2D Bose and Non-Fermi Liquid "Metals"

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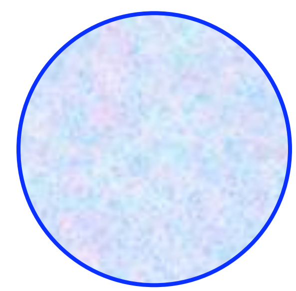
KITP Cold Atoms Workshop
10/5/2010

Interest: A class of exotic gapless 2D Many-Body States

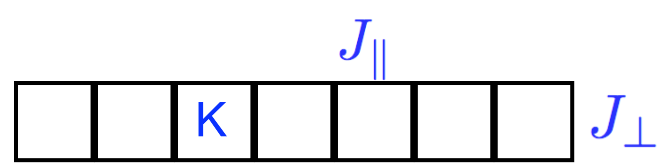
a) What are these "strange-metals"? Singular surfaces in momentum space (eg. ***Bose- surfaces***)

b) Variational wavefunctions?

c) 2D Bose-metal in cold atoms?



- 2D "Strange-Metals" have ***tractable quasi-1D descendents***
- Approach 2D via quasi-1D "ladders" with DMRG



What is a "Bose-Metal"?

First: Bose Condensate in Free Bose Gas

Superfluid in interacting Bose Gas

2D Free Bose Gas

Free particle Hamiltonian

$$H_0 = \sum_j \frac{\mathbf{p}_j^2}{2m}$$

Equal time Boson
Green's function

$$G_b(\mathbf{r}) = \langle b^\dagger(\mathbf{r}) b(\mathbf{0}) \rangle$$

Momentum
distribution fucntion

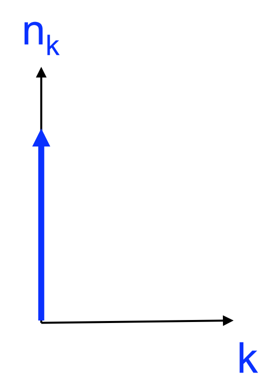
$$n^b_{\mathbf{k}} = G_b(\mathbf{k}) = \langle b^\dagger_{\mathbf{k}} b_{\mathbf{k}} \rangle$$

Off-diagonal
long-ranged order

$$G_b(\mathbf{r} \to \infty) = \rho$$

BEC condensate

$$n^{BEC}_{\mathbf{k}} = N\delta_{\mathbf{k},\mathbf{0}}$$



2D Interacting Superfluid

Interacting Hamiltonian

$$H = \sum_j \frac{\mathbf{p}_j^2}{2m} + \sum_{i,j} V(\mathbf{r}_i - \mathbf{r}_j)$$

Green's function

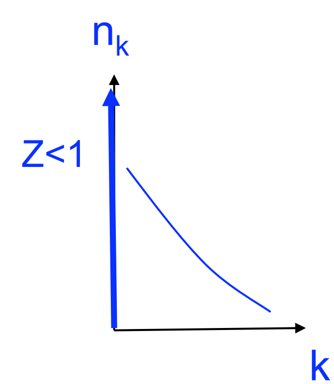
$$G_b(\mathbf{r}) = \langle b^\dagger(\mathbf{r}) b(\mathbf{0}) \rangle$$

Off-diagonal
long-ranged order

$$G_b(\mathbf{r} \to \infty) = \rho_c = Z\rho; \qquad Z < 1$$

Depleted Condensate
density in
Interacting Superfluid

$$n_{\mathbf{k}}^{SF} = ZN\delta_{\mathbf{k},0} + \delta n_{\mathbf{k}}^{SF}$$



2D Bose-Metal

$$G_b(\mathbf{r}) = \langle b^\dagger(\mathbf{r}) b(\mathbf{0}) \rangle$$

(Equal time Boson Green's function)

- A ***stable liquid phase*** of bosons that is not a superfluid

- Real space Green's function has oscillatory power law decay (***not*** a Bose condensate)

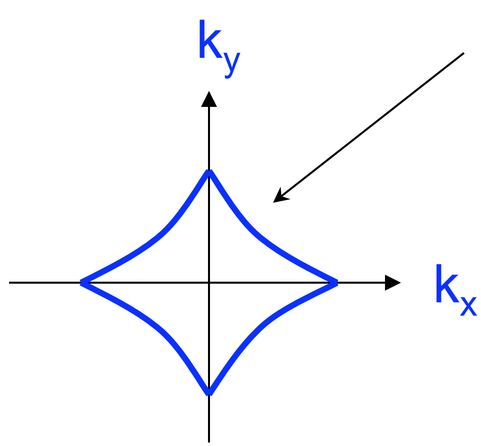
$$G_{BM}(\mathbf{r}) \sim \frac{\cos[\mathbf{k}_B(\hat{\mathbf{r}}) \cdot \mathbf{r}]}{|\mathbf{r}|^{\alpha(\hat{\mathbf{r}})}}$$

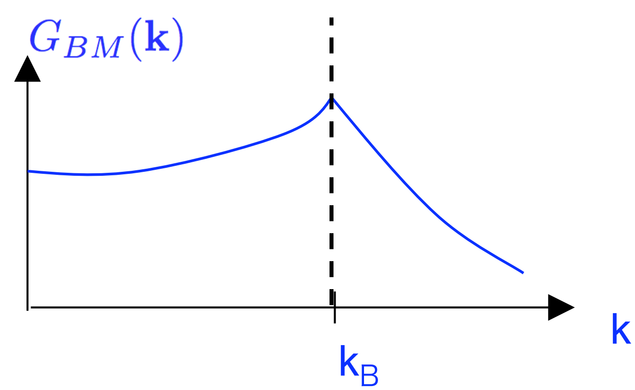
- ***Singularities*** in momentum distribution function

- Singular momentum on a ***"Bose surface"***

$$G_b(\mathbf{k}) = \langle b^\dagger_{\mathbf{k}} b_{\mathbf{k}} \rangle$$

$$\mathbf{k}_B(\hat{\mathbf{r}})$$





Angular dependent anomalous dimension $\alpha(\hat{\mathbf{r}})$

What is a “Non-Fermi-liquid metal”?

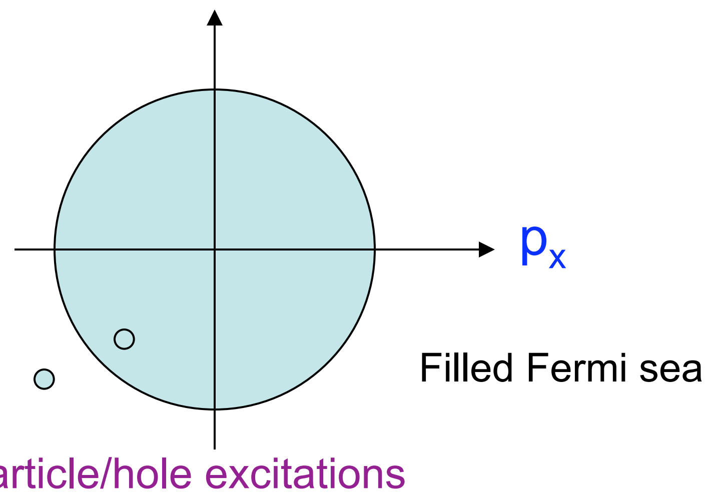
First: What is a Fermi Liquid Metal

2D Free Fermi Gas

Free Fermions

$$H_0 = \sum_j \frac{\mathbf{p}_j^2}{2m}$$

$$\epsilon_k = \frac{k^2}{2m} - \mu$$

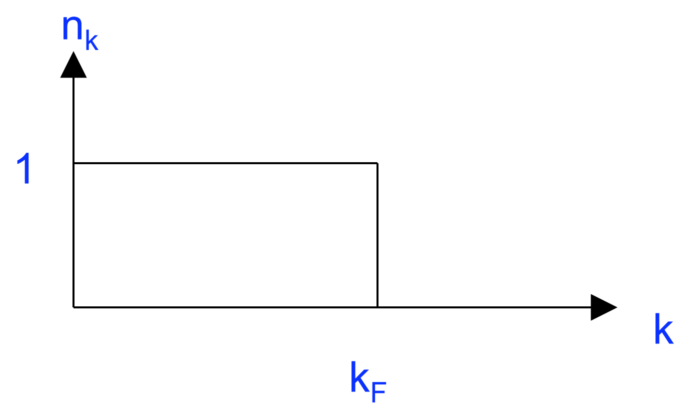


$$\mathcal{H}_0 = \sum_k \epsilon_k c_k^\dagger c_k$$

Momentum Distribution Function:

$$n_k = \langle c_k^\dagger c_k \rangle$$

$$n_{\mathbf{k}}^{FF} = \Theta(k_F - |\mathbf{k}|)$$



Volume of Fermi sea determined by the density of particles $\rho = k_F^2/4\pi$

Fermion Spectral function:

Sharp quasiparticle excitations:

$$A_0(k,\omega) = ImG_0(k,\omega) = \delta(\omega - \epsilon_k)$$

2D Fermi-liquid Metal

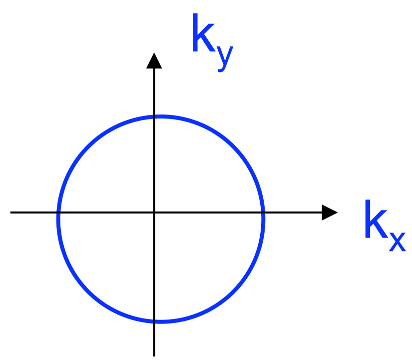
Equal time Green's function: $G(\mathbf{r}-\mathbf{r}') = \langle c^\dagger(\mathbf{r}) c(\mathbf{r}') \rangle$

Oscillatory decay $G_{FL}(\mathbf{r}) \sim \frac{\cos(k_F|\mathbf{r}| - 3\pi/4)}{|\mathbf{r}|^{\alpha_{FL}}}; \quad \alpha_{FL} = 3/2$

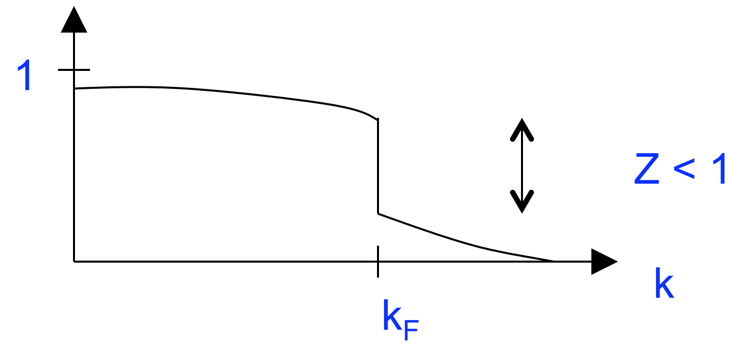
Momentum distribution function $n_{\mathbf{k}}^{FL} = Z \cdot n_{\mathbf{k}}^{FF} + \delta n_{\mathbf{k}}^{FL} \qquad Z < 1$

