Light-induced gauge potentials for cold atoms: An introduction

Gediminas Juzeliūnas

Institute of Theoretical Physics and Astronomy,

Vilnius University , Vilnius, Lithuania

November 4, 2010

Kavli Institute for Theoretical Physics, UC Santa Barbara,

Collaboration

- **P. Öhberg & group,** Heriot-Watt University, *Edinburgh*
- ***M. Fleischhauer & group,*** *TU Kaiserslautern,*
- ***L. Santos & group,*** *Universität Hannover*
- ***J. Dalibard & group, ENS, Paris***
- ***I. Spielman, D. Campbel, C. Clark and J. Vaishnav, NIST***
- ***M. Lewenstein, ICFO, Spain***

Quantum Optics Group @ ITPA, Vilnius University

V. Kudriasov, J. Ruseckas, G. J., A. Mekys & T. Andrijauskas

+ V. Pyragas & S. Grubinskas (not in the picture)

Quantum Optics Group @ ITPA, Vilnius University

- Light-induced gauge potentials
- Slow light (with OAM, multi-component, …)
- Cold atoms in optical lattices
- Graphene
- Metamaterials

OUTLINE

- Background
  - Creating B_{eff} via rotation
  - Creating B_{eff} in optical lattices
- Origin of geometric potentials
- Specific schemes for generating
  - Abelian gauge potentials
  - Non-Abelian gauge potentials (for cold atoms)
- Conclusions
- P.S. Latest realistic proposal by Ian Spielman to produce Rashba SO coupling.

BEC and Degenerate Fermi gases

- Atomic physics meets solid state physics
  - Degenerate Fermi gas ↔ Electrons in solids
  - Atoms in optical lattices
- Trapped atoms - **electrically neutral** particles →
- No Lorentz force and thus direct analogy with **magnetic** properties due to electrons in solids
- → Various attempts to produce artificial magnetic fields for cold atoms

BEC and Degenerate Fermi gases

- Usual method to create an **effective** magnetic field: **Rotation of a trap**

Trap rotation

Hamiltonian in the rotating frame

[see e.g. A. Fetter, RMP 81, 647 (2009)]

$$H_0' = p^2/(2M) + \tfrac{1}{2}M\omega_\perp^2 r^2 - \boldsymbol{\Omega}\cdot\boldsymbol{r}\times\boldsymbol{p}$$

or **rotation vector**

$$H_0' = \frac{(\boldsymbol{p} - M\boldsymbol{\Omega}\times\boldsymbol{r})^2}{2M} + \frac{1}{2}M(\omega_\perp^2 - \Omega^2)r^2$$

Effective vector potential (constant B_{eff})

Centrifugal potential (anti-trapping)

Coriolis force (equivalent to Lorentz force)

Trap rotation

- Constant B_{eff}: $B_{eff} \sim \Omega$
- Trapping frequency: $\omega_{eff} = \sqrt{\omega_\perp^2 - \Omega^2}$
- $\Omega \to \omega_\perp$ → Landau problem

Features

- *B_{eff} is constant*
- *B_{eff} acts on all atoms in the same way*
- *Applies usually to cylindrically symmetric traps*
- *One can not rotate the trap at large frequencies*
- *→ Other methods of producing B_{eff} without rotation are desirable*

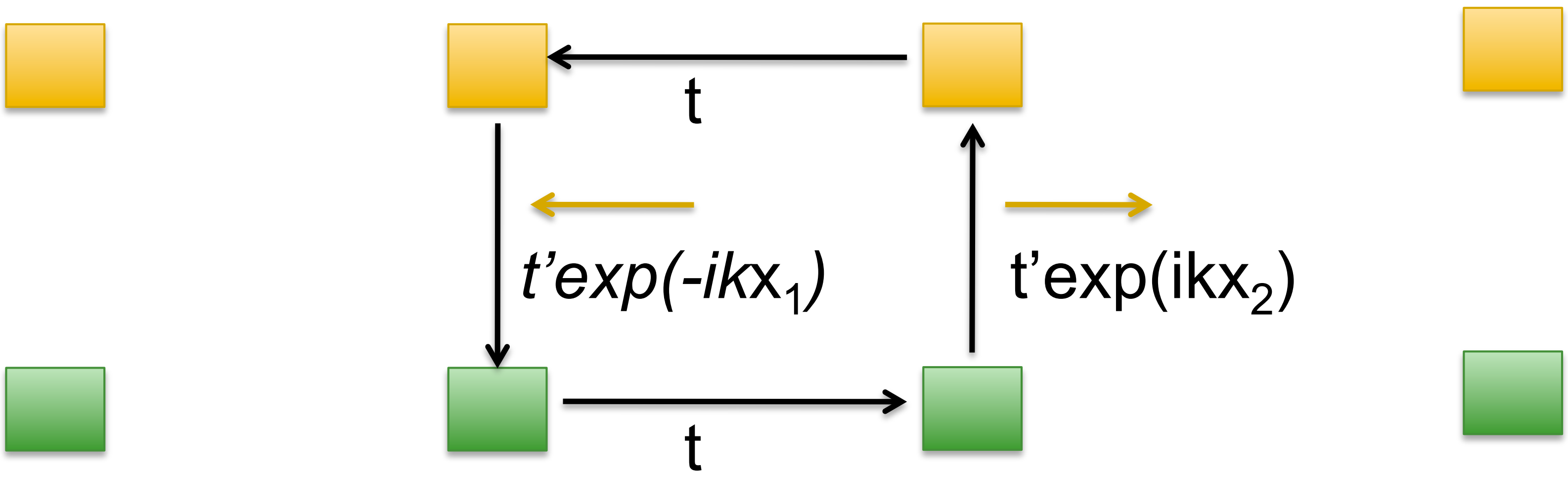
Effective magnetic fields without rotation

- **<u>Optical lattices</u>**

  Initial proposals:
  - D. Jaksch and P. Zoller, New J. Phys. **5**, 56 (2003)
  - E. Mueller, Phys. Rev. A **70**, 041603 (R) (2004)
- B_{eff} *is produced by inducing an* assymmetry in atomic transitions between the lattice sites.
- Non-vanishing phase for atoms moving along a closed path on the lattice
- → Non-zero magnetic flux → $B_{eff} \neq 0$

Effective magnetic fields without rotation

- **Optical lattices**
  - D. Jaksch and P. Zoller, New J. Phys. **5**, 56 (2003)
  - J. Dalibard and F. Gerbier, NJP **12**, 033007 (2010).
  - -Laser-assisted tunneling between different internal states along *y* axis (**with recoil** along x).
  - -Ordinary tunneling along *x* direction (t).



  - Non-vanishing phase for the atoms moving along a plaquette: $S=k(x_2-x_1)=ka$

Effective magnetic fields without rotation

- **Optical lattices**:

The method can be extended to create Non-Abelian gauge potentials

(Laser assisted, state-sensitive tunneling)

- K. Osterloh, M. Baig, L. Santos, P. Zoller and M. Lewenstein, Phys. Rev. Lett. **95**, 010403 (2005)

Here: Creation of B_{eff} using geometric potentials

- Features:
  - No rotation is necessary
  - No lattice is needed

Geometric potentials

- Emerge in various areas of physics (molecular, condensed matter physics etc.)
- First considered by Mead, Berry, Wilczek and Zee and others in the 80’s.
- More recently – in the context of motion of cold atoms affected by laser fields.
- Advantage of such atomic systems: possibilities to control and shape gauge potentials by choosing proper laser fields.

Creation of B_{eff} using geometric potentials

Atomic dynamics taking into account both internal and center of mass motion.

- The full atomic Hamiltonian

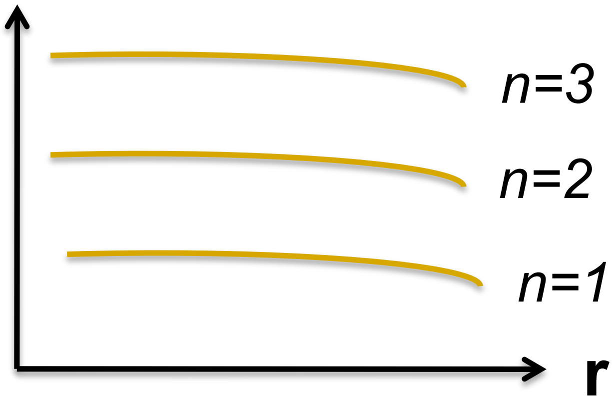
$$\hat{H} = \frac{\hat{p}^2}{2M} + \hat{V}(\mathbf{r}) + \hat{H}_0(\mathbf{r}, t).$$

  - $\hat{H}_0(\mathbf{r}, t)$ — the Hamiltonian for the electronic (fast) degrees of freedom, ← (including **r**-dependent atom-light coupling)
  - $\hat{p}^2/2M + \hat{V}(\mathbf{r})$ — the Hamiltonian for center of mass (slow) degrees of freedom.
  - $\hat{V}(\mathbf{r})$ — the external trapping potential. (for c.m. motion)
- $\hat{H}_0(\mathbf{r}, t)$ has eigenfunctions $|\chi_n(\mathbf{r}, t)\rangle$ with eigenvalues $\varepsilon_n(\mathbf{r}, t)$.
- Full atomic wave function ↖ (**r**-dependent "dressed" eigenstates)

$$|\Phi\rangle = \sum_n \Psi_n(\mathbf{r}, t)|\chi_n(\mathbf{r}, t)\rangle.$$

Non-degenerate state with n=1

- Adiabatic atomic energies $\varepsilon_n(\mathbf{r})$



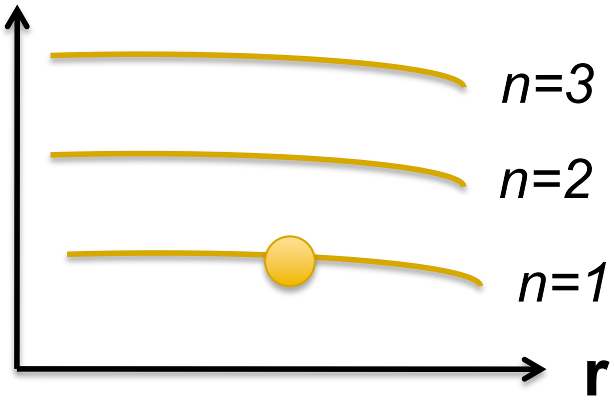
- Full state vector:

$$|\Phi\rangle = \sum_{n=1}^{N} |\chi_n(\mathbf{r})\rangle \Psi_n(\mathbf{r},t)$$

$\Psi_n(\mathbf{r},t)$ – wave-function of the atomic centre of mass motion in the n-th atomic internal "dressed" state $|\chi_n(\mathbf{r})\rangle$

Non-degenerate state with n=1

- Adiabatic atomic energies $\varepsilon_n(\mathbf{r})$



- Full state vector:

$$|\Phi\rangle = \sum_{n=1}^{N} |\chi_n(\mathbf{r})\rangle \Psi_n(\mathbf{r},t)$$

$\Psi_n(\mathbf{r},t)$ – wave-function of the atomic centre of mass motion in the n-th atomic internal "dressed" state $|\chi_n(\mathbf{r})\rangle$

- Adiabatic approximation

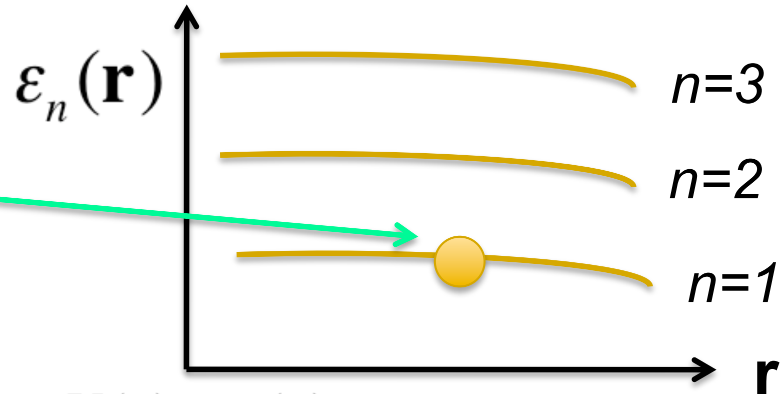
$$|\Phi\rangle \approx |\chi_1(\mathbf{r})\rangle \Psi_1(\mathbf{r},t)$$

- What is the equation of motion for $\Psi_1(\mathbf{r},t)$?

Non-degenerate state with n=1

Adiabatic approximation:

$$|\Phi\rangle \approx |\chi_1(\mathbf{r})\rangle \Psi_1(\mathbf{r},t)$$



Equation of motion:

- $i\hbar\partial_t|\Phi\rangle = H|\Phi\rangle$ with $\hat{H} \approx \frac{\hat{p}^2}{2M} + V(\mathbf{r}) + \varepsilon_1(\mathbf{r})$

$$\hat{p} = -i\hbar\nabla$$

We have:

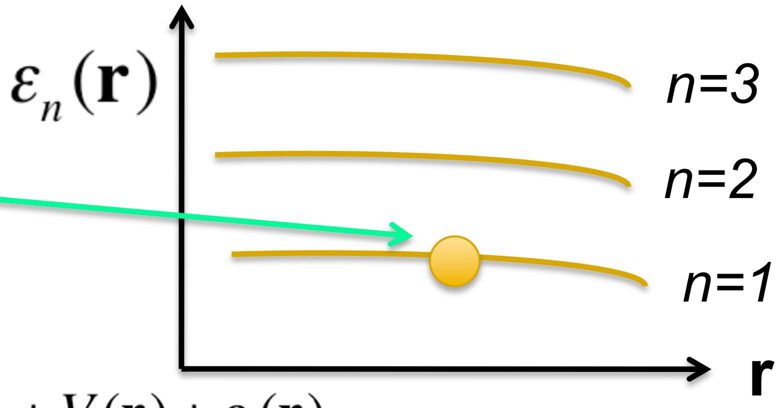
$$\hat{p}|\Phi\rangle = -i\hbar|\chi_1(\mathbf{r})\rangle\nabla\Psi_1(\mathbf{r},t) - i\hbar|\nabla\chi_1(\mathbf{r})\rangle\Psi_1(\mathbf{r},t)$$

Projecting onto the state $|\chi_1(\mathbf{r})\rangle$

Non-degenerate state with n=1

Adiabatic approximation:

$$|\Phi\rangle \approx |\chi_1(\mathbf{r})\rangle \Psi_1(\mathbf{r},t)$$



Equation of motion:

- $i\hbar\partial_t|\Phi\rangle = H|\Phi\rangle$ with $\hat{H} \approx \frac{\hat{p}^2}{2M} + V(\mathbf{r}) + \varepsilon_1(\mathbf{r})$

$$\hat{p} = -i\hbar\nabla$$

We have:

$$\hat{p}|\Phi\rangle = -i\hbar|\chi_1(\mathbf{r})\rangle\nabla\Psi_1(\mathbf{r},t) - i\hbar|\nabla\chi_1(\mathbf{r})\rangle\Psi_1(\mathbf{r},t)$$

Brings a vector potential

and thus $\hat{p}|\Phi\rangle \approx |\chi_1(\mathbf{r})\rangle(-i\hbar\nabla - \mathbf{A}_{11})\Psi_1(\mathbf{r},t)$

$$\mathbf{A}_{11} = i\hbar\langle\chi_1(\mathbf{r})|\nabla\chi_1(\mathbf{r})\rangle$$

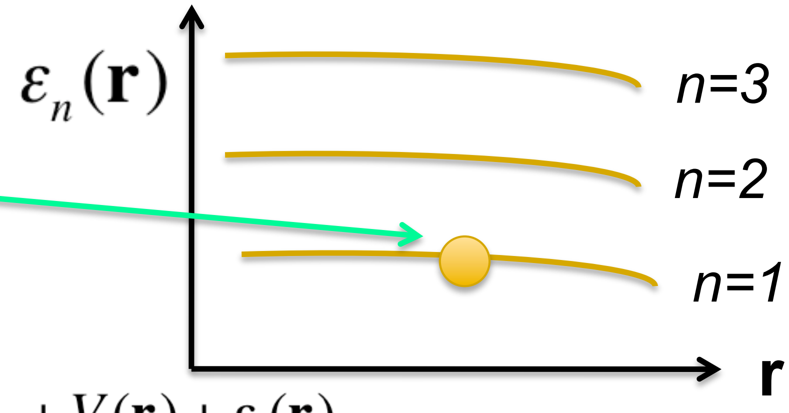
→ $i\hbar\partial_t\Psi_1(\mathbf{r},t) = H\Psi_1(\mathbf{r},t)$ $\qquad \hat{H} = \frac{(\mathbf{p} - \mathbf{A}_{11})^2}{2M} + V(\mathbf{r}) + \varepsilon_1(\mathbf{r})$

Vector potential $\mathbf{A}_{11} \equiv \mathbf{A}$ appears.

