Solids and supersolids in dipolar quantum gases

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UNIVERSITY OF INNSBRUCK

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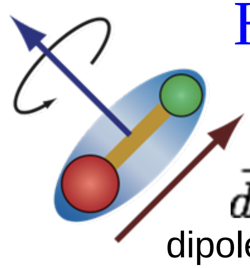
AUSTRIAN ACADEMY OF SCIENCES

AFOSR

EOARD

polar molecules

rotation, N



$$H = B\,\mathbf{N}^2$$

dipole moment

$X^1\Sigma$ closed shell molecules (e.g., RbCs) *in the electronic and vibrational groundstate*

- **What molecules?**

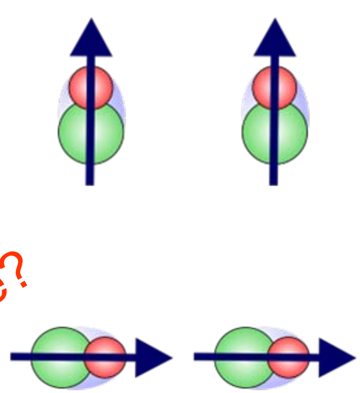
- Heteronuclear molecules,
- $X^1\Sigma$ closed shell (RbCs, KRb, …)
- Electronic and vibrational ground-state

- Preparation by:
  - -Feshbach-res./Photoassociation+STIRAP.
  - -Buffer-gas cooling

Why polar molecules?

- coupling to optical and microwave fields (cooling/trapping)

-permanent dipole moment:
- possibility for strong dipole-dipole interactions



repulsion

unstable?

attraction

- **Two molecules in E-field: Dipole-dipole interactions**

- Long-range
- Anisotropic

$$V(\mathbf{r}) = D\left[\frac{1}{r^3} - 3\frac{z^2}{r^5}\right]$$

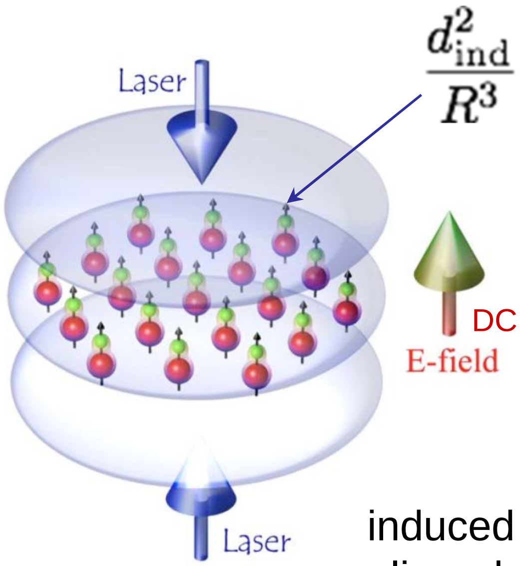
(Stability)

previously..

- Confined polar molecules

  - Stability in 2D

  - Many-body phases

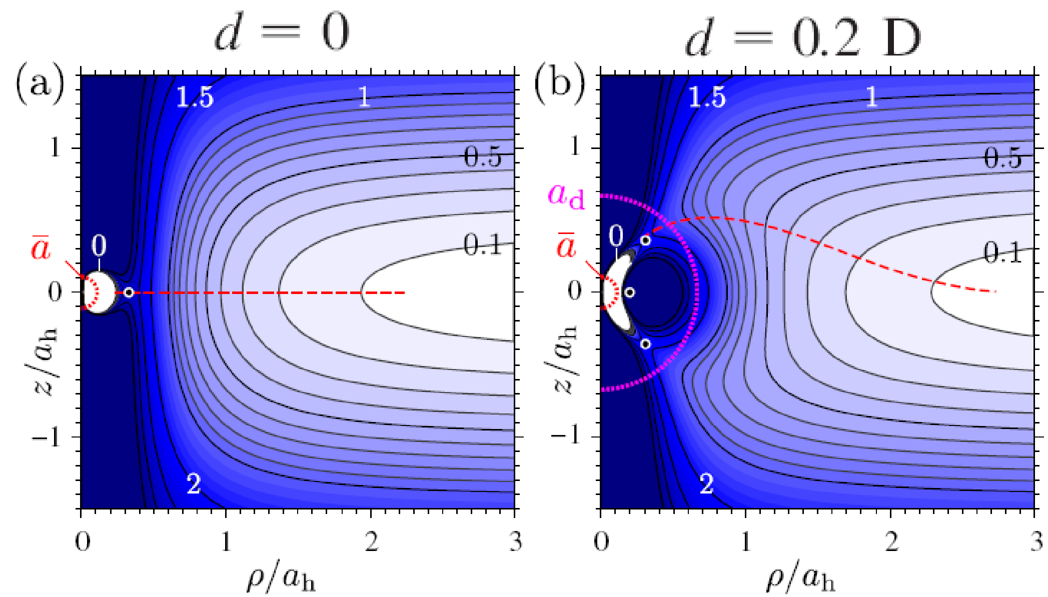


induced & aligned dipoles

Buechler, Demler, Lukin, Micheli, Prokof'ev, Pupillo, Zoller, PRL 2007

Micheli, Pupillo, Buechler, Zoller PRA 2007

- stability in 2D



"p"-wave barrier (|m|=1)

dipole-dipole interaction $a_d = \mu d^2/\hbar^2$

$$V = \frac{\mu\Omega^2 z^2}{2} + \frac{\hbar^2(m^2 - 1/4)}{2\mu\rho^2} - \frac{C_6}{r^6} + \frac{d^2}{r^3}\left(1 - \frac{3z^2}{r^2}\right)$$

Transverse confinement

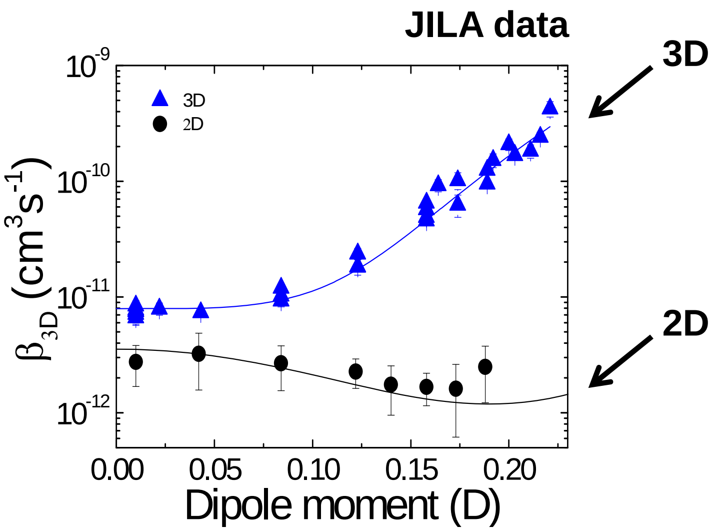
$a_h = \sqrt{\hbar/\mu\Omega}$

vdW attraction $\bar{a} \propto (2\mu C_6/\hbar^2)^{1/4}$

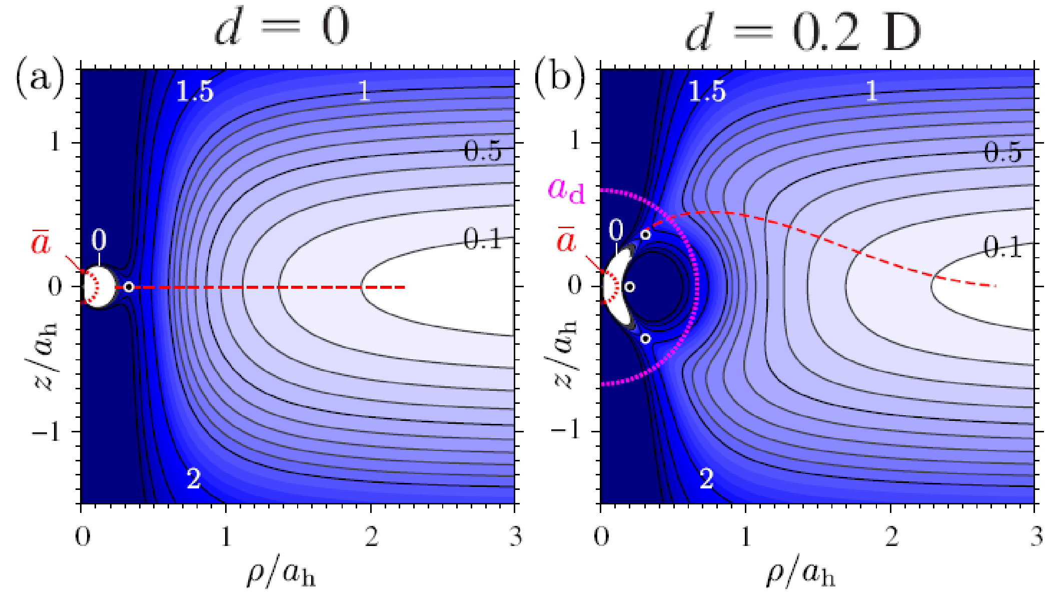
- **Tunneling rate**
  ("large" dipoles d>1D)

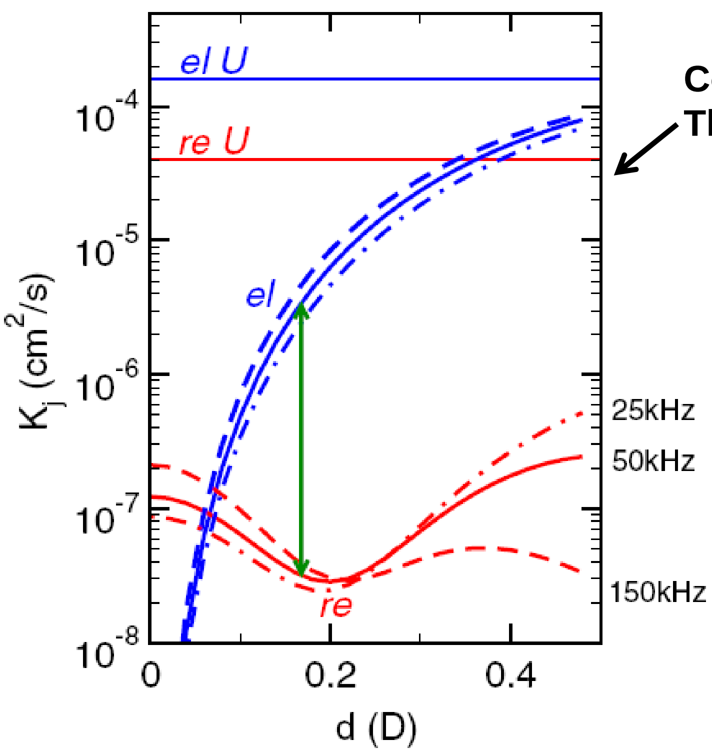
$$\Gamma = \omega_p e^{-\left(\frac{a_d}{a_h}\right)^{2/5}}$$

quantum degeneracy..?



- Stability in 2D





Colision Rates, 2D Theory

- Loss suppression: measured

- Evaporative cooling?

Micheli, Idziaszek, Pupillo, Baranov, Zoller, Julienne, PRL 105, 073202 (2010)

Quemener & Bohn, PRA 81, 060701(R) (2010)

previously..

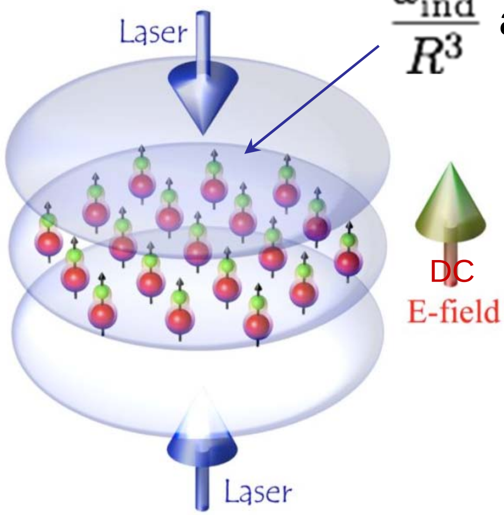
- Confined polar molecules

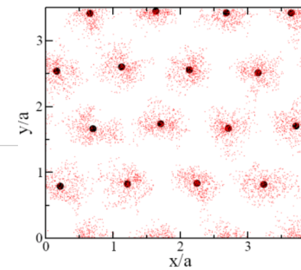
  - Stability in 2D

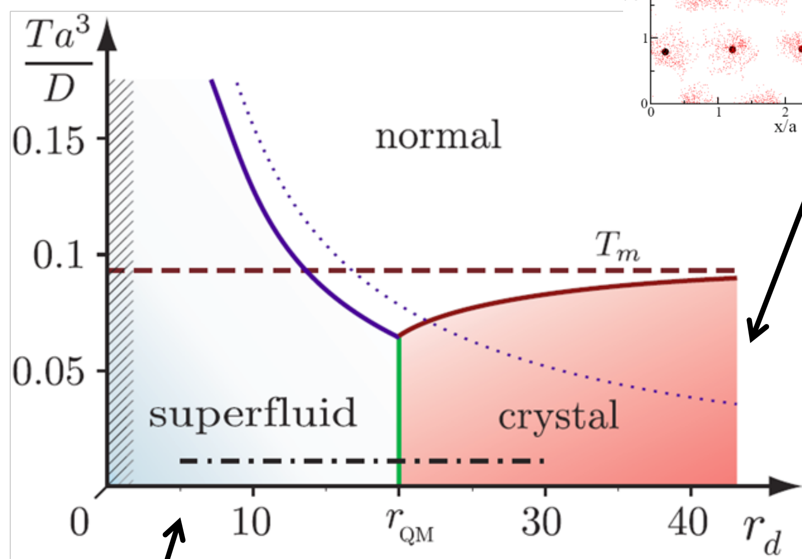
    See talks by Andrea!