Luttingers Thm: Volume inside Fermi surface set by total density of fermions

$\rho = k_F^2 / 4\pi$



$G(\mathbf{k}) = \langle c_{\mathbf{k}}^\dagger c_{\mathbf{k}} \rangle$

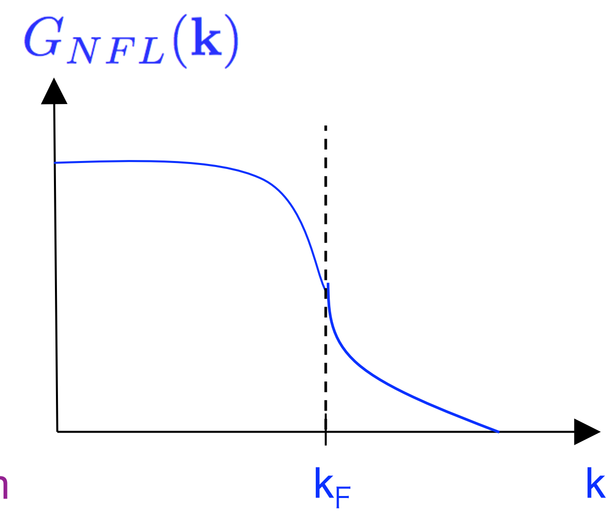


Quasi-particle excitation are (infinitely) long-lived on the Fermi surface

$A(k,\omega) = Z\delta(\omega - \epsilon_k) + A_{inc}(k,\omega)$

2D Non-Fermi Liquid Metal

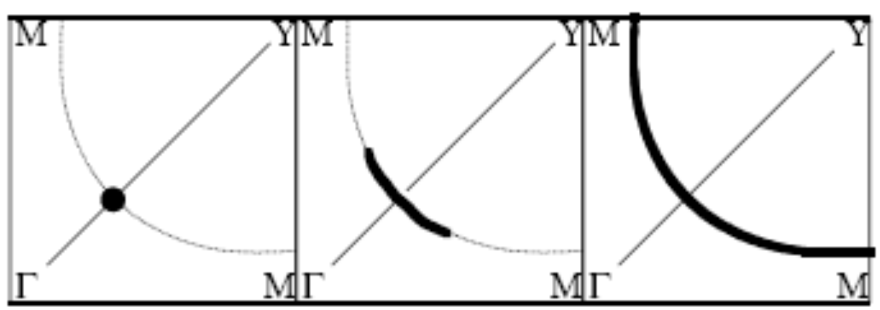
Various possibilities:



1) A singular "Fermi surface" that satisfies Luttinger's theorem but without a jump discontinuity in momentum distribution function

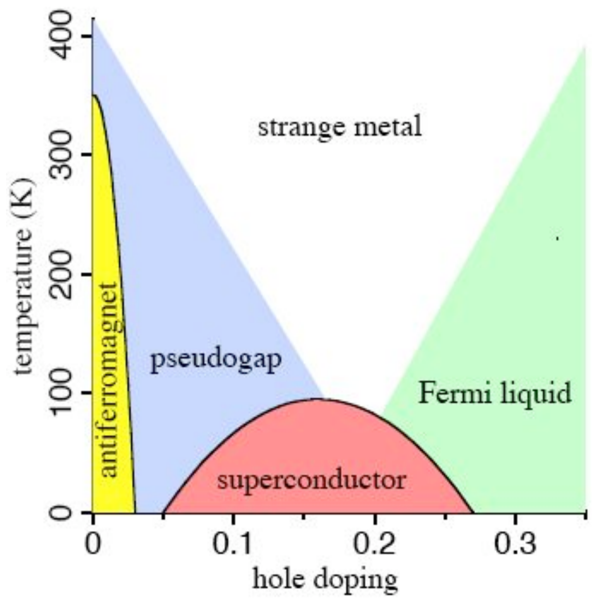
2) A singular Fermi surface that violates Luttinger's theorem
(eg. volume "x" rather than "1-x")

3) A singular "Fermi surface" with ``arc"



Motivation for Non-Fermi-Liquid Metal: “Abnormal” state of High T_c Superconductors

Phase Diagram



Strange metal: “Fermi surface” but quasiparticles are not “sharp”
Spectral function measured with ARPES suggests Z=0

Strategy: Construct candidate Non-Fermi liquid quantum states as putative strange metals

Wavefunction for 2D Bose-Metal?
Wavefunction for 2D Non-Fermi liquid Metal?

First: Wavefunction for BEC and Superfluid phase of Bosons

Wavefunction for Free Fermions and a Fermi liquid

Wavefunctions for Bose BEC and Superfluid

Bose Einstein Condensate (BEC)

$$\Psi_{BEC} = 1$$

Wavefunction is everywhere positive
ie. nodeless

Interacting Superfluid (SF)

Maintain the same nodeless structure,
put in a factor to keep the particles apart

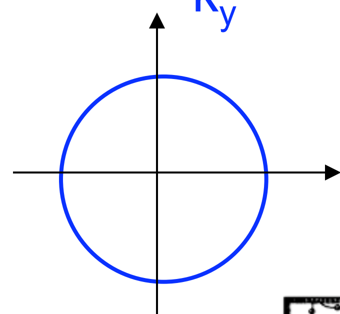
$$\Psi_{SF} = e^{-\sum_{i<j} u(\mathbf{r}_i - \mathbf{r}_j)} \geq 0$$

Jastrow form $u(\mathbf{r})$ is a variational parameter (function)

Wavefunction for 2D Free Fermi gas

(N spinless fermions in 2D)

Free Fermion determinant: (eg with 2D circular Fermi surface)

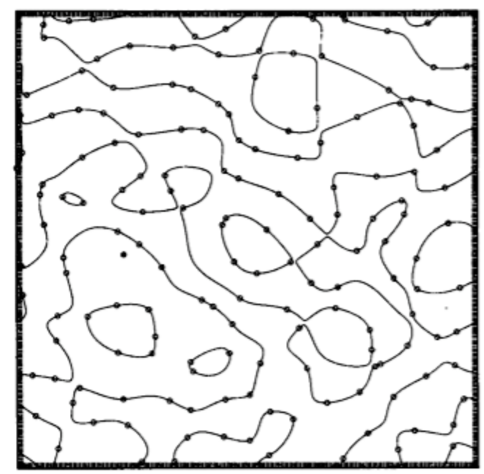


$$\Psi_{FF}(\{\mathbf{r}_i\}) = det[e^{i\mathbf{k}_i\cdot\mathbf{r}_j}]$$

Real space ***"nodal structure"***

Define a ``relative single particle function"

$$\Phi_{\mathbf{r}_2,\ldots,\mathbf{r}_N}(\mathbf{r}) \equiv \Psi(\mathbf{r},\mathbf{r}_2,\ldots,\mathbf{r}_N)$$



Nodal lines:
Ultraviolet and infrared "locking"

Wavefunction for interacting Fermi liquid?

Keep the sign (nodal) structure of free fermions, modifying the amplitude of the wavefunction, eg to keep the particles apart.

Common form: Multiply the free fermion wavefunction by a Jastrow factor,

$$\Psi_{Jastrow} = e^{-\sum_{i<j} u(\mathbf{r}_i - \mathbf{r}_j)} \geq 0$$

Proposed Fermi liquid wavefunction, with u(r) as a variational function

$$\Psi_{FL} = e^{-\sum_{i<j} u(\mathbf{r}_i - \mathbf{r}_j)} \psi_{FF}$$

Open question: Does the momentum distribution function that follows from this class of wavefunctions have a jump discontinuity on a Fermi surface with volume set by the density of particles?? Most probably yes!

$$G(\mathbf{r} - \mathbf{r}') = \int_{\mathbf{r}_2,..,\mathbf{r}_N} \Psi^*_{FL}(\mathbf{r}, \mathbf{r}_2, .., \mathbf{r}_N) \Psi_{FL}(\mathbf{r}', \mathbf{r}_2, .., \mathbf{r}_N) \quad \longrightarrow \quad \langle c^\dagger_{\mathbf{k}} c_{\mathbf{k}} \rangle = G(\mathbf{k})$$

Wavefunction for a 2D (D-wave) Bose-Metal

O. Motrunich/ MPAF Phys. Rev. B (2007)

Wavefunctions:

N bosons moving in 2d: $\Psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N)$

Define a ``relative single particle function" $\Phi_{\mathbf{r}_2,\ldots,\mathbf{r}_N}(\mathbf{r}) \equiv \Psi(\mathbf{r}, \mathbf{r}_2, \ldots, \mathbf{r}_N)$.

"Known" example of
boson non-superfluid:
Laughlin nu=1/2 Bosons:

$$\Psi_{\nu=1/2}(z_1, z_2, \ldots, z_N) = \prod_{i<j} (z_i - z_j)^2 \ .$$

Point nodes in ``relative particle function"
Relative d+id 2-particle correlations

$$\Phi_{\nu=1/2}(z) \sim (z - z_i)^2$$

Goal: Construct time-reversal invariant analog of Laughlin,
(with relative d_{xy} 2-particle correlations)

Wavefunction for D-wave Bose-Metal (DBM)

Hint: nu=1/2 Laughlin is a determinant squared

$$\Psi_{\nu=1/2} = [\Psi_{\nu=1}]^2$$

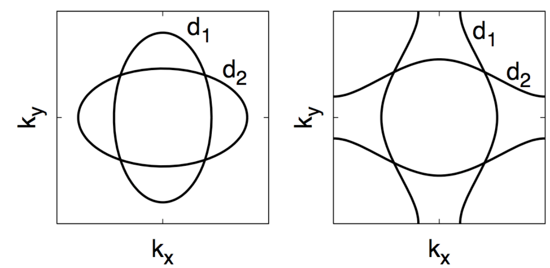
$$\Phi_{\nu=1}(z) \sim (z - z_i)$$

p+ip 2-body

Try squaring Fermi sea wf:
No, "s-wave" with ODLRO

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) = (\det e^{i\mathbf{k}_i \cdot \mathbf{r}_j})^2 \ , \quad \text{(S-type)}.$$

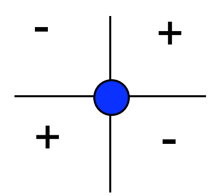
``D-wave" Bose-Metal:
Product of 2 different Fermi sea determinants, elongated in the x or y directions



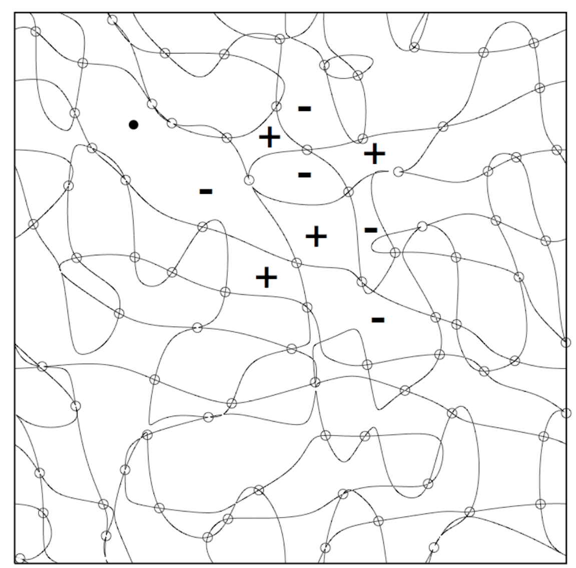
$$\Psi_{D_{xy}}(\mathbf{r}_1, \ldots, \mathbf{r}_N) = (det)_x \times (det)_y$$

Nodal structure of DBM wavefunction:

$$\Phi_{D_{xy}}(\mathbf{r}) \sim (x - x_i)(y - y_i)$$



D_{xy} relative
2-particle correlations



Gauge Theory for D-wave Bose Metal phase

Slave Fermion decomposition for lattice bosons: $b^\dagger(\mathbf{r}) = d_1^\dagger(\mathbf{r}) d_2^\dagger(\mathbf{r})$

