Strongly-interacting Quantum Gases in One-dimensional Geometry

Hanns-Christoph Nägerl

"Frontiers of Ultracold Atoms and Molecules"
October 2010, Santa Barbara

FWF Der Wissenschaftsfonds.

START Project
Y227-N02

EuroQUASAR
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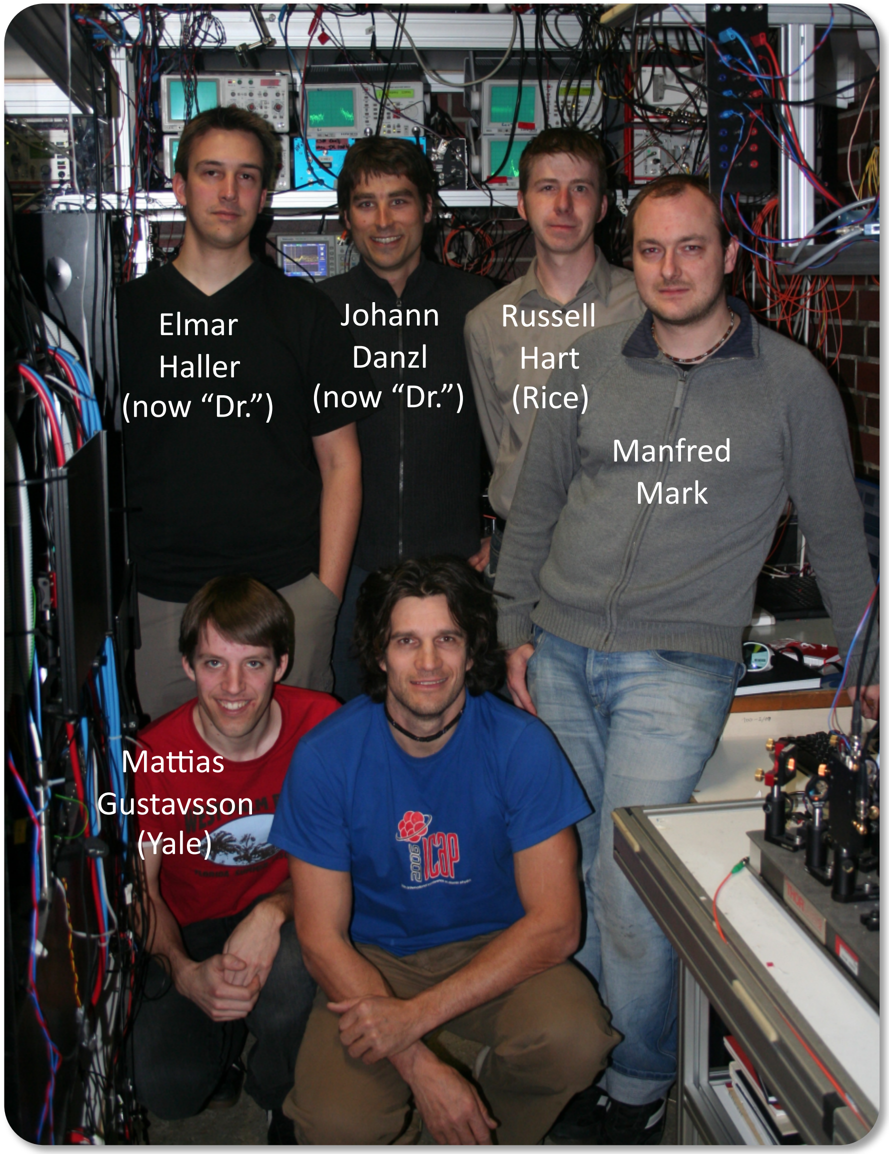
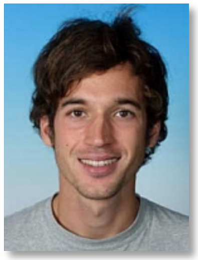
European Science Foundation
Setting Science Agendas for Europe

Team



diploma students:


Lukas Reichsöllner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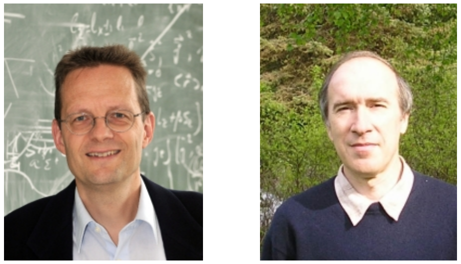
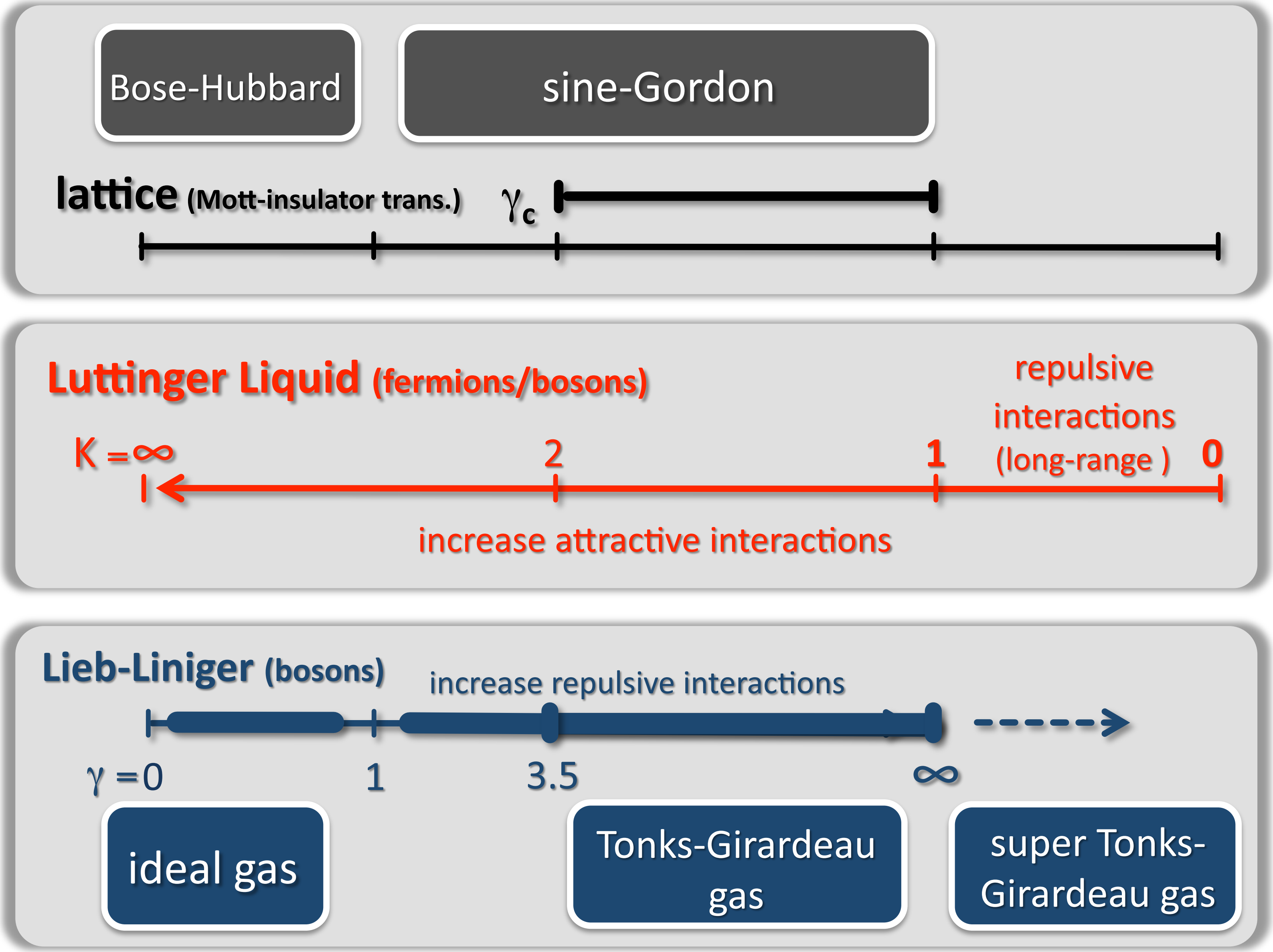
Peter Schmelcher / Vladimir Melezhik



and thanks to:

H.-P. Büchler
A. Daley
H. Ritsch
W. Zwerger

Overview: Interaction regimes of 1D quantum gases



Bose-Hubbard

sine-Gordon

lattice (Mott-insulator trans.)

γ_c

Luttinger Liquid (fermions/bosons)

$K = \infty$

2

1

0

repulsive interactions (long-range)

increase attractive interactions

Lieb-Liniger (bosons)

increase repulsive interactions

$\gamma = 0$

1

3.5

∞

ideal gas

Tonks-Girardeau gas

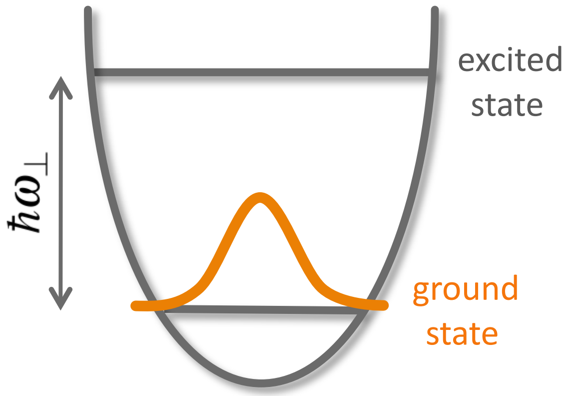
super Tonks-Girardeau gas

Low-dimensional systems

“quasi” low-dimensional systems

- **strong confinement** along “transversal” directions
- the particles are in the **transversal ground state**
- transversal motion is “**frozen out**”

energy of particles $\ll$ energy gap

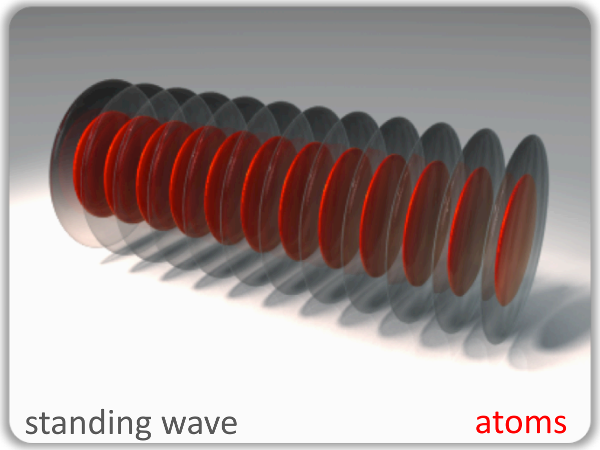


$\omega_\perp$ - transversal trap frequency

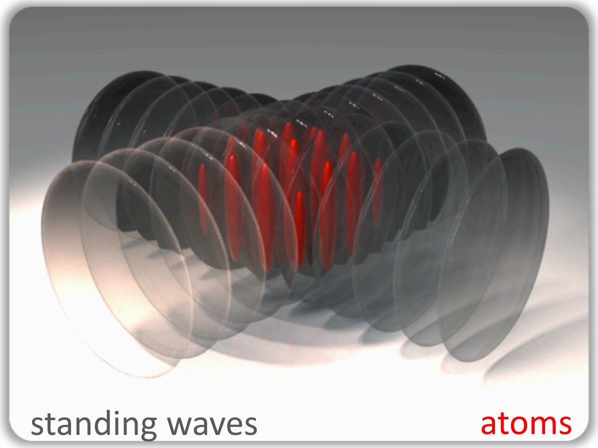
Standard optical lattices

- tight confinement
- parallel investigation of low-dimensional systems
- however: inhomogeneous

quasi-2D systems



quasi-1D systems

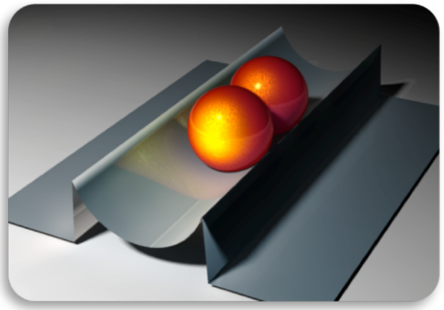


Outline

Strongly-interacting Quantum Gases in One-dimensional Geometry

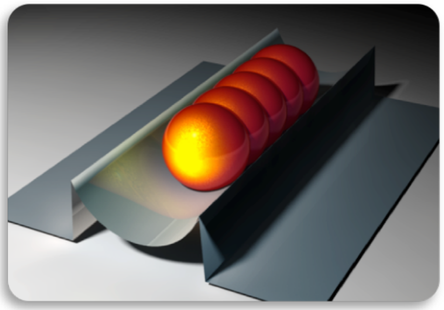
two-body physics

- confinement-induced scattering resonances



many-body physics

- excited 1D quantum phase
  (super Tonks-Girardeau phase)
- 1D quantum phase transition
  (pinning phase transition)
- Outlook: Transport in 1D



Scattering with confinement

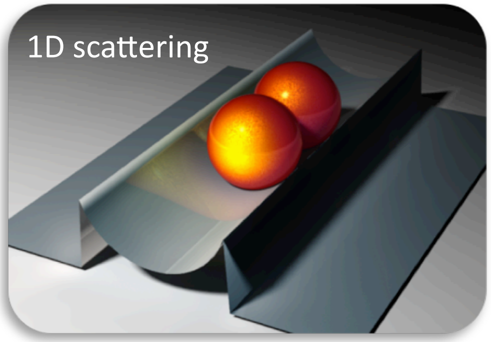
Hamiltonian with transversal confinement

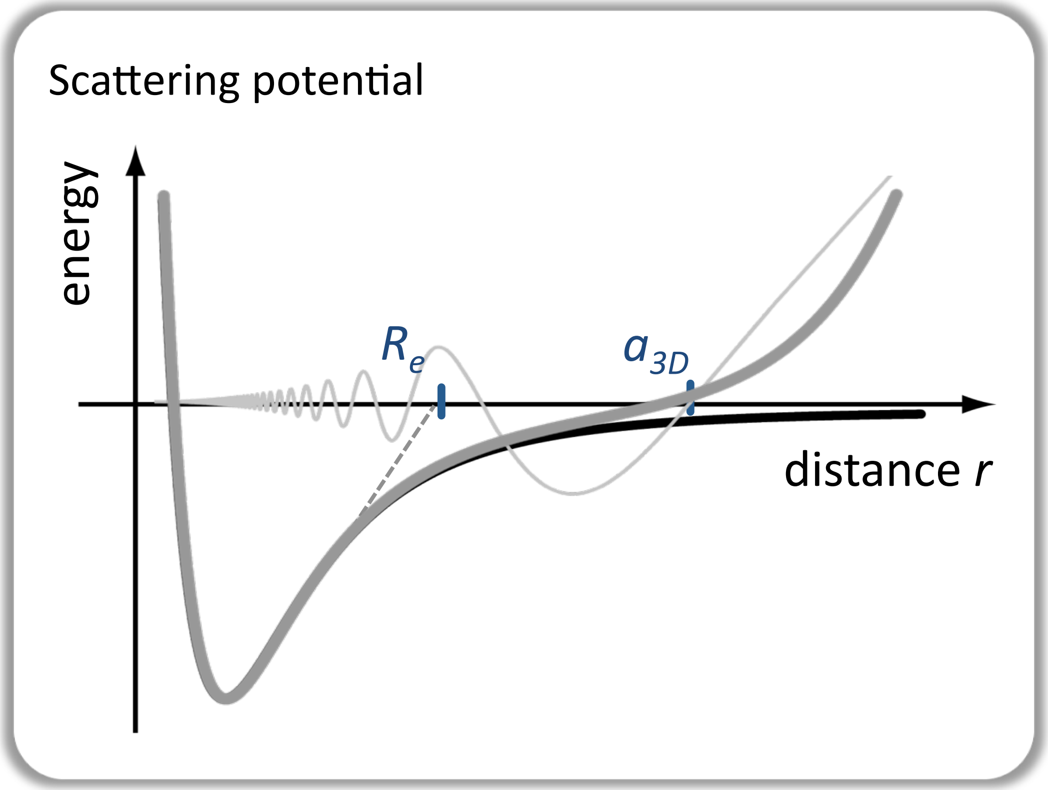
$$H = \frac{p_z^2}{2m} + g_{3D}\,\delta(r_z, r_\perp) + H_\perp(p_\perp, r_\perp)$$

longitudinal

contact interaction

transversal




Scattering potential

Length scales:

- range R_e
- s-wave scattering length a_{3D}
- confinement length $a_\perp$

$$a_\perp = \sqrt{\frac{\hbar}{m\omega_\perp}}$$

regime: $R_e \ll a_{3D} \sim a_\perp$

1D coupling constant

Hamiltonian with transversal confinement

$$H = \frac{p_z^2}{2m} + \boxed{g_{3D}\,\delta(r_z, r_\perp)} + H_\perp(p_\perp, r_\perp)$$

longitudinal

contact interaction

transversal

⇨

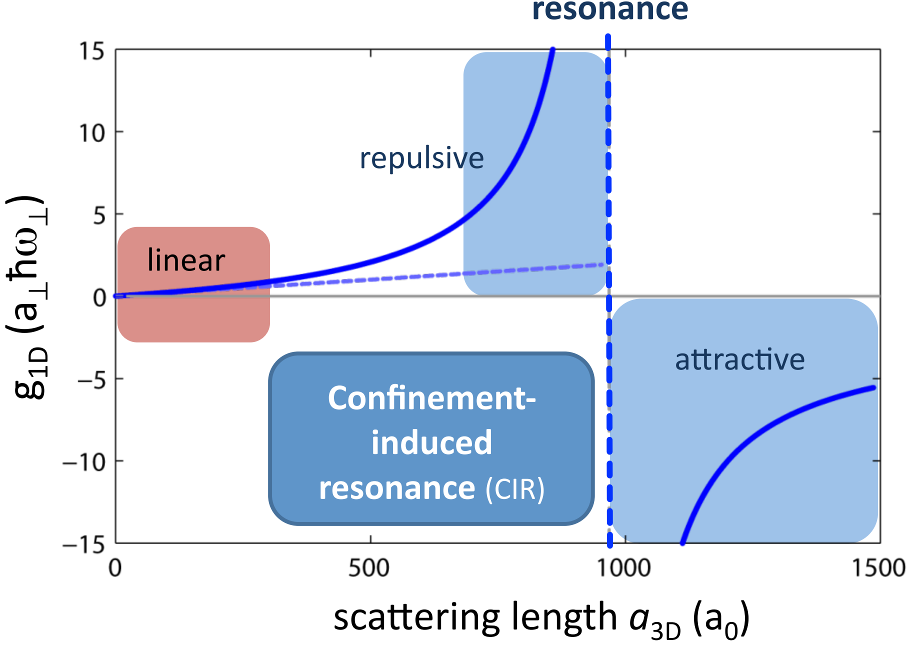
Construct a **1D coupling constant**?

$$H = \frac{p_z^2}{2m} + g_{1D}\,\delta(r_z)$$

longitudinal

transversal

1D coupling constant



M. Olshanii,
Phys. Rev. Lett. **81**, 938 (1998)

$$g_{1D} = 2\hbar\omega_\perp a_{3D}\left(1 - \overset{\nearrow}{C}\,\frac{a_{3D}}{a_\perp}\right)^{-1}$$

constant close to 1

linear regime:

g_{1D} is proportional to a_{3D}

$$g_{1D} = 2\hbar\omega_\perp a_{3D}$$

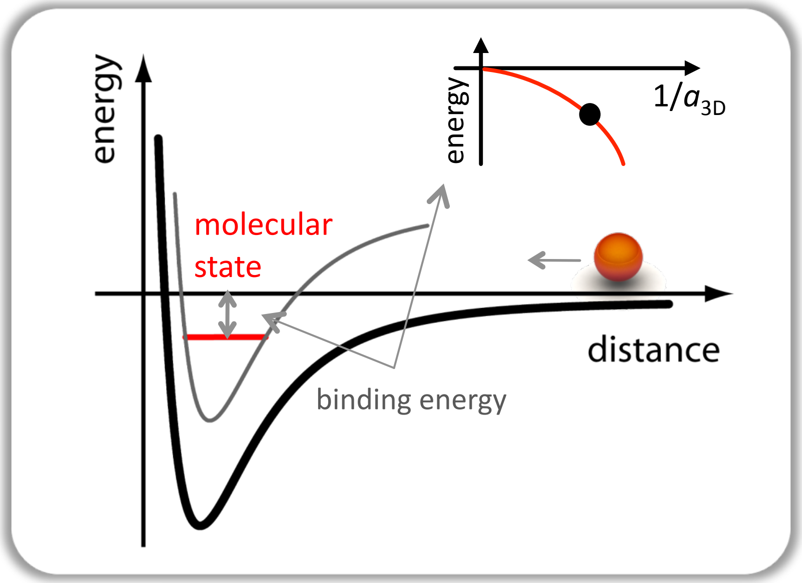
resonance:

g_{1D} divergences for $\frac{a_{3D}}{a_\perp} \approx 1$

Scattering resonances

Magnetic Feshbach resonance (3D)

- scattering particles couple to a molecular state
- FBR: energy of molecular state matches energy of scattering particles



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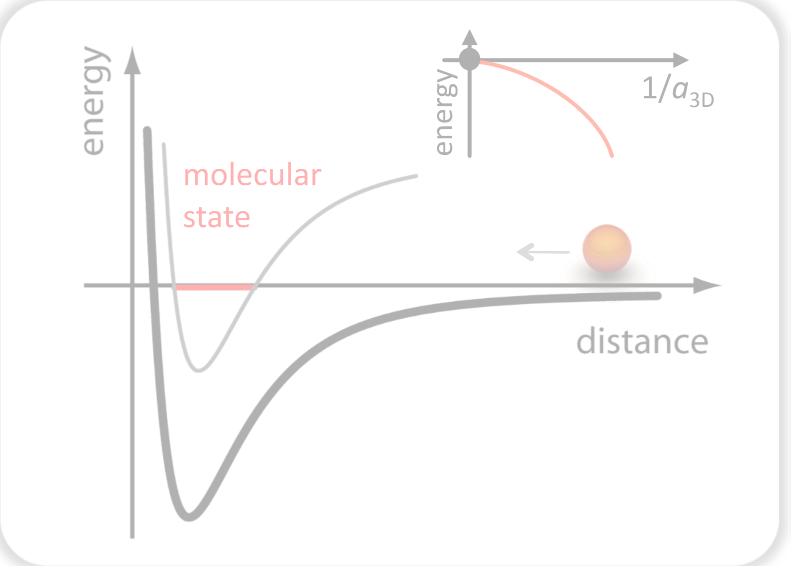
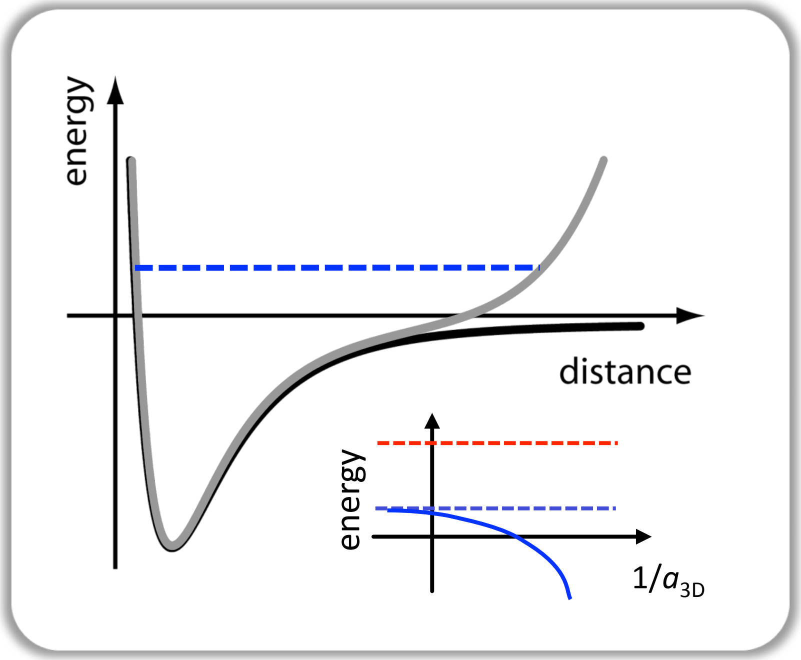
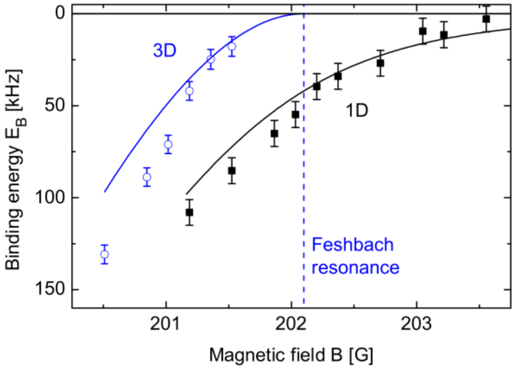




Changes due to the confinement

- shift of zero energy
- change of binding energy

(group T. Esslinger, PRL 94, 210401)



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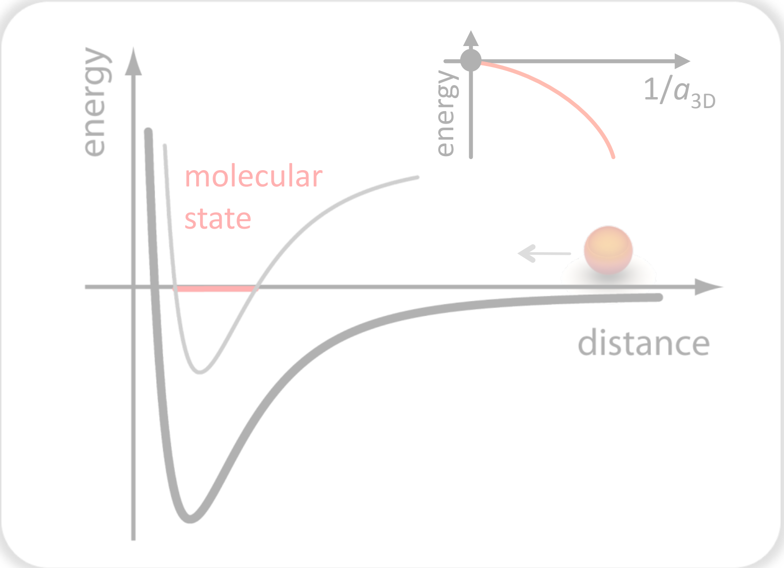
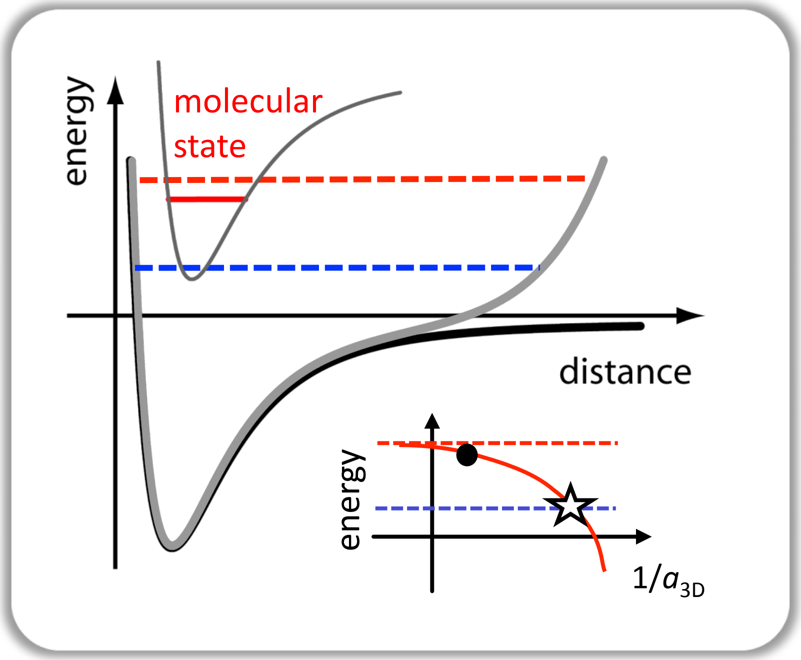




Changes due to the confinement

- shift of zero energy
- change of binding energy
- additional excited states
- scattering particles couple to **molecular state in transverally excited level**

CIR condition:

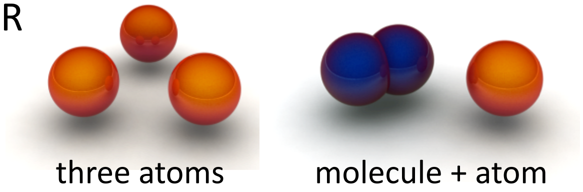
energy of **excited molecular state** matches the **zero energy**

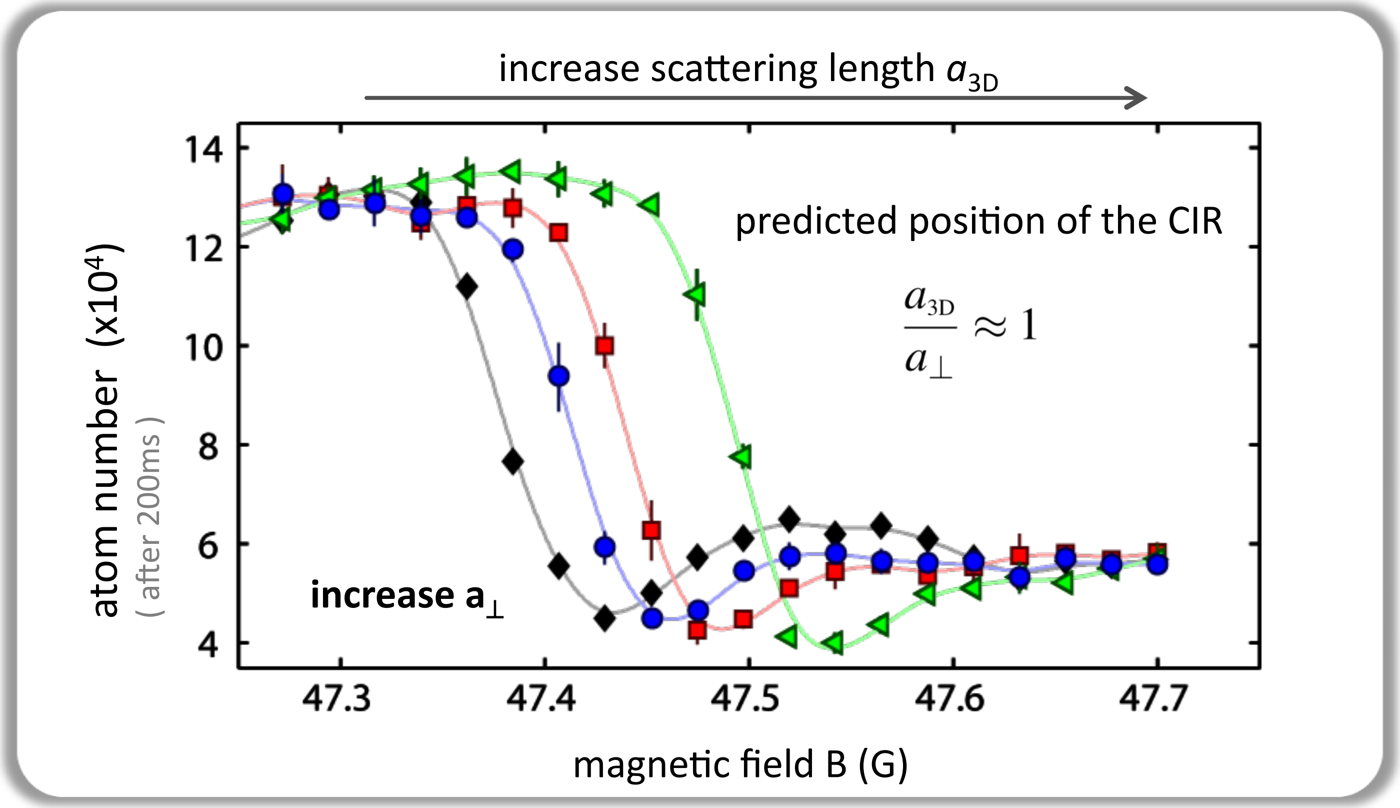
Confinement-induced resonance (CIR)

