Quantum Information Processing and Quantum Simulation with Ultracold Alkaline-Earth Atoms in Optical Lattices

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Quantum Simulation: Nature Phys. 6, 289 (2010)

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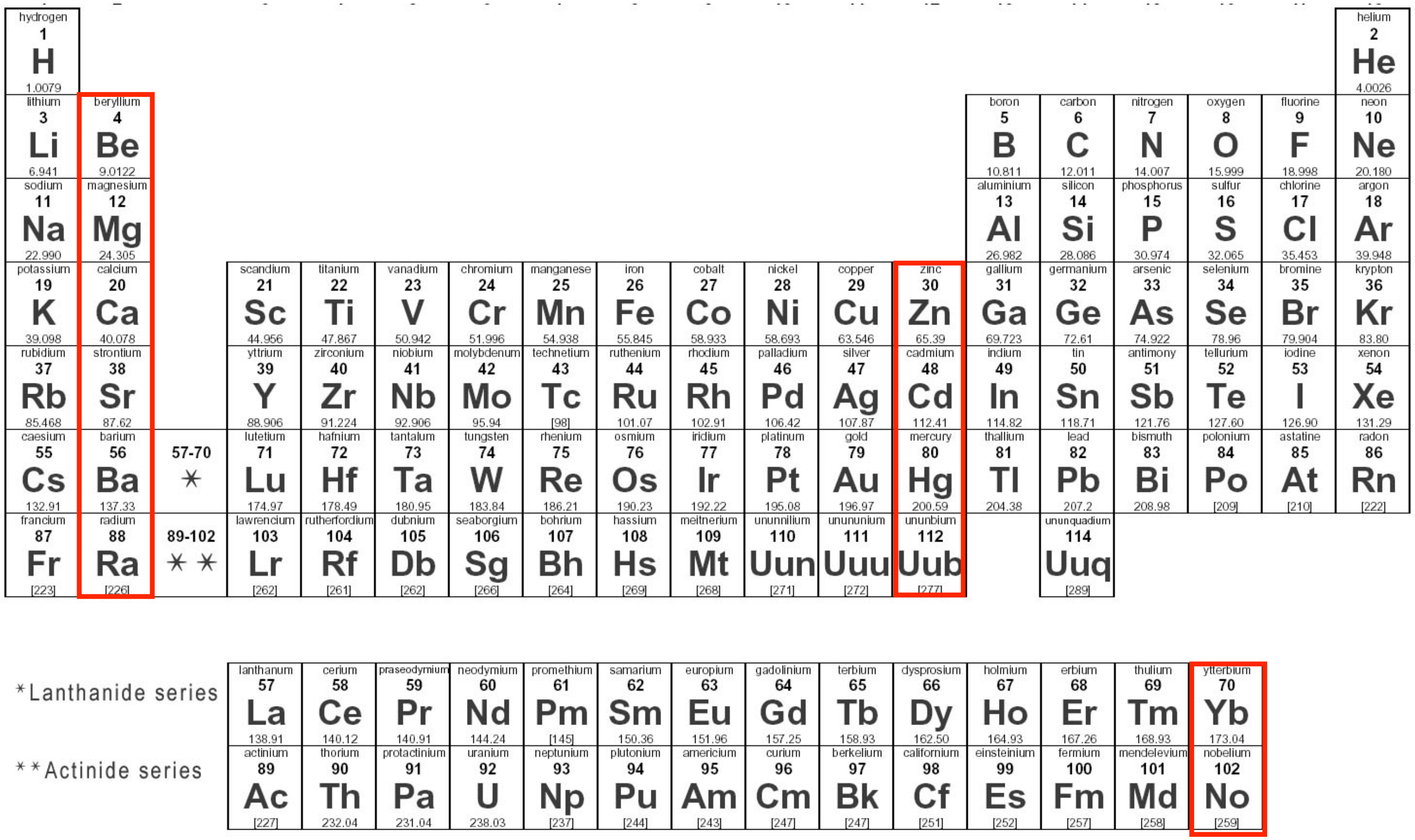
Alkaline-earth(-like) atoms

