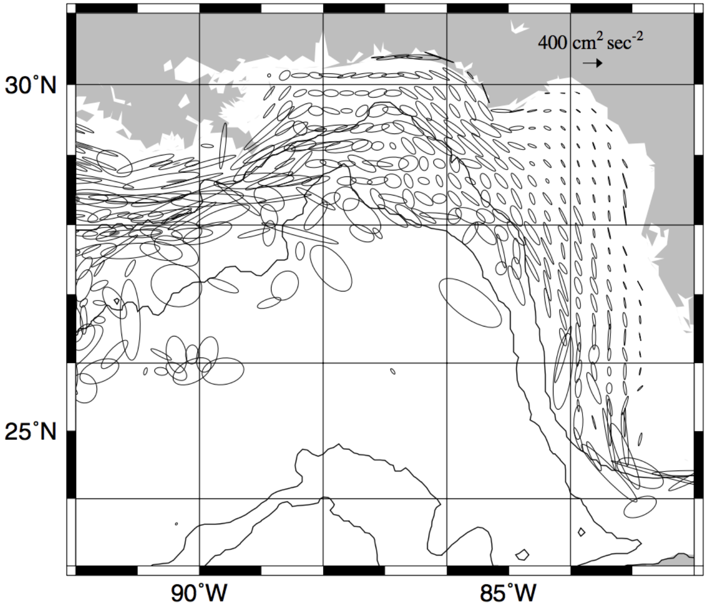

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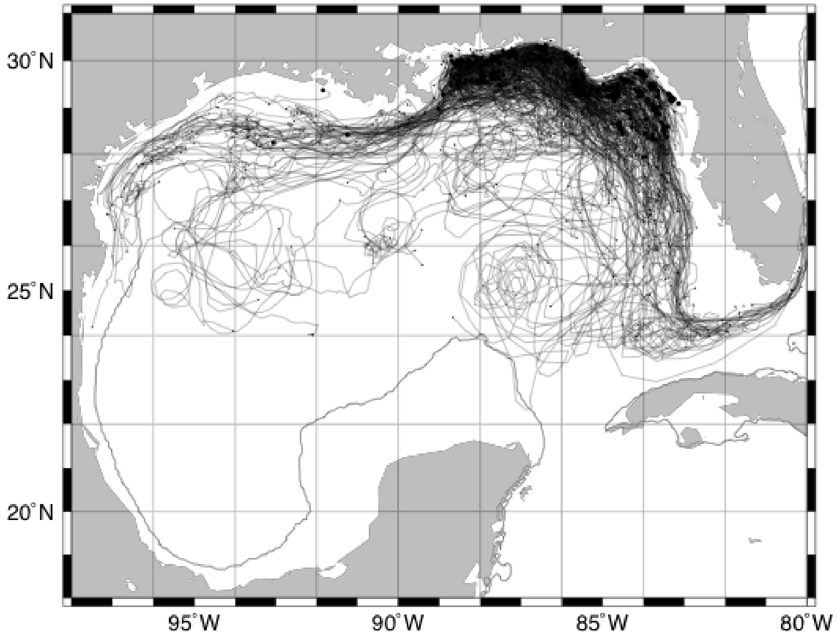


Corresponding **Lagrangian** trajectories (from which Eulerian mean derives) can show significant displacements. All trajectories begin in region of red star and have significant displacements.

Figures from Ohlmann and Niiler, 2005

Advection-Diffusion equation

$$\frac{\partial C}{\partial t} + \mathbf{U}.\nabla C = -\nabla. < \mathbf{u}'c' >= \nabla.(\mathbf{K}\nabla C)$$

$$< \mathbf{u}'c' >=< \mathbf{u}'\mathbf{l}' > \nabla C$$

Mixing Length

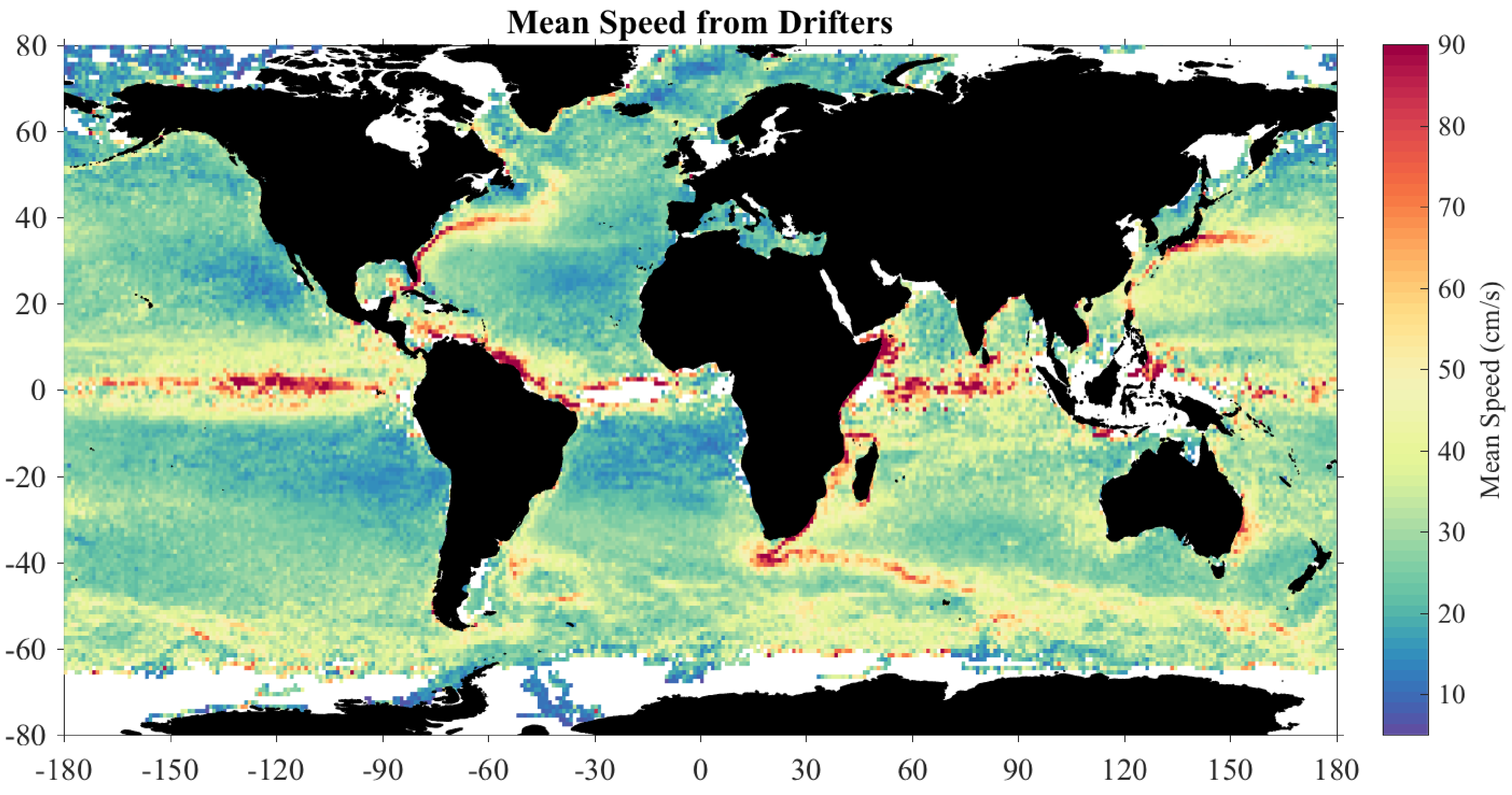
$$\mathbf{K}(t) = -\int_0^t < \mathbf{u}'(t)\mathbf{u}'(\tau) > d\tau$$

Integrate over many eddy turnover times.

Single Particle Statistics

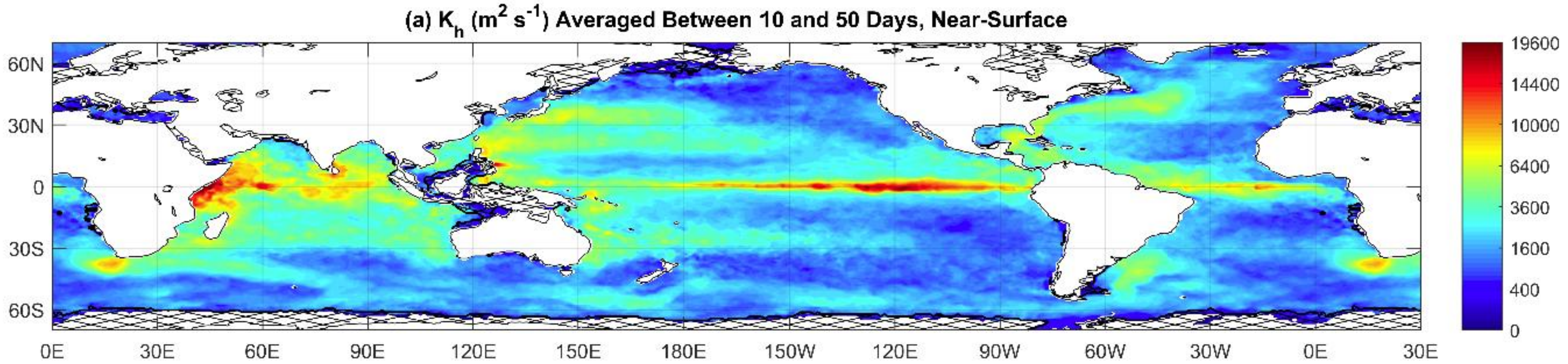
Taylor 1921, Davis 1987, 1991

|U|



Mean Speed from Drifters
80
60
40
20
0
-20
-40
-60
-80
-180
-150
-120
-90
-60
-30
0
30
60
90
120
150
180
90
80
70
60
50
40
30
20
10
Mean Speed (cm/s)

