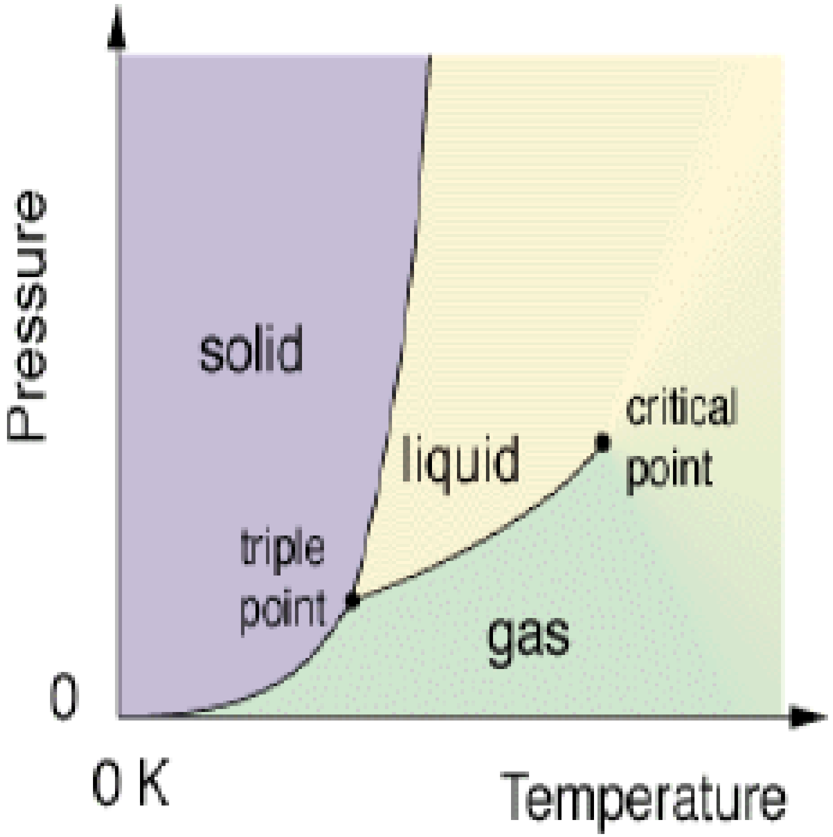
Quantum phase transition from a Luttinger liquid to a gas of cold polar molecules