Detection of a CIR by means of atom loss

- tune the interactions strength (a_{3D}) with a magnetic Feshbach resonance

- observe three-body losses close to the CIR





E. Haller *et al.*,
Phys. Rev. Lett. **104**, 200403 (2010)

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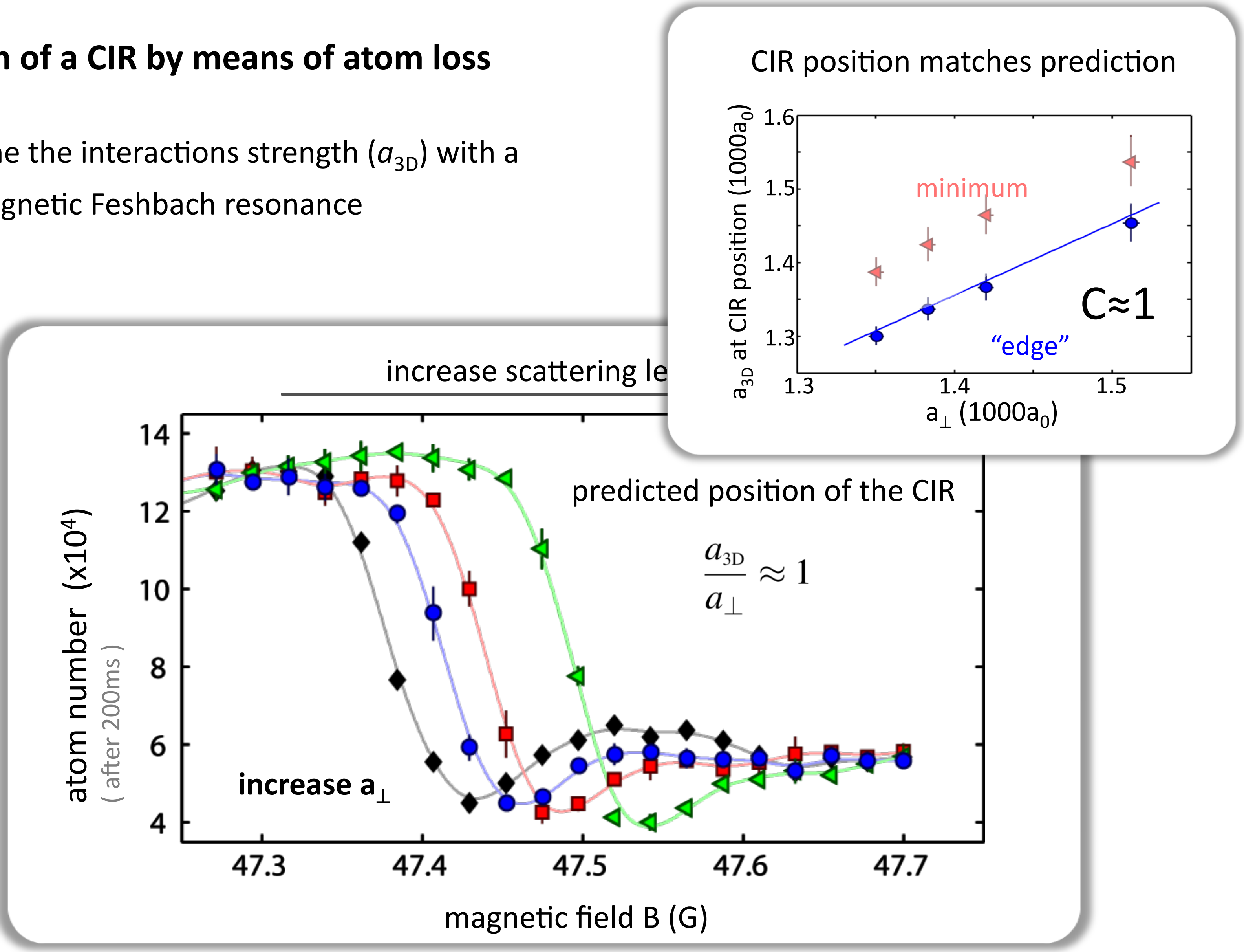


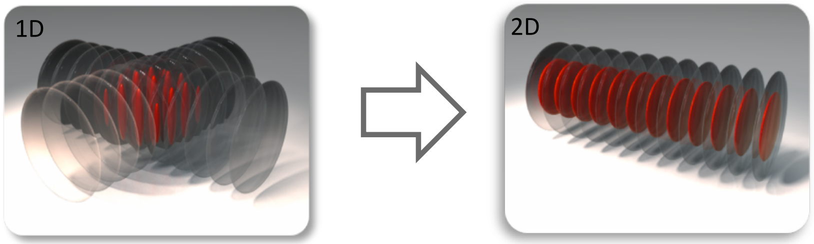
CIR position matches prediction

predicted position of the CIR

$$\frac{a_{3D}}{a_\perp} \approx 1$$

E. Haller *et al.*,
Phys. Rev. Lett. **104**, 200403 (2010)

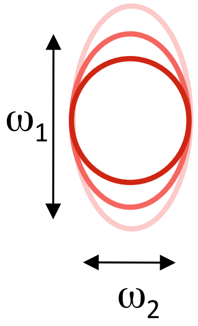
1D to 2D system



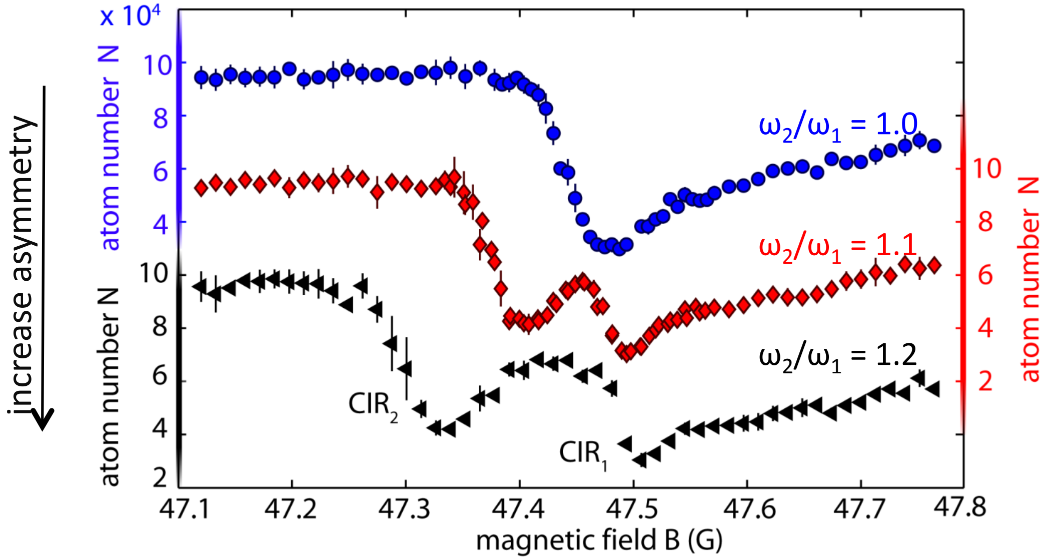
Are there confinement-induced resonances in 2D systems?

change the power in one beam

single tube



Double resonance for small asymmetry



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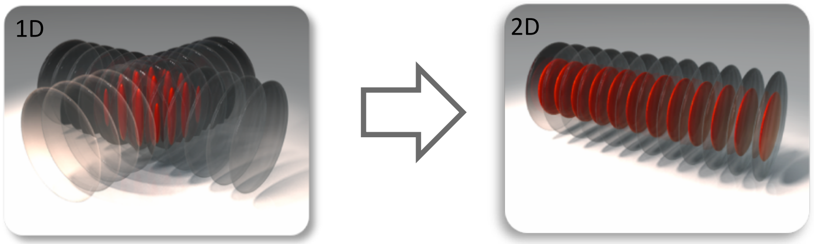

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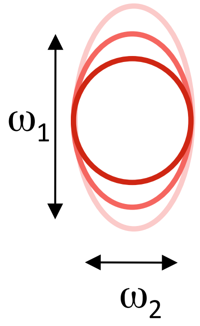
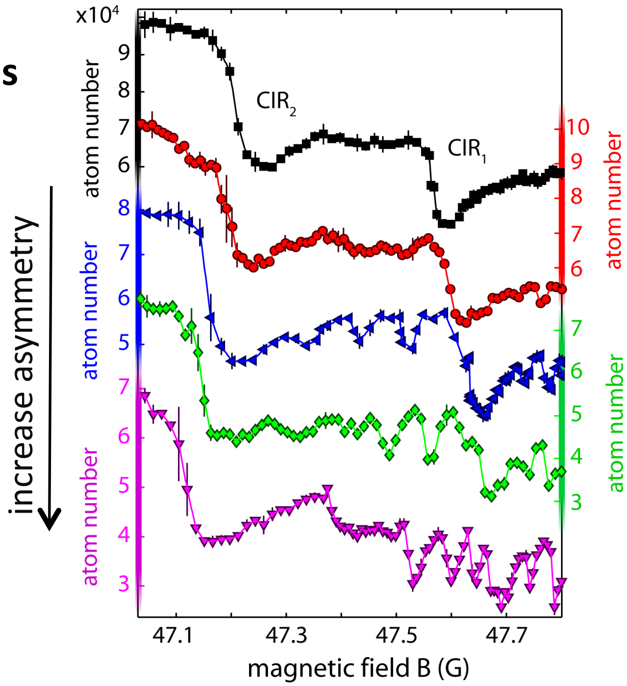


Spectrum of resonances

for large asymmetry



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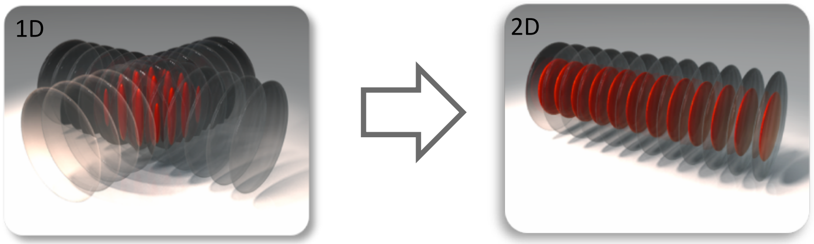

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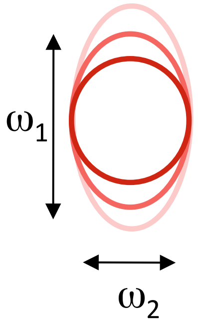
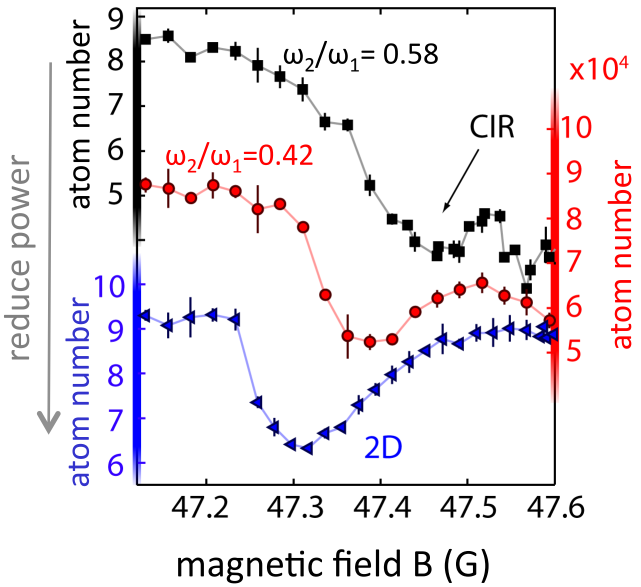


One resonance persists in the 2D system

observed
for $a_{3D} > 0$

predicted
for $a_{3D} < 0$

open question



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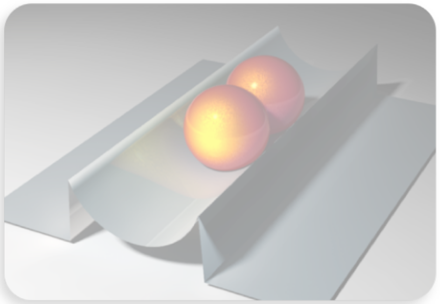
Strongly-interacting Quantum Gases in One-dimensional Geometry

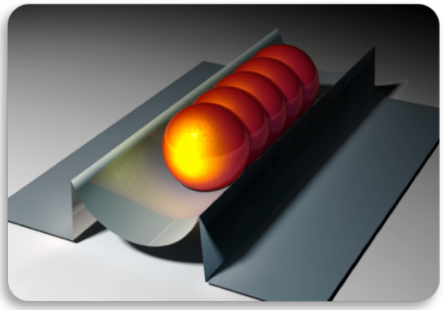
two-body physics

- confinement-induced scattering resonances



many-body physics

- super Tonks-Girardeau phase
  - Bose-Fermi mapping
  - Tonks-Girardeau gas
  - super Tonks-Girardeau phase



- 1D quantum phase transition

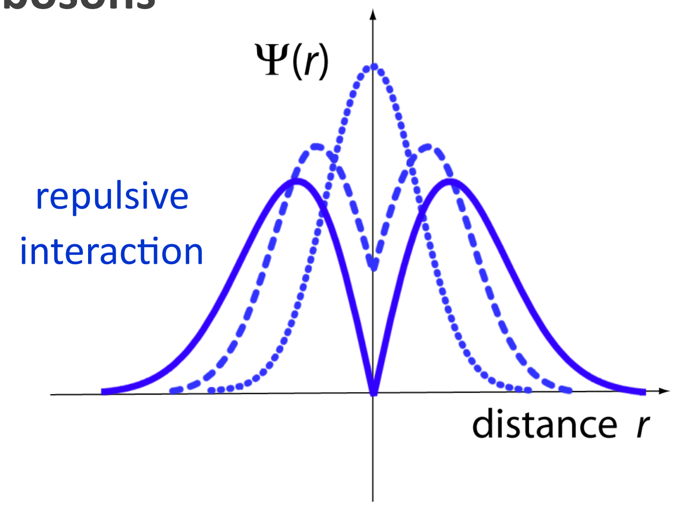
  (pinning phase transition)

Bose-Fermi mapping

Bose-Fermi mapping:

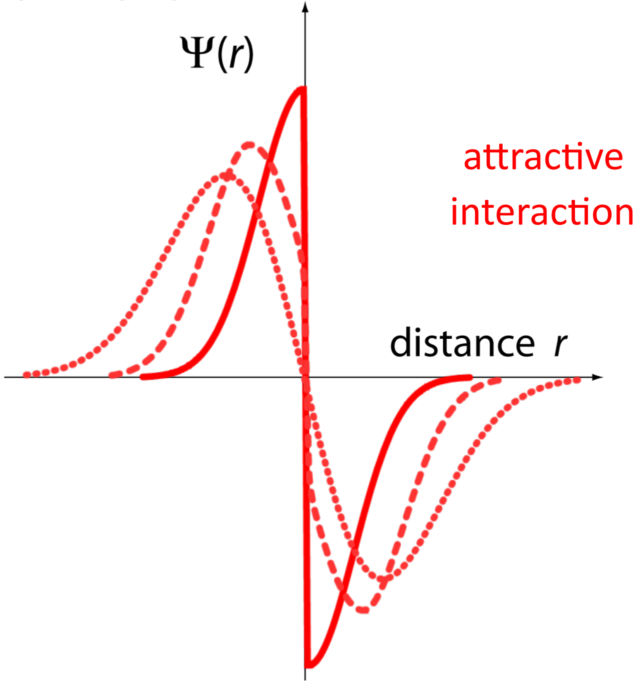
bosons and fermions in 1D show similar density distributions

bosons



sketch: wave functions for two particles
in harmonic trap

fermions



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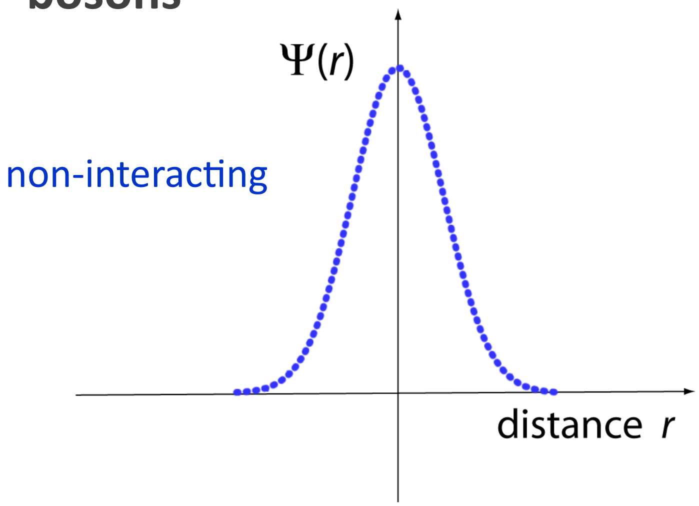

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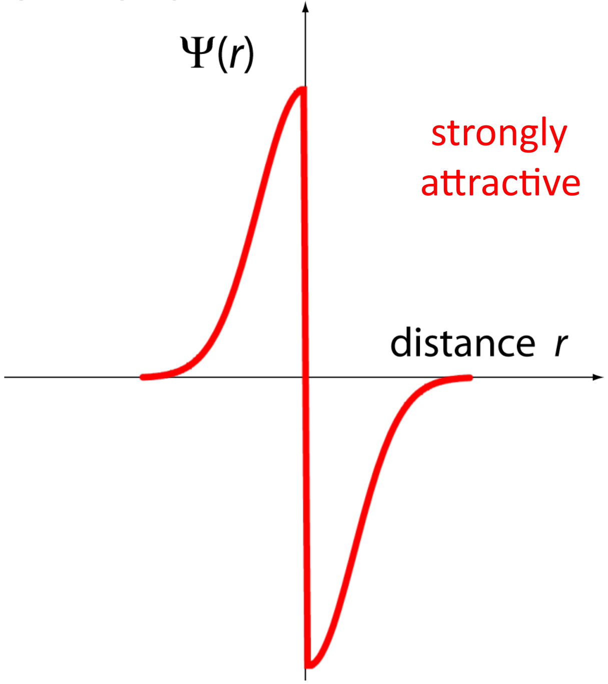

