Frontiers of Ultracold Atoms and Molecules, KITP, Oct. 14, 2010

Ultracold Fermi Gases in One and Quasi-one Dimension

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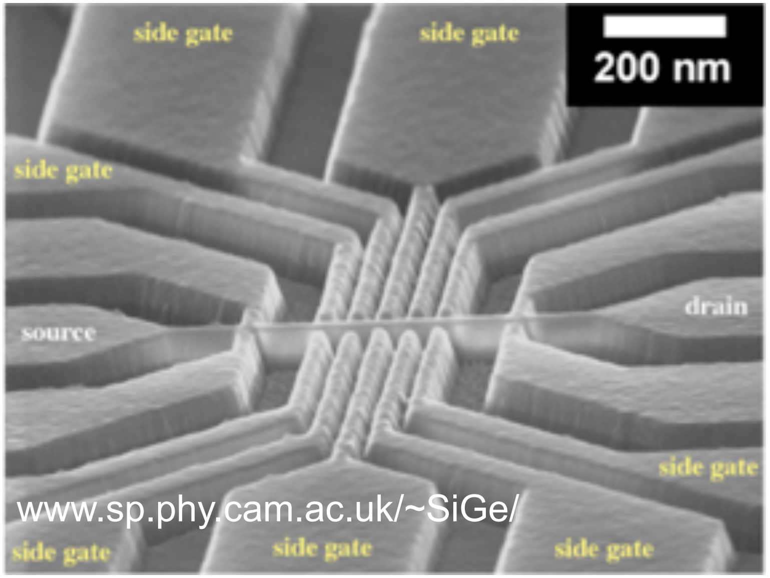
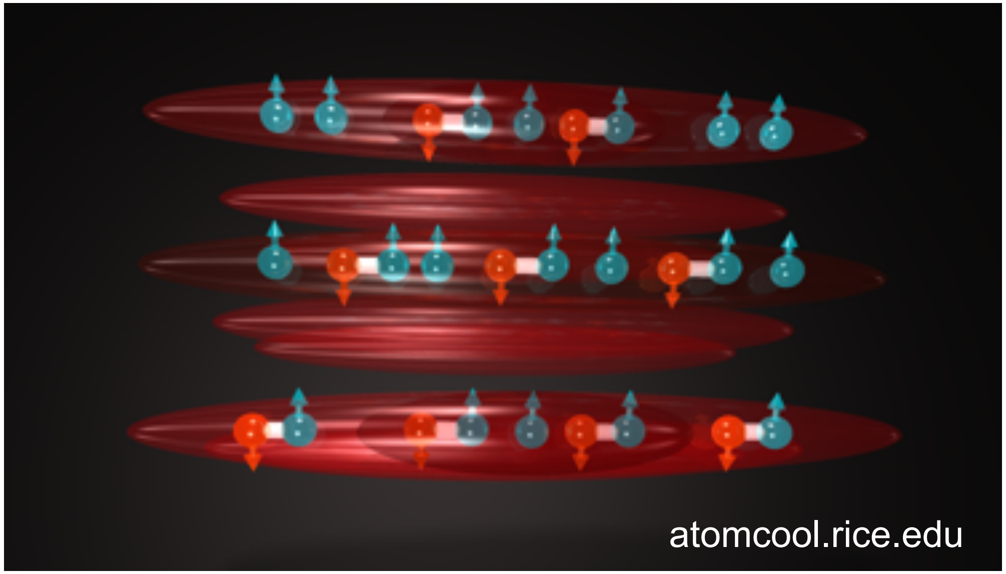
New addition to the 1D family

Electrons in quantum wire



Atoms in optical lattice (1D tubes)



Recent cold atoms experiments has brought us

- New 1D systems;
- Known 1D models in new parameter regimes;
- New theoretical challenges.

I will be very specific and focus only on one example: the equilibrium properties of 1D Fermi gases with attractive interaction and spin imbalance.

Motivation: the Rice experiment

doi:10.1038/nature09393

Spin-imbalance in a one-dimensional Fermi gas

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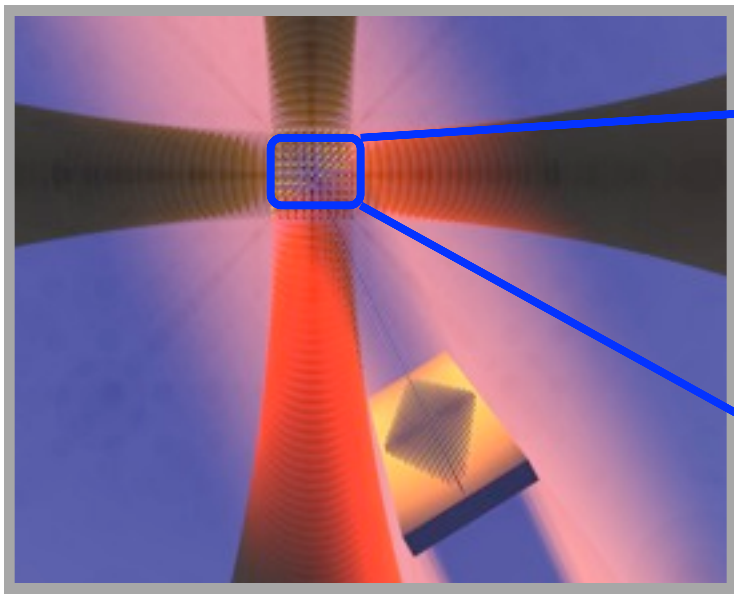
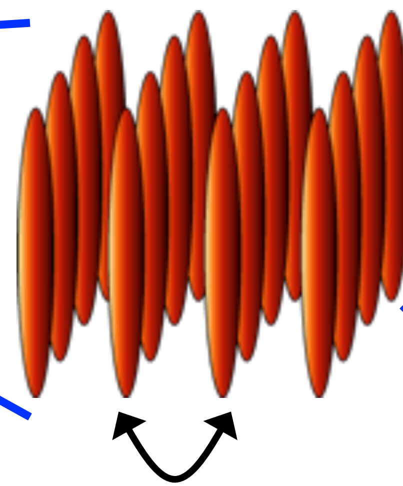
Talk by Prof. Hulet on Monday.

