Impurities in a Fermi sea: polarons vs. molecules

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SETTING SCIENCE AGENDAS FOR EUROPE

FerMix - EuroQuam

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Kayvan Sadegzadeh (Cambridge)

Many-body systems

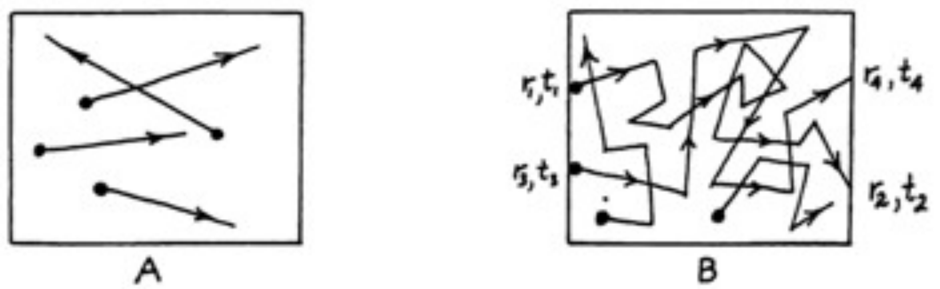


Fig. 0.2 A. *Non-interacting Particles*
B. *Interacting Particles*

(from Richard Mattuck's book)