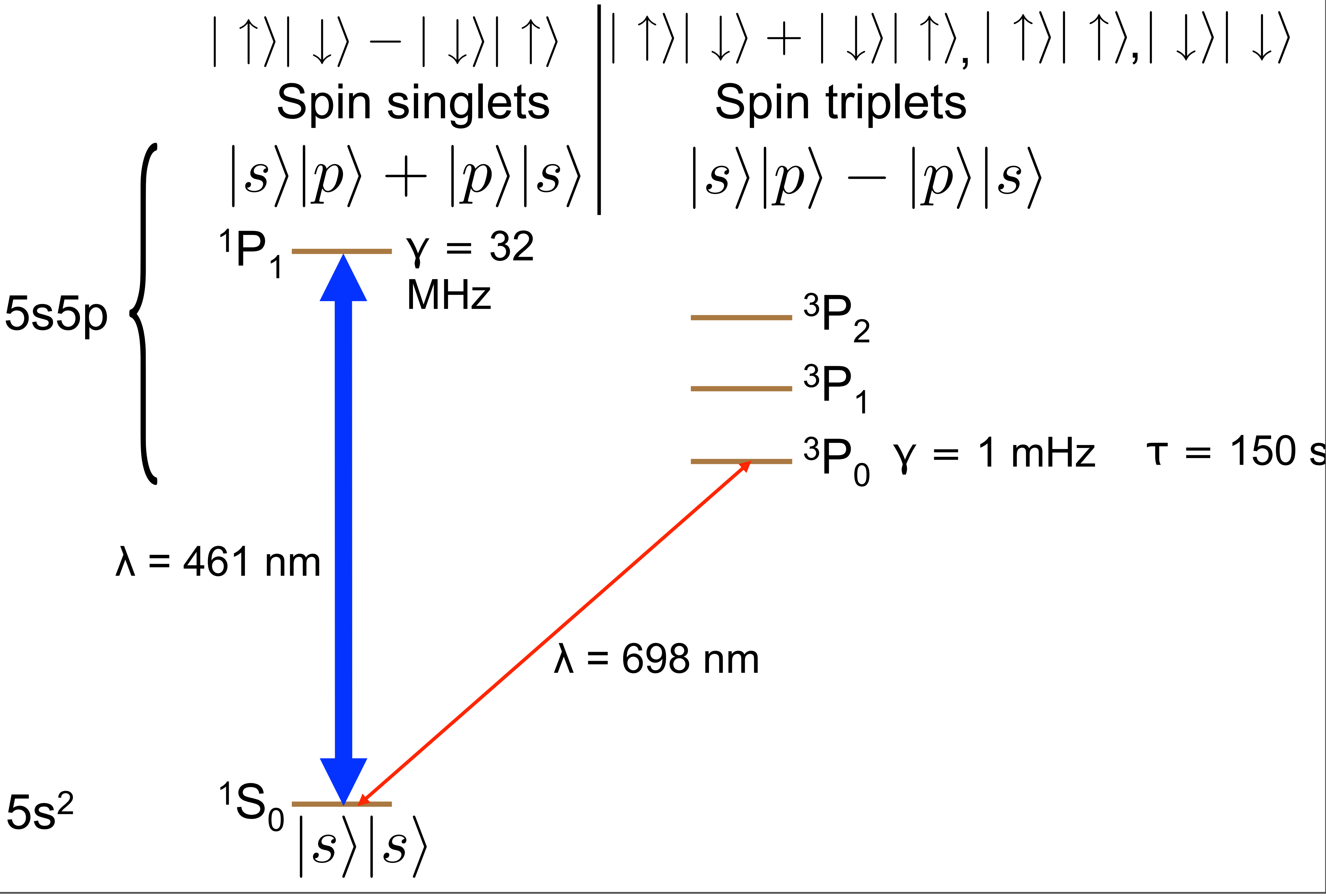


hydrogen 1 H 1.0079																		helium 2 He 4.0026
lithium 3 Li 6.941	beryllium 4 Be 9.0122												boron 5 B 10.811	carbon 6 C 12.011	nitrogen 7 N 14.007	oxygen 8 O 15.999	fluorine 9 F 18.998	neon 10 Ne 20.180
sodium 11 Na 22.990	magnesium 12 Mg 24.305												aluminium 13 Al 26.982	silicon 14 Si 28.086	phosphorus 15 P 30.974	sulfur 16 S 32.065	chlorine 17 Cl 35.453	argon 18 Ar 39.948
potassium 19 K 39.098	calcium 20 Ca 40.078		scandium 21 Sc 44.956	titanium 22 Ti 47.867	vanadium 23 V 50.942	chromium 24 Cr 51.996	manganese 25 Mn 54.938	iron 26 Fe 55.845	cobalt 27 Co 58.933	nickel 28 Ni 58.693	copper 29 Cu 63.546	zinc 30 Zn 65.39	gallium 31 Ga 69.723	germanium 32 Ge 72.61	arsenic 33 As 74.922	selenium 34 Se 78.96	bromine 35 Br 79.904	krypton 36 Kr 83.80
rubidium 37 Rb 85.468	strontium 38 Sr 87.62		yttrium 39 Y 88.906	zirconium 40 Zr 91.224	niobium 41 Nb 92.906	molybdenum 42 Mo 95.94	technetium 43 Tc [98]	ruthenium 44 Ru 101.07	rhodium 45 Rh 102.91	palladium 46 Pd 106.42	silver 47 Ag 107.87	cadmium 48 Cd 112.41	indium 49 In 114.82	tin 50 Sn 118.71	antimony 51 Sb 121.76	tellurium 52 Te 127.60	iodine 53 I 126.90	xenon 54 Xe 131.29
caesium 55 Cs 132.91	barium 56 Ba 137.33	57-70 *	lutetium 71 Lu 174.97	hafnium 72 Hf 178.49	tantalum 73 Ta 180.95	tungsten 74 W 183.84	rhenium 75 Re 186.21	osmium 76 Os 190.23	iridium 77 Ir 192.22	platinum 78 Pt 195.08	gold 79 Au 196.97	mercury 80 Hg 200.59	thallium 81 Tl 204.38	lead 82 Pb 207.2	bismuth 83 Bi 208.98	polonium 84 Po [209]	astatine 85 At [210]	radon 86 Rn [222]
francium 87 Fr [223]	radium 88 Ra [226]	89-102 * *	lawrencium 103 Lr [262]	rutherfordium 104 Rf [261]	dubnium 105 Db [262]	seaborgium 106 Sg [266]	bohrium 107 Bh [264]	hassium 108 Hs [269]	meitnerium 109 Mt [268]	ununnilium 110 Uun [271]	unununium 111 Uuu [272]	ununbium 112 Uub [277]		ununquadium 114 Uuq [289]				

*Lanthanide series	lanthanum 57 La 138.91	cerium 58 Ce 140.12	praseodymium 59 Pr 140.91	neodymium 60 Nd 144.24	promethium 61 Pm [145]	samarium 62 Sm 150.36	europium 63 Eu 151.96	gadolinium 64 Gd 157.25	terbium 65 Tb 158.93	dysprosium 66 Dy 162.50	holmium 67 Ho 164.93	erbium 68 Er 167.26	thulium 69 Tm 168.93	ytterbium 70 Yb 173.04
* *Actinide series	actinium 89 Ac [227]	thorium 90 Th 232.04	protactinium 91 Pa 231.04	uranium 92 U 238.03	neptunium 93 Np [237]	plutonium 94 Pu [244]	americium 95 Am [243]	curium 96 Cm [247]	berkelium 97 Bk [247]	californium 98 Cf [251]	einsteinium 99 Es [252]	fermium 100 Fm [257]	mendelevium 101 Md [258]	nobelium 102 No [259]

- fermionic alkaline-earths have nuclear spin I > 0

Electronic level structure of ^{87}Sr (nuclear spin levels not shown)



$|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle$

Spin singlets

$|s\rangle|p\rangle + |p\rangle|s\rangle$

$|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle, |\uparrow\rangle|\uparrow\rangle, |\downarrow\rangle|\downarrow\rangle$

Spin triplets

$|s\rangle|p\rangle - |p\rangle|s\rangle$

5s5p

$^1P_1$ γ = 32 MHz

$^3P_2$

$^3P_1$

$^3P_0$ γ = 1 mHz τ = 150 s

λ = 461 nm

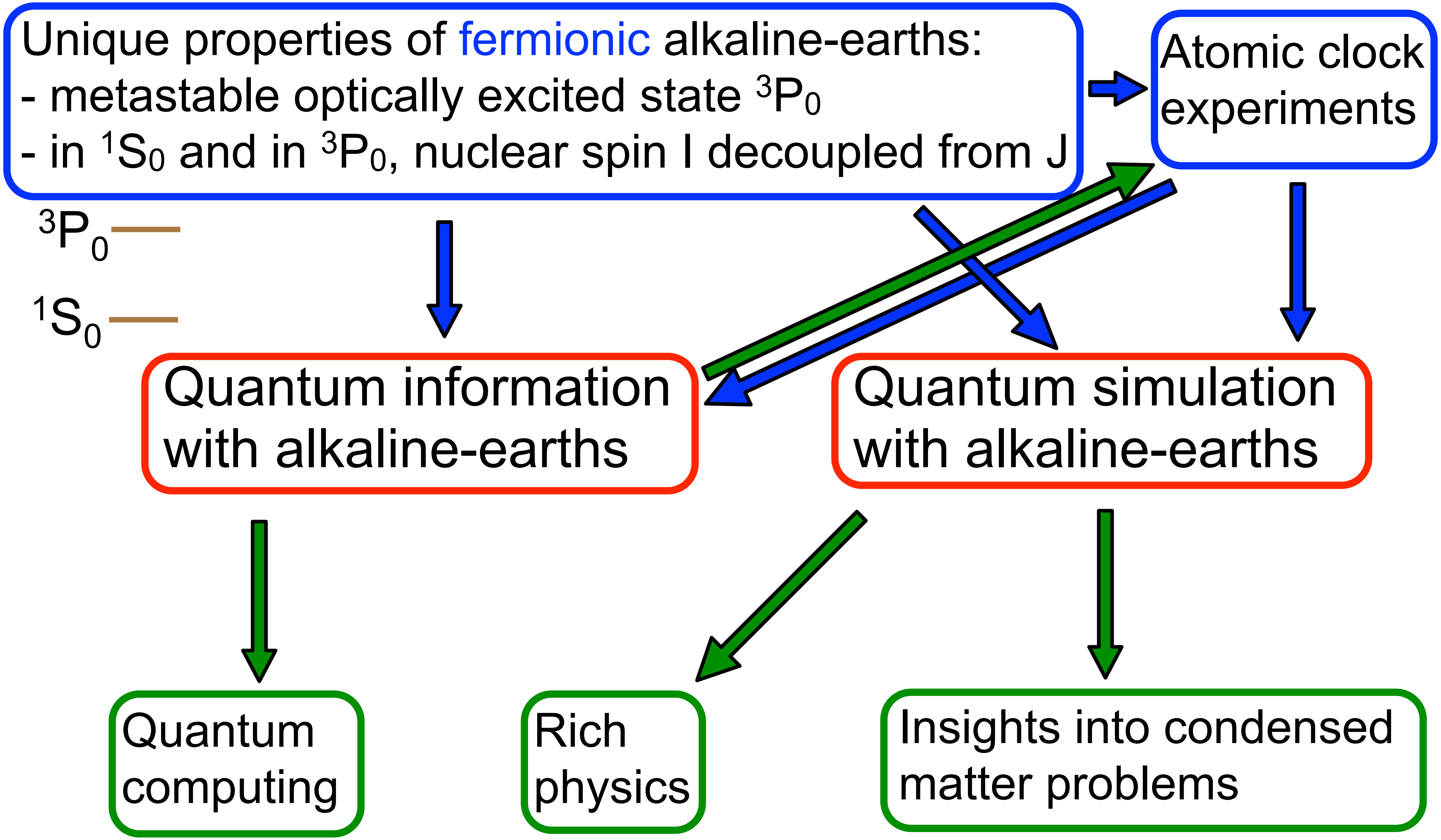
λ = 698 nm

$5s^2$

$^1S_0$

$|s\rangle|s\rangle$

Brief Motivation and Overview



Unique properties of fermionic alkaline-earths:
- metastable optically excited state $^3P_0$
- in $^1S_0$ and in $^3P_0$, nuclear spin I decoupled from J