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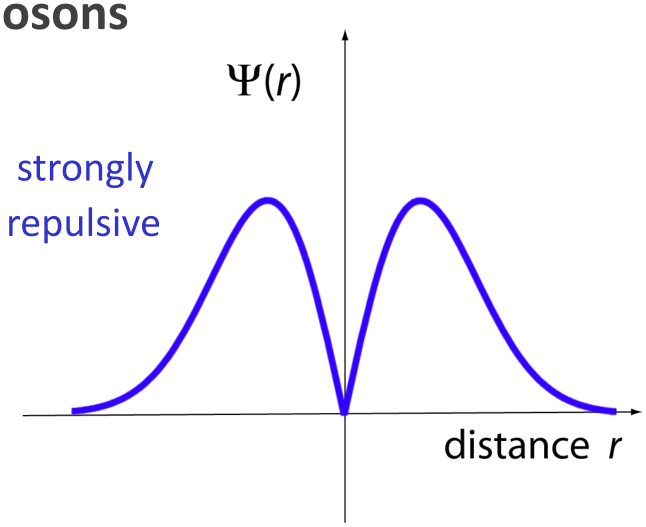

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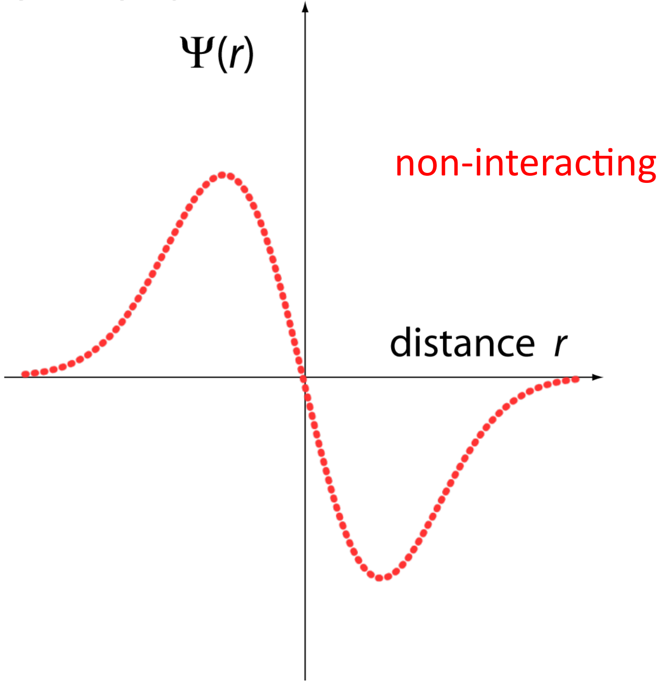


Lieb - Liniger model

Model: E. Lieb and W. Liniger,
Phys. Rev. **130**, 1605 (1963)

- bosons in uniform 1D system
- repulsive contact potential

Hamilton operator:

$$H = -\sum_i \frac{\partial^2}{\partial x_i^2} + c\,\gamma \sum_{<i,j>} \delta(x_i - x_j)$$

kinetic energy

interaction energy

c - constant
γ - interaction strength

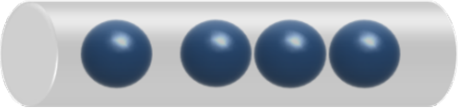
$$\gamma = \frac{m\, g_{1\mathrm{D}}}{\hbar^2 n}$$

Ideal gas $\gamma = 0$

(non-interacting bosons)

Tonks-Girardeau gas (TG)

(non-interacting fermions)
(hard spheres) $\gamma \to \infty$



γ − parameter

Tonks-Girardeau gas

Experimental realizations to reach γ > 1

I) Approach (increase confinement strength)

- γ depends on the **density** and **confinement strength**
- **reached γ ~ 5 to 10**

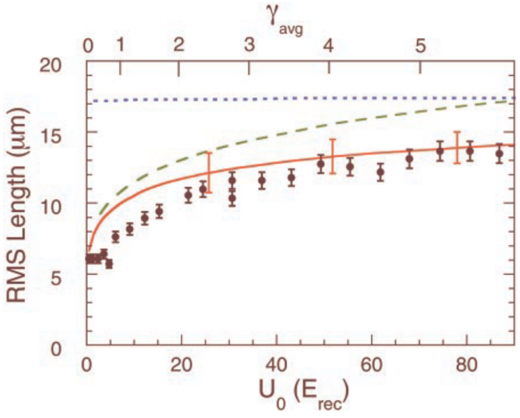
2004); 305 Scie

a Ki

$$\gamma = \frac{m g_{1D}}{\hbar^2 n} \sim \frac{\omega_\perp}{n}$$

O

T. Kinoshita *et al.*, Science **305,** 1125 (2004)



II) Our approach (use CIR)

- tune interactions with a confinement-induced resonance
- **reached γ ~ 500**

$$\gamma = \frac{m g_{1D}}{\hbar^2 n}$$

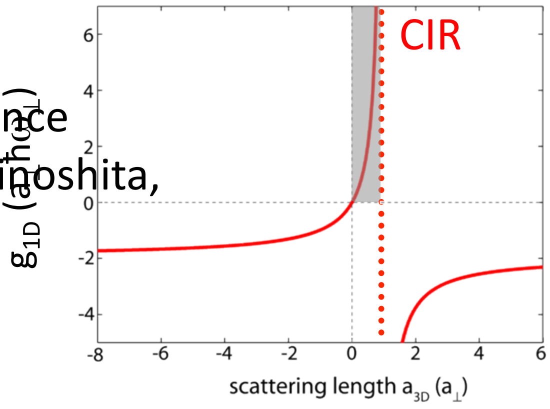
2... 305 Science

et al. Toshiya Kinoshita,

Gas

O

E. Haller *et al.*, Science **325** 1224 (2009)

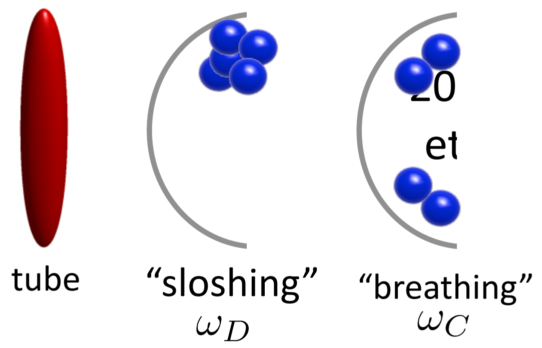


other approaches: B. Paredes *et al.*, Nature **429**, 277 (2004).
N. Syassen *et al.*, Science **320**, 1329 (2008).

Detection method

Collective oscillations

the oscillation frequency depends on the interaction regime.



interaction regimes	$(\omega_C/\omega_D)^2$
5 Science	
iiya Kinoshita,	
s	

C. Menotti, S. Stringari, PRA **66,** 043610 (2002)

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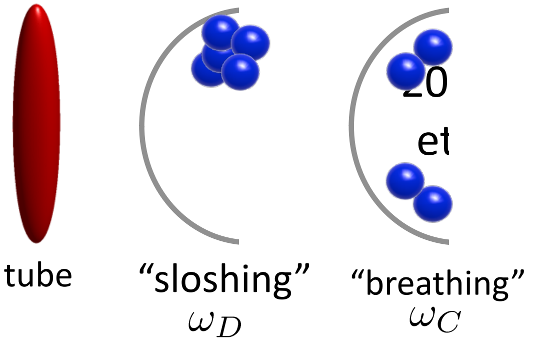
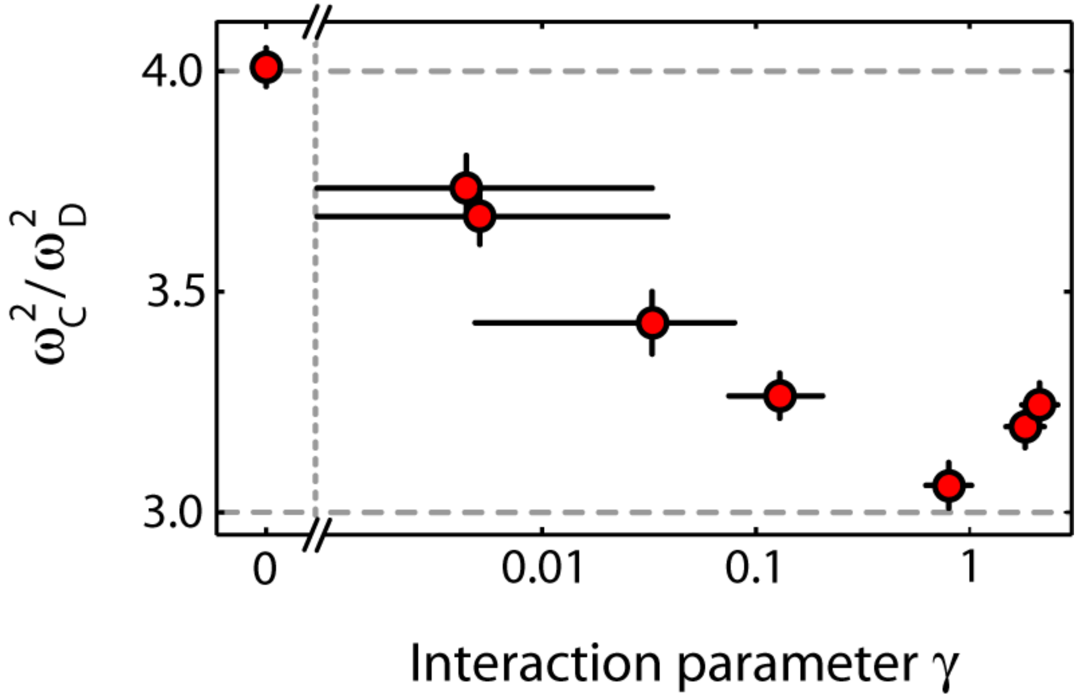


interaction regimes	$(\omega_C/\omega_D)^2$
1D mean field	3
Non-interacting	4

C. Menotti, S. Stringari, PRA **66,** 043610 (2002)



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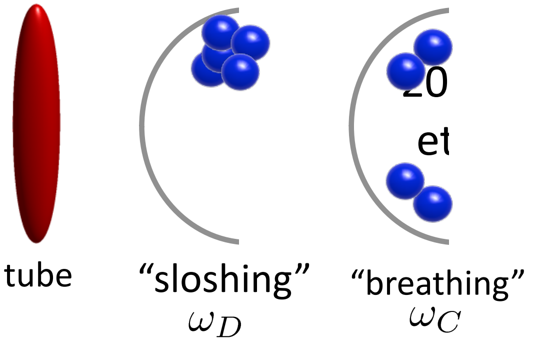

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5 Science 1D mean field ıiya Kinoshita,	3
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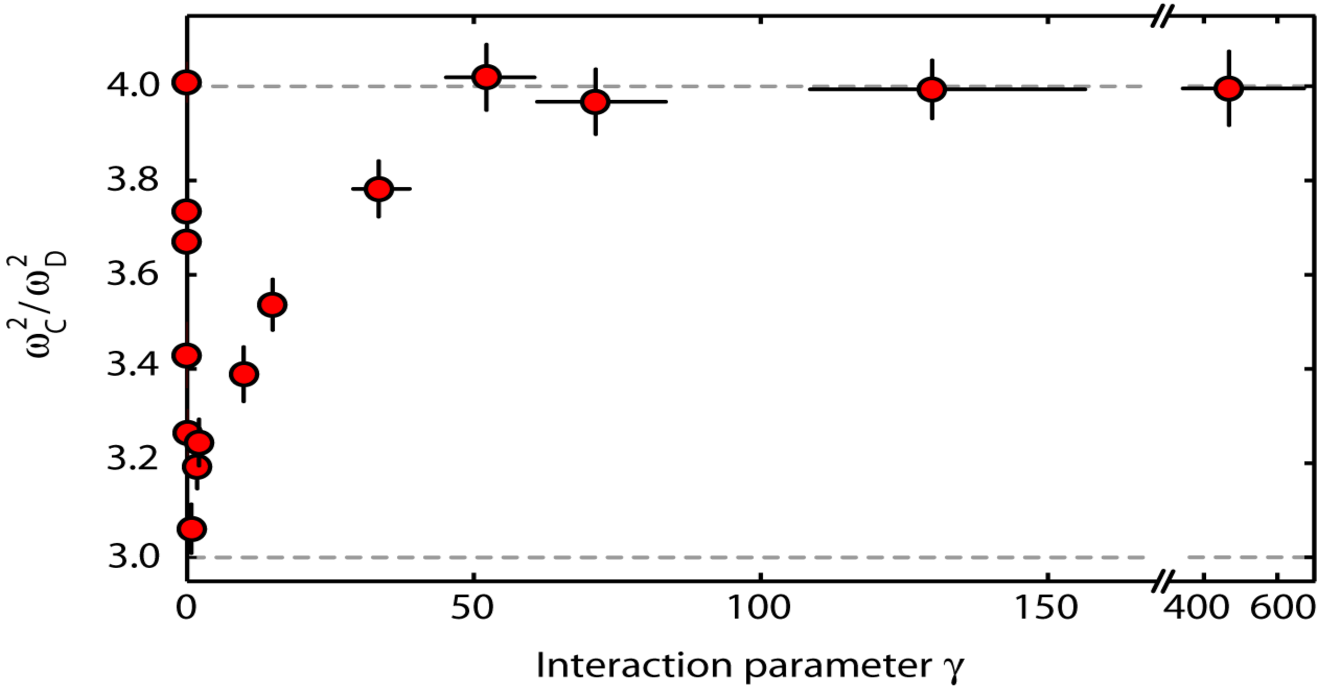

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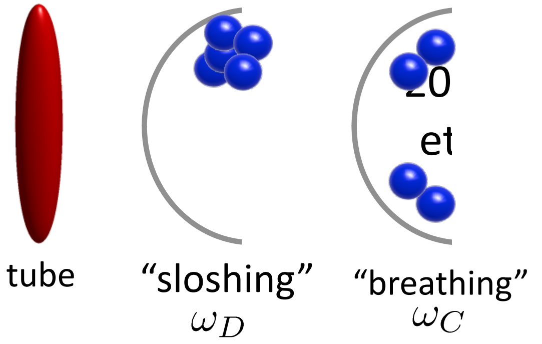

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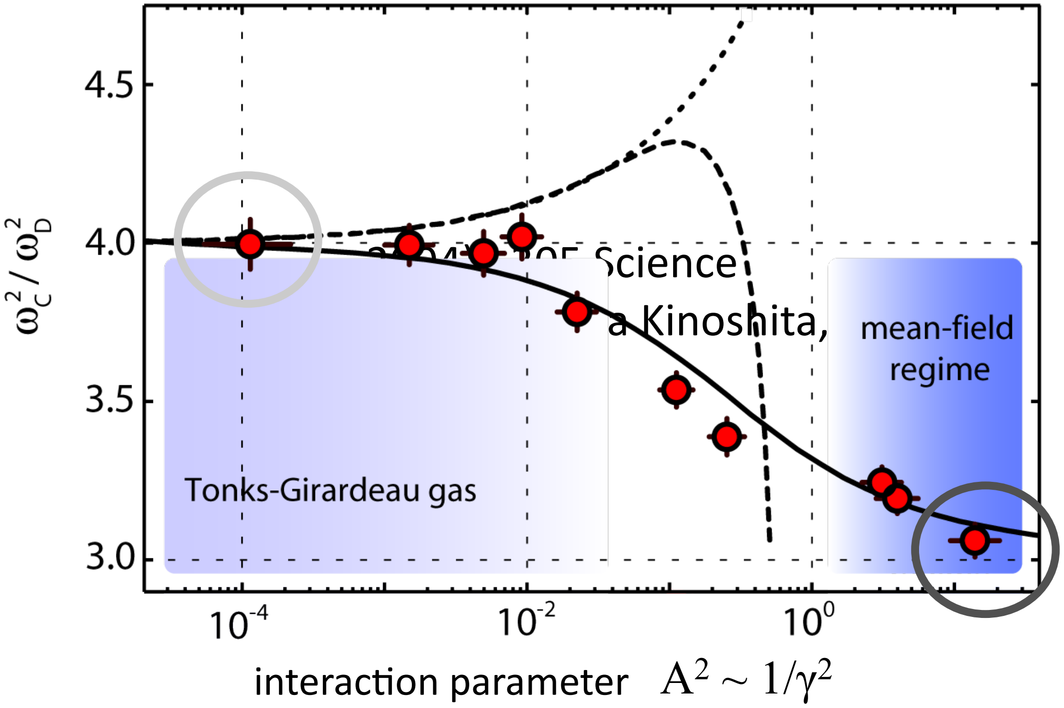


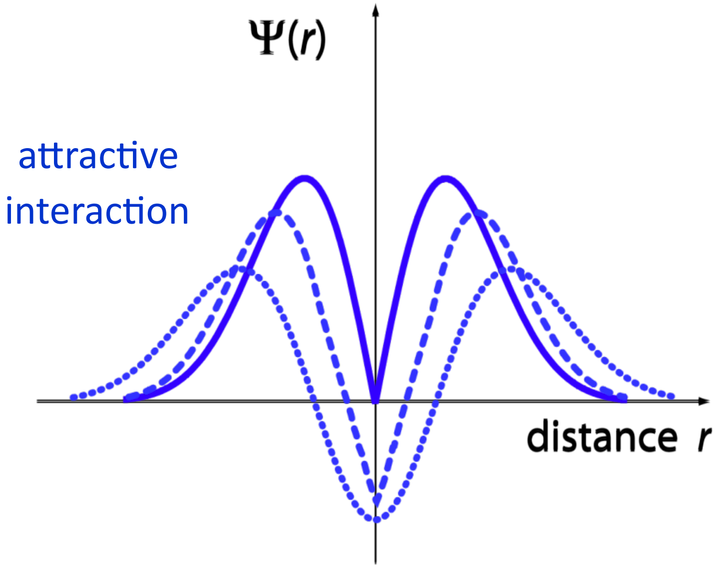
Extension of the Bose-Fermi mapping

Extended Bose-Fermi mapping:

Astrakharchik *et al.*,
PRL **95** 190407 (2005)

Excited Bosons with attractive interactions and ground state Fermions with repulsive interactions show the same density distribution.

bosons, excited



sketch: wave functions for two particles
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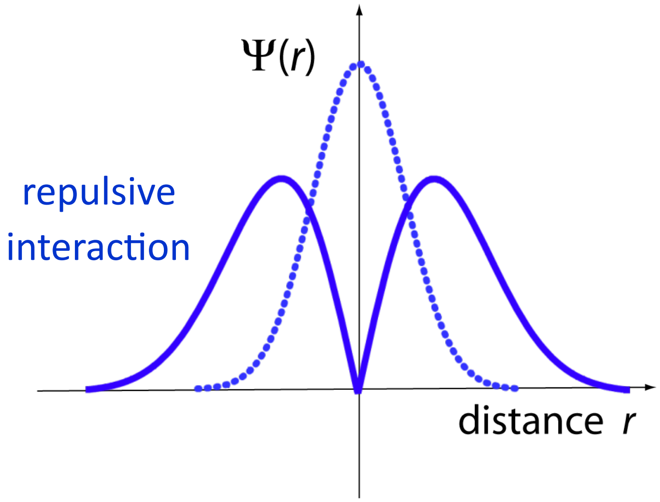
bosons, ground state

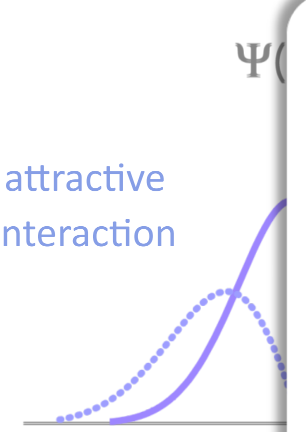


Extension of the Bose-Fermi mapping

Matching wave functions

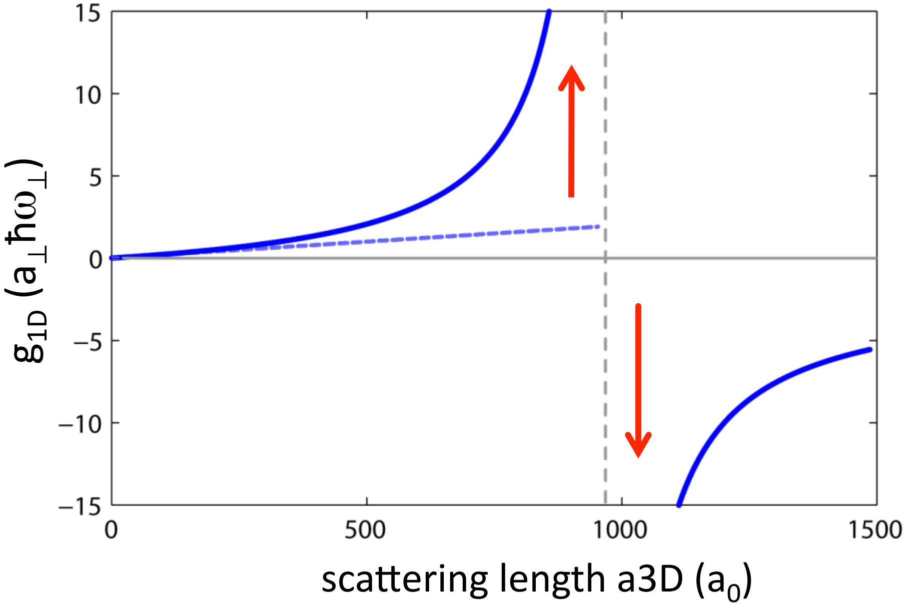
on both sides of the confinement-induced resonance

Bosons, excited



sketch: wave func
in harmon

Confinement-induced resonance



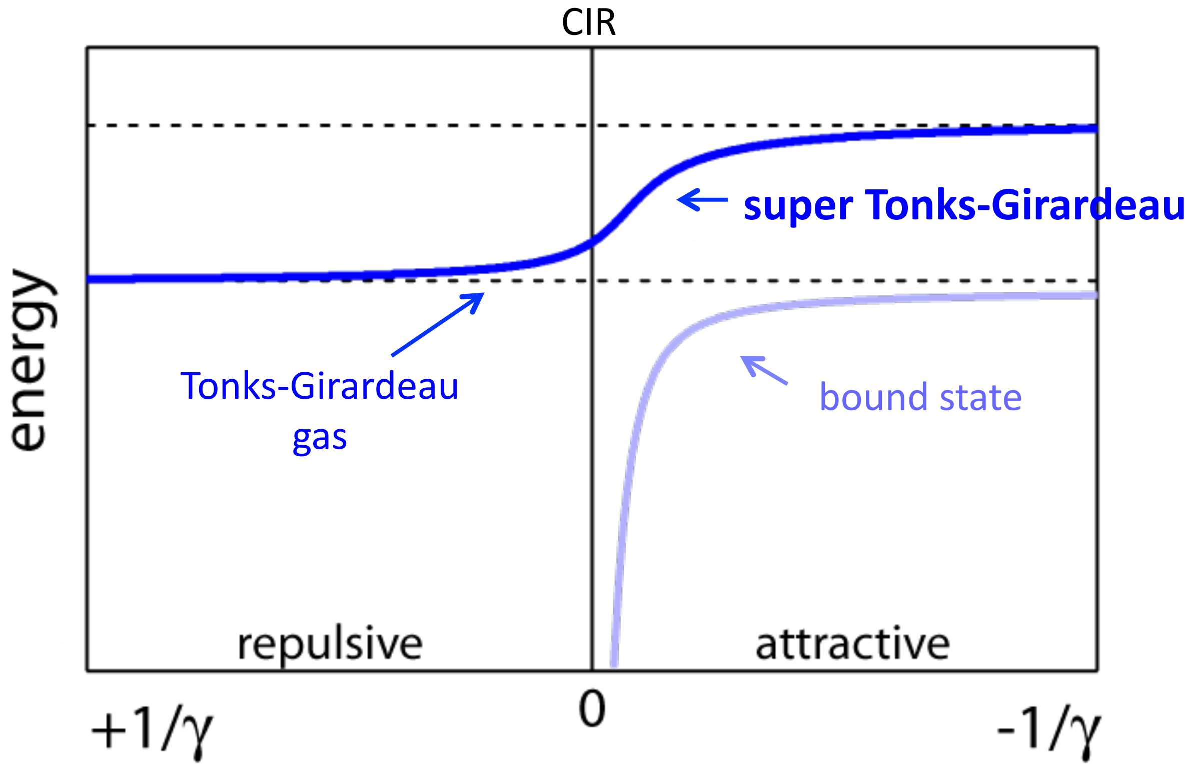
Bosons, ground state

Super Tonks-Girardeau gas

Matching wave functions

on both sides of the confinement-induced resonance

Energy levels at the confinement-induced resonance

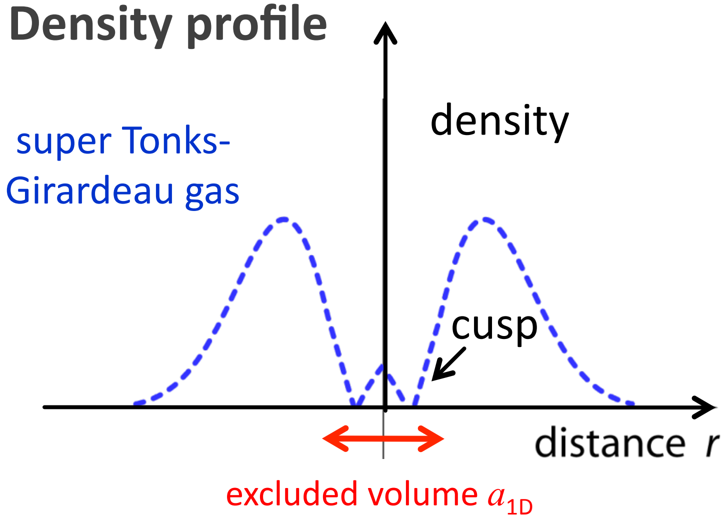


Super Tonks-Girardeau gas

$$a_{1D} = -\frac{2\hbar^2}{mg_{1D}}$$

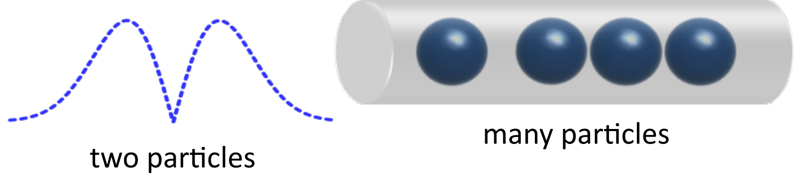
Properties of the super Tonks-Girardeau gas

Density profile

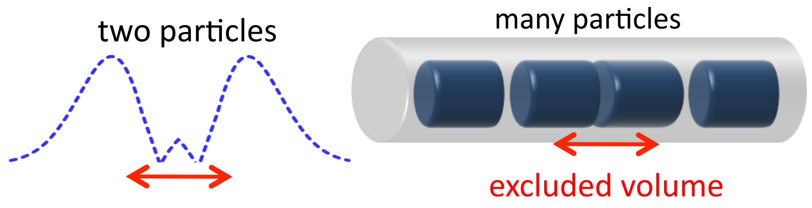


sketch: density for two particles in a harmonic trap

Tonks-Girardeau gas **(hard spheres)**



super Tonks-Girardeau gas (**hard rods**)



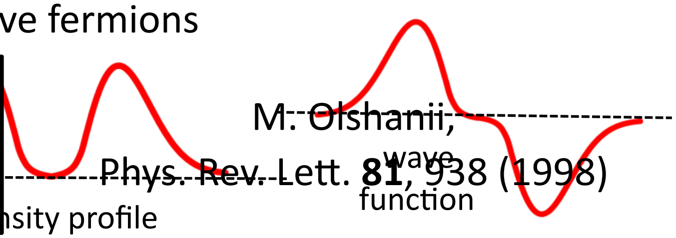
excited bosons,
+ attractive interactions

⇔

ferr
+ re
inte

$$a_{1D} = -\frac{2\hbar^2}{mg_{1D}}$$

repulsive fermions

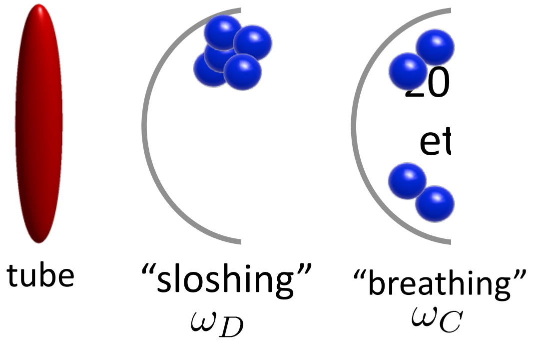


M. Olshanii, Phys. Rev. Lett. **81**, 938 (1998)

Detection method

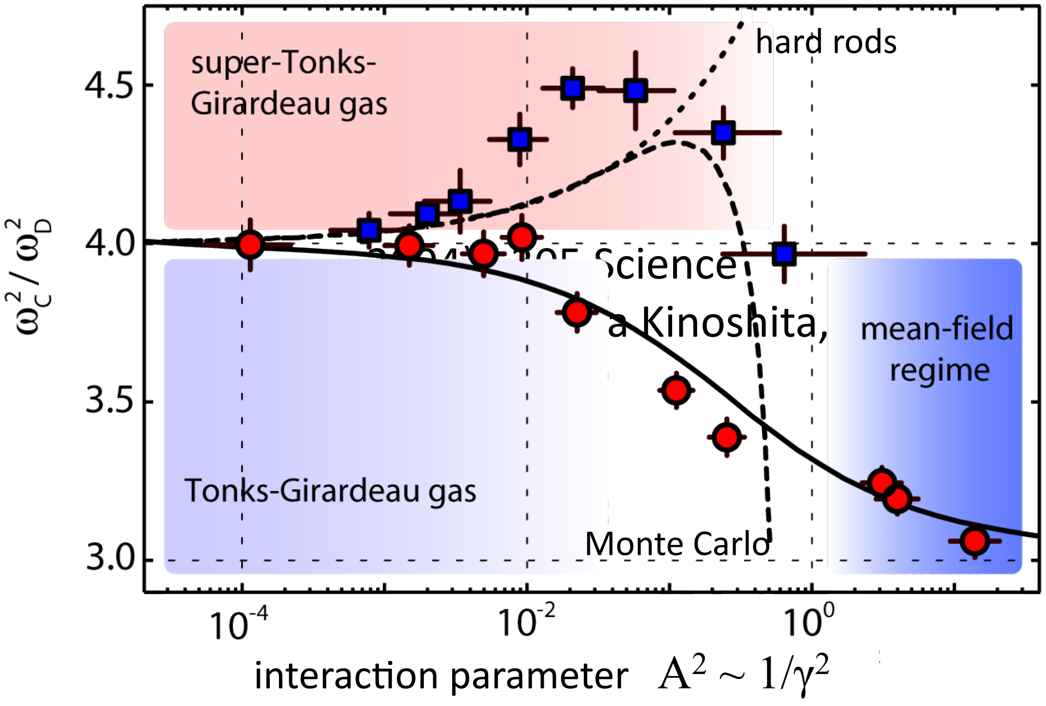
Collective oscillations

the oscillation frequency depends on the interaction regime.



interaction regimes	$(\omega_C/\omega_D)^2$
1D mean field	3
Tonks-Girardeau gas	4
Super-Tonks Girardeau	> 4

