Convection & Entrainment in Stars

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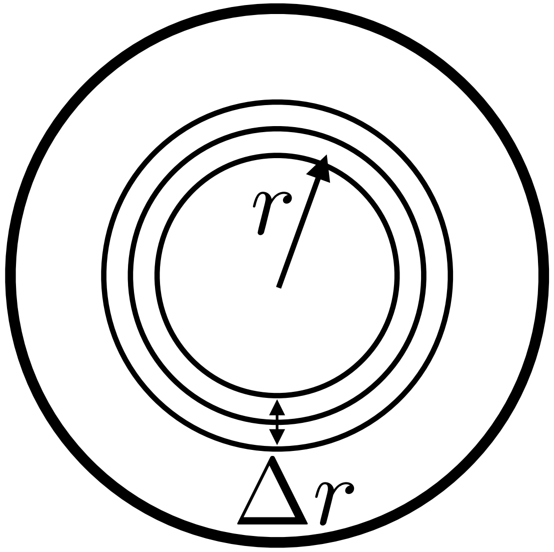
What is a star?

self-gravitating, supported by nuclear energy

How does energy get out?

1. Radiation



$$F_{BB} = \sigma T(r)^4$$

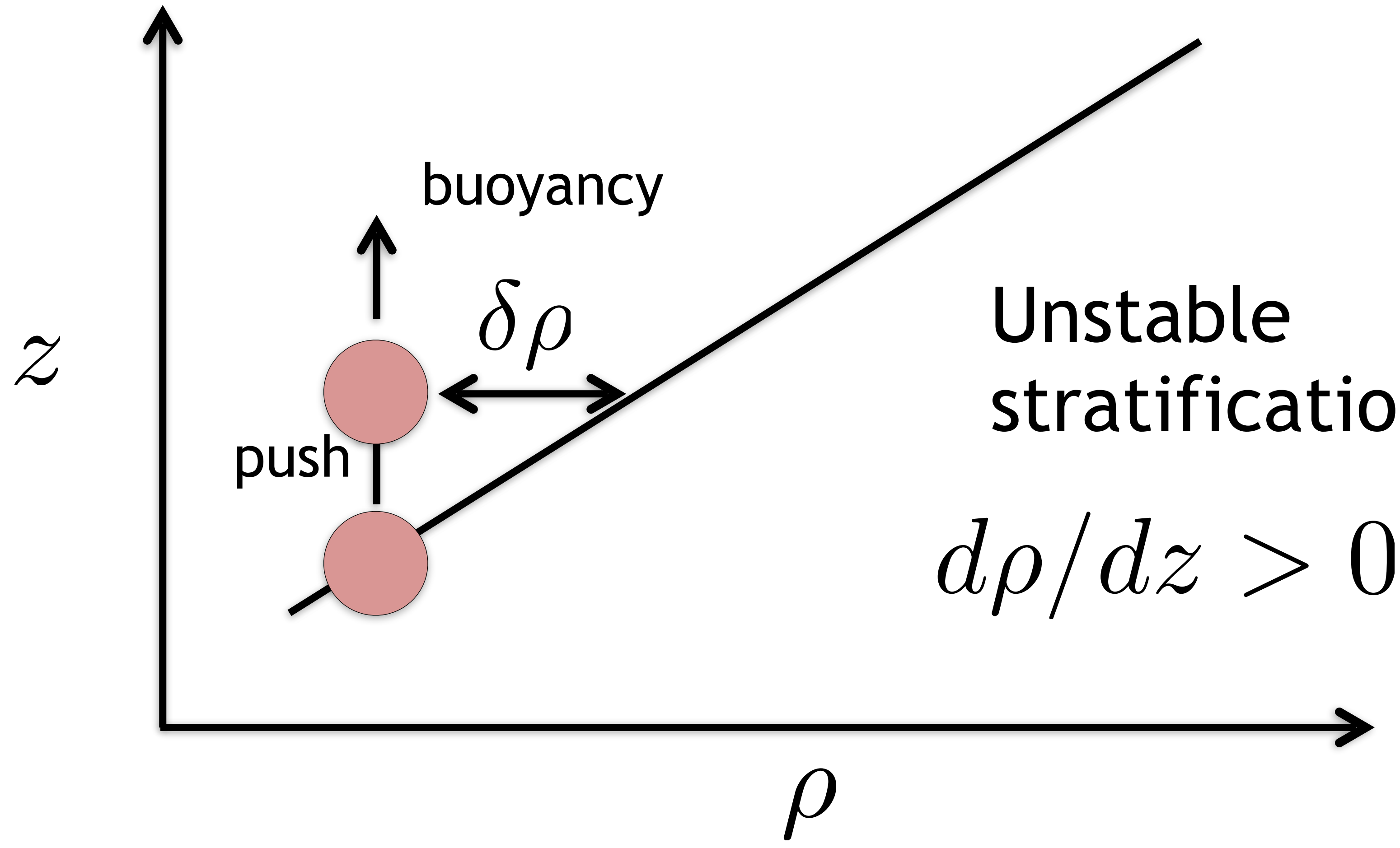
$$F_{BB} = \sigma T(r + \Delta r)^4$$

$$L = \frac{F}{4\pi r^2} \sim \frac{4T^3}{4\pi r^2 \kappa \rho} \frac{dT}{dr}$$

How does energy get out?

1. Radiation
2. Convection

Convection (Boussinesq)



Unstable stratification

$$d\rho/dz > 0$$

Convection (Compressible)

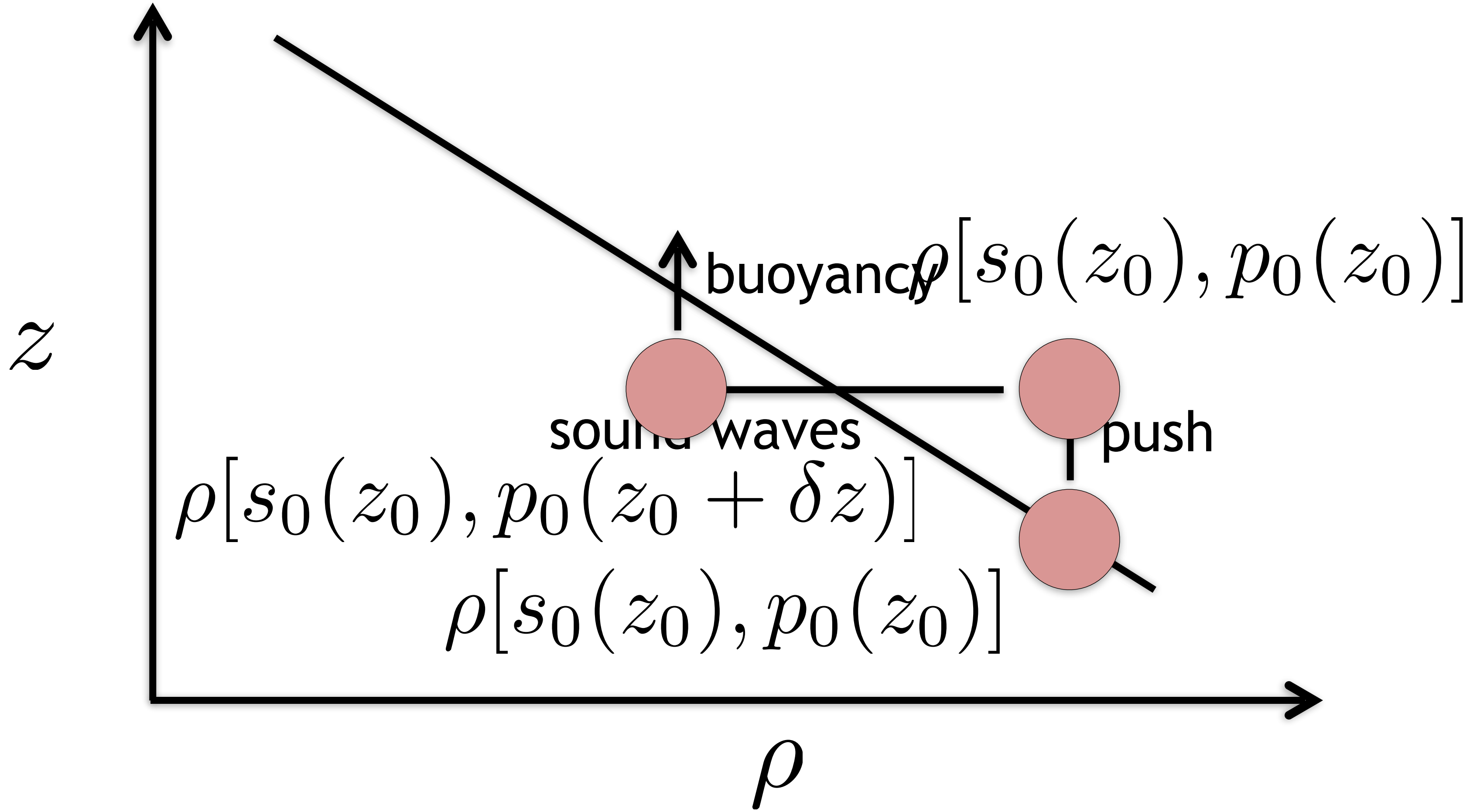
$$t_{\rm heat} \gg t_{\rm buoy} \gg t_{\rm sound}$$

adiabatic
(constant entropy)

pressure
equilibrium with
background

Convection (Compressible)

$$p_0(z_0 + \delta z) < p_0(z_0)$$



Convection (Compressible)

density of background

density of fluid element

$$\rho[s_0(z_0+\delta z), p_0(z_0+\delta z)] \quad > \quad \rho[s_0(z_0), p_0(z_0+\delta z)]$$

unstable

$$\left(\frac{\partial \rho}{\partial s}\right)_p = -\frac{\rho T}{c_p}\alpha_T < 0$$

$$\left(\frac{\partial \rho}{\partial z}\right)_p = \left(\frac{ds_0}{dz}\right)\left(\frac{\partial \rho}{\partial s}\right)_p$$

$$\Longrightarrow \boxed{\frac{\mathrm{d}s_0}{\mathrm{d}z} < 0}$$

Convection (Compressible)

density of background

density of fluid element

$$\rho[s_0(z_0+\delta z), p_0(z_0+\delta z)] \quad < \quad \rho[s_0(z_0), p_0(z_0+\delta z)]$$

stable

$$\left(\frac{\partial\rho}{\partial s}\right)_p = -\frac{\rho T}{c_p}\alpha_T < 0$$

$$\left(\frac{\partial\rho}{\partial z}\right)_p = \left(\frac{ds_0}{dz}\right)\left(\frac{\partial\rho}{\partial s}\right)_p$$

$$\Longrightarrow \boxed{\frac{\mathrm{d}s_0}{\mathrm{d}z} > 0}$$

	boussinesq	compressible
stable	$\frac{\mathrm{d}\rho_0}{\mathrm{d}z} < 0$	$\frac{\mathrm{d}s_0}{\mathrm{d}z} > 0$
unstable	$\frac{\mathrm{d}\rho_0}{\mathrm{d}z} > 0$	$\frac{\mathrm{d}s_0}{\mathrm{d}z} < 0$

How does energy get out?