Non-degenerate state with n=1

Adiabatic approximation:

$$|\Phi\rangle \approx |\chi_1(\mathbf{r})\rangle \Psi_1(\mathbf{r},t)$$



Equation of motion:

- $i\hbar\partial_t|\Phi\rangle = H|\Phi\rangle$ with $\hat{H} \approx \frac{\hat{p}^2}{2M} + V(\mathbf{r}) + \varepsilon_1(\mathbf{r})$ $\quad \hat{p} = -i\hbar\nabla$

We have:

$$\hat{p}|\Phi\rangle = -i\hbar|\chi_1(\mathbf{r})\rangle\nabla\Psi_1(\mathbf{r},t) - i\hbar|\nabla\chi_1(\mathbf{r})\rangle\Psi_1(\mathbf{r},t)$$

Brings a vector potential

and thus $\hat{p}|\Phi\rangle \approx |\chi_1(\mathbf{r})\rangle(-i\hbar\nabla - \mathbf{A}_{11})\Psi_1(\mathbf{r},t)$

$$\mathbf{A}_{11} = i\hbar\langle\chi_1(\mathbf{r})|\nabla\chi_1(\mathbf{r})\rangle$$

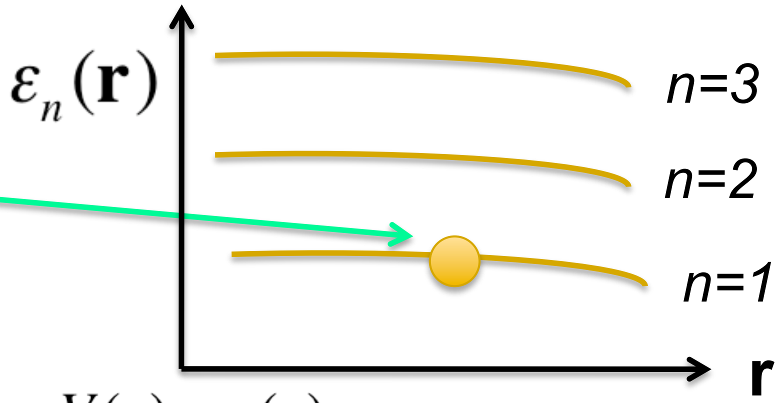
→ $i\hbar\partial_t\Psi_1(\mathbf{r},t) = H\Psi_1(\mathbf{r},t)$ $\quad \hat{H} = \frac{(\mathbf{p}-\mathbf{A}_{11})^2}{2M} + V(\mathbf{r}) + \varepsilon_1(\mathbf{r})$

Vector potential $\mathbf{A}_{11} \equiv \mathbf{A}$ appears. **What is missing?**

Non-degenerate state with n=1

Adiabatic approximation:

$$|\Phi\rangle \approx |\chi_1(\mathbf{r})\rangle \Psi_1(\mathbf{r},t)$$



Equation of motion:

- $i\hbar\partial_t|\Phi\rangle = H|\Phi\rangle$ with $\hat{H} \approx \dfrac{\hat{p}^2}{2M} + V(\mathbf{r}) + \varepsilon_1(\mathbf{r})$ $\qquad \hat{p} = -i\hbar\nabla$

We have:

$$\hat{p}|\Phi\rangle = -i\hbar|\chi_1(\mathbf{r})\rangle\nabla\Psi_1(\mathbf{r},t) - i\hbar|\nabla\chi_1(\mathbf{r})\rangle\Psi_1(\mathbf{r},t)$$

Using completeness !!!

and thus

$$\hat{p}|\Phi\rangle = \left[|\chi_1(\mathbf{r})\rangle(-i\hbar\nabla - \mathbf{A}_{11}) - \sum_{n=2}^{N}|\chi_n(\mathbf{r})\rangle\mathbf{A}_{n1}\right]\Psi_1(\mathbf{r},t)$$

$$\mathbf{A}_{n1} = i\hbar\langle\chi_n(\mathbf{r})|\nabla\chi_1(\mathbf{r})\rangle$$

→ $i\hbar\partial_t\Psi_1(\mathbf{r},t) = H\Psi_1(\mathbf{r},t)$ $\qquad \hat{H} = \dfrac{(\mathbf{p}-\mathbf{A}_{11})^2}{2M} + V(\mathbf{r}) + \varepsilon_1(\mathbf{r}) + \Phi(\mathbf{r})$!!!

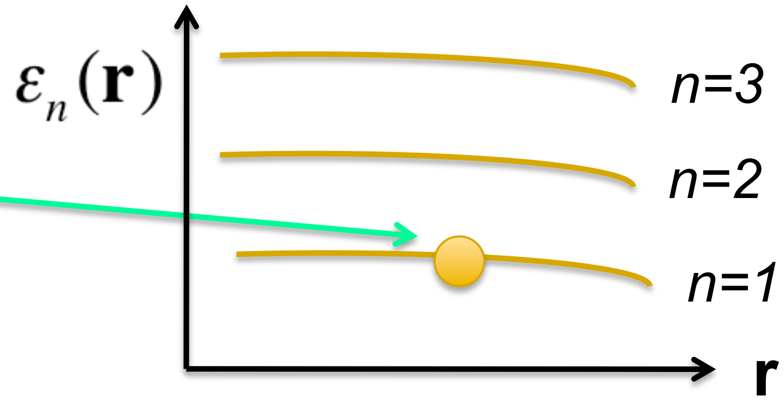
!!!

Vector potential $\mathbf{A}_{11} \equiv \mathbf{A}$ & scalar potential $\Phi \equiv \Phi(\mathbf{r}) = \dfrac{1}{2M}\sum_{n=2}^{N}\mathbf{A}_{1n}\mathbf{A}_{n1}$

Non-degenerate state with n=1

Adiabatic approximation:

$$|\Phi\rangle \approx |\chi_1(\mathbf{r})\rangle \Psi_1(\mathbf{r},t)$$



Equation of motion:

$$i\hbar\partial_t \Psi_1(\mathbf{r},t) = H\Psi_1(\mathbf{r},t)$$

$$\hat{p} = -i\hbar\nabla$$

$$\hat{H} = \frac{(\mathbf{p}-\mathbf{A})^2}{2M} + V(\mathbf{r}) + \varepsilon_1(\mathbf{r}) + \Phi(\mathbf{r})$$

$$|\chi_1(\mathbf{r})\rangle$$

$\mathbf{A} \equiv \mathbf{A}_{11} = i\hbar\langle\chi_1(\mathbf{r})|\nabla\chi_1(\mathbf{r})\rangle$ - effective vector potential (Berry connection)

$\Phi \equiv \Phi(\mathbf{r}) = \dfrac{1}{2M}\sum_{n=2}^{N}\mathbf{A}_{1n}\mathbf{A}_{n1}$ - effective scalar potential

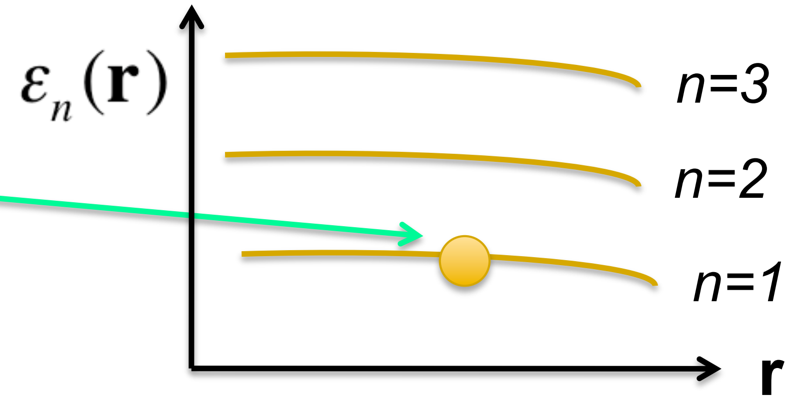
$\mathbf{B} = \nabla \times \mathbf{A}$ - effective magnetic field (non-trivial situation if $\mathbf{B} \neq 0$)

Both $\mathbf{A}$ & Φ appear due to position-dependence of $|\chi_1(\mathbf{r})\rangle$

Non-degenerate state with n=1

Adiabatic approximation:

$$|\Phi\rangle \approx |\chi_1(\mathbf{r})\rangle \Psi_1(\mathbf{r},t)$$



Equation of motion:

$$i\hbar\partial_t \Psi_1(\mathbf{r},t) = H\Psi_1(\mathbf{r},t)$$

$$\hat{p} = -i\hbar\nabla$$

$$\hat{H} = \frac{(\mathbf{p}-\mathbf{A})^2}{2M} + V(\mathbf{r}) + \varepsilon_1(\mathbf{r}) + \Phi(\mathbf{r})$$

Momentum associated with $|\chi_1(\mathbf{r})\rangle$

$\mathbf{A} \equiv \mathbf{A}_{11} = i\hbar\langle\chi_1(\mathbf{r})|\nabla\chi_1(\mathbf{r})\rangle$ - effective vector potential (Berry connection)

Kinetic energy of micro-motion

$\Phi \equiv \Phi(\mathbf{r}) = \frac{1}{2M}\sum_{n=2}^{N}\mathbf{A}_{1n}\mathbf{A}_{n1}$ - effective scalar potential

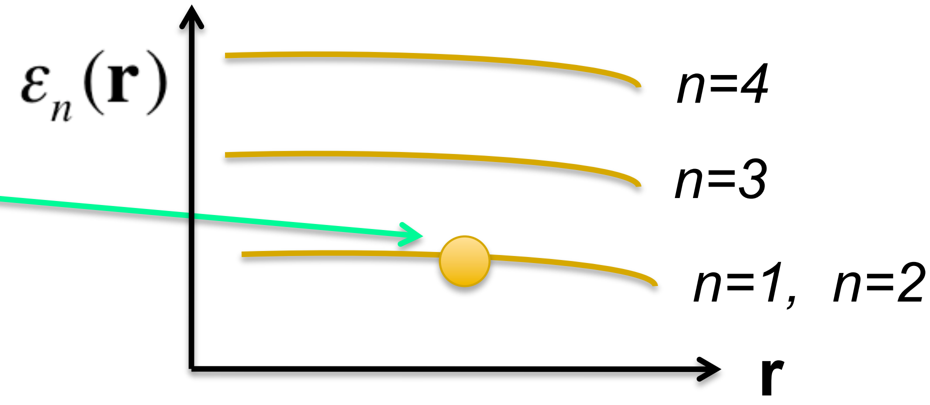
$\mathbf{B} = \nabla\times\mathbf{A}$ - effective magnetic field (non-trivial situation if $\mathbf{B}\neq 0$)

Both $\mathbf{A}$ & Φ appear due to position-dependence of $|\chi_1(\mathbf{r})\rangle$

Degenerate states with n=1 and n=2

Adiabatic approximation:

$$|\Phi\rangle \approx \sum_{n=1}^{2} |\chi_n(\mathbf{r})\rangle \Psi_n(\mathbf{r},t)$$

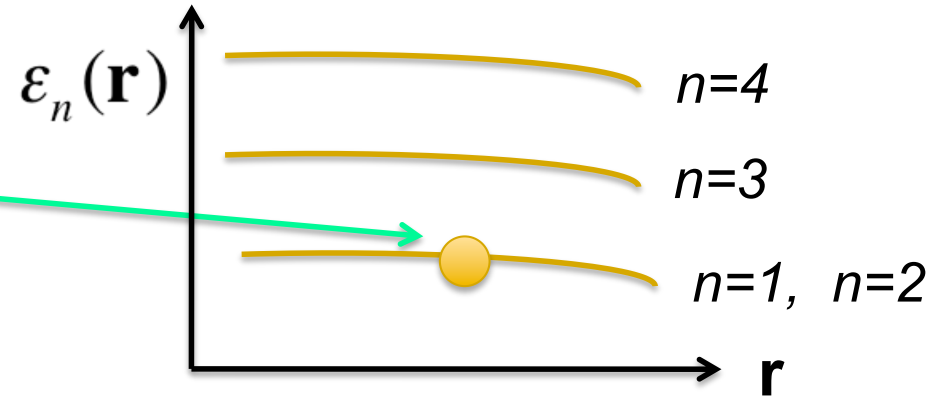


Repeating the same procedure …

Degenerate states with n=1 and n=2

Adiabatic approximation:

$$|\Phi\rangle \approx \sum_{n=1}^{2} |\chi_1(\mathbf{r})\rangle \Psi_1(\mathbf{r},t)$$



Equation of motion:

$$i\hbar\partial_t \Psi(\mathbf{r},t) = H\Psi(\mathbf{r},t)$$

$\Psi(\mathbf{r},t)$ - two-component atomic wave-function

$$\hat{H} = \frac{(\mathbf{p}-\mathbf{A})^2}{2M} + V(\mathbf{r}) + \varepsilon_1(\mathbf{r}) + \Phi(\mathbf{r})$$

2x2 matrix

$$\mathbf{A}_{lj} = i\hbar\langle \chi_l(\mathbf{r}) | \nabla \chi_j(\mathbf{r})\rangle, \qquad (l,j=1,2)$$ - effective vector potential

2x2 matrix

$$\Phi_{lj} = \frac{1}{2M}\sum_{n=3}^{N} \mathbf{A}_{ln}\mathbf{A}_{nj}$$ - effective scalar potential

$$\mathbf{B} = \nabla\times\mathbf{A} + \frac{1}{i\hbar}\mathbf{A}\times\mathbf{A}$$ - effective magnetic field (curvature)

Both $\mathbf{A}$ & Φ appear due to position-dependence of $|\chi_j(\mathbf{r})\rangle$

Adiabatic approximation

Degenerate states

The first q dressed states are degenerate and these levels are well separated from the remaining $N-q$

$$i\hbar\frac{\partial}{\partial t}\tilde{\Psi} = \left[\frac{1}{2M}(-i\hbar\nabla - \mathbf{A})^2 + V + \phi\right]\tilde{\Psi},$$

↑ (q-component wave-function)

where $\mathbf{A}$ and V are truncated $q \times q$ matrices,

$$\phi_{n,n'} = \frac{1}{2M}\sum_{m=q+1}^{N} \mathbf{A}_{n,m} \cdot \mathbf{A}_{m,n'}$$

- $\mathbf{A}$ is now a q×q matrix! ↗ $\mathbf{A}_{n,m} = i\hbar\langle\chi_n|\nabla\chi_m\rangle$
- Non-Abelian case if A_x, A_y, A_z do not commute.