Gauge Theory Hamiltonian: $H_{U(1)} = H_t + H_a$

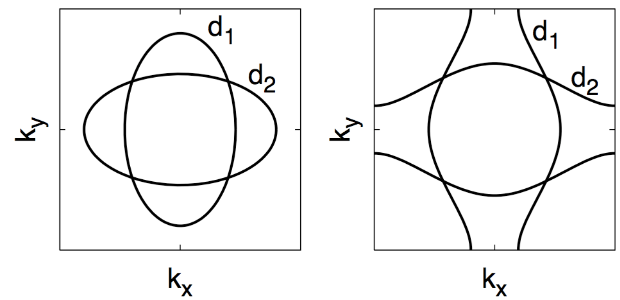
$$\begin{aligned} H_t &= -\sum_{\mathbf{r}} \left[t_\parallel e^{i a_x(\mathbf{r})} d_1^\dagger(\mathbf{r}) d_1(\mathbf{r}+\hat{\mathbf{x}}) + t_\perp e^{i a_y(\mathbf{r})} d_1^\dagger(\mathbf{r}) d_1(\mathbf{r}+\hat{\mathbf{y}}) + h.c. \right] \\ &\quad - \sum_{\mathbf{r}} \left[t_\perp e^{-i a_x(\mathbf{r})} d_2^\dagger(\mathbf{r}) d_2(\mathbf{r}+\hat{\mathbf{x}}) + t_\parallel e^{-i a_y(\mathbf{r})} d_2^\dagger(\mathbf{r}) d_2(\mathbf{r}+\hat{\mathbf{y}}) + h.c. \right] \end{aligned}$$

$$H_a = h \sum_{\mathbf{r}} \sum_{\mu = x,y} e_\mu^2(\mathbf{r}) - K \sum_{\mathbf{r}} \cos[(\nabla \times a)_{\mathbf{r}}]$$

$$(\nabla \cdot e)_{\mathbf{r}} = d_1^\dagger(\mathbf{r}) d_1(\mathbf{r}) - d_2^\dagger(\mathbf{r}) d_2(\mathbf{r})$$

Strong coupling: h>>K,t integrate out gauge field gives Boson Hamiltonian: $\mathcal{H}(\hat{b}, \hat{b}^\dagger)$

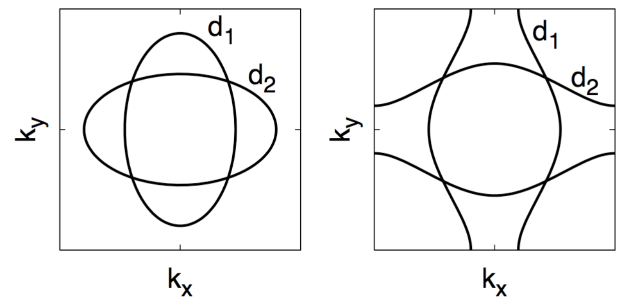
Weak Coupling: K>> h,t Anisotropic Fermi surfaces of d_1 and d_2 minimally coupled to a (non-compact) U(1) gauge field



Bose Surfaces in D-wave Bose-Metal

Mean Field Green's functions factorize:

$$G_b^{MF}(\mathbf{r},\tau) = G_{d_1}^{MF}(\mathbf{r},\tau)G_{d_2}^{MF}(\mathbf{r},\tau)/\bar{\rho}$$



$$G_{d_\alpha}^{MF}(\mathbf{r}) \approx \frac{1}{2^{1/2}\pi^{3/2}} \frac{\cos(\mathbf{k}_{F_\alpha}\cdot\mathbf{r} - 3\pi/4)}{c_\alpha^{1/2}|\mathbf{r}|^{3/2}}.$$

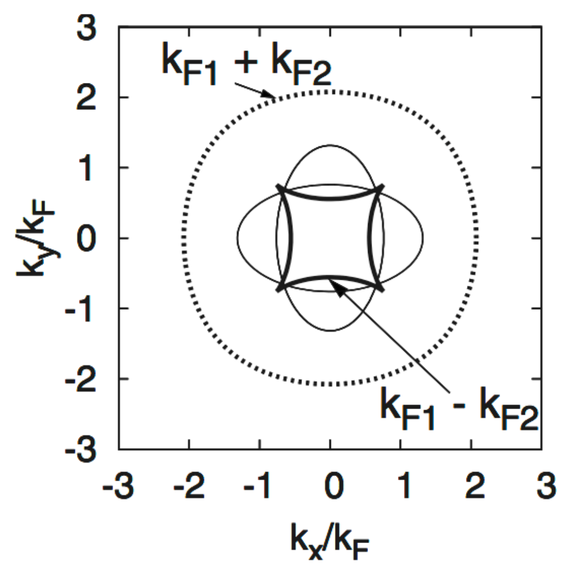
$$(\partial\epsilon_\alpha/\partial\mathbf{k})_{\mathbf{k}_{F\alpha}(\hat{\mathbf{r}})} = (const)\hat{\mathbf{r}}$$

Momentum distribution function:

$$n_b(\mathbf{k}) = \int G_b(\mathbf{r})e^{i\mathbf{k}\cdot\mathbf{r}}d\mathbf{r}$$

Two singular lines in momentum space, Bose surfaces:

$$\mathbf{k}_{F_1}(\hat{\mathbf{r}}) \pm \mathbf{k}_{F_2}(\hat{\mathbf{r}})$$



Gutzwiller wavefunction for Electron NFL “metal”

Decompose the electron:
spinless charge e boson
and s=1/2 neutral fermionic spinon

$$c^\dagger_{\mathbf{r}\alpha} = b^\dagger_{\mathbf{r}} f^\dagger_{\mathbf{r}\alpha}$$

Mean Field Theory

Treat “Spinons” and Bosons as Independent: $\mathcal{H} = \mathcal{H}_f + \mathcal{H}_b$

Wavefunctions $\psi_f(\mathbf{x}_{i\uparrow}, \mathbf{x}_{i\downarrow})$ $\psi_b(\mathbf{r}_j)$

(enlarged Hilbert space - twice as many particles)

“Fix-up” Mean Field Theory

Gutzwiller projection: “glue” together Fermion and Boson “partons”

$$\Psi_G \equiv \psi_f(\mathbf{x}_{i\alpha}) \times \psi_b(\mathbf{r}_i \rightarrow \mathbf{x}_{i\alpha})$$

Project back into physical Hilbert space

Fermi and Non-Fermi Liquids?

Put the Spinons in a filled Fermi sea

$$\psi_f = \det[e^{i\mathbf{k}_i\cdot\mathbf{x}_{j\uparrow}}] \times \det[e^{i\mathbf{k}_i\cdot\mathbf{x}_{j\downarrow}}]$$

Fermi Liquid: Put the Bosons into a superfluid

$$\psi_b^{SF} = e^{-\sum_{i<j} u(\mathbf{r}_i - \mathbf{r}_j)}$$

$$\Psi_{FL} = \mathcal{P}_G[\psi_f^{FF} \times \psi_b^{SF}]$$

Non-Fermi Liquid: Put Bosons into an *uncondensed* fluid - a "Bose metal"

$$\Psi_{NFL} = \mathcal{P}_G[\psi_f^{FF} \times \psi_b^{BoseMetal}]$$

D-wave NFL Metal: Product of Fermi sea and D-wave Bose-Metal

Bose-Metals in Cold Atoms?

Bosonic Atoms in an optical Lattice

(1) Bose-Hubbard model:

$$\mathcal{H}_B = -J \sum_{\langle ij \rangle} (b_i^\dagger b_j + h.c.) + U \sum_i n_i^2$$

Unfrustrated;

Superfluid away from commensurate filling

Fermionic Atoms in an optical Lattice

(2) ***Attractive U Fermion Hubbard model***

$$\mathcal{H} = -t \sum_{ij} (c_{i,\alpha}^\dagger c_{j,\alpha} + h.c.) - U \sum_i n_{i\uparrow} n_{i\downarrow}$$

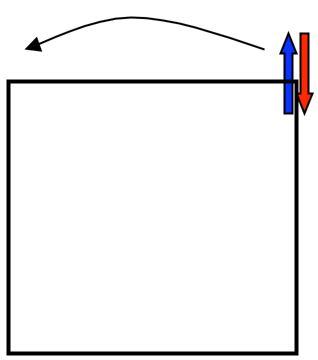
Cooper-Pair Hopping model

Attractive U Hubbard model for Fermionic atoms

$$\mathcal{H}_F = -t \sum_{ij} (c^\dagger_{i,\alpha} c_{j,\alpha} + h.c.) - U \sum_i n_{i\uparrow} n_{i\downarrow}$$

$$b_i^\dagger = c^\dagger_{i\uparrow} c^\dagger_{i\downarrow}$$

Hard core boson (Cooper pair)



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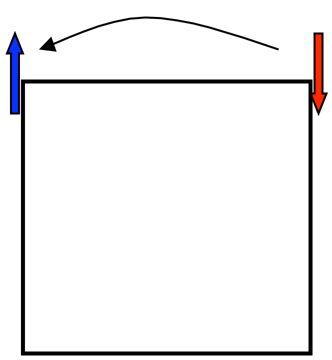

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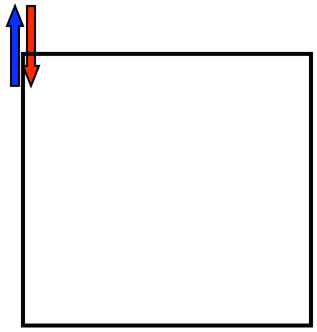

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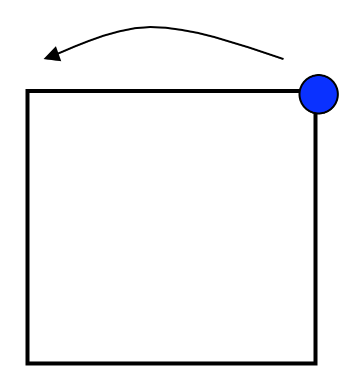


Attractive U Hubbard: Paired Superfluid phase

Attractive U Hubbard model for Fermionic atoms

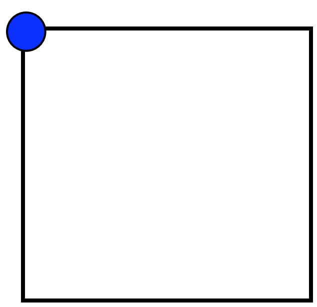
$$\mathcal{H}_F = -t \sum_{ij} (c^\dagger_{i,\alpha} c_{j,\alpha} + h.c.) - U \sum_i n_{i\uparrow} n_{i\downarrow}$$

$$b^\dagger_i = c^\dagger_{i\uparrow} c^\dagger_{i\downarrow}$$

Hard core boson (Cooper pair)

$$\mathcal{H}_B = -J \sum_{\langle ij \rangle} (b^\dagger_i b_j + h.c.) + V \sum_{\langle ij \rangle} n_i n_j$$

$$J \sim V \sim t^2/U$$

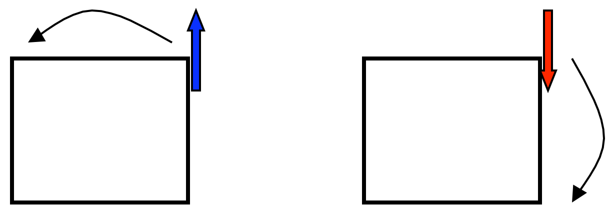


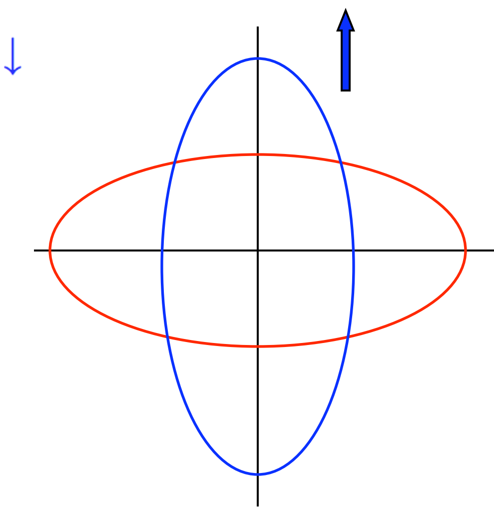
Unfrustrated (hard-core) ***Bose-Hubbard model*** -

Paired superfluid phase for arbitrary U/t

Generate frustration?
Anisotropic attractive U Hubbard model

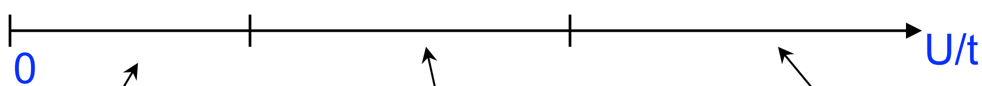
$$\mathcal{H}_F = -\sum_{ij} t^{\alpha}_{ij}(c^{\dagger}_{i,\alpha}c_{j,\alpha} + h.c.) - U\sum_i n_{i\uparrow}n_{i\downarrow}$$





Phase diagram??