  - Many-body phases

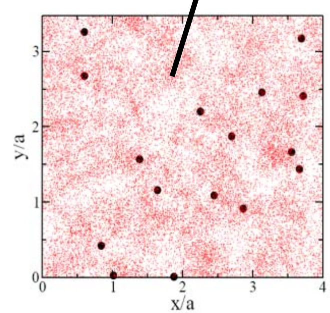
$\frac{d_{\mathrm{ind}}^2}{R^3}$ induced & aligned dipoles







$$r_d = \frac{E_{int}}{E_{kin}} = \frac{D/a^3}{\hbar^2/ma^2} = \frac{Dm}{\hbar^2 a}$$



Buechler, Demler, Lukin, Micheli, Prokof'ev, GP, Zoller, PRL 2007
Micheli, Pupillo, Buechler, Zoller PRA 2007

exact Path Integral Monte-Carlo simulations

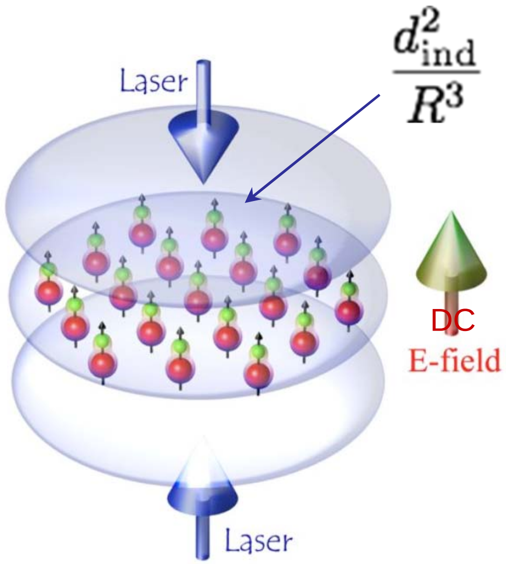
(Prokof'ev, Svistunov Boninsegni..)

previously..

- Confined polar molecules

  - Stability in 2D
  - Dipolar crystal

induced &
aligned dipoles

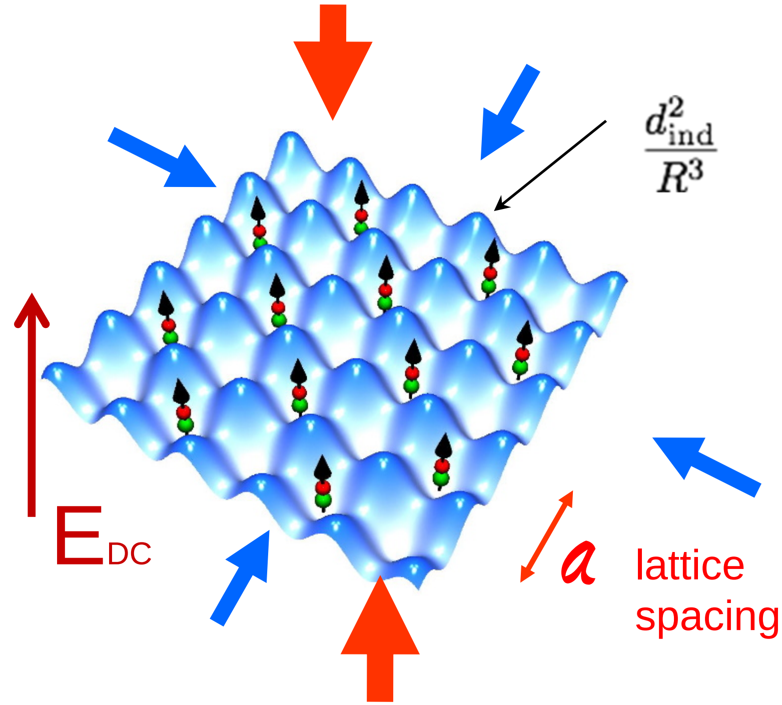


Buechler, Demler, Lukin, Micheli, Prokof'ev, GP, Zoller, PRL 2007

Micheli, Pupillo, Buechler, Zoller PRA 2007

- Bosonic polar molecules on a 2D lattice

  - "hard-core"molecules on a 2D lattice
  - phase diagram with long-range interactions?
    - so far: nearest-neighbor interactions

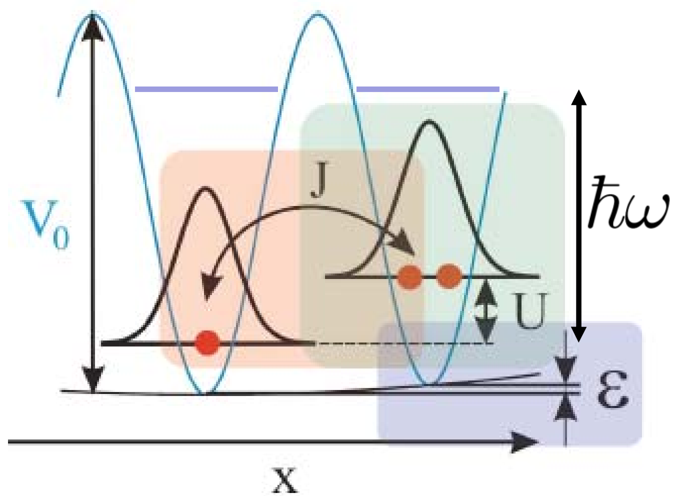


Capogrosso-Sansone, Trefzger, Lewenstein, Zoller, GP, PRL 2010

See also: L. Pollet, J. D. Picon, H.P. Buechler, M. Troyer, PRL2010
Goral, Santos, Lewenstein, PRL 2000

in contrast to.. cold atoms in optical lattices, contact interactions

- The Bose-Hubbard model



Onsite interaction $\frac{U}{E_R} \alpha \frac{a_s}{\lambda}$

$$\hat{H} = -J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j + \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1) + \sum_i \epsilon_i \hat{n}_i$$

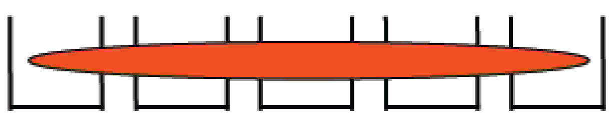
Kinetic energy:
hopping

External parabolic
confinement..

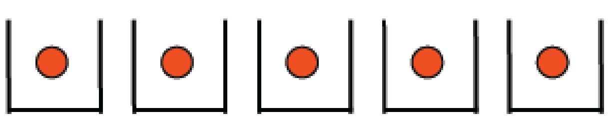
Jaksch *et al.*, PRL 1998

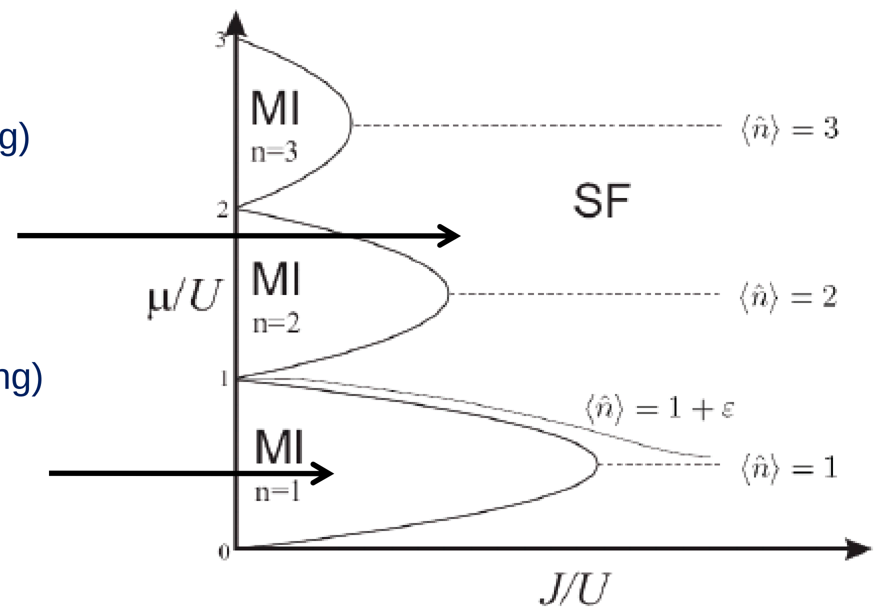
- Phase diagram

Superfluid: J >U (or just non-integer lattice filling)



Mott insulator: J < U (_and_ integer lattice filling)

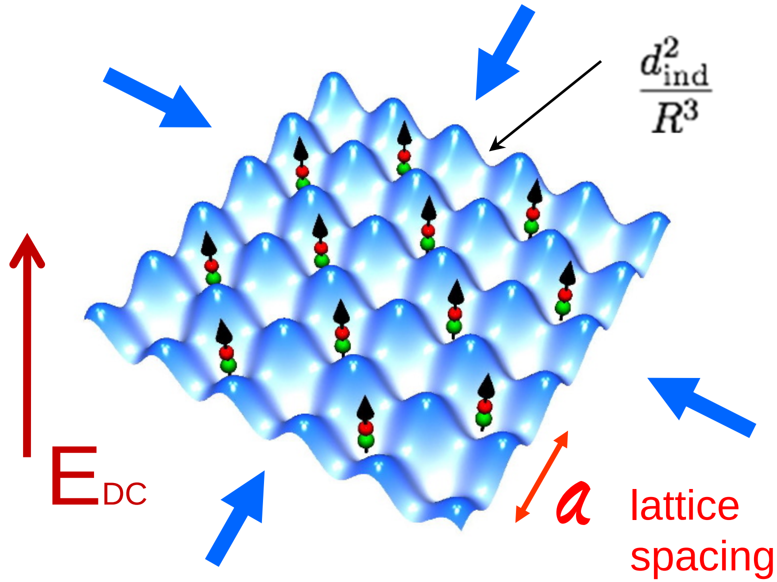




Th.: M.P.A. Fisher et al, PRB 1989, Exp.: Munich, NIST/JQI, ETH, MIT, Mainz, Chicago, Innsbruck…

with bosonic polar molecules on a 2D lattice..

- Polar molecules on a 2D lattice



- 2D lattice
- Polarized molecules: Long-range, repulsive dipole-dipole interactions
-Tight-binding $T, d^2/a^3 < \hbar\omega \quad \cdots$
- Hard-core particles: $\rho < 1$

- Extended Hubbard-like Model

local chemical potential

$$H = -J \sum_{<i,j>} \left[b_i^\dagger b_j + b_i b_j^\dagger \right] + V \sum_{i<j} \frac{n_i n_j}{r_{ij}^3} - \sum_i \mu_i n_i$$

Kinetic energy: hopping

dipole-dipole interactions $V = \frac{d^2}{a^3}$

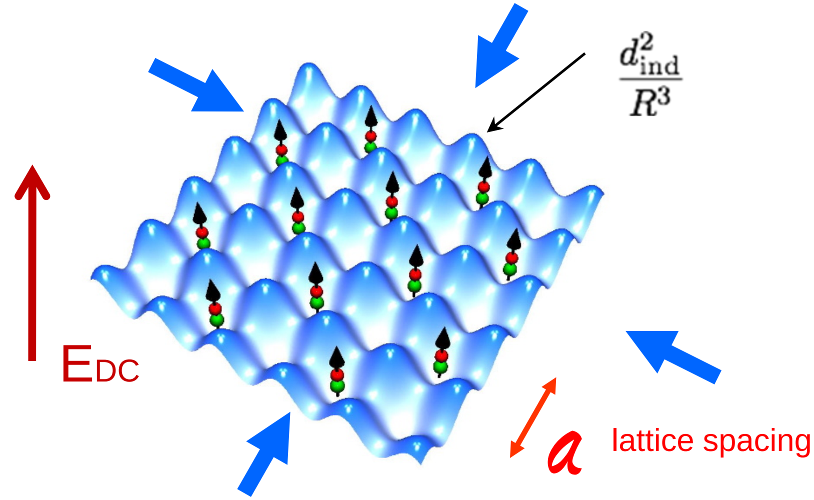
$r_{ij} = |i - j|$

$\mu_i = \mu - \Omega i^2$

parabolic confinement

- *Exact* Quantum Monte-Carlo simulations, Worm-Algorithm, Prokof'ev/Svistunov

with bosonic polar molecules on a 2D lattice…

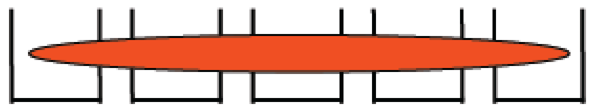


- Single-band extended Hubbard model

$$H = -J \sum_{<i,j>} \left[b_i^\dagger b_j + b_i b_j^\dagger \right] + V \sum_{i<j} \frac{n_i n_j}{r_{ij}^3} - \sum_i \mu_i n_i$$

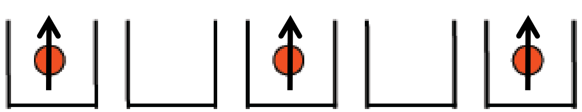
Novel phases!