2 Transport Mechanisms

1. Radiation

$$\frac{dT}{dr} = -\frac{3\kappa\rho L}{16\pi a c r^2 T^3}$$

2. Convection

$$\frac{ds}{dr} = 0 \qquad \frac{dT}{dr} = -\frac{g}{c_p}$$

convective if:

$$\frac{3\kappa\rho L}{16\pi acr^2T^3} > \left|\frac{dT}{dr}\right|_{\mathrm{ad}}$$

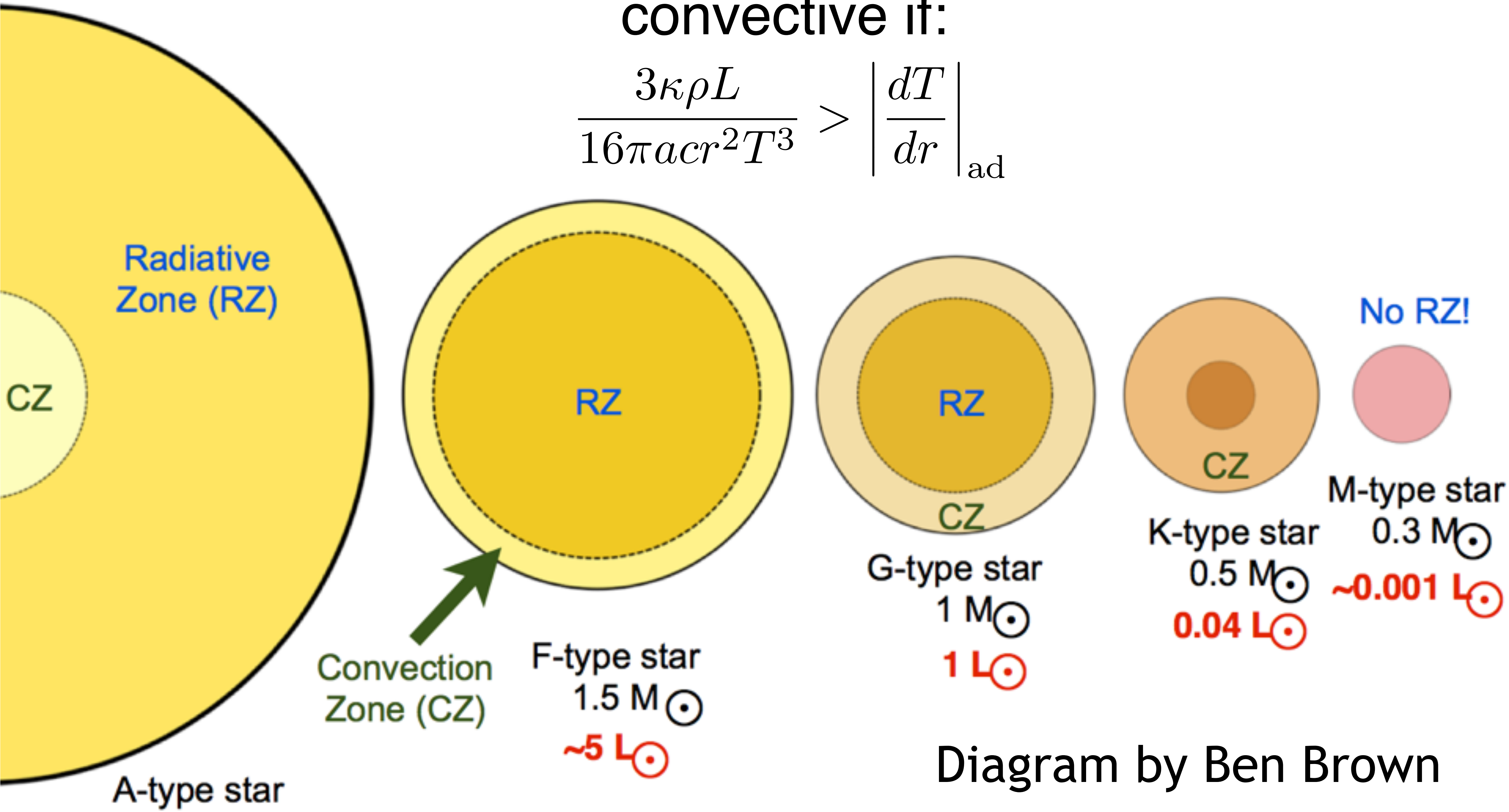


Diagram by Ben Brown

Mixing in Convection Zone

$$\partial_t c + \boldsymbol{u} \cdot \nabla c = D \nabla^2 c$$

$$D \text{ small}$$

$$\mathrm{Pe} = \frac{UL}{D} \sim \frac{\boldsymbol{u} \cdot \nabla c}{D \nabla^2 c}$$

$$\mathrm{Pe} \sim 10^{10}$$

Mixing in Convection Zone

$$\boldsymbol{u} \cdot \nabla c = D_T \nabla^2 c \qquad D_T = \frac{1}{3}\alpha_{MLT} u_c H_p$$

MLT

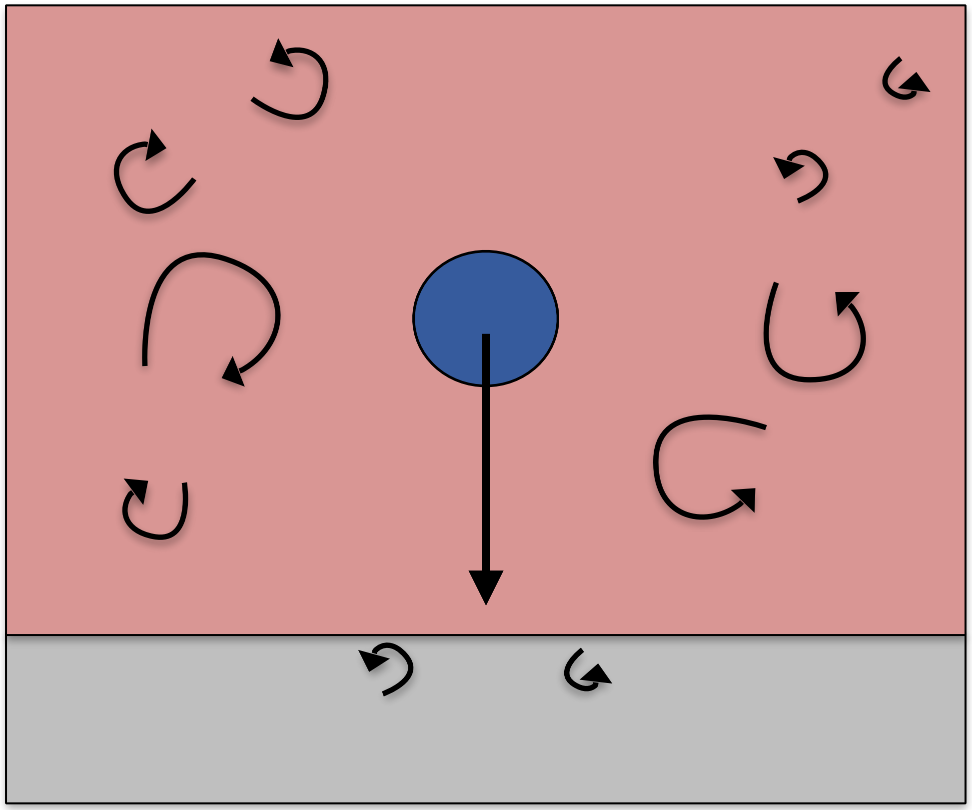
$$u_c \sim \left(\frac{L_{\rm conv}}{\rho}\right)^{1/3}$$

Good approximation?
NO! But doesn't really matter — diffusion fast

Convective Overshoot

What happens past convection zone?

Convection



Stably stratified

Convective Overshoot

Why is it important?

1. Changes amount of fuel if burning in convection zone
2. Changes Li abundance in solar-type stars
3. Can be seen in asteroseismology

Convective Overshoot

How far does thermal penetrate?

$$\boldsymbol{u} \cdot \nabla \boldsymbol{u} \sim \frac{\rho'}{\rho_0} g$$

$$\frac{\rho'}{\rho_0} \sim \ell_{\rm ov} \frac{\partial_z \rho_0}{\rho_0}$$

$$\frac{u_c^2}{\ell_{\rm ov}} \sim \frac{g \ell_{\rm ov}}{H}$$

$$\frac{\partial_z \rho_0}{\rho_0} = \frac{1}{H}$$

$$\frac{\ell_{\rm ov}}{H} \sim \frac{u_c}{c_s} = \mathrm{Ma}$$

$$\frac{\rho'}{\rho_0} \sim \frac{\ell_{\rm ov}}{H}$$

Convective Overshoot

How far does thermal penetrate?

$$\frac{\ell_{\mathrm{ov}}}{H} \sim \frac{u_c}{c_s} = \mathrm{Ma}$$

$$c_s \sim 200\,\mathrm{km/s}$$

$$u_c \sim 100\,\mathrm{m/s}$$

$$\frac{\ell_{\mathrm{ov}}}{H} \sim 10^{-3}$$

Convective Overshoot

How far does thermal penetrate?

$$\frac{\ell_{\rm ov}}{H} \sim 10^{-3}$$

likely underestimate

Seems like a small amount, but even small ell$_{\rm ov}$ can be important:

$$\mathrm{Pe} \sim 10^{10}$$

Convective Overshoot

How far does thermal penetrate?

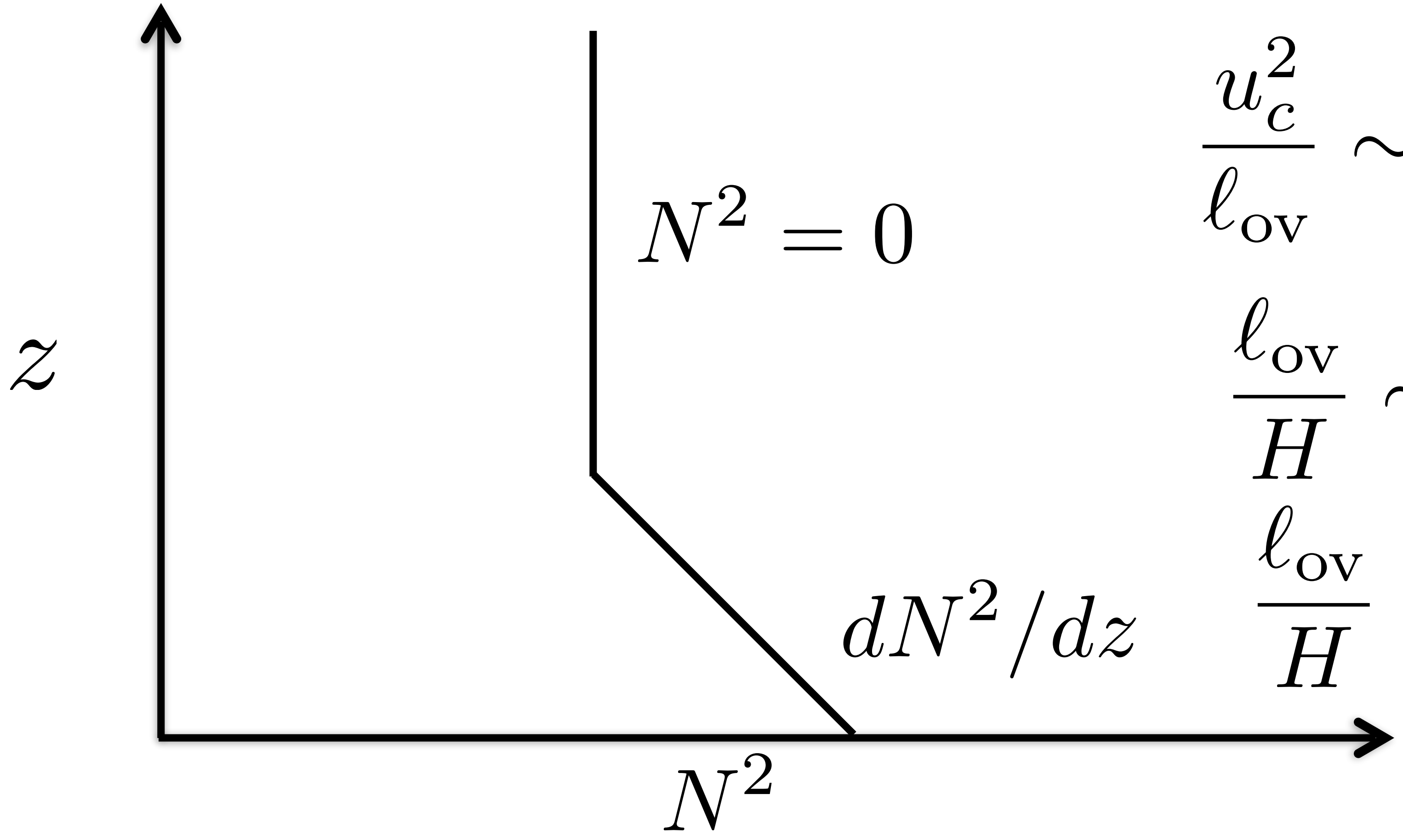
$$\boldsymbol{u} \cdot \nabla \boldsymbol{u} \sim \frac{\rho'}{\rho_0} g \qquad \frac{\rho'}{\rho_0} \sim \ell_{\rm ov} \frac{\partial_z \rho_0}{\rho_0}$$