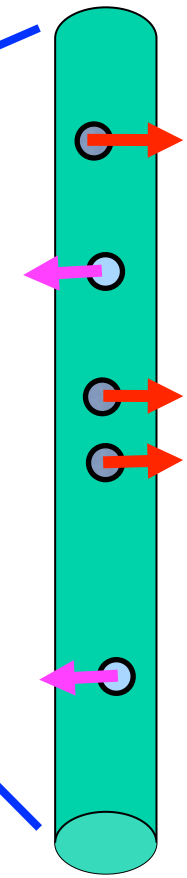
Liao et al, Nature 467, 567 (2010)



Crossed beam optical trap



2D lattice array



Spin mixtures of ^{6}Li atoms trapped in an array of 1D tubes.

Earlier experiments: Moritz et al, PRL 94, 210401 (2005) and others as reviewed in, e.g., Bloch, Dalibard, Zwerger, RMP 80, 885 (2008); Giorgini, Pitaevskii, Stringari, RMP 80, 1215 (2008)

The Gaudin-Yang model

One-dimensional Fermi gas with contact interaction described by Gaudin-Yang model

$$H = -\int dx \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger(x) \left[\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \mu_\sigma\right] \psi_\sigma(x) - g \int dx \psi_\uparrow^\dagger(x)\psi_\downarrow^\dagger(x)\psi_\downarrow(x)\psi_\uparrow(x).$$

M. Gaudin, *Phys. Lett. A* 24 (1967); C. N. Yang, *PRL* 19 (1967).

1. We focus on attractive interaction.
2. Spin up and down fermions can have different chemical potentials, say $\mu_\uparrow > \mu_\downarrow$

   define (the average) chemical potential $\mu = (\mu_\uparrow + \mu_\downarrow)/2$

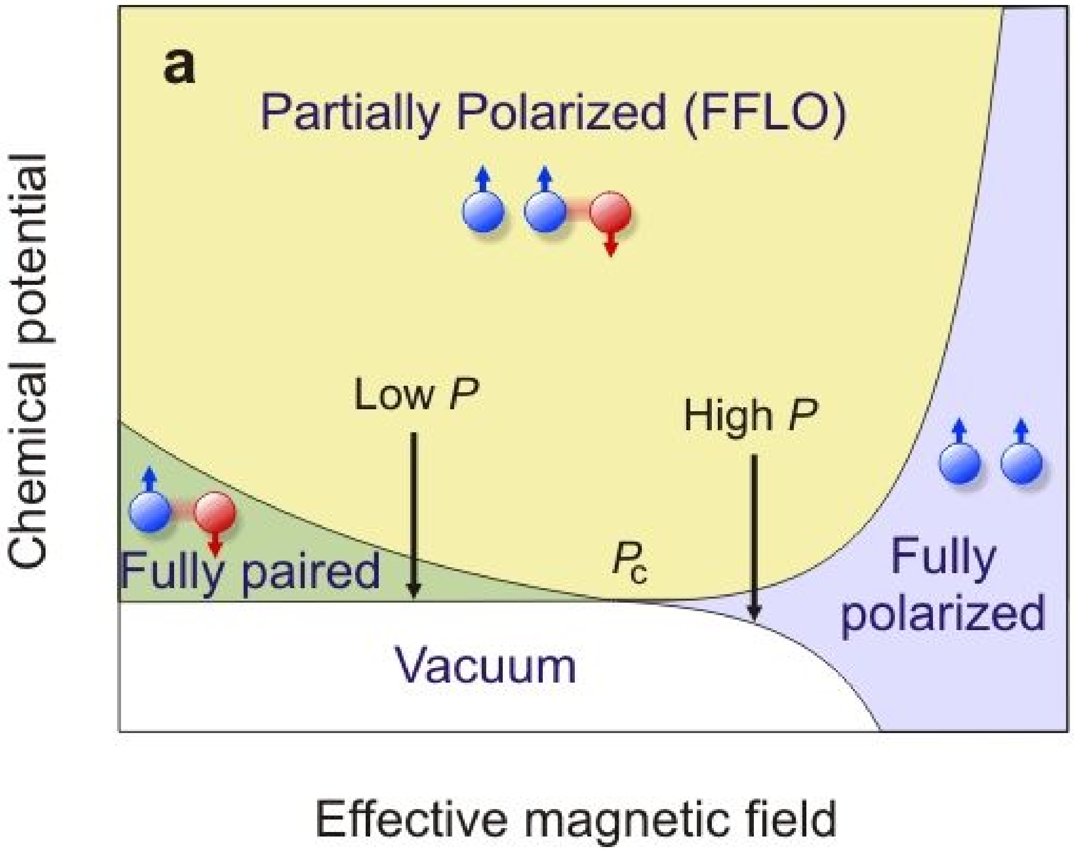
   and the effective magnetic field $h = (\mu_\uparrow - \mu_\downarrow)/2$

In canonical ensemble, this corresponds to having total density $n = n_\uparrow + n_\downarrow$

magnetization (spin imbalance) $M = n_\uparrow - n_\downarrow$ and polarization $p = M/n$

The zero temperature phase diagram

The GY model is exactly solvable by Bethe ansatz both at zero and finite T.