?????



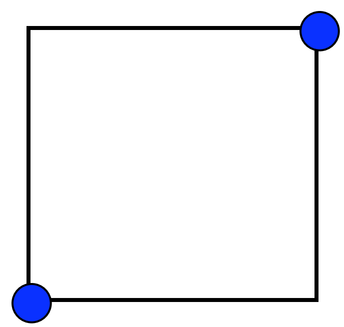
Metal
(no weak coupling
BCS instability)

Intermediate coupling
phase??

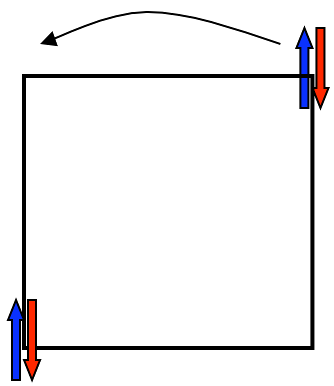
Cooper-Pair Bose-Metal ??

Superconductor (paired superfluid)

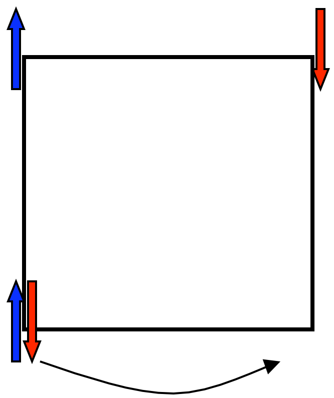
Generate Cooper pair ring exchange



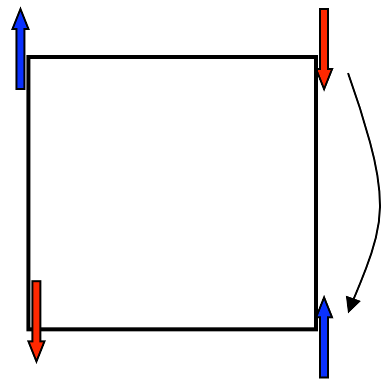
Generate Cooper pair ring exchange



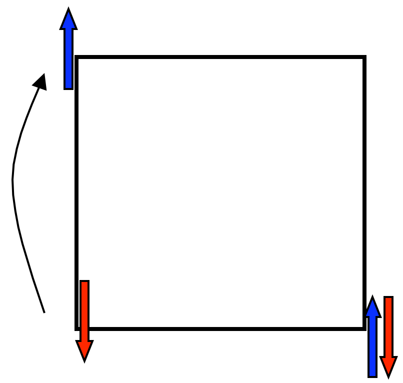
Generate Cooper pair ring exchange



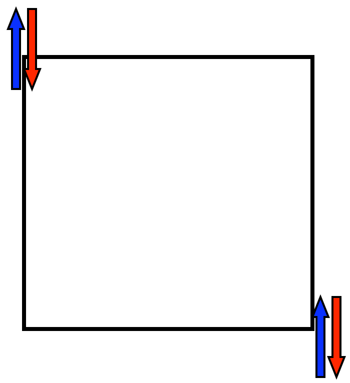
Generate Cooper pair ring exchange



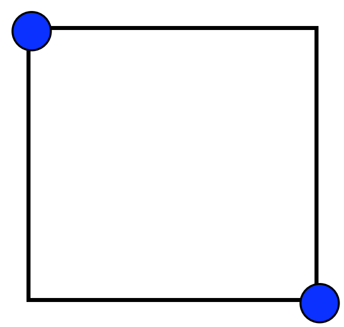
Generate Cooper pair ring exchange



Generate Cooper pair ring exchange

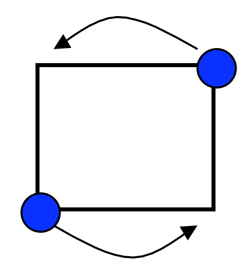


Generate Cooper pair ring exchange



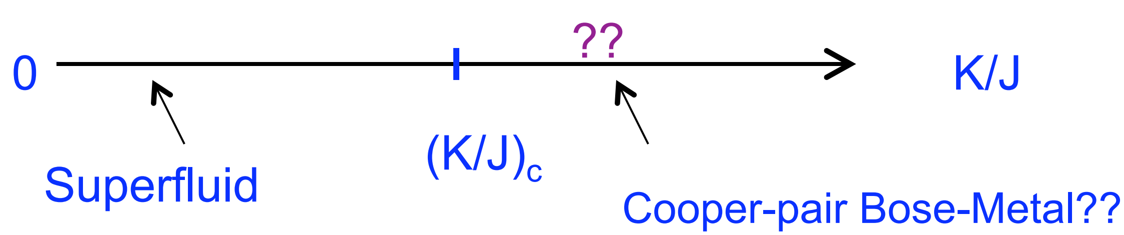
Cooper-Pair Ring Exchange Model

$$\mathcal{H}_{JK} = -J \sum_{ij} (b_i^\dagger b_j + h.c.) + K \sum_{plackets} (b_1^\dagger b_2 b_3^\dagger b_4 + h.c.)$$



Cooper-pair ring term

Phase diagram: K/J and density of Cooper pairs bosons



Boson J-K Model support 2D Cooper-pair Bose-Metal?

$$\mathcal{H}_{JK} = -J \sum_{ij} (b_i^\dagger b_j + h.c.) + K \sum_{plackets} (b_1^\dagger b_2 b_3^\dagger b_4 + h.c.)$$

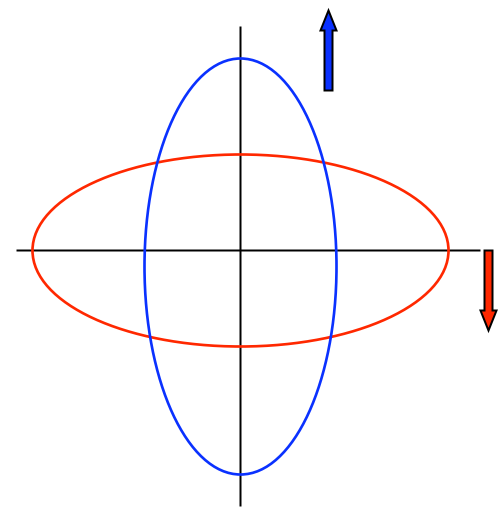
ED is too small for putative gapless phase
Sign problem for Quantum Monte Carlo
Variational Monte Carlo is biased
DMRG only works well in 1D

Exploit Bose Surface in Cooper-Pair Bose Metal

"Slave Fermion" decomposition: $b_i^\dagger = c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger$

Mean Field Green's functions factorize:
(no gauge fluctuations)

$$G_b^{MF}(\mathbf{r}) = G_{c_\uparrow}(\mathbf{r}) G_{c_\downarrow}(\mathbf{r})$$

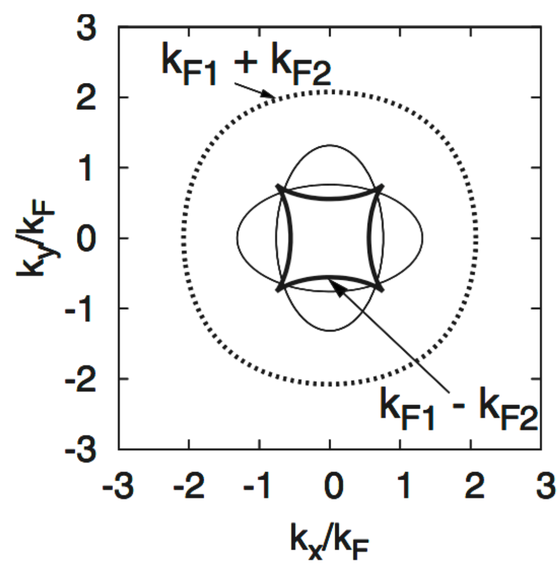


Momentum distribution function:

$$\langle b_k^\dagger b_k \rangle$$

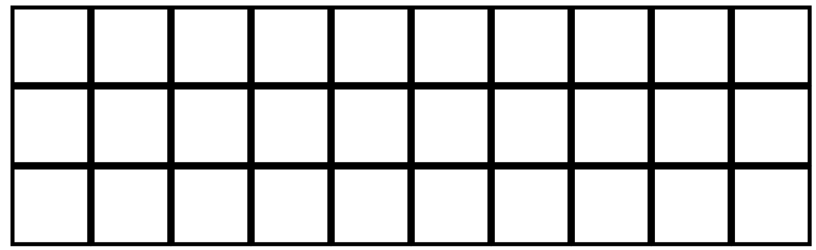
Two singular lines in
momentum space, Bose surfaces:

$$\mathbf{k}_{F_1}(\hat{\mathbf{r}}) \pm \mathbf{k}_{F_2}(\hat{\mathbf{r}})$$

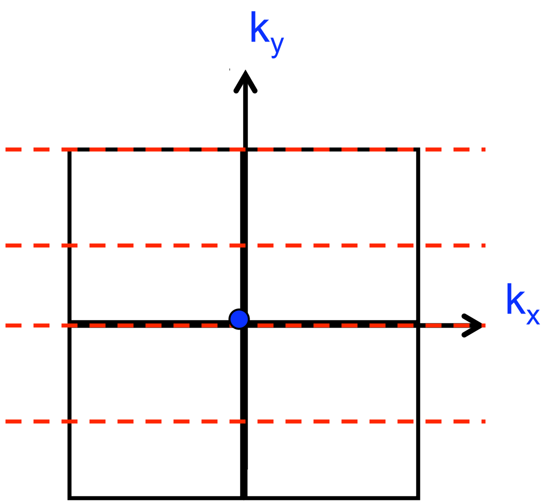


Ladders to the Rescue

Transverse y-components of momentum become quantized

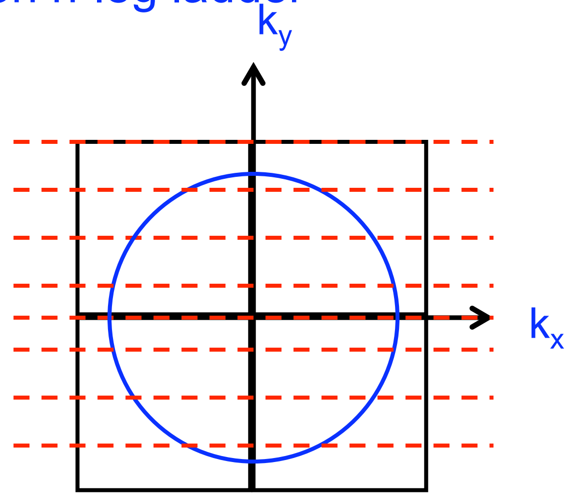


Put Bose superfluid on n-leg ladder



Single gapless 1d mode

Put Cooper-pair Bose-Metal on n-leg ladder



Many gapless 1d modes, one for each “Bose point”

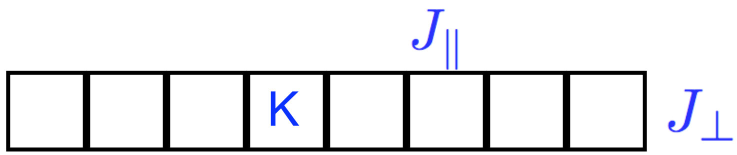
Expectation: Signature of Bose surface in Bose-Metal on n-leg ladders!!