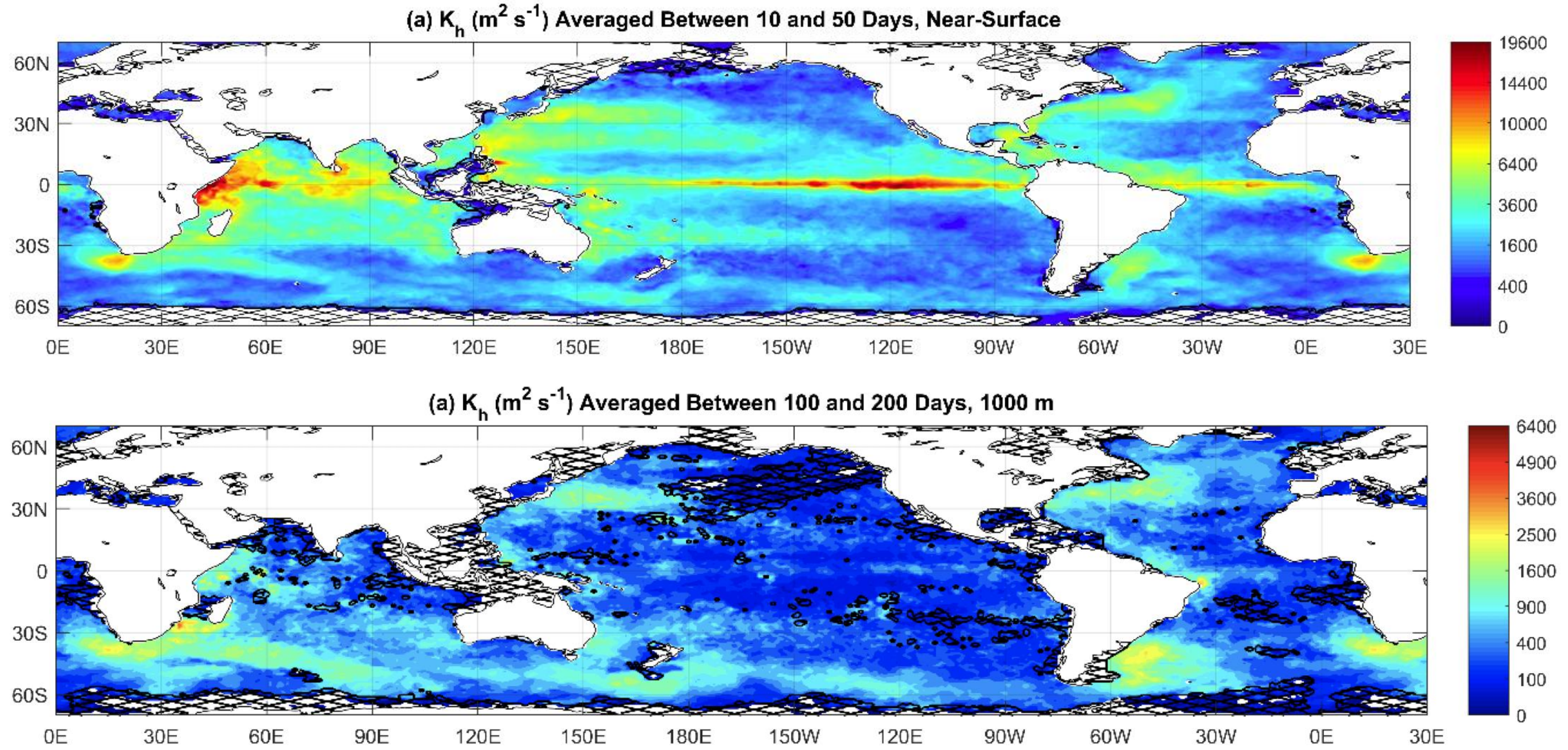
K



Roach, Balwada and Speer 2018 (submitted)

Smaller eigenvalue of the symmetric part of the **K** tensor.

K



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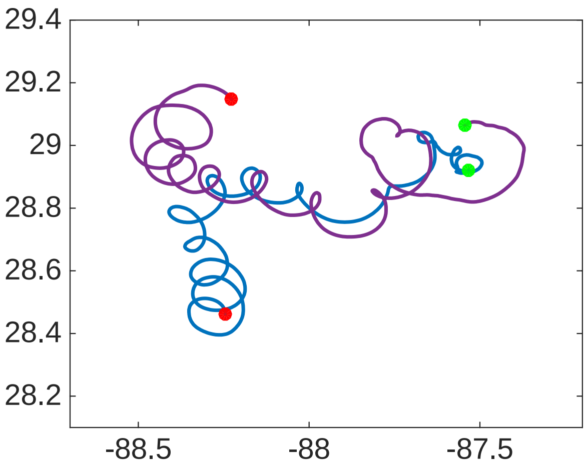
Time series analysis

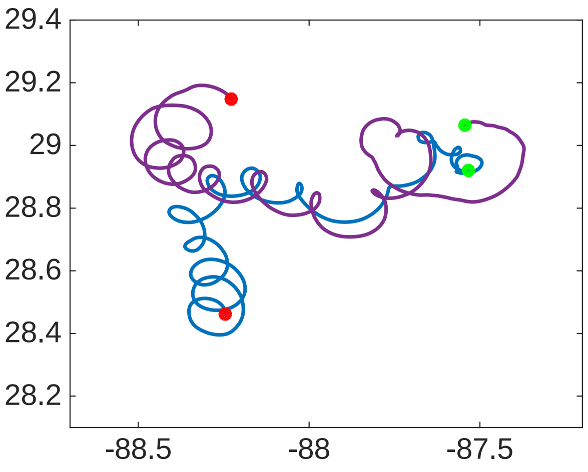


$$z(t) = u(t) + iv(t)$$

Rotary spectra is the power spectrum of z(t).

Positive and negative frequencies indicate the cyclonic and anticyclonic features.

Time series analysis



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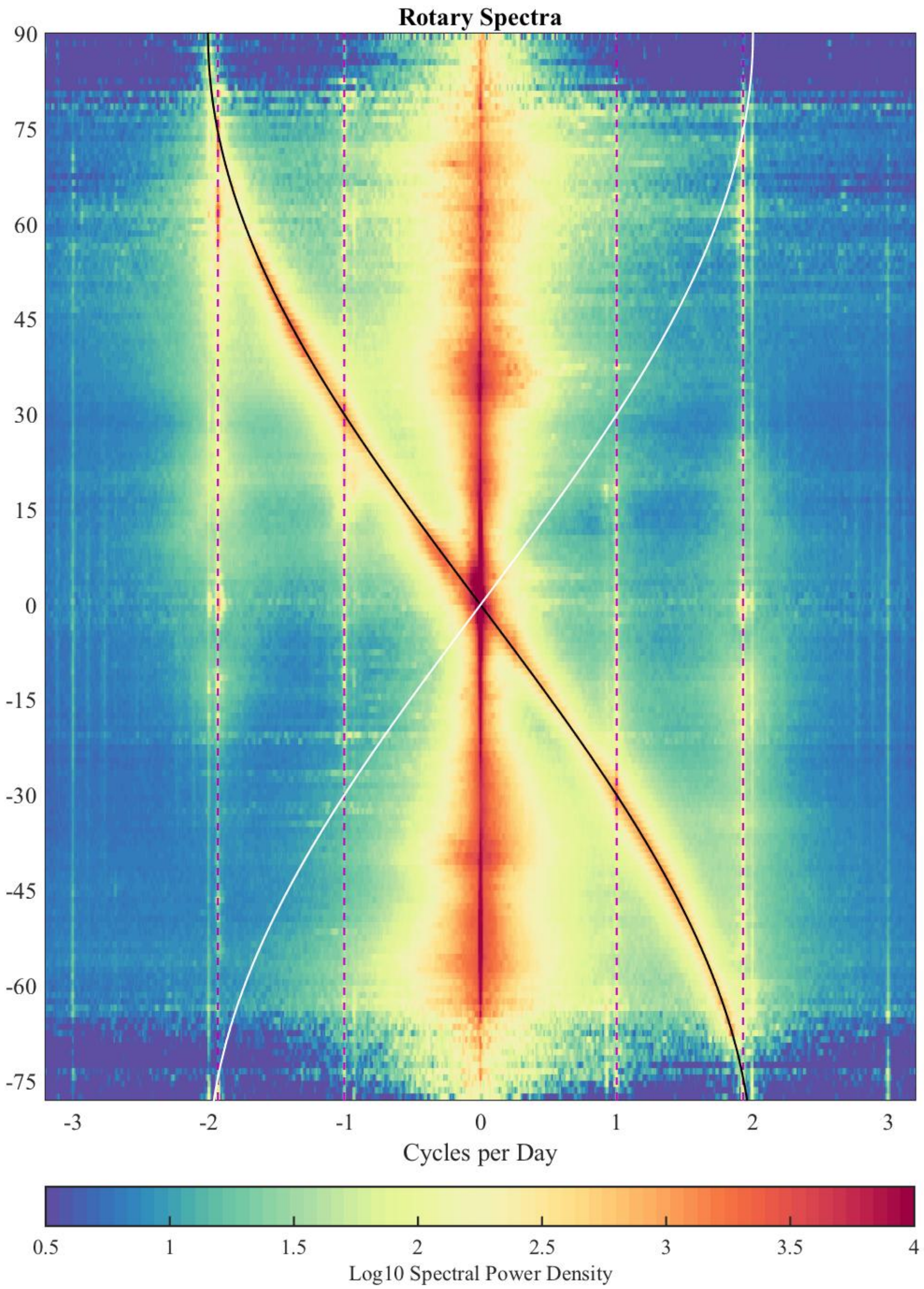
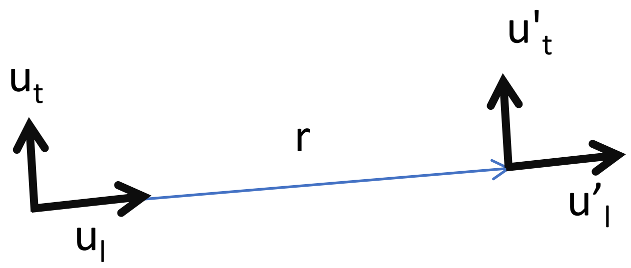