Degenerate states

The first q dressed states are degenerate and these levels are well separated from the remaining $N-q$

$$i\hbar\frac{\partial}{\partial t}\tilde{\Psi} = \left[\frac{1}{2M}(-i\hbar\nabla - \mathbf{A})^2 + V + \phi\right]\tilde{\Psi},$$

↑ (q-component wave-function)

where $\mathbf{A}$ and V are truncated $q \times q$ matrices,

$$\phi_{n,n'} = \frac{1}{2M}\sum_{m=q+1}^{N} \mathbf{A}_{n,m} \cdot \mathbf{A}_{m,n'}$$

- Non-Abelian case if A_x, A_y, A_z do not commute

$$B_i = \frac{1}{2}\epsilon_{ikl}F_{kl}, \qquad F_{kl} = \partial_k A_l - \partial_l A_k - \frac{i}{\hbar}[A_k, A_l].$$

B – curvature

- → **B**≠0 even if **A** is constant !!!

$$\mathbf{F} = (\mathbf{v}\times\mathbf{B} - \mathbf{B}\times\mathbf{v})/2$$

- artificial “Lorentz force”

VOLUME 52, NUMBER 24 PHYSICAL REVIEW LETTERS 11 JUNE 1984

Appearance of Gauge Structure in Simple Dynamical Systems

Frank Wilczek and A. Zee[a]
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106
(Received 9 April 1984)

- But first – Abelian gauge potentials

(a single degenerate dressed state)

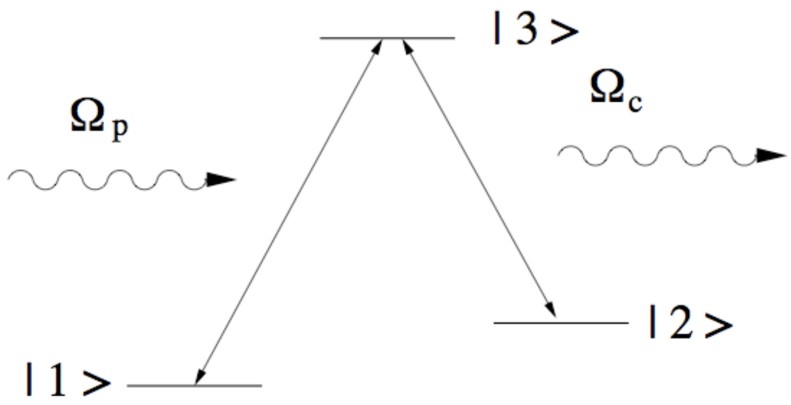
$$\mathbf{A} \equiv \mathbf{A}_{11} = i\hbar \langle \chi_1(\mathbf{r}) | \nabla \chi_1(\mathbf{r}) \rangle$$

Large possibilities to control and shape the effective vector potential by changing the light beams

Light-induced gauge potentials for Λ-type atoms

Initial idea: R. Dum and M. Olshanii, Phys. Rev. Lett. **76**, 1788 (1996).

Λ-type Atoms



Probe beam: $\Omega_p = \mu_{13} E_p$
Control beam: $\Omega_c = \mu_{23} E_c$

Dark state

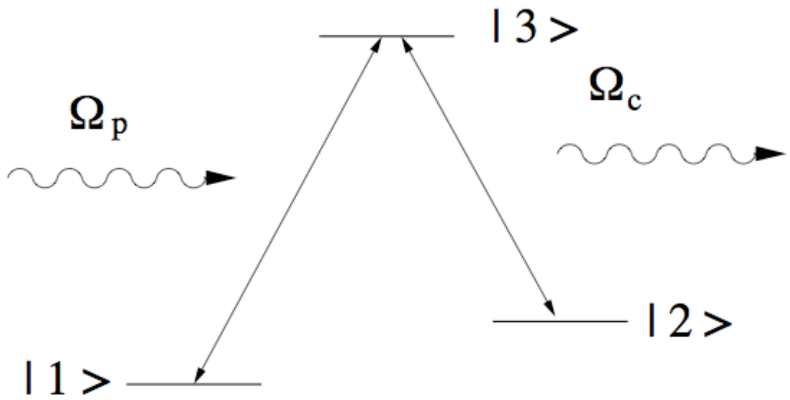
$$|D\rangle \sim \Omega_c|1\rangle - \Omega_p|2\rangle$$

Destructive interference, cancelation of absorbtion — EIT

↖(First observed in the 70's, – G. Alzetta et al. 1976, E. Arimondo, …)

Gauge potentials using light beams with OAM

Λ-type Atoms



Dark state

$$|D\rangle \sim \Omega_c|1\rangle - \Omega_p|2\rangle$$

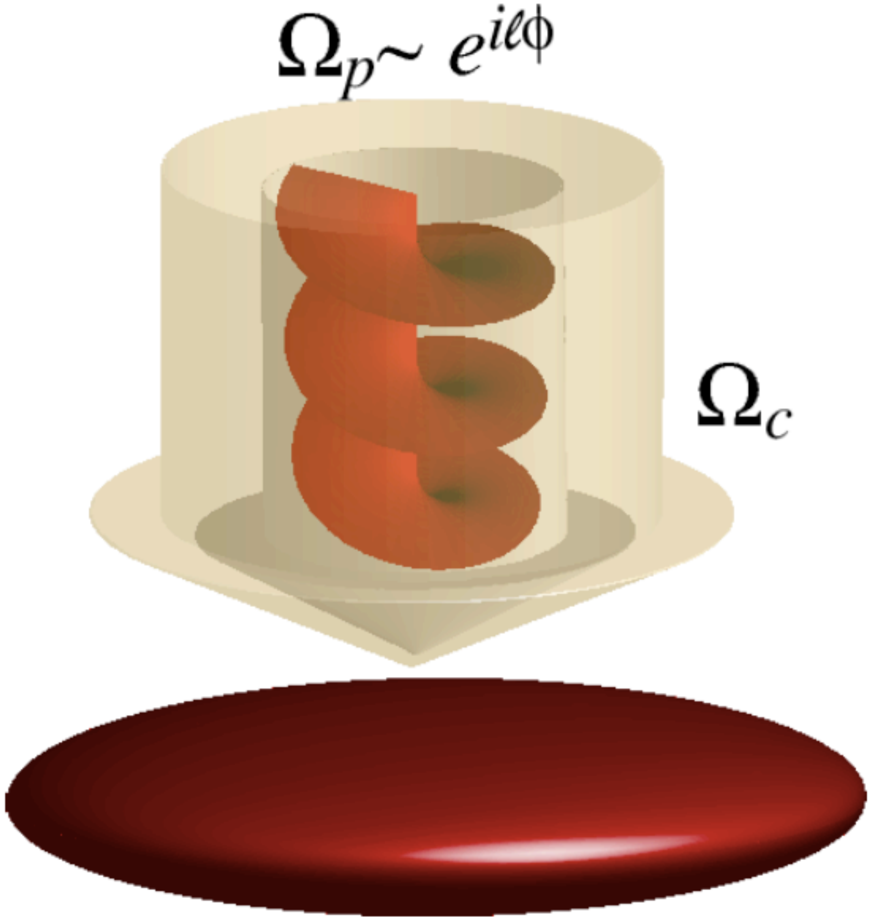
Destructive interference, cancelation of absorbtion — EIT

Probe beam: $\Omega_p = \mu_{13} E_p$
Control beam: $\Omega_c = \mu_{23} E_c$

Atomic dark state – position dependent: $|D\rangle = |D(\mathbf{r})\rangle$

→ $\mathbf{A} = i\hbar\langle D(\mathbf{r})|\nabla D(\mathbf{r})\rangle$ (Abelian vector potential)

Creation of B_{eff} using light vortices (OAM)



Light vortex

Light vortex — light beam with phase

$$\Omega_p \sim e^{ikz+il\varphi},$$

where φ is azimuthal angle, l — winding number.
Light vortices have orbital angular momentum (OAM) along the propagation axis $M_z = \hbar l$.

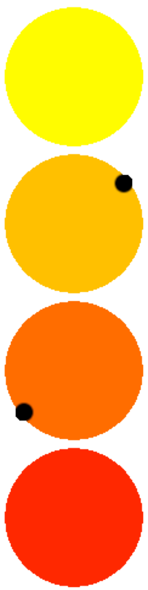
(Twisted beams of light)

G. Juzeliūnas and P. Öhberg, Phys. Rev. Lett. **93**, 033602 (2004);
G. Juzeliūnas, P. Öhberg, J. Ruseckas, and A. Klein, PRA **71**, 053614 (2005).

<u>Dark state:</u> $|D\rangle \sim |1\rangle\Omega_c - |2\rangle\Omega_p,$

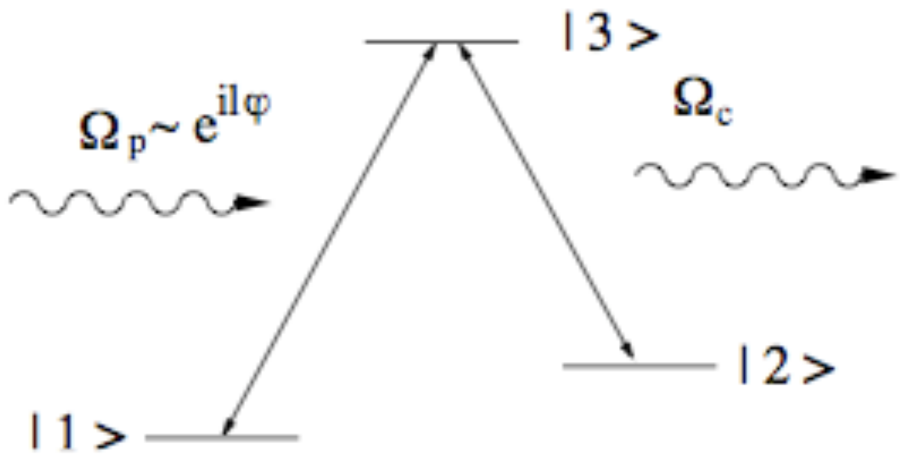
φ - azimuthal angle

- Atomic **dark-state** is **position-dependent**:
- $|D\rangle = |D(\mathbf{r})\rangle \quad \rightarrow$



$|D\rangle = |1\rangle$

$|D\rangle = |2\rangle$



$$\mathbf{A}(\mathbf{r}) = i\hbar\langle D|\nabla|D\rangle$$

Non-trivial case: $\mathbf{A}(\mathbf{r}) \neq \nabla U \quad \rightarrow \quad \mathbf{B} = \nabla \times \mathbf{A} \neq 0$

Effective Magnetic Field

$$\mathbf{A} = -\hbar \frac{|\zeta|^2}{1+|\zeta|^2} \nabla S, \qquad \mathbf{B} = \hbar \frac{\nabla S \times \nabla |\zeta|^2}{(1+|\zeta|^2)^2},$$

$$\phi = \frac{\hbar^2}{2M} \frac{(\nabla |\zeta|)^2 + |\zeta|^2 (\nabla S)^2}{(1+|\zeta|^2)^2},$$

where

$$\zeta = \frac{\Omega_p}{\Omega_c} = |\zeta| e^{iS},$$

- The vector potential **A** is determined by:
  - the gradient of phase of the probe beam,
  - the ratio between the intensities of the control and probe beams.

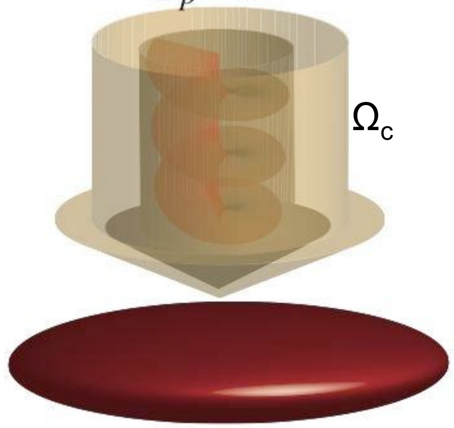
B≠0 if $\nabla|\zeta|^2 \times \nabla S \neq 0$

(the gradients $\nabla|\zeta|^2$ and ∇S are not parallel)

Effective Magnetic Field

$$\mathbf{A} = -\hbar \frac{|\zeta|^2}{1+|\zeta|^2}\nabla S, \qquad \mathbf{B} = \hbar \frac{\nabla S \times \nabla|\zeta|^2}{(1+|\zeta|^2)^2},$$

$$\phi = \frac{\hbar^2}{2M}\frac{(\nabla|\zeta|)^2 + |\zeta|^2(\nabla S)^2}{(1+|\zeta|^2)^2},$$



where

$$\zeta = \frac{\Omega_p}{\Omega_c} = |\zeta| e^{iS}, \qquad S = l\phi$$

- The vector potential **A** is determined by:
  - the gradient of phase of the probe beam,
  - the ratio between the intensities of the control and probe beams.

B≠0 if $\nabla|\zeta|^2 \times \nabla S \neq 0$

(the gradients $\nabla|\zeta|^2$ and ∇S are not parallel)

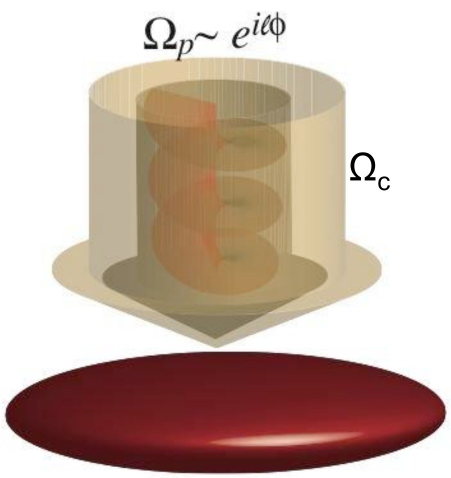
Effective Magnetic Field

$$\mathbf{A} = -\hbar \frac{|\zeta|^2}{1+|\zeta|^2}\nabla S, \qquad \mathbf{B} = \hbar \frac{\nabla S \times \nabla|\zeta|^2}{(1+|\zeta|^2)^2},$$

$$\phi = \frac{\hbar^2}{2M}\frac{(\nabla|\zeta|)^2 + |\zeta|^2(\nabla S)^2}{(1+|\zeta|^2)^2},$$

where

$$\zeta = \frac{\Omega_p}{\Omega_c} = |\zeta| e^{iS}, \qquad S = l\phi$$



- Light beams with OAM can introduce an effective magnetic field which acts on the electrically neutral atoms.
- The vector potential **A** is determined by:
  - the gradient of phase of the probe beam,
  - the ratio between the intensities of the control and probe beams.