2 A GUIDE TO FEYNMAN DIAGRAMS [0.0

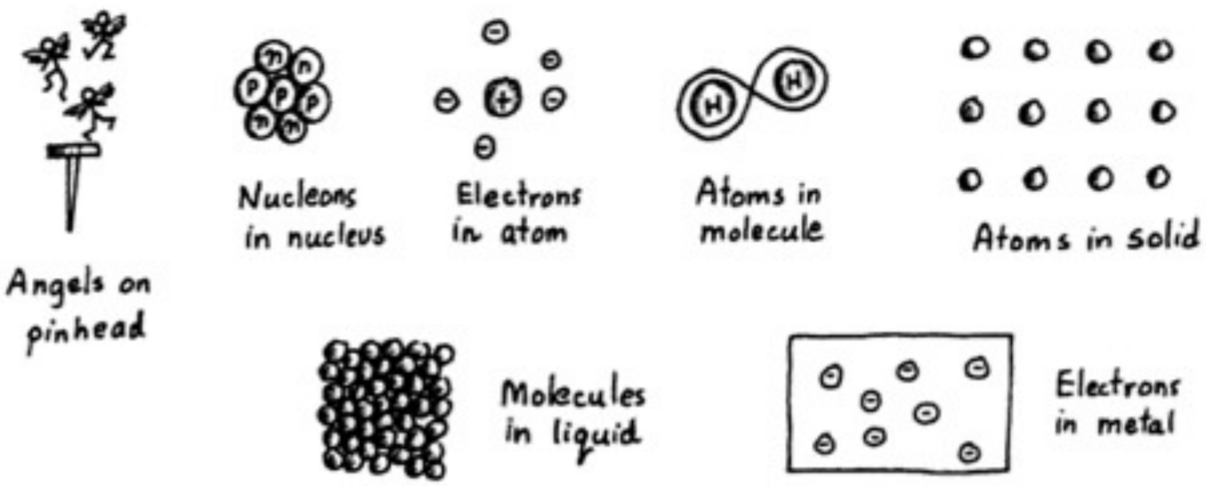


Fig. 0.1 *Some Many-body Systems*

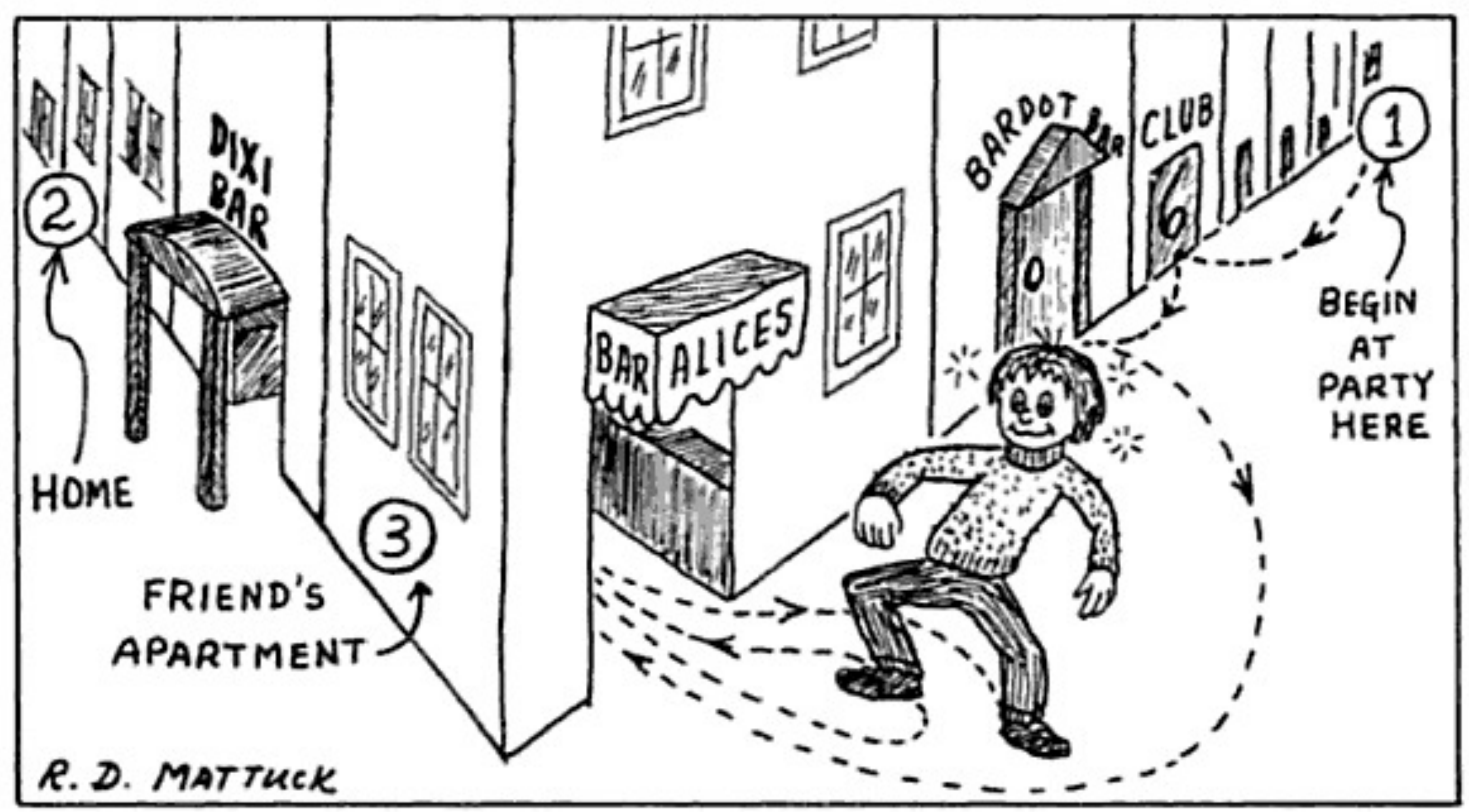


Fig. 1.1 *Propagation of Drunken Man*

Quasi-Particles

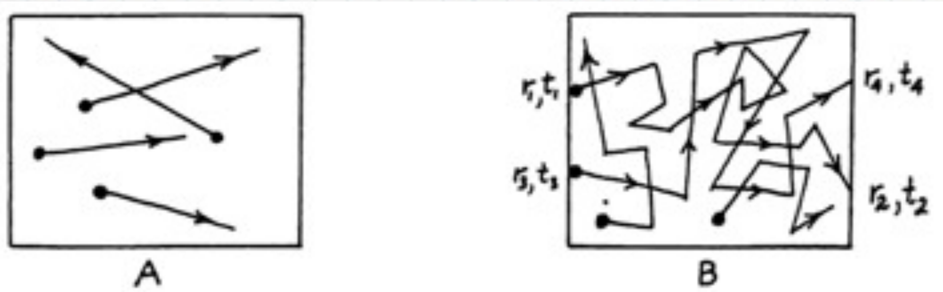


Fig. 0.2 A. *Non-interacting Particles*
B. *Interacting Particles*

Landau's idea:
who cares about real particles?

Of importance are the excitations,
which behave
as quasi-particles!