Atomic clock experiments

Quantum information with alkaline-earths

Quantum simulation with alkaline-earths

Quantum computing

Rich physics

Insights into condensed matter problems

Quantum Information Processing with Ultracold Alkaline-Earth Atoms

Quantum information with ultracold fermionic alkaline-earths

Proposals:

- Derevianko, Cannon PRA ’04
- Hayes, Julienne, Deutsch PRL ’07
- Reichenbach, Deutsch PRL ’07
- Daley, Boyd, Ye, Zoller PRL ’08
- AG, Rey, Daley, Boyd, Ye, Zoller, Lukin PRL ’09
- Reichenbach, Julienne, Deutsch PRA ’09
- Shibata, Kato, Yamaguchi, Uetake, Takahashi App. Phys. B ’09
- etc...

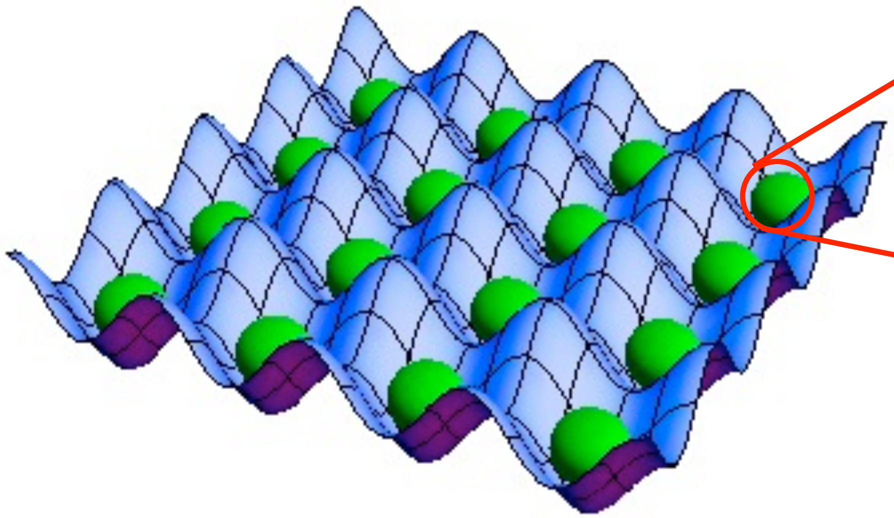
Alkaline-Earth Atoms as Few-Qubit Quantum Registers

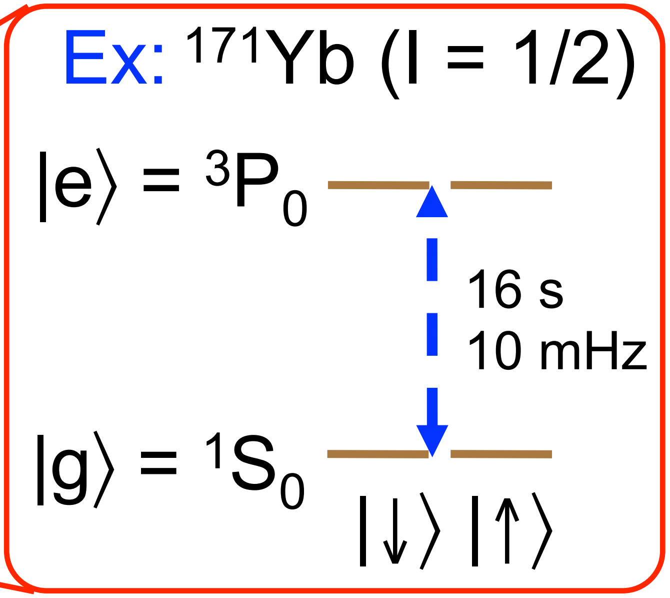
The Idea

Need for accurate manipulation of large quantum systems:

- quantum computation, precision measurements, etc...

Array of few-qubit quantum registers!





Ex: ^{171}Yb (I = 1/2)

$|e\rangle = {}^3P_0$

16 s
10 mHz

$|g\rangle = {}^1S_0$

$|\downarrow\rangle$ $|\uparrow\rangle$

Need to:

1) accurately manipulate single register
2) detect $|g\rangle$-$|e\rangle$ qubit without destroying $|\downarrow\rangle$-$|\uparrow\rangle$ qubit
3) couple registers

Related: Cirac et al., Dur & Briegel, Sorensen & Molmer, Jiang et al., Monroe, Saffman, Hayes et al, Daley et al, Stock et al, Strauch et al...

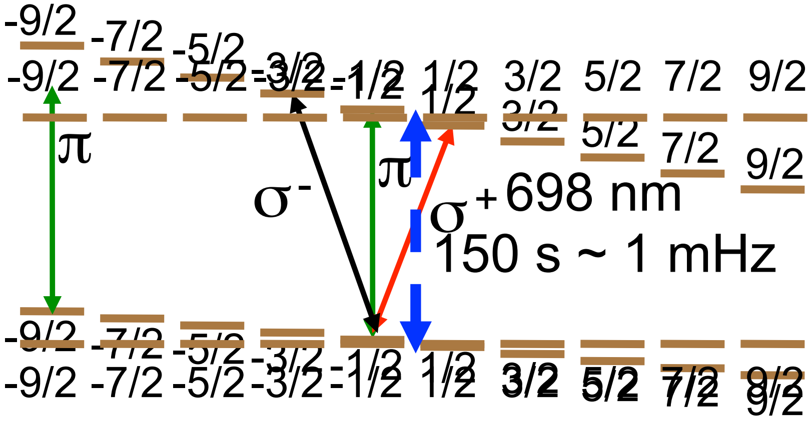
Manipulate an Individual Register: Easy

Ex: ^{87}Sr (I = 9/2) [1 electronic + up to 3 nuclear qubits]

- apply a magnetic field
- different g-factors

$|e\rangle = {}^3P_0$

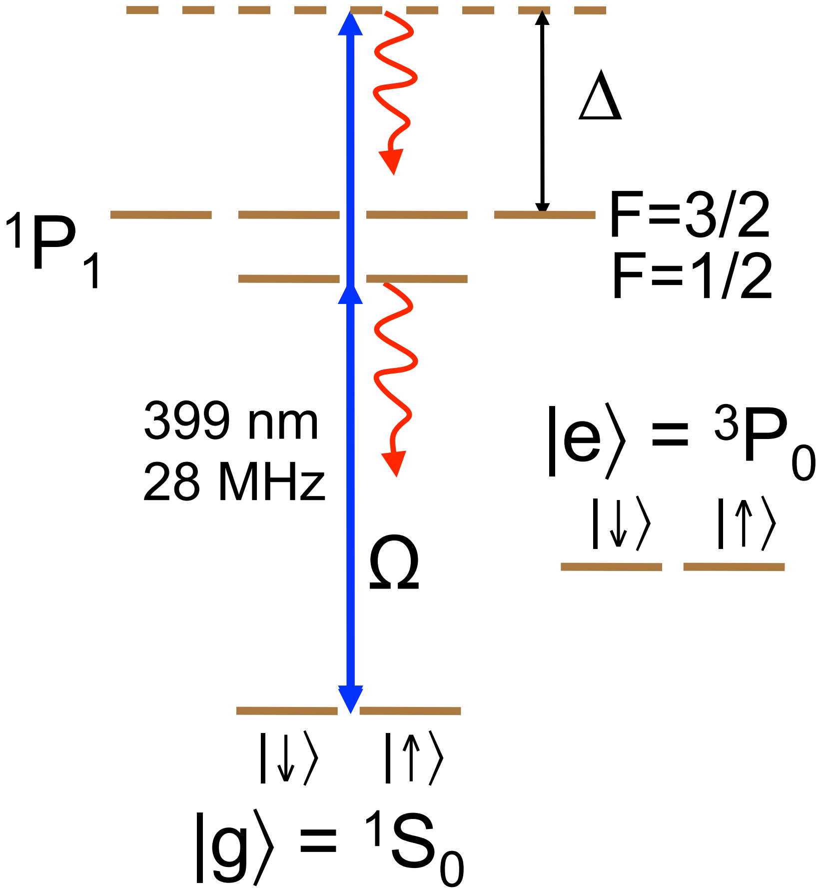
$|g\rangle = {}^1S_0$



- Transitions resolved in experiments [Boyd et al., Science (2006)].