$$\boldsymbol{u} \cdot \nabla \boldsymbol{u} \sim \frac{s'}{c_p} g \qquad s' \sim \ell_{\rm ov} \partial_z s_0$$

$$\partial_z s_0 = 0$$

Convective Overshoot

How far does thermal penetrate?



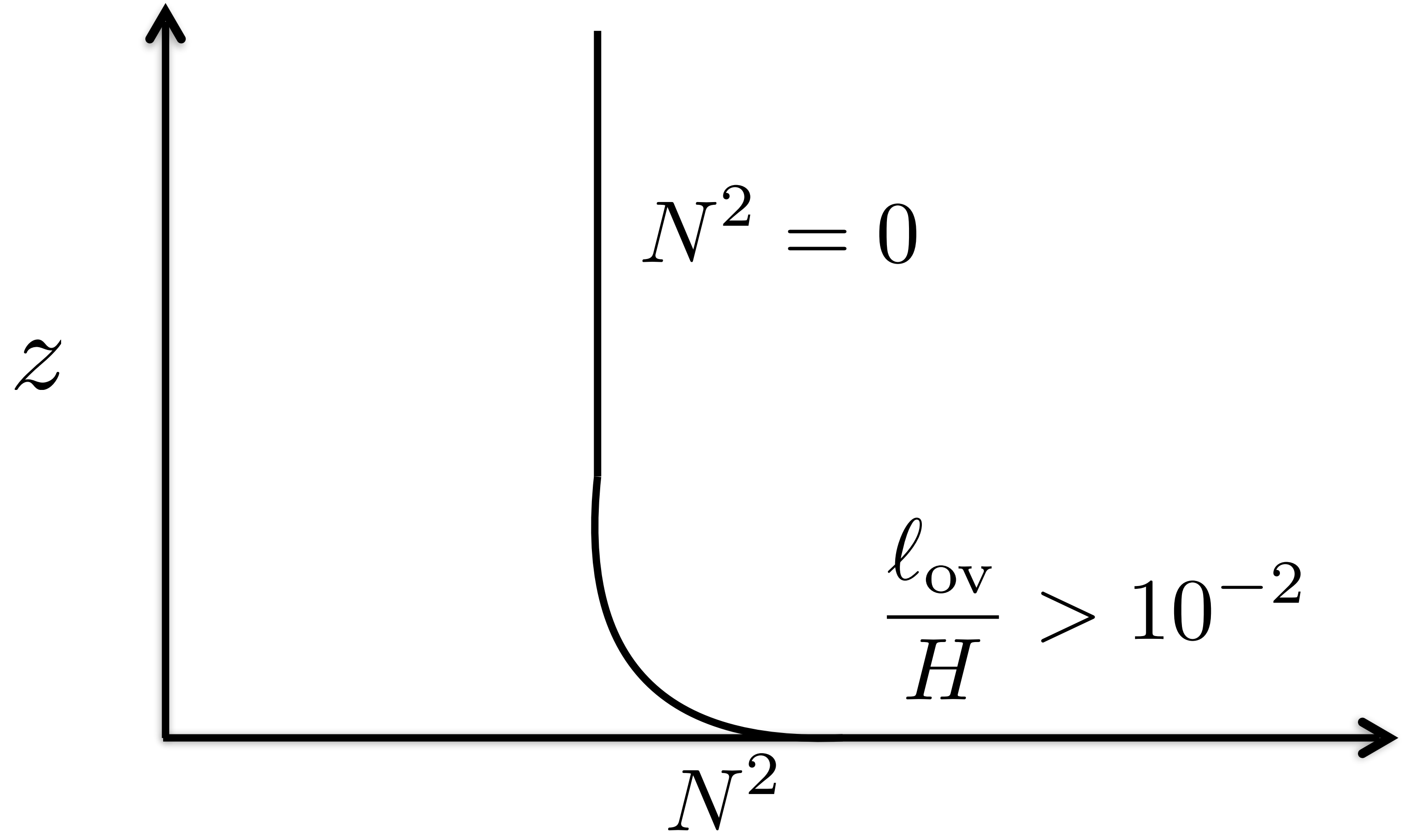
$$\frac{u_c^2}{\ell_{\rm ov}} \sim \ell_{\rm ov}^2 \frac{dN^2}{dz}$$

$$\frac{\ell_{\rm ov}}{H} \sim {\rm Ma}^{2/3}$$

$$\frac{\ell_{\rm ov}}{H} \sim 10^{-2}$$

Convective Overshoot

How far does thermal penetrate?



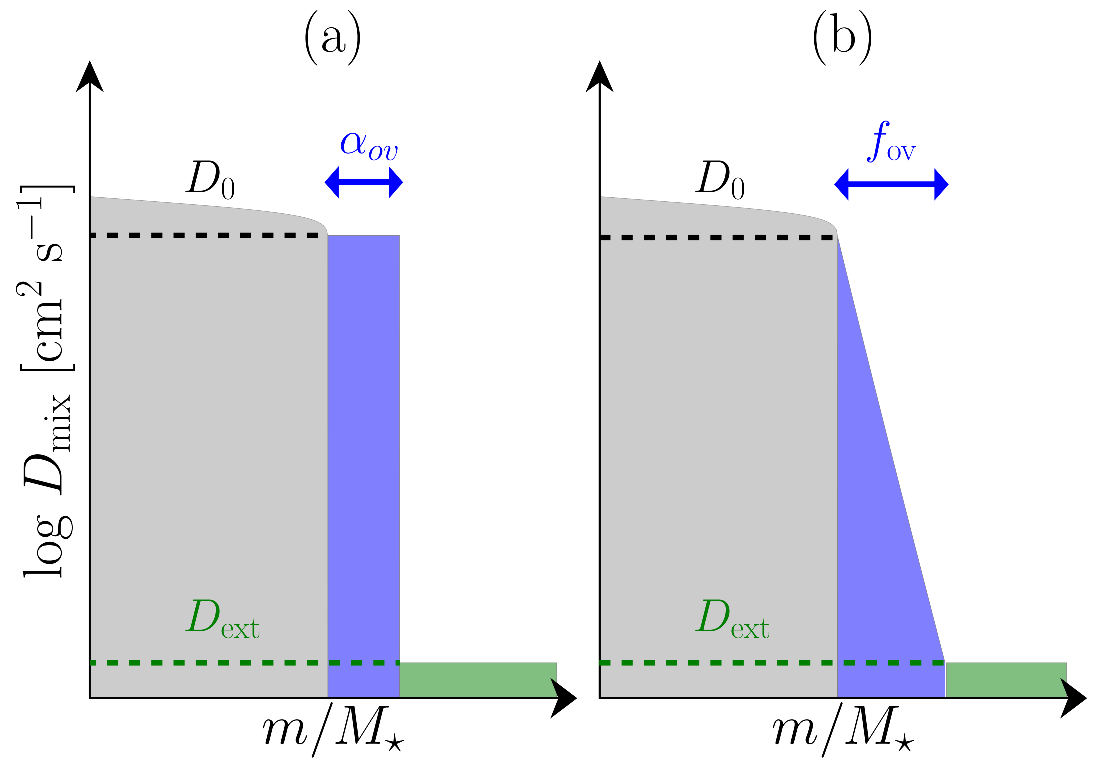
Convective Overshoot

How far does thermal penetrate?

What sets N^2 near rad-conv boundary?
Probably opacity or geometric effects
Not studied carefully yet

Convective Overshoot

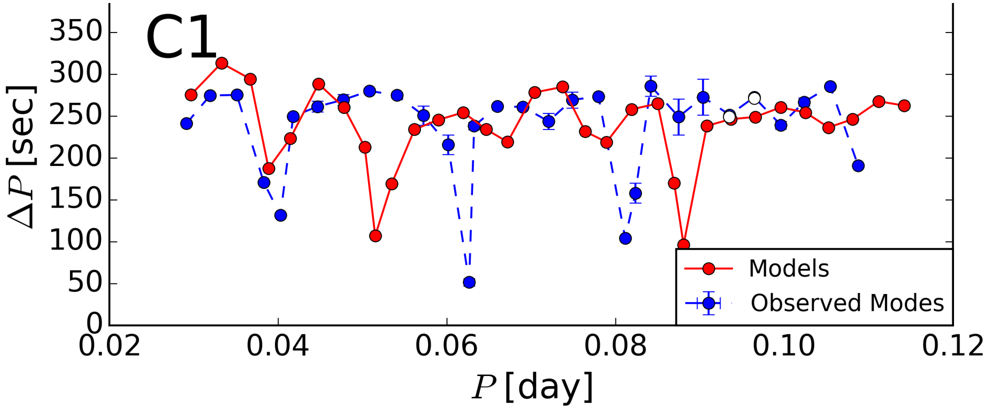
How to model?