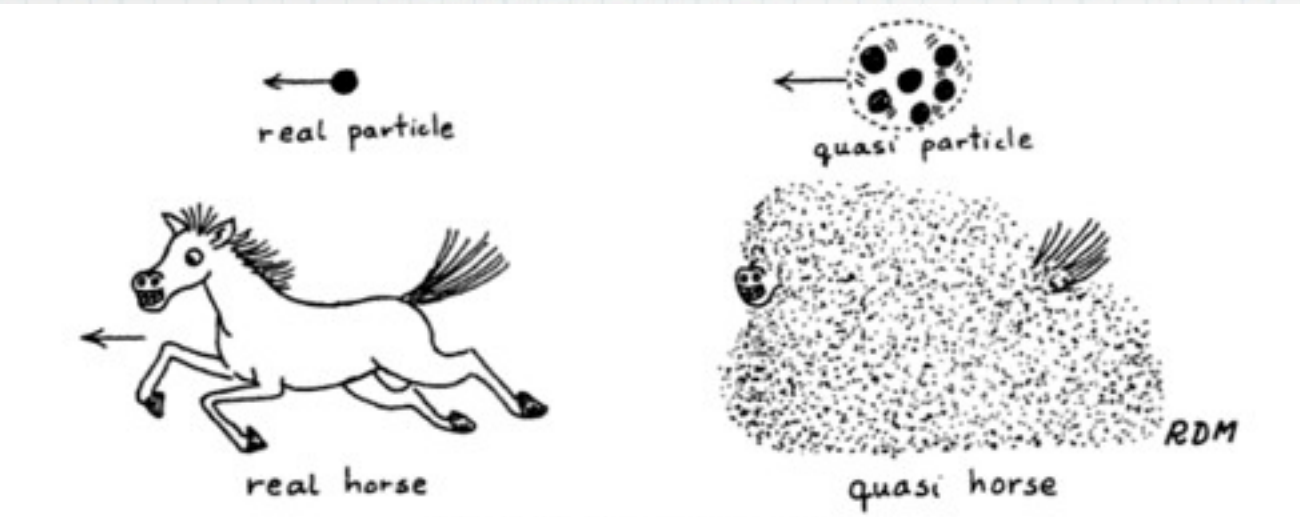


Fig. 0.4 *Quasi Particle Concept*

a QP is a "free particle" with:
@ chemical potential
@ renormalized mass
@ shielded interactions
@ lifetime

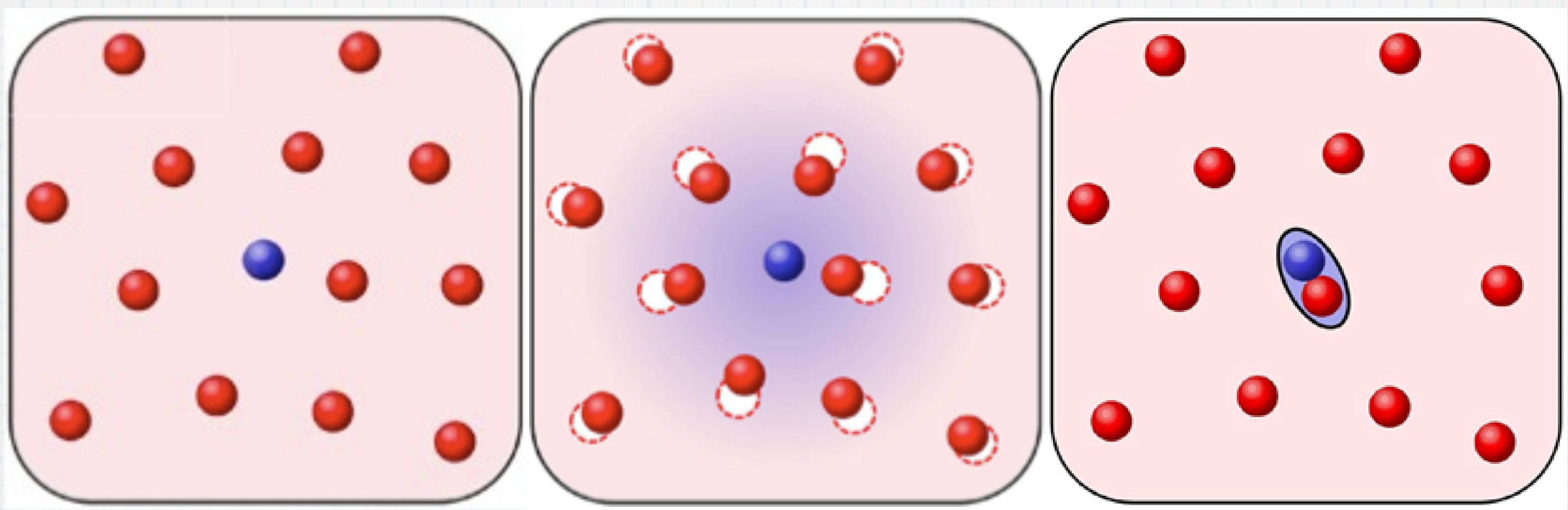
The MIT experiment

Schirotzek, Wu, Sommer & Zwierlein, PRL 2009

- non-interacting ↑ Fermi sea (N>>1)
- a single ↓ impurity

BCS — Attraction strength → BEC

$(k_F a)^{-1} < 0$



$(k_F a)^{-1} > 0$

free particle — QP (polaron) — QP (molecule)