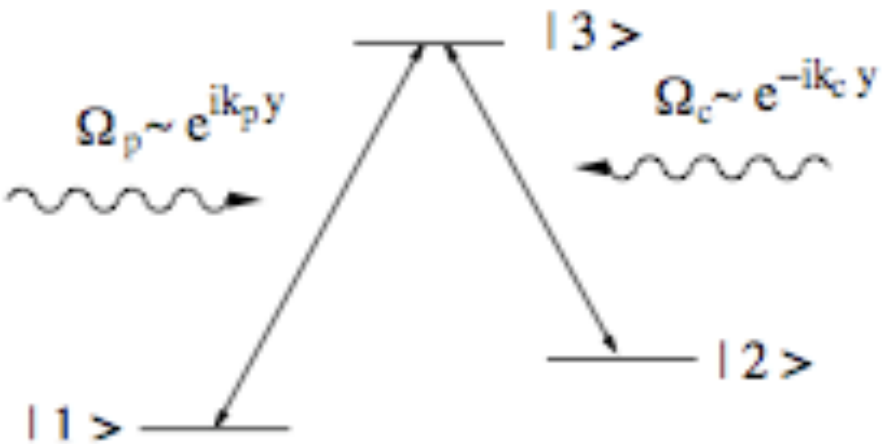
B≠0 if $\nabla|\zeta|^2 \times \nabla S \neq 0$: **B** ~ $\mathbf{e}_z$

(the gradients $\nabla|\zeta|^2$ and ∇S are not parallel)

Counter-propagating beams with spatially shifted profiles

[G. Juzeliūnas, J. Ruseckas, P. Öhberg, and M. Fleischhauer, Phys. Rev. A **73**, 025602 (2006).]

$$\zeta = \Omega_p / \Omega_c \propto |\zeta| e^{iS}$$

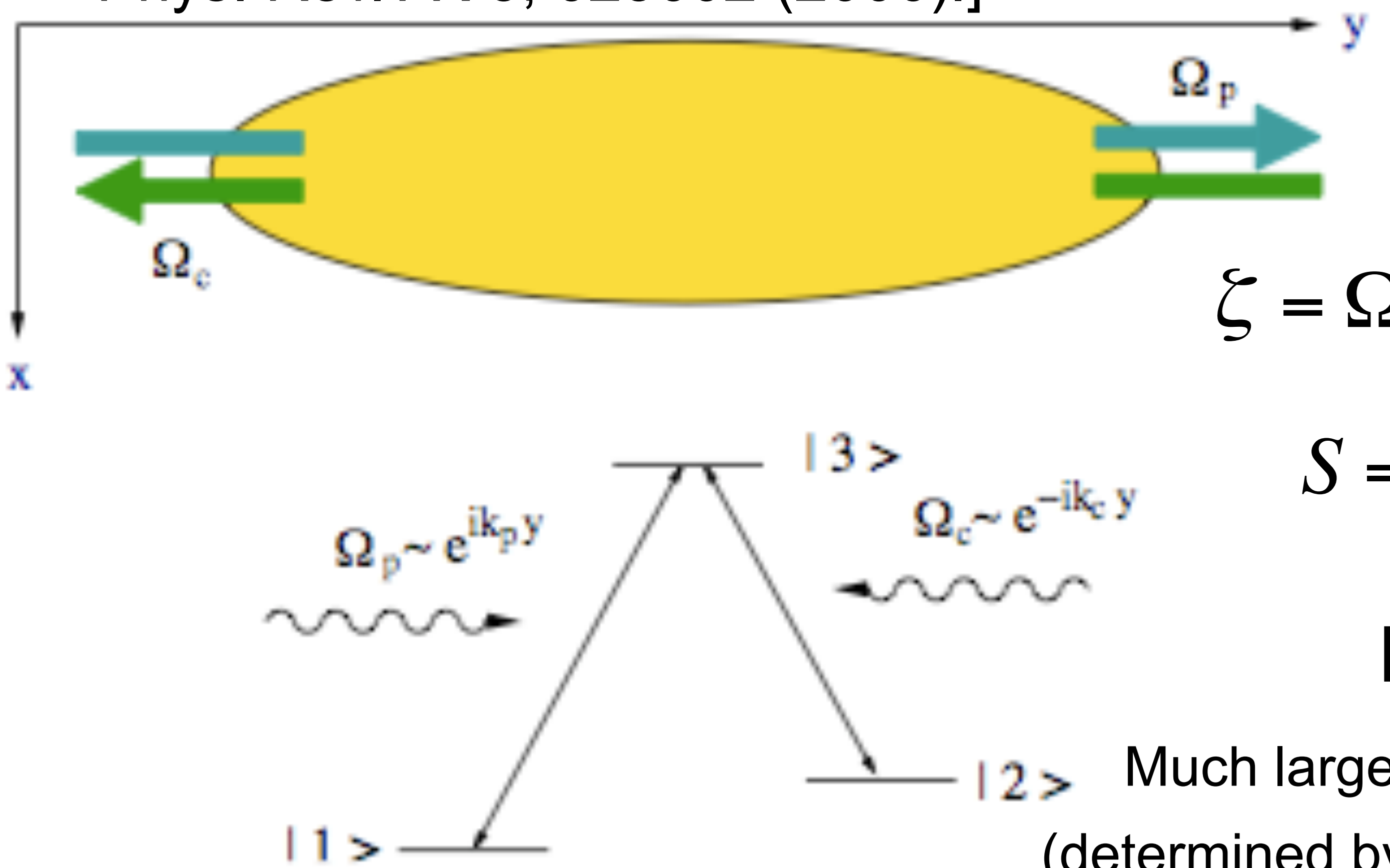


$$S = (k_p + k_c) y$$

(The gradients $\nabla|\zeta|^2$ and ∇S are not parallel)

Counter-propagating beams with spatially shifted profiles

[G. Juzeliūnas, J. Ruseckas, P. Öhberg, and M. Fleischhauer, Phys. Rev. A **73**, 025602 (2006).]



$$\zeta = \Omega_p / \Omega_c \propto |\zeta| e^{iS}$$

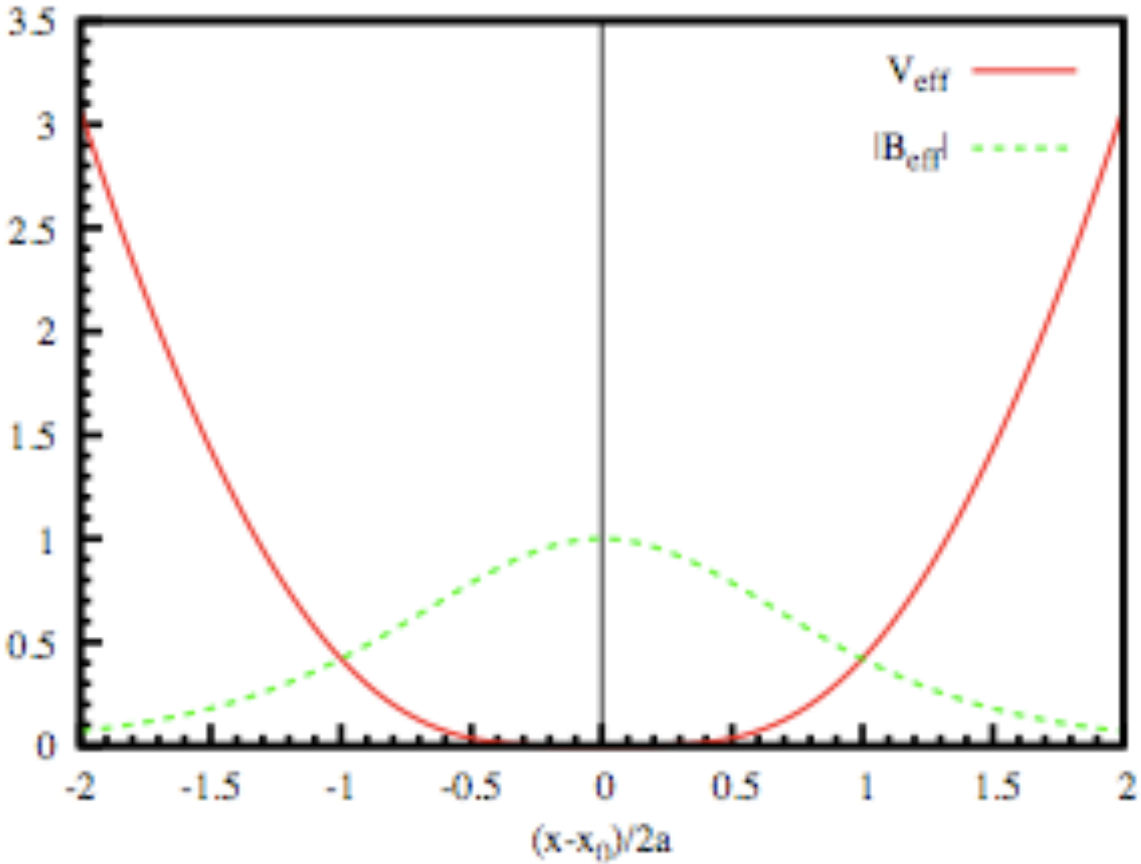
$$S = (k_p + k_c) y$$

$$\mathbf{B} \sim \mathbf{e}_z$$

Much larger Dirac fluxes Φ_{Dirac} (determined by the sample length)

(The gradients $\nabla|\zeta|^2$ and ∇S are not parallel)

Counter-propagating beams with spatially shifted profiles

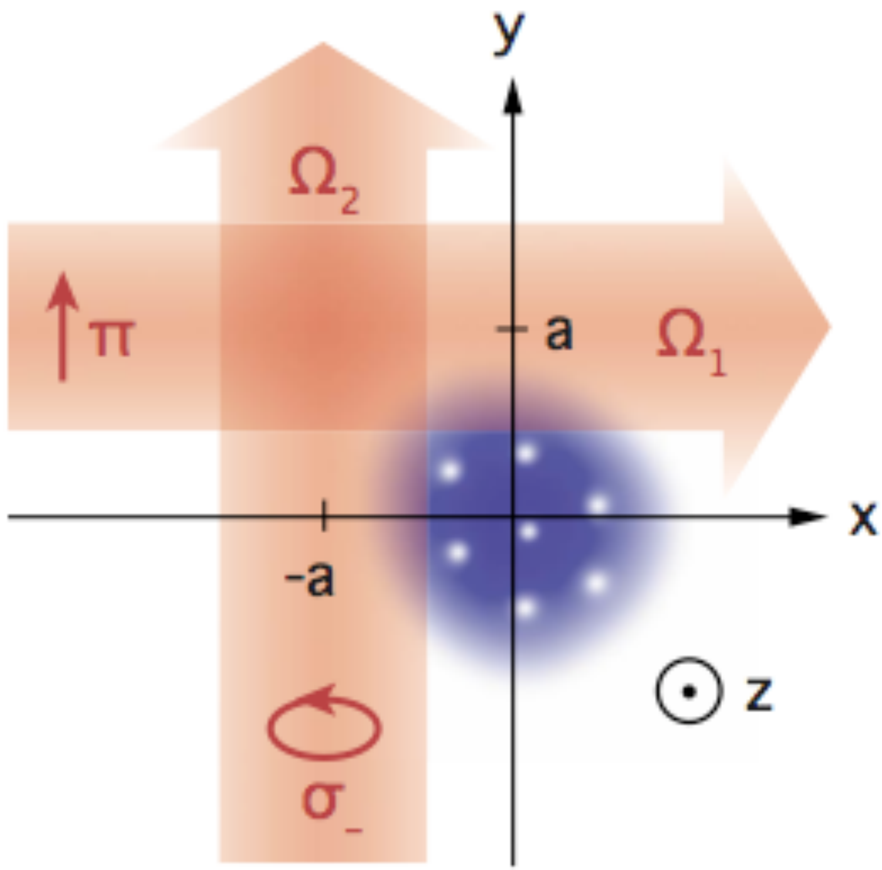


(Effective magnetic field – green)

PHYSICAL REVIEW A **79**, 011604(R) (2009)

Practical scheme for a light-induced gauge field in an atomic Bose gas

Kenneth J. Günter,* Marc Cheneau, Tarik Yefsah, Steffen P. Rath, and Jean Dalibard
Laboratoire Kastler Brossel and CNRS, Ecole Normale Supérieure, 24 rue Lhomond, 75005 Paris, France
(Received 24 November 2008; published 21 January 2009)



Selected for a Viewpoint in *Physics*
PRL **102**, 130401 (2009) PHYSICAL REVIEW LETTERS week ending 3 APRIL 2009

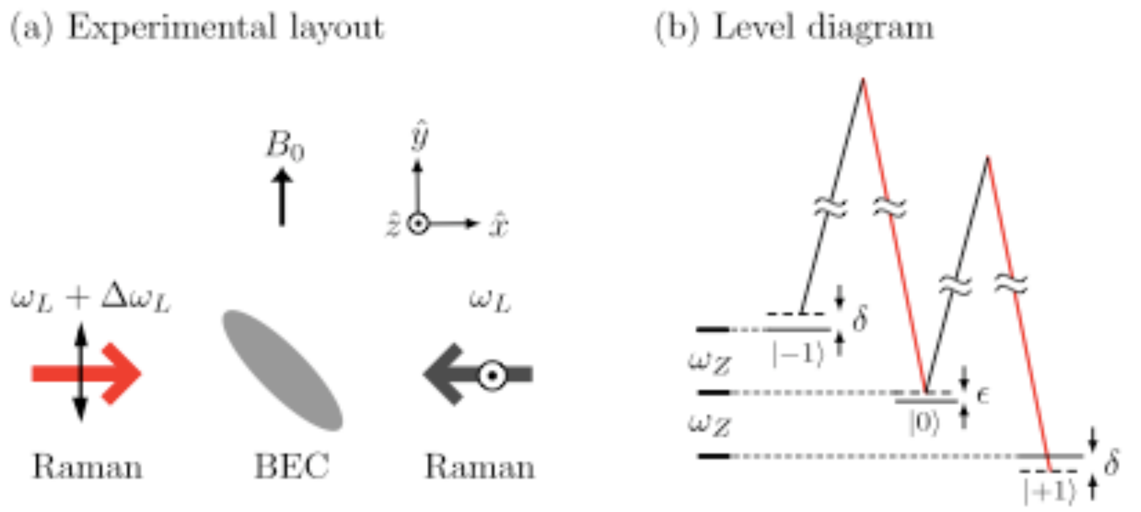
Bose-Einstein Condensate in a Uniform Light-Induced Vector Potential

Y.-J. Lin, R. L. Compton, A. R. Perry, W. D. Phillips, J. V. Porto, and I. B. Spielman*
Joint Quantum Institute, National Institute of Standards and Technology, and University of Maryland, Gaithersburg, Maryland, 20899, USA
(Received 17 September 2008; published 30 March 2009)

Selected for a Viewpoint in *Physics*
PRL **102,** 130401 (2009) PHYSICAL REVIEW LETTERS week ending 3 APRIL 2009