Pedersen et al 2018

Convective Overshoot

exponential
overshoot

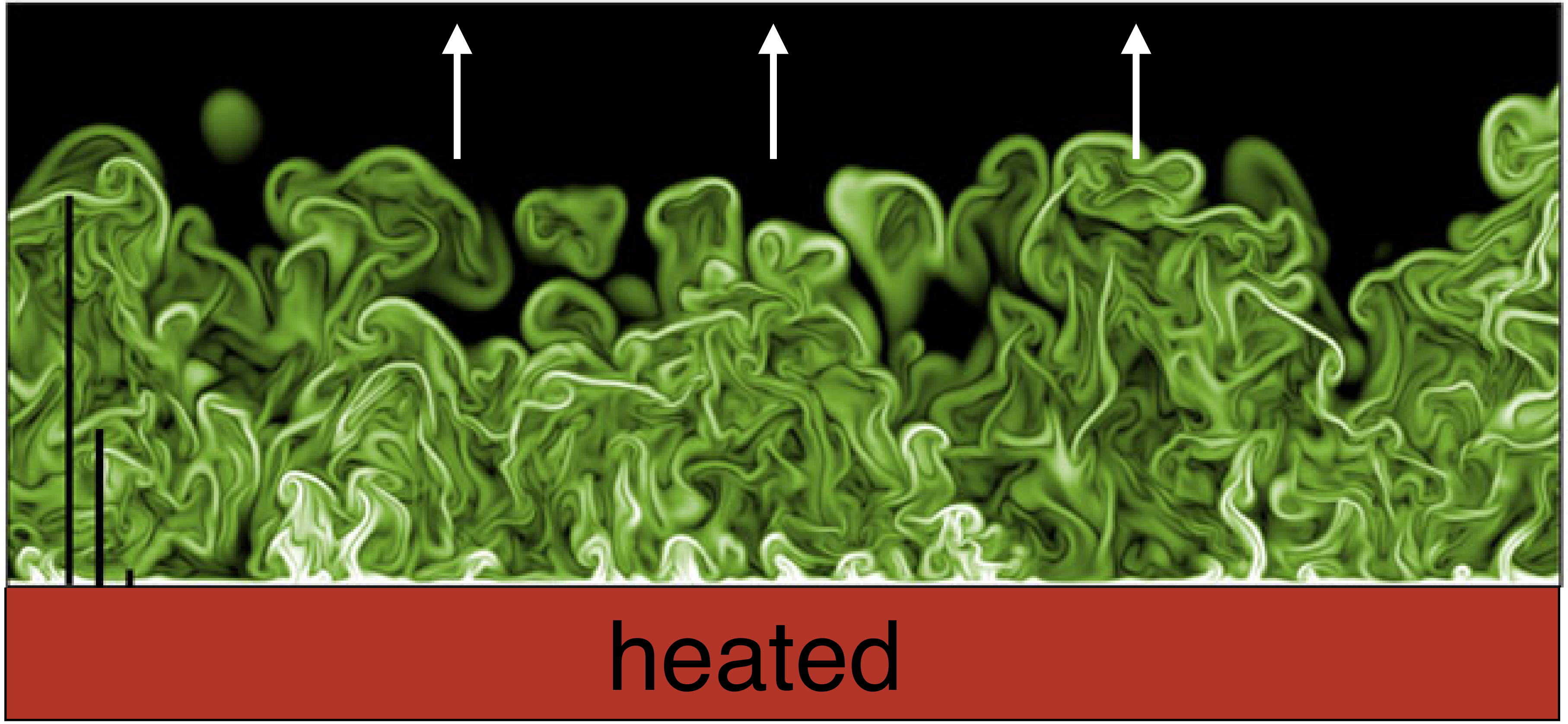


Ghasemi et al 2017

Convective Overshoot

ABL

log(dissipation rate)