P-M transition: Prokof'ev&Svistunov, PRB 2008

Polaron: variational Ansatz

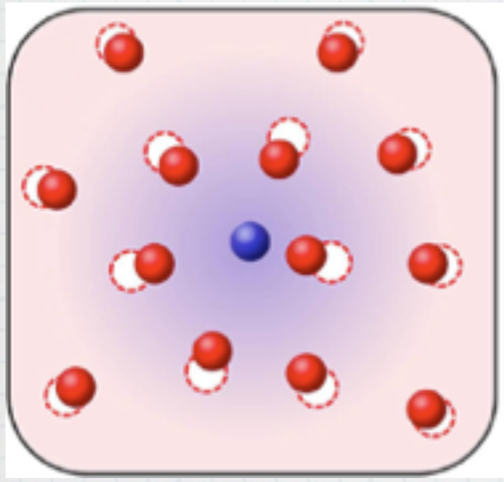
(F. Chevy, PRA 2006)

the $\downarrow$ impurity

$$|\psi_{\mathbf{p}}\rangle = \phi_0 c^\dagger_{\mathbf{p}\downarrow} |0\rangle_\uparrow + \sum_{q<k_F}^{k>k_F} \phi_{\mathbf{qk}}\, c^\dagger_{\mathbf{p+q-k}\downarrow} \boxed{c^\dagger_{\mathbf{k}\uparrow}\, c_{\mathbf{q}\uparrow}\, |0\rangle_\uparrow}$$

non-interacting $\uparrow$ Fermi sea

Particle-Hole dressing



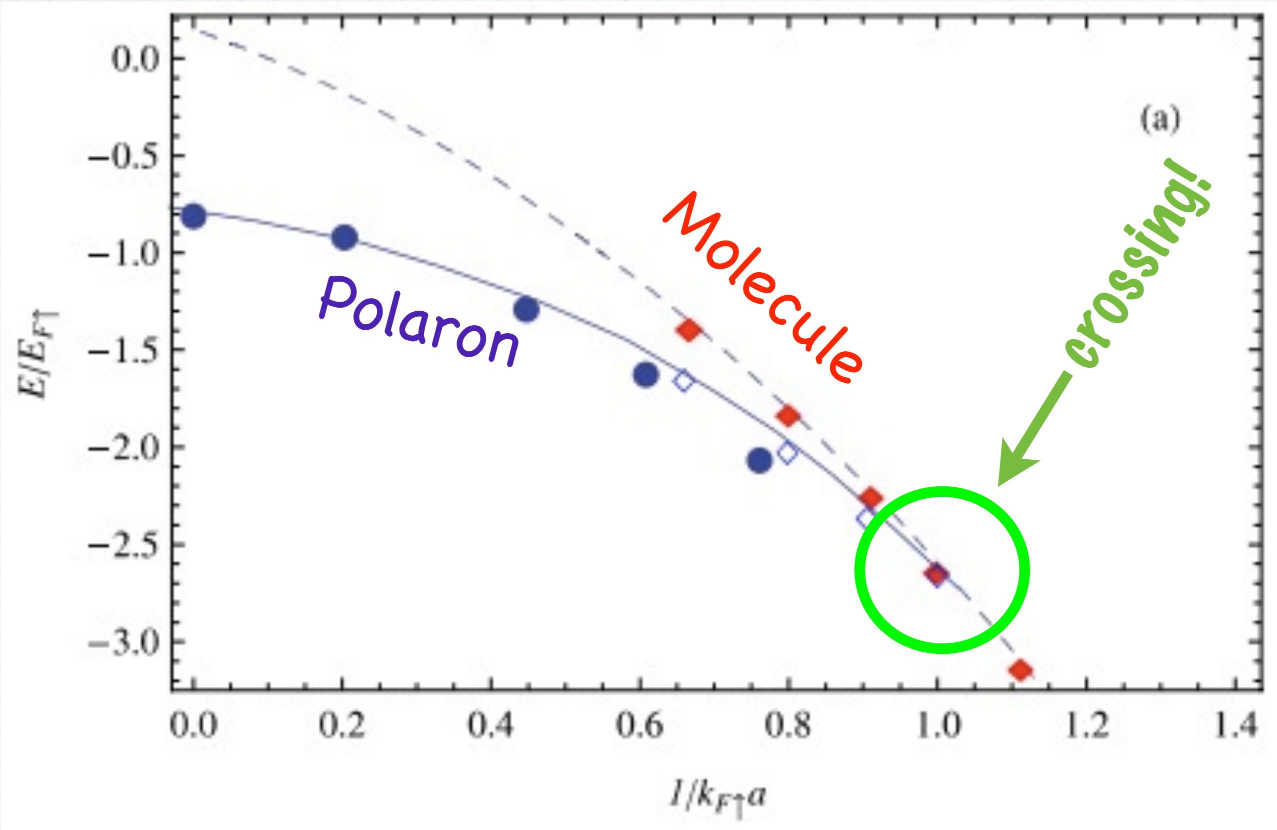
Very good agreement with QMC results for $\mu_\downarrow$ and m^*