Elipot et al. 2016

Structure Functions

$$Sp = < (u - u')^p >$$



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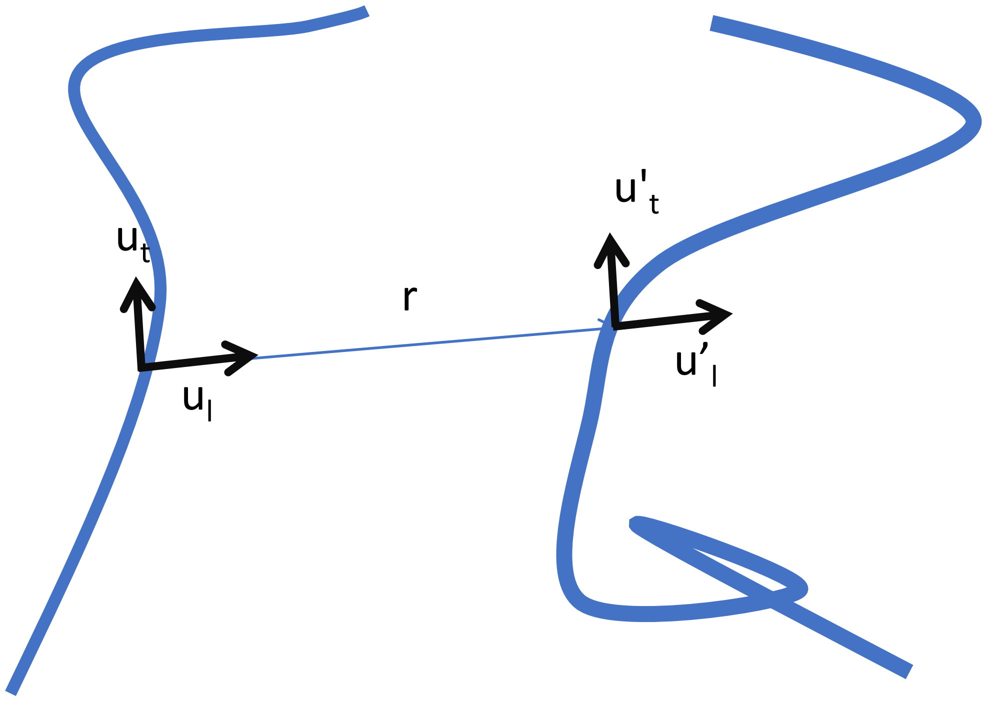


2^{nd} Order Structure Functions

$$S2(r) =< (u(x+r) - u(x))^2 >$$
$$= 2 < u^2 > -2 < u(x+r)u(x) >$$

2^{nd} Order Structure Functions

$$S2(r) =< (u(x+r) - u(x))^2 >$$
$$= 2 < u^2 > -2 < u(x+r)u(x) >$$

Properties:

1. At large r, S2 is the kinetic energy in the system.
2. Relationship to Kinetic Energy Spectrum

$$S2 = 2\int_o^\infty E(k)(1 - J_o(kr))dk$$ Bennett 1984

$$E(k) \sim k^{-n} -> S2(r) \sim r^{n-1}$$ $1 < n < 3$, for long inertial ranges

$$\sim r^2$$ $n > 3$, a locally linear function

3. Related to the instantaneous relative dispersion coefficient. (Babiano et al 1990)

$$\chi(r) = \{< \frac{1}{2}\frac{d}{dt}\mathbf{r}.\mathbf{r} > |_r^2\}^{1/2} = [S2_{ll}]^{1/2} r$$

2^{nd} Order Structure Functions

$$S2(r) =< (u(x+r) - u(x))^2 >$$

Properties …

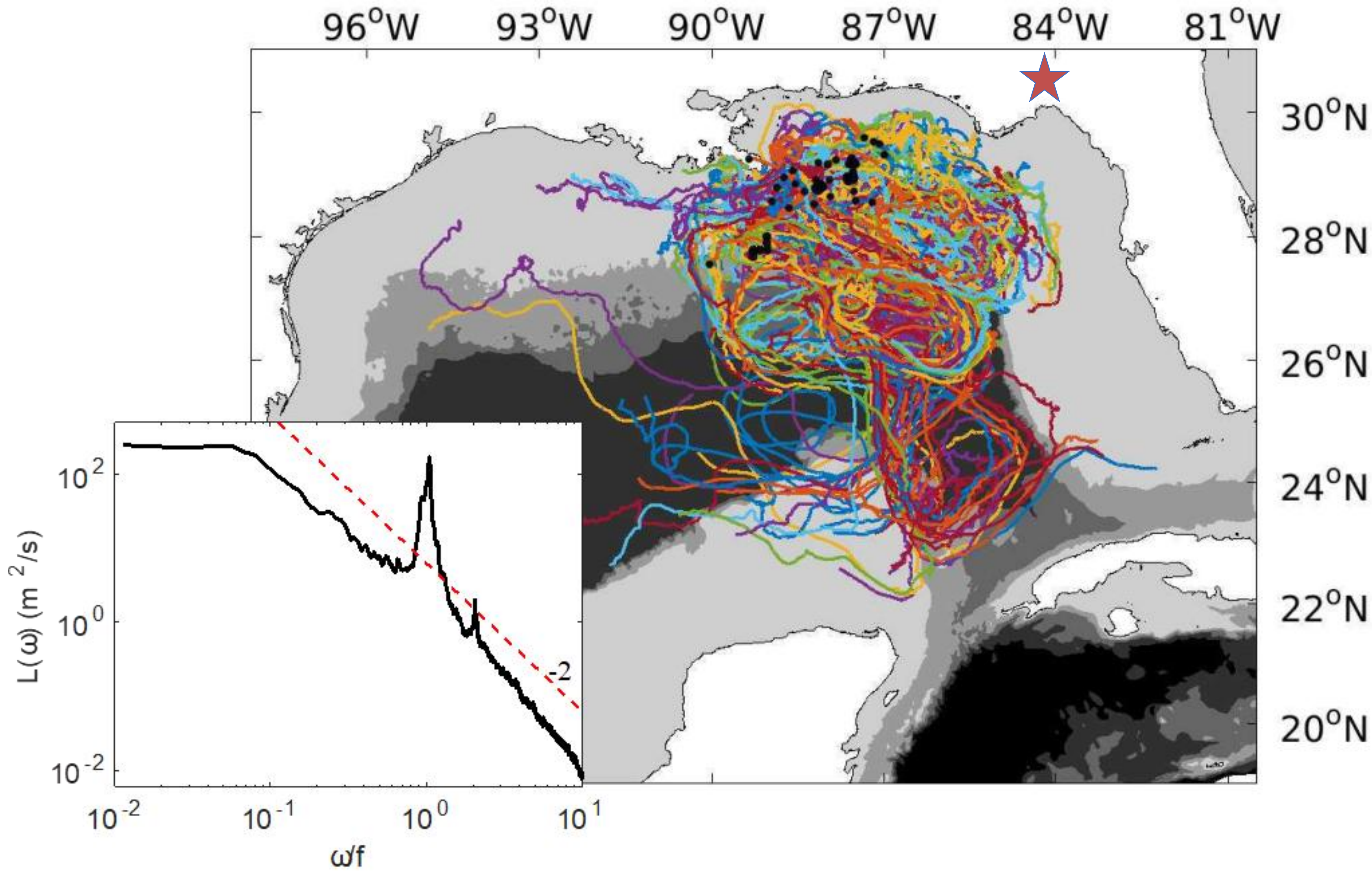
4. Can be decomposed into rotational and divergent components.