Superfluid **(SF)**: J >>V



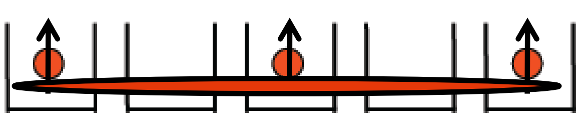
Insulators: J << V, ***for all rational lattice filling***

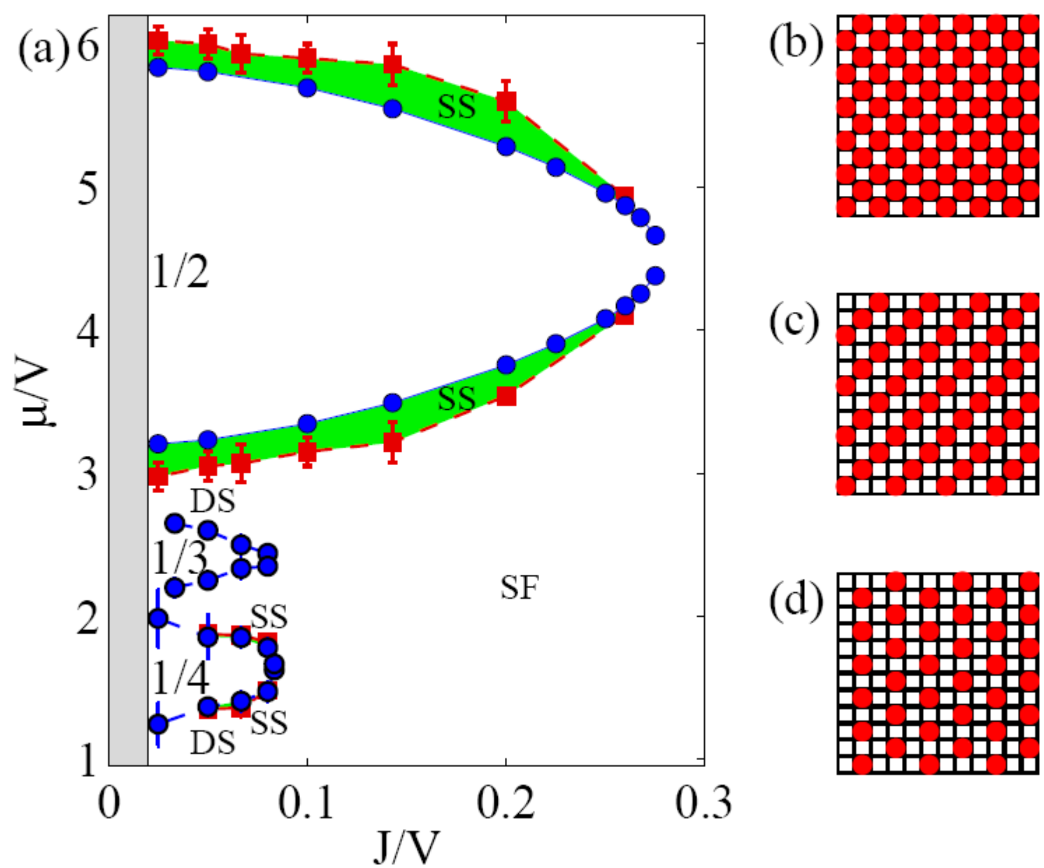
- Devil's staircase of lattice solids **(DS)**
- Metastability



Supersolids **(SS)**: J ~V

- coexistence of superfluid and crystalline orders!
- **condensation of vacancies and interstitials**

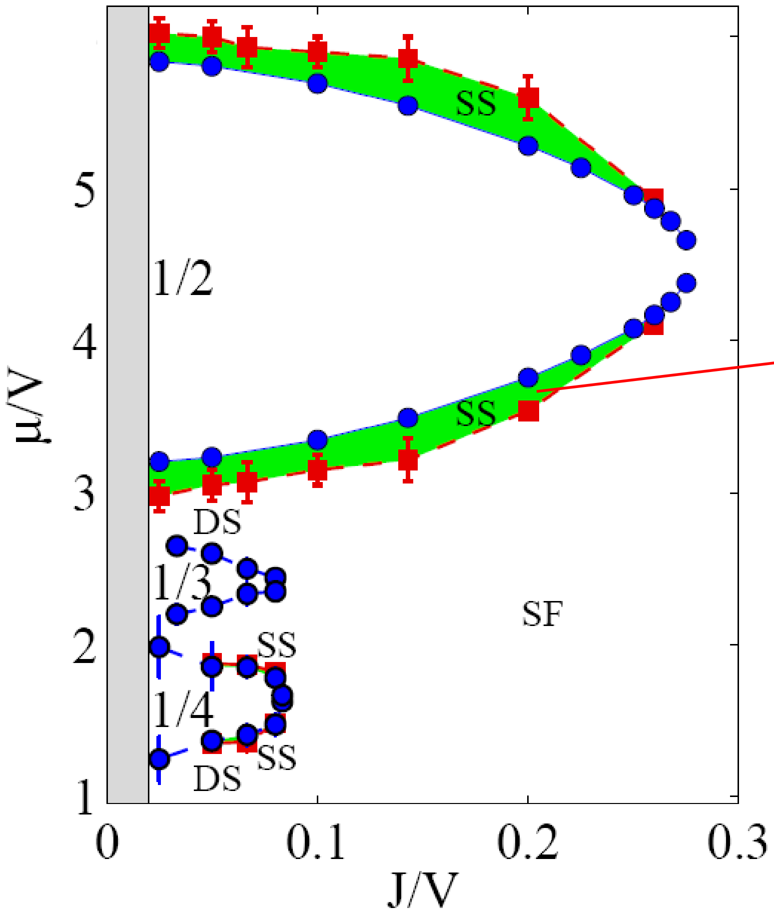




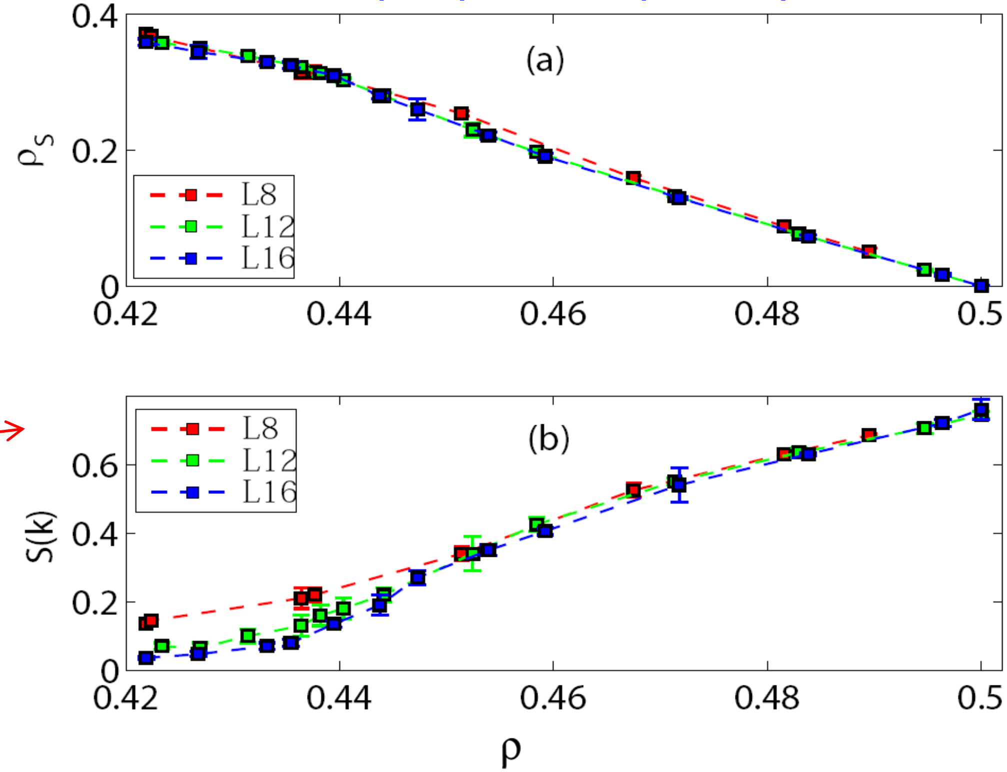
See also: Goral, Santos, lewenstein PRL 2002
Bruder, R. Fazio, G Schoen, PRB 1993

Capogrosso-Sansone, Trefzger, Lewenstein, Zoller, GP, PRL 2010

Supersolids



Vacancy supersolid (V/J=5)



- Supersolids: vacancies and impurities
  - -finite superfluid density over wide range of densities
  - - finite structure factor S(π,π) over wide range of densities

RbCs $(d = 1.25\text{Debye})$

$V_{0,\perp}/E_{\mathrm{R}} = 40$

$V_0/E_{\mathrm{R}} = 4$ $\quad a = 400\text{nm}$

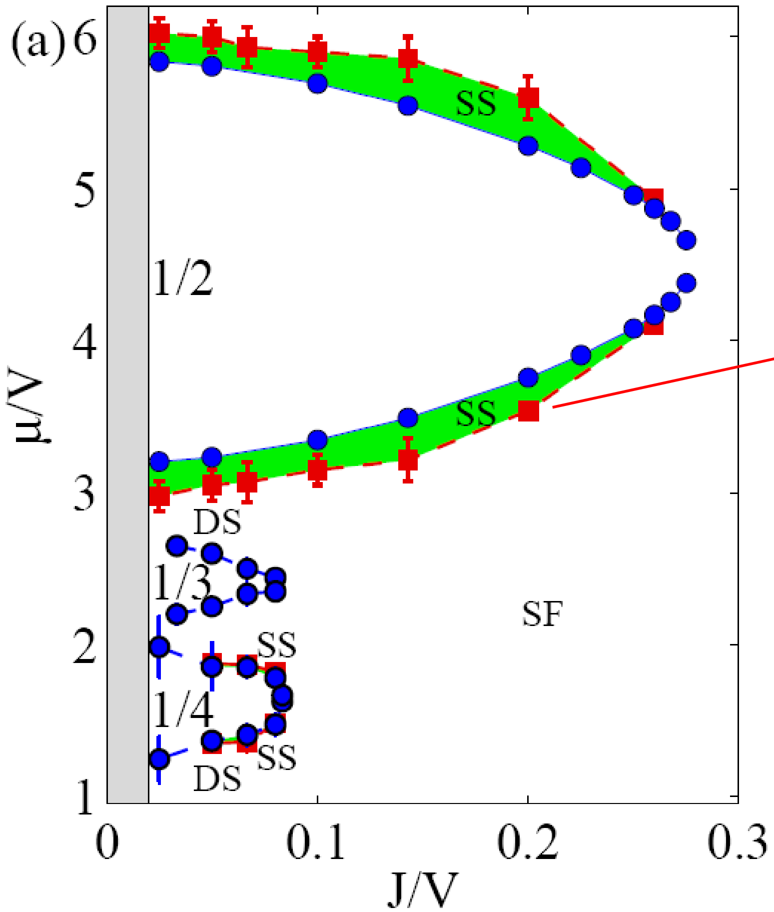
$\omega_\perp/2\pi \approx 18\text{kHz}$ $\quad \omega/2\pi \approx 6\text{kHz}$

$D/(a^3 h) \simeq 3.5\text{kHz}$

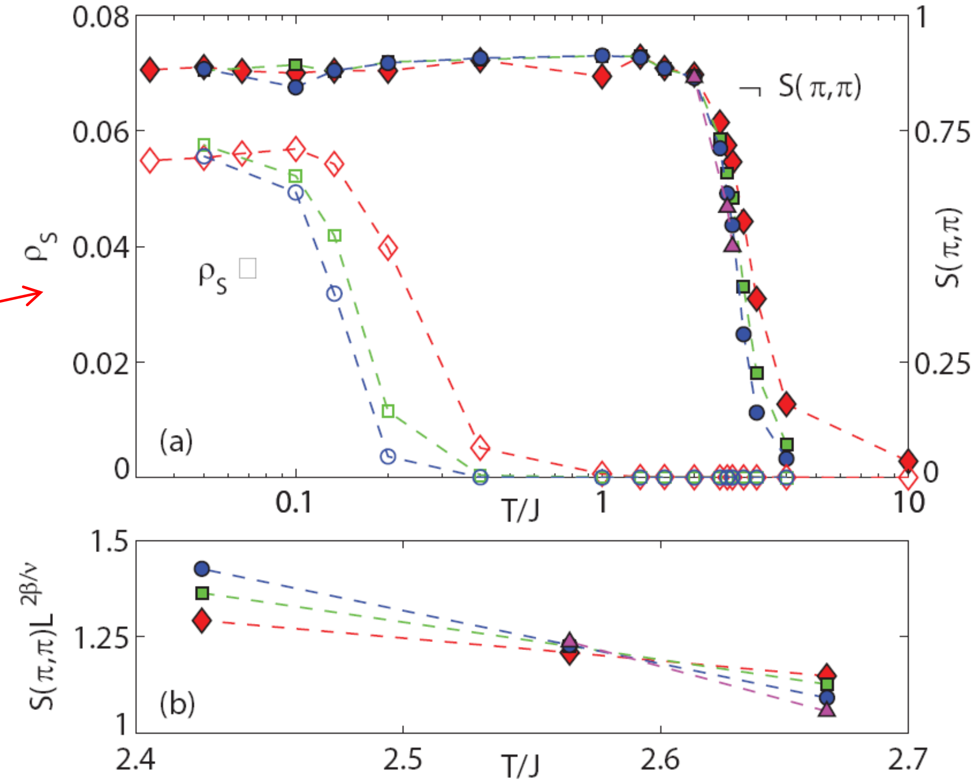
$J/h \simeq 120\text{Hz}$ $\quad J/V \gtrsim 0.03$

Finite-T melting of a supersolid

- Phase-Diagram



Finite-T (V/J=10):



- Generic 2-step transition:
  - SuperSolid / “Liquid crystal”: KT
  - “Liquid-crystal”/ Normal fluid: 2D-Ising

Supersolids in lattices, see: S. Yi, T. Li, C.P. Sun, PRL 2007
C. Bruder, R. Fazio, G Schoen, PRB 1993

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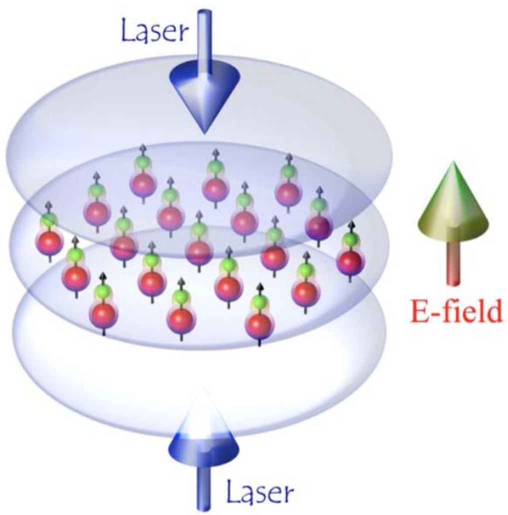
this talk…

- Can we do this with cold atoms?



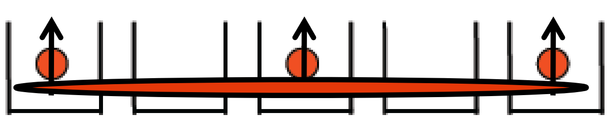
- Can we do this "better"?

Supersolids: J ~V

coexistence of superfluid and crystalline orders!

- **condensation of vacancies and interstitials (Andreev-Lifshits scenario)**

Free space? (no optical lattice..) see He4!