This variational Ansatz has a diagrammatic equivalent:
the forward scattering, or ladder, approximation.

(Combescot et al., PRL 2007)

QP parameters

Chemical potential $\mu_\downarrow$



◇, ◆ : QMC

—, ⋯ :variat, diagr

● : MIT expmt

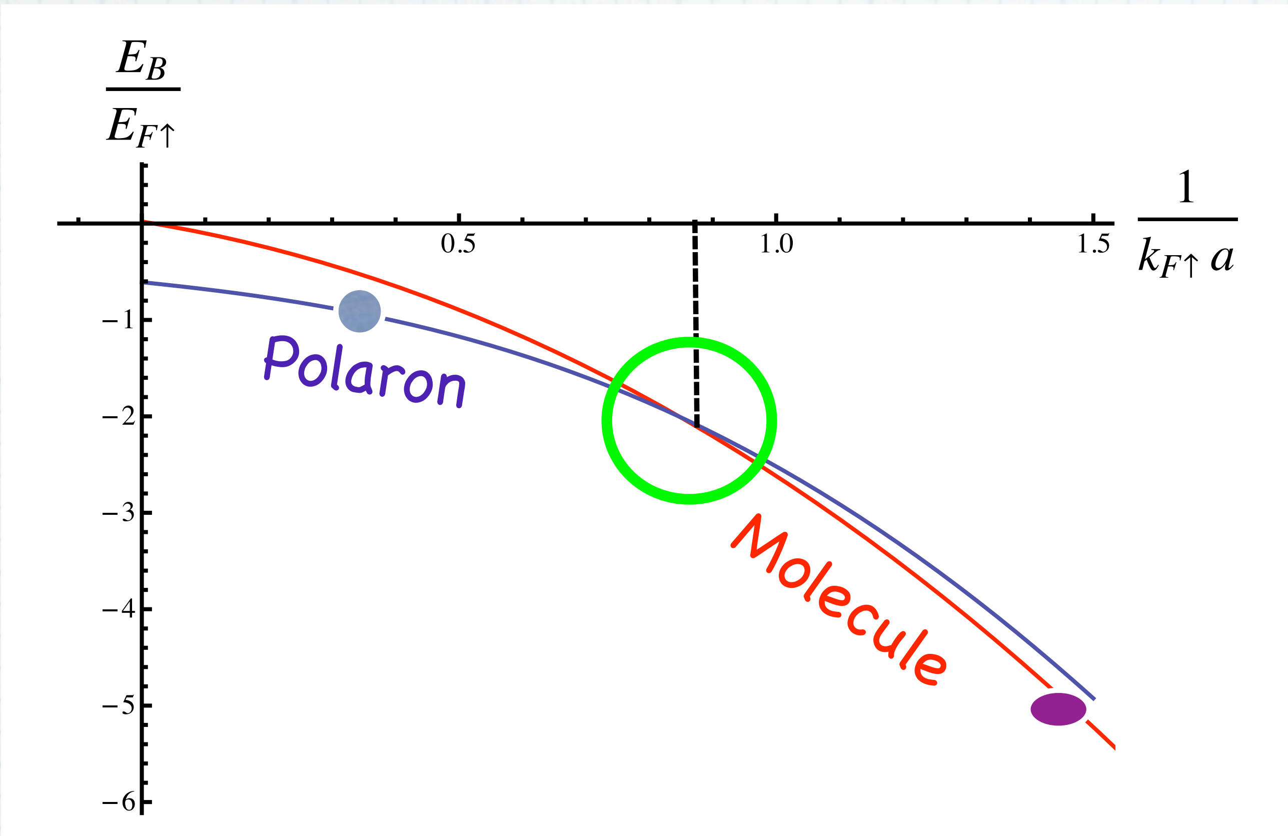
QMC: Prokof'ev&Svistunov

Variational and diagrammatic: Chevy, Recati, Lobo, Stringari, Combescot, Leyronas
Massignan&Bruun, Zwerger, Punk, Stoof, Mora,...