Boson ring model on the 2-Leg Ladder

- Exact Diagonalization (2 x 18)
- Variational Monte Carlo
- DMRG (2 x 50)

E. Gull, D. Sheng, S. Trebst,
O. Motrunich and MPAF,
Phys. Rev. B 78, 54520 (2008)

$$H = H_J + H_4 \ ,$$
$$H_J = -J \sum_{\mathbf{r};\, \hat{\mu}=\hat{\mathbf{x}},\hat{\mathbf{y}}} (b_{\mathbf{r}}^\dagger b_{\mathbf{r}+\hat{\mu}} + h.c.) \ ,$$
$$H_4 = K_4 \sum_{\mathbf{r}} (b_{\mathbf{r}}^\dagger b_{\mathbf{r}+\hat{\mathbf{x}}} b_{\mathbf{r}+\hat{\mathbf{x}}+\hat{\mathbf{y}}}^\dagger b_{\mathbf{r}+\hat{\mathbf{y}}} + h.c.) \ ,$$



Correlation Functions:

1) Momentum Distribution function

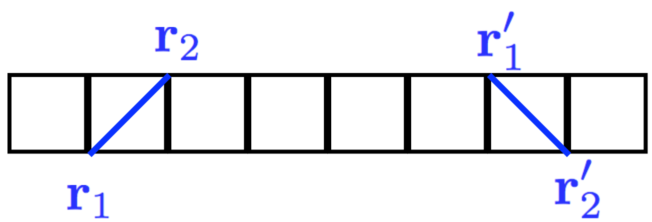
$$n(k_x, k_y) = \langle b_{\mathbf{k}}^\dagger b_{\mathbf{k}} \rangle; \quad k_y = 0, \pi$$

2) Density-density structure factor

$$\mathcal{D}(\mathbf{k}) = \sum_{\mathbf{r}} e^{i\mathbf{k}\cdot\mathbf{r}} \langle n_{\mathbf{r}} n_{\mathbf{0}} \rangle \qquad n_{\mathbf{r}} = b_{\mathbf{r}}^\dagger b_{\mathbf{r}}$$

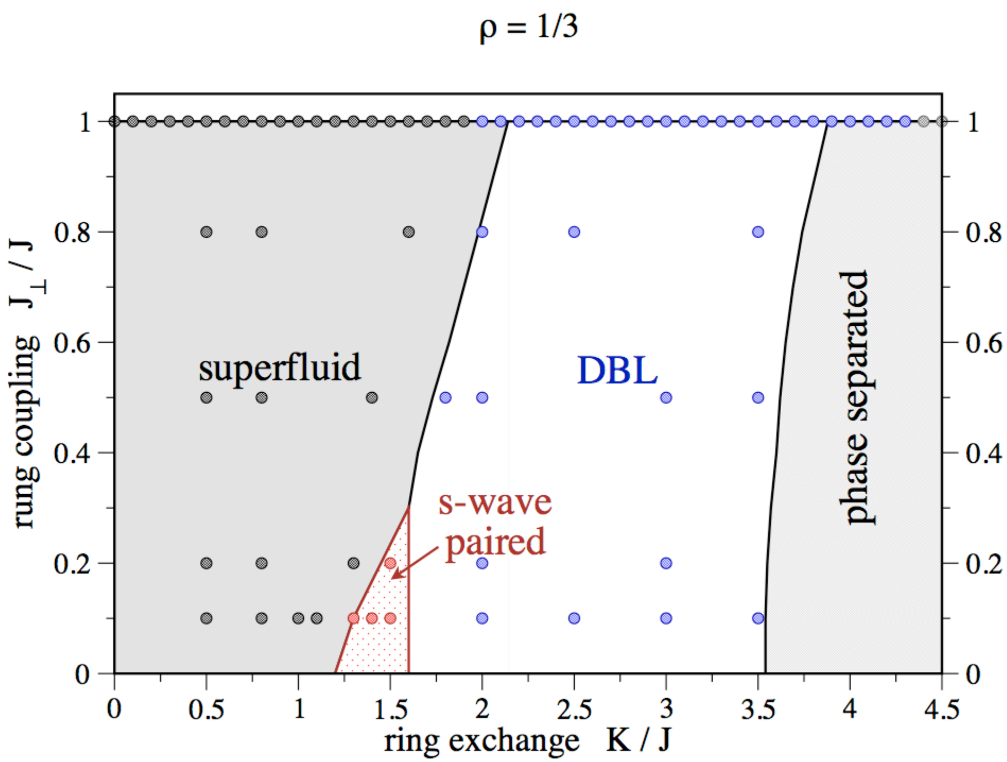
3) Pair-boson correlator

$$P(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_1', \mathbf{r}_2') = \langle b_{\mathbf{r}_1}^\dagger b_{\mathbf{r}_2}^\dagger b_{\mathbf{r}_1'} b_{\mathbf{r}_2'} \rangle$$



Ladder descendant of 2D Bose-metal??

Phase Diagram for 2-leg ladder



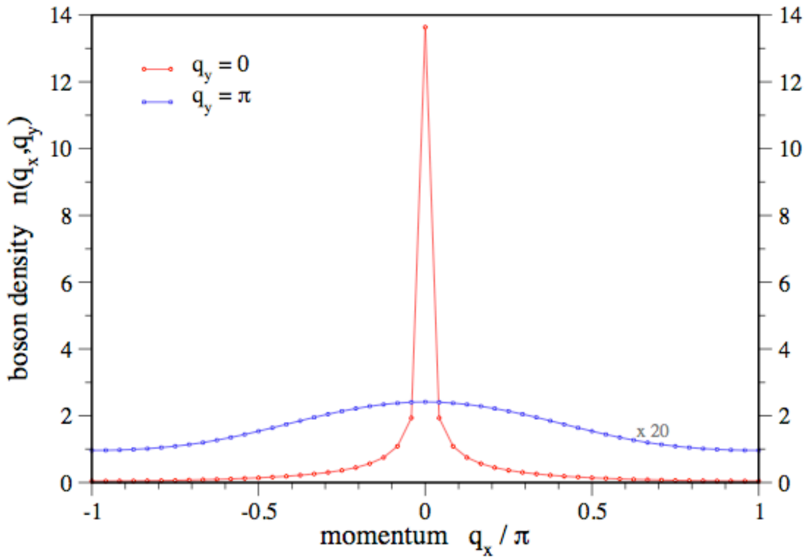
Phases:

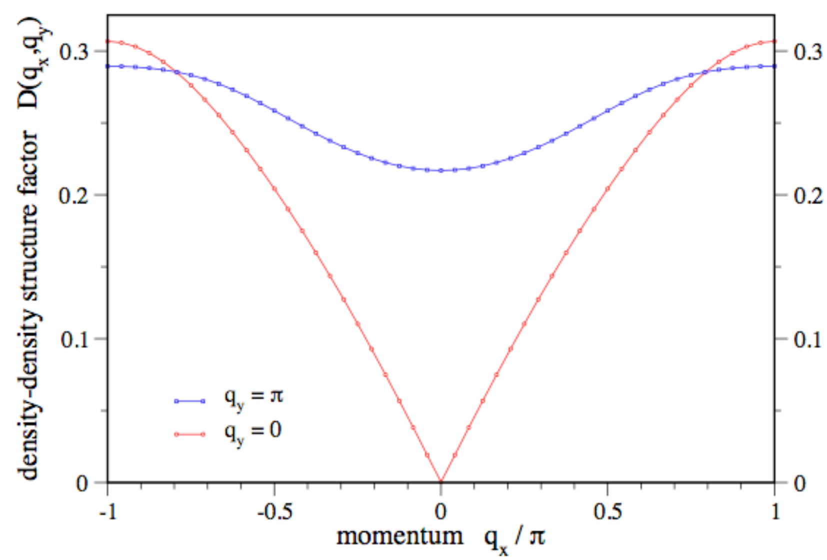
1) Superfluid – "Bose condensate"
2) D-Wave Bose Metal - DBL
3) s-wave Pair-Boson "condensate"

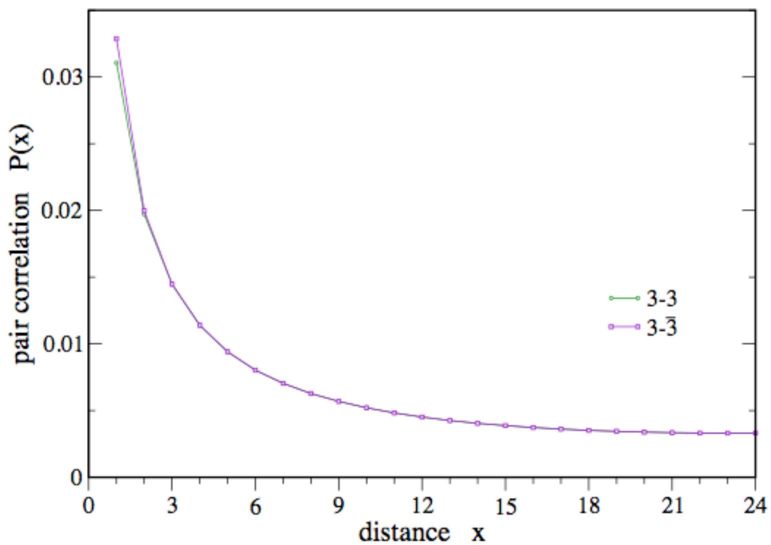
D-wave Bose-Metal occupies large region of phase diagram

Superfluid (DMRG)