Kim and Chan, Nature 2004 ; Prokof'ev, Adv. Phys. 2007

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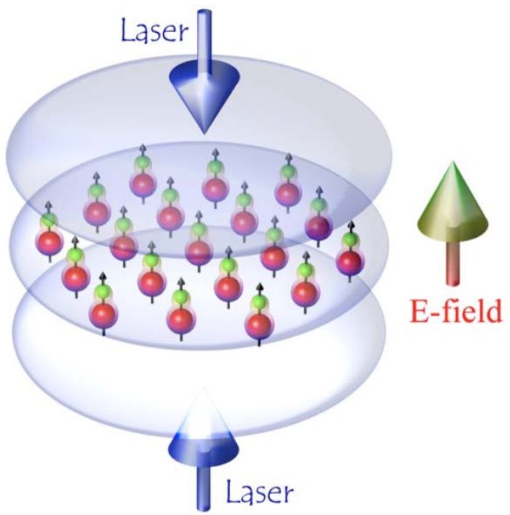

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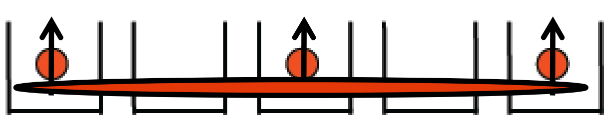
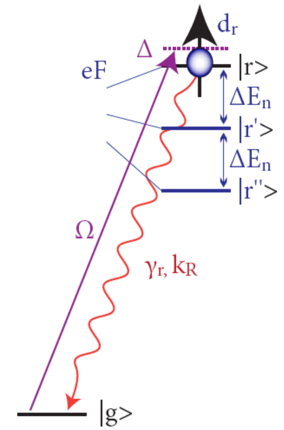


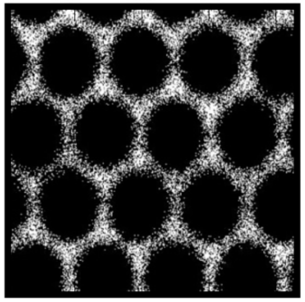
Kim and Chan, Nature 2004 ; Prokof'ev, Adv. Phys. 2007

- Dipole-blockaded gases
  - Rydberg-dressed atoms



GP, Micheli, Boninsegni, Lesanovsky, Zoller, PRL 2010

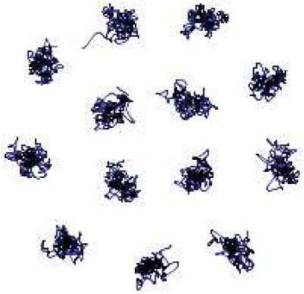
- Novel quantum phases?



Cinti, Jain, Boninsegni, Micheli, Zoller, GP, arXiv:1005:2403, PRL in press

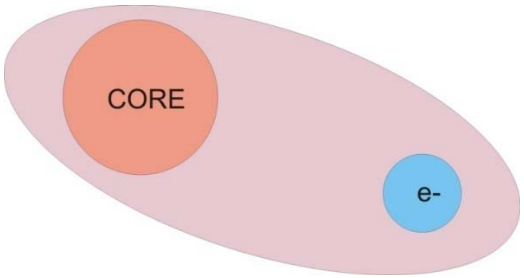
See: Henkel, Nath and Pohl, PRL 2010

Strongly correlated phases with Rydberg gases

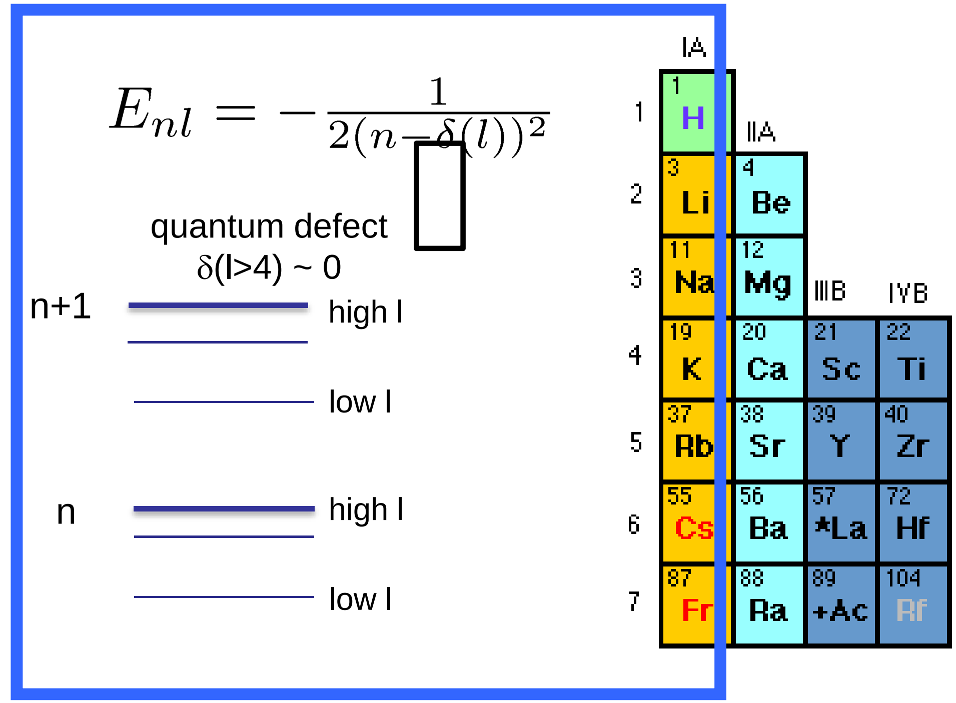


Pupillo, Micheli, Boninsegni, Lesanovsky, Zoller, PRL 2010

- Rydberg atoms?



Rydberg states: highly-excited electronic states of Alkali atoms

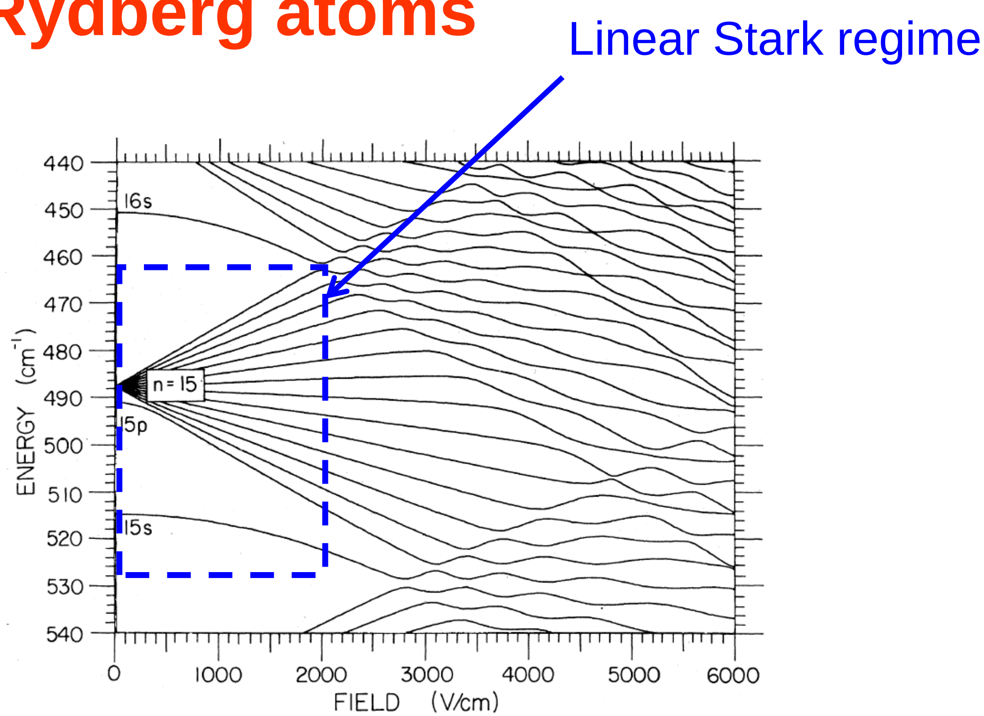


Rydberg atoms:

- hydrogen-like atoms
- simple level structure
- radiative lifetime $\tau \sim n^3$ (τ ~ ms)
- large displacement of core and electron $\langle r \rangle \sim n^2$
- mesoscopic objects ($\langle r \rangle$ ~ μm)
- highly susceptible to external fields
- strong long-ranged interaction

external dynamics is usually **FROZEN**

Rydberg atoms



- Energy $\mathbf{E} = \mathbf{d} \cdot \mathbf{F}$

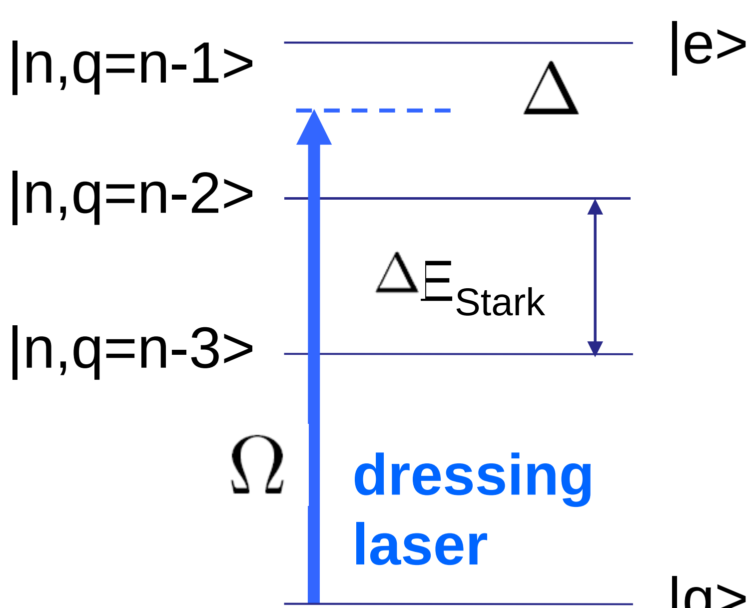
- dipole moment of the uppermost Stark state

$$d_{\max} = \frac{3}{2} e\, a_0 n(n-1)$$

is of the order of several kDebyes

Idea

- don't need such large dipole moment
- use coherent weak admixture of the Rydberg state to the ground state



advantages: 1) prolonged lifetime
2) interactions compatible with (optical) trapping

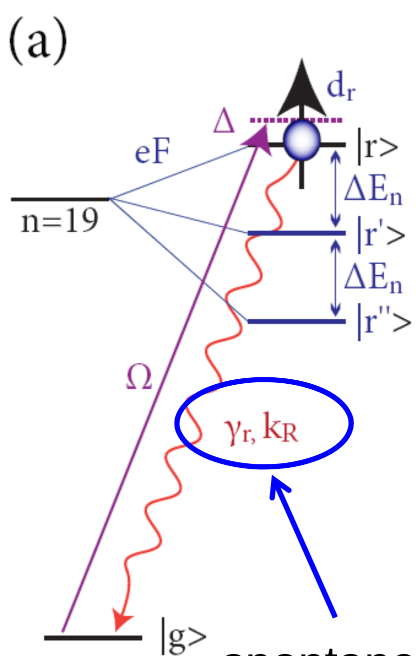
$$|G\rangle \approx \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \frac{\Omega}{\Delta} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |g\rangle + \frac{\Omega}{\Delta} |e\rangle$$

dressed ground state acquires character of the excited state

See also: Santos, Shlyapnikov, et al., PRL 2000

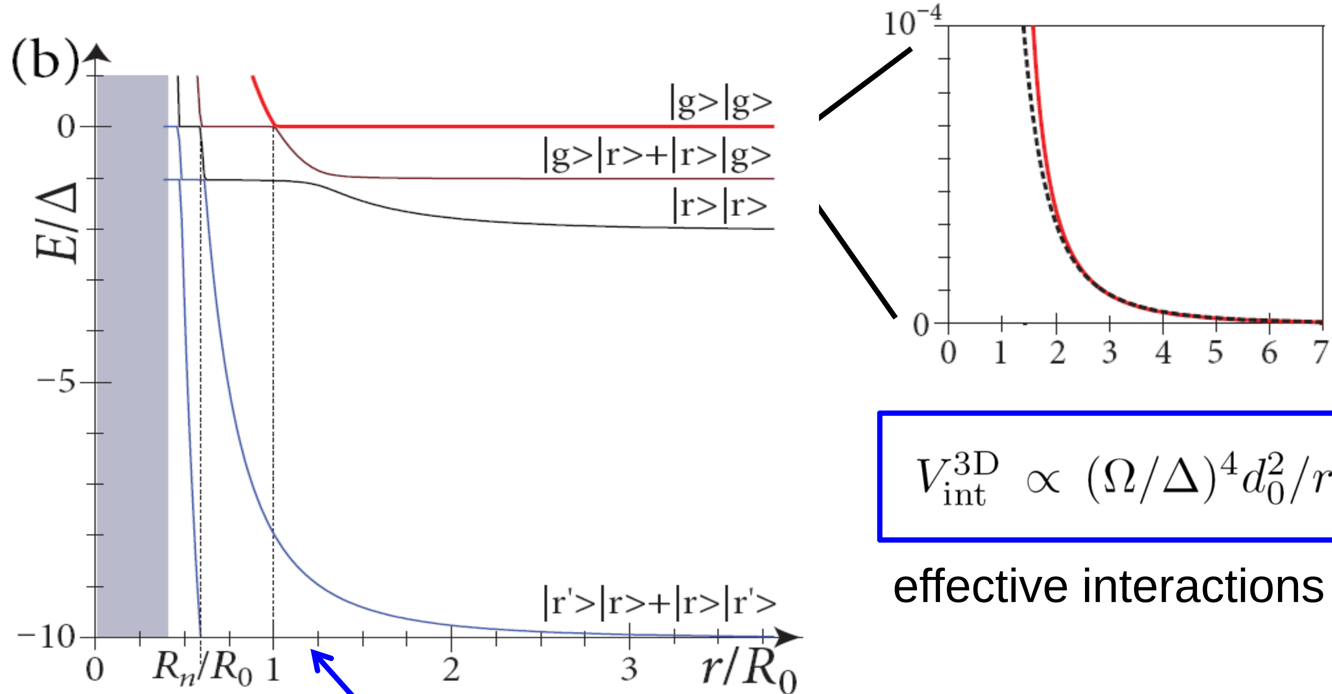
Blue-shielding: effective interactions

- Laser Dressing



spontaneous emission

- Dressed Born-Oppenheimer potentials



$$V_{\text{int}}^{3D} \propto (\Omega/\Delta)^4 d_0^2/r^3$$

effective interactions

Condon point ~ 500 nm

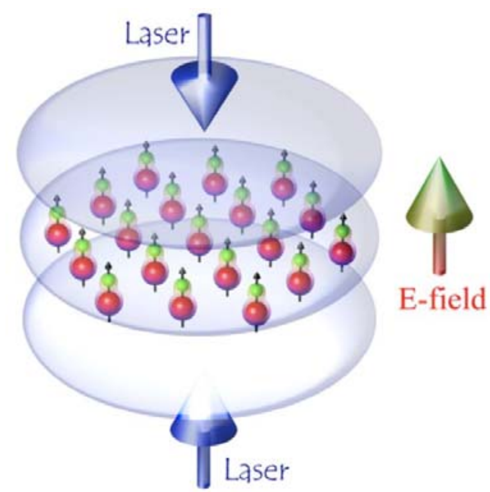
$$R_0 \simeq (D/\hbar\Delta)^{1/3}$$
$$D = d_0^2/(4\pi\epsilon_0)$$