$$(S2_l, S2_t) -> (S2_R, S2_D)$$ Buhler et al 2014, Lindborg 2015

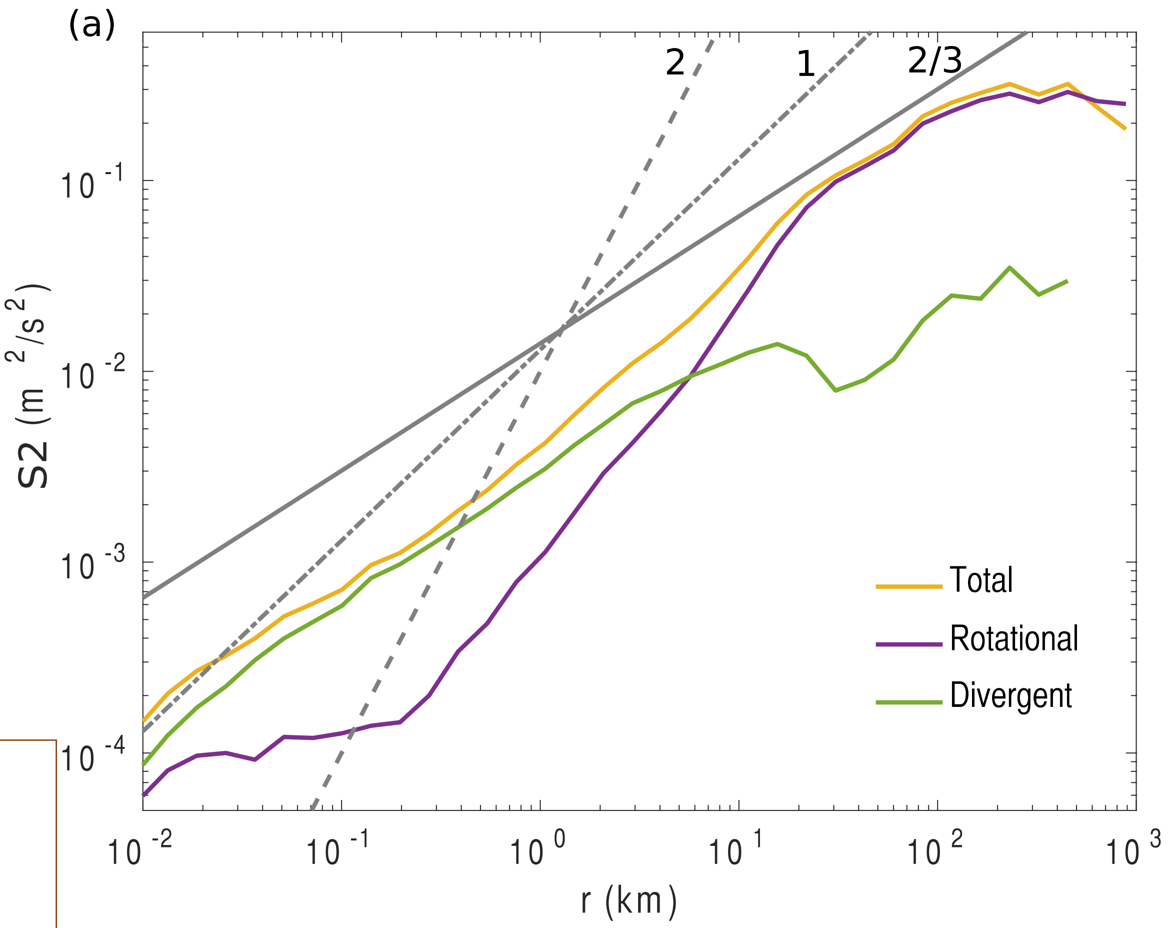
$$S_{rr} = S_{tt} + \int_0^r \frac{1}{r}(S_{tt} - S_{ll})\, dr,$$

$$S_{dd} = S_{tt} - \int_0^r \frac{1}{r}(S_{tt} - S_{ll})\, dr.$$

Grand Lagrangian Deployment (GLAD)



