Matrix Models for ETH and random CFT

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Gravity from Algebra @ KITP
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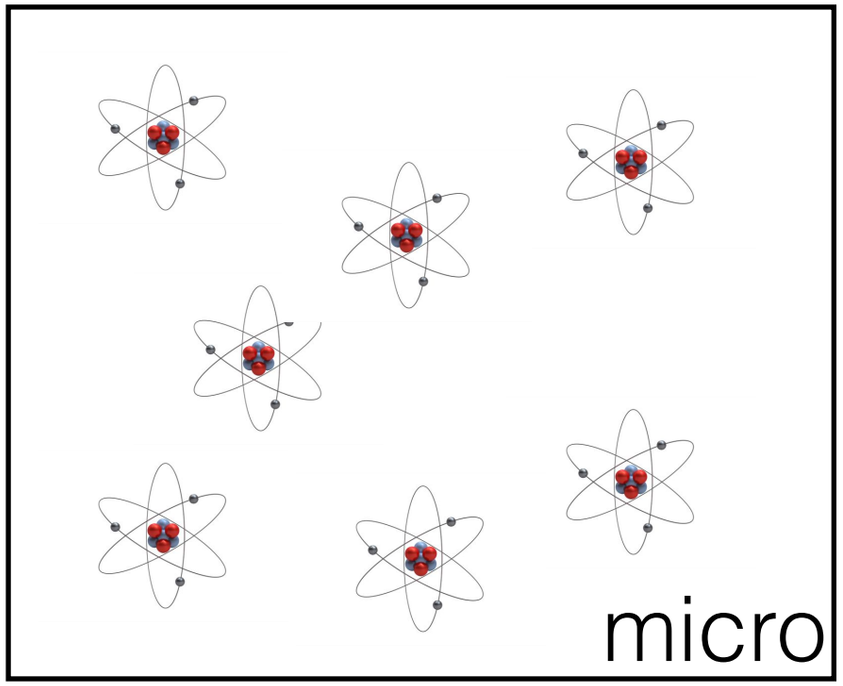
Introduction and background

The question

vs.



$$S_{\mathrm{BH}} = \frac{A}{4G_N} < \infty$$

$$S_{\mathrm{micro}} = \log \Omega$$

What is the microstructure of $\mathcal{H}_{\mathrm{BH}}$?

The plan

I Matrix models for quantum chaos and gravity

II An ETH matrix model

III JT + matter as an ETH matrix model

IV The statistics of the crossing equation & random CFT

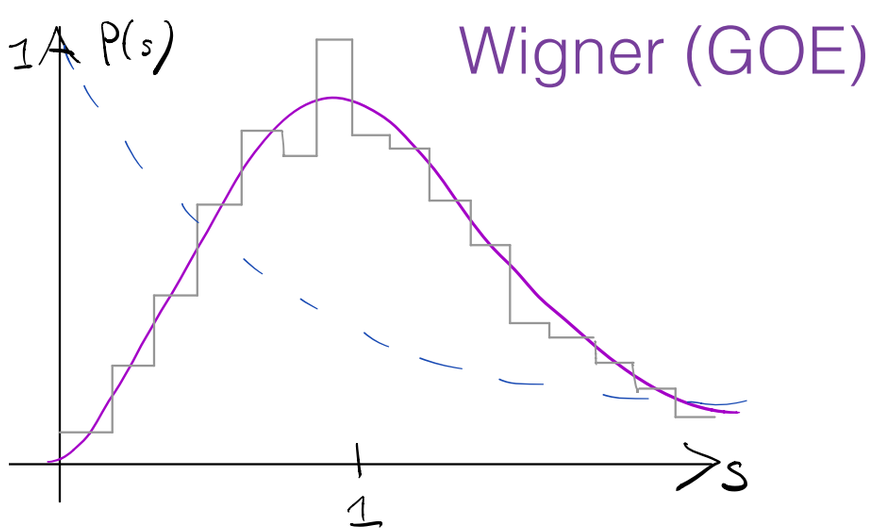
Based on 2209.02130 & 2209.02131 with [**Jafferis, Kolchmeyer, Mukhametzhanov**] and work in progress with [**Belin, de Boer, Jafferis & Nayak**]