From Liao et al, Nature 467, 567 (2010)

Experiments are in the

strong coupling limit

$$\gamma \gg 1, \; i.e., \; \epsilon_B \gg \mu$$

at "low" temperatures

$$T \ll \epsilon_B$$

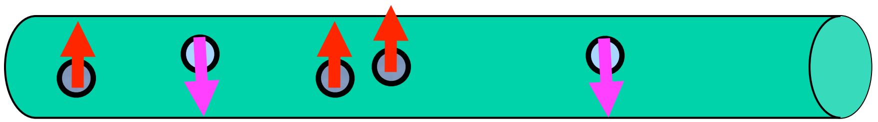
$$\gamma = 2/(na_1)$$

$$a_{1D} = -\frac{2\hbar^2}{mg_{1D}}, \quad \epsilon_B = \frac{\hbar^2}{ma_{1D}^2}.$$

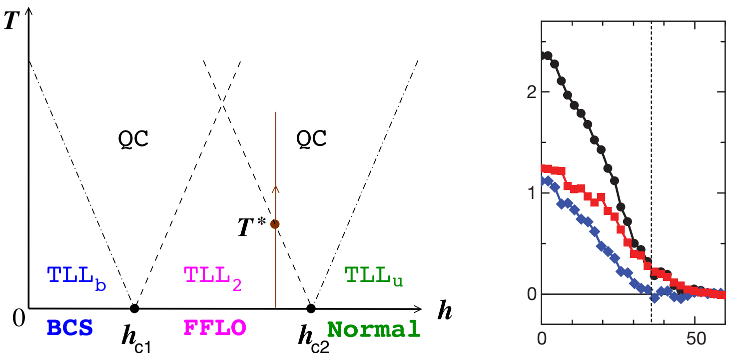
Obtained by Orso, PRL 98 (2007); Hu et al PRL (2007); Guan et al, PRB (2007)

Outline

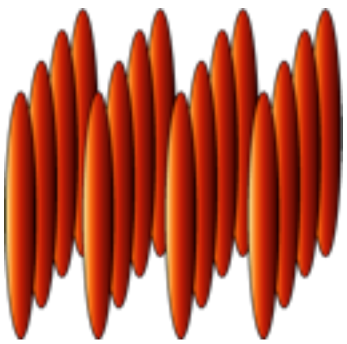
1. Understanding the 1D FFLO phase



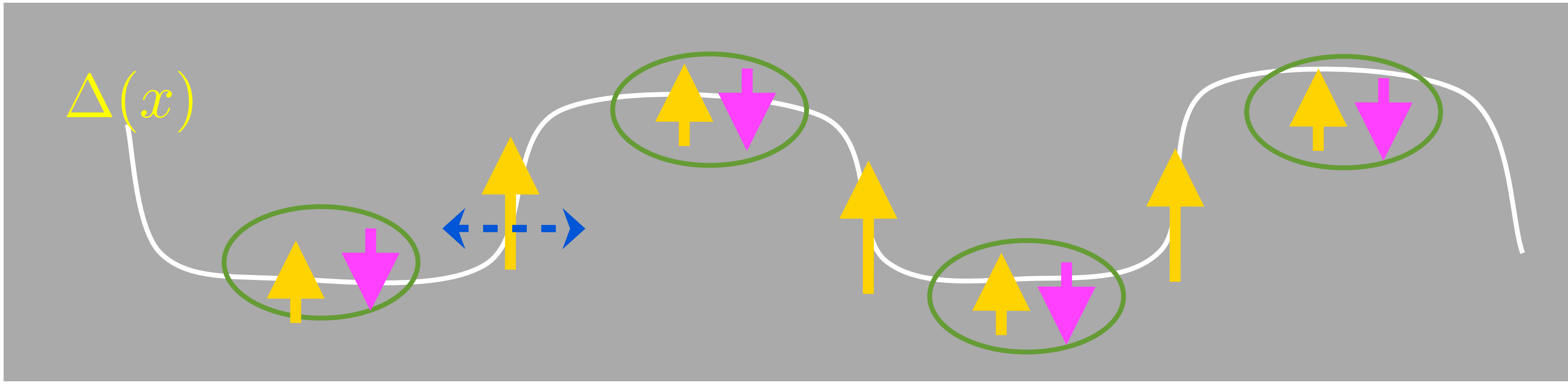
2. Exact thermodynamics



3. Phase diagram of weakly coupled tubes



What is the nature of the "1D FFLO" phase?



In 1D, quantum fluctuations will melt any crystalline structure of Δ. So 1D FFLO has to be a quantum critical phase with only algebraic order.

- What kind of quantum liquid is it?
  (Neither a Luther-Emery liquid nor a spin-charge separated Luttinger liquid)

- What are the elementary excitations?

- What are the (analytic form of) correlation functions at zero and finite T?

Bethe ansatz alone cannot answer all these questions. We need an effective field theory [in the spirit of Haldane, J. Phys. A 15, 507 (82)] to describe the new 1D quantum fluid.