2nd Order Velocity Structure Functions



Enstrophy Cascade

$$r^2 \sim k^{-3}$$

Energy Cascade, wave cascade, SQG

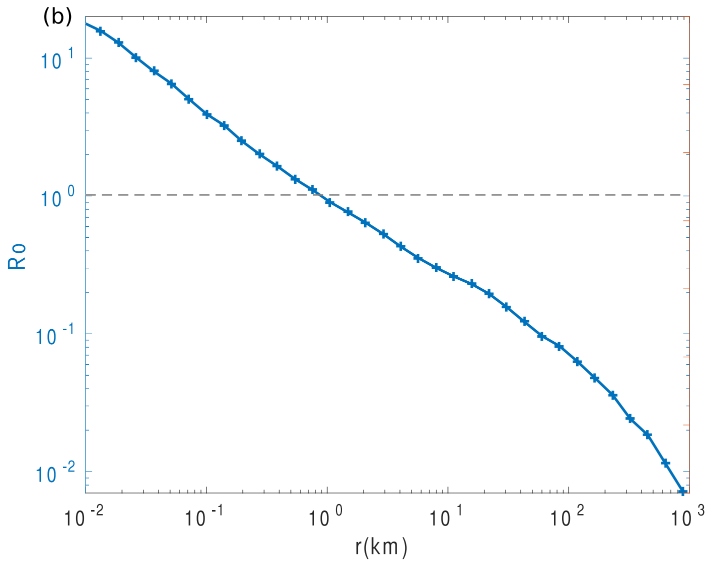
$$r^{2/3} \sim k^{-5/3}$$

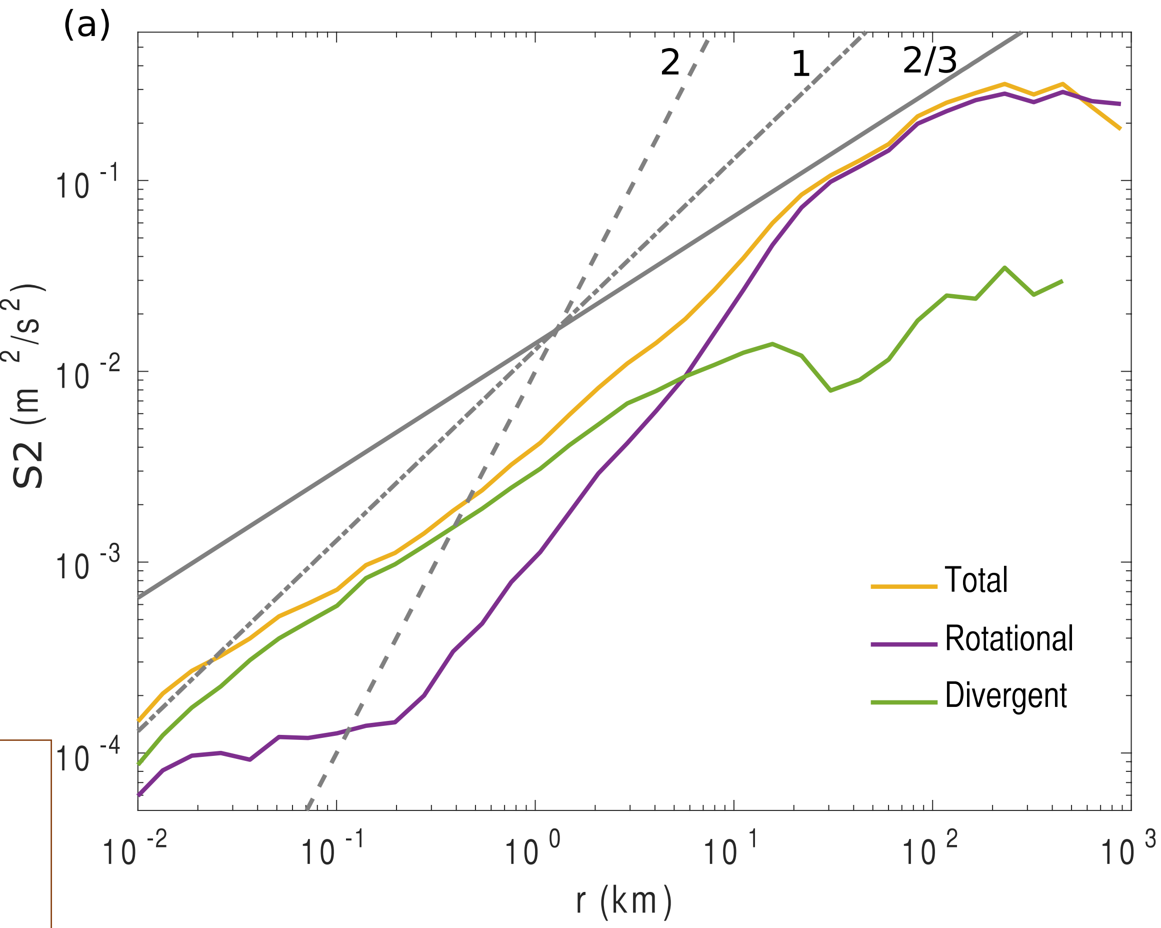
Mixed Cascade, no cascade

$$r^1 \sim k^{-2}$$

Balwada et al 2016

2nd Order Velocity Structure Functions





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$$r^2 \sim k^{-3}$$

Energy Cascade, wave cascade, SQG

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Mixed Cascade, no cascade

$$r^1 \sim k^{-2}$$

Quantifying differential kinematic properties

Method 1 - least squares

$$u_{rel_i} = A\, x_{rel_i} + B\, y_{rel_i} + \varepsilon u_i$$

$$v_{rel_i} = C\, x_{rel_i} + D\, y_{rel_i} + \varepsilon v_i$$

divergence = $A + D$

vorticity = $C - B$

Method 2 - area change

divergence $= A^{-1} \frac{dA}{dt}$

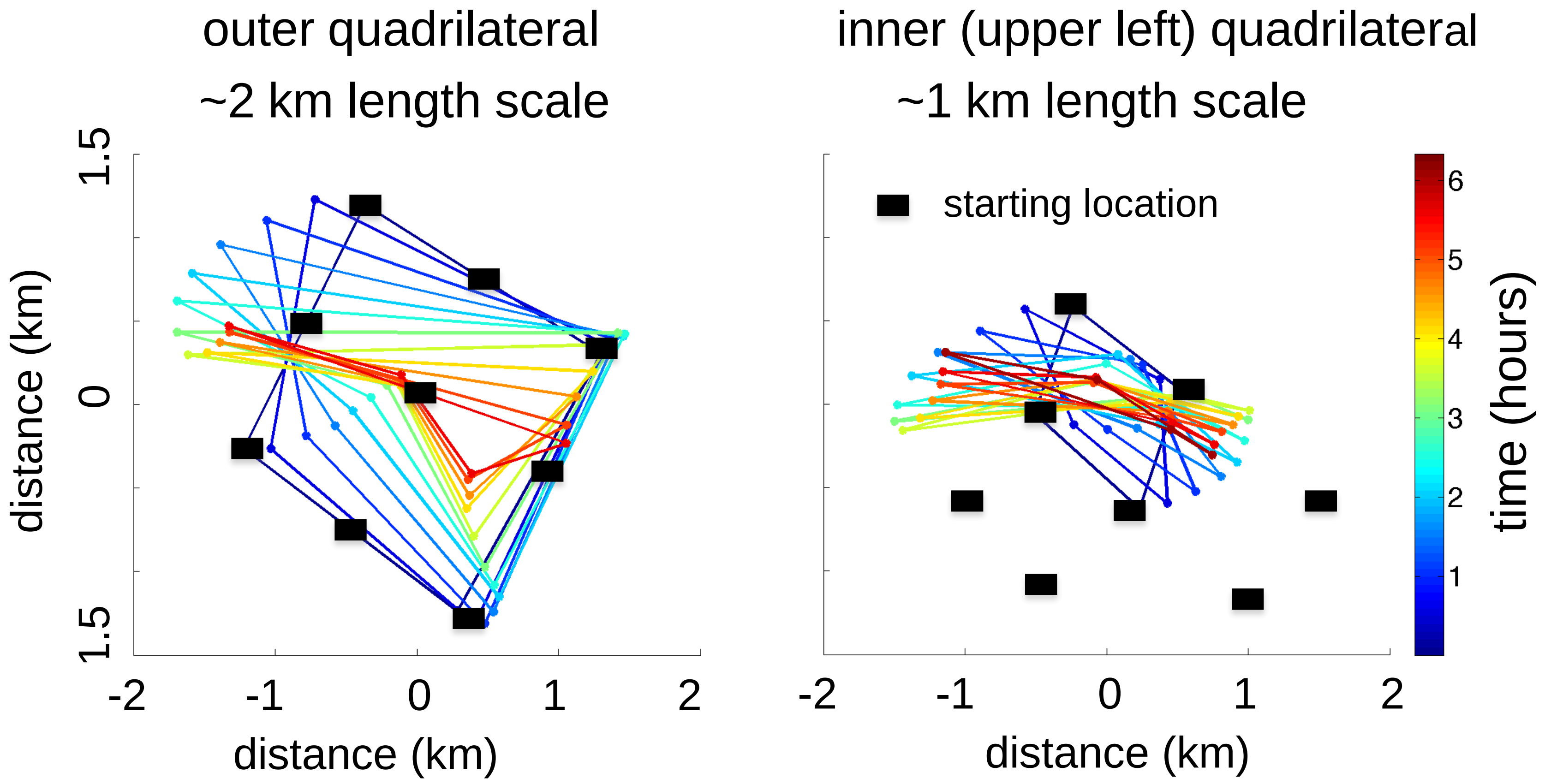
vorticity with 90° rotation transform

$u \rightarrow -v'$

$v \rightarrow u'$

Molinari and Kirwan 1975, Okubo and Ebbesmeyer 1976

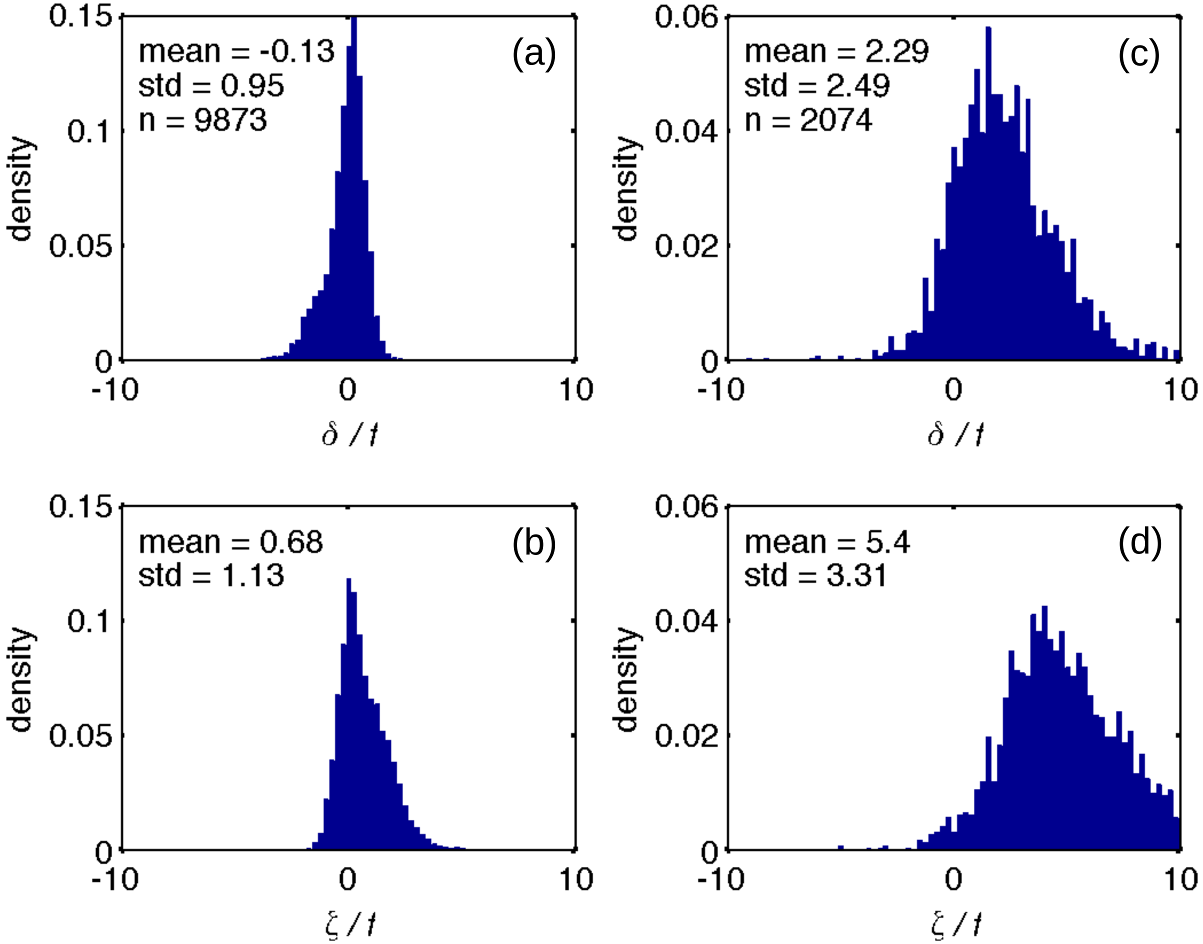
Relative motion for different length scales



- relative motion varies with length scale
- drifters tend to "line up" after a few hours