Ia) Matrix Models and Quantum Chaos

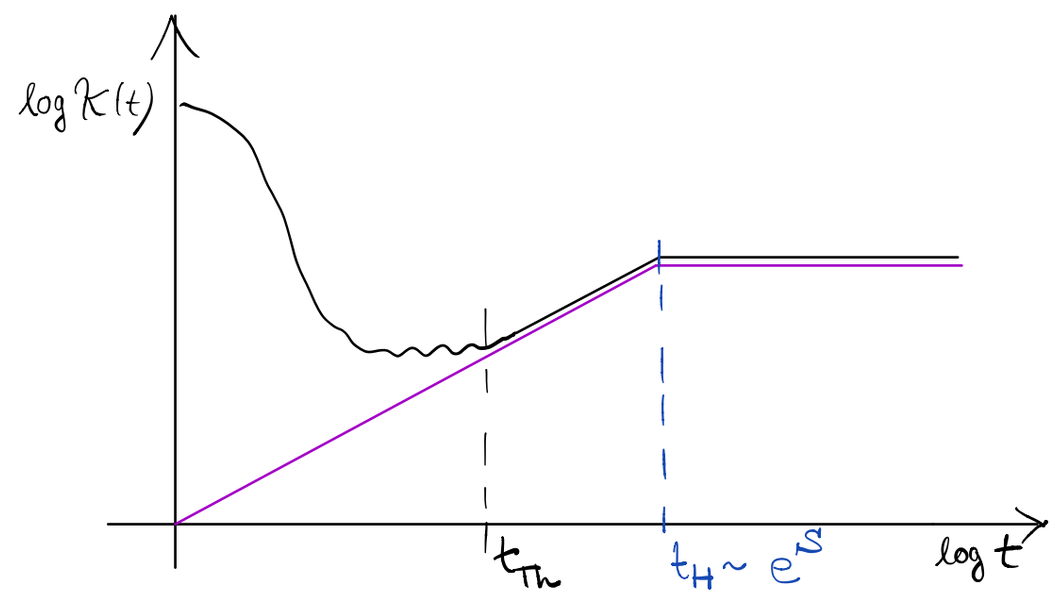
Matrix models = random matrix theory (RMT) = matrix integrals
[Wigner, Dyson,…]

Quantum chaotic H(amiltonian) $\cong$ RMT

Exemplified, e.g., by spectral probes:



2-level correlation



Spectral from factor

Chaotic Hamiltonians show RMT statistics (Gaussian, in absence of further inputs / information)

Ib) Including operators: Eigenstate Thermalization (ETH)

Attempts to answer: How do unitary quantum systems thermalize?

$$\langle i|\mathcal{O}|j\rangle = \overline{\mathcal{O}}(\overline{E})\delta_{ij} + e^{-S/2} f(E_i, E_j) R_{ij}$$

[Deutsch; Srednicki,…]

Smooth function of average energy

smooth function theory dept.

random matrix (Gaussian, for now)

we will see that finer-grained analysis ⇒ non-Gaussianities
[JS, Vielma; Foini, Kurchan; Dymarsky,…]