Detect the Electronic Qubit without Erasing the Nuclear Qubits

Ex: ^{171}Yb (I = 1/2)



Problem: hyperfine coupling in $^1P_1$

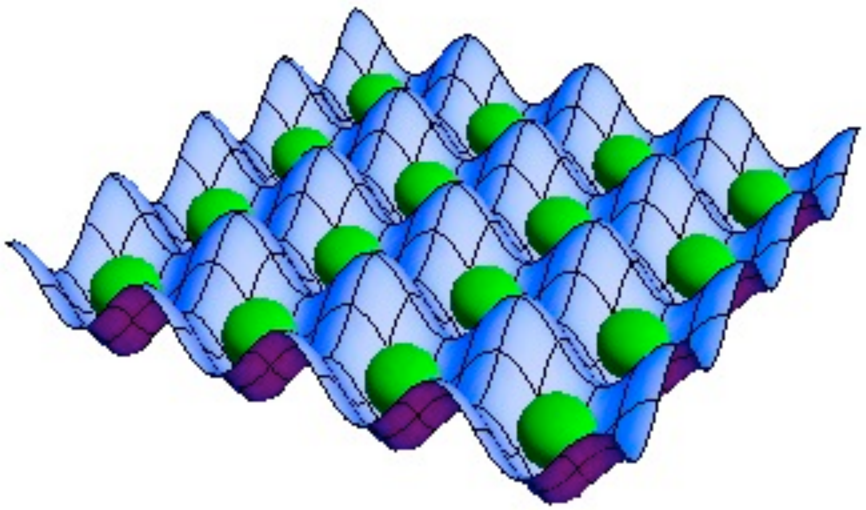
$$\hat{H} = A\hat{\mathbf{I}}\cdot\hat{\mathbf{J}}$$

Solution: Use off-resonant fluorescence
[Childress et al, PRA (2005)]

- destructive interference prevents nuclear spin flips

Related work: Reichenbach & Deutsch, PRL (2007)

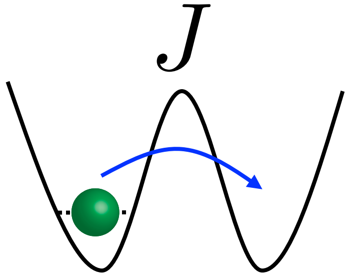
Coupling two registers



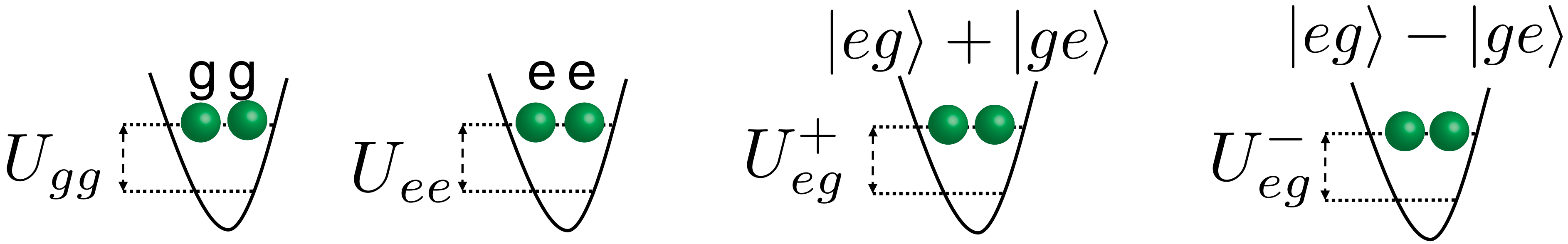
$|e\rangle = {}^3P_0$ —

$|g\rangle = {}^1S_0$ —

- Tunneling rate (assume same for g and e):

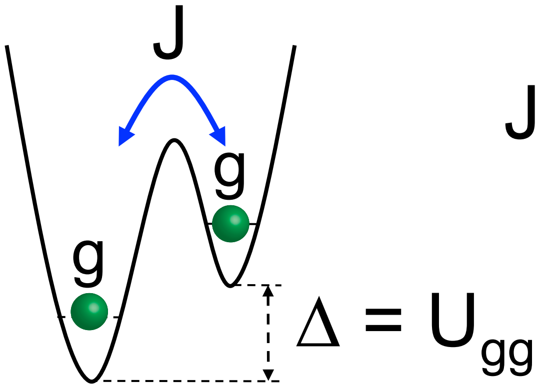


- 4 different interaction energies:

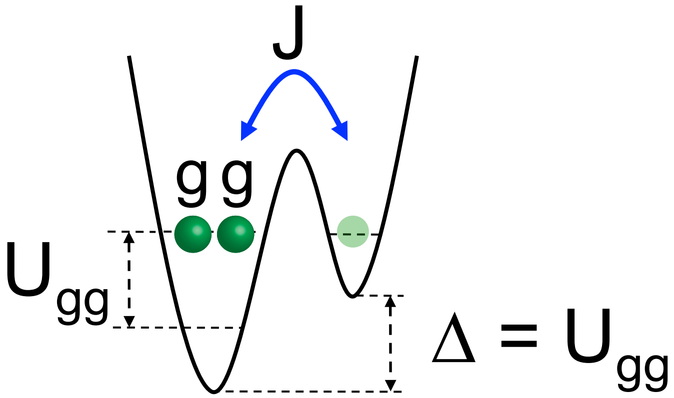


Coupling two registers

Ex: ^{171}Yb (I = 1/2): spin states ↑ and ↓



J << U's



- resonantly driven two-level system [Folling et al, Nature (2007), Cheinet et al, PRL (2008)]
- after time T ∝ 1/J, have a 2π pulse:

$$|g,g\rangle(|\uparrow,\downarrow\rangle-|\downarrow,\uparrow\rangle) \rightarrow -|g,g\rangle(|\uparrow,\downarrow\rangle-|\downarrow,\uparrow\rangle)$$

- repeat for all nuclear Bell states

=> two-qubit phase gate on the electrons:

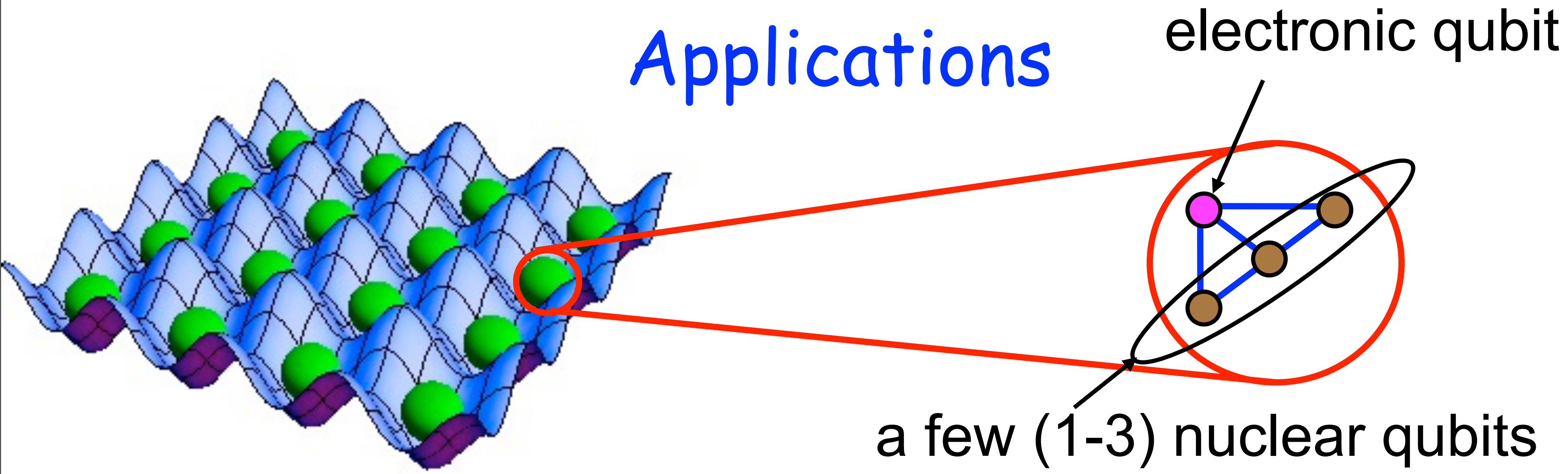
$$|g,g\rangle \rightarrow -|g,g\rangle \quad |g,e\rangle \rightarrow |g,e\rangle \quad |e,g\rangle \rightarrow |e,g\rangle \quad |e,e\rangle \rightarrow |e,e\rangle$$