Divergence and Vorticity from 4-drifter clusters

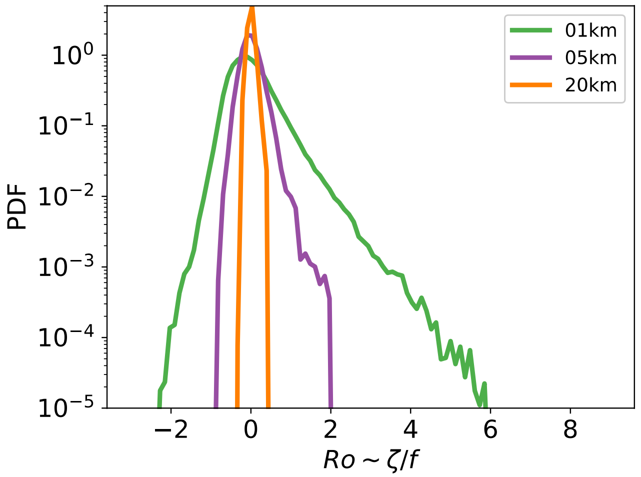
fronts **eddies**



divergence

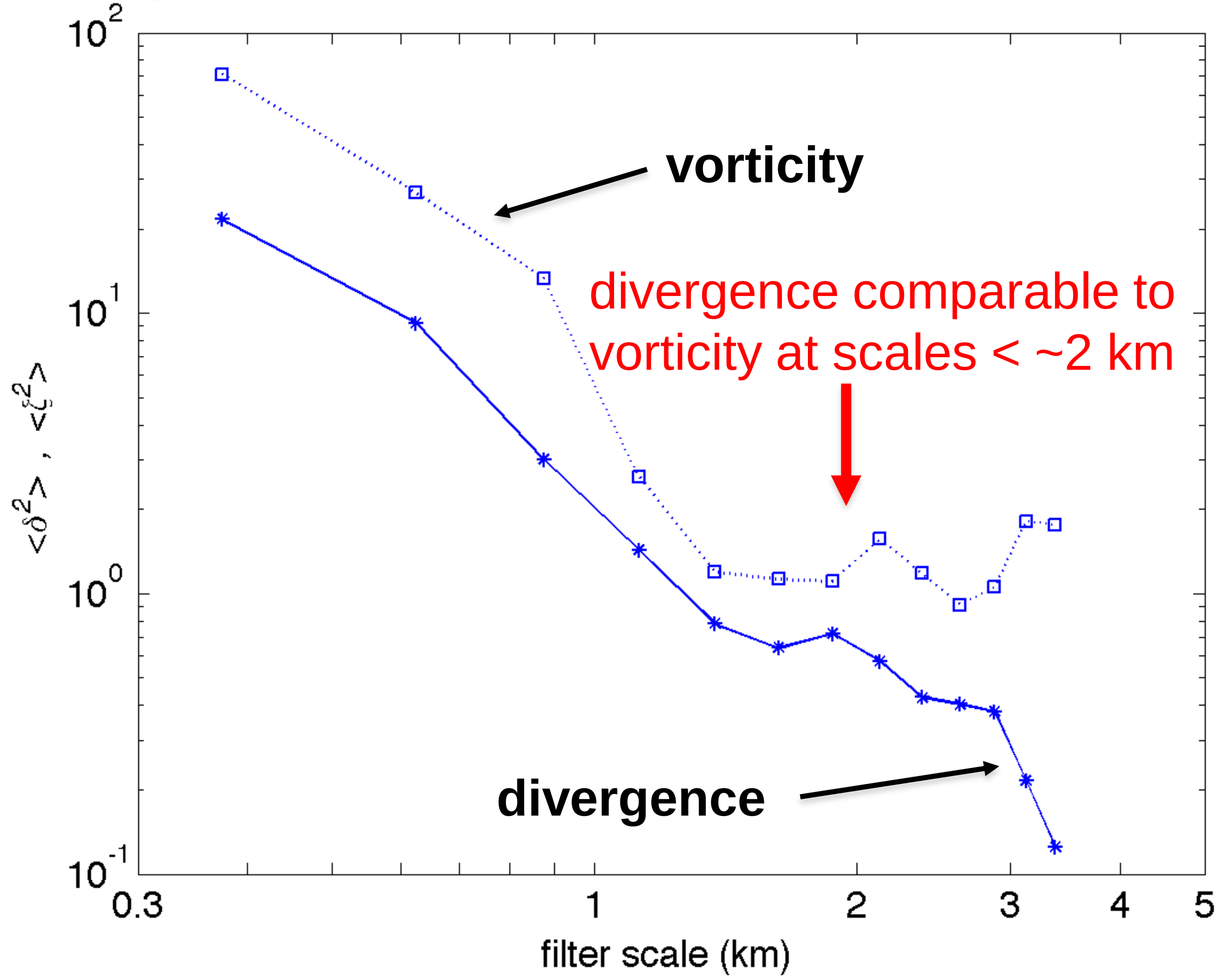
vorticity

MITgcm Model



Div and vor on scales from 0.25 – 4 km can be many times f.

Scale dependence of div and vor from 4-drifter clusters



Ro (r) ~ zeta (r) / fr

Ro(k) ~ f k^3 E(k)

Should be flat for an enstrophy cascade.

RMS divergence (solid line) and vorticity (dashed line) as a function of length scale (Ohlmann et al. 2017).

3rd Order Structure Functions

$$S3 =< \delta u_l^3(r) > + < \delta u_l(r)\delta u_t^2(r) >$$

This is the quantity that shows up in the Karmin-Howarth-Monin (KHM) equation. It gives a measure of flow of energy or enstrophy through scales, or spectral energy flux.

In inertial ranges

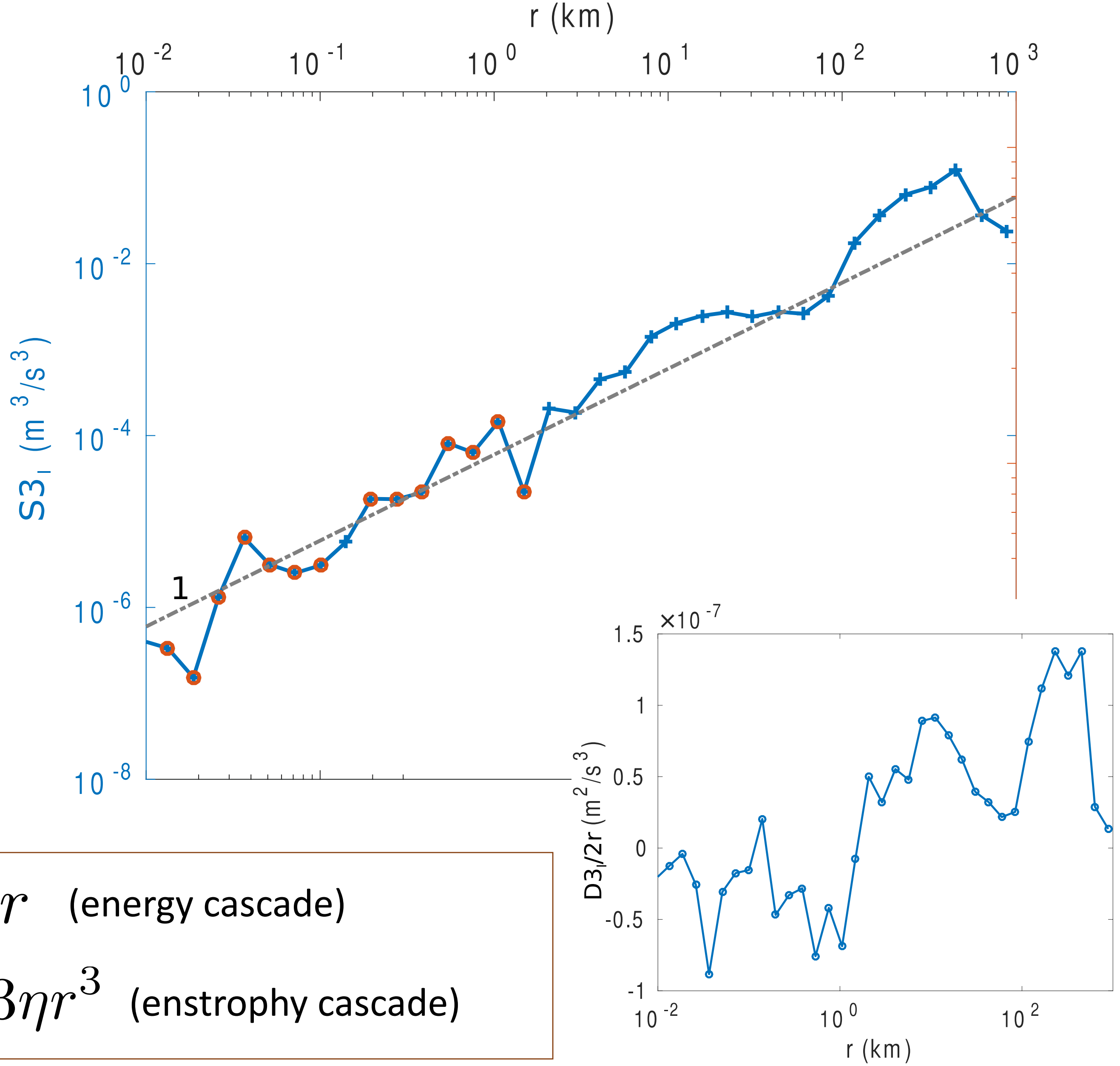
$$S3 = -\frac{4}{3}\epsilon r$$ 3D forward energy cascade.

$$S3 = 2\epsilon r$$ 2D inverse energy cascade.

$$S3 = 1/3\eta r^3$$ 2D forward enstrophy cascade.

$$S3 = -2\epsilon r$$ Forward energy cascade in quasi 2D. (Lindborg and Cho 2001)

3rd Order Velocity Structure Functions



$$S3 = 2\epsilon r$$ (energy cascade)