Bosonization of GY model

For weak attraction, linearize the spectrum at two Fermi surfaces,

$$\bar{v}_f = \frac{v_{f,\uparrow} + v_{f,\downarrow}}{2}, \quad \delta v_f = v_{f,\uparrow} - v_{f,\downarrow}, \quad \delta k_f = k_{f,\uparrow} - k_{f,\downarrow}.$$

and the charge and (shifted) spin field,

$$\phi_c = \frac{\phi_\uparrow + \phi_\downarrow}{\sqrt{2}}, \quad \phi_s = \frac{\phi_\uparrow - \phi_\downarrow - \delta k_f x}{\sqrt{2}}.$$

Then the bosonized Hamiltonian becomes, $H = \int dx h(x)/2\pi$ with

$$\begin{aligned} h(x) \quad &= \boxed{\bar{v}_f (\nabla\theta_c)^2 + (\bar{v}_f + \frac{g}{\pi})(\nabla\phi_c)^2} + \delta v_f \delta k_f \frac{\nabla\phi_c}{\sqrt{2}} \\ &+ \boxed{\bar{v}_f (\nabla\theta_s)^2 + (\bar{v}_f - \frac{g}{\pi})(\nabla\phi_s)^2} + \delta k_f (\bar{v}_f - \frac{g}{\pi})\sqrt{2}\nabla\phi_s \\ &+ \delta v_f (\nabla\theta_c \nabla\theta_s + \nabla\phi_c \nabla\phi_s) \\ &+ \boxed{\frac{g}{\pi a^2}\cos(\sqrt{8}\phi_s)} \end{aligned}$$

sine-Gordon

effective magnetic field

spin-charge mixing (marginal)

Non-perturbative bosonization

Gaudin-Yang model H_{GY},

Lattice regularization ⇓ ⇑ Continuum limit

Attractive Hubbard model H_U

⇓ Particle-hole transformation

Repulsive Hubbard model H'_U

Conformal field theory ⇓ Bethe ansatz

Frahm & Korepin (91)

Penc & Solyom (93)

Two-component Luttinger H_{TL}

1) Bosonization ⇓ 2) Diagonalization
3) P-h transform back

Free energy ⇐ H_{FFLO} ⇒ Correlation functions

Affleck (86); Blote et al (86)

H. Frahm, T. Vekua (08)

Bethe ansatz

Microscopic Hamiltonian → Long wave length, low energy description

Field-theory

Effective field theory of 1D FFLO

1D FFLO is a two-component Luttinger liquid with **spin-charge mixing,**

$$H_{\mathrm{FFLO}} = \sum_{i=1,2} \int \frac{dx}{2\pi} u_i \left[(\nabla \vartheta_i)^2 + (\nabla \varphi_i)^2 \right]$$

- Two gapless excitations (normal modes). Each is admixture of spin and charge excitations. The degree of mixing is described by the dressed charge matrix *Z*.
- Coefficients/parameters (u_i, Z_{ij}) calculated from the *Bethe ansatz*.
- The theory applies to arbitrary interaction strength, and any 0<p<1.
- Correlation functions and thermodynamics can be computed easily.

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = \left(\bar{Z}^{\mathrm{T}}\right)^{-1} \begin{pmatrix} -\phi_\uparrow \\ \phi_\downarrow \end{pmatrix} \qquad \begin{pmatrix} \vartheta_1 \\ \vartheta_2 \end{pmatrix} = \bar{Z} \begin{pmatrix} -\theta_\uparrow \\ \theta_\downarrow \end{pmatrix} \qquad \bar{Z} = \begin{bmatrix} Z_{cc} - Z_{sc} & Z_{sc} \\ Z_{ss} - Z_{cs} & -Z_{ss} \end{bmatrix}$$

Kun Yang developed a field theory for 1D superconductor in magnetic field, Phys. Rev. B **63**, 140511(R) (2001), where spin-charge mixing was neglected.

Correlation functions

The effective field theory enables us to compute the single particle propagator and pair correlation function analytically:

$$G_\sigma(x,\tau) = -\langle T_\tau \psi_{R,\sigma}(x,\tau)\psi^\dagger_{R,\sigma}(0,0)\rangle,$$

$$\chi_{(}x,\tau) = -\langle T_\tau \Delta(x,\tau)\Delta^\dagger(0,0)\rangle.$$

$$\Delta(x) = \psi_{R,\uparrow}(x)\psi_{L,\downarrow}(x)$$

At T=0 and for $x \rightarrow \infty$, we find

$$G_\uparrow(x) \sim \frac{e^{ik_{f\uparrow}x}}{x^{2\delta_\uparrow}},$$

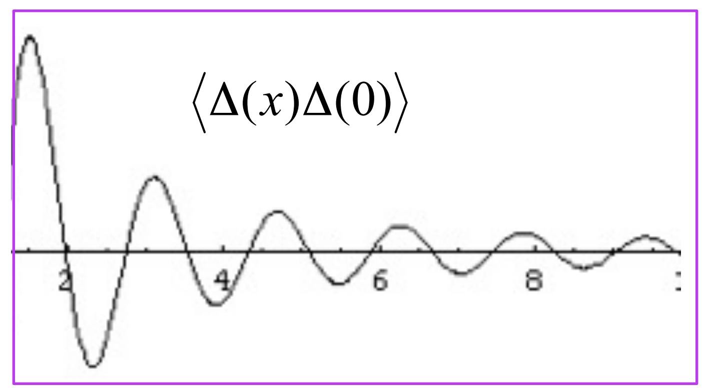
$$\boxed{\chi(x) \sim \frac{e^{iq_\star x}}{x^{2\delta_\Delta}},}$$