Dima Feldman and Kam Tuen Law

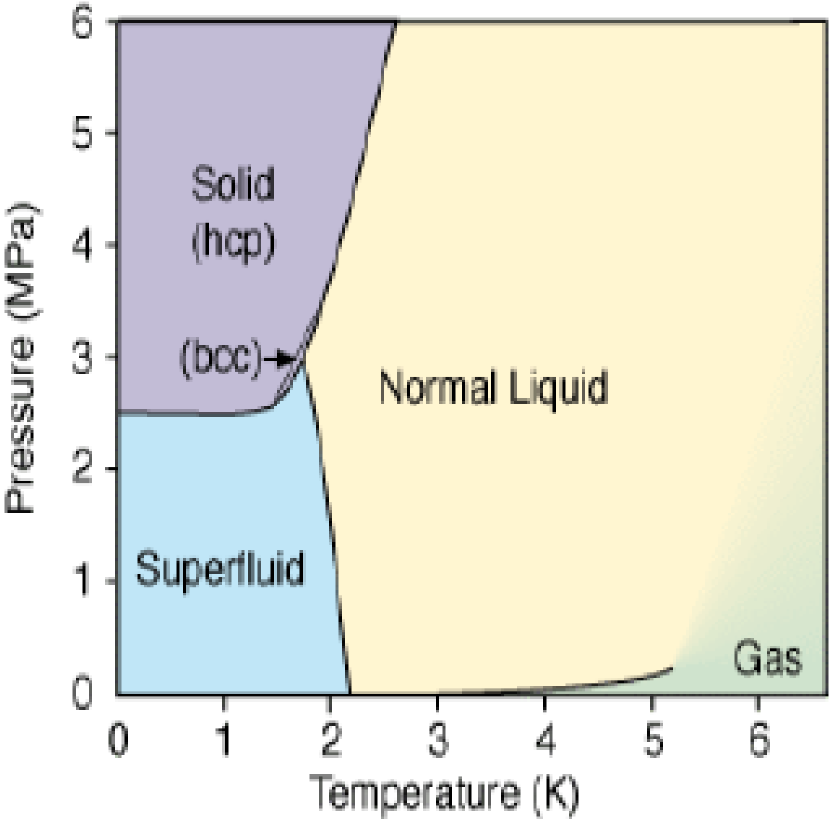
Brown University

K. T. Law and D. E. Feldman, Phys. Rev. Lett. **101**, 096401 (2008)

Liquid-gas transition



Typical phase diagram



Helium

Solid: maintains volume and shape

Liquid: maintains volume

Gas: fills all available volume

Outline

Zero-temperature liquid-gas transition:

- polar molecules in a helical optical trap

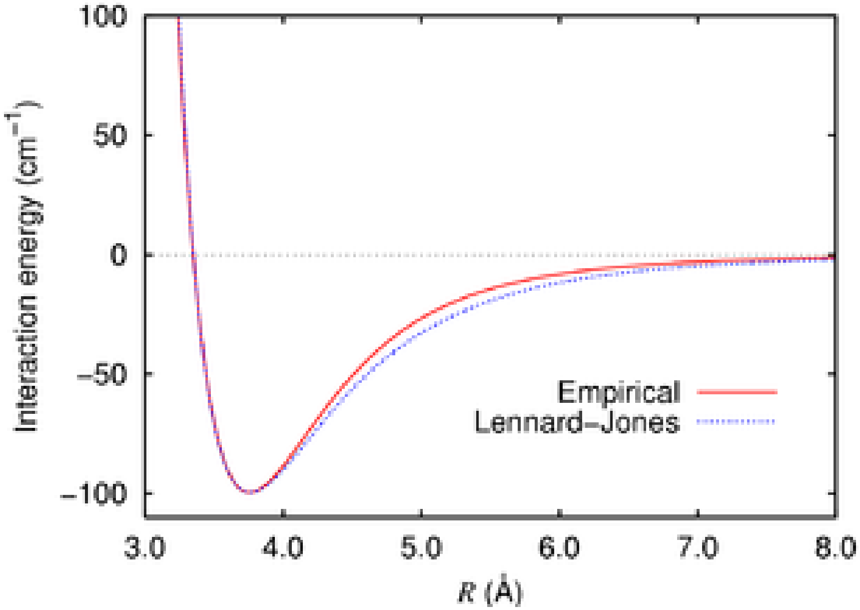
Phase diagram:

- no multi-atomic gases
- finite pressure

Second order transition

Zero-temperature liquid-gas transition

Cold gases exist at T=0 at weak intermolecular interaction = low density



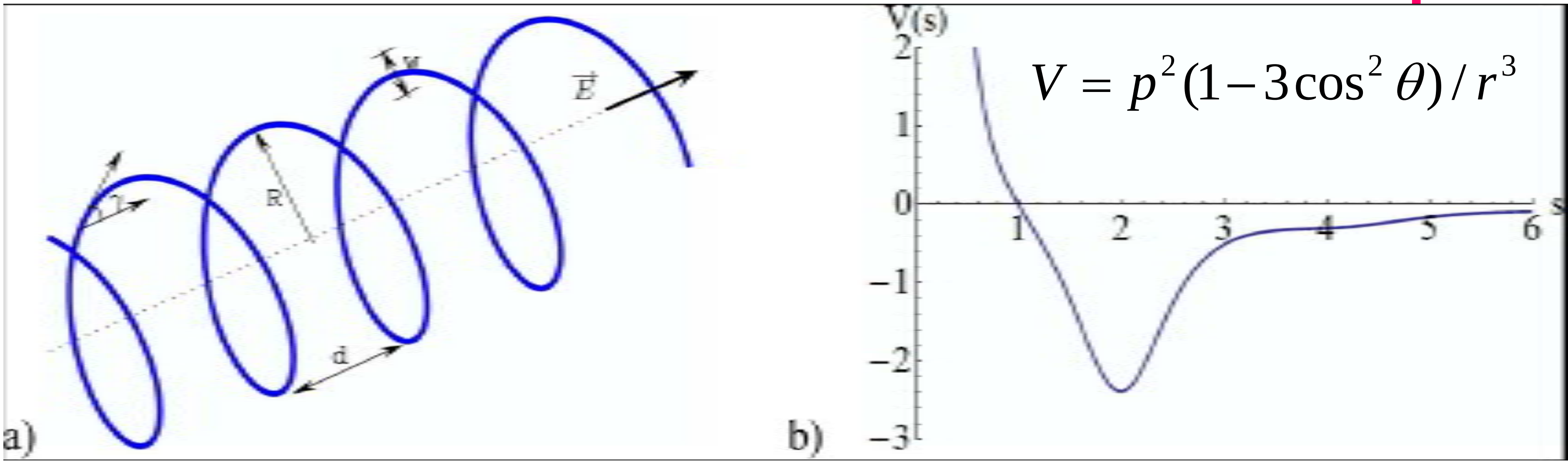
A liquid would emerge in the presence of a Lennard-Jones type interaction

Interaction engineering with cold atoms

Feshbach resonance: modeled by a delta-function

Dipole forces: sign does not depend on distance

Polar molecules in a helical trap



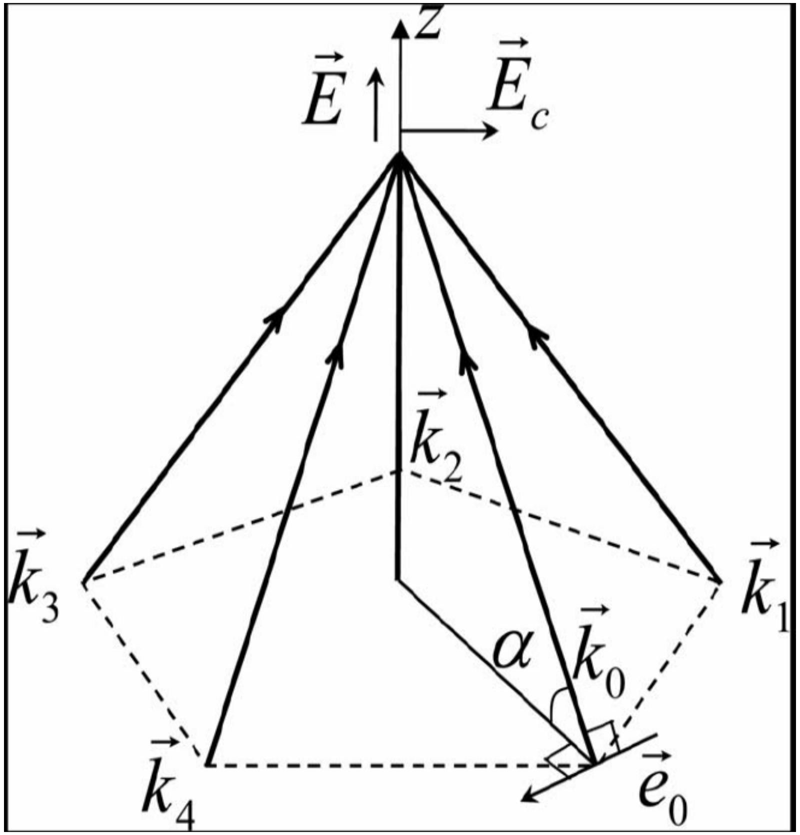
a) Helical lattice

b) Interaction *V(s)* in units of $p^3/(\pi R)^3$ as a function of distance in units of πR

Electric field polarizes molecules with dipole moment *p* along the axis of the helix. Phase transition at a critical electric field.

Gas occupies all available volume. The volume of the liquid depends on the interaction strength.

Helical lattice



Circular polarized beam + linearly polarized side beams

Y. K. Pang et al., Opt. Express **13**, 7615 (2005);
S. P. Gorkhali et al., J. Soc. Inf. Display **15**, 553 (2007).
See also M. Bhattacharya, Otp. Comm. **279**, 219 (2007).