=> universal manipulation of the full multiregsiter system

- similarly for I > 1/2

Related ideas: Hayes et al, PRL (2007); Daley et al, PRL (2008); Stock et al, PRA (2008); Strauch et al, PRA (2008)

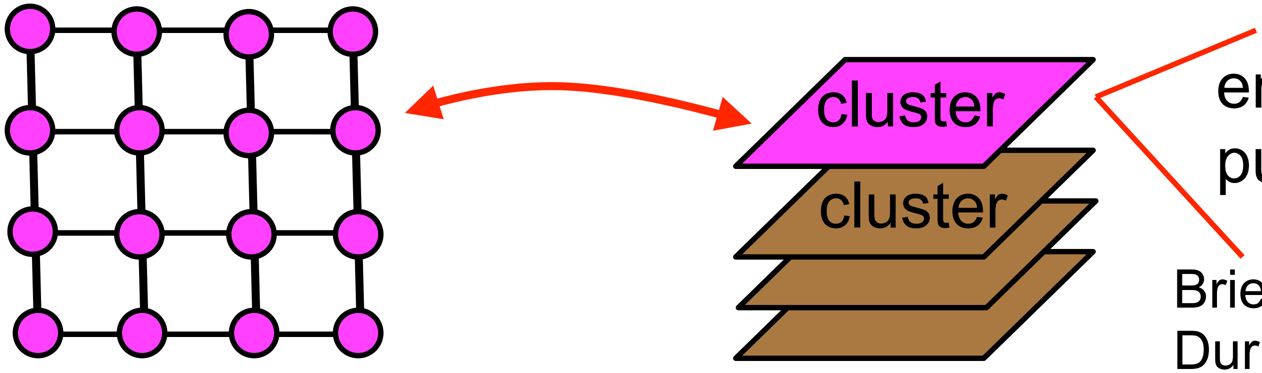
Applications



=> circuit-based quantum computation [Jiang et al., PRA (2007)]

=> generation of high-fidelity entangled many-body states [Dur & Briegel, PRL (2003)]

- cluster states (for measurement-based quantum computation)
- GHZ states (for precision measurements)



$|+\rangle|+\rangle\cdots|+\rangle$

$|+\rangle = |g\rangle + |e\rangle$

Briegel, Dur, Cirac, Zoller, PRL (1998)
Dur & Briegel, PRL (2003)
Aschauer, Dur, Briegel, PRA (2005)

Details: PRL 102, 110503 (2009)

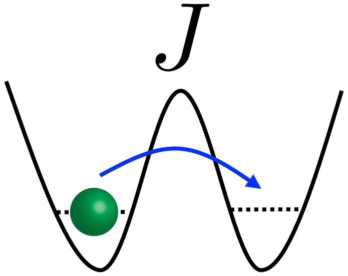
Quantum Simulation with Alkaline-Earth Atoms in Optical Lattices: Two-Orbital SU(N) Magnetism

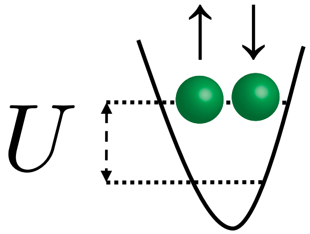
Outline

- Two orbitals and SU(N) symmetry
- Symmetries of the Hamiltonian in more detail
- Four examples:
  - Examples 1 & 2: SU(N) symmetry
  - Examples 3 & 4: two orbitals

Reminder: the usual Hubbard model

$$H = -J \sum_{\langle j,i \rangle ,\sigma} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_j n_{j\uparrow} n_{j\downarrow}$$



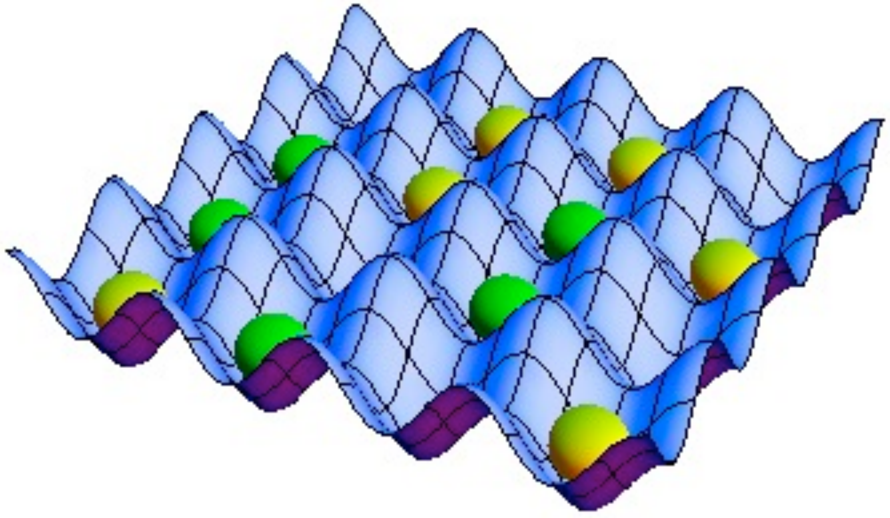


$c^\dagger_{j\sigma}$ - fermion on site j, with spin $\sigma = \uparrow, \downarrow$

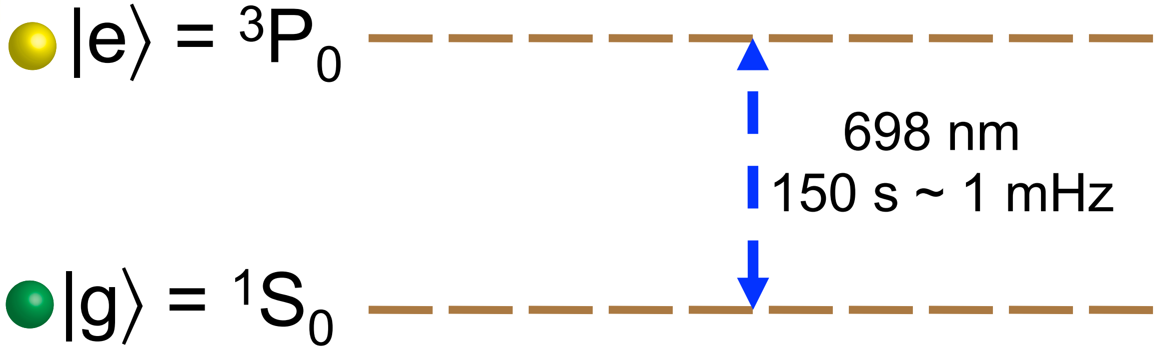
$$n_{j\sigma} = c^\dagger_{j\sigma} c_{j\sigma}$$

- high-temperature superconductivity
- cold-atom realizations: Esslinger, Bloch, ...