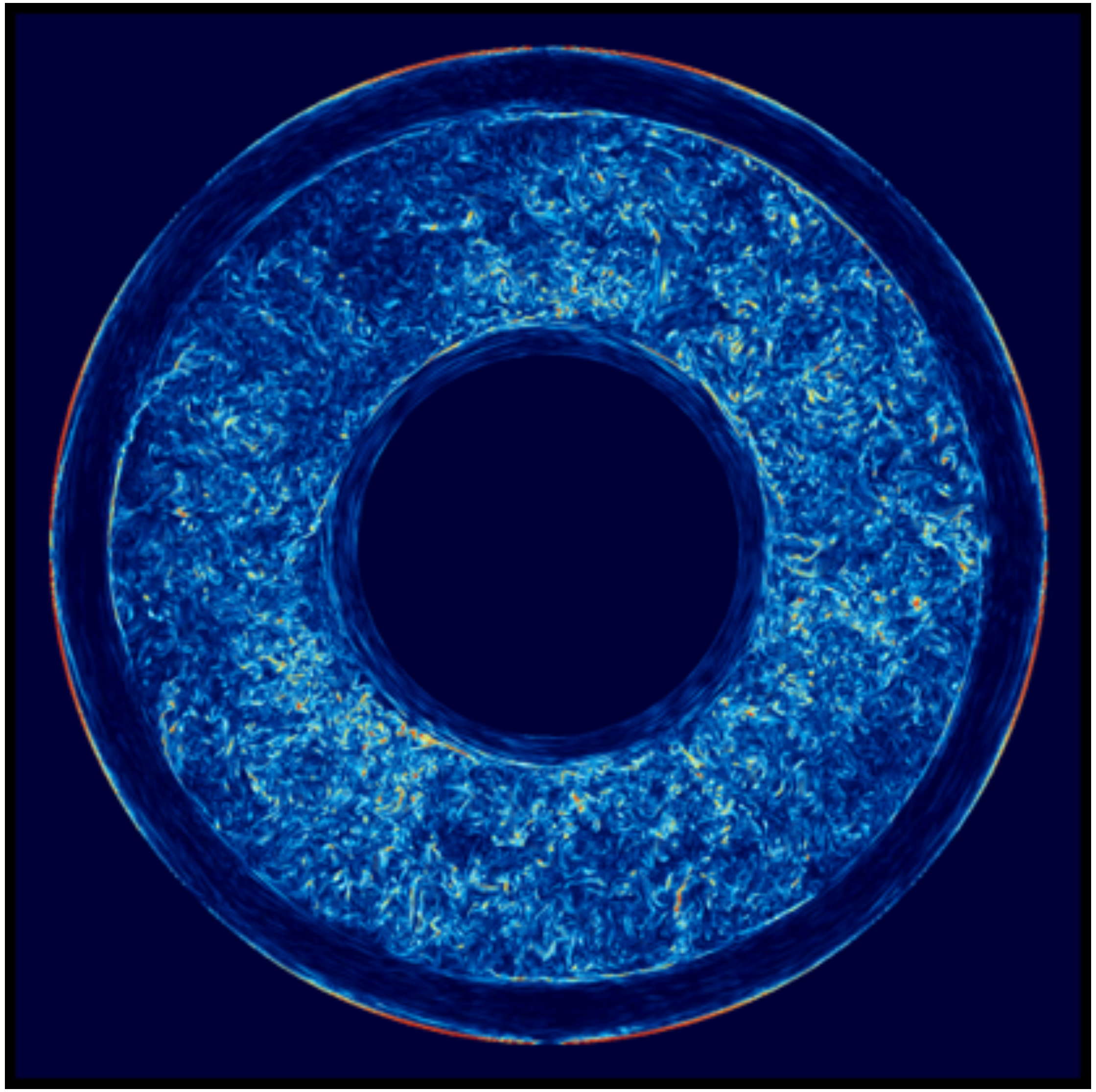


$$\dot{M} \sim \mathrm{Ri}^a$$

$$\mathrm{Ri} = \frac{N^2}{(dU/dz)^2}$$

Mellado 2012

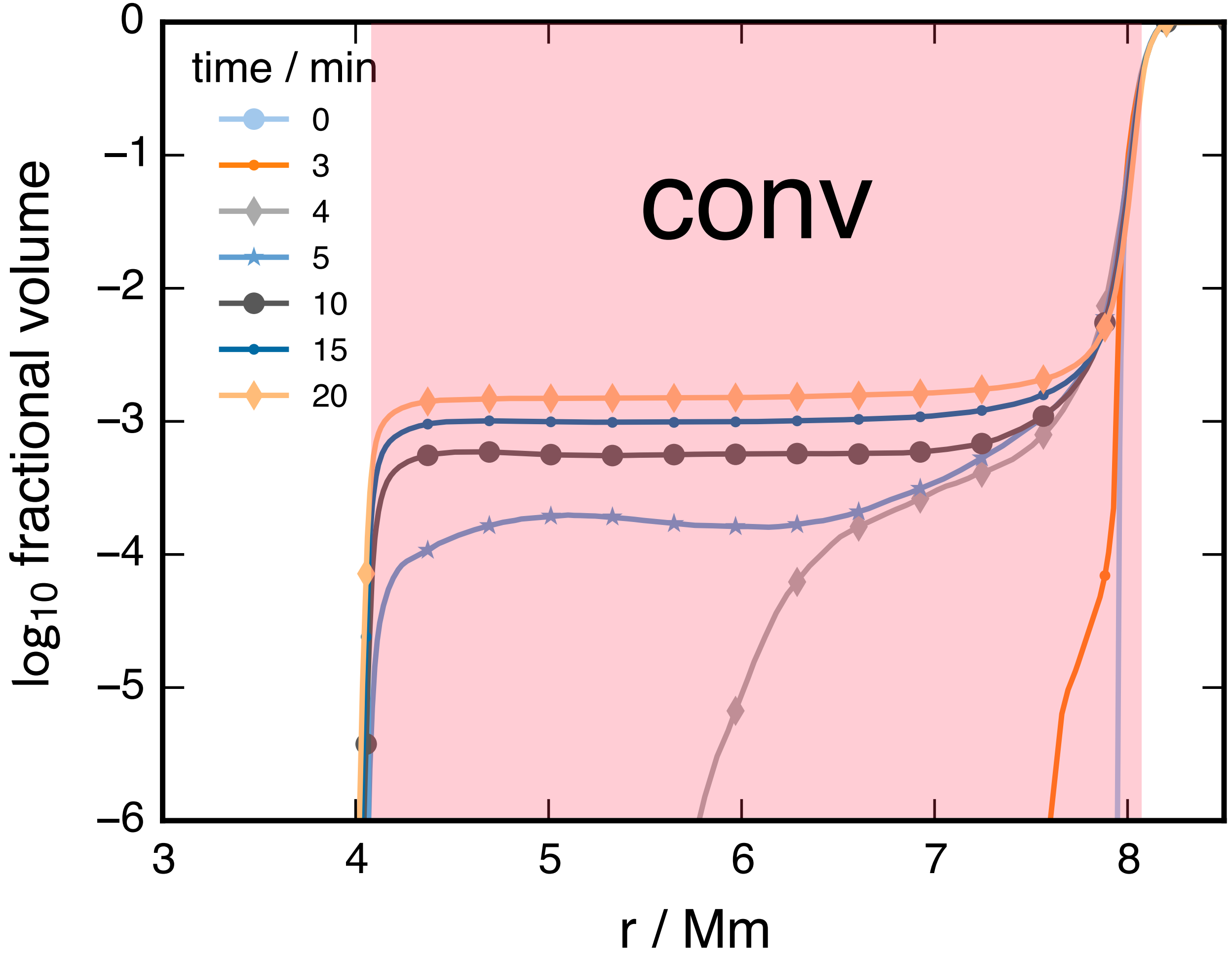
Diffusion Approximation



Oxygen burning in massive star
~10 days before SN

Jones et al 2016

Diffusion Approximation

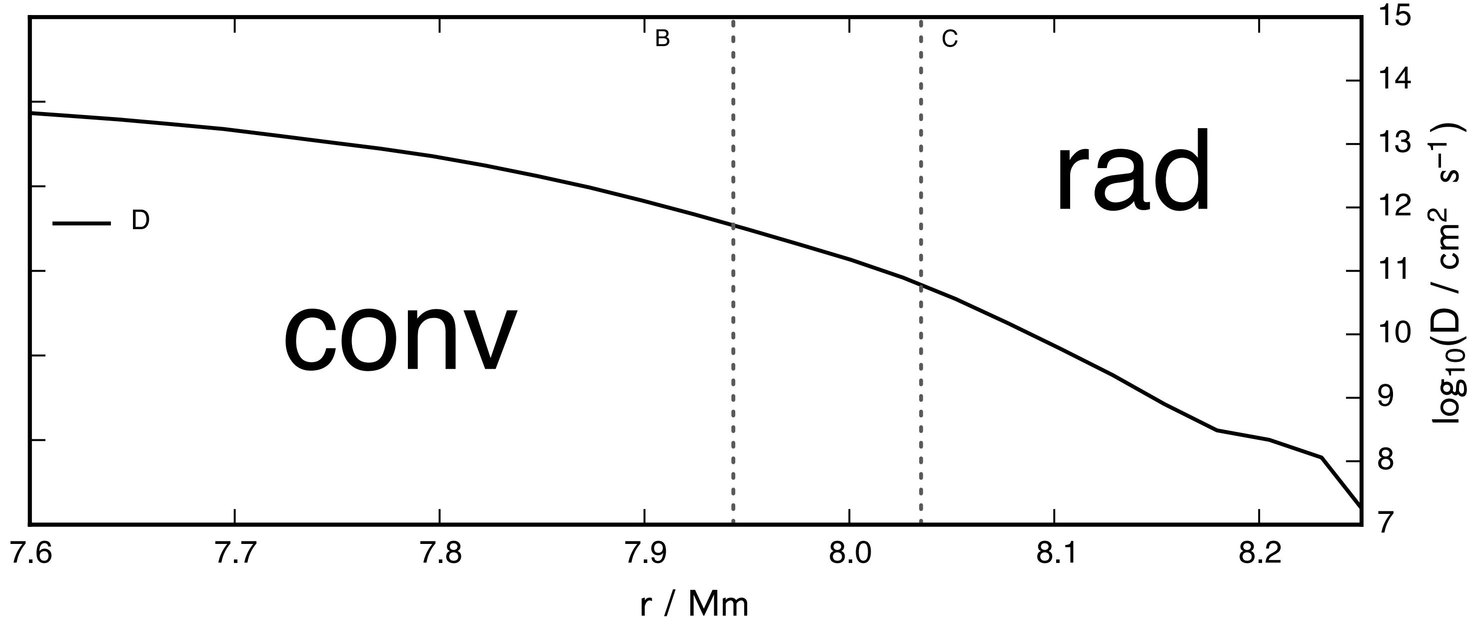


$$\partial_t c = D_T \nabla^2 c$$

$$D_T = \frac{\Delta c}{\nabla^2 \overline{c}}$$

Jones et al 2016

Diffusion Approximation



$$\partial_t c = D_T \nabla^2 c$$

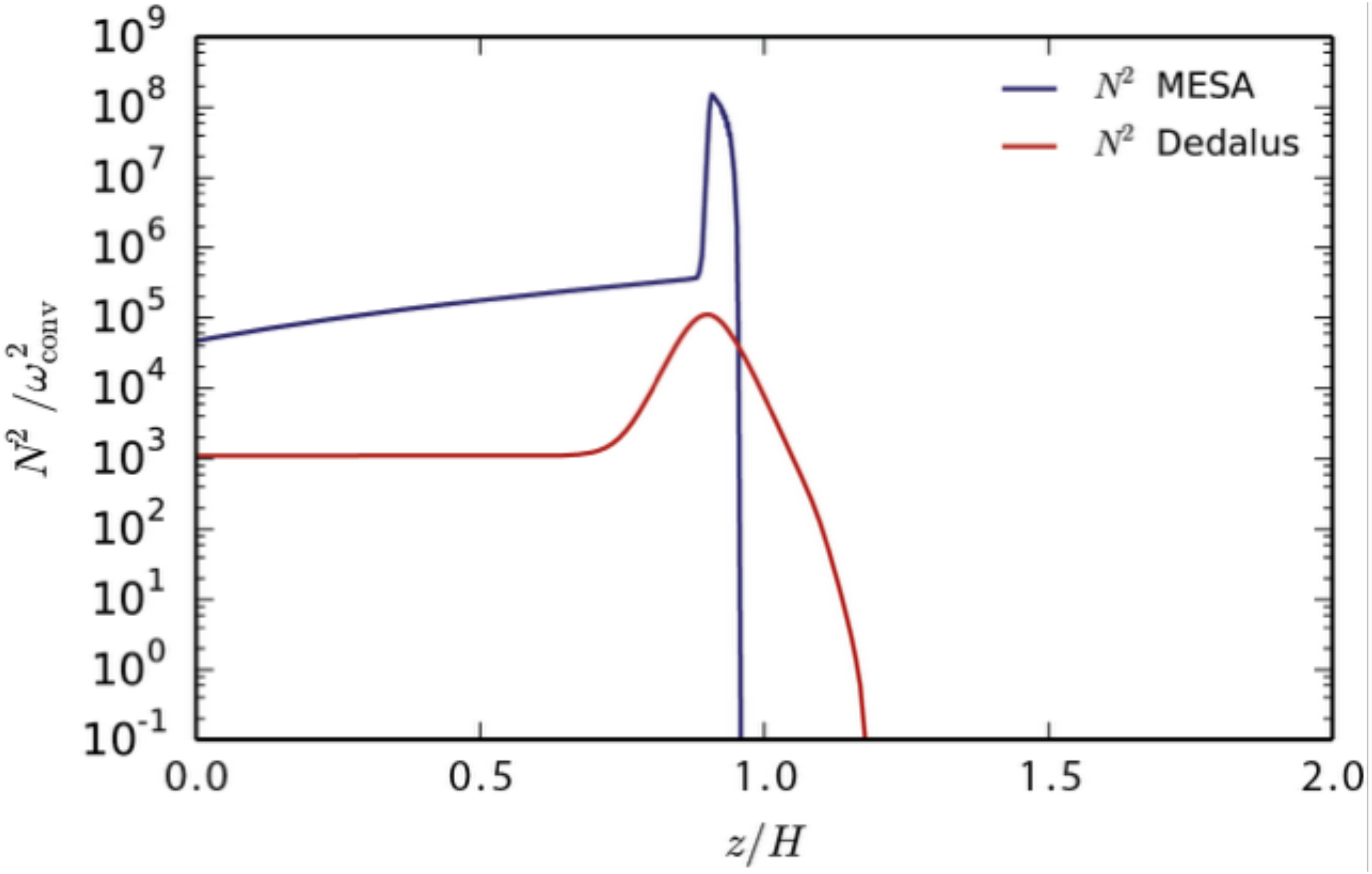
$$D_T = \frac{\Delta c}{\nabla^2 \overline{c}}$$

need to check intermediate times

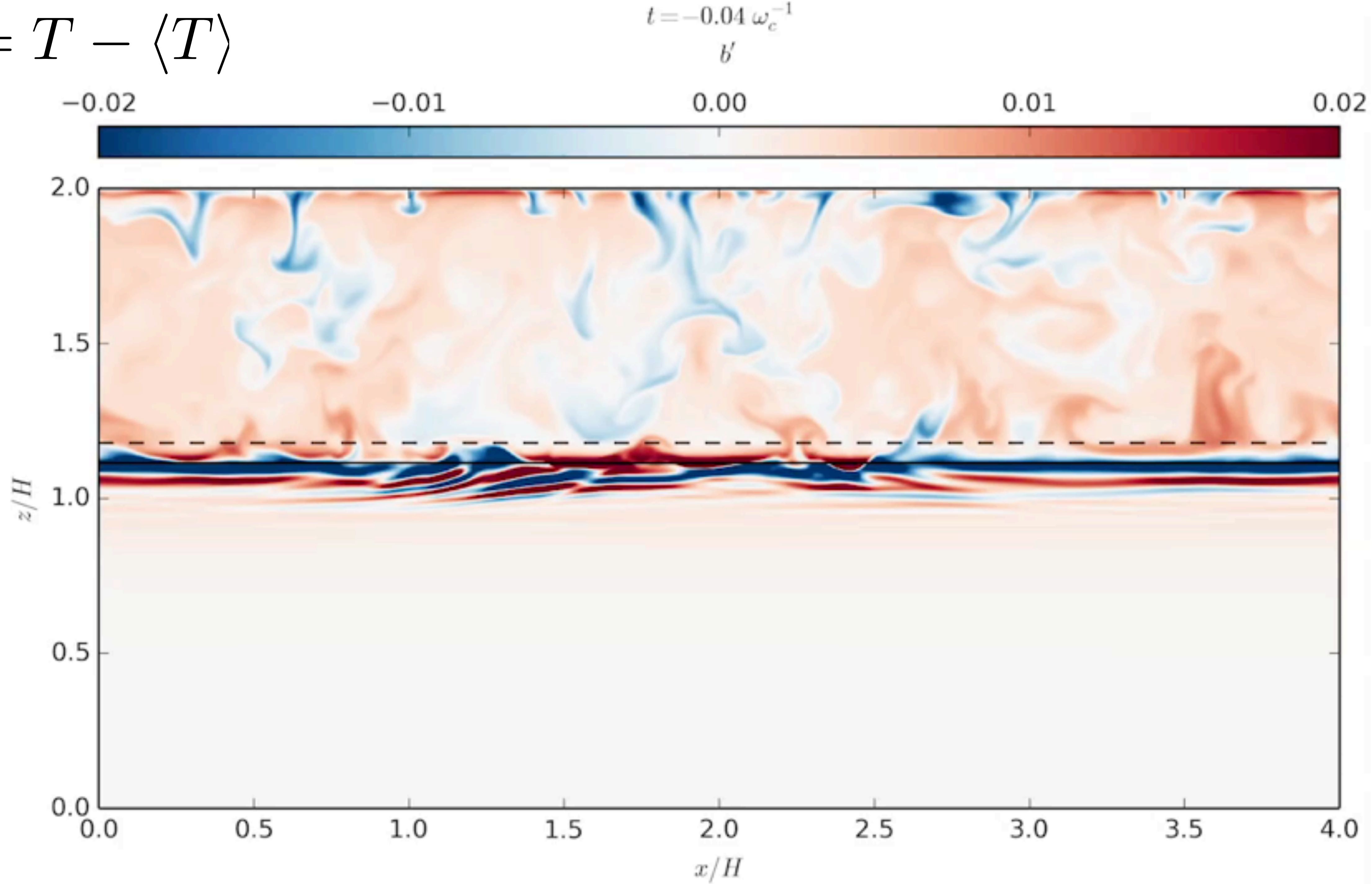
Jones et al 2016

Is Diffusion Model Even Applicable??