$q_\star = k_{f,\uparrow} - k_{f,\downarrow}$ FFLO wave vector

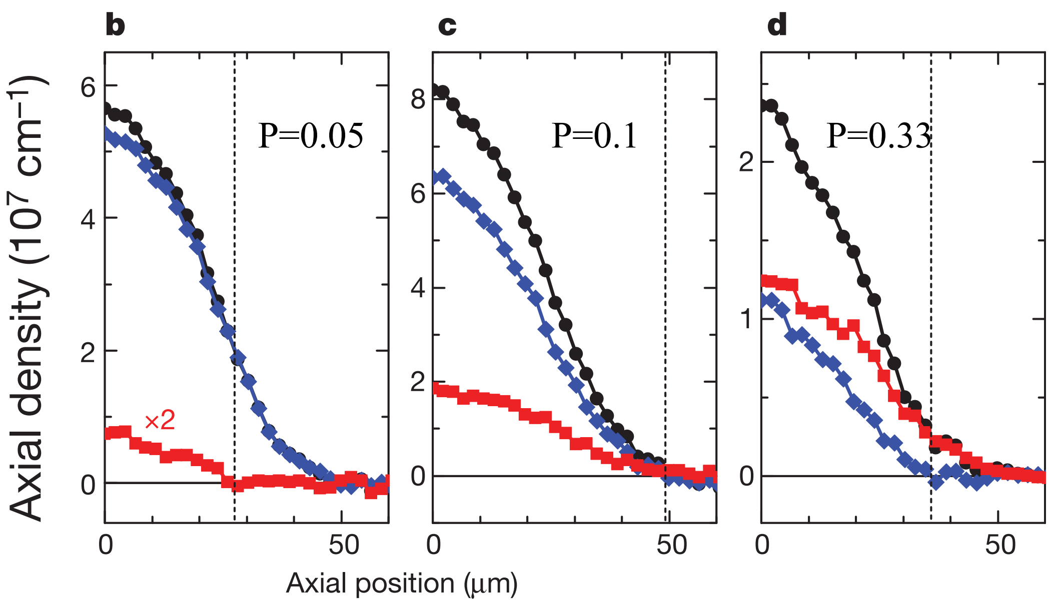
$\delta_\uparrow$ Scaling dimension of single particle field operator

δ_Δ Scaling dimension of pair operator

Signatures of 1D FFLO



Thermodynamics of Gaudin-Yang gas



black: $n_\uparrow$, blue: $n_\downarrow$, red: $n_\uparrow - n_\downarrow$ Liao et al, Nature 467, 567 (2010)

Goal:

To find a cheap/efficient way to compute the finite T thermodynamics of GY gas to help interpreting the data and determine T.

The basic equations for the exact thermodynamics of GY model have been known for quite some time. See for example, Takahashi (99); Guan et al, PRB 76, 085120 (07).

The so-called thermodynamic Bethe ansatz equations can only be solved numerically.
P. Kakashvili and C. J. Bolech, PRA 79, 041603 (09).

But the procedure is complicated and expensive.

Excitations in the strong coupling limit

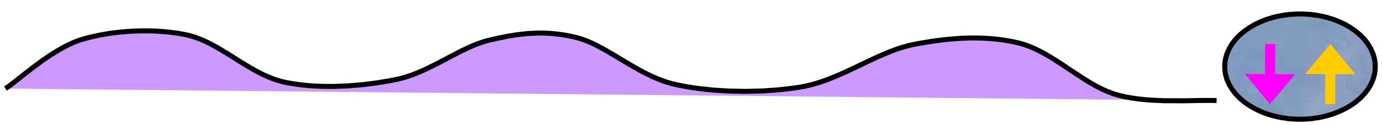
In the strong coupling limit (realized in experiments), simplification occurs:

The 1D FFLO phase becomes two almost free gases:

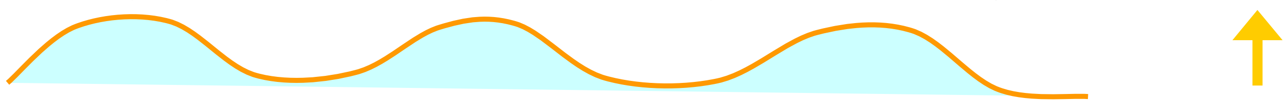
1) a gas of hard-core “molecules” → free gas of spinless fermions;
2) a free gas of leftover fermions.

Three kinds of excitations:

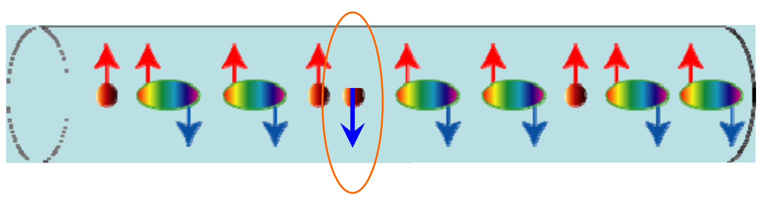
1. Density fluctuations of bound pairs — gapless



2. Density fluctuations of unpaired fermions — gapless



3. Spin fluctuations (wrong spins) — cost finite energy, gapped



(broken pairs or flipped spins)