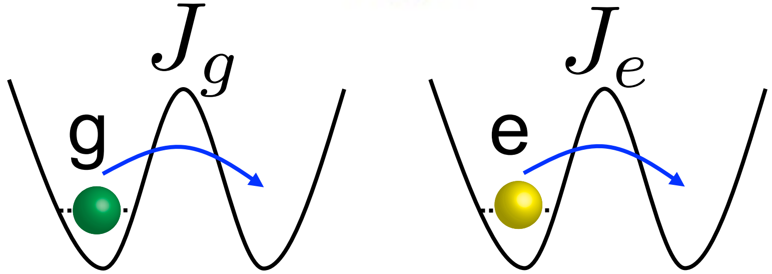
Two orbitals & SU(N) symmetry

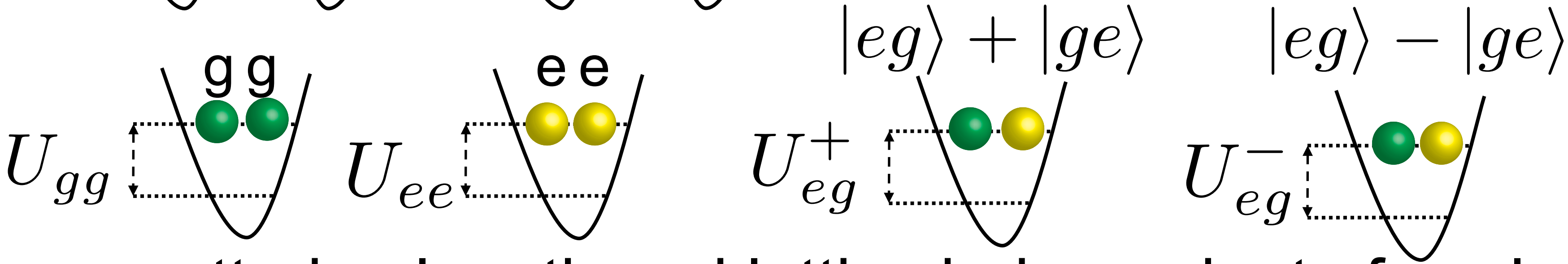


Ex: ^{87}Sr (I = 9/2)



N = 2 I + 1 = 10



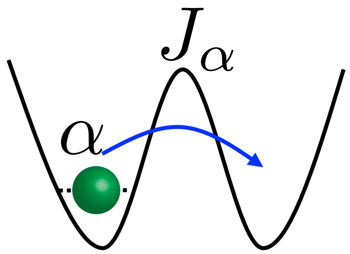


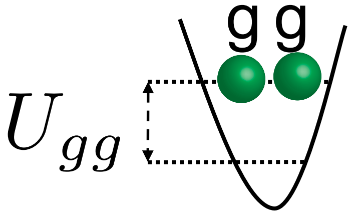
Key: scattering length and lattice independent of nuclear spin (aside from fermionic statistics)

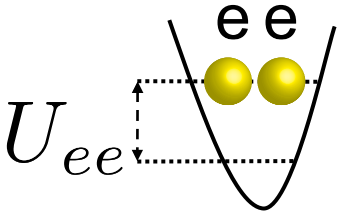
=> SU(N) symmetry

See also Cazalilla, Ho, Ueda, New J. Phys. (2009): SU(6) symmetry in ^{173}Yb

Two-orbital SU(N)-symmetric single-band Hubbard model







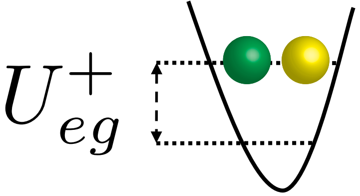
$$H = -\sum_{\langle j,i\rangle\alpha,m} J_\alpha c^\dagger_{i\alpha m} c_{j\alpha m} + \sum_{j,\alpha} \frac{U_{\alpha\alpha}}{2} n_{j\alpha}(n_{j\alpha}-1)$$

$$+V\sum_j n_{je}n_{jg} + V_{ex}\sum_{j,m,m'} c^\dagger_{jgm}c^\dagger_{jem'}c_{jgm'}c_{jem}$$

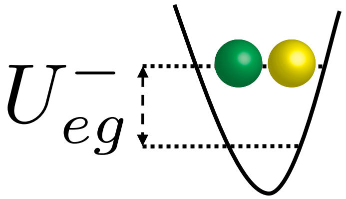
$$V = (U^+_{eg}+U^-_{eg})/2$$

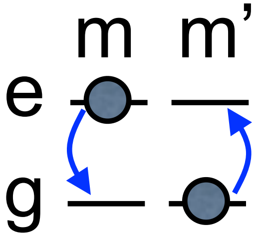
$$V_{ex} = (U^+_{eg}-U^-_{eg})/2$$

$|eg\rangle + |ge\rangle$



$|eg\rangle - |ge\rangle$





$c^\dagger_{j\alpha m}$ - site j, electronic state α (g or e), nuclear spin m (-I ... I)

$$n_{j\alpha} = \sum_m n_{j\alpha m} = \sum_m c^\dagger_{j\alpha m}c_{j\alpha m}$$

Symmetries of the Hamiltonian

$$H = -\sum_{\langle j,i\rangle \alpha,m} J_\alpha c^\dagger_{i\alpha m} c_{j\alpha m} + \sum_{j,\alpha} \frac{U_{\alpha\alpha}}{2} n_{j\alpha}(n_{j\alpha}-1)$$
$$+V\sum_j n_{je} n_{jg} + V_{ex} \sum_{j,m,m'} c^\dagger_{jgm} c^\dagger_{jem'} c_{jgm'} c_{jem}$$

SU(2) pseudospin algebra

$$T^x = \frac{1}{2}\sum_{jm}(c^\dagger_{jem}c_{jgm} + c^\dagger_{jgm}c_{jem})$$
$$T^y = \frac{1}{2}\sum_{jm}(-ic^\dagger_{jem}c_{jgm} + ic^\dagger_{jgm}c_{jem}),$$
$$T^z = \frac{1}{2}\sum_{j}(n_{je} - n_{jg}),$$

$[T^x, T^y] = iT^z$ etc...

SU(N=2I+1) algebra generating nuclear spin rotations

$$S^m_n = \sum_{j,\alpha} c^\dagger_{j\alpha n} c_{j\alpha m}$$
$$[S^m_n, S^p_q] = \delta_{mq} S^p_n - \delta_{pn} S^m_q$$

$$[T^z, H] = 0$$

U(1)xSU(N)

$$[S^m_n, H] = 0 \quad \forall m, n$$

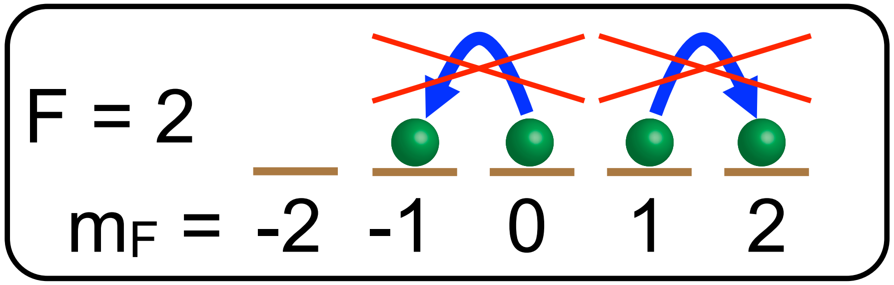
Symmetries of the Hamiltonian

$$H = -\sum_{\langle j,i\rangle \alpha,m} J_\alpha c^\dagger_{i\alpha m} c_{j\alpha m} + \sum_{j,\alpha} \frac{U_{\alpha\alpha}}{2} n_{j\alpha}(n_{j\alpha}-1)$$

$$+V\sum_{j} n_{je} n_{jg} + V_{ex} \sum_{j,m,m'} c^\dagger_{jgm} c^\dagger_{jem'} c_{jgm'} c_{jem}$$

Notice: $S^m_m = \sum_{j\alpha} n_{j\alpha m}$

$$[S^m_m, H] = 0$$



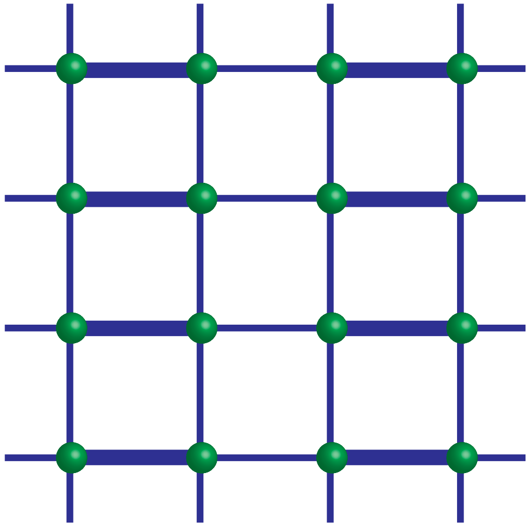
=> if some nuclear Zeeman levels are unoccupied, they will stay unoccupied.