Diffusion Approximation



$$b' = T - \langle T \rangle$$

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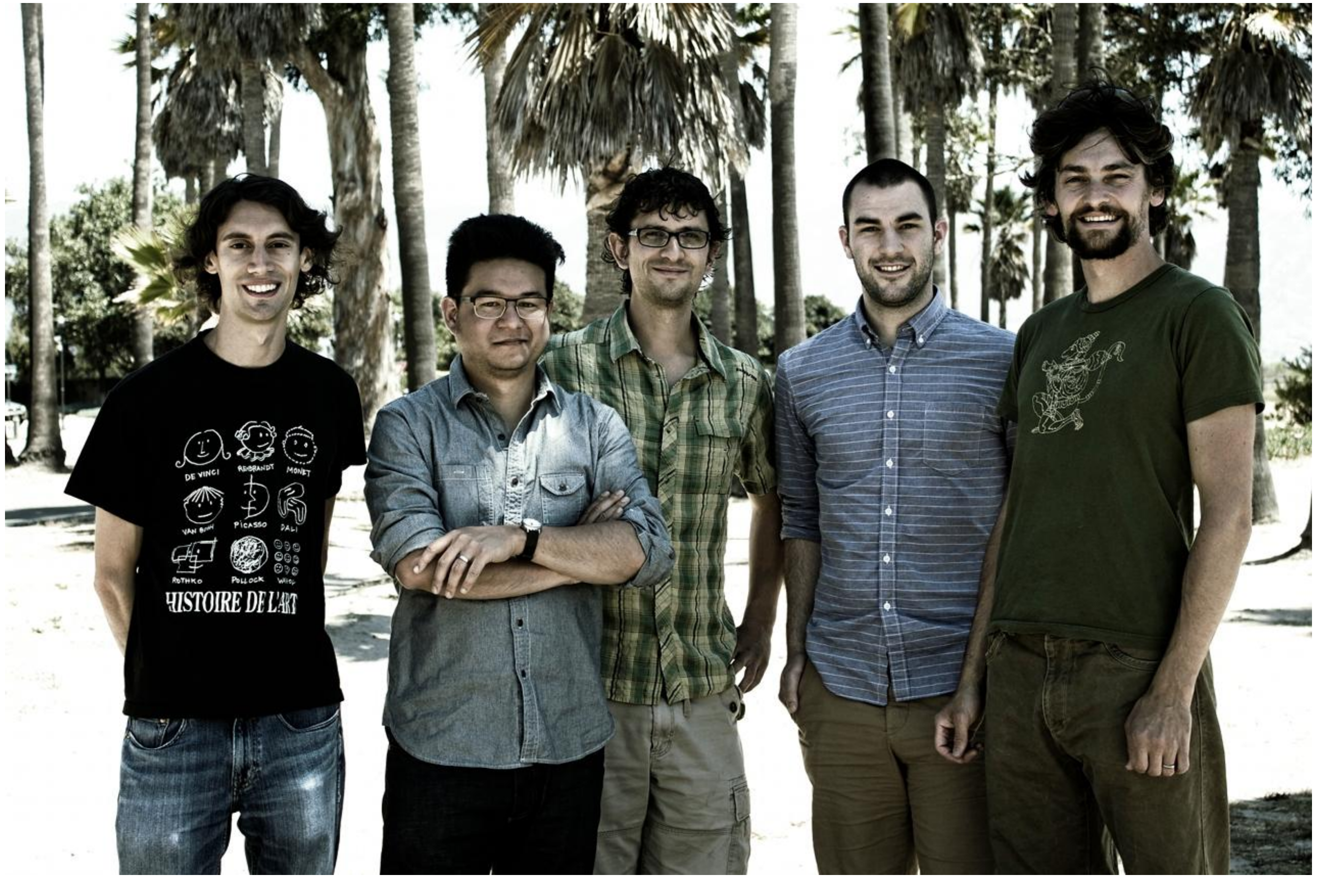
The team so far



Australian Government

Australian Research Council

NSF

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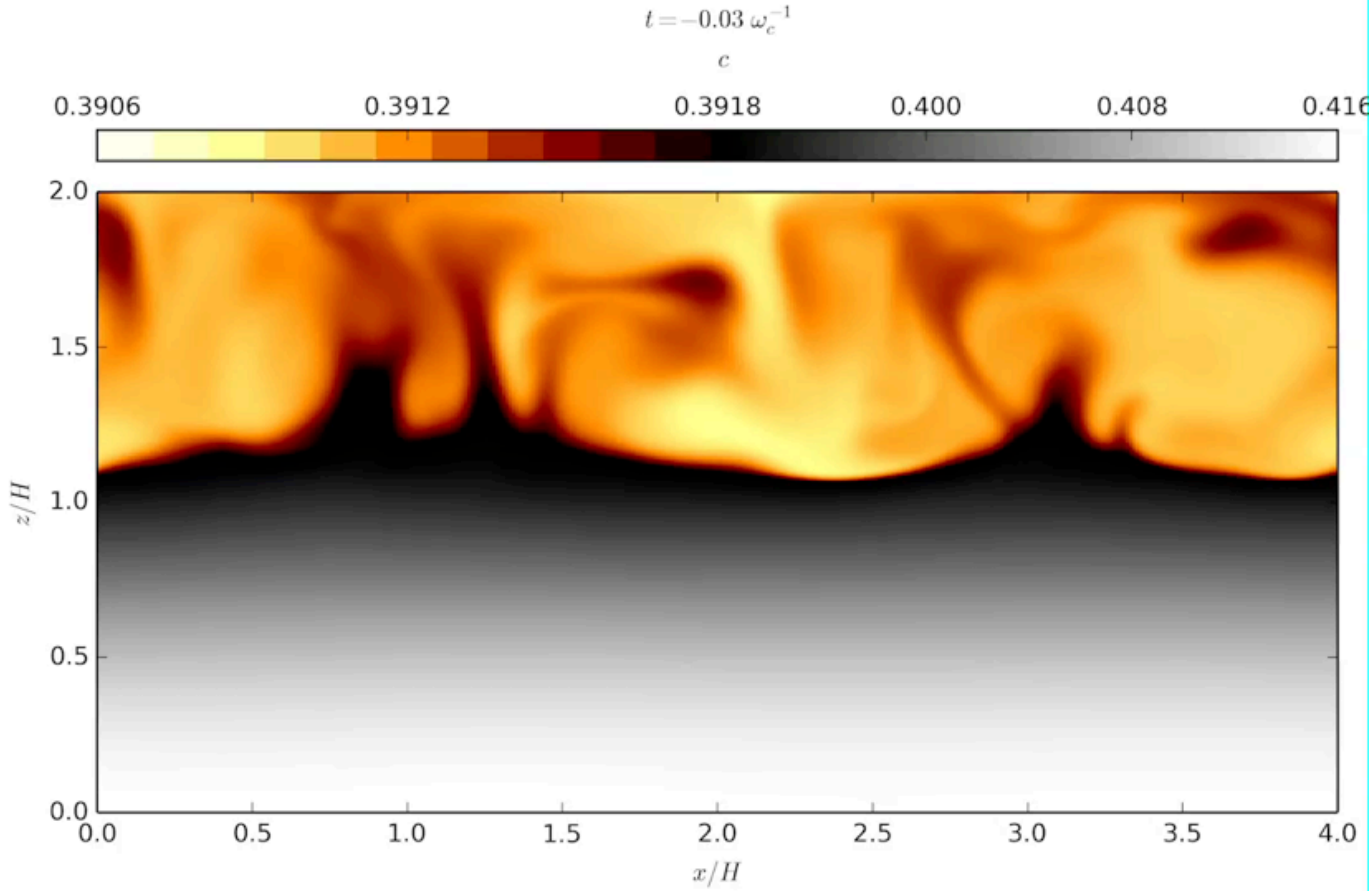
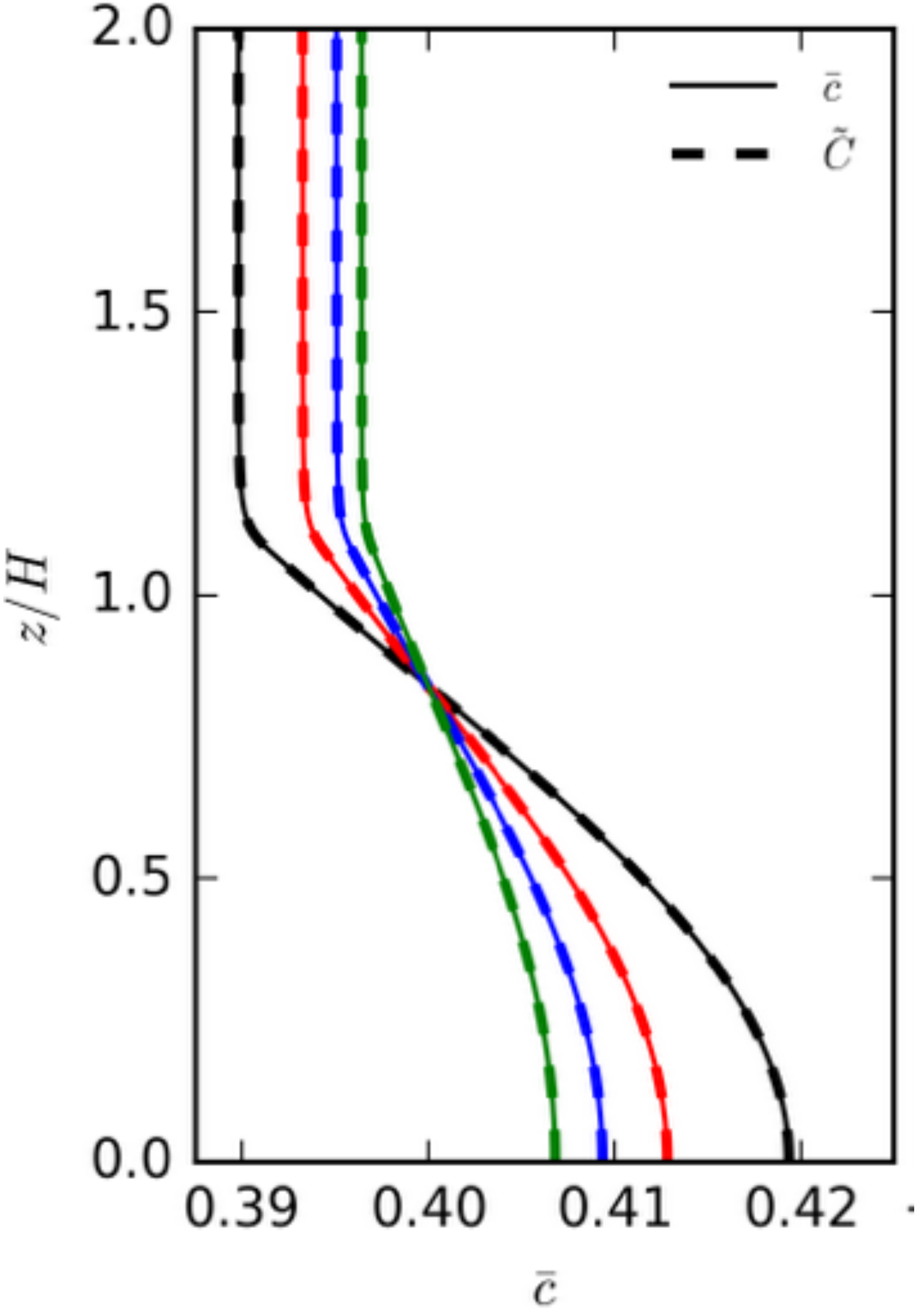


$t = -0.03\ \omega_c^{-1}$
c
0.3906
0.3912
0.3918
0.400
0.408
0.416
2.0
1.5
1.0
0.5
0.0
z/H
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
x/H