- 2D collisional stability for: $\hbar\omega_\perp > V_{\text{int}}^{3D}(R)$
- 2D effective interactions: $V_{\text{int}}^{2D} = (\Omega/\Delta)^4 D/\rho^3$

~ polar molecules

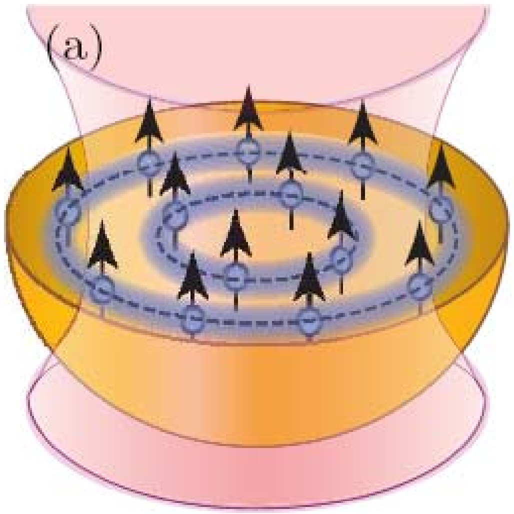
- Residual spontaneous emission: $\gamma_{\text{eff}} \sim (\Omega/\Delta)^2\gamma_r$

intrinsic heating source



2D Effective Many-Body Hamiltonian

- Setup (scheme)



- 2D Hamiltonian:

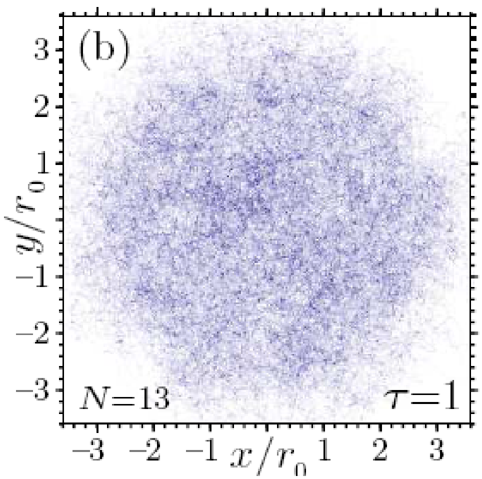
$$\frac{H}{\epsilon_0} = \sum_{i=1}^{N} \left[-\frac{1}{2\tau^2} \frac{\partial^2}{\partial \boldsymbol{\rho}_i^2} + \frac{1}{2} \boldsymbol{\rho}_i^2 \right] + \sum_{i>j} \frac{1}{|\boldsymbol{\rho}_i - \boldsymbol{\rho}_j|^3}$$

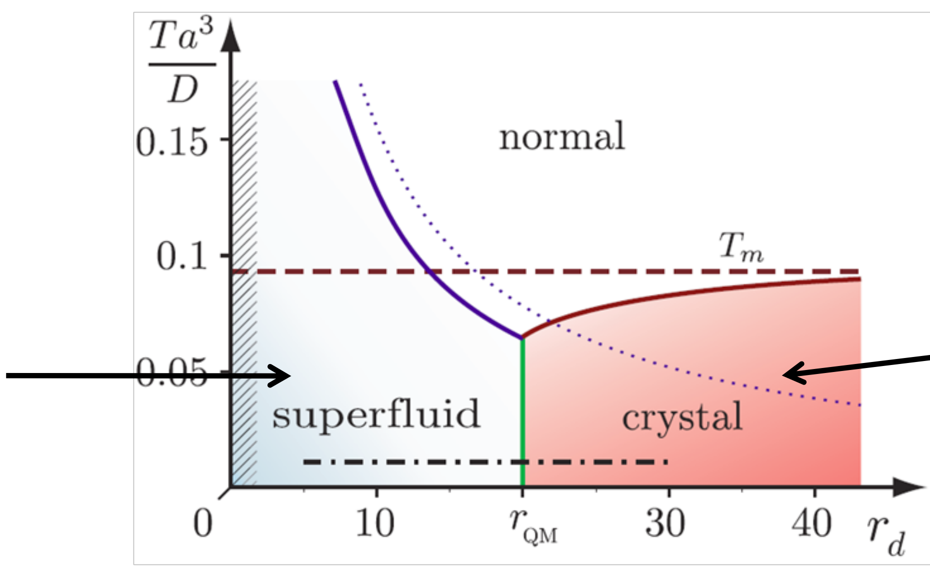
$$\tau \equiv \epsilon_0 / \hbar\omega = (r_0/\ell)^2$$ effective mass

$$\epsilon_0 = m\omega^2 r_0^2 = D/r_0^3 = (m^3 \omega^6 D^2)^{1/5}$$

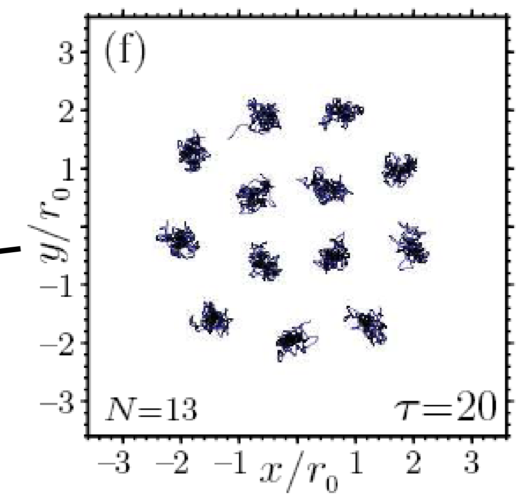
$$\tau_c = (r_{QM} a/\ell)^{2/5} \simeq 3$$

Superfluid



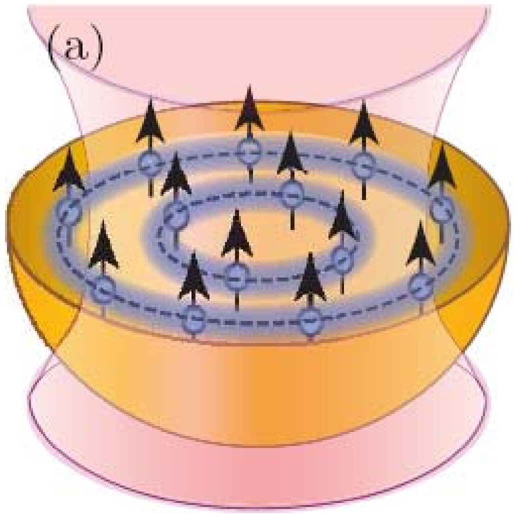


Classical crystal



2D Effective Many-Body Hamiltonian

- Setup (scheme)



- 2D Hamiltonian:

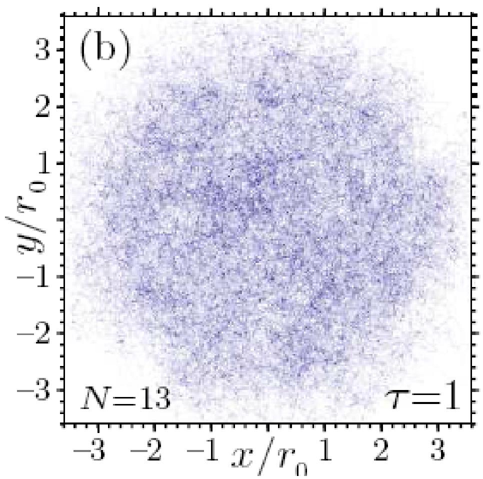
$$\frac{H}{\epsilon_0} = \sum_{i=1}^{N} \left[-\frac{1}{2\tau^2} \frac{\partial^2}{\partial \boldsymbol{\rho}_i^2} + \frac{1}{2} \boldsymbol{\rho}_i^2 \right] + \sum_{i>j} \frac{1}{|\boldsymbol{\rho}_i - \boldsymbol{\rho}_j|^3}$$

$$\tau \equiv \epsilon_0 / \hbar\omega = (r_0/\ell)^2$$ effective mass

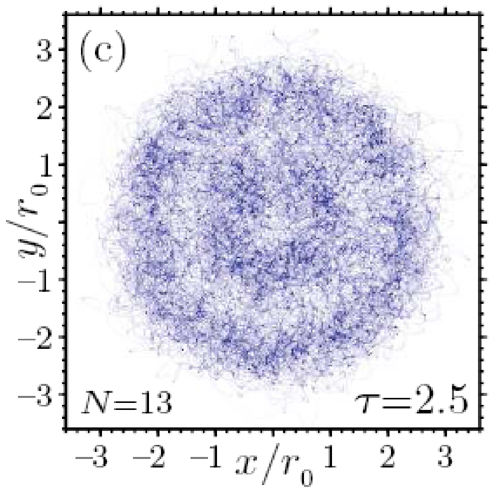
$$\epsilon_0 = m\omega^2 r_0^2 = D/r_0^3 = (m^3\omega^6 D^2)^{1/5}$$

$$\tau_c = (r_{QM} a/\ell)^{2/5} \simeq 3$$

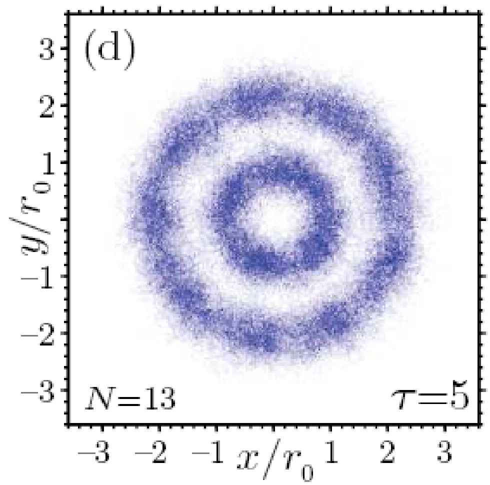
Superfluid



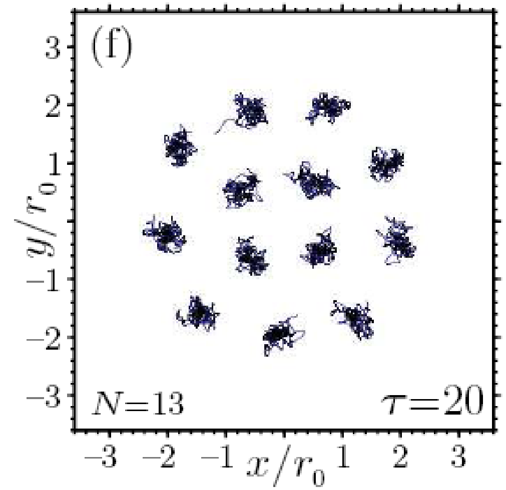
Supersolid



Ring crystal

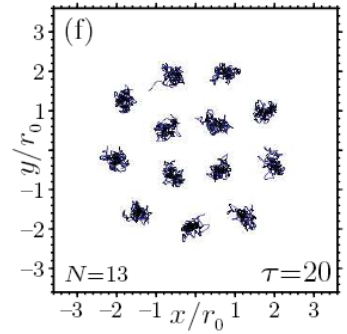


Classical crystal



Observable with Rydberg-dressed atoms?

- Superfluids: $\tau < \tau_c$
- Supersolids: $\tau \lesssim \tau_c$
- Ring-shaped crystals $\tau \gtrsim \tau_c$
- Classical crystals $\tau \gg \tau_c$



- Atoms: ^{87}Rb atoms

$n = 20$ $\quad d_r \approx 1.45\text{kD}$

$\Gamma_r/2\pi \sim 100\text{kHz}$

$\omega/2\pi = 1\text{kHz}$

- Electric field & Laser

$\mathcal{E}_{DC} = 25\text{kV/m}$

$\Omega/2\pi = 80\text{MHz}$

$\Delta/2\pi = 1\text{GHz}$

- Effective parameters

$d \approx 9.1\text{Debye}$

$r_0 \approx 1.07\mu\text{m}$

$\tau \approx 10$

$\tau \gg \tau_c$

$\Gamma/2\pi \approx 640\text{Hz}$

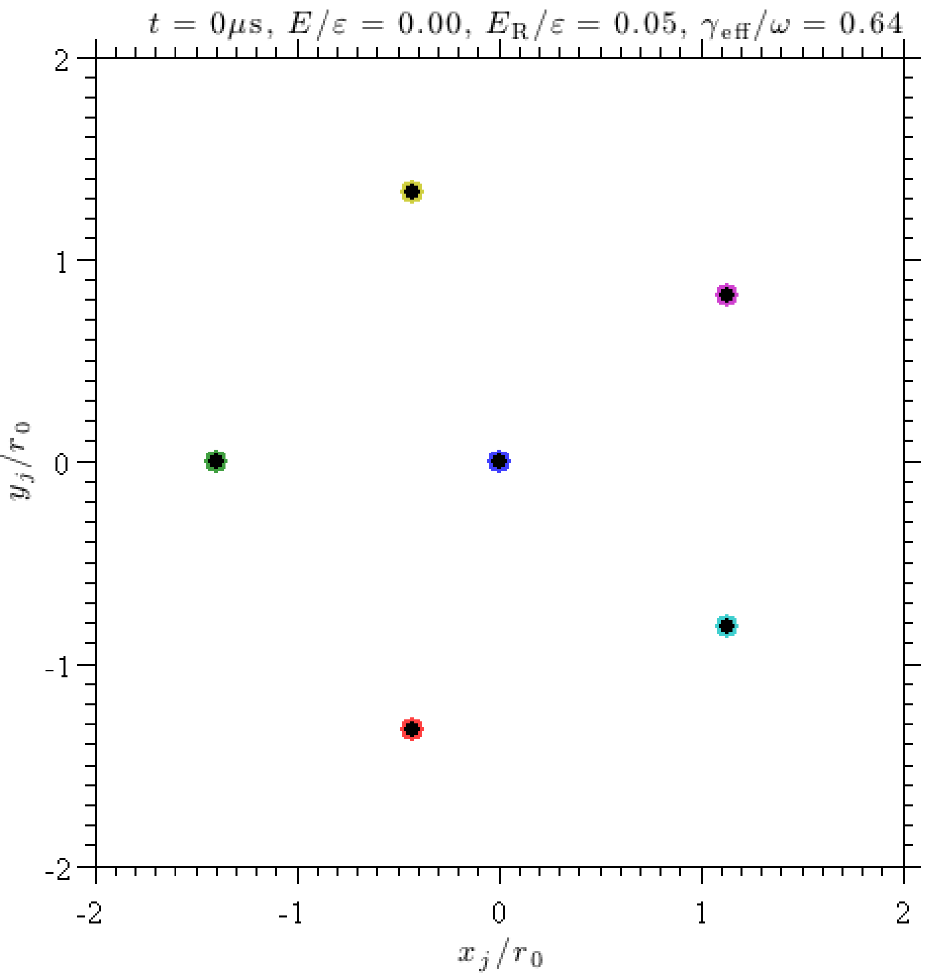
Crystals observable!