=> I = 9/2 atom (N = 10) can reproduce behavior of all lower I (N < 10) just by choosing initial state

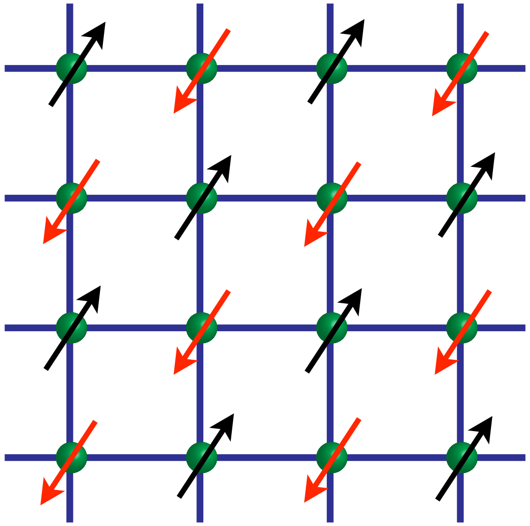
Example 1:

Exotic Valence Bond Solid Phases

$N = 2$ (2 nuclear spin components) one g atom per site

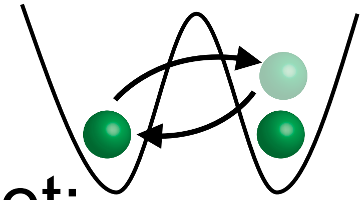


array of singlets?



magnetic (Neel) order?

$J_g \ll U_{gg}$ => spin Hamiltonian

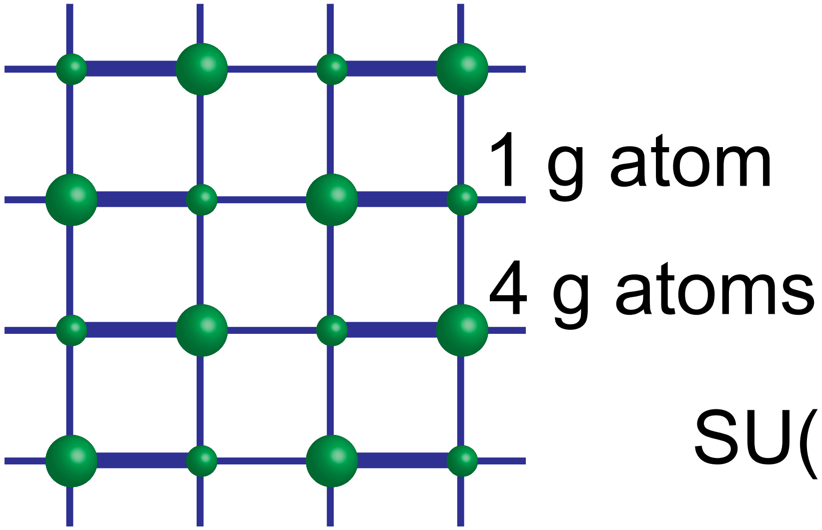


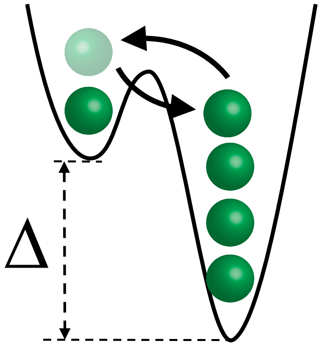
SU(2)-symmetric Heisenberg antiferromagnet:

$$H = \frac{4J_g^2}{U_{gg}} \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

Example 1:
Exotic Valence Bond Solid Phases

$N = 5$ (5 nuclear spin components)





$J_g \ll U_{gg}$

SU(5)-symmetric spin Hamiltonian

Tune Δ to get anti-ferromagnetic interaction

SU(N) singlets: N atoms, energy $\sim -NJ_g^2/U_{gg}$

=> exotic valence bond solid (VBS) order is expected

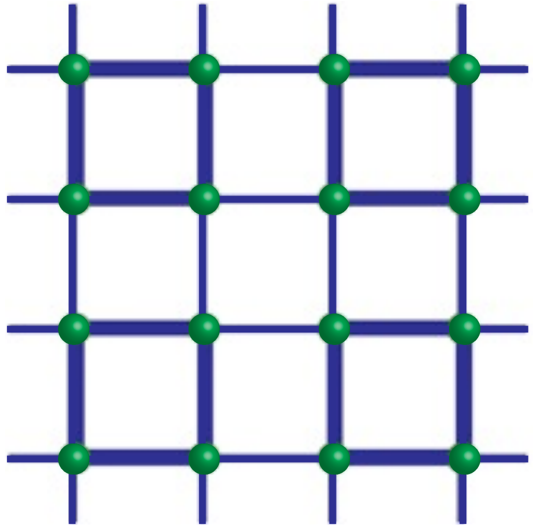
=> gapped & broken rotational symmetry

Melting experiment.

[Read & Sachdev (1990), Harada et al (2003), Assaad (2005),...]

Example 2:
Exotic Valence Plaquette Solid Phases

$N = 4$ (4 nuclear spin components)



1 g atom per site

[Li et al, PRL (1998); Bossche et al, EPJ B, (2000); Xu & Wu, PRB (2008)]

Similar arrangements => exotic spin liquid phases.
[Hermele, Rey, Gurarie, PRL 103, 135301 (2009)]

Example 3: Kugel-Khomskii model

One atom per site (both g and e allowed)

$$H = \sum_{\langle i,j\rangle}\Big[(A+BS_{ij}^2)(T_i^xT_j^x+T_i^yT_j^y)$$

$$+[C+DS_{ij}^2](T_i^zT_j^z+\frac{1}{4})$$

$$+E(1-S_{ij}^2)(T_i^z+T_j^z)+FS_{ij}^2\Big]$$

~ spin-dependent XXZ in pseudospin

- competing orders

$|mm\rangle$ & $|mn\rangle + |nm\rangle$ N = 2 => triplet

$$S_{ij}^2 = \sum_{mn} S_m^n(i) S_n^m(j)$$

= 1 if symmetric under i-j exchange

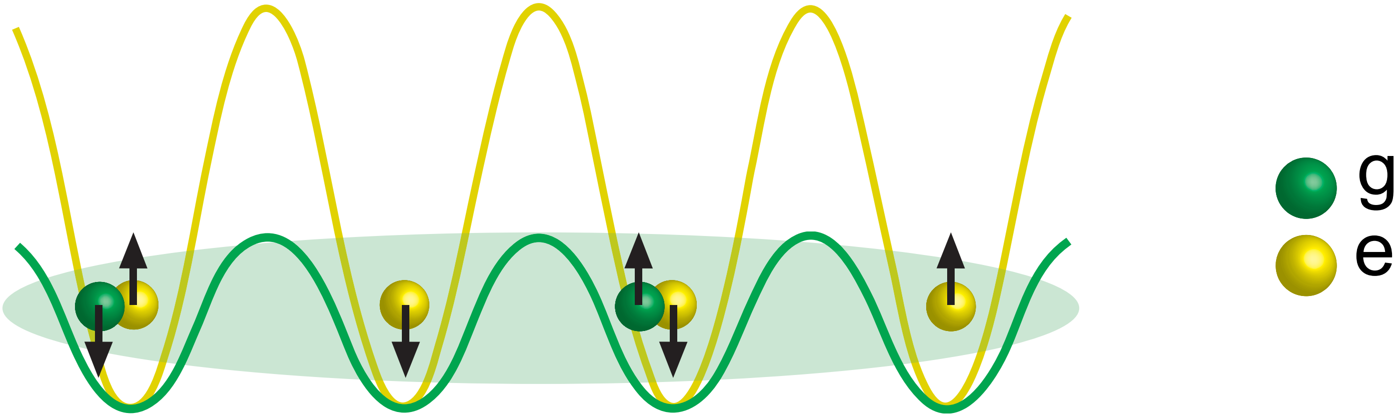
= -1 if antisymmetric under i-j exchange