2.0
1.5
1.0
0.5
0.0
z/H
0.39
0.40
0.41
0.42
$\bar{c}$
$\bar{c}$
$\tilde{C}$

Diffusion Approximation

$$\partial_t c - D\nabla^2 c = -\boldsymbol{u}\cdot\boldsymbol{\nabla} c$$

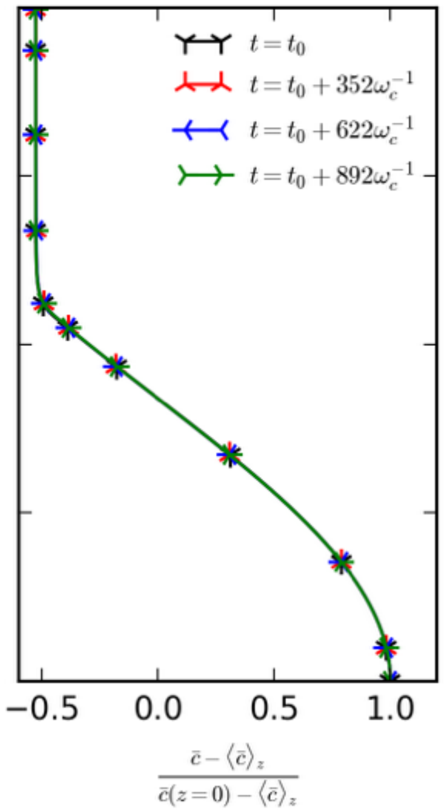
$$\bar{c}(z,t) - \langle \bar{c} \rangle_z \rightarrow c_{\mathrm{ss}}(z,t) = A(t)C(z)$$

$$-\lambda C - D\partial_z^2 C = -\left\langle \boldsymbol{u}\cdot\boldsymbol{\nabla}\frac{c}{A}\right\rangle_{x,y,t}$$

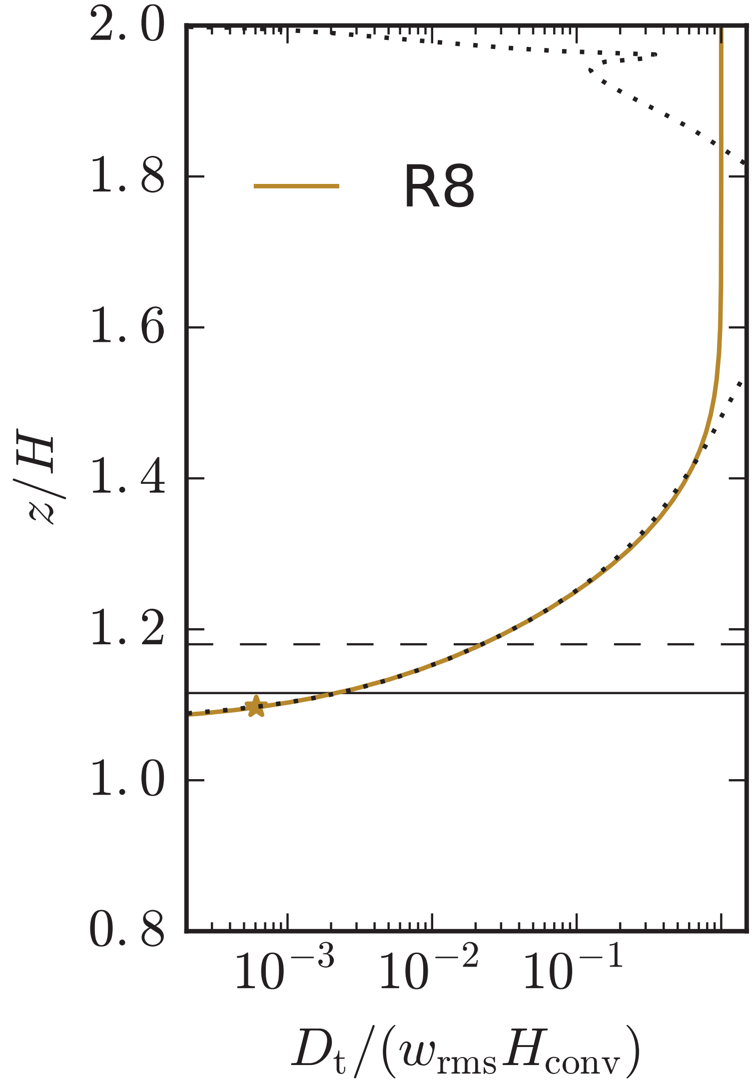
Ansatz:

$$-\lambda C = \partial_z[(D + D_{\mathrm{t}})\partial_z C]$$

$$-\left\langle \boldsymbol{u}\cdot\boldsymbol{\nabla}\frac{c}{A}\right\rangle_{x,y,t} = \partial_z(D_{\mathrm{t}}\partial_z C)$$



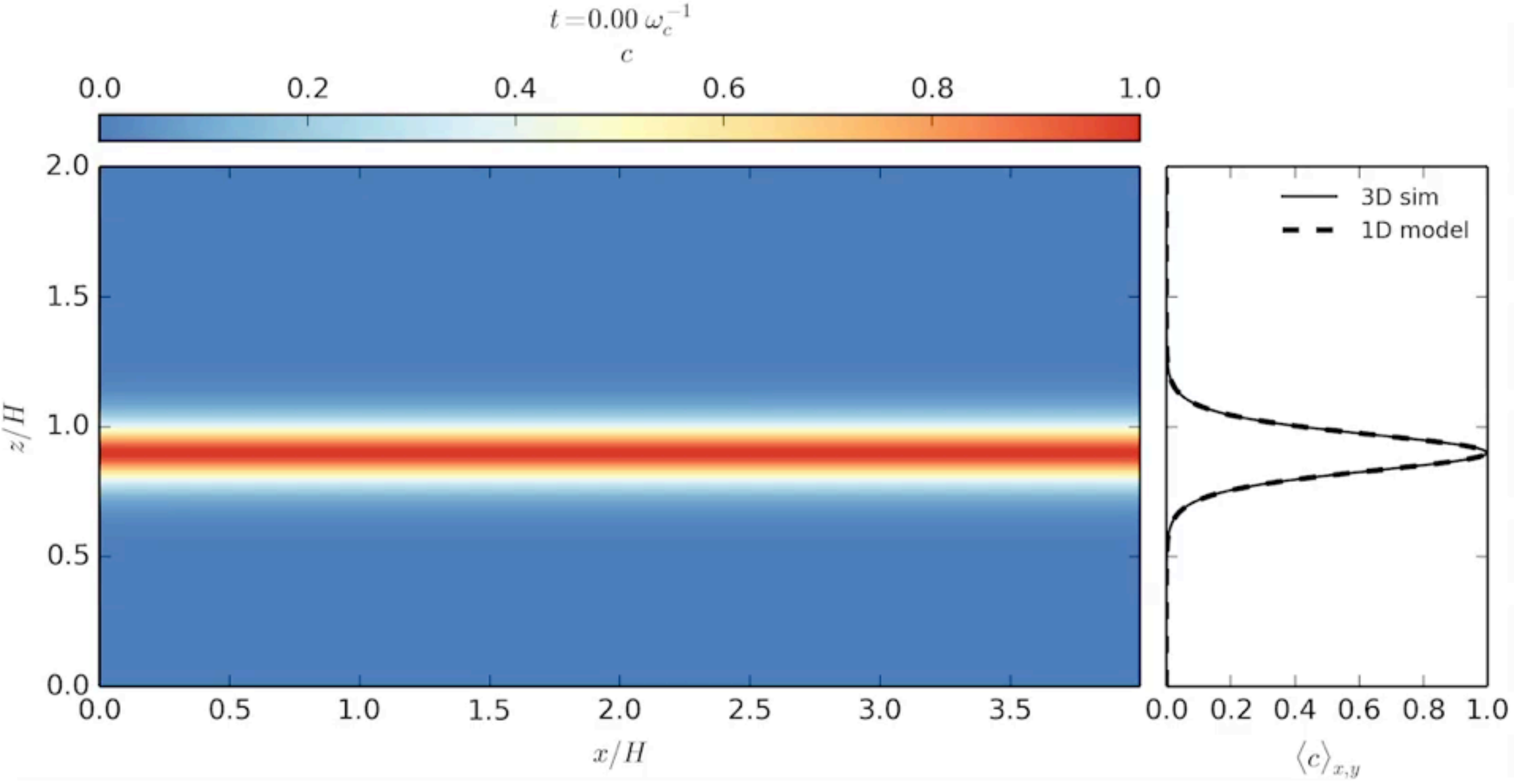
Diffusion Approximation



Solve:

$$\partial_t C = \partial_z[(D + D_{\mathrm{t}})\partial_z C]$$

Does it work?



$t = 0.00\ \omega_c^{-1}$
c
0.0
0.2
0.4
0.6
0.8
1.0
2.0
1.5
1.0
0.5
0.0
z/H
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
x/H
3D sim
1D model
0.0 0.2 0.4 0.6 0.8 1.0
$\langle c \rangle_{x,y}$

Is Diffusion Model Even Applicable??

Yes!
(At least for current parameters)

Why Turbulent Diffusivity?

$$\partial_t c - D\nabla^2 c = -\boldsymbol{u}\cdot\boldsymbol{\nabla} c$$

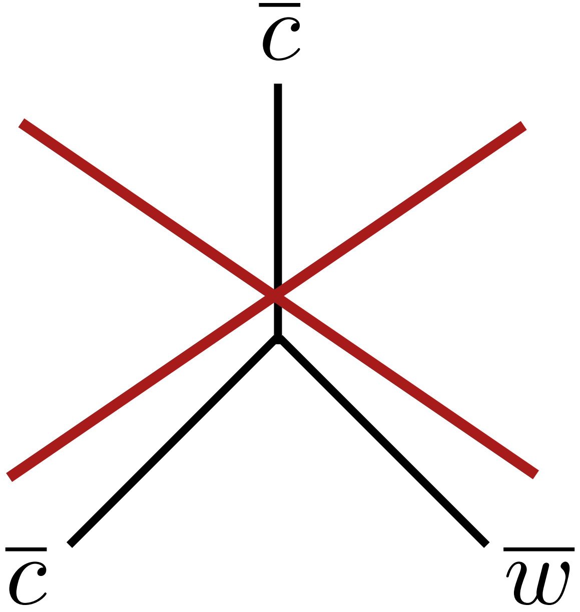
$$c(x,y,z,t) = \overline{c}(z,t) + \tilde{c}(x,y,z,t)$$

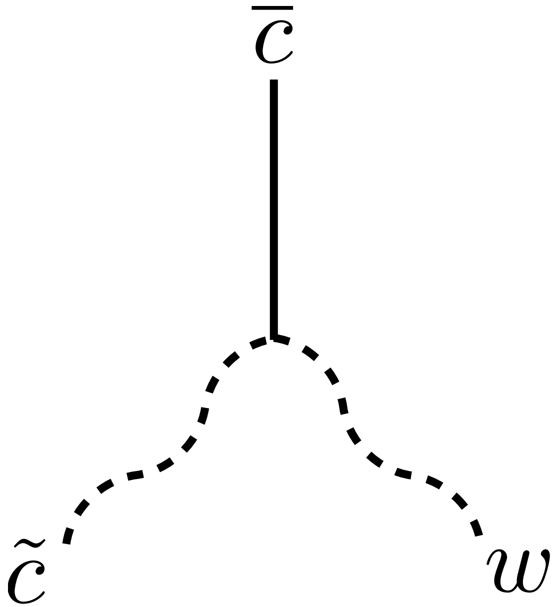
$$\partial_t \overline{c} - D\partial_z^2 \overline{c} = -\left\langle \boldsymbol{\nabla}\cdot(\boldsymbol{u}\tilde{c})\right\rangle = -\partial_z \left\langle w\tilde{c}\right\rangle$$

$$\partial_t \tilde{c} - D\nabla^2 \tilde{c} = -w\partial_z \overline{c} - \cancel{\partial_z\left(w\tilde{c} - \left\langle w\tilde{c}\right\rangle\right)}$$