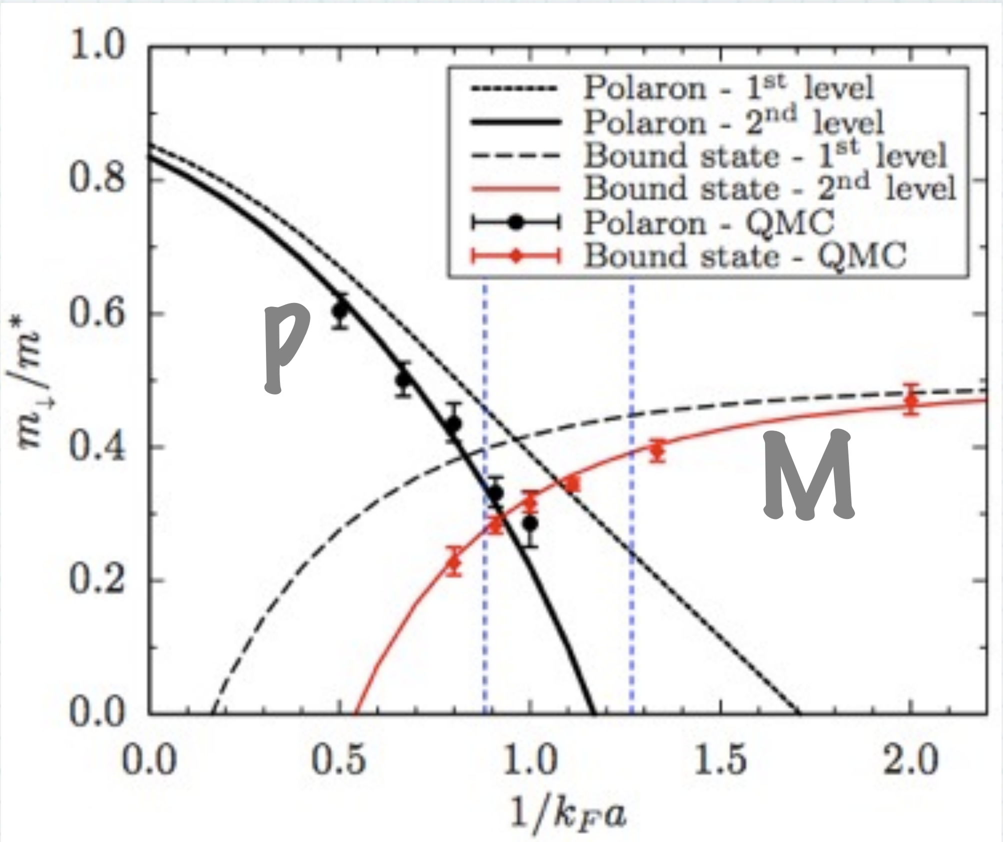
Experiments: MIT, ENS

QP parameters

Chemical potential $\mu_\downarrow$



Effective mass m*



P-P Interactions: Mora&Chevy, PRL 2010
Zhenhua, Zöllner & Pethick, PRL 2010

Equation of state of a unitary Fermi gas

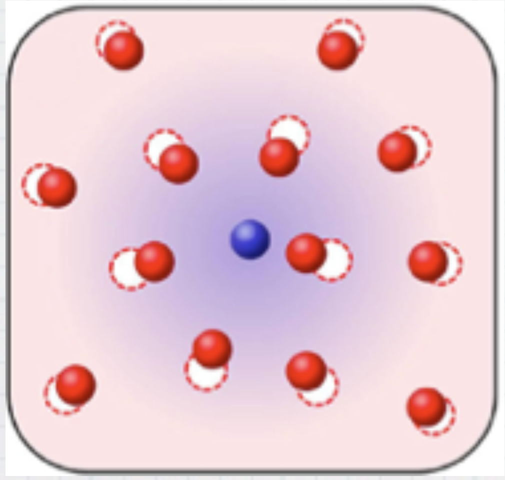
In the normal phase at T=0,

$$P = \boxed{\frac{1}{15\pi^2}\left(\frac{2m_\uparrow}{\hbar^2}\right)^{3/2} \mu_\uparrow^{5/2}} + \boxed{\frac{1}{15\pi^2}\left(\frac{2m^*_\downarrow}{\hbar^2}\right)^{3/2} (\mu_\downarrow - A\mu_\uparrow)^{5/2}}$$