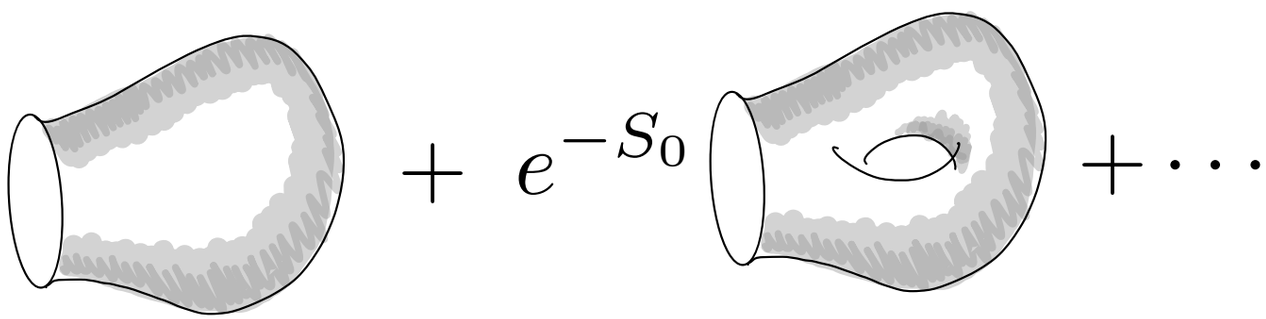
Goal: fit ETH and RMT into a joint framework, the "ETH matrix model"

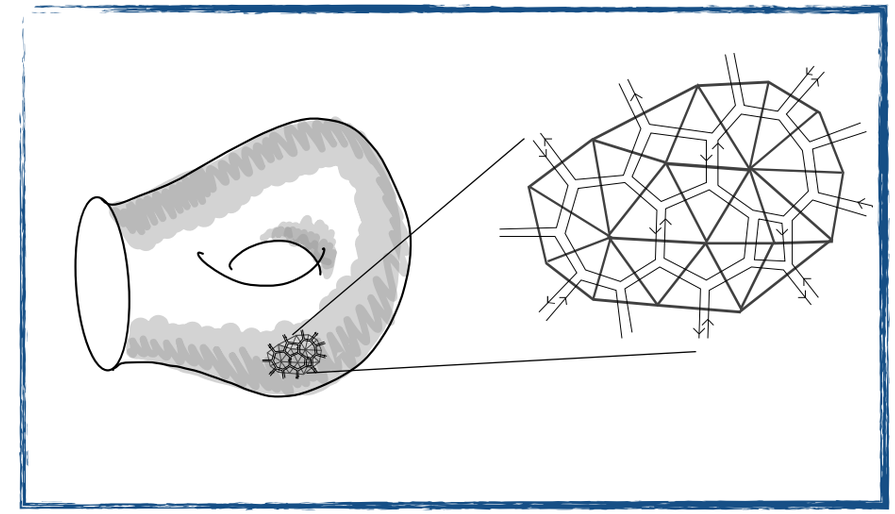
Remark: (universal) emergence of RMT and operator statistics can be understood in terms of a Goldstone EFT: [Wegner; Efetov; Altland, JS]

$$U(2|2) \longrightarrow U(1|1) \times U(1|1)$$

II) Random matrices and quantum gravity

2D gravity can be defined from random triangulations of surfaces:





[Kazakov; Gross, Migdal; Moore, Shenker;… SSS]

$$\sum_{\text{top}} \int \mathcal{D}h e^{-S[h]} \quad \leftrightarrow \quad \int d\mu[H] e^{-\text{tr}V(H;\lambda,N)}$$

RMT = discrete triangulation of 2D universes, continuum gravity path integral emerges from double-scaling limit

Recent developments suggest unity of points I & II

[SSS; Altland, Bagrets, JS, Nayak; Johnson,….]

An ETH matrix model

De-Gaussing ETH

It is usually said that R_{ij} is a Gaussian random matrix, but it cannot be so

- Would lead to trivial OTO 2n-point functions, i.e. fail to capture Lyapunov-type chaos
- More generally, fails to produce crossing-invariant 4-pt function

⋮

For example, to capture 4pt OTO (Lyapunov exponent), need

$$\overline{R_{ij}R_{jk}R_{kl}R_{li}}\Big|_{\text{conn}} = e^{-3S(\overline{E})}g^{(4)}(E_1, E_2, E_3, E_4)$$

Contributes to OTOC at O(1) due to Hilbert-space sums, determines Lyapunov exponent

[JS, Vielma; Foini, Kurchan, Murthy, Srednicki]