Conditions

$$\frac{\hbar^2}{2MR^2} \propto \frac{p^2}{R^3}$$

for realistic parameters: *M* on the order of 100 a.u., *p* on the order of 1 Debye, *R* on the order of a micron.

$$\Delta_E \propto \frac{(pE)^2}{E_0 - \hbar\omega + i\Gamma}$$

laser optical intensity on the order of ten kW per square cm. [S. Kotochigova and E. Tiesinga,Phys. Rev. A **73**, 041405 (2006)]

$$T < \hbar^2 / MR^2 \propto 10\,\mathrm{nK}$$

Effective Hamiltonian

$$H = -\sum_i \frac{\hbar^2}{2M}\frac{\partial^2}{\partial s_i^2} + \sum_{i>j} V(s_i - s_j)$$

from adiabatic approximation

$2\pi L_z + dp_z$ plays the role of the 1D momentum

Luttinger liquid

We focus on P=0

- Weak interaction: dilute gas of independent particles
- Strong interaction: nearest neighbor interaction dominates and can be approximated by a harmonic potential

$$S_0 = \frac{1}{2}\int d\tau \left[\sum_k M\dot{s}_k^2 + \sum_k K(s_k - s_{k+1} - h)^2 \right]$$

$$\left\langle (s_{n+k}(t) - s_n(t) - kh)^2 \right\rangle = \frac{\hbar \ln k}{\pi\sqrt{KM}}$$

Phase transition

S. Sachdev, T. Senthil, R. Shankar, Phys. Rev. B **50**, 258 (1994)

$$S = \int d\tau dx [\hbar \Psi^* \partial_\tau \Psi - \frac{\hbar^2}{2m} \Psi^* \nabla^2 \Psi - \mu |\Psi|^2 + g |\Psi|^4]$$

Second order transition. Finite pressure.
Perturbation theory fails for bound state formation.

Exact results

$$V(x) = +\infty, x < a; V(x) = -AU(x), x > a$$

No bound states for small nonzero *A*. Variational proof. Number all particles from left to right. Set masses of particles with even numbers to infinity and prove that there is still no bound states. This can be reduced to a single-particle problem.

Exact results

$$H = -\sum_i \frac{\hbar^2}{2M}\frac{\partial^2}{\partial s_i^2} + \sum_{i>j} V(s_i - s_j)$$

$$H_{12} = -\frac{\hbar^2}{M}\frac{d^2}{d\Delta_1^{\;2}} + V(\Delta_1);\, \Delta_1 = s_2 - s_1$$

$$H_{12}\psi_2(\Delta_1) = \varepsilon_2\psi_2;\ \text{two-particle ground state, } \varepsilon_2 < 0$$

$$H_{123} = -\sum_{k=1}^{3}\frac{\hbar^2}{2M}\frac{\partial^2}{\partial s_k^2} + V(\Delta_1) + V(\Delta_2) + V(\Delta_1 + \Delta_2)$$

$$\psi_3(s_1, s_2, s_3) = \psi_2(\Delta_1)\psi_2(\Delta_2)$$

$$\left\langle \psi_3 \,|\, H_{123} \,|\, \psi_3 \right\rangle = \int d\Delta_1 d\Delta_2 \psi_2(\Delta_1)\psi_2(\Delta_2)[V(\Delta_1) + V(\Delta_2) -$$

$$\frac{\hbar^2}{M}\left(\frac{\partial^2}{\partial\Delta_1^2} + \frac{\partial^2}{\partial\Delta_2^2}\right) + \frac{\hbar^2}{M}\frac{\partial^2}{\partial\Delta_1\partial\Delta_2} + V(\Delta_1 + \Delta_2)]\psi_2(\Delta_1)\psi_2(\Delta_2)$$

$$= 2\varepsilon_2 + \text{square of the integral of a full derivative} + \text{negative}$$

Lower energy per particle in a three-particle state than in the two-particle ground state. A generalization of this argument proves that the transition occurs directly from a monoatomic gas to a condensed state.