$|mn\rangle - |nm\rangle$ N = 2 => singlet

Generalization to any N of SU(N=2)-symmetric Kugel-Khomskii model:

- used to model spin-orbital interactions in transition metal oxides with perovskite structure [Kugel,Khomskii, JETP (1973); Tokura,Nagaosa, Science (2000)]

Example 4:
SU(N)-symmetric Kondo Lattice Model



$$H_{KL} = -\sum_{\langle j,i\rangle \alpha,m} J_g c^\dagger_{igm} c_{jgm} + V_{ex}\sum_{j,m,m'} c^\dagger_{jgm} c^\dagger_{jem'} c_{jgm'} c_{jem}$$

- N = 2 case is important in the study of
  - colossal magnetoresistance
  - heavy fermion materials

- Temperature requirement is favorable:
  e.g. heavy Fermi liquid may be accessed with $k_B T \sim U$ (not J^2/U)

[Proposals to study N = 2 Kondo physics with cold atoms:
Duan, Europhys. Lett. '04; Paredes, Tejedor, Cirac, PRA '05]

Detailed study: Foss-Feig, Hermele, Gurarie, Rey PRA '10; arxiv '10

Conclusions

Alkaline-earth atoms for quantum information processing [PRL 102, 110503 (2009)]:

- few-qubit quantum registers
- applications to quantum computing and precision measurements

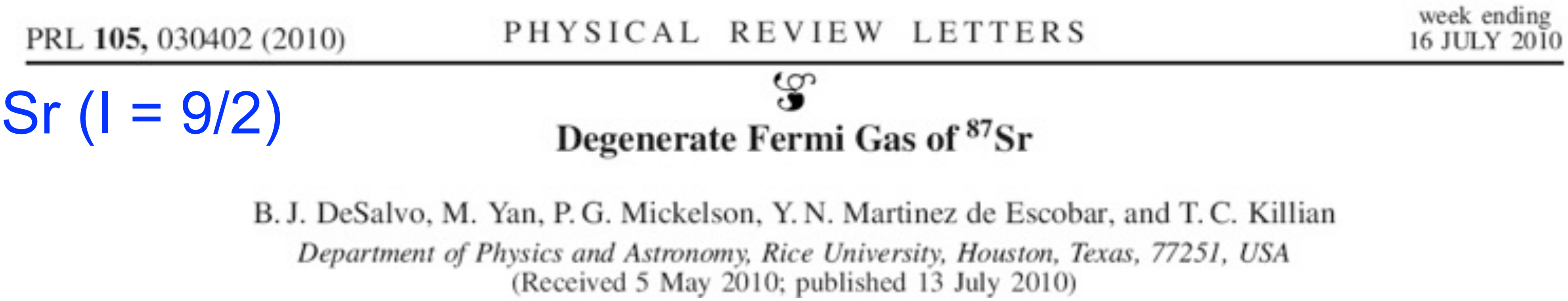
Alkaline-earth atoms for quantum simulation [Nature Phys. 6, 289 (2010)]:

- two-orbital SU(N) Hubbard model
  - the Kugel-Khomskii model
  - SU(N) antiferromagnets
  - SU(N) Kondo lattice model
- few-body (skipped) & many-body experiments
- possible insights into strongly-correlated systems in condensed matter

Outlook

- Analyze new models
- Alkali dimers, ions
- Clock shifts: PRL 103, 260402 (2009)

^{87}Sr (I = 9/2)


PRL **105,** 030402 (2010) PHYSICAL REVIEW LETTERS week ending 16 JULY 2010

Degenerate Fermi Gas of ^{87}Sr

B. J. DeSalvo, M. Yan, P. G. Mickelson, Y. N. Martinez de Escobar, and T. C. Killian
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(Received 5 May 2010; published 13 July 2010)


PHYSICAL REVIEW A **82**, 011608(R) (2010)

Double-degenerate Bose-Fermi mixture of strontium

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[1]*Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, 6020 Innsbruck, Austria*
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(Received 6 June 2010; published 20 July 2010)

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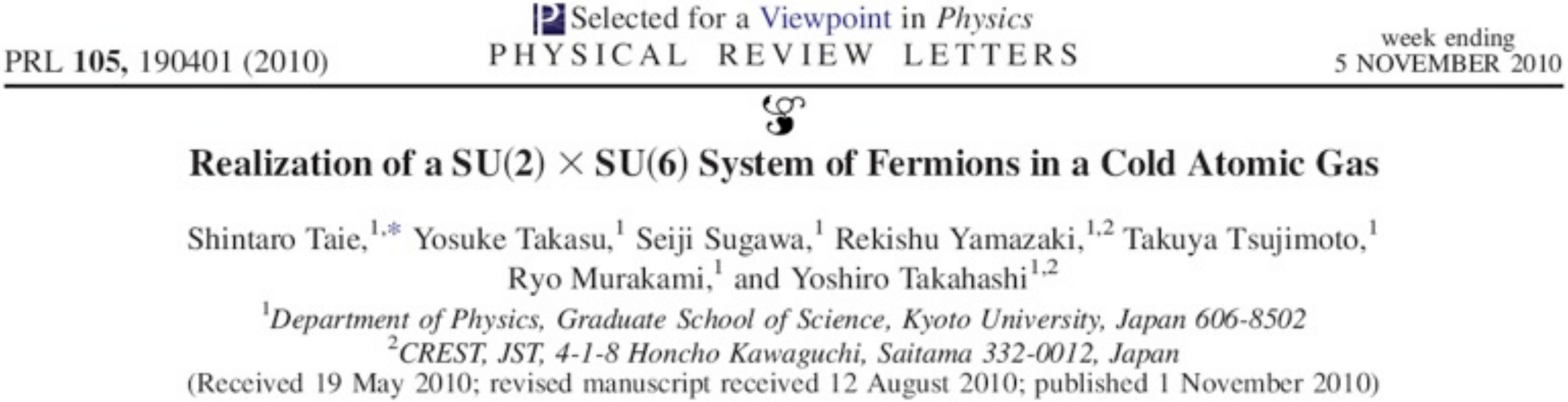
^{171}Yb (I = 1/2) and ^{173}Yb (I = 5/2)


Selected for a Viewpoint in *Physics*
PRL **105,** 190401 (2010) PHYSICAL REVIEW LETTERS week ending 5 NOVEMBER 2010

Realization of a SU(2) × SU(6) System of Fermions in a Cold Atomic Gas

Shintaro Taie,[1,*] Yosuke Takasu,[1] Seiji Sugawa,[1] Rekishu Yamazaki,[1,2] Takuya Tsujimoto,[1] Ryo Murakami,[1] and Yoshiro Takahashi[1,2]
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(Received 19 May 2010; revised manuscript received 12 August 2010; published 1 November 2010)

Conclusions

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- applications to quantum computing and precision measurements

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- two-orbita
  - the Kug
  - SU(N) a
  - SU(N) Kondo lattice model
- few-body (skipped) & many-body experiments
- possible insights into strongly-correlated systems in condensed matter

Thank you

Outlook

- Analyze new models
- Alkali dimers, ions

- Clock shifts: PRL 103, 260402 (2009)