non-interacting ↑ non-interacting QP

$A = -0.615$

$m^*_\downarrow = 1.2m$



Same thermodynamics for:
- ultracold atoms
- dilute neutron matter

What's left?

✓ chemical potential

✓ renormalized mass

✓ shielded interactions

✓ lifetime

this work!

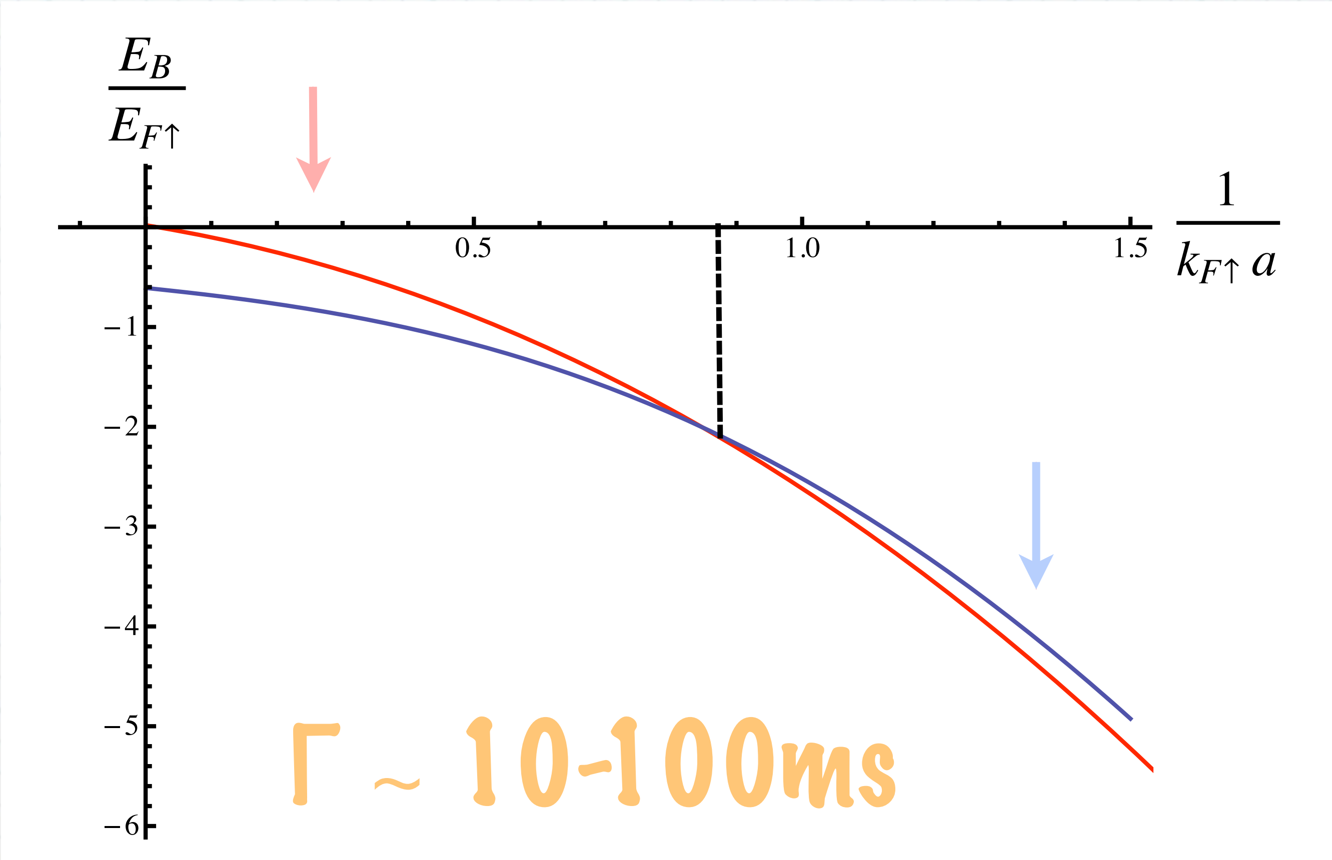
Very long QP lifetimes!