Packaging as a (two) matrix model

We have two inputs:

- Energy-level statistics: leads to random matrix H_{ij}
- Operator matrix-element statistics: leads to random matrix R_{ij}

$$\mathcal{Z} = \int d\mu[H] d\mu[\mathcal{O}] e^{-\mathrm{tr} V(H,O)}$$

In energy eigenbasis we have

$$V = -\sum_i V(E_i) - e^{S_0} \sum_{i,j} \tfrac{1}{2} F_{ij} \mathcal{O}_{ij} \mathcal{O}_{ji} + e^{S_0} \sum_{i_1,\ldots i_n} G^{(n)}_{i_1 \ldots i_n} \mathcal{O}_{i_1 i_2} \cdots \mathcal{O}_{i_n i_1}$$

"Gaussian ETH"

Determined by matching $\boldsymbol{\rho}$(E)

Determined by matching hierarchy of $g^{(n)}$

The ETH matrix model

Comments:

- V(H) determines $\boldsymbol{\rho}(E)$, $\boldsymbol{\rho}^{(2)}(E, E')$,… gives connected energy correlations
- F_{ij}, $G^{(n)}_{ij}$…. non-Gaussianities of operator statistics

Like in standard ETH, these are free functions: how to determine them?

Input more knowledge by imposing constraints:

- EFT of quantum chaos (quantum-chaos universality)
- Determine non-Gaussianities by imposing additional constraints (e.g. modular crossing, s/t crossing,…)

⋮

An ETH matrix model
For JT with matter

Worked example: JT + scalar

Suppose we are interested in constructing a matrix model for

$$S[g,\phi] = S_{\rm JT} + \int_{\mathcal{M}} d^2x\sqrt{g}\left((\partial\varphi)^2 + m^2\varphi^2\right)$$

The naive Gaussian ETH ansatz would suggest

$$V(H,\mathcal{O}) = \tilde{V}_{\rm SSS}(H) + e^{S_0}\sum_{i,j}\frac{1}{2}F_{ij}\mathcal{O}_{ij}\mathcal{O}_{ji}$$

with $F_{ij} = F(E_i,E_j) = \left(\frac{\Gamma(\Delta \pm i\sqrt{E_i} \pm i\sqrt{E_j})}{\Gamma(2\Delta)}\right)^{-1}$ (disc 2-pt function)

However: this leads to a 4-pt function that is not crossing invariant

Introducing the constraint

Fix the problem by introducing a large non-Gaussianity

We need to feed more information to the ETH matrix model:

$$\mathcal{O}(\tau)\mathcal{O}(0) = \frac{1}{\tau^{2\Delta}} + \sum_n \tau^{2n} \left[\mathcal{O}\mathcal{O}\right]_n + \text{descendants}$$

"OPE constraint"

2**Δ** + 2n double-trace primary

We can impose this as a constraint $M^i_{jk} = 0$ on the matrix-model

$$\mathcal{Z} = \int d\mu[H] d\mu[\mathcal{O}] \exp\left(-V(H) + \frac{\Lambda}{2} \sum_{i,j,k} |M^i_{jk}|^2\right)$$

Giving rise to a quartic non-Gaussianity in the $\mathcal{O}$-potential

Successes, issues & conjecture(s)

The resulting quartic matrix model captures (careful 2x-scaling!)

- Disc 2n-point functions of JT + matter
- JT + matter energy and operator correlations at higher genus:
  - "Empty" double trumpet
  - Double trumpet with $\mathcal{O}$ - insertions
  - Pair of pants
  - Iterative construction of - potential matches constraint potential

Conjecture: matches JT + matter @ all genera

Issue: UV issue related to small wormhole identified with matrix-model saddle destabilisation $\Longrightarrow$ find new stable saddle?