DMRG 2 x 48







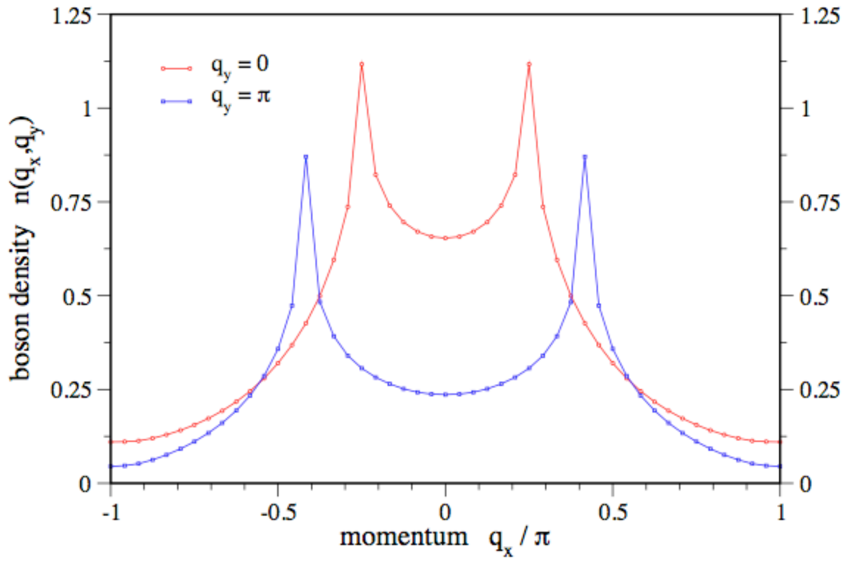
$$\rho = 1/3$$

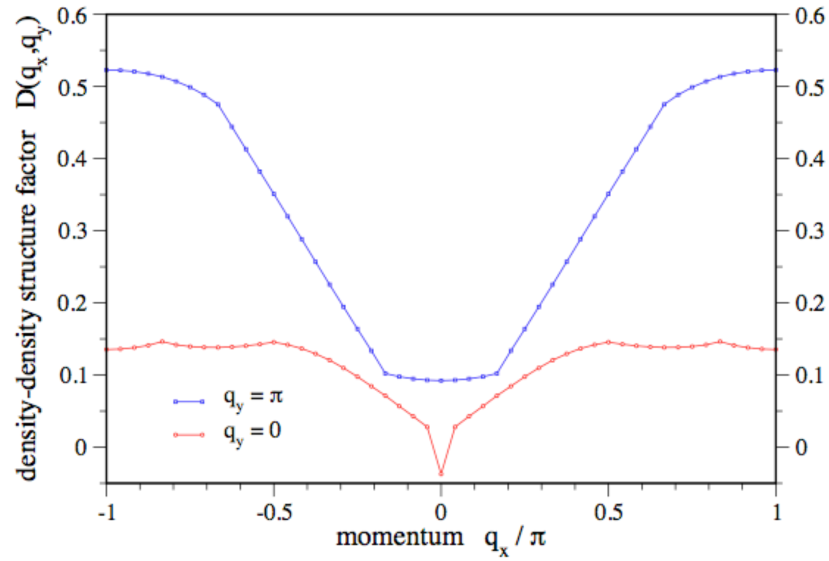
$$K/J = 1$$

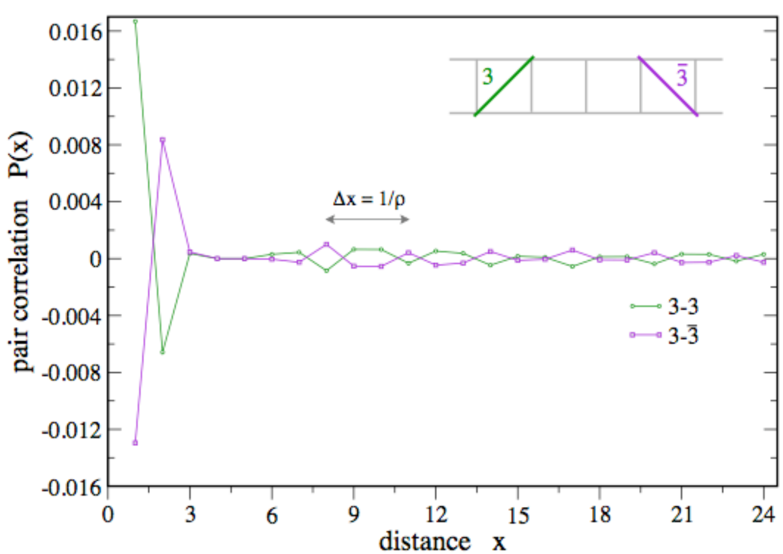
$$J_\perp/J = 1$$

D-Wave Bose Metal (from DMRG)

DMRG 2 x 48





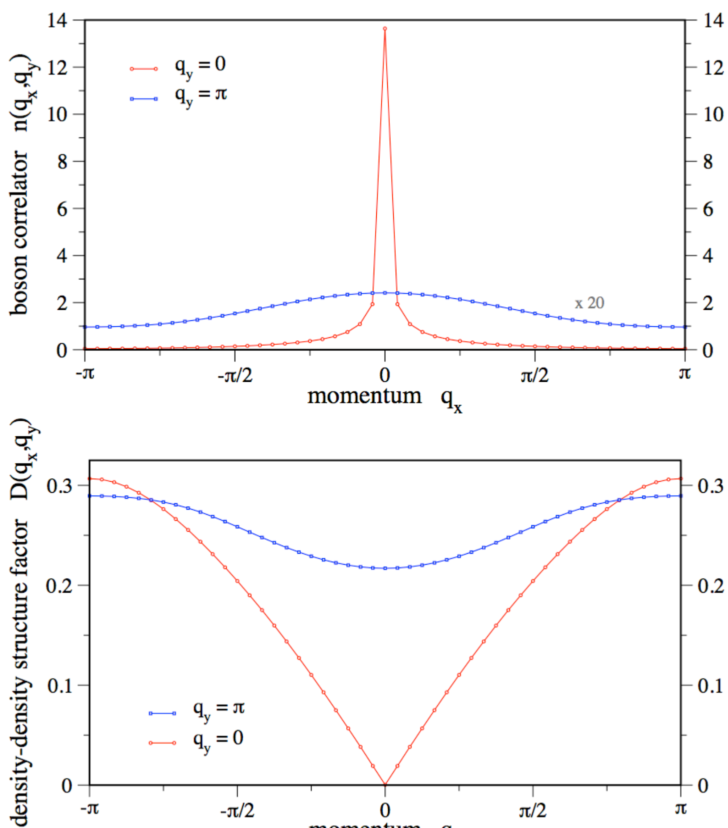


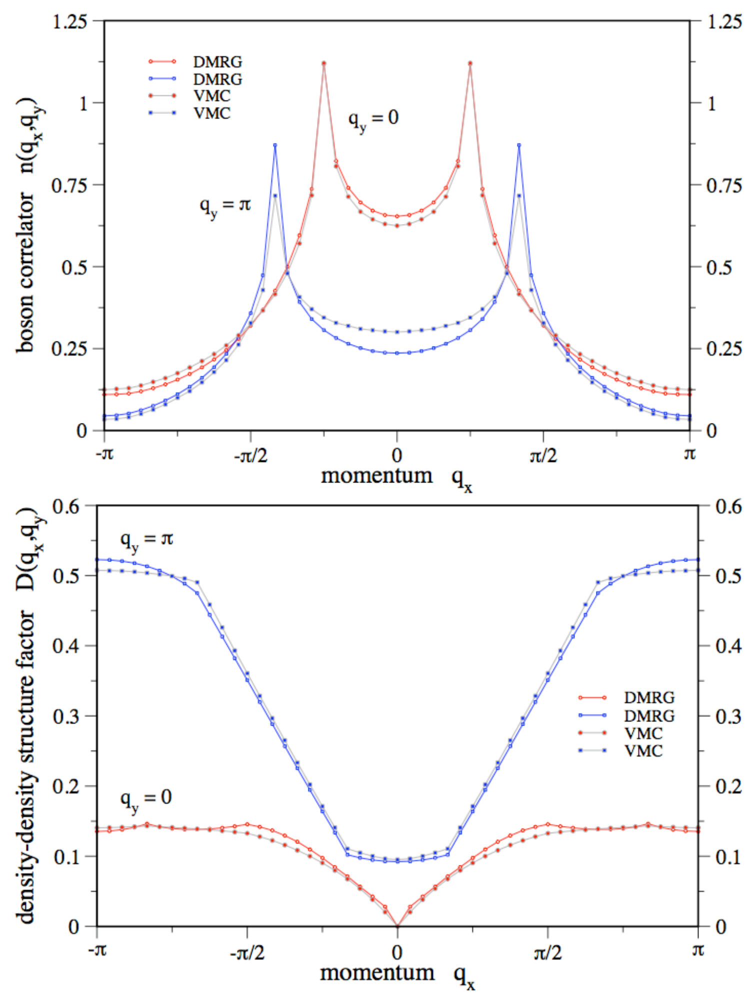
$$\rho = 1/3$$

$$K/J = 3$$

$$J_\perp/J = 1$$

Superfluid versus D-wave Bose-Metal

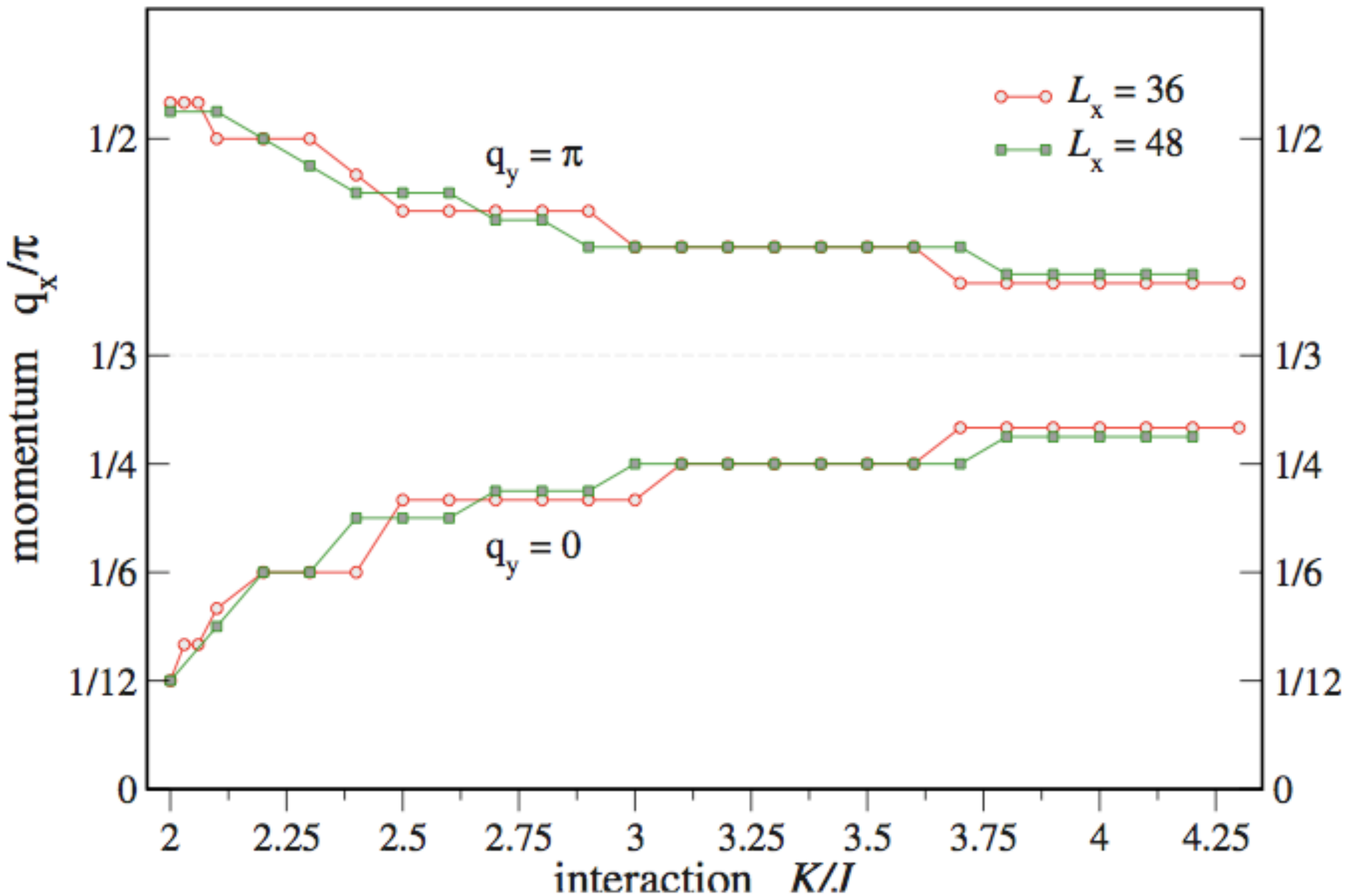




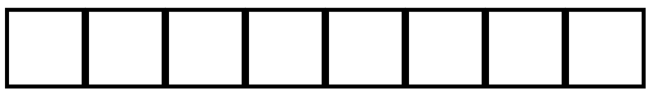
Superfluid - “condensed”
at zero momentum

D-wave Bose-Metal; Singular
“Bose points” at $q_y = 0, \pi$

Singular Momentum in D-wave Bose-Metal
(Bose “surface”)



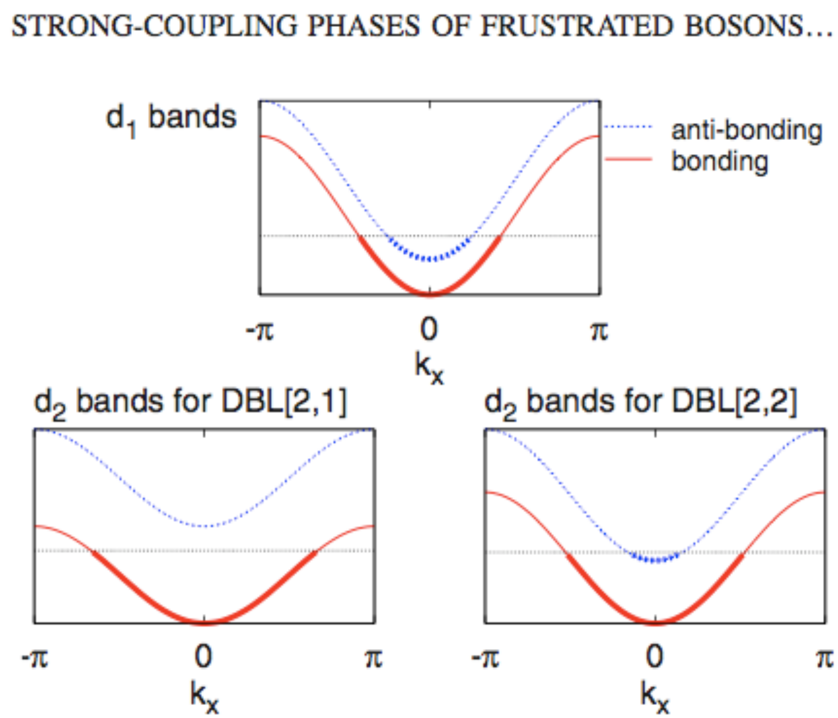
Variational Wavefunction for ladder



In DBM:

Bonding/Antibonding occupied
For d_1 Fermion

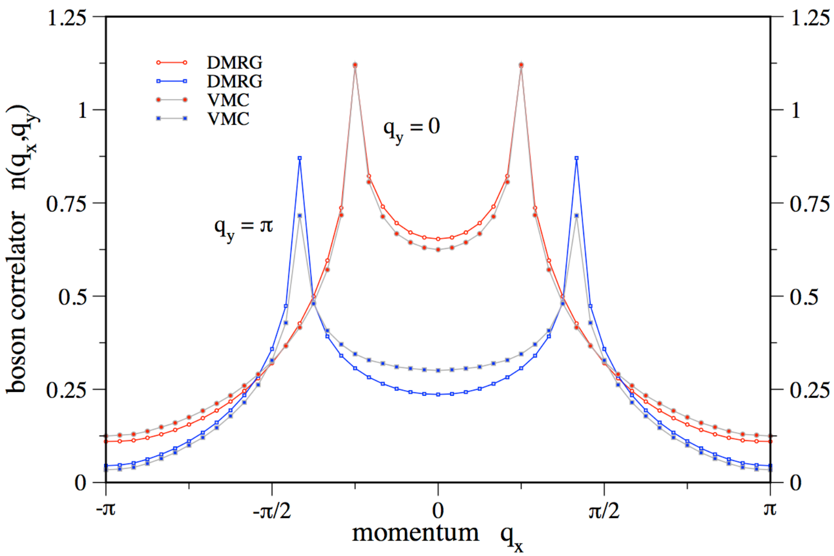
Just Bonding occupied
For d_2 Fermion

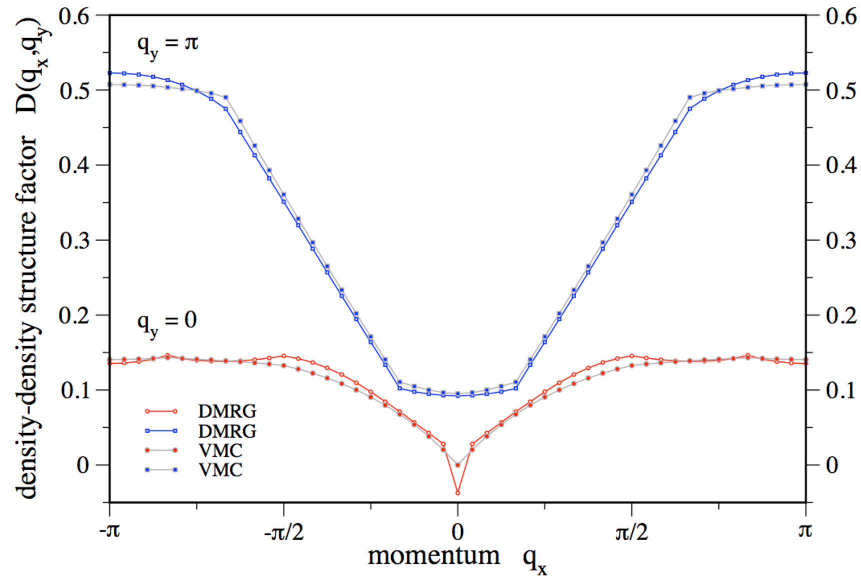


Variational parameter:
Fermi wavevectors in d_1 bands

$$\Psi_{\mathrm{bos}}(r_1,r_2,\ldots)=\Psi_{d_1}(r_1,r_2,\ldots)\cdot\Psi_{d_2}(r_1,r_2,\ldots).$$

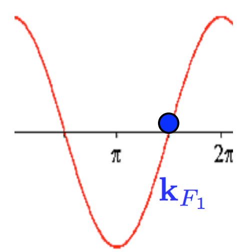
DBM: How good is ladder variational wavefunction?



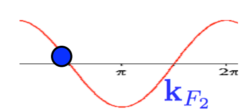


Gauge mean field theory predicts singularities in momentum distribution function at: $\mathbf{k}_{F_1} \pm \mathbf{k}_{F_2}$

Both DMRG and $\det_1 \times \det_2$ Wavefunction show singular cusps *only* at: $\mathbf{k}_{F_1} - \mathbf{k}_{F_2}$



d_1



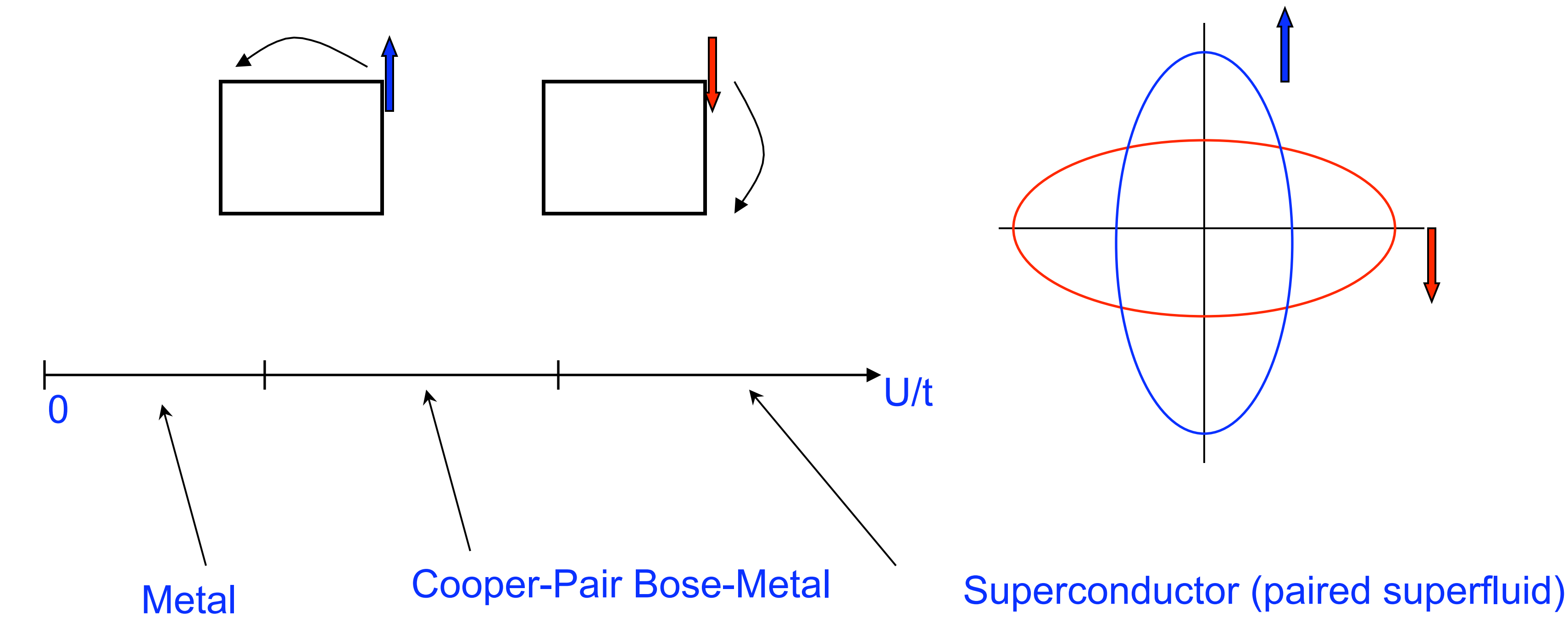
d_2

Why? Ampere's Law - Parallel currents attract

d_1 and d_2 Fermions have opposite gauge charge, so right moving d_1 attracts left moving d_2 to form boson at momentum: $\mathbf{k}_{F_1} - \mathbf{k}_{F_2}$

(Conjectured) Phase-Diagram for Anisotropic attractive U Hubbard model

$$\mathcal{H}_F = -\sum_{ij} t_{ij}^{\alpha}(c_{i,\alpha}^{\dagger} c_{j,\alpha} + h.c.) - U \sum_i n_{i\uparrow} n_{i\downarrow}$$



Direct analysis of anisotropic Attractive U Hubbard model

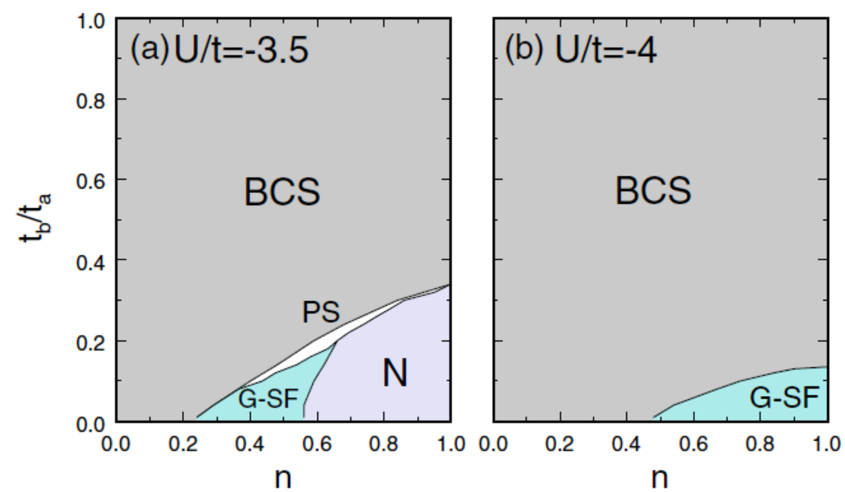
A.E. Feiguin and MPAF, PRL 103, 25303 (2009).