Bose-Einstein Condensate in a Uniform Light-Induced Vector Potential

Y.-J. Lin, R. L. Compton, A. R. Perry, W. D. Phillips, J. V. Porto, and I. B. Spielman*
Joint Quantum Institute, National Institute of Standards and Technology, and University of Maryland, Gaithersburg, Maryland, 20899, USA
(Received 17 September 2008; published 30 March 2009)



nature Vol 462 | 3 December 2009 | doi:10.1038/nature08609

LETTERS

Synthetic magnetic fields for ultracold neutral atoms

Y.-J. Lin[1], R. L. Compton[1], K. Jiménez-García[1,2], J. V. Porto[1] & I. B. Spielman[1]

Characteristic features of the method

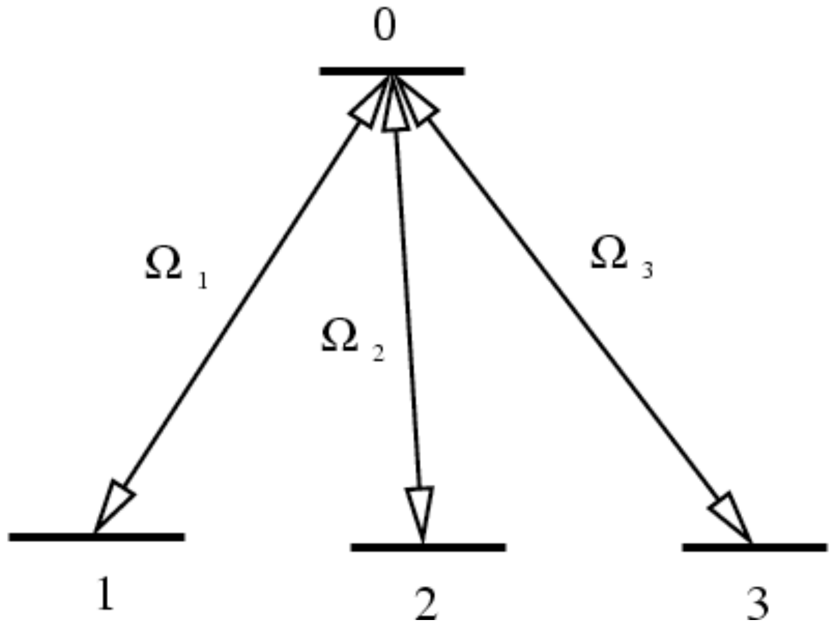
- No rotation of atomic gas
- No lattice is necessary
- Effective magnetic field can be shaped by choosing proper control and probe beams
- Acts on the selected atoms

→ Extension to the non-Abelian gauge potentials

Non-Abelian gauge potentials

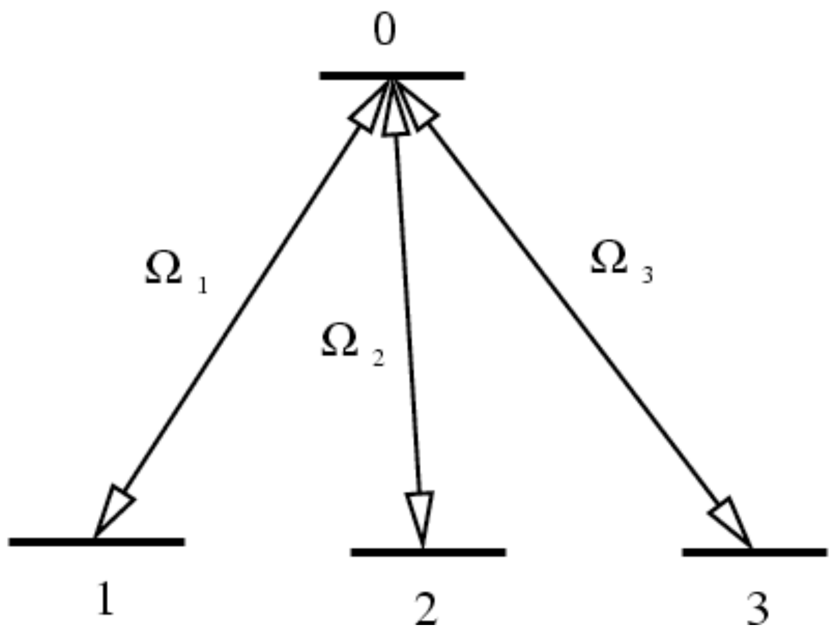
More than one degenerate dressed state

Tripod configuration



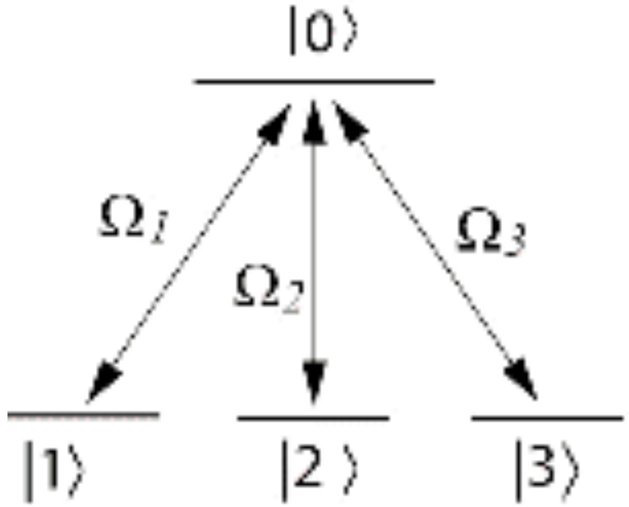
- M.A. Ol'shanii, V.G. Minogin, Quant. Optics 3, **317** (1991)
- *R. G. Unanyan, M. Fleischhauer, B. W. Shore, and K. Bergmann, Opt. Commun.* ***155****, 144 (1998)*
- ***Two*** *degenerate* ***dark states***
- *(Superposition of atomic ground states* ***immune*** *from the atom-light coupling)*

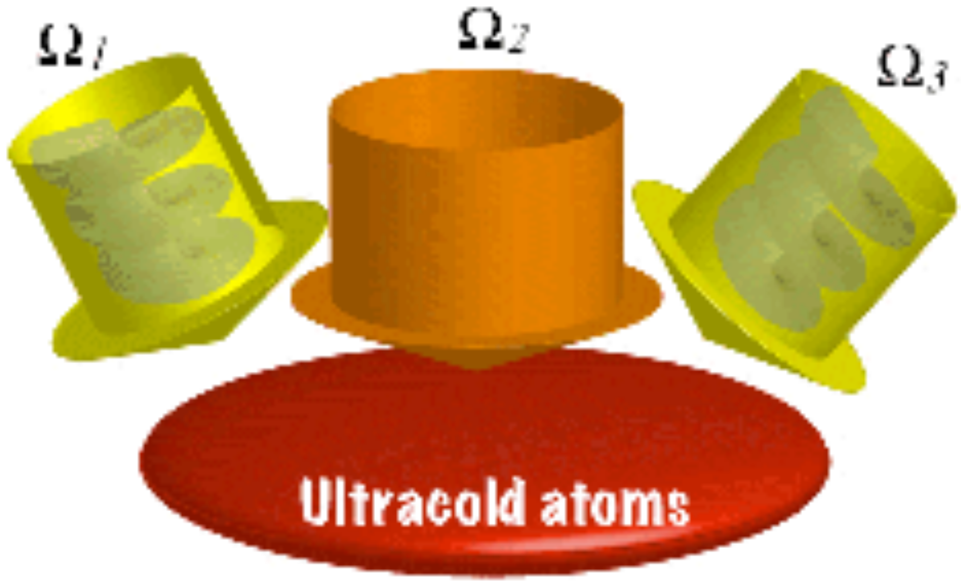
Tripod configuration



- ***Two*** *degenerate* ***dark states***
- *(Superposition of atomic ground states* ***immune*** *from the atom-light coupling)*
- ***Dark states:*** *destructive interference for transitions to the excited state*
- *Lasers keep the atoms in these* ***dark*** *(dressed)* ***states***

(Non-Abelian) light-induced gauge potentials



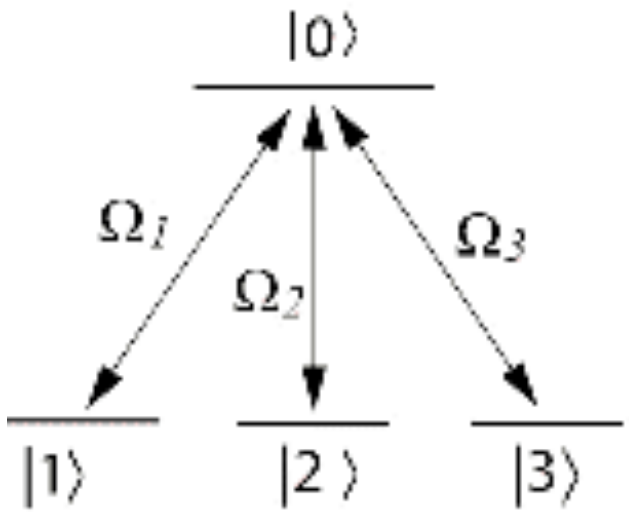


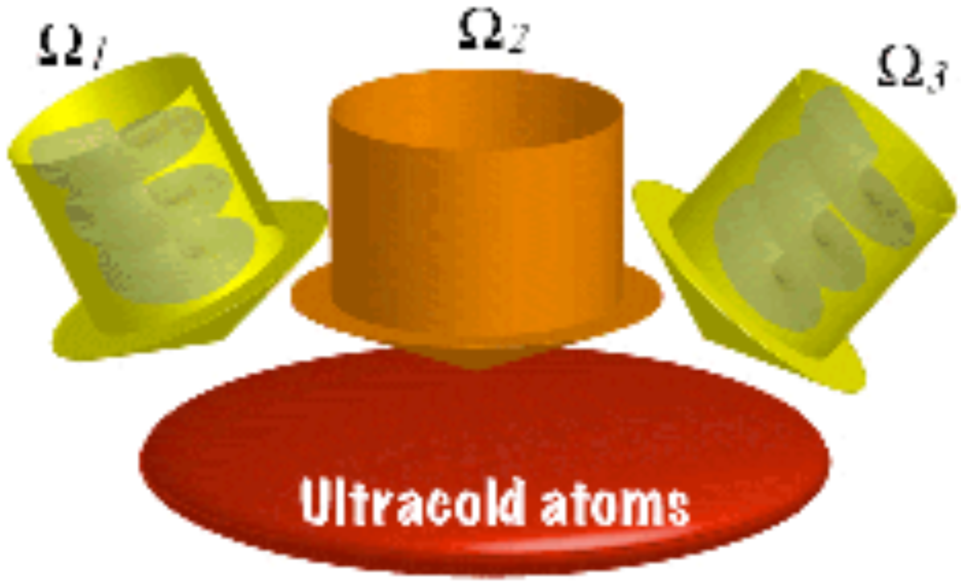
for centre of mass motion of dark-state atoms:
(Due to the spatial dependence of the dark states)

↖ (two dark states)

J. Ruseckas, G. Juzeliūnas and P.Öhberg, and M. Fleischhauer, Phys. Rev. Letters 95, 010404 (2005).

(Non-Abelian) light-induced gauge potentials





Centre of mass motion of dark-state atoms:

$$i\hbar\frac{\partial}{\partial t}\Psi = \left[\frac{1}{2m}(-i\hbar\nabla - \mathbf{A})^2 + V + \Phi\right]\Psi,$$

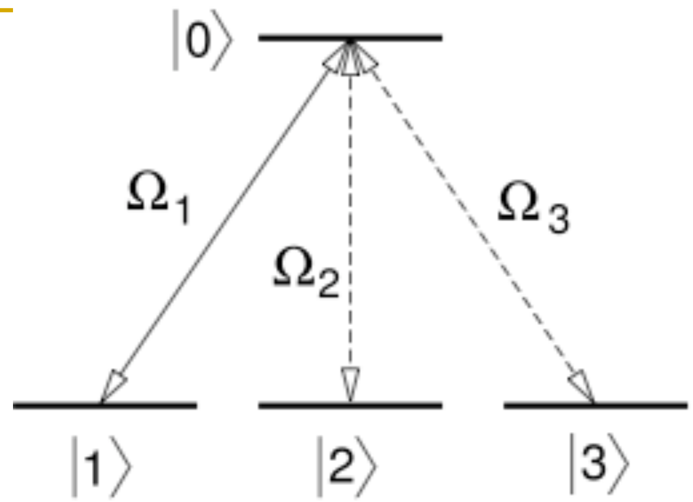
← Two component atomic wave-function

$$\mathbf{A}_{n,m} = i\hbar\langle D_n(\mathbf{r})|\nabla D_m(\mathbf{r})\rangle$$

- 2x2 matrix

A - effective vector potential (Mead-Berry connection)

Tripod scheme



- Two degenerate dark states:

$$|D_1\rangle = \sin\phi e^{iS_{31}}|1\rangle - \cos\phi e^{iS_{32}}|2\rangle,$$

$$|D_2\rangle = \cos\theta\cos\phi e^{iS_{31}}|1\rangle + \cos\theta\sin\phi e^{iS_{32}}|2\rangle - \sin\theta|3\rangle,$$

where

$$\Omega_1 = \Omega\sin\theta\cos\phi\, e^{iS_1}, \quad \Omega_2 = \Omega\sin\theta\sin\phi\, e^{iS_2}, \quad \Omega_3 = \Omega\cos\theta\, e^{iS_3}.$$

- Vector gauge potential:

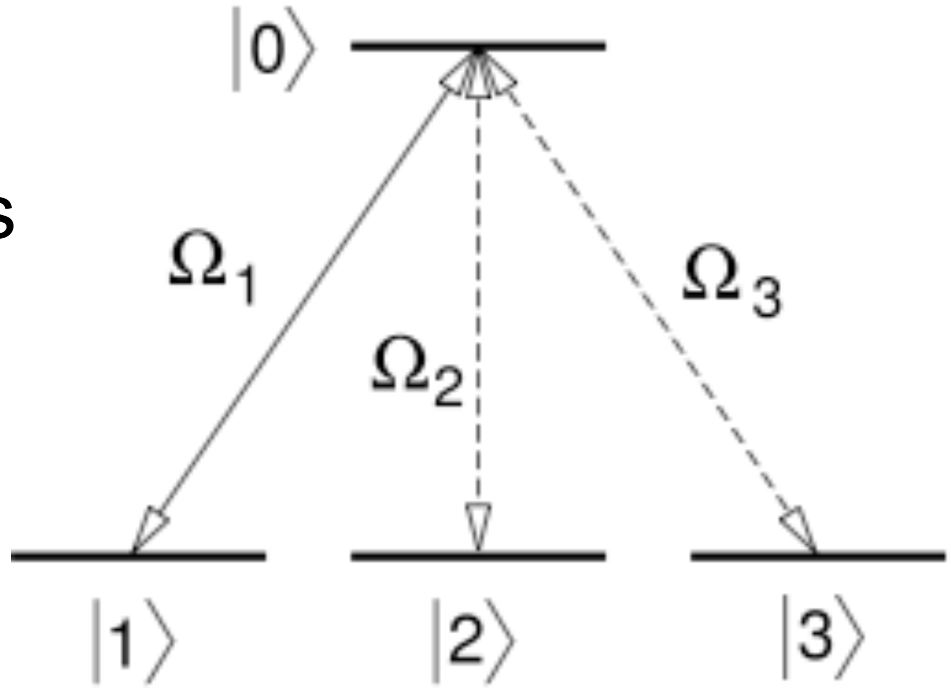
$$\mathbf{A}_{11} = \hbar\left(\cos^2\phi\nabla S_{23} + \sin^2\phi\nabla S_{13}\right),$$

$$\mathbf{A}_{12} = \hbar\cos\theta\left(\frac{1}{2}\sin(2\phi)\nabla S_{12} - i\nabla\phi\right),$$

$$\mathbf{A}_{22} = \hbar\cos^2\theta\left(\cos^2\phi\nabla S_{13} + \sin^2\phi\nabla S_{23}\right).$$