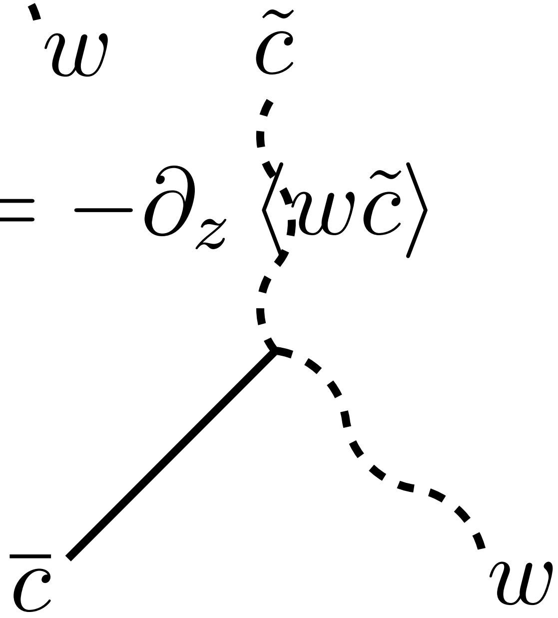
"pain in the neck"

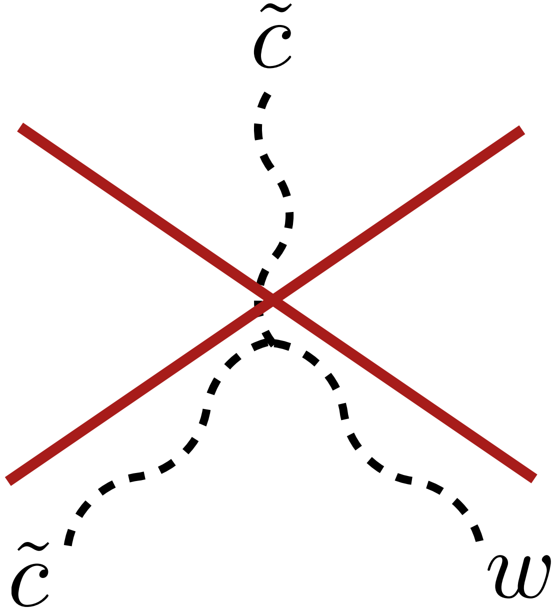
Why Turbulent Diffusivity?





$$\partial_t \tilde{c} - D\nabla^2 \tilde{c} = -w\partial_z \bar{c} - \partial_z \left(w\tilde{c} - \langle w\tilde{c} \rangle \right)$$

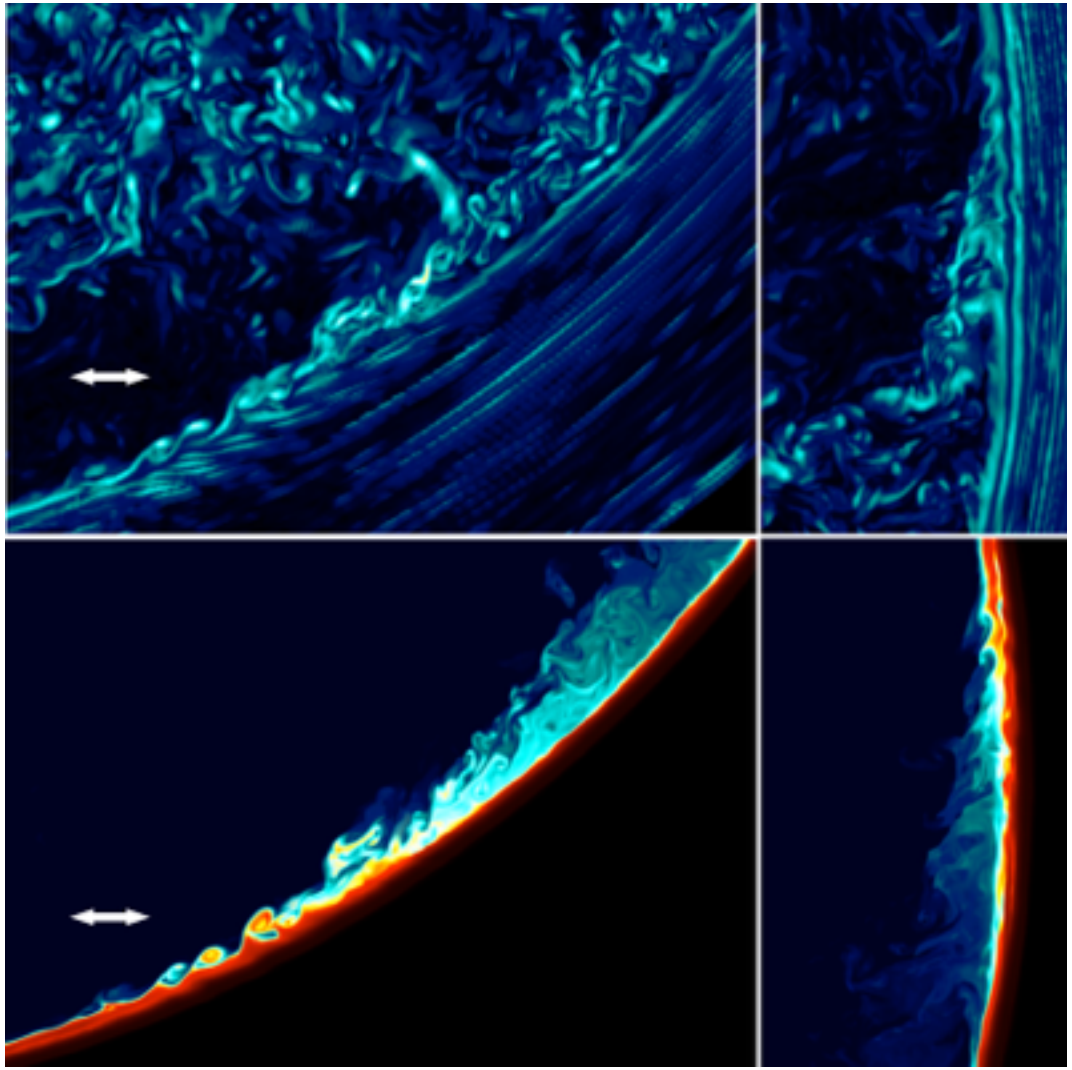




$$\partial_t \bar{c} - D\partial_z^2 \bar{c} = -\langle \boldsymbol{\nabla} \cdot (\boldsymbol{u}\tilde{c}) \rangle = -\partial_z \langle w\tilde{c} \rangle$$

Quasilinear Approximation

Physical Model of Overshoot



Kelvin-Helmholtz instability?

$$\mathrm{Ri} = \frac{N^2}{(dU/dz)^2}$$

$$\mathrm{Ri} \sim 10^3$$

Woodward et al 2014

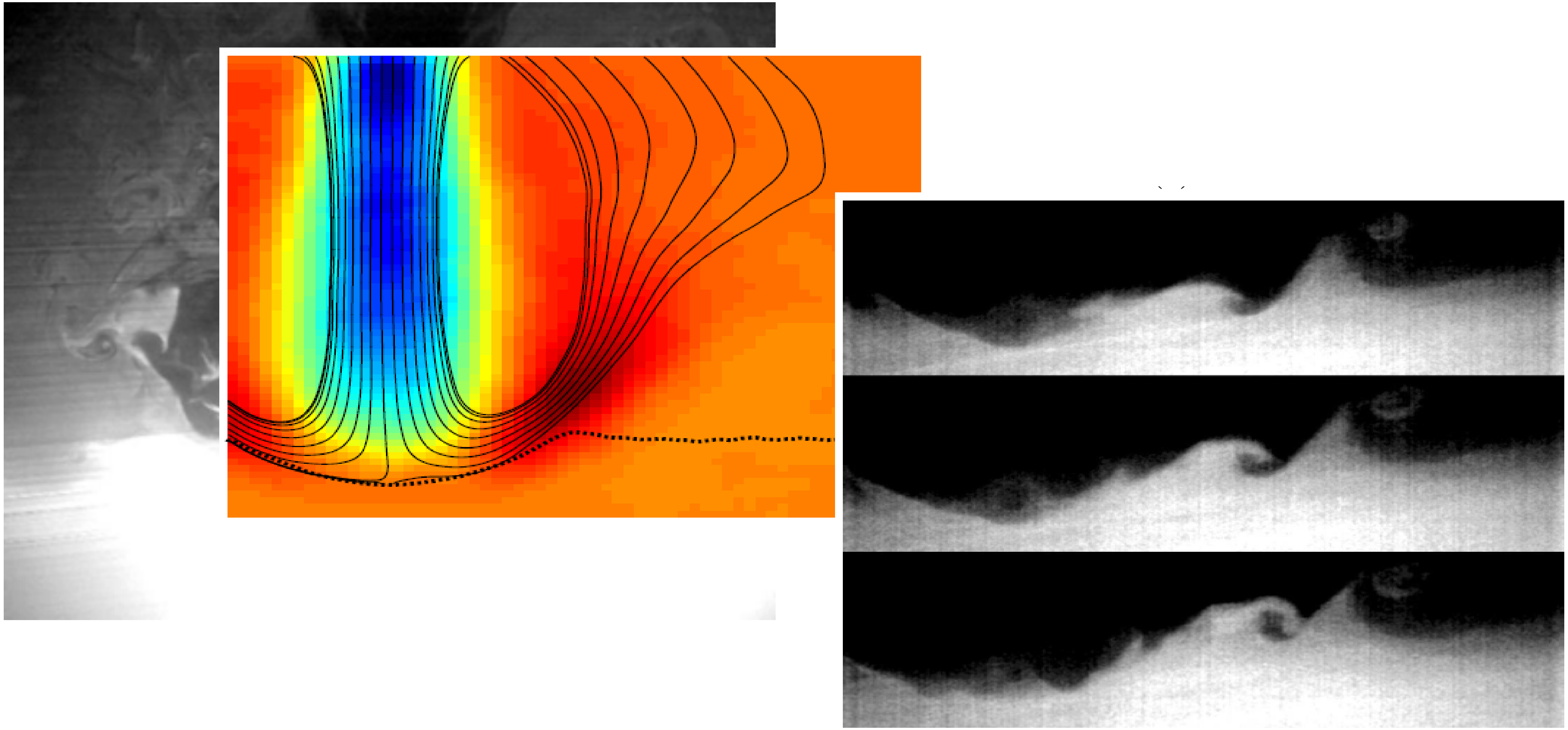
Physical Model of Overshoot

Miles instability

$$U(z_0) = c_p$$

resonance

Physical Model of Overshoot



Herault et al 2018

Summary

1. Convection much weaker than stable stratification -> weak overshoot
2. Overshoot mixes like a diffusion — decreases rapidly with depth
3. May work via Miles instability??