Spontaneous emission
effective heating rate

Classical melting temperature of a dipolar crystal $T_M \sim 0.1\epsilon_0$

Lifetime of a crystal: $\mathcal{T} \sim T_M/(\Gamma_{\text{eff}} E_R)$

Spontaneous emission: classical dynamics



^{87}Rb atoms $\quad n = 20$

$d_r \approx 1.45\text{kD} \quad \Gamma_r/2\pi \sim 100\text{kHz}$

$\mathcal{E}_{DC} = 25\text{kV/m}$

$\Omega/2\pi = 80\text{MHz} \quad \Delta/2\pi = 1\text{GHz}$

$\Gamma/2\pi \approx 640\text{Hz}$

$d \approx 9.1\text{Debye} \quad \mathcal{T} \approx 62.5\mu s$

- Numerics (classical molecular dynamics)

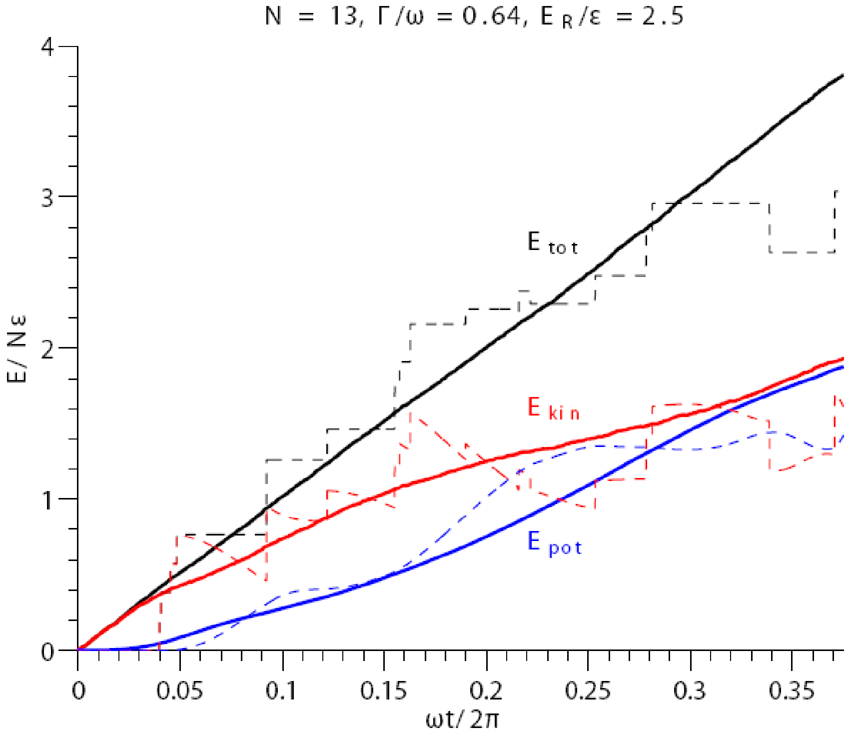
$\mathcal{T} \simeq 200\mu s$

Rydberg Crystal: out-of-equilibrium stability

now also cooling!
See Outlook: Zhao, Glaetzle, ...

$$d_{\text{max}} = \frac{3}{2} e\, a_0 n(n-1)$$

Classical molecular dynamics



^{87}Rb atoms $n = 20$

$d_r \approx 1.45\text{kD}$ $\Gamma_r/2\pi \sim 100\text{kHz}$

$\mathcal{E}_{\text{DC}} = 25\text{kV/m}$

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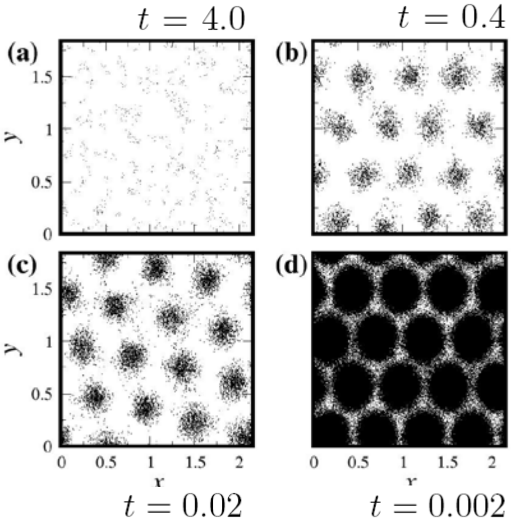
$\Gamma/2\pi \approx 640\text{Hz}.$

$d \approx 9.1\text{Debye}$ $\mathcal{T} \approx 62.5\mu\text{s}.$

- Numerics (classical molecular dynamics)

$\mathcal{T} \simeq 200\mu\text{s}.$

Modified dipolar interactions: new phases of matter with Rydberg-dressed atoms



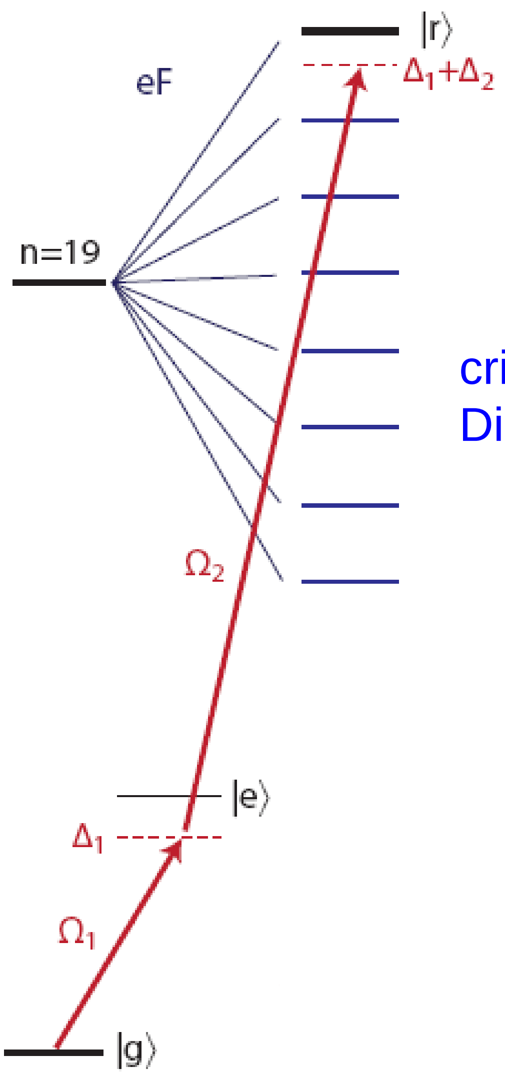
Cinti, Jain, Boninsegni, Micheli, Zoller, Pupillo,

PRL 105, 135301 (2010)

Red-detuning: „dipole-blockade‘

Single atom:

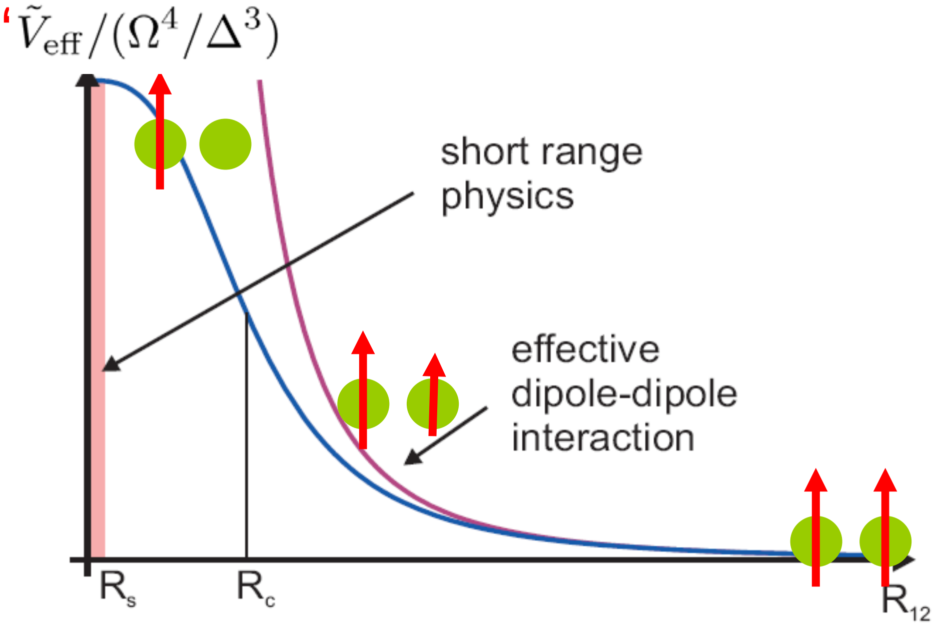
Dressing of ground-state
by Rydberg-states (in electric field)



$$R_c = \left[\frac{d_0^2}{8\pi\epsilon_0\Delta}\right]^{1/3}$$

critical radius >= 500nm

Dipole-dipole interaction for R > Rc



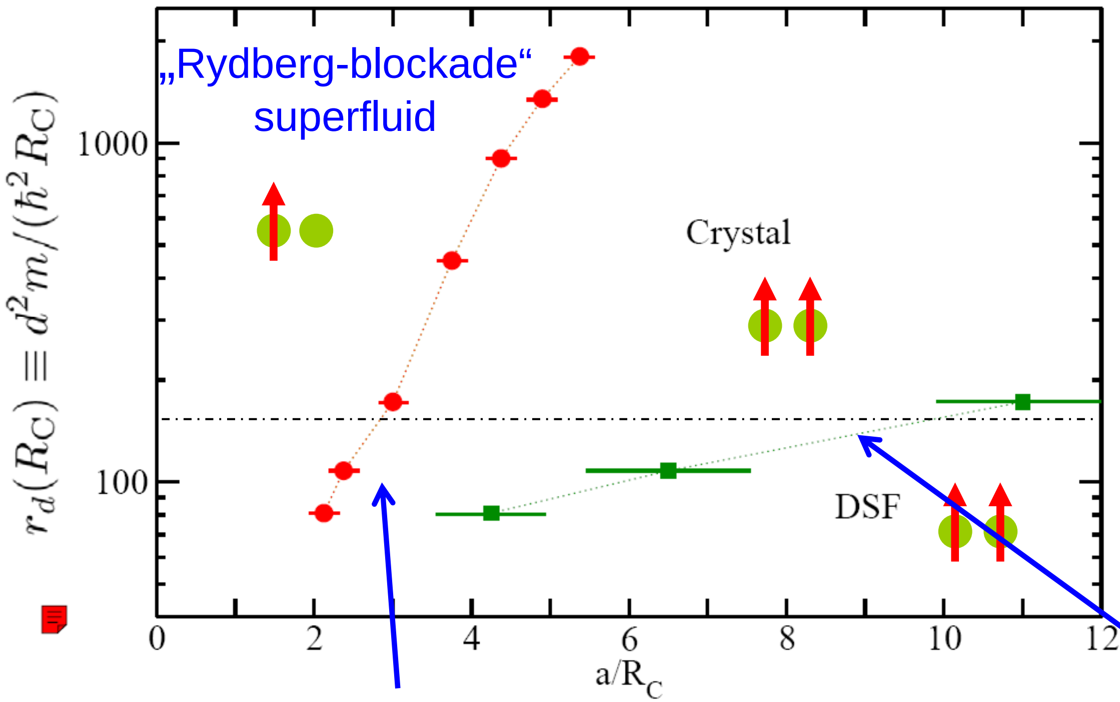
effective two-body interaction

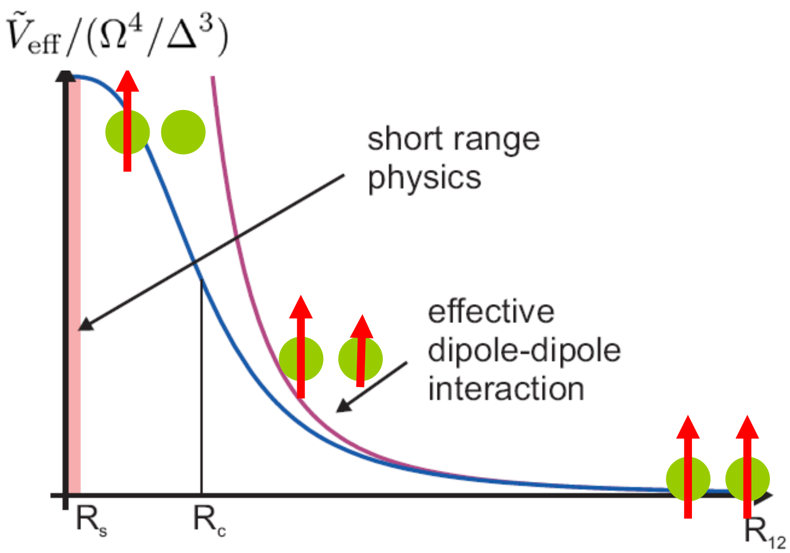
$$V_{\mathrm{ad},12} = -2\frac{\Omega^4}{\Delta^3}\left[1+\left(\frac{R_c}{R_{12}}\right)^3\right]^{-1}$$

see: I. Lesanovsky, T. Pohl, ...