Tripod scheme

Two degenerate dark states



- **A** is 2×2 matrix

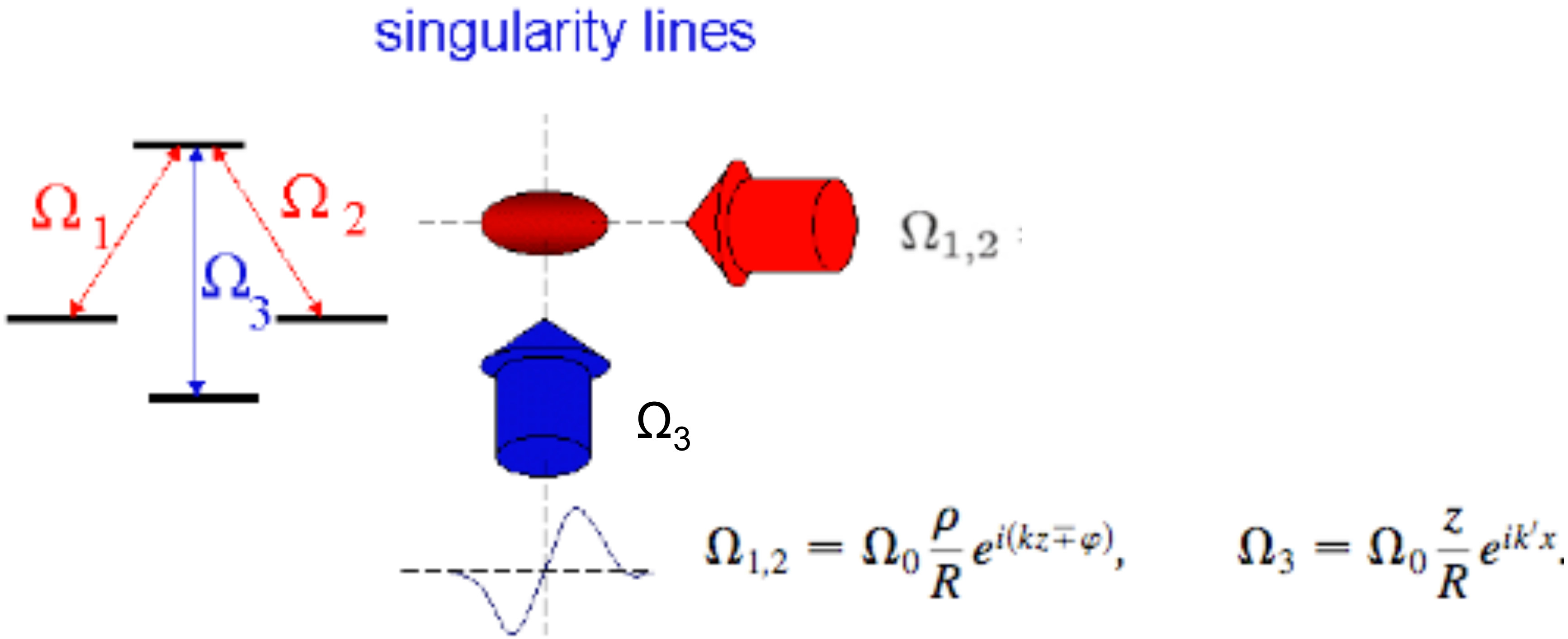
- Non-Abelian case if A_x, A_y, A_z do not commute

$$B_i = \frac{1}{2}\epsilon_{ikl}F_{kl}, \qquad F_{kl} = \partial_k A_l - \partial_l A_k - \frac{i}{\hbar}[A_k, A_l].$$

B - curvature

- $\mathbf{F} = (\mathbf{v}\times\mathbf{B} - \mathbf{B}\times\mathbf{v})/2$ - artificial “Lorentz force”

Non-Abelian magnetic monopole



J. Ruseckas, G. Juzeliūnas, P. Öhberg, and M. Fleischhauer, Phys. Rev. Lett. 95, 010404 (2005).

V. Pietilä and M. Möttönen, Phys. Rev. Lett. 102, 080403 (2009).

Non-Abelian magnetic monopole

- Laser fields:

$$\Omega_{1,2} = \Omega_0 \frac{\rho}{R} e^{i(kz \mp \varphi)}, \qquad \Omega_3 = \Omega_0 \frac{z}{R} e^{ik'x}.$$

- The effective magnetic field

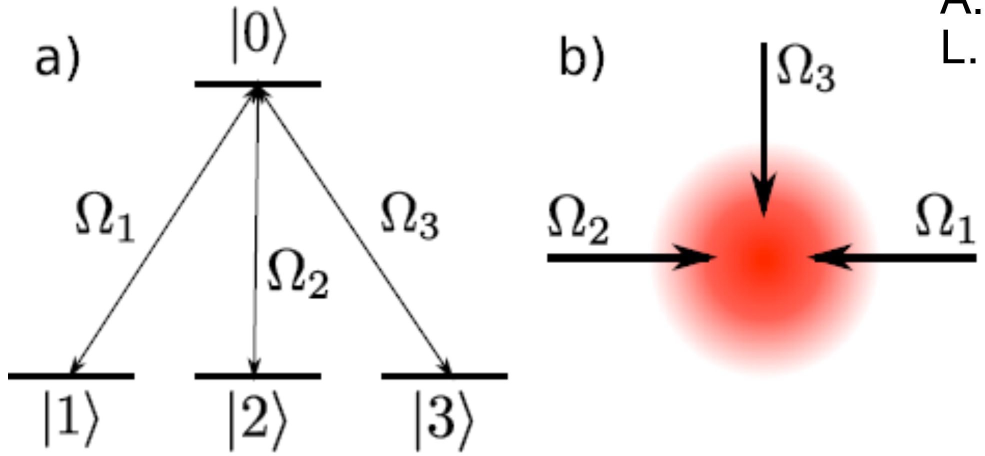
Monopole fied →

$$\mathbf{B} = \frac{\hbar}{r^2} \mathbf{e}_r \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \cdots .$$

J. Ruseckas, G. Juzeliūnas, P. Öhberg, and M. Fleischhauer, Phys. Rev. Lett. 95, 010404 (2005).

V. Pietilä and M. Möttönen, Phys. Rev. Lett. 102, 080403 (2009).

Three plane wave setup



A. Jacob, P. Öhberg, G. Juzeliūnas and L. Santos, *Appl. Phys. B.* **89,** 439 (2007).

(centre of mass motion, dark-state atoms):

$$H = \frac{1}{2m}(-i\hbar\nabla - \hbar\kappa\sigma_\perp)^2 \cdot$$ (→ Rashba-type Hamilltonian)

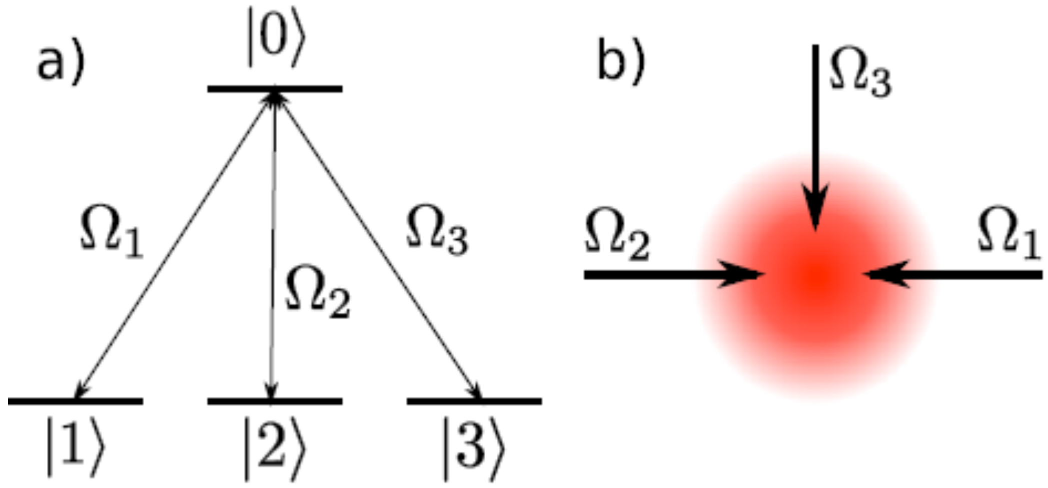
$\boldsymbol{\sigma}_\perp = \mathbf{e}_x\sigma_x + \mathbf{e}_y\sigma_y$ – spin ½ operator

(acting on the subspace of atomic dark states)

→ Rashba-type Spin-Orbit coupling

Constant non-Abelian **A** with $[A_x, A_y] \sim \sigma_z$ → **B** ~ $\mathbf{e}_z$

Three plane wave setup



Centre of mass motion of dark-state atoms:

$$H = \frac{1}{2m}(-i\hbar\nabla - \hbar\kappa\sigma_\perp)^2$$

(→ Rashba-type Hamilltonian)

Plane-wave solutions: $\Psi^{\pm}_{\mathbf{k}}(\mathbf{r}) = g^{\pm}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}$ $\qquad \sigma_{\mathbf{k}} g^{\pm}_{\mathbf{k}} = \pm g^{\pm}_{\mathbf{k}}$

→ two dispersion branches with positive or negative chirality

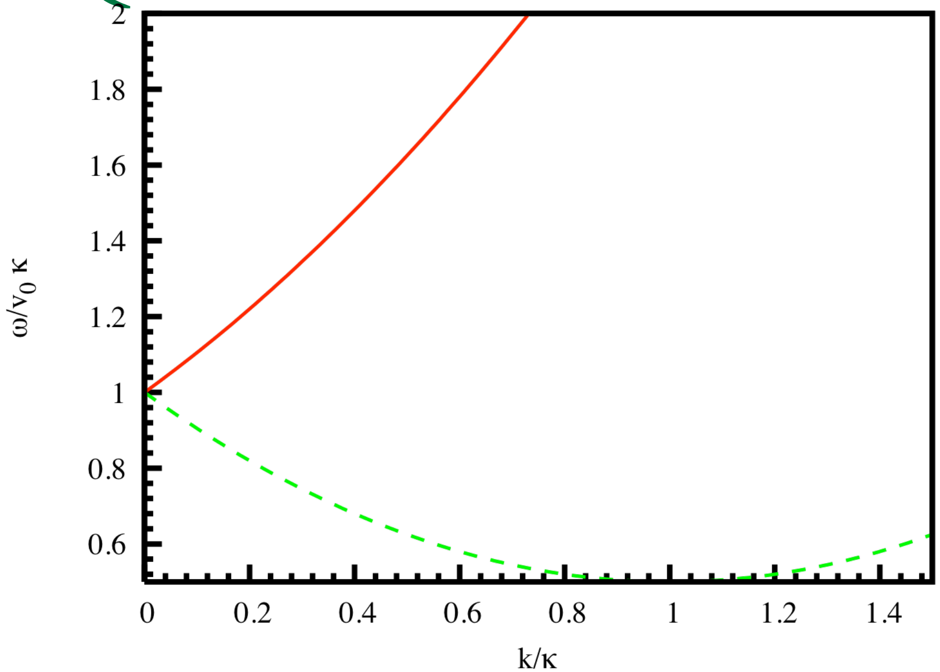
$$\hbar\omega^{\pm}_k = \frac{\hbar^2}{2M}(k \pm \kappa)^2 \qquad \sigma_{\mathbf{k}} = \sigma \cdot \mathbf{k}/k$$

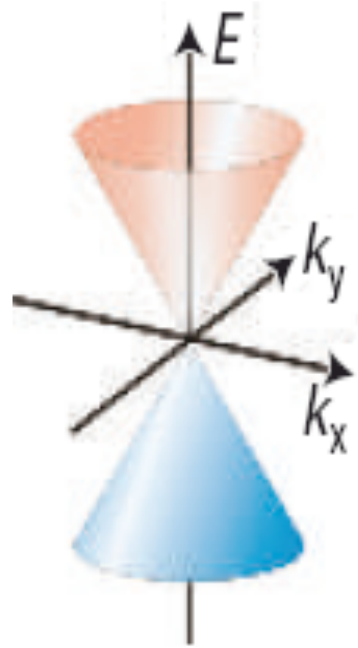
For small k: Similarities to graphene:

Dirac-type Hamiltonian: $H_{\mathbf{k}} = \hbar v_0 \mathbf{k} \cdot \sigma_\perp$

Two dispersion cones: $\hbar\omega_k^\pm = \pm\hbar v_0 k$.

→ Quasirelativistic behaviour or cold atoms



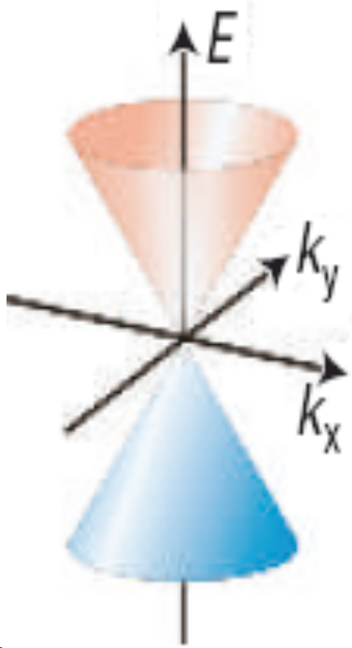


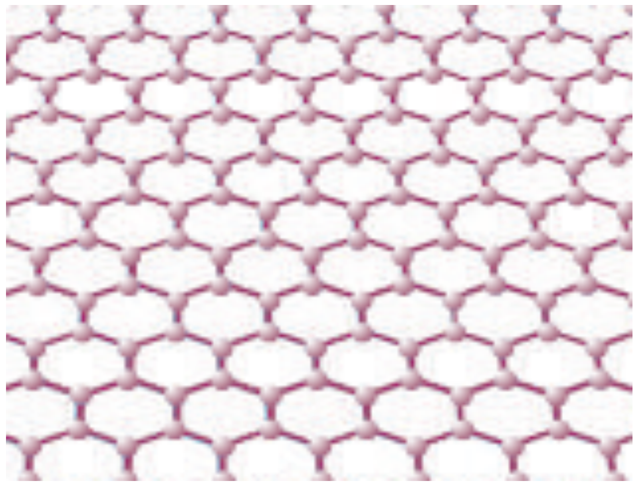
G. Juzeliūnas, J. Ruseckas, L. Santos, M. Lindberg and P. Öhberg, Phys. Rev. A 77, 011802(R) (2008)

$v_o \approx 1 cm/s$

Similar to electrons in graphene

- Graphene – hexagonal 2D crystal of carbon atoms
- Electron energy spectrum near E_F





- Near E_F:
  - Linear dispersion,
  - (Two cones with positive and negative $v_g^{\pm} = \pm v_0$)
  - Electrons behave like relativistic massless particles
  - (Dirac type effective Hamiltonian)