C. Menotti, S. Stringari, PRA **66** 043610 (2002)

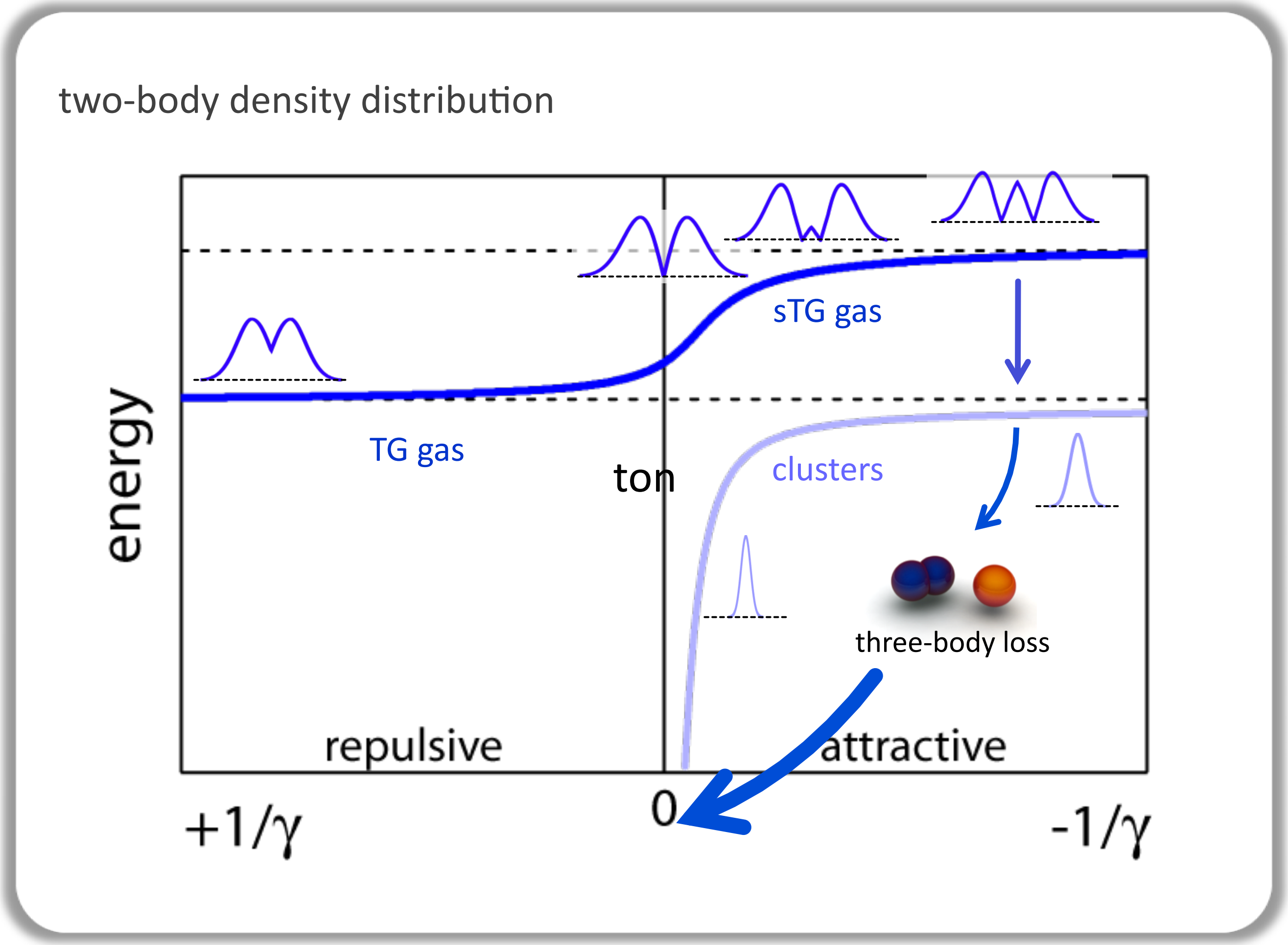


E. Haller *et al.*, Science **325** 1224 (2009)

Super Tonks-Girardeau gas

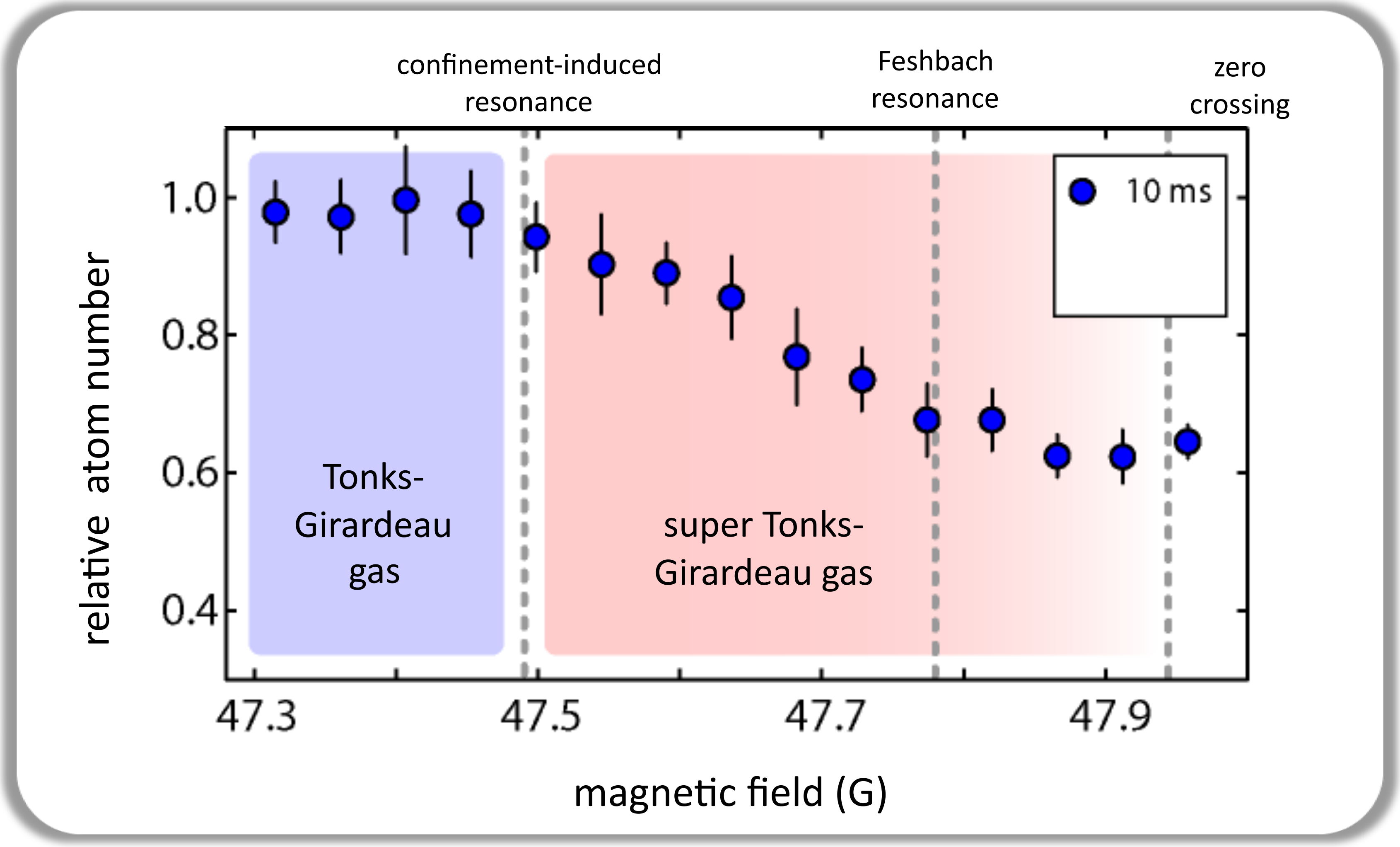
Stability of the super Tonks-Girardeau gas

Strong attractive interactions stabilize the state



Super Tonks-Girardeau gas

Estimated lifetime of the sTG state   10 < τ < 50 ms



Overview

Interaction regimes of 1D quantum gases

attractive fermions

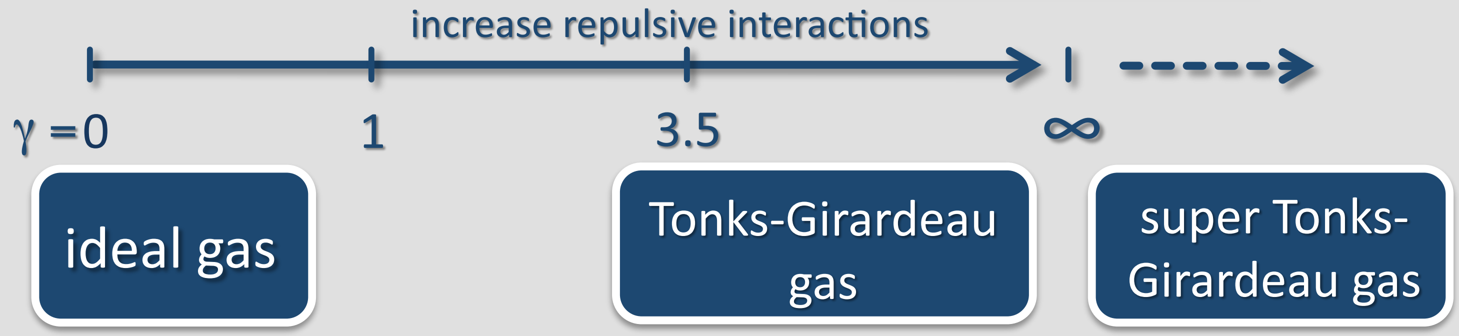
non-interacting

n

bosons

repulsive bosons

Lieb-Liniger (bosons)

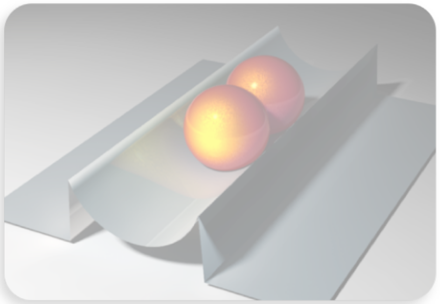


Outline

Strongly-interacting Quantum Gases in One-dimensional Geometry

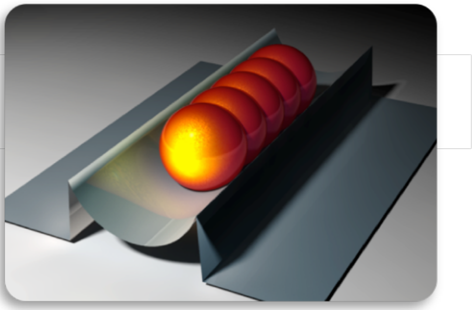
two-body physics

- confinement-induced scattering resonances



many-body physics

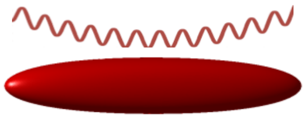
- super Tonks-Girardeau phase
- 1D quantum phase transition
  - pinning transition
  - amplitude modulation spectroscopy
  - transport properties



Sine-Gordon model

Sine-Gordon model

- add a periodic perturbation to a Luttinger liquid



for commensurate density

$n \approx 2/\lambda$ (lattice spacing)

$$H = \frac{\hbar v}{2\pi}\int dx \left[K\left(\frac{\partial}{\partial x}\phi(x)\right)^2 + \frac{1}{K}\left(\frac{\partial}{\partial x}\theta(x)\right)^2\right] + \frac{V n}{2}\int dx \cos[2\theta(x)]$$

V - perturbation strength

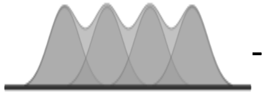
Ground states of the **Sine-Gordon Hamiltonian**

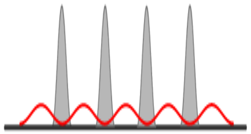
(depending on ***K*** and the perturbation strength ***V***)

phase transition

⟷

- **Superfluid**
  - delocalized atoms
  - phase coherent sites
  - continuous excitation spectrum





- **Mott-insulator**
  - localized atoms
  - incoherent phase
  - gaped excitation spectrum

Mott-insulator transitions

Mott – insulator phase transition

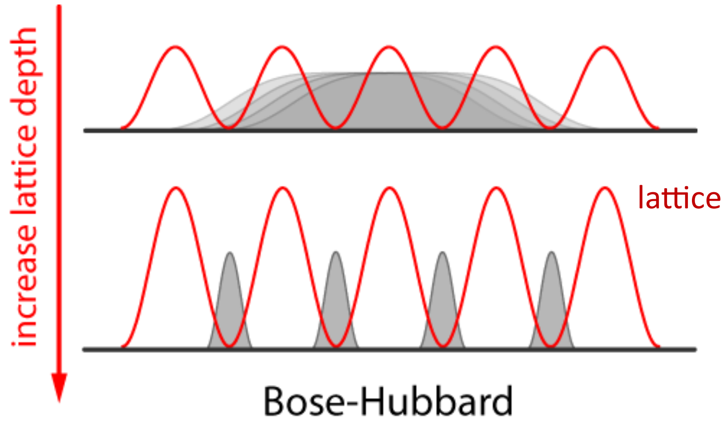
“metal - insulator transition”

Mott-Hubbard transition

- **deep lattice**, tight-binding approximation
- connects ground states of the

Bose-Hubbard model

superfluid ⟷ Mott insulator

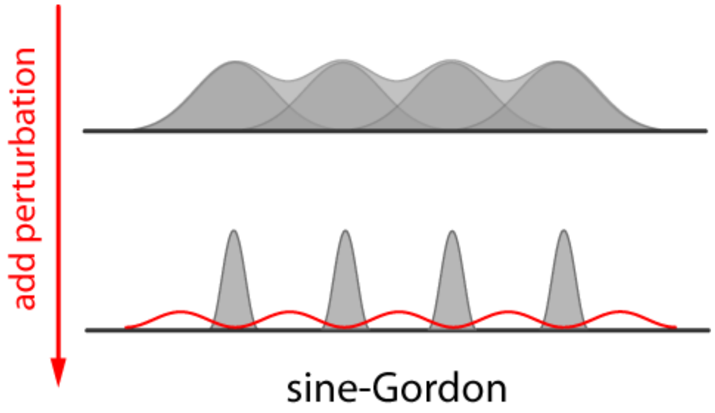


Pinning transition

- add **shallow lattice** (perturbation)
- connects ground states of the

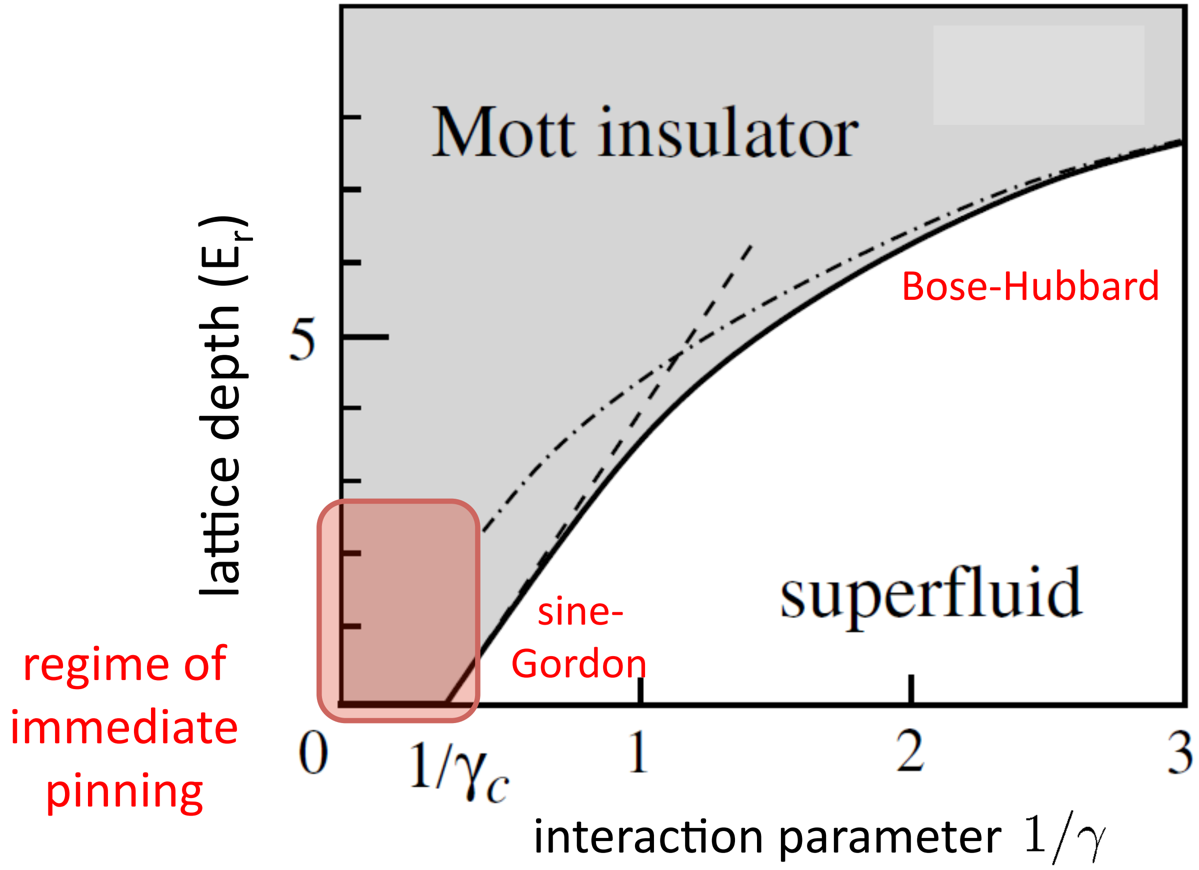
sine-Gordon model

Tonks gas ⟷ Mott insulator



Complete phase diagram

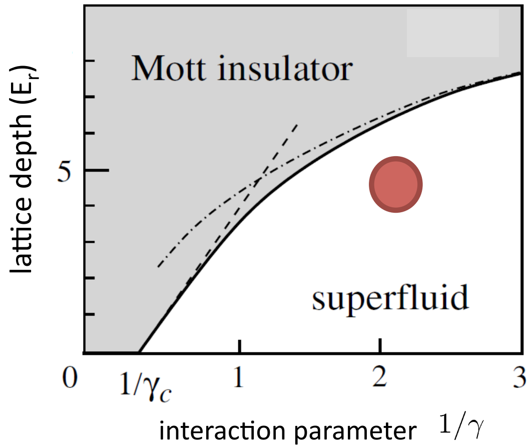
phase diagram **Mott-Hubbard transition** and **pinning transition**



H.P. Büchler, G. Blatter,
W. Zwerger, PRL **90,** 130401 (2003)