Transition point $p \propto \hbar / \sqrt{M\lambda}$

p controlled by the dc electric field

Variational method

Neglect all interactions except nearest neighbors.
We expect universal behavior near the transition.
For analytical calculations use the Morse potential

$$V(s) = A\{\exp(-2\alpha[s-h]) - 2\exp(-\alpha[s-h])\}$$

$$\psi_{\mathrm{VAR}} = \prod_{k=1}^{N-1} \psi_k(\Delta_k)$$

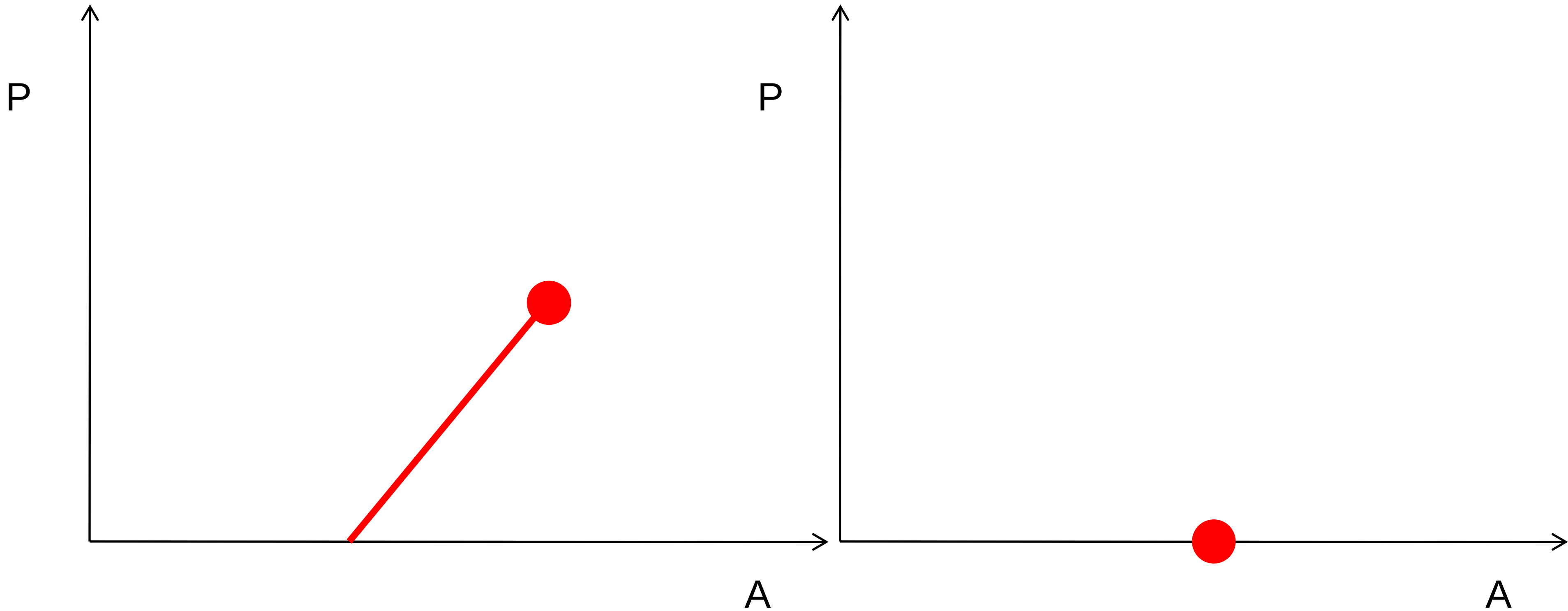
$$E = -A[1-\alpha\hbar/\sqrt{4MA}]^2 \propto (A-A_c)^2,\ A > A_c$$