Honer, Weimer, Pfau, Buechler arXiv:1004.2099

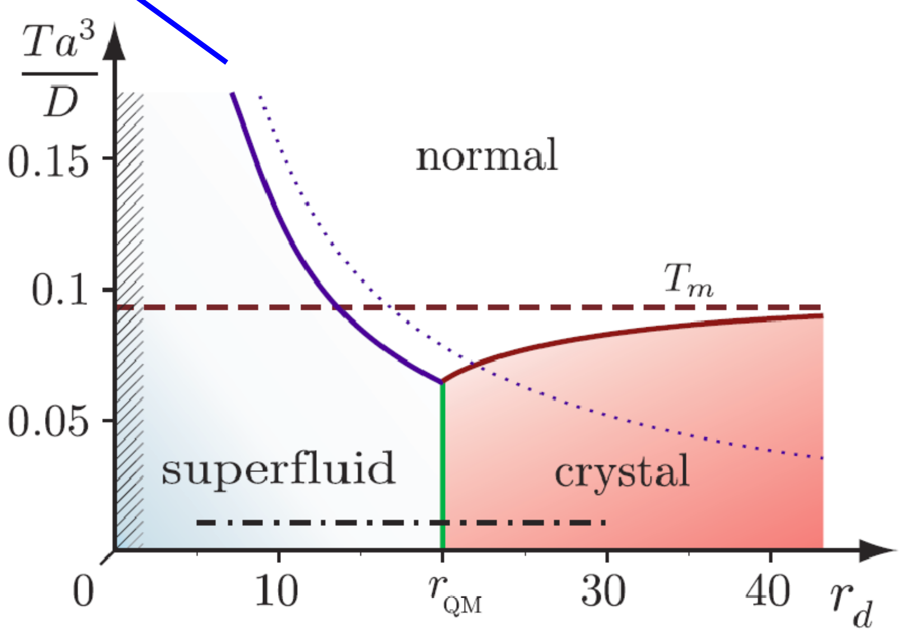
Quantum phases in red-dressed, dipole-blockaded gases



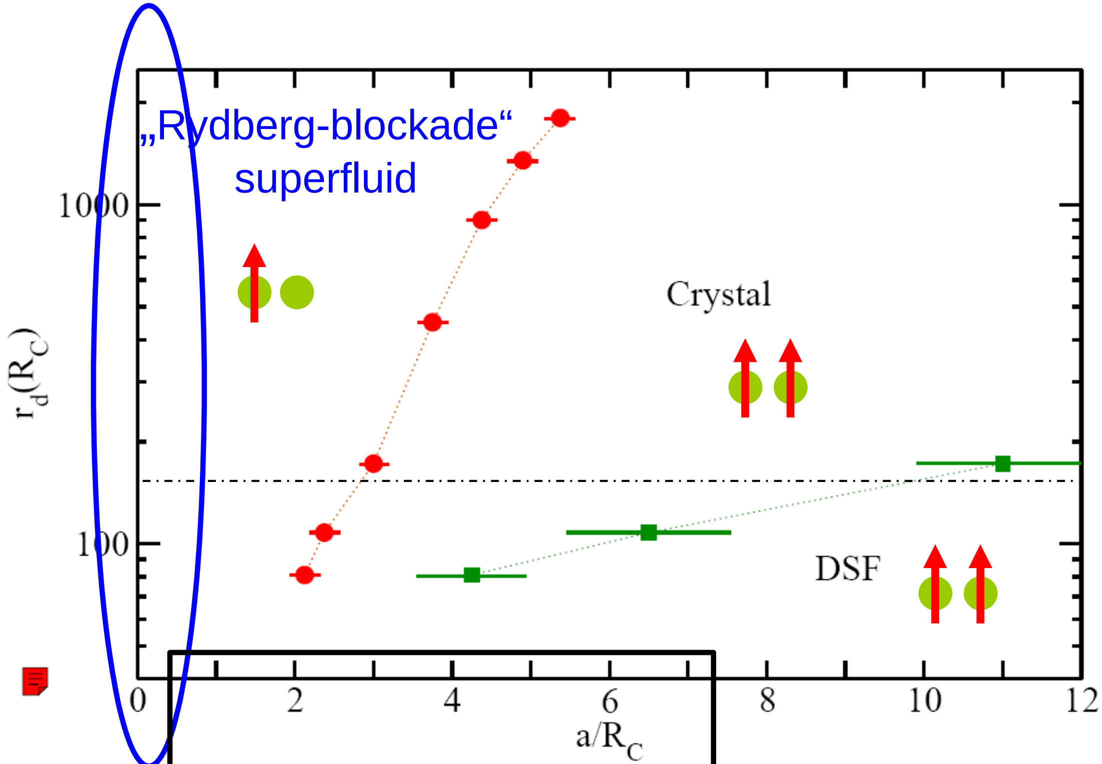


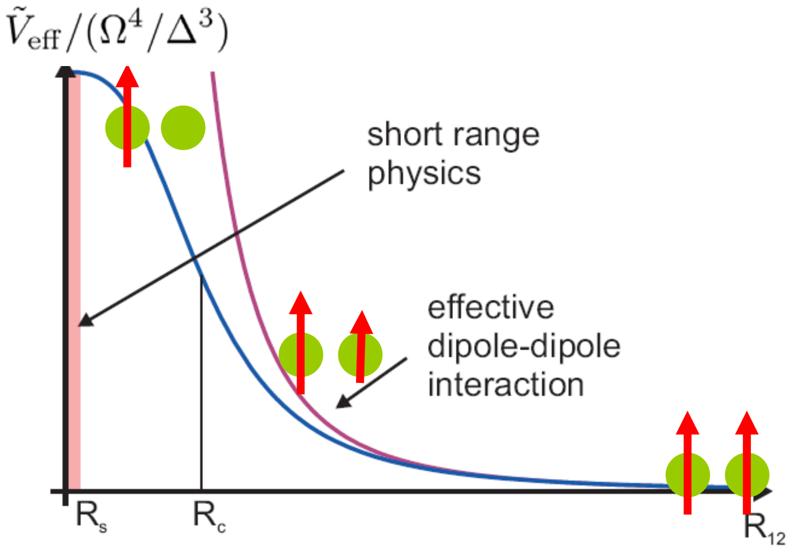
Intermediate-density: Re-entrant phase transition

Low-density: SF/C QFT (PRL 2007)

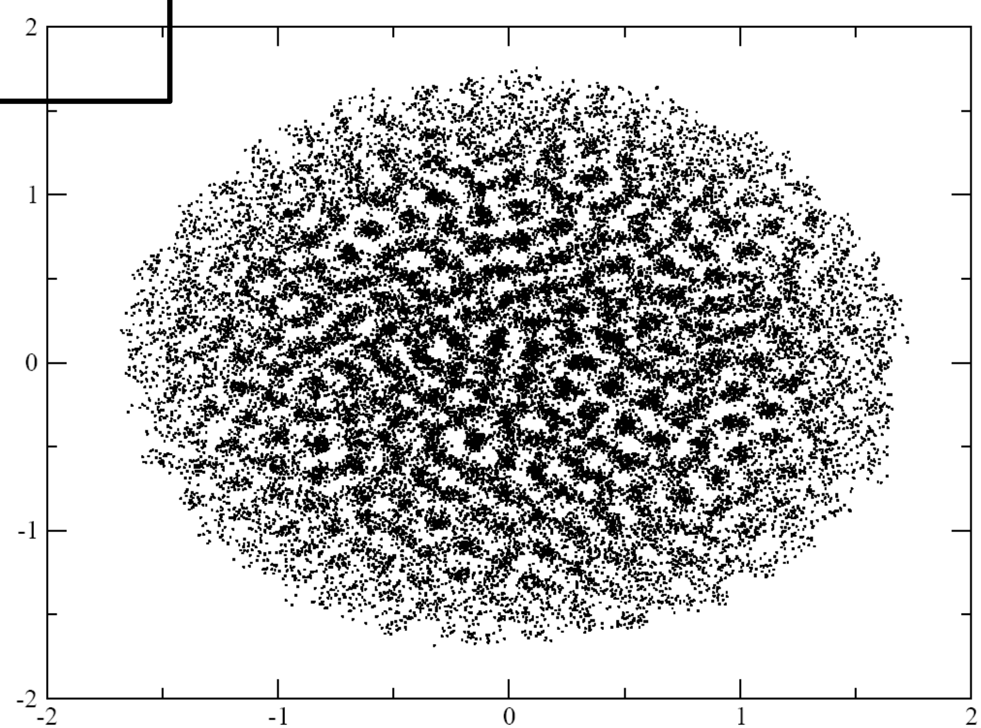


Quantum phases in red-dressed, dipole-blockaded gases





High-density: ..?



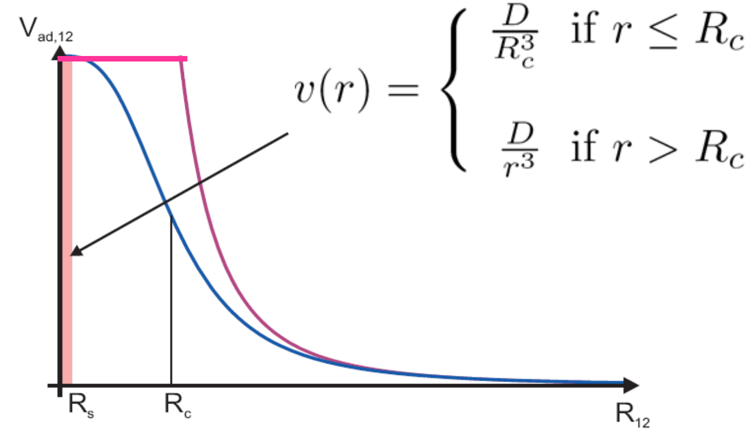
Droplet formation!

N=1000 particles

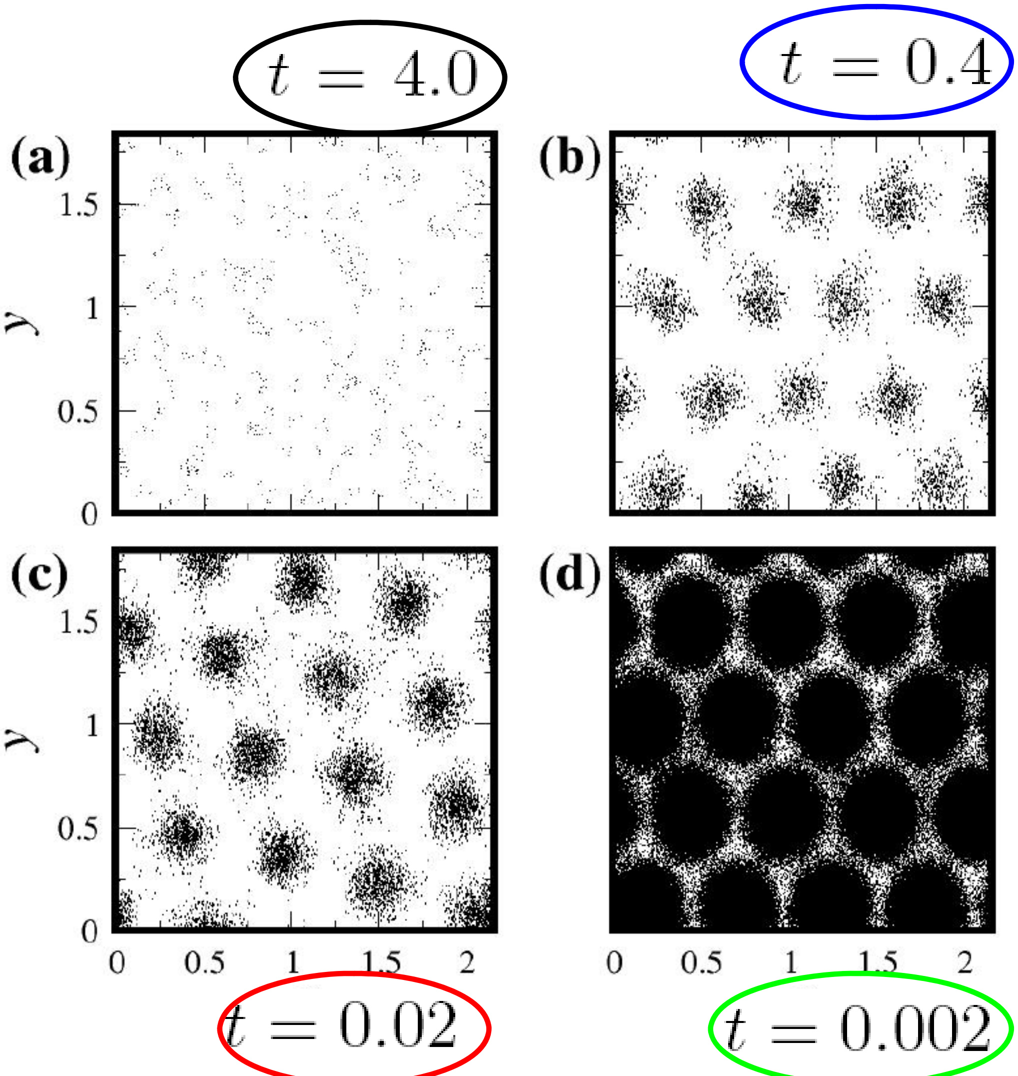
New model potential

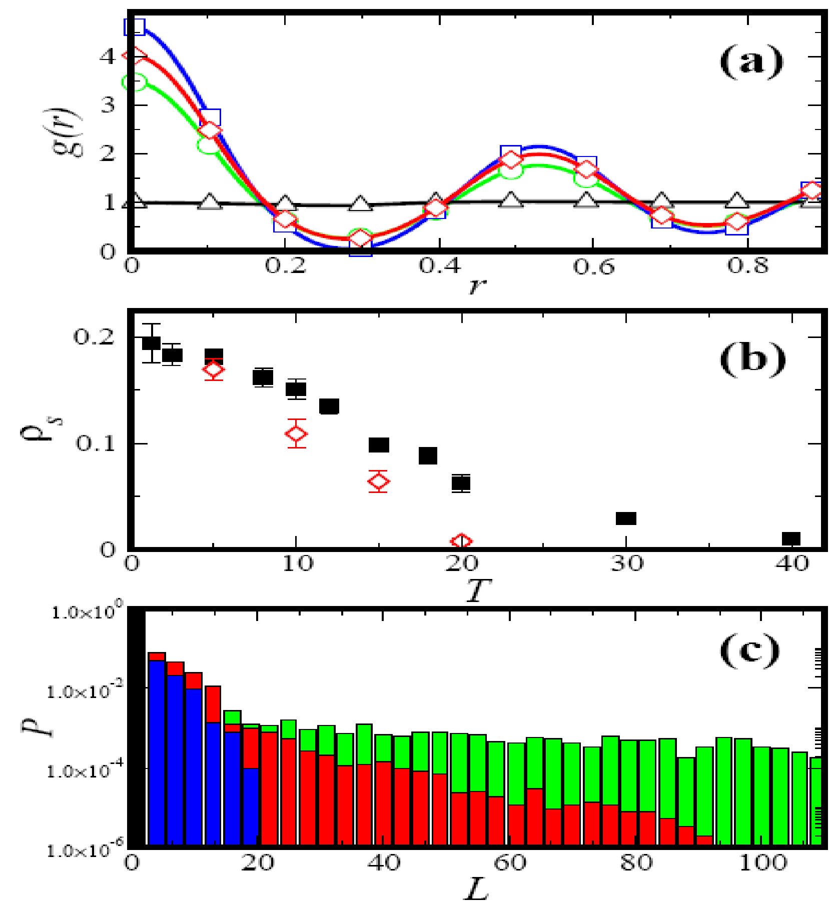
High-density: ..?