BCS theory – Phase diagram

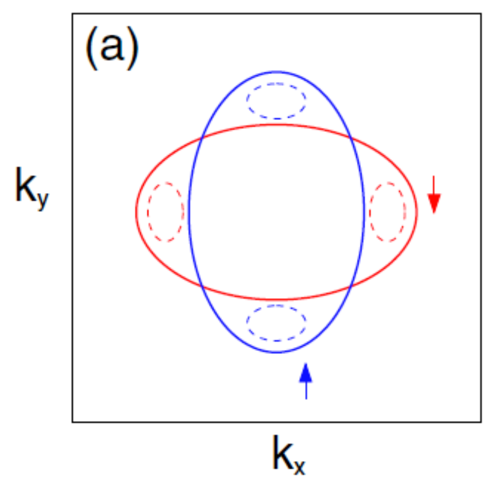
t_b/t_a = anisotropy in the hopping
n = Fermion density

$$\epsilon_\uparrow(k_x, k_y) = -2t_a\cos(k_x) - 2t_b\cos(k_y) - \mu,$$

$$\epsilon_\downarrow(k_x, k_y) = -2t_b\cos(k_x) - 2t_a\cos(k_y) - \mu,$$



N= normal metal
BCS = fully gapped superfluid
G-SF = "gapless superfluid",
condensate at Q=0 with gapless
unpaired Fermi pockets



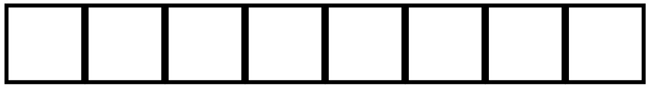
Cooper-pair Bose metal not accessible in BCS theory

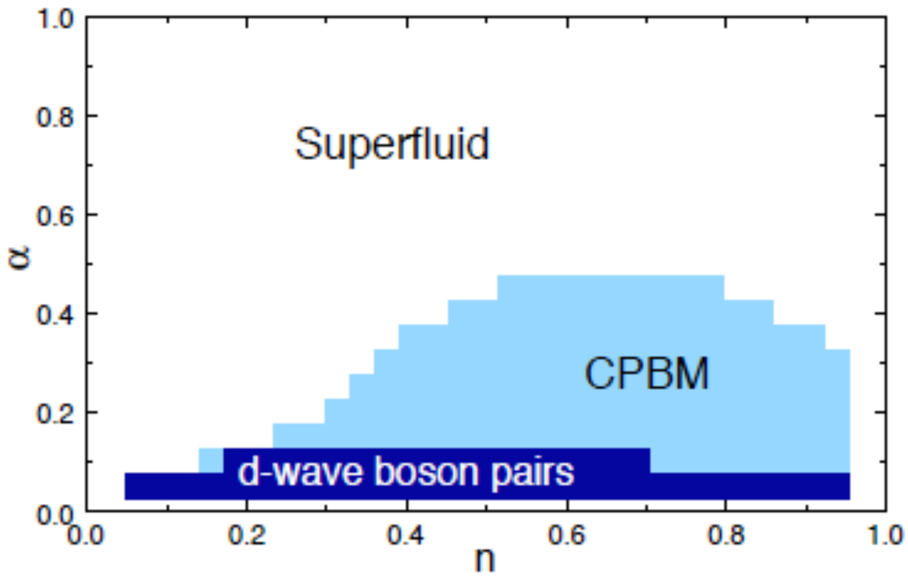
Anisotropic attractive U Hubbard on 2-leg ladder

A.E. Feiguin, MPAF, cond-mat/1007.5251

$$H = -\sum_{i,\lambda,\sigma} t_{x,\sigma}\left(c^\dagger_{i,\lambda,\sigma}c_{i+1,\lambda,\sigma} + \mathrm{H.c.}\right) - \sum_{i,\sigma} t_{y,\sigma}\left(c^\dagger_{i,1,\sigma}c_{i,2,\sigma} + \mathrm{H.c.}\right) + U\sum_{i,\lambda} n_{i,\lambda,\uparrow}n_{i,\lambda,\downarrow}.$$

$$t_{x\uparrow} = t_{y\downarrow} |= 1, t_{x\downarrow} = t_{y\uparrow} = \alpha$$

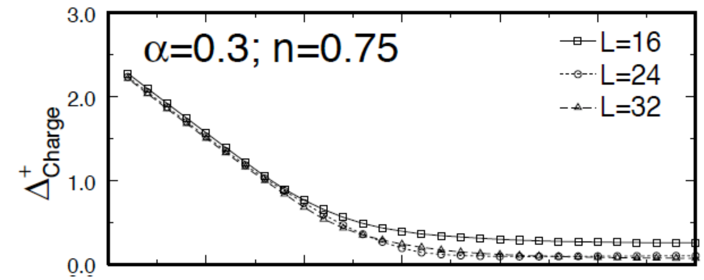




U=-4

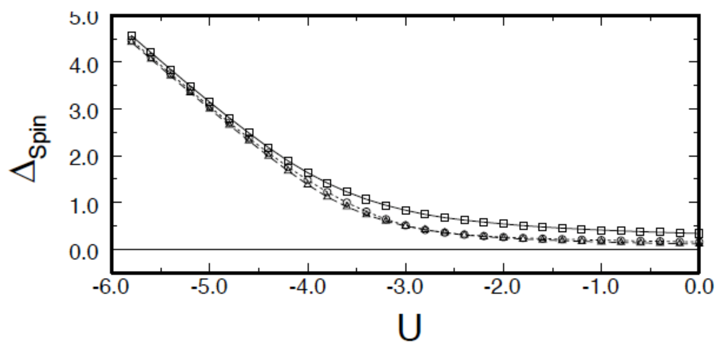
Evidence for CPBM

$$b_i^\dagger = c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger$$



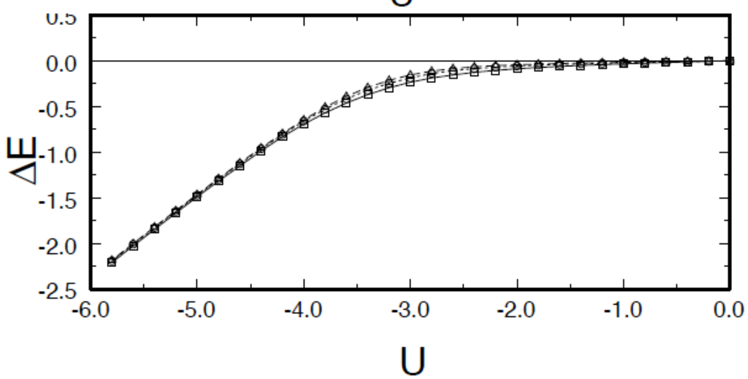
Fermion gap

$$\Delta_c^+ = E_{(N+1,S+\frac{1}{2})} + E_{(N-1,S-\frac{1}{2})} - 2E_{(N,S)}$$



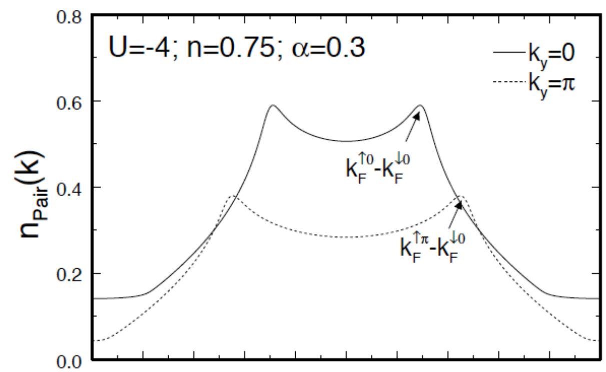
Spin gap

$$\Delta_s^+ = E_{(N,S+1)} - E_{(N,S)}.$$



Pair binding energy

$$\Delta E = [E_{(N-2,S)} - E_{(N,S)}] - [E_{(N-1,S+\frac{1}{2})} - E_{(N,S)}] - [E_{(N-1,S-\frac{1}{2})} - E_{(N,S)}]$$



$$n_{\mathrm{Pair}}(\mathbf{k}) = (1/L)\sum_{ij} \exp[i\mathbf{k}(\mathbf{r}_i - \mathbf{r}_j)]\, \langle b_i^\dagger b_j \rangle$$

Cooper-pair momentum
Distribution function

Resembles Bose-metal,
Definitely not a superfluid

Fermions are gapped,
But Cooper-pair “sees” residual
effects of Fermi surfaces

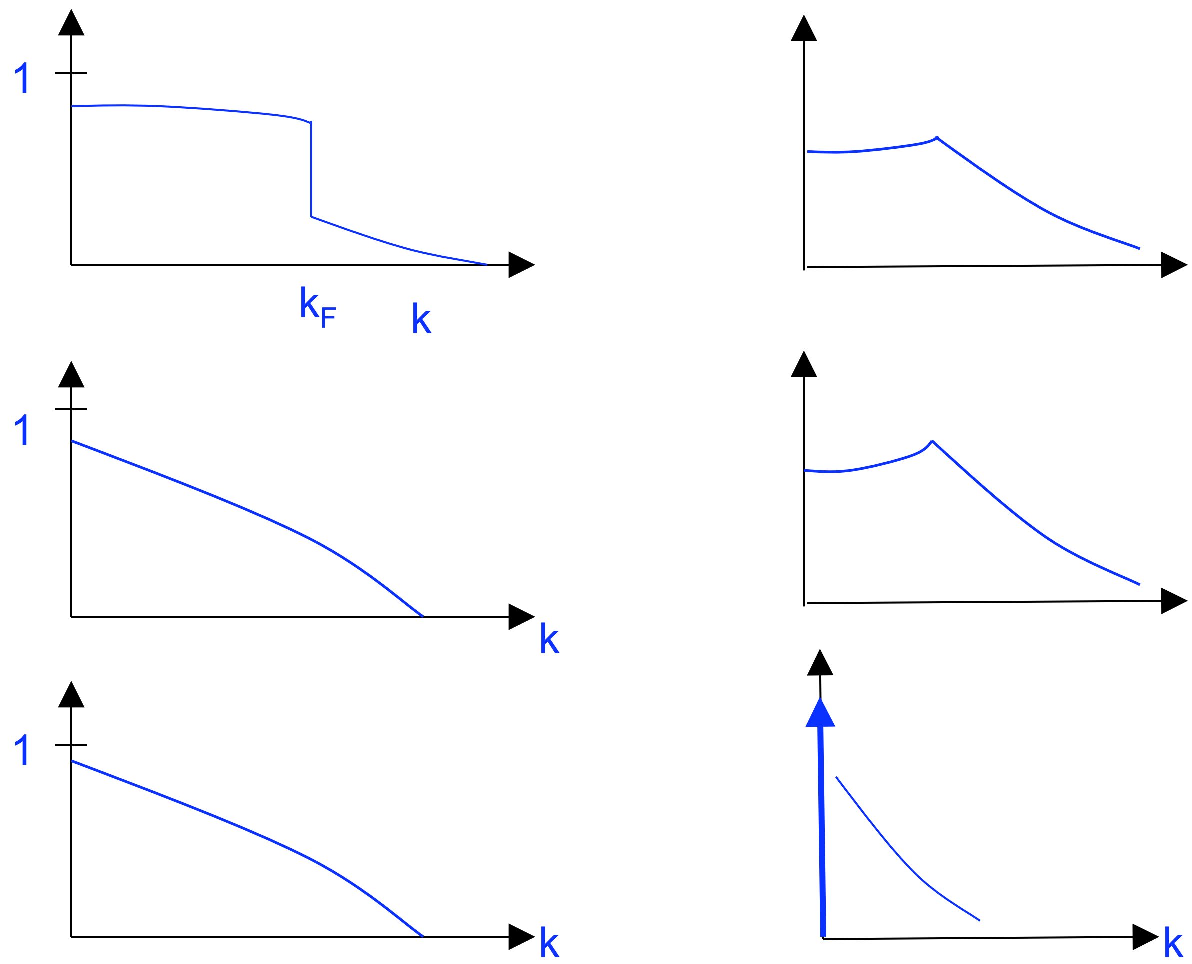
Fermion and Cooper-pair momentum distribution functions

$$\langle c_k^\dagger c_k \rangle \qquad \langle b_k^\dagger b_k \rangle \qquad b_i = c_{i\uparrow} c_{i\downarrow}$$

Metal

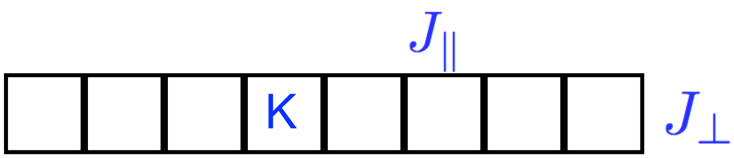
Cooper-pair Bose-metal

superfluid



Summary & Outlook

- Bose-Metals are 2D gapless liquids with singular "Bose" surfaces
- Cooper-pair Bose-metal accessible in cold atoms with anisotropic attractive Hubbard model?
- Every 2D Bose-Metal has a distinct set of quasi-1D descendents states which should be numerically accessible via DMRG
- Hard core bosons with 4-site ring term on the 2-leg ladder has a quasi-1D descendant Bose-Metal ground state over a large part of phase diagram



Future generalizations (DMRG, VMC, gauge theory):

- Boson Ring exchange models on 3-leg (in progress), 4-leg ladders
- Quasi-1D descendents of 2D non-Fermi liquids of itinerant electrons?
  (D-Wave Metal on the n-leg ladder?)
- Other Hamiltonians with Bose-Metal or non-Fermi-liquid states???