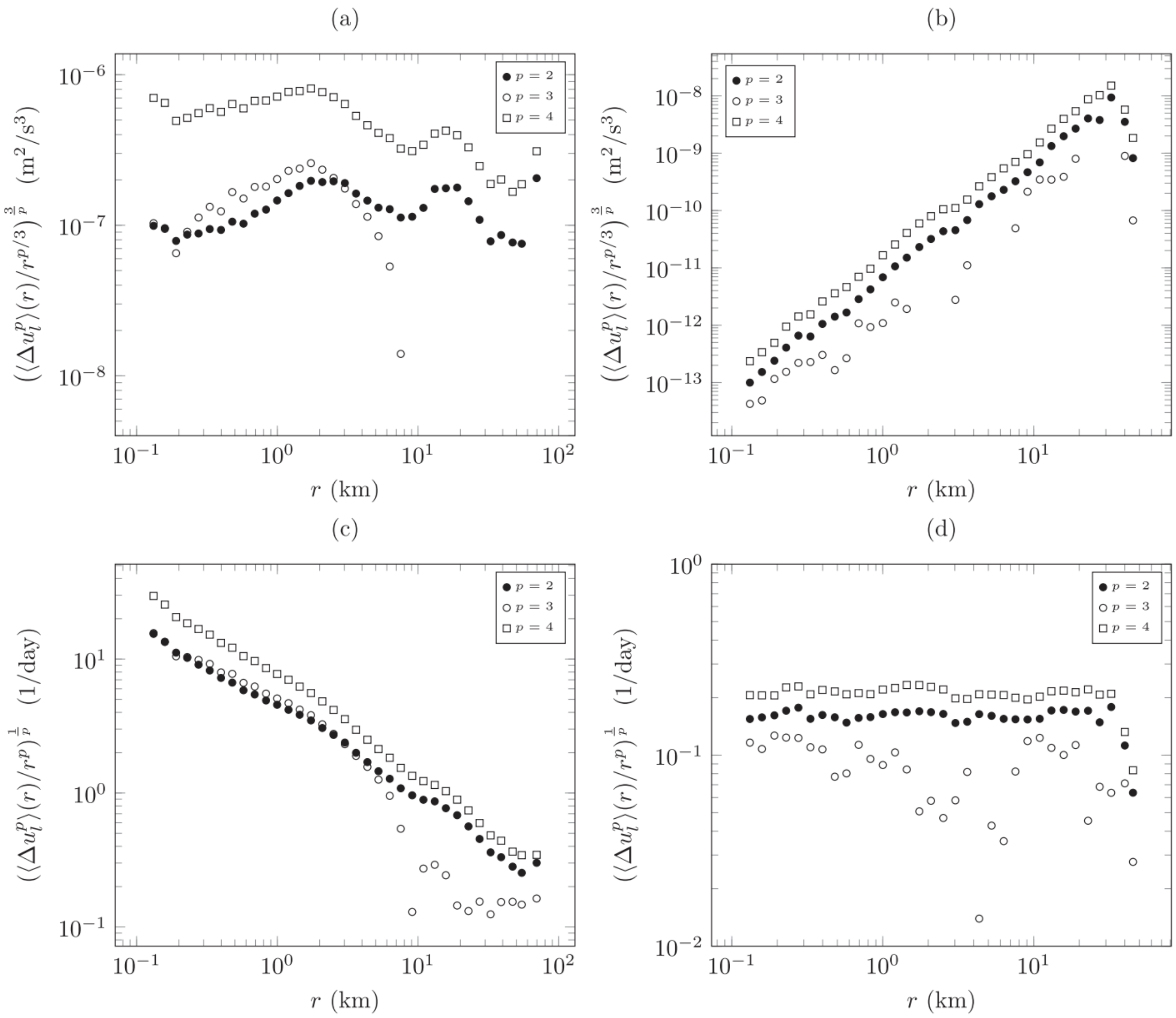
$$S3 = 1/3\eta r^3$$ (enstrophy cascade)

Balwada et al 2016,
Poje et al 2017

$$S_{\alpha}^{n}(r) = C_n(\varepsilon r)^{n/3}.$$

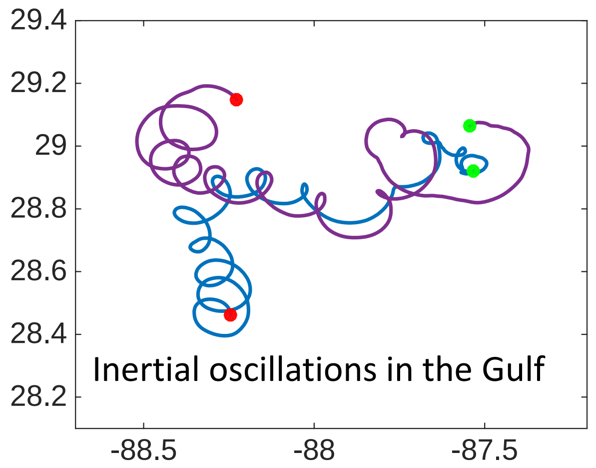
$$S_{\alpha}^{n}(r) = \tilde{C}_n \nu^{n/3} r^n.$$

GLAD AVISO



Poje et al 2017

Relative Dispersion



Relative Diffusivity

$$K_2(r,t) = \frac{d\overline{r^2}}{dt}$$

$$K_2(r) \sim r\sqrt{S2(r)}$$

$K_2 \sim r^2; \overline{r^2} \sim e^{\lambda t}$ (Enstrophy cascade, non-local)

$K_2 \sim r^{4/3}; \overline{r^2} \sim t^3$ (Energy cascade, local)

$K_2 = constant; \overline{r^2} \sim t$

Finite Scale Lyapunov Exponent (FSLE)

$$r = r_0 e^{\lambda(r)t}$$

$\lambda \sim r^0$ (Exponential, non-local)

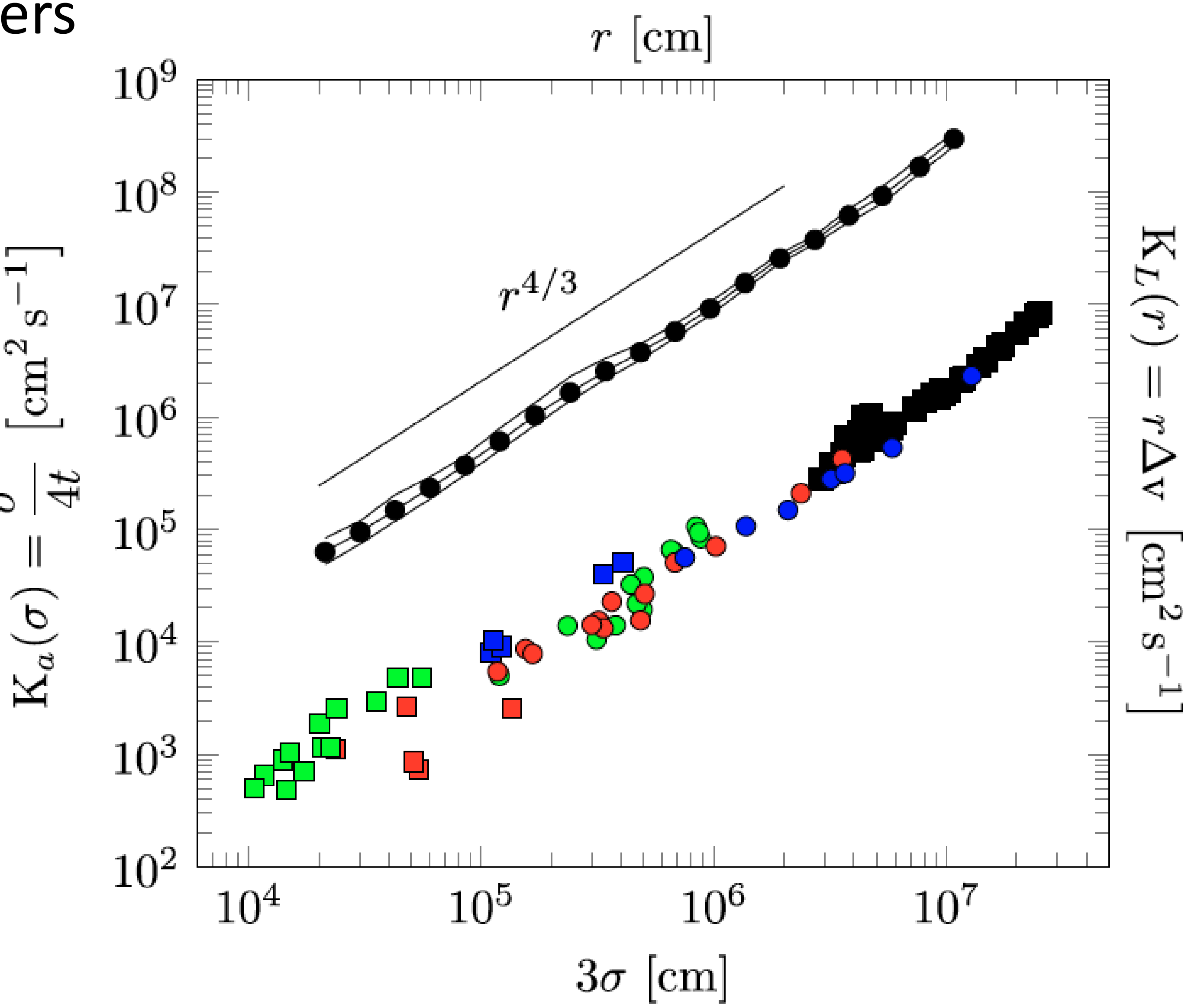
$\lambda \sim r^{-2/3}$ (Cubic, local)

$\lambda \sim r^{-2}$ (Linear, diffusion)