They are suppressed for $T \ll \varepsilon_B, 2h$

Thermodynamic Bethe Ansatz simplified

In the experimental regime, $T \ll \epsilon_B, 2h; \quad \gamma \gg 1$

We find the TBA equations simplify to 4 coupled algebraic equations:

$$
\begin{aligned}
\mu_b &= \mu + \frac{\varepsilon_B}{2} - \frac{a_1}{4} p_b - a_1 p_u, \\
\mu_u &= \mu + T \ln(2 \cosh \frac{h}{T}) - a_1 p_b - \frac{a_1}{4} p_u \mathrm{sech}^2 \frac{h}{T},
\end{aligned}
$$

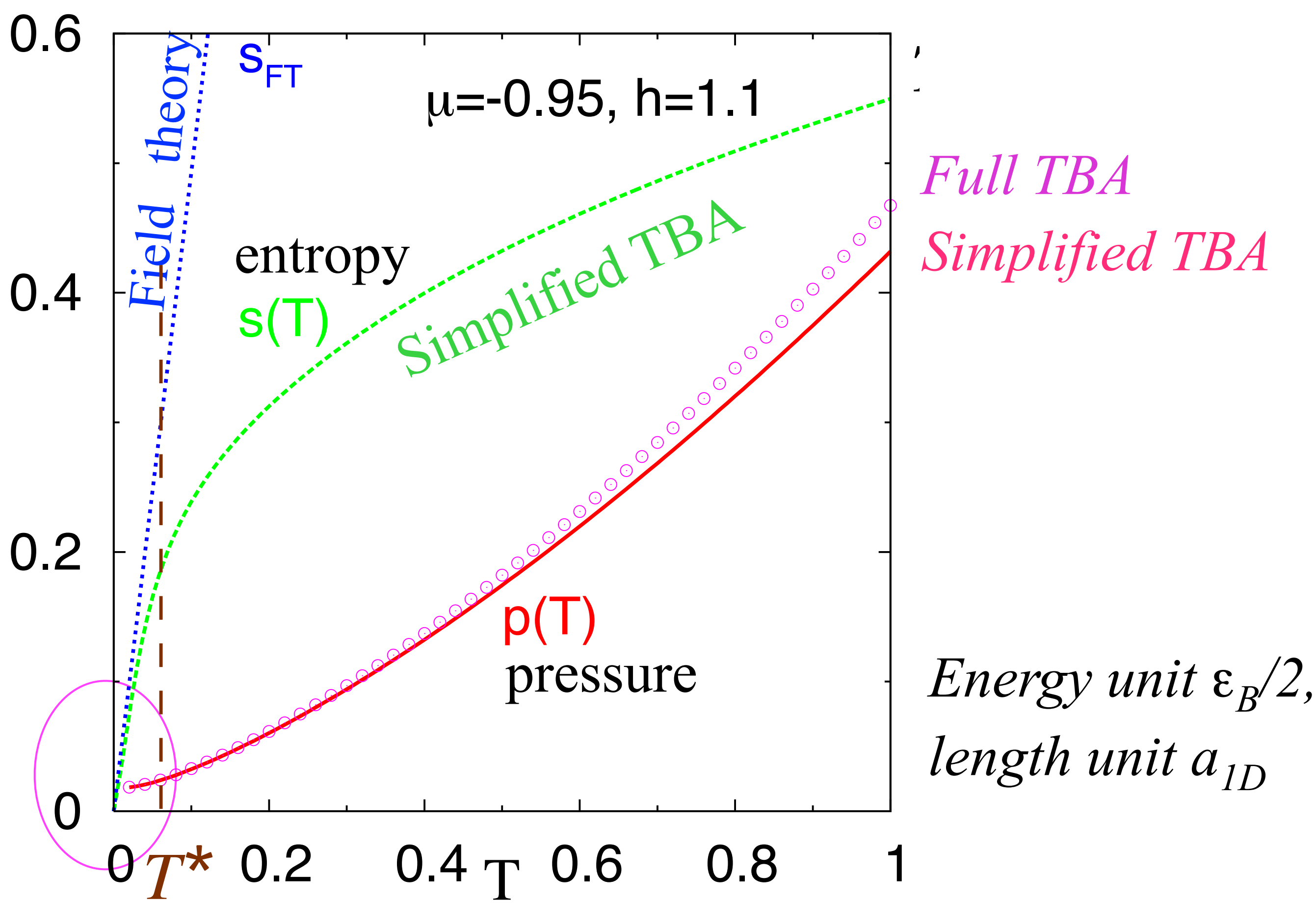
$$
\left.
\begin{aligned}
p_b &= -\sqrt{\frac{m}{\pi \hbar^2}} T^{3/2} \mathrm{Li}_{\frac{3}{2}}(-e^{2\mu_b/T}), \\
p_u &= -\sqrt{\frac{m}{2\pi \hbar^2}} T^{3/2} \mathrm{Li}_{\frac{3}{2}}(-e^{\mu_u/T}).
\end{aligned}
\right\} \textit{partial pressure}
$$

Furthermore, for when $T \ll h$, $\mu_u = \mu + h - a_1 p_b$.