$$E = 0,\ A < A_c$$

$$\rho \propto (A - A_c)$$

Second order liquid-gas transition? Critical point at zero pressure

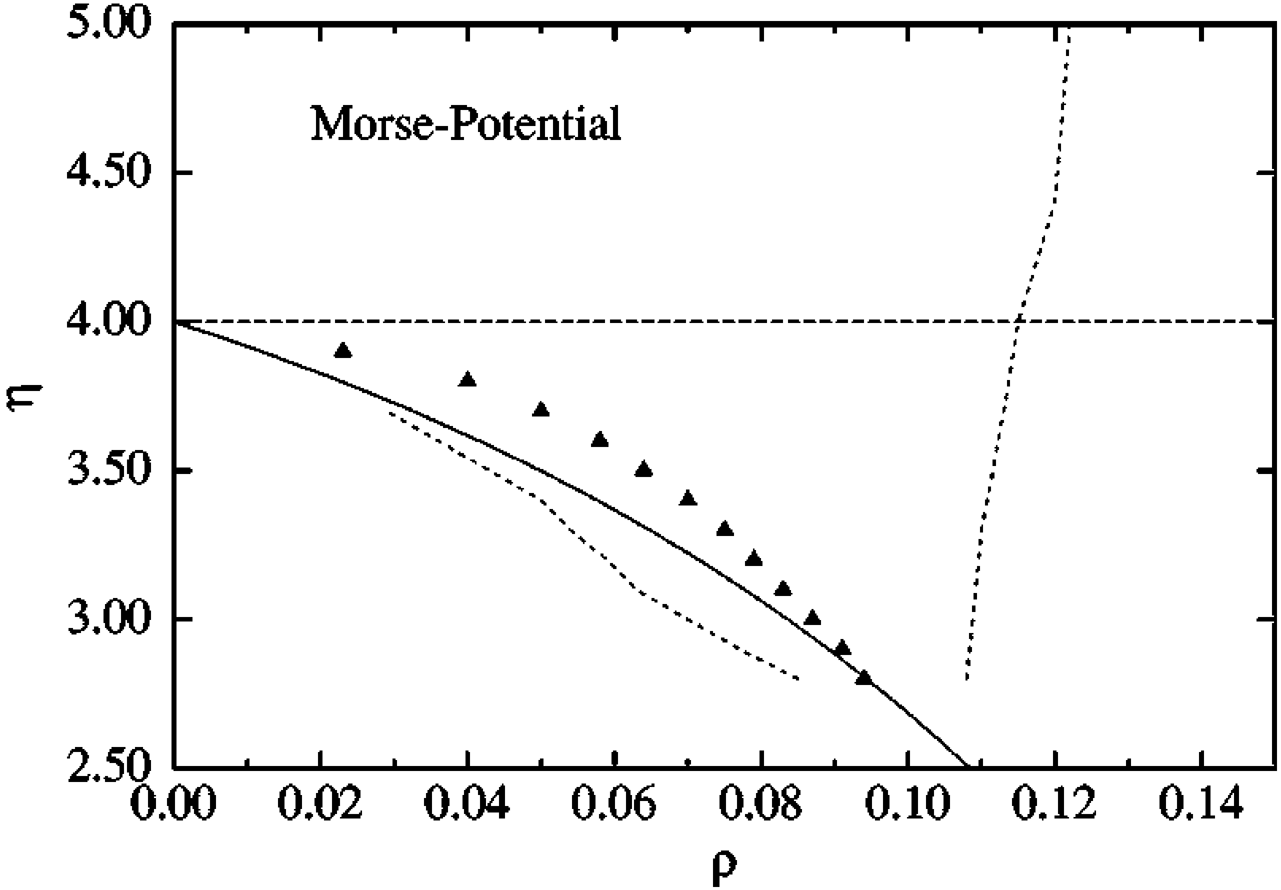
Critical point



$$H \to H - P(s_N - s_1)$$

Second order liquid-gas transition? Critical point at zero pressure

Numerical results



E. Krotscheck, M. D. Miller, and J. Wojdylo, Variational approach to the many-boson problem in one dimension, Phys. Rev. B **60**, 13028 (1999)

Summary

- Cold polar molecules in helical optical lattices exhibit quantum liquid-gas transition at critical electric field
- Direct transition between a monoatomic gas and a liquid
- Second order?
- Is the transition at the dimer formation threshold?