1) Choose: a < Rc
2) Start from a high temperature *t*, and lower it..
3) Energy scale: $\epsilon_\circ = D/r_\circ^3 = \hbar^2/mr_\circ^2$
4) Choose weak interactions (no single-particle crystals)
   $r_\circ/a < r_{QM} \simeq 18$



Density modulations



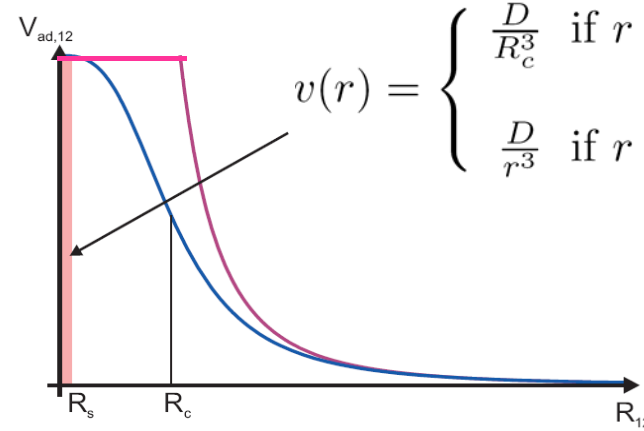


Off-diagonal long-range order..

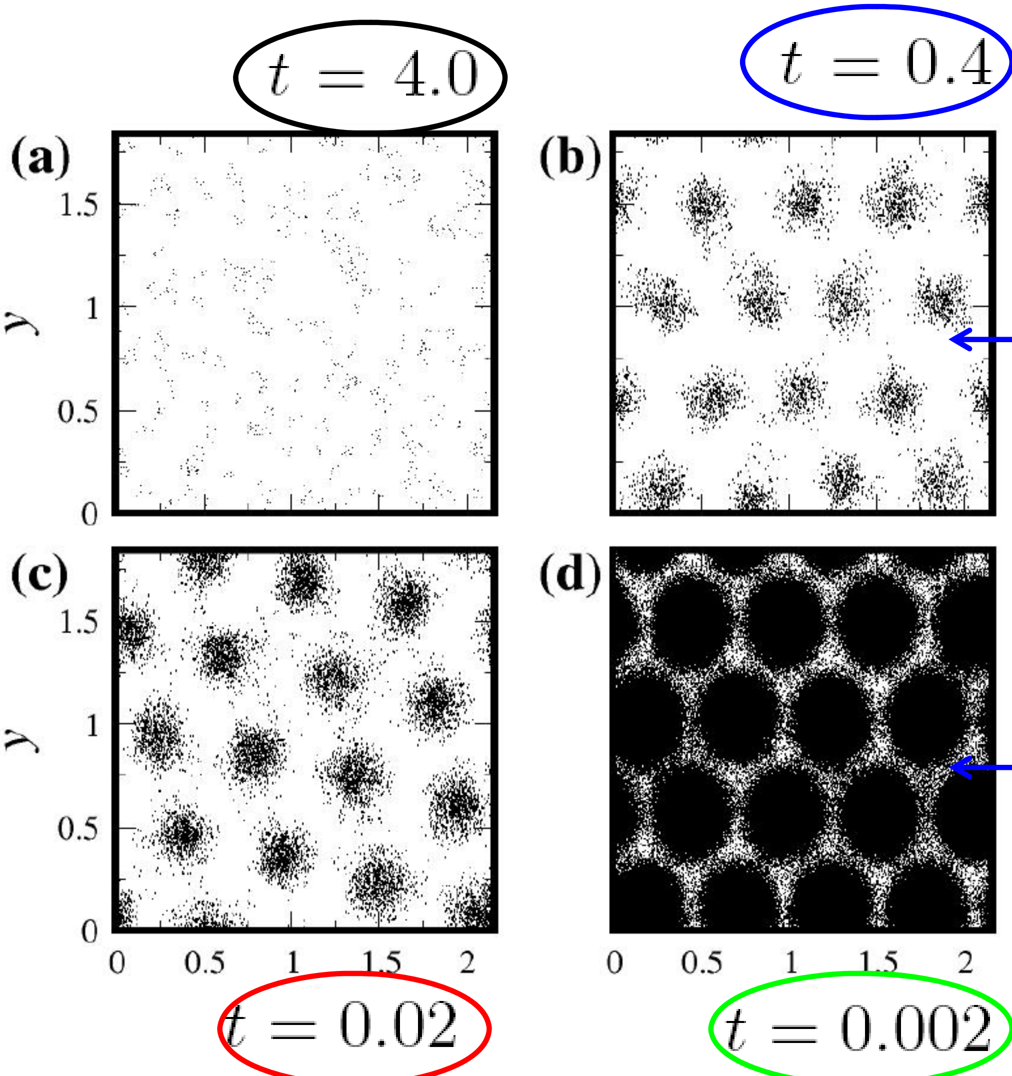
New model potential

High-density: ..?

1) Choose: a < Rc (~500-1000 nm)
2) Start from a high temperature *t*, and lower it..
3) Energy scale: $\epsilon_\circ = D/r_\circ^3 = \hbar^2/m r_\circ^2$ (~10-100 nK)
4) Choose weak interactions (no single-particle crystals) $r_\circ/a < r_{QM} \simeq 18$



$$v(r) = \begin{cases} \frac{D}{R_c^3} & \text{if } r \leq R_c \\ \frac{D}{r^3} & \text{if } r > R_c \end{cases}$$



2 NEW PHASES:

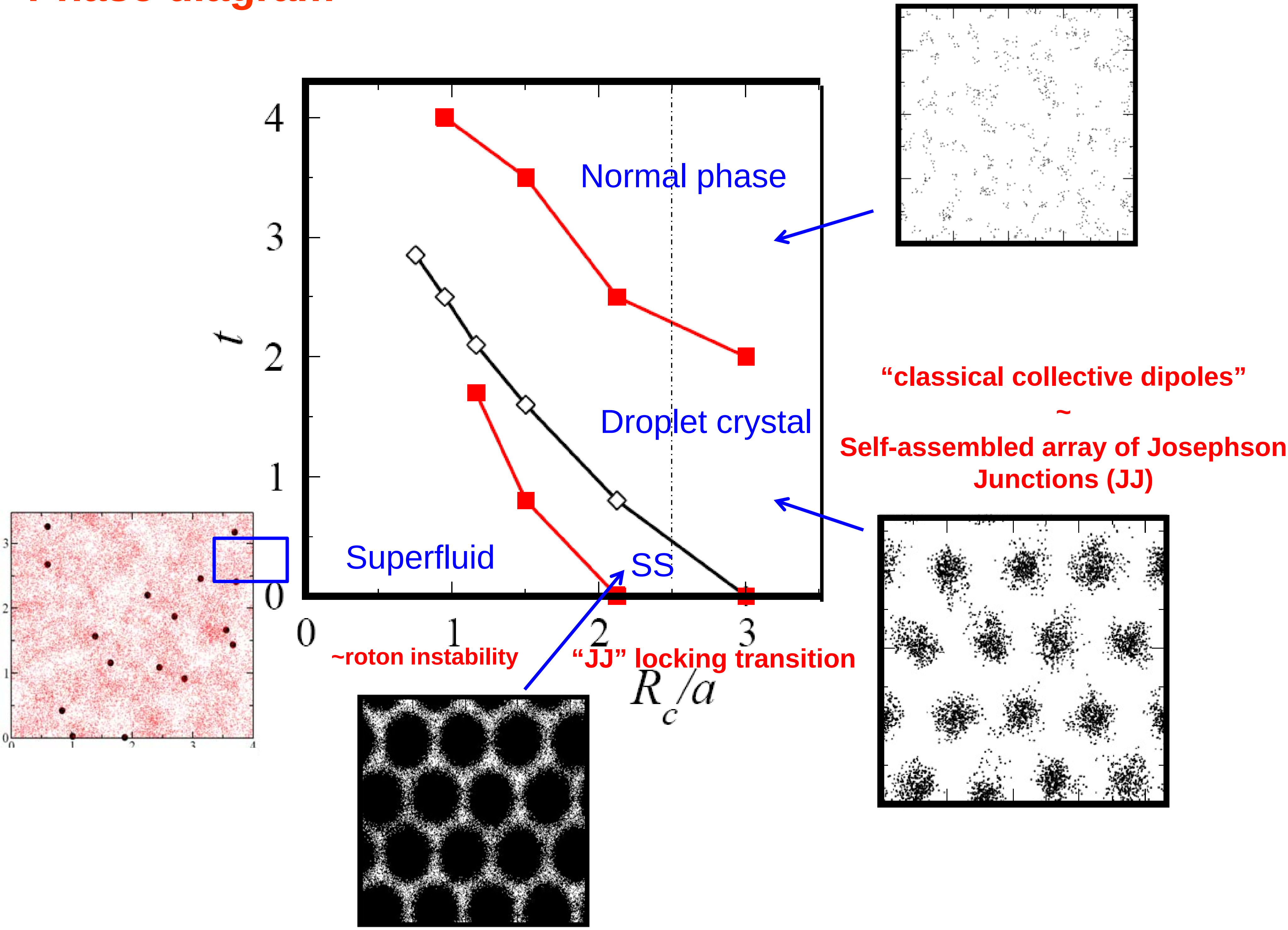
Crystal of mesoscopic superfluid droplets

Droplet crystal + finite superfluid fraction
=
Supersolid phase (SS)

see also Henkel, Nath, Pohl, PRL 2010
Li, Liu, Lin, arXiv:1005.4027

Santos, Shlyapnikov, et al., PRL 2003
O'Dell , Giovanazzi, Kurizki..

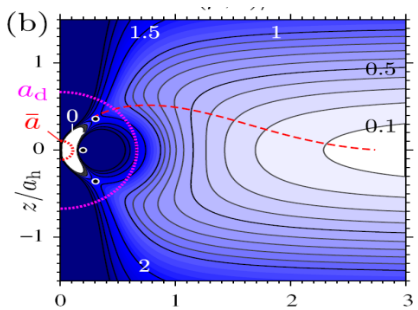
Phase diagram



Conclusions

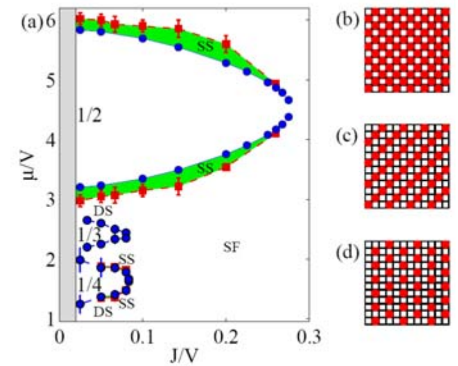
Stability and cooling in 2D

PRL 2010



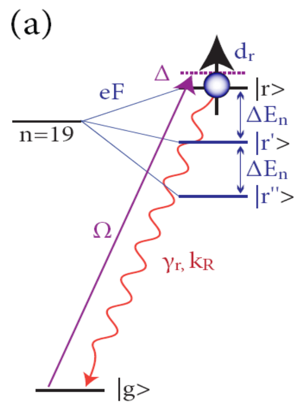
Phase-diagram of 2D lattice dipoles

PRL 2010



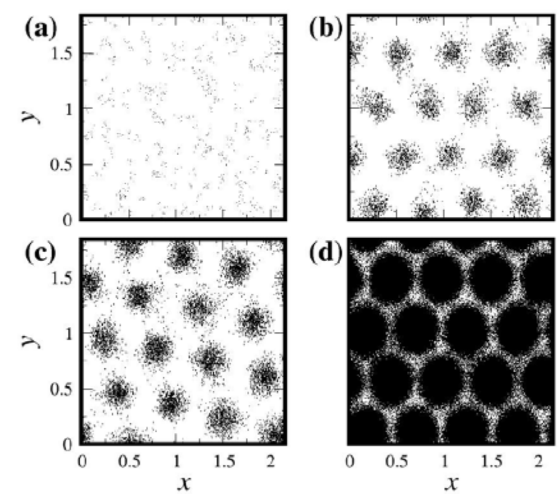
Rydberg-dressed atoms

PRL 2010



Supersolid droplet crystal

PRL 2010



Outlook

- **Bi/multi-layered structures of molecules?**
- **Dynamic properties of a supersolid?**
- **Heating and cooling of a Rydberg gas?**