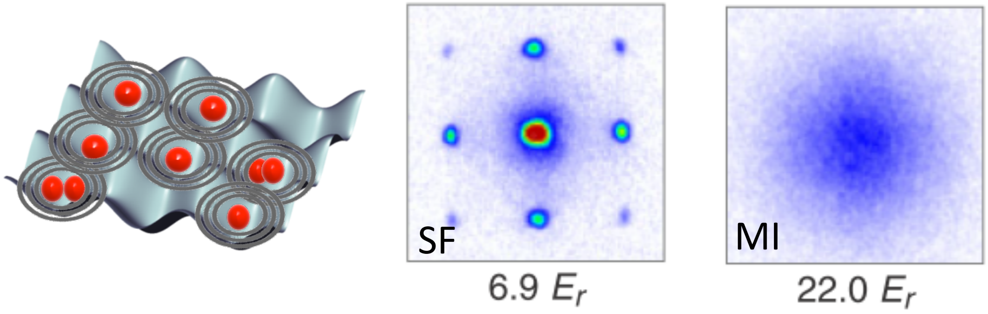
Experimental probe

Probe a property, which is present in only one phase



superfluid phase: **phase coherence**

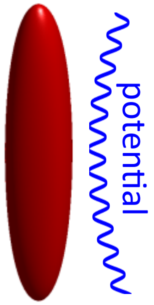
momentum profile, 3D lattice

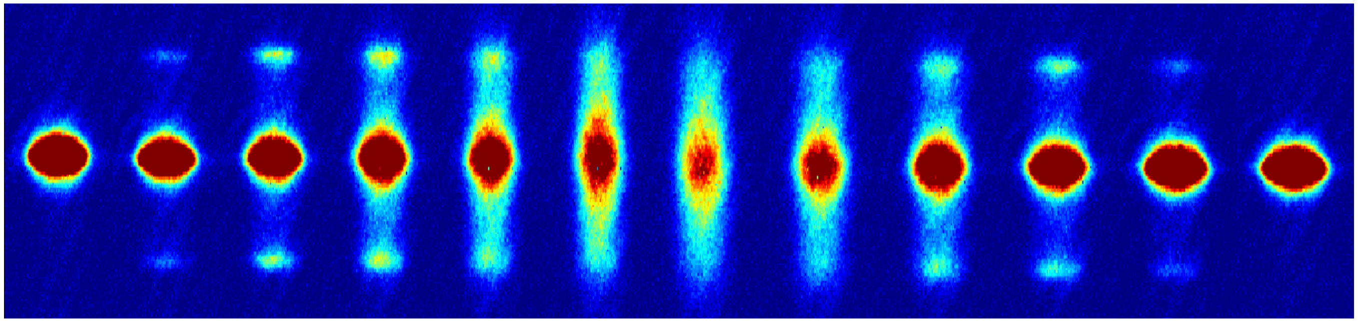


M. Greiner *et al.*, Nature **415**, 39 (2002)

1D system, weak interactions

fails for a **Tonks-Girardeau gas**

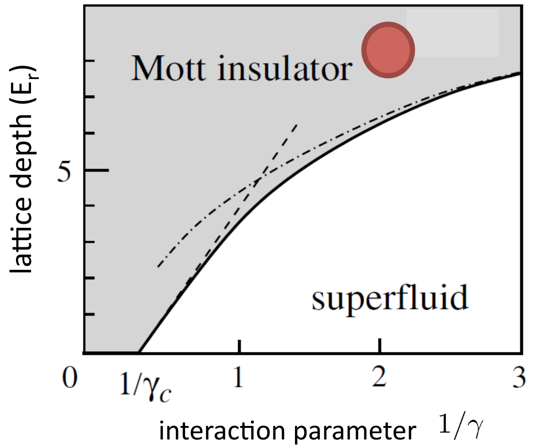




a_{3D} =40 a_0, lattice depth varied from 0 to 15 to 0 E_R

Experimental probe

Probe a property, which is present
in only one phase

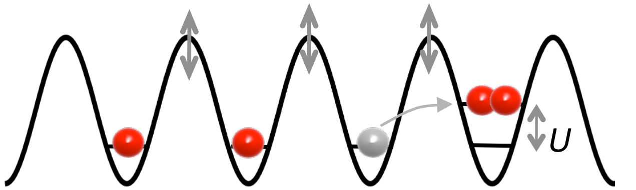


Mott insulating phase:

energy gap in excitation spectrum

method: **amplitude modulation spectroscopy**

T. Stöferele et al.,
Phys. Rev. Lett. **92**,
130403 (2003)
(Esslinger group)



amplitude modulation

energy gap U

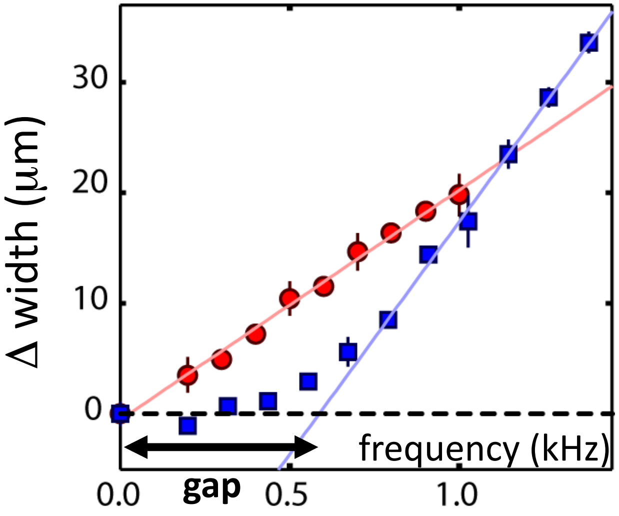
typical excitation spectra

superfluid

excitation spectrum is **gapless**

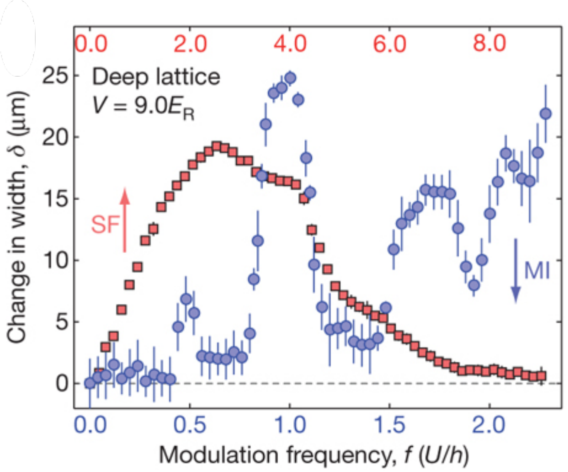
Mott insulator

excitation spectrum is **gapped**

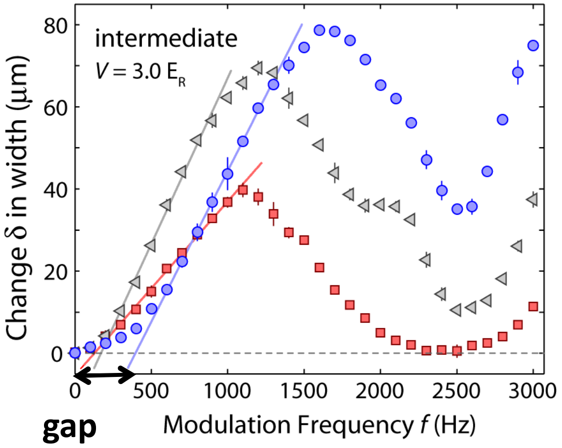


Excitation spectrum

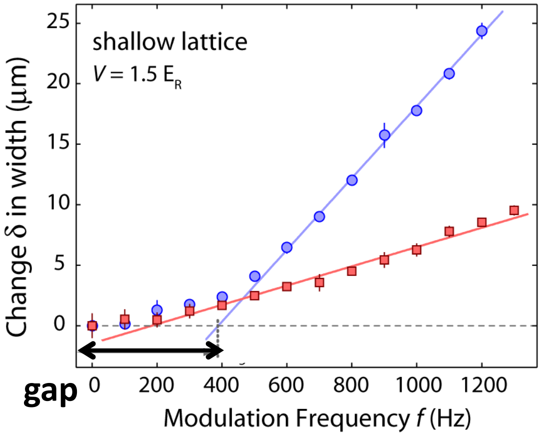
deep lattice depth



intermediate lattice depth



shallow lattice depth



basic idea:

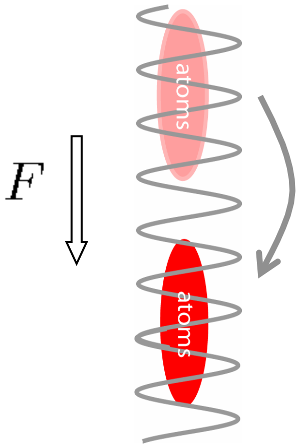
- start in a Mott-insulator and **determine the gap energy**
- reduce γ until the **gap disappears**

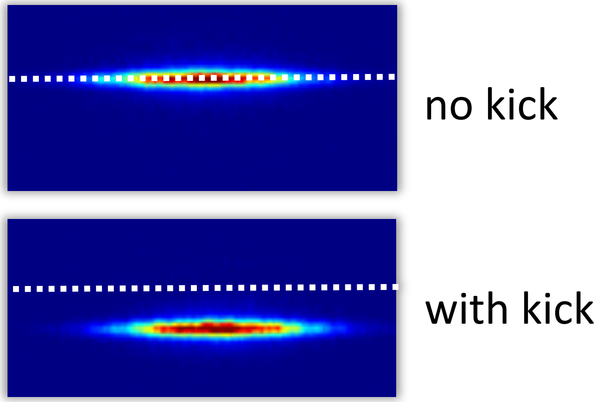
⟶ γ at the transition point

E. Haller et al., Nature 66, 597 (2010)

Transport properties

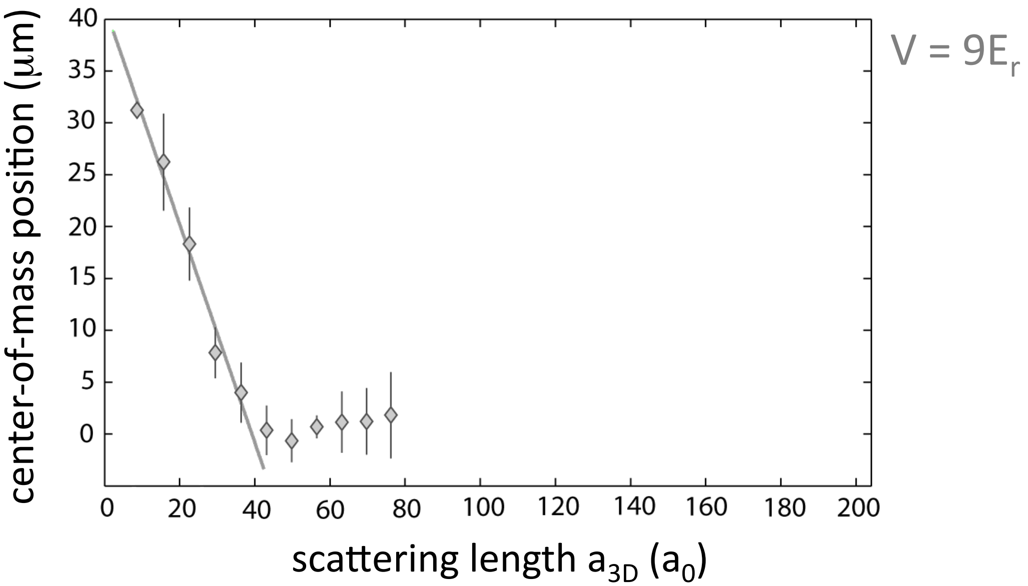
accelerate atoms
with „gentle kick“





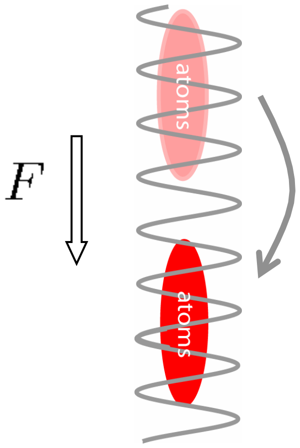
momentum space

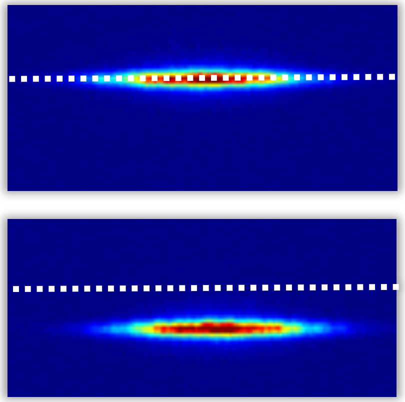
displacement after
expansion depends
on interactions



Transport properties

accelerate atoms
with „gentle kick“



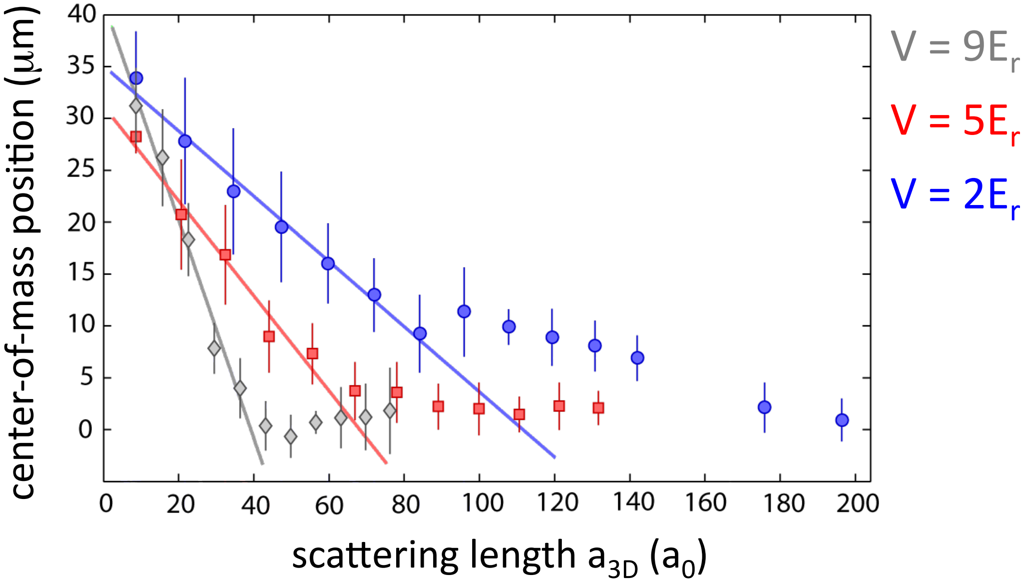


no kick

with kick

momentum space

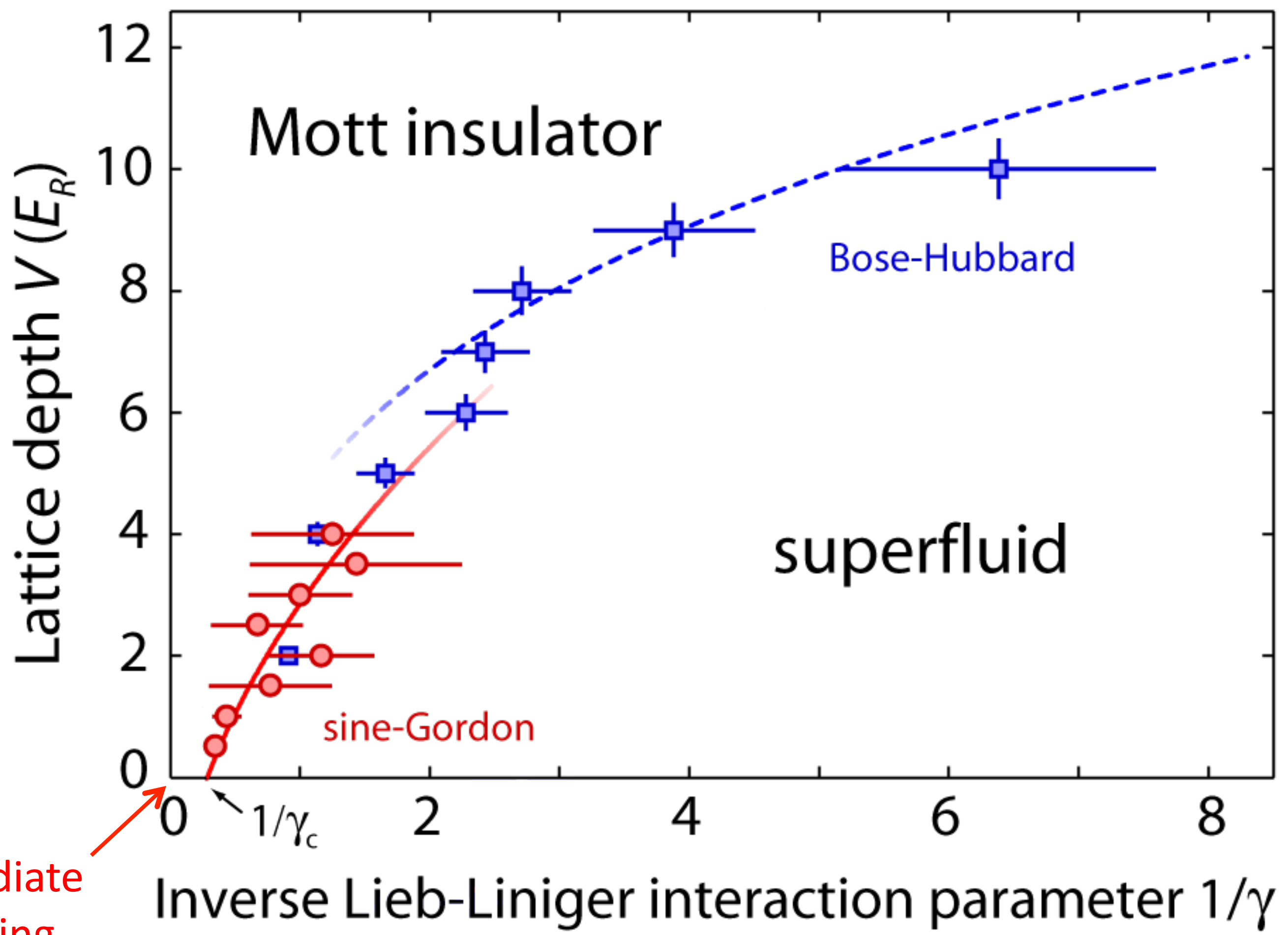
displacement after
expansion depends
on interactions