Relative diffusivity at the surface from GLAD drifters

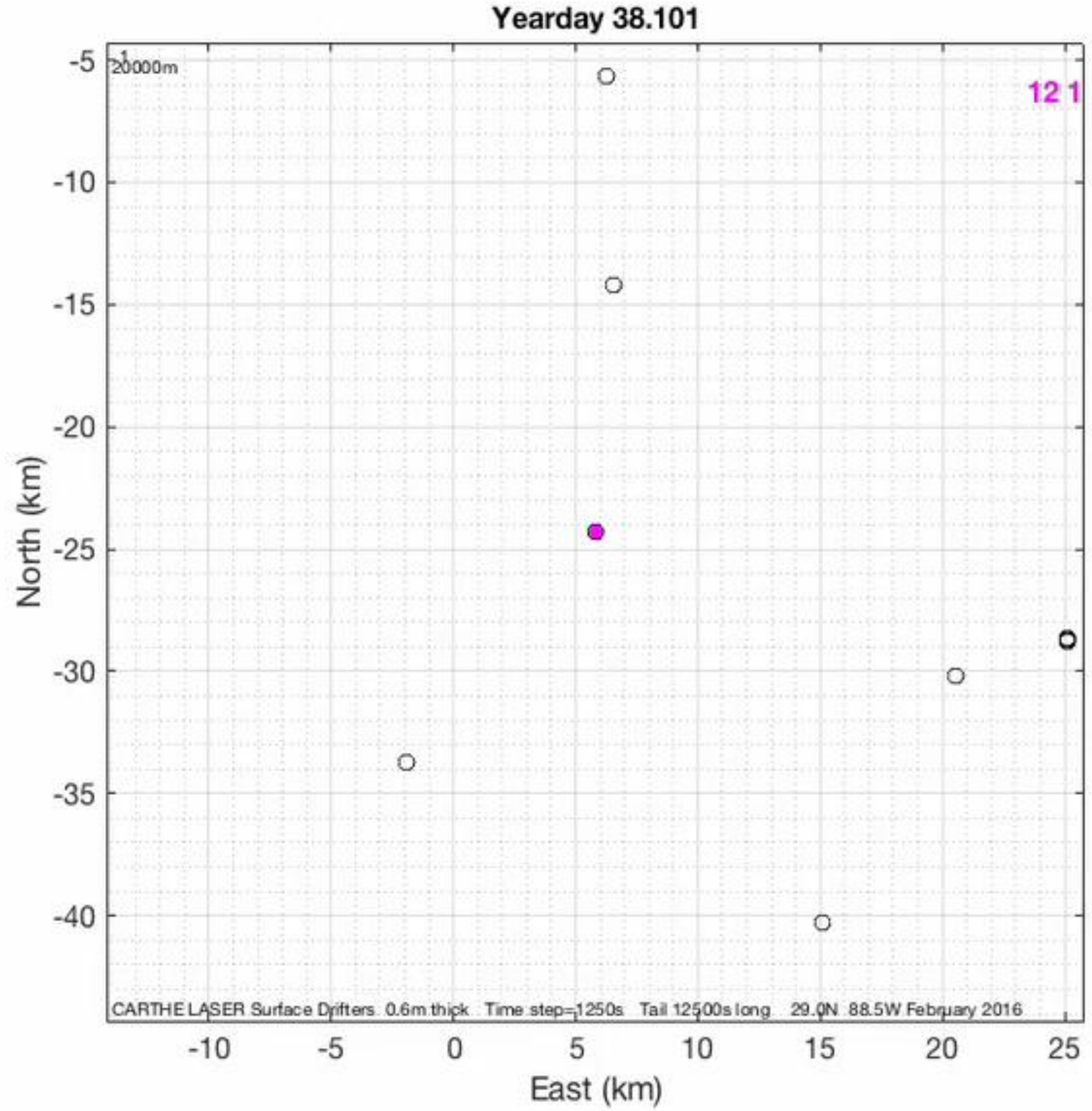
$$K_2(r) \sim r\sqrt{S2(r)}$$

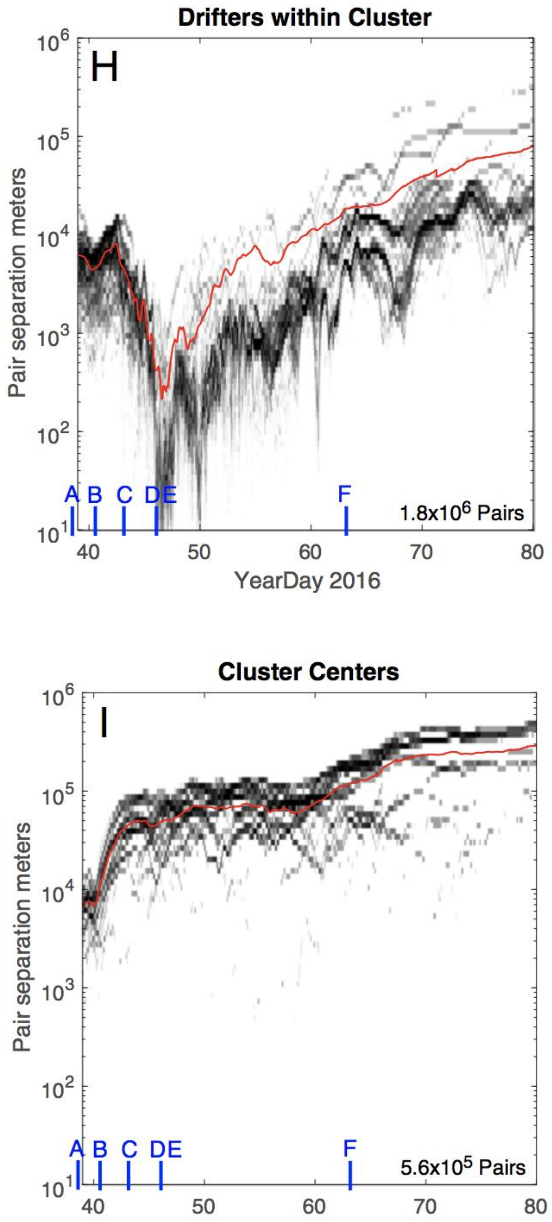
Richardson like dispersion acting from 100m to 1000km.



Poje et al 2014

LASER Experiment – Relative dispersion in a sea of convergence.

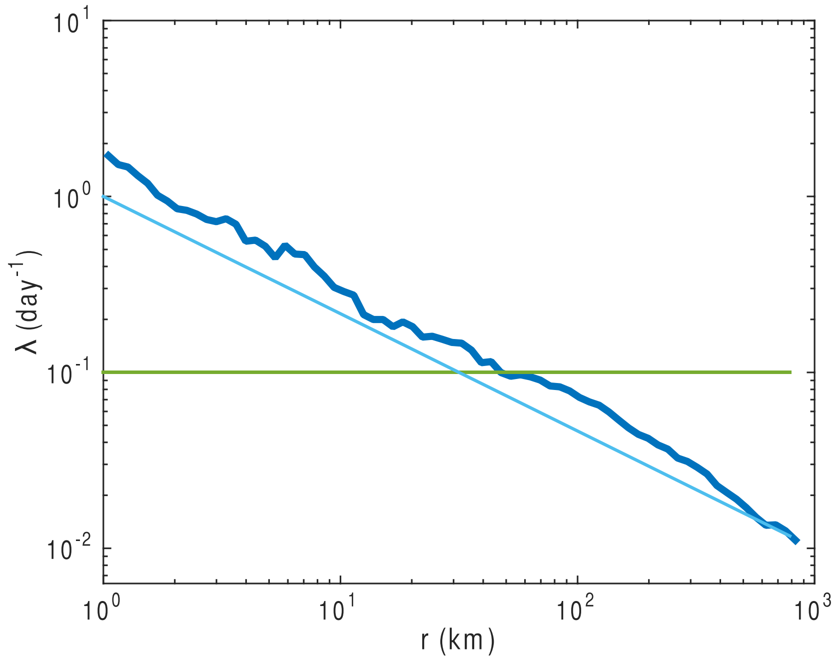




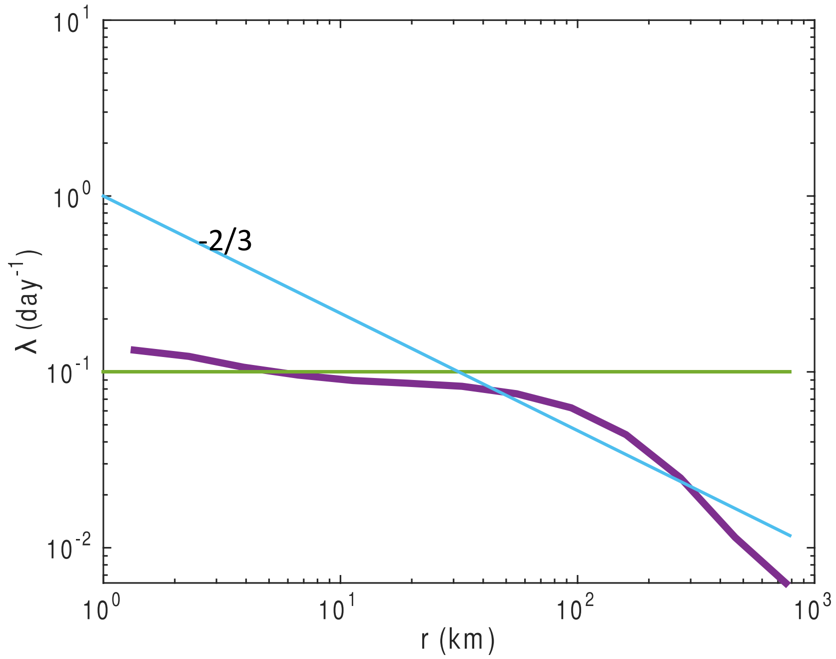
D'Asaro et al. 2018

FSLE at 1500m from DIMES floats.

Observations :
Local stirring



Model :
Non-local stirring



Richardson like dispersion acting in the ocean at depths of 1500m. (:O ???)

Balwada et al. (in prep)

Summary

- Lagrangian observations are natural.
- They provide an extremely powerful way of measuring the ocean – guidance by theory and analysis using powerful methods is needed.
- Single trajectories
  - Describe the ocean as a mean flow + diffusion.
  - Time series analysis - wealth of information about temporal evolution.
- Multiple trajectories
  - Spatial correlation functions (structure functions).
    - Spatial energy distribution.
    - Energy flux.
  - Estimate kinematic properties (vorticity, divergence, shear, strain).
  - Quantify relative dispersion.

Unanswered questions/ Open problems.

- Data homogeneity – Lagrangian observations measure what the flow demands, rather than what scientists demand.
  - How biased are the observations? (Palmer et al - in prep)
  - Can we account for them?
- Why does a Richardson-Obukhov scaling working at such wide range of length scales at the ocean? Obviously it is not 3D turbulence.
- We think that the deep ocean has a smooth velocity field, yet observations show otherwise.

?

The End.