A tensor model for random CFT

Ensembles for random (2D)CFT

Let us think about the idea of “ETH matrix models” for (random) CFT:

do not expect straight-forward Wigner (“Altland-Zirnbauer”)-type RMTs

These should be joint statistical models of

[Cotler, Jensen; de Boer, Belin, Nayak, JS; Chandra, Collier, Maloney, Hartman]

$$\{\Delta_i \, , \, C_{ijk}\}$$

random matrix
$\Delta_i = \mathrm{eig}(M_{ij})$

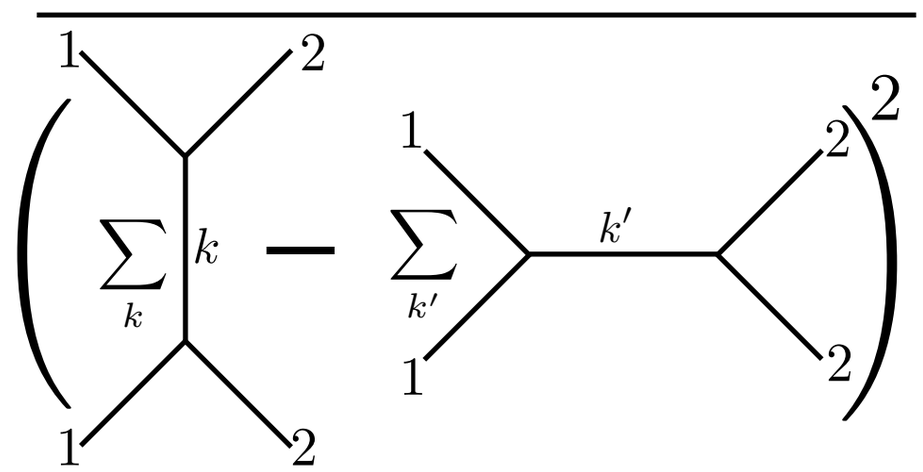
random tensor

We can construct the resulting tensor model by imposing approximate bootstrap constraints (→ notion of approximate CFT)

Crossing implies non-Gaussianity

Suppose crossing in the ensemble is satisfied on average, then imposing the variance of crossing on average to be ≈ 0

$$\overline{\left(\langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_2\mathcal{O}_1\rangle_s - \langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_2\mathcal{O}_1\rangle_t\right)^2} =$$



$$\approx 0$$

implies the existence of large non-Gaussianities among OPE coefficients

$$\overline{C^2_{12k}C_{11k'}C_{22k'}} = \left\{ \begin{array}{ccc} \mathcal{O}_1 & \mathcal{O}_2 & \mathcal{O}_k \\ \mathcal{O}_2 & \mathcal{O}_1 & \mathcal{O}_{k'} \end{array} \right\} \overline{C^2_{11k}}\,\overline{C^2_{22k}}$$

Virasoro 6j (crossing kernel)

~ DOZZ formula (Tom's talk)

A tensor model for random CFT

Builds “ETH tensor model”, with potential given by the square of the crossing equation

$$\mathcal{Z} = \int d\mu[H] d\mu[C] \exp\left(-V(H,C)\right)$$

with quartic non-linearity implementing the square of crossing

$$V_{(4)} = \sum \left(C_{ijm} C_{mkl} C^*_{ijn} C^*_{ijn} \delta(\Delta_m - \Delta_n) - C_{ijm} C_{mkl} C^*_{ikn} C^*_{nlj} \left\{ \begin{array}{ccc} i & j & m \\ l & n & k \end{array} \right\} \right)$$

More terms in the potential come from modular crossing to fix $\rho(\Delta)$

A new role for “simplicial-gravity” in 3D? [Boulatov; M. Gross, Ambjørn,…]

Discussion and outlook

Summary

Chaotic nature of BH-Hilbert space: re-interpret matrix-type models as approximate ensembles of generalised Wigner type