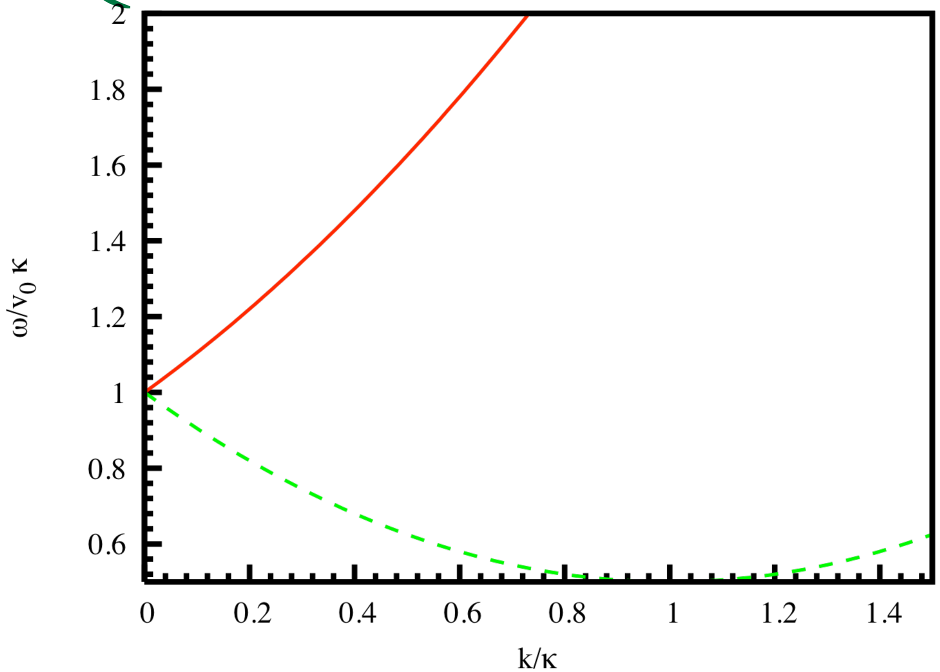
$v_o \approx 10^6$m/s

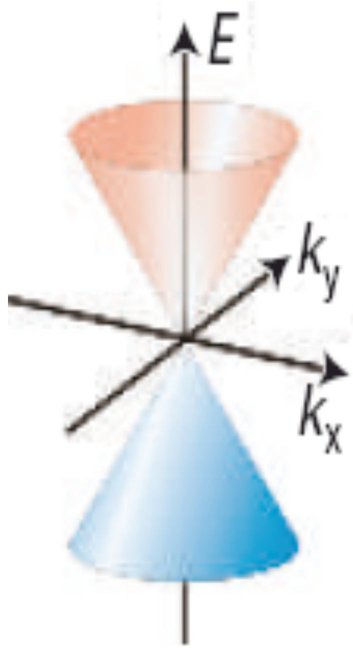
For small k: Similarities to graphene:

Dirac-type Hamiltonian: $H_{\mathbf{k}} = \hbar v_0 \mathbf{k} \cdot \sigma_\perp$

Two dispersion cones: $\hbar\omega_k^\pm = \pm\hbar v_0 k$.

→ Quasirelativistic behaviour or cold atoms





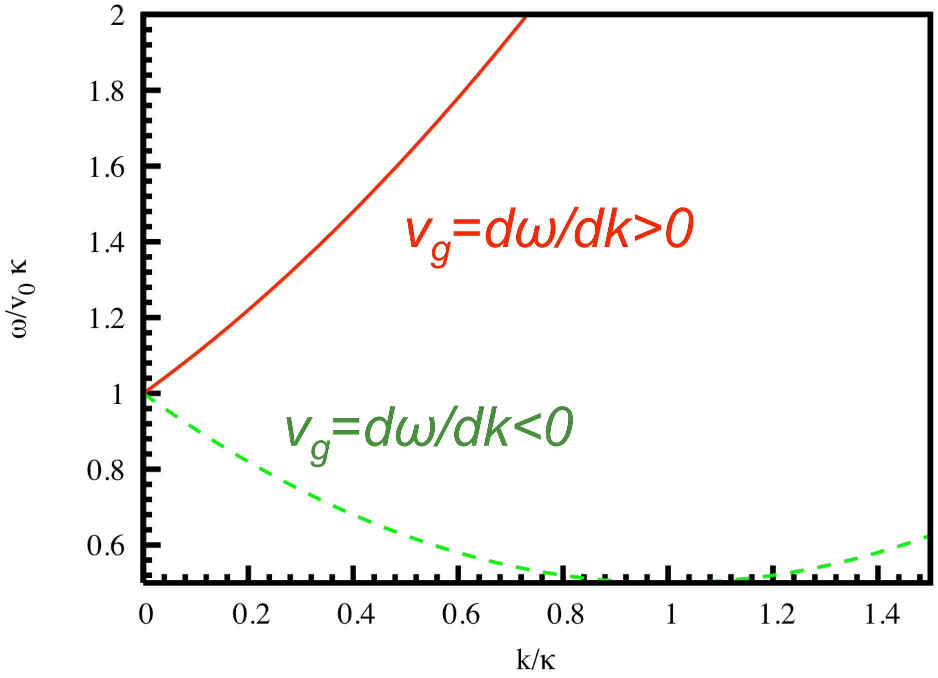
G. Juzeliūnas, J. Ruseckas, L. Santos, M. Lindberg and P. Öhberg, Phys. Rev. A 77, 011802(R) (2008)

$v_o \approx 1$cm/s

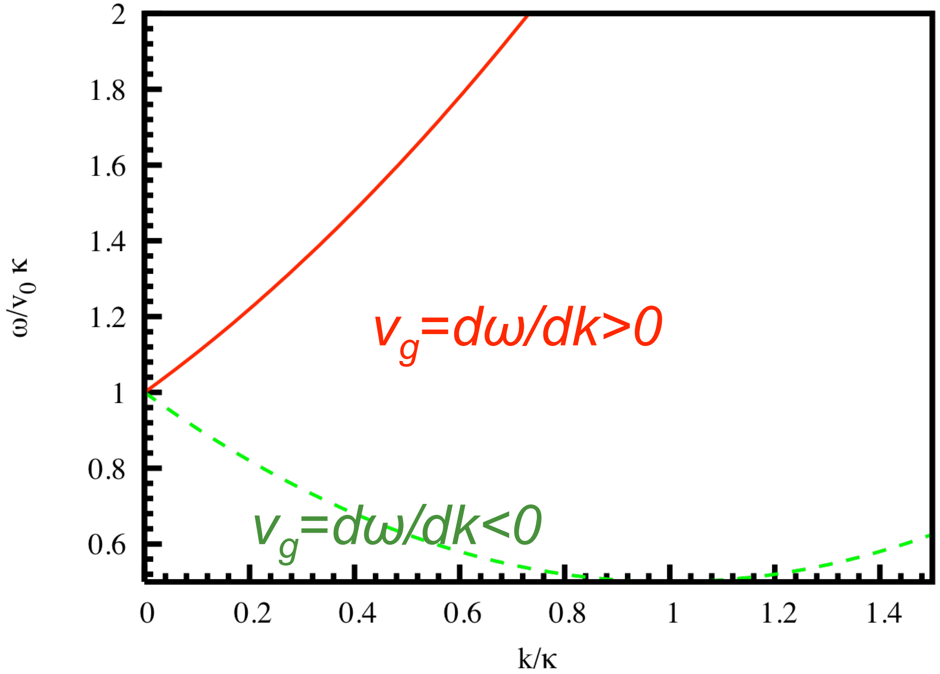
Zittenbewegung of cold atoms

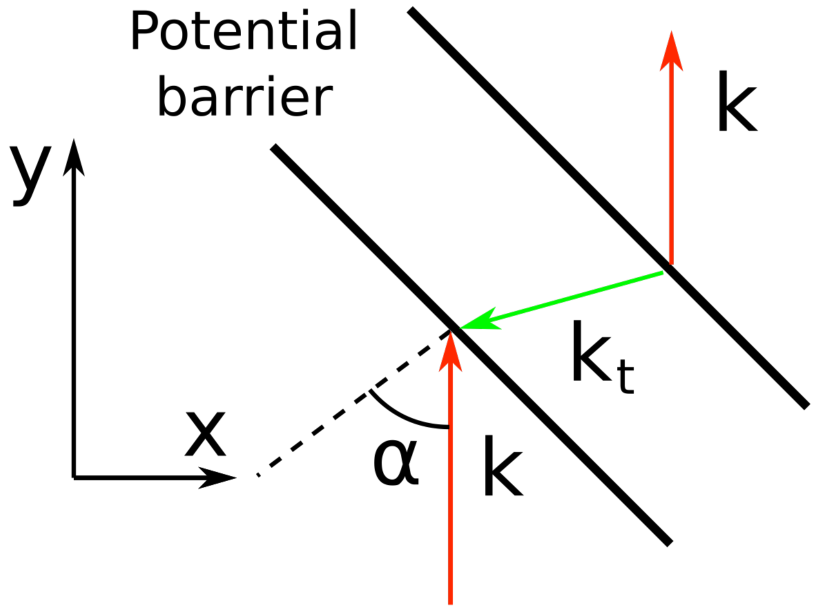
- J. Y. Vaishnav and C. W. Clark, Phys. Rev. Lett. 100, 153002 (2008).
- M. Merkl, F. E. Zimmer, G. Juzeliūnas, and P. Öhberg, Europhys. Lett. 83, 54002 (2008).

Dispersion of centre of mass motion for cold atoms in light fields



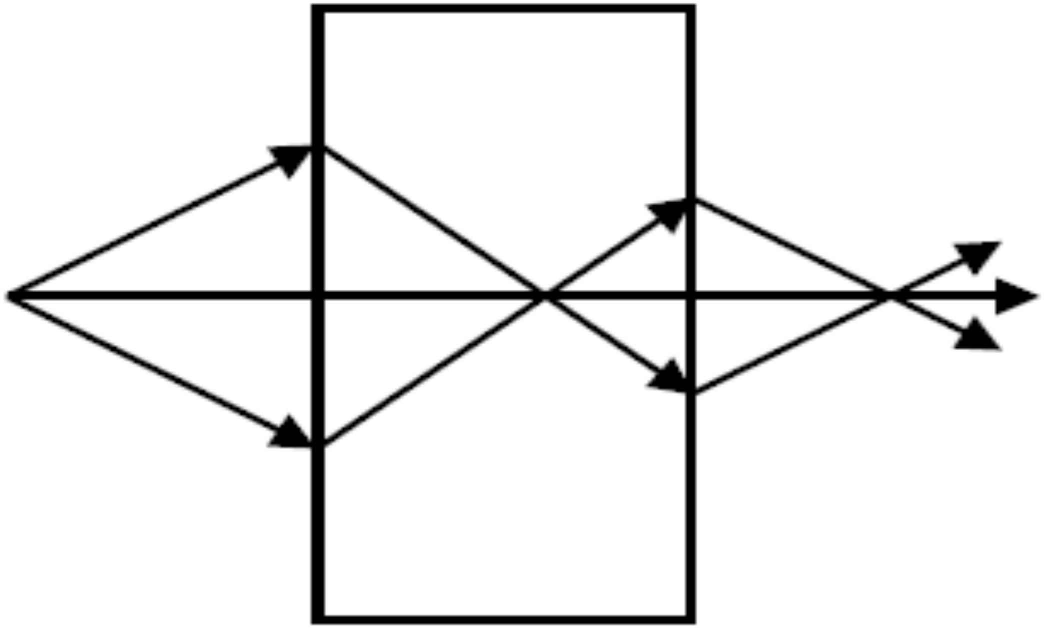
Negative refraction of cold atoms at a potential barrier



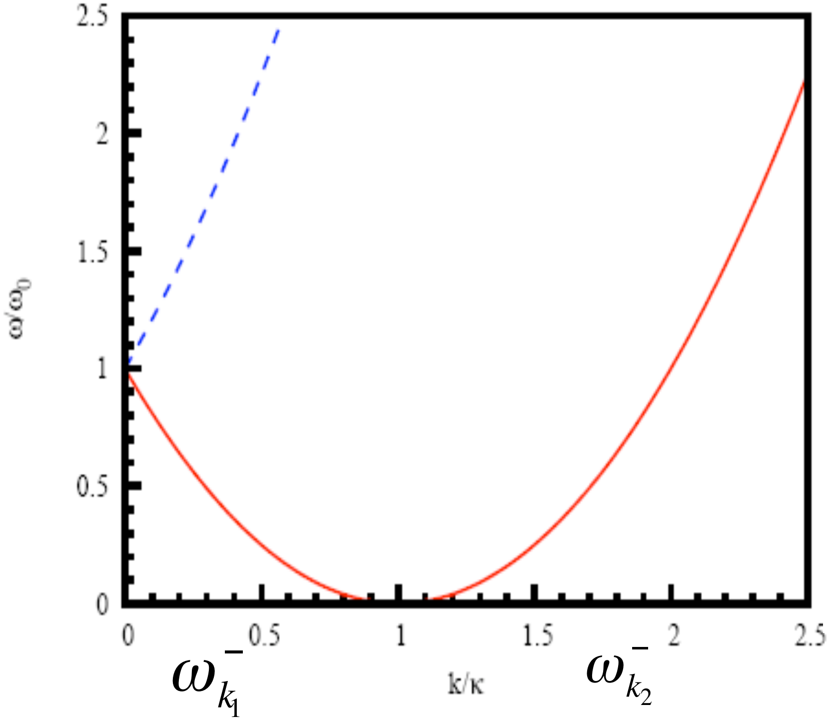


- Veselago-type lenses for cold atoms

Veselago-type lenses for ultra-cold atoms



Double and negative reflection of atoms



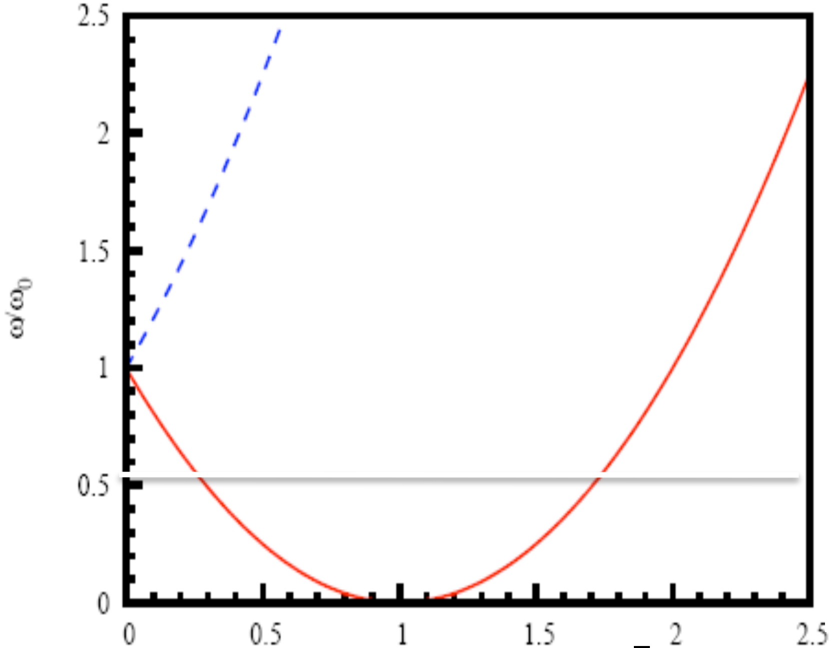
G, Juzeliunas, J. Ruseckas, A. Jacob, L. Santos, P. Ohberg, Phys. Rev. Lett. **100**, 200405 (2008).