The thermodynamic potential: $\mathcal{G} = -p = -(p_b + p_u)$

^{6}Li gas described by $\mathrm{Li}_{3/2}(x)$

How good is the simplified TBA

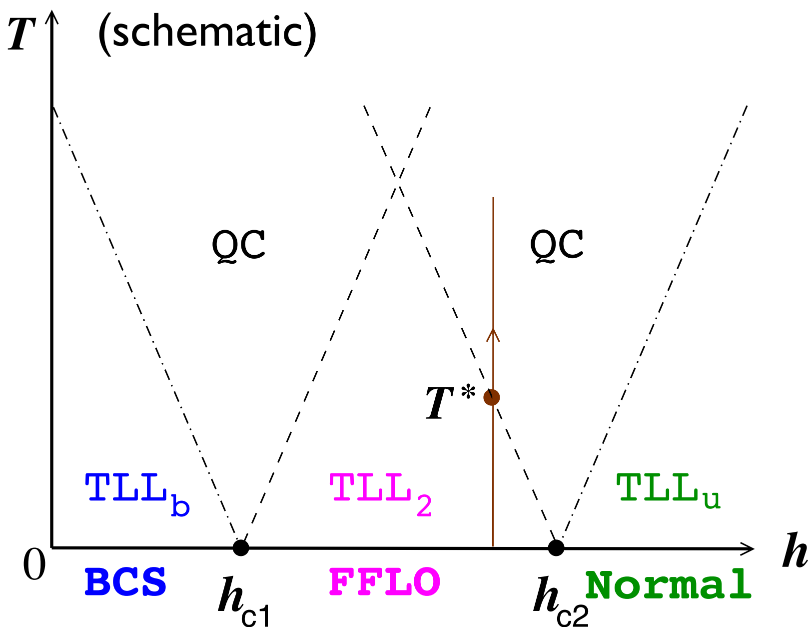


TBA **agrees** with field theory when $T \ll \mu_u, \mu_b$

$$s_{FT} = \frac{\pi}{3\hbar}\left(\frac{1}{v_b} + \frac{1}{v_u}\right)T + \ldots$$

Similar truncation approximation was employed by Bauer and Mueller et al to fit the data of Rice experiment. Liao et al, Nature 467, 567 (2010)

Crossover at finite temperature



Smooth crossover at finite T

TLL description valid for T<T*, away from the quantum critical points.

3 phases, 2 quantum critical points at T=0

EZ et al, *PRL* 103, 140404 (2009)

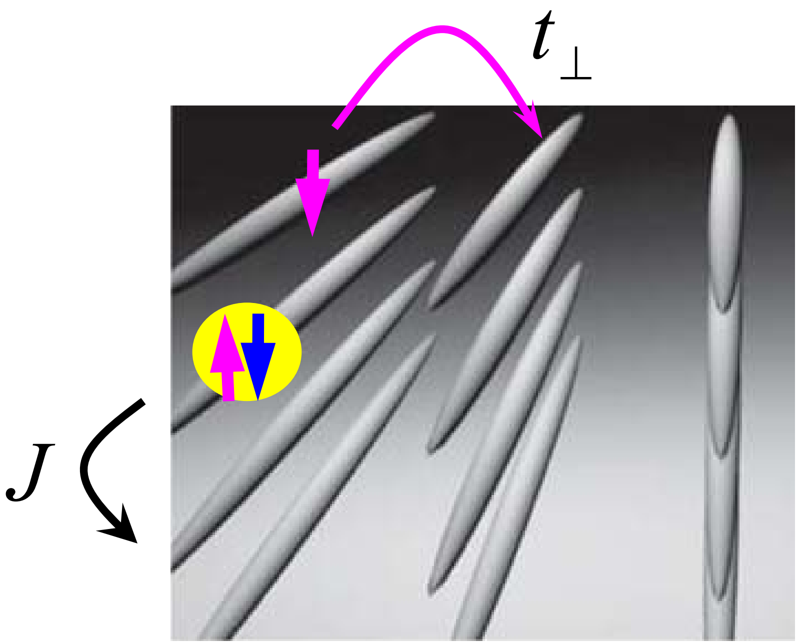
TLL: Tomonaga-Luttinger(-like) liquid
QC: quantum critical region
T*: crossover temperature from relativistic to non-relativistic dispersion

Very recent work includes higher order corrections to simplified TBA to discuss the equation of state and quantum criticality of the same model.

Xiwen Guan and Tin-Lun Ho, arXiv:1010.1301 (2010).

Beyond 1D: inter-tube coupling

Single particle inter-tube tunneling tends to confine the fractional excitations in 1D to give birth to well defined quasiparticles.



Momentum scale

Scaling dimension of single particles

$$\kappa dt_\perp(\kappa)/d\kappa = (2\delta_\uparrow - 2)t_\perp(\kappa)$$
$$\kappa dJ(\kappa)/d\kappa = (2\delta_\Delta - 2)J(\kappa)$$

Scaling dimension of pairs

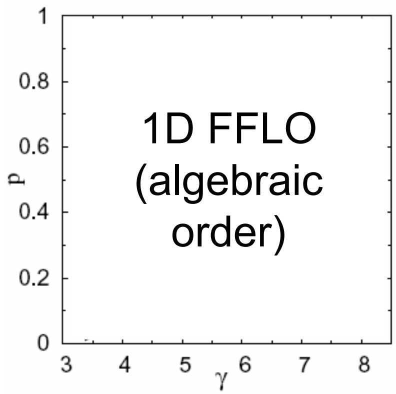
Pair tunneling (Josephson coupling) tends to lock the phases of individual tubes to produce a genuine superfluid state with long range order.

What does the 1D FFLO turn into in the presence of these perturbations?

Answer this question by a renormalization group analysis of our effective theory.