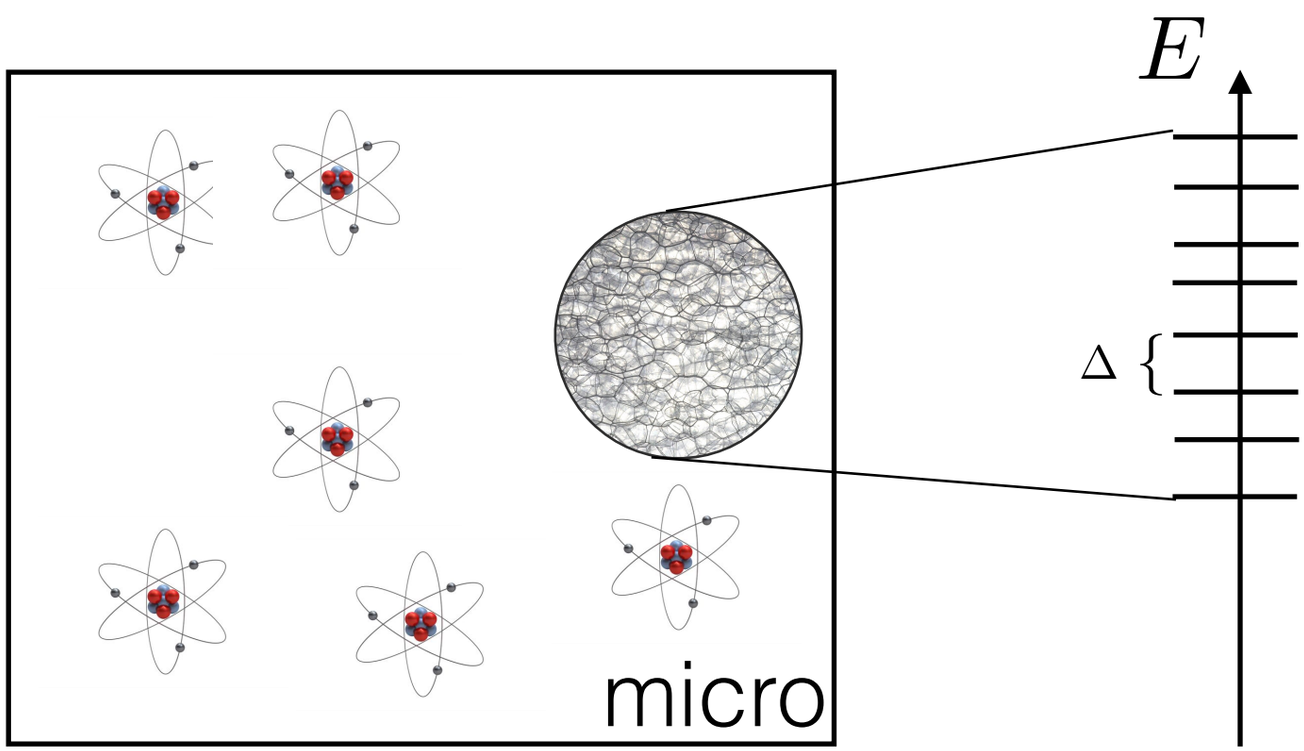
Appropriate structure: “ETH matrix models”. Closely related to recent developments in statistical physics

Apply to gravity through double-scaled ETH-matrix models (did not go into detail in this talk)

Low-energy supergravity as a moment-generating gadget refining/ updating statistical information

$\Longrightarrow$ eventually localise matrix/tensor potentials to land on fixed UV theory?

The microstructure of $\mathscr{H}_{\mathrm{BH}}$



Quantum chaos implies dense quasi-crystalline structure of BH micro states

(Partial) microscopic information fed into matrix model via constraints, giving rise to non-Gaussian RMTs

Outlook

Establish JT+matter matrix model @ all genera through some generalisation of topological recursion to 2-matrix models?

Find non-perturbatively regulated version of JT + matter by going to stable saddle?

Revisit tensor-models of 3D gravity from Wigner perspective?

Statistical models of higher-D CFTs:“The random CFT bootstrap”. Same tensor model applies, but explicit expression harder to come by

Statistical physics applications of ETH matrix models?

Thanks for your attention!