ω^-_k - sharp Mexican hat

$$\hbar\omega^{\pm}_k = \frac{\hbar^2}{2M}(k \pm \kappa)^2$$

$$v^-_{k_2} = -v^-_k = -v^-_{k_1}$$

Double and negative reflection of atoms



$\omega^{-}_{k_1}$ $\omega^{-}_{k_2}$

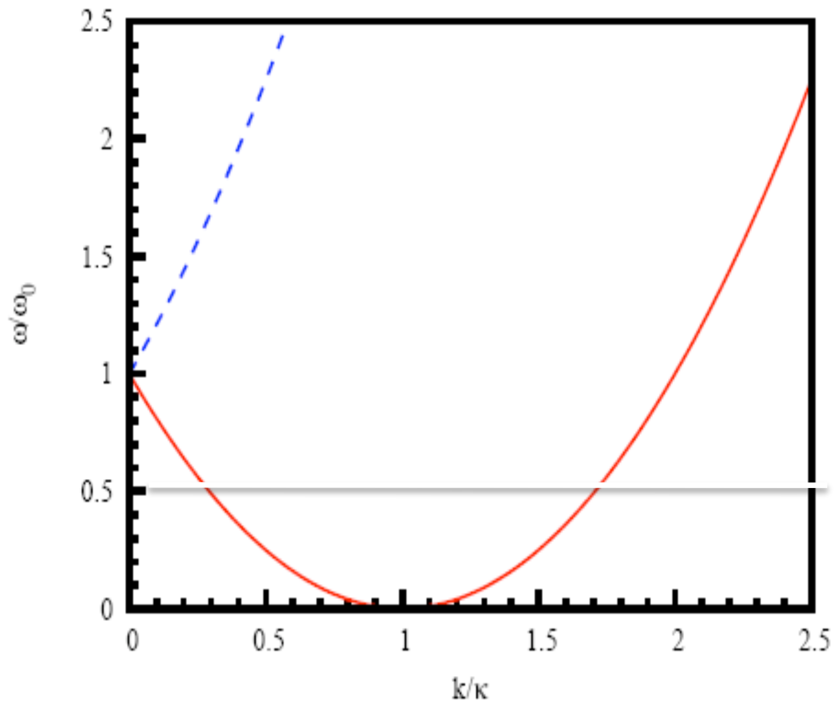
G, Juzeliunas, J. Ruseckas, A. Jacob, L. Santos, P. Ohberg, Phys. Rev. Lett. **100**, 200405 (2008).

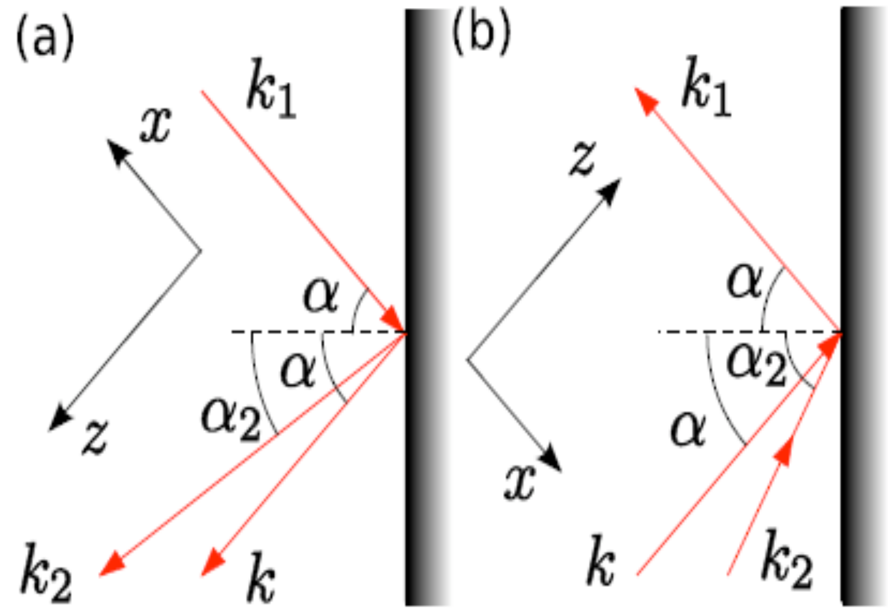
ω_k^- - sharp Mexican hat

$$\hbar\omega_k^{\pm} = \frac{\hbar^2}{2M}(k \pm \kappa)^2$$

$$v_{k_2}^- = -v_k^- = -v_{k_1}^-$$

Double and negative reflection of atoms





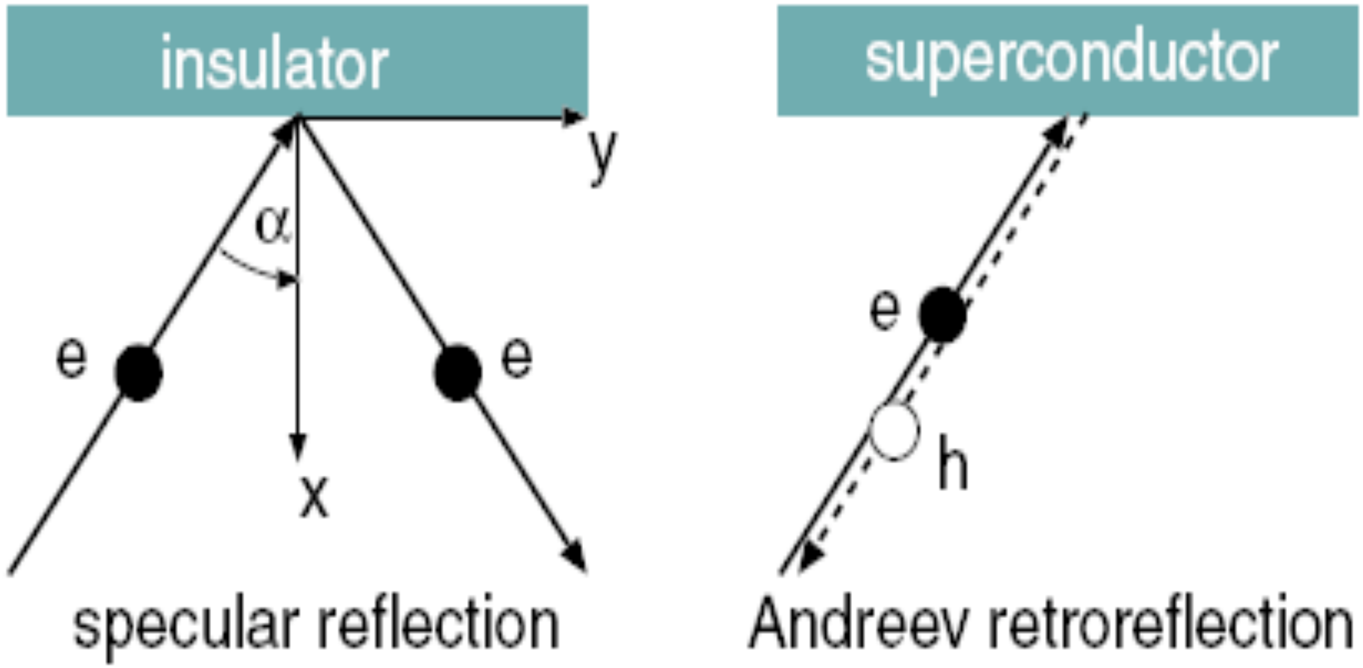
Reflection of atoms with negative chi-

(b) Negatively reflected wave – closer to the surface (k_2<k)

(a) $\mathbf{k}_2$ – closer to the normal (k_2>k)

Resembles Andreev reflection

- Electron is converted into a hole with a negative effective mass upon reflection



Double and negative reflection of atoms: Wave-packet simulations

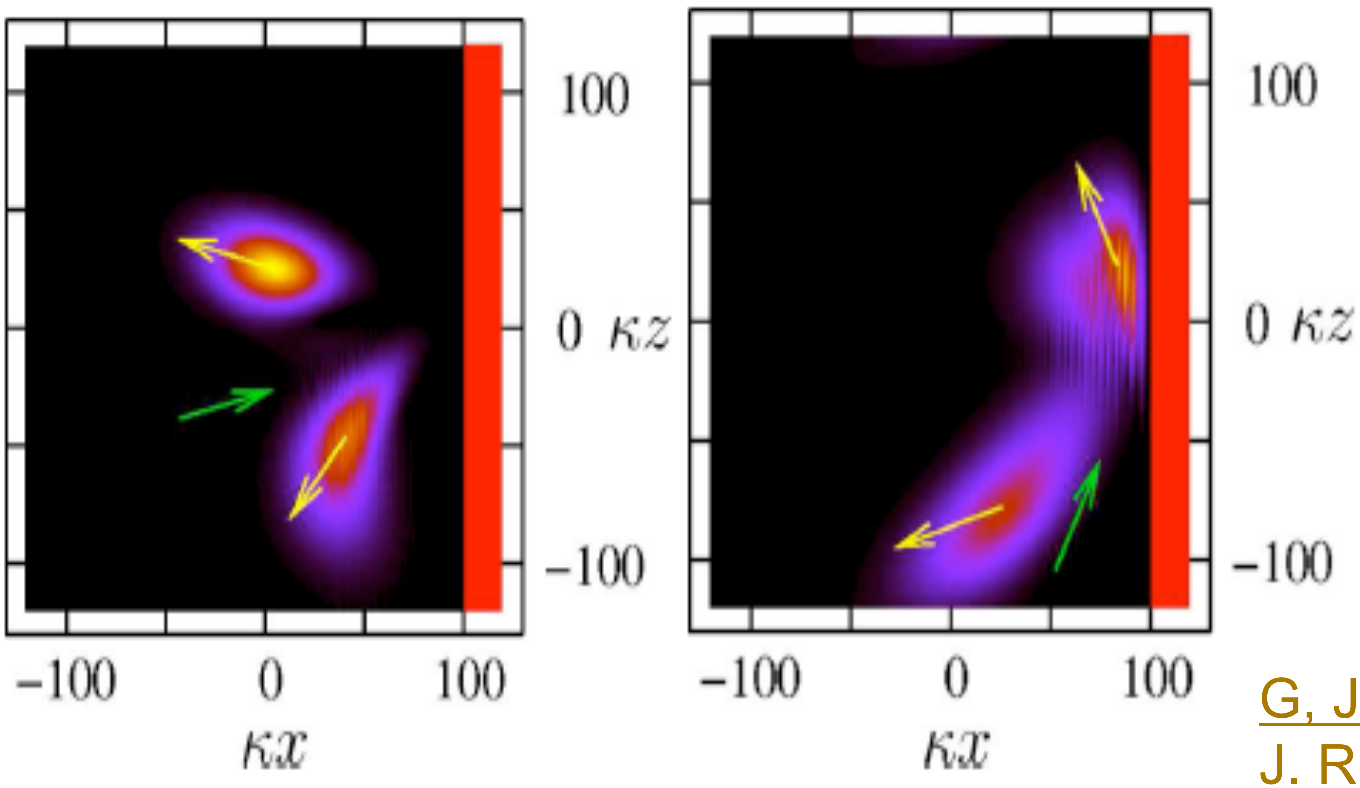


FIG. 5: (Color online) Reflection of an atomic wave-packet with a negative chirality for $\alpha = 15^\circ$, $k = 1.5\kappa$ (left) and $\alpha = 65^\circ$, $k = 0.5\kappa$ (right). The incident wave-packet is taken to be Gaussian with momentum width $\Delta k = 0.1\kappa$. An additional arrow indicates the incident direction.

G, Juzeliunas, J. Ruseckas, A. Jacob, L. Santos, P. Ohberg, Phys. Rev. Lett. **100**, 200405 (2008).

Conclusions

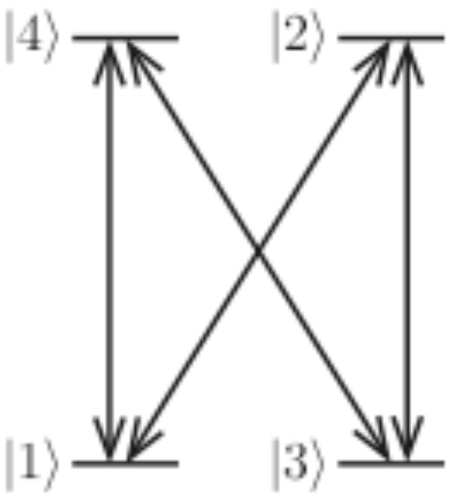
- Abelian gauge potentials appear if there is non-trivial spatial dependence of amplitudes or phases of laser fields (or spatial variation of atomic levels).
- Non-Abelian fields can be formed using the plane-wave setups. They can simulate the spin 1/2 Rashba-type Hamiltonian for cold atoms.
- Spin 1 Rashba coupling can also be generated [PRA 81, 053403 (2010)].

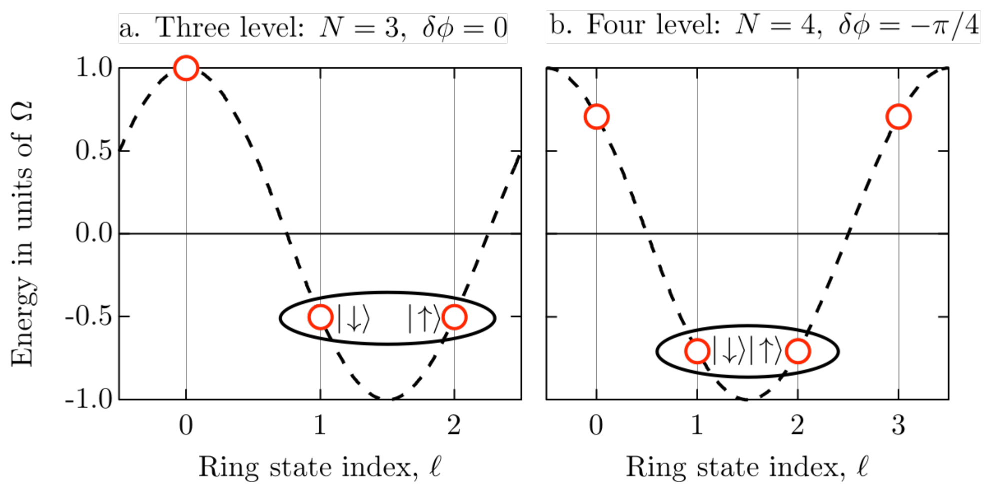
P.S. Latest proposal by Ian Spielman to produce Rashba SO coupling

- A drawback of the tripod scheme: degenerate dark states are not the ground atomic dressed states → collision-induced loses

P.S. Latest proposal by Ian Spielman to produce Rashba SO coupling

- A drawback of the tripod scheme: degenerate dark states are not the ground atomic dressed states → collision-induced loses
- Ian's closed loop setup overcomes this drawback:





[D. L. Campbell, G. Juzeliūnas and I. B. Spielman, in preparation]

Thank you!