RG phase diagram of weakly coupled tubes

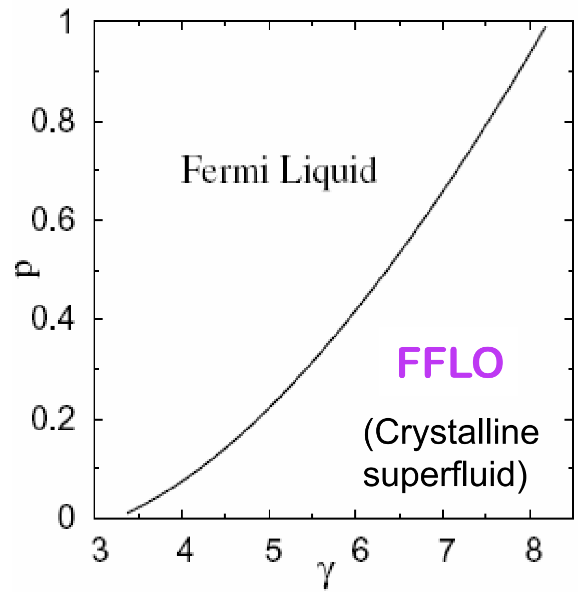
In the limit of weak transverse tunneling, $t_\perp \to 0$.



Splitting of 1D FFLO

3D FFLO $\delta_\Delta < \delta_\uparrow$

Fermi liquid $\delta_\Delta > \delta_\uparrow$



Polarization

$$p = (n_\uparrow - n_\downarrow)/n$$

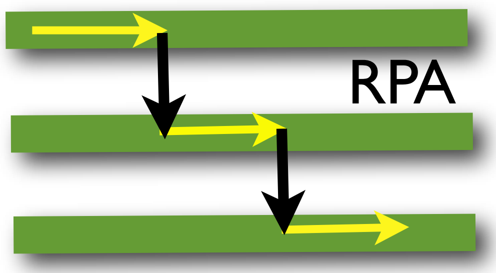
As $t_\perp$ grows, the region of stability for FFLO shrinks

Interaction strength $\gamma = 2/(na_{1D})$

Experiments are in the (dilute and) strong coupling regime, $\gamma \gg 1$

Perturbation theory in quasi-1D

For finite $t_\perp$, RG looses its prediction power. According to RPA,

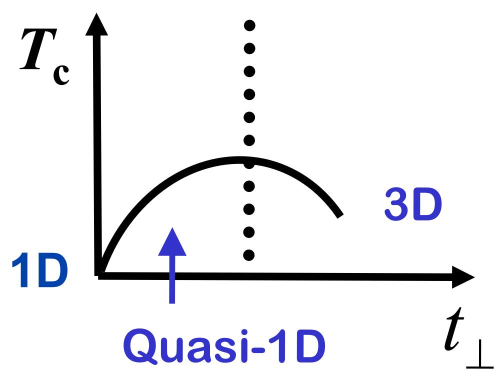


Beyond RPA:
E. Arrigoni, PRB **61**, 7909 (00)
S. Biermann et al, PRL 87, 276405 (01)
M. Bocquet, PRB 65, 184415(02)
...

The 3D pair susceptibility is related to the exact 1D susceptibility by

$$\mathscr{X}^{-1}(q_\star, k_\perp = 0, \omega = 0) = \chi_0^{-1} - z_\perp J,$$

Qualitative picture of the 1D-to-3D dimensional crossover:



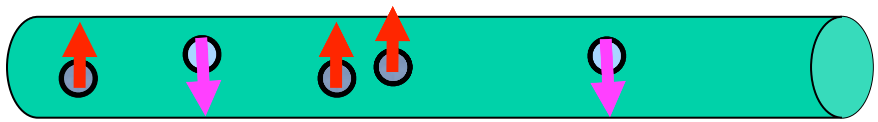
$T_c \propto J^{1/\eta_\Delta}$

At strong coupling,

$T_c \propto J \propto t_\perp^2$

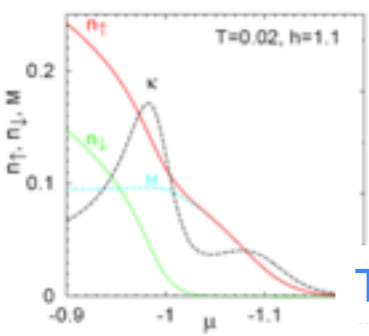
Summary

1. 1D FFLO as a special (spin-charge mixed) quantum liquid

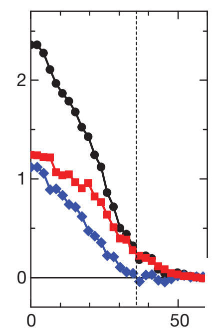


Bethe ansatz → effective field theory, correlation functions

2. Thermodynamics: simplified TBA

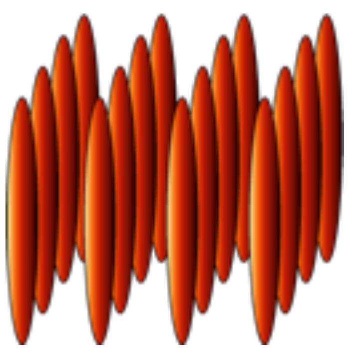


Theory ↔



Experiments

3. Phase diagram of weakly coupled tubes



Stability region of FFLO in quasi-1D.

Mean field BdG analysis of anisotropic lattice systems:
Parish et al, PRL **99**, 250403 (2007) and lot of others.

Acknowledgement

Collaborators:




Many helpful discussions with Randy Hulet, Jason Ho, Carlos Bolech, Qi Zhou

Reference:

EZ and Liu, *J. Low. Temp. Phys.*, 158, 36 (2010)
EZ, Guan, Liu, Batchelor, and Oshikawa, *PRL* 103, 140404 (2009)
EZ and Liu, *PRA* 78, 063605 (2008)