Complete phase diagram

universität innsbruck

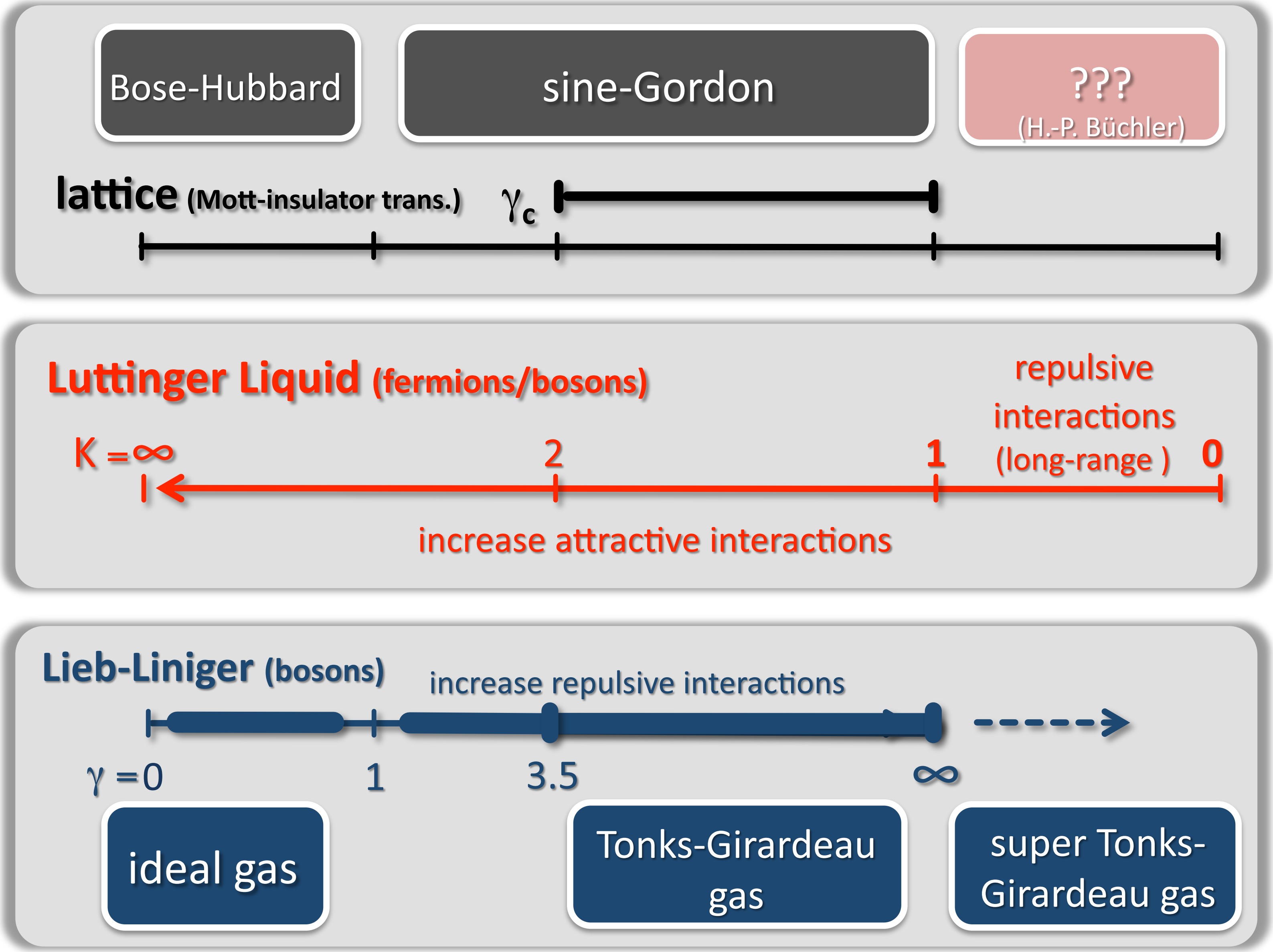
Amplitude modulation spectroscopy and transport measurement



immediate pinning

E. Haller et al., Nature 466, 597 (2010)

Summary: Interaction regimes of 1D quantum gases



Bose-Hubbard

sine-Gordon

???
(H.-P. Büchler)

lattice (Mott-insulator trans.)

γ_c

Luttinger Liquid (fermions/bosons)

repulsive interactions (long-range)

$K = \infty$

2

1

0

increase attractive interactions

Lieb-Liniger (bosons)

increase repulsive interactions

$\gamma = 0$

1

3.5

∞

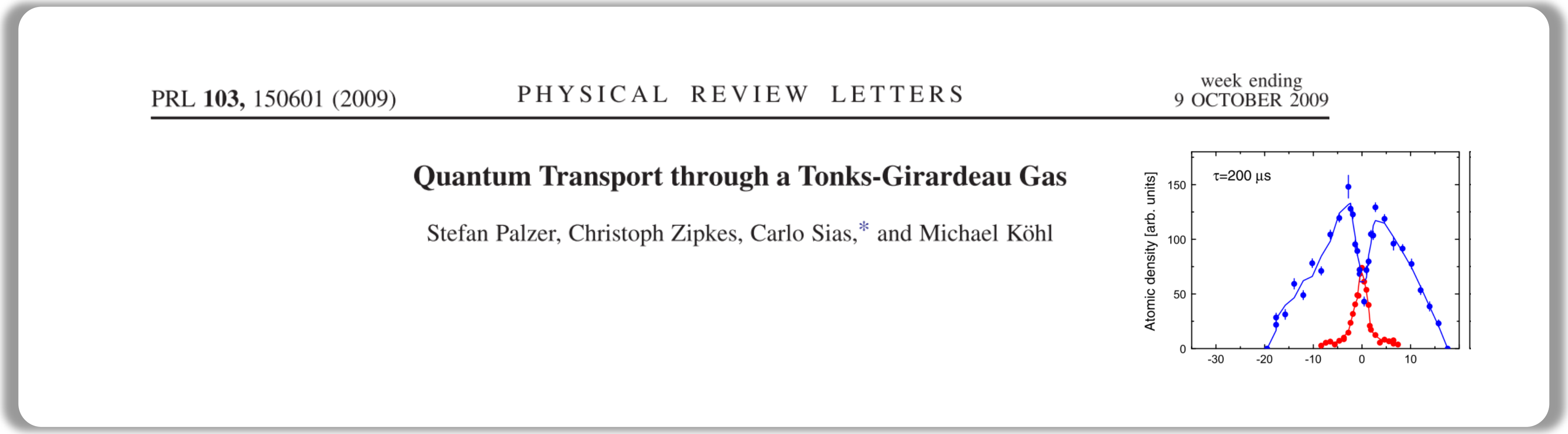
ideal gas

Tonks-Girardeau gas

super Tonks-Girardeau gas

Part III (current work)

Defects in interacting 1D gases: transport



PRL **103,** 150601 (2009) PHYSICAL REVIEW LETTERS week ending 9 OCTOBER 2009

Quantum Transport through a Tonks-Girardeau Gas

Stefan Palzer, Christoph Zipkes, Carlo Sias,* and Michael Köhl

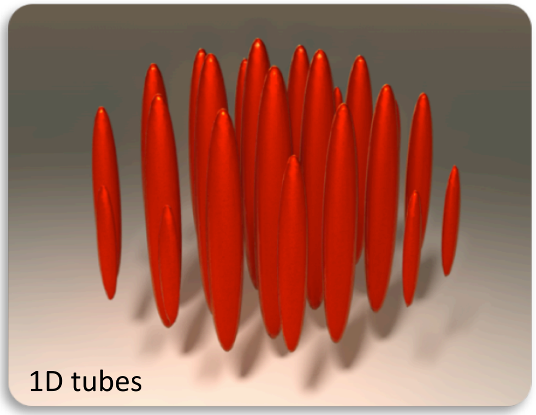
PRL **102,** 070402 (2009) PHYSICAL REVIEW LETTERS week ending 20 FEBRUARY 2009

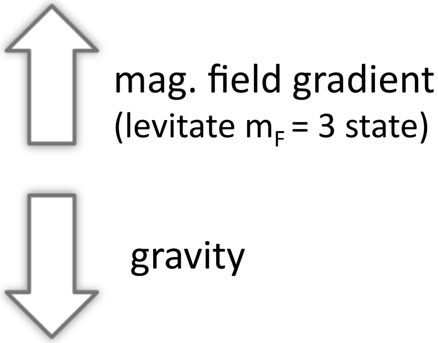
Bloch Oscillations in a One-Dimensional Spinor Gas

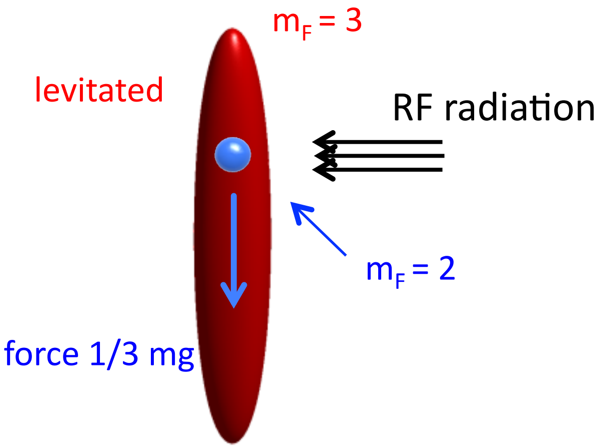
D. M. Gangardt[1,*] and A. Kamenev[2]

Defects in 1D, setup

Setup





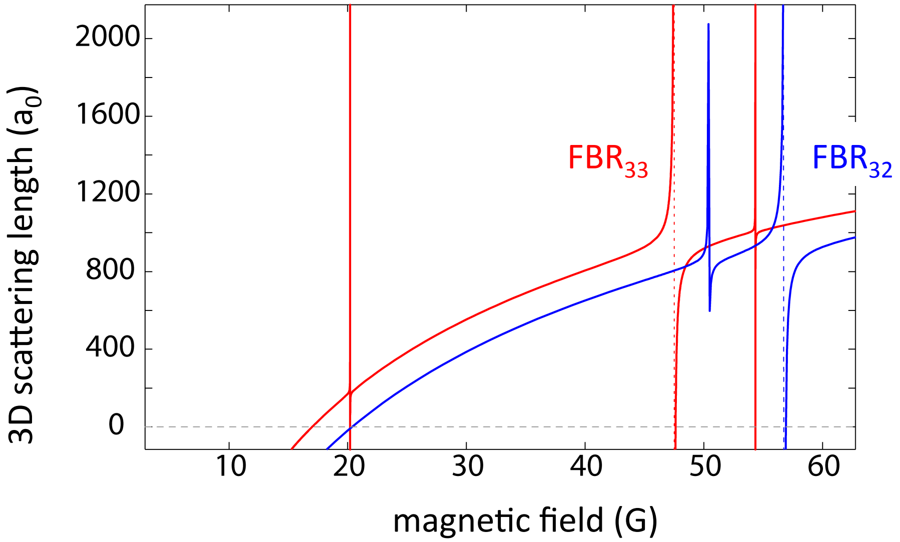


Scattering length

tune scattering length for collisions

$m_F = 3$ and $m_F = 3$ (a_{33})

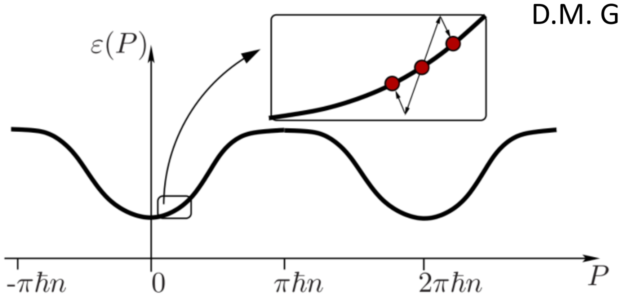
$m_F = 3$ and $m_F = 2$ (a_{32})



Oscillations in 1D (without a lattice)

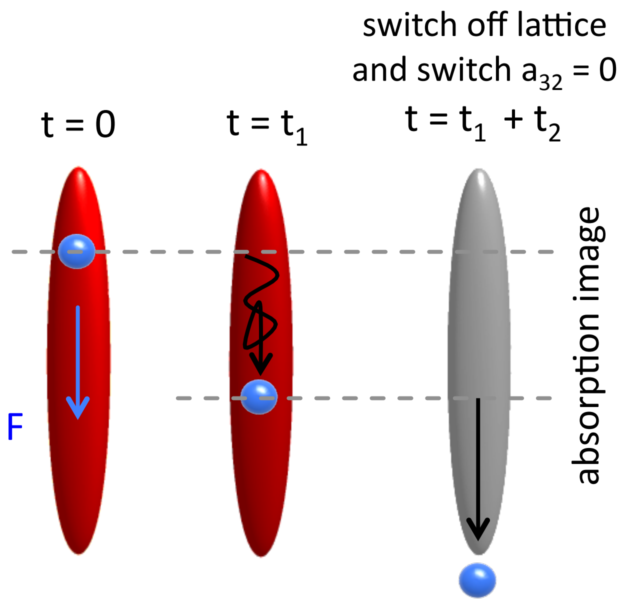
Oscillations in position space?

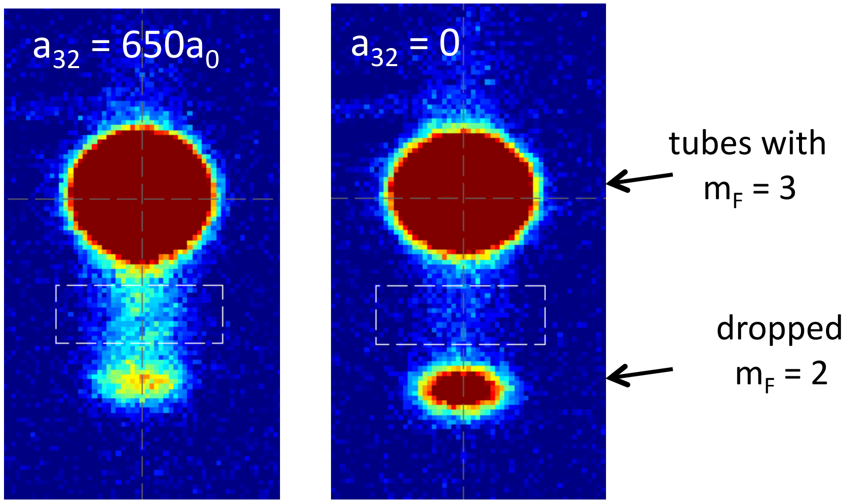
amplitude estimated from
“band width”
($x_0 < 3\ \mu m$)
→ to small for detection?



D.M. Gangardt and A. Kamenev,
PRL **102**, 070402 (2009)

Oscillations in momentum space?





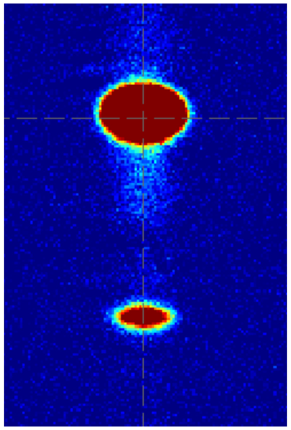
For large scattering length a_{32}, m_F=2 atoms are “stuck” even after switching off the 2D lattice.

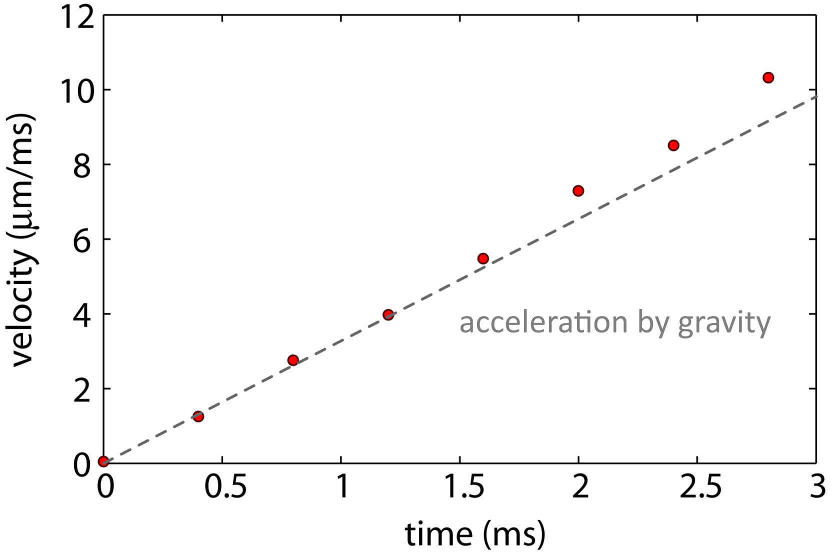
Oscillations in 1D (without a lattice)

weak interactions

$a_{32} = 0\ a_0$ and $a_{33} = 220\ a_0$

defects are not effected by 1D system





intermediate interaction strength

$a_{32} = 285\ a_0$ and $a_{33} = 470\ a_0$

some of the defects oscillate