G. Bruun & P. Massignan, PRL 2010

$$\Gamma_P \sim Z_M \left(\Delta\omega\right)^{9/2}$$

$$\Delta\omega = \omega_P - \omega_M$$

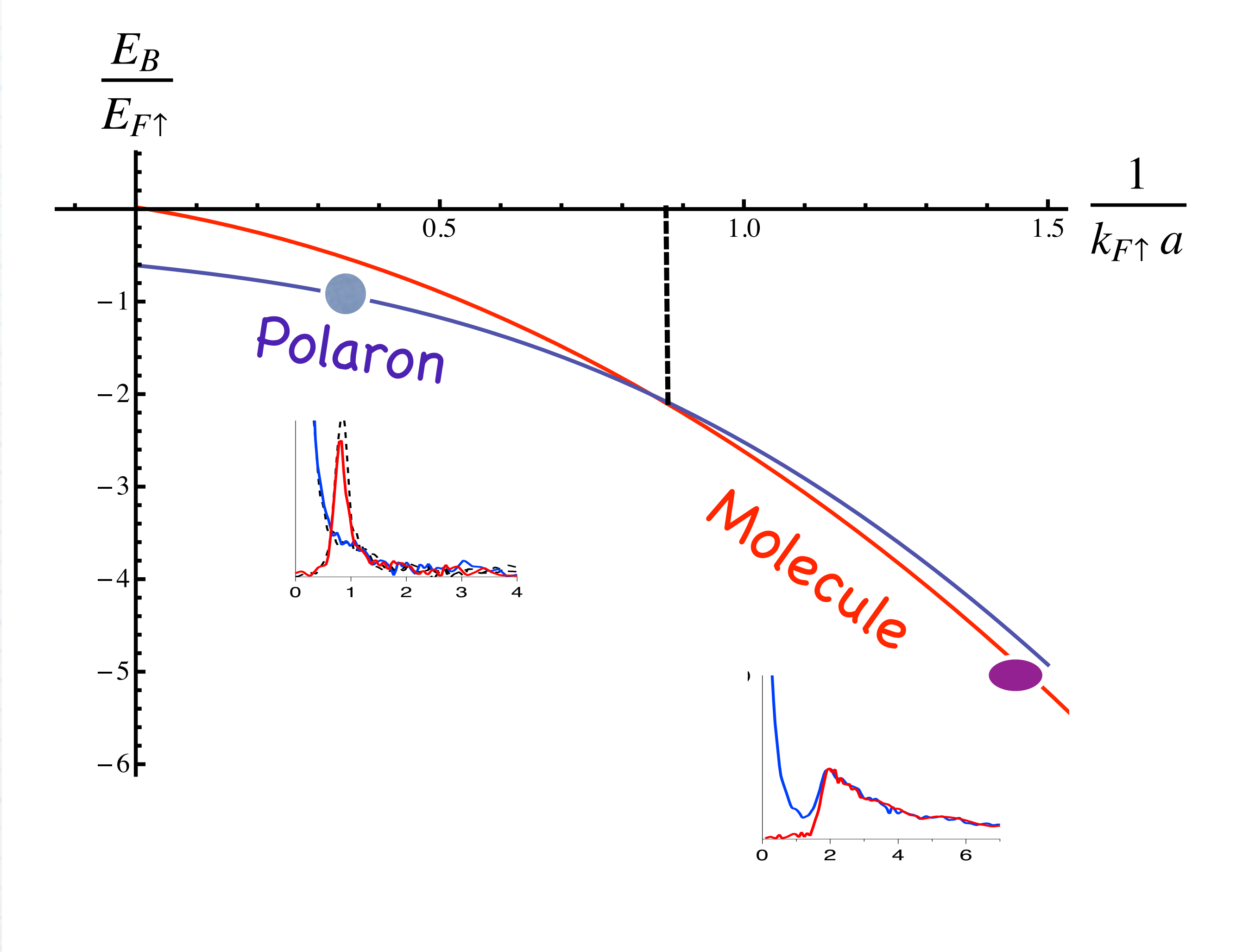
$$\Gamma_M \sim Z_P \left(-\Delta\omega\right)^{9/2}$$



Experimental observation

Methods:

- RF spectra
- Collective modes
  to measure m* vs. time
- Density profiles in the trap



Issues:

- Phase separation?
  - stabilized by finite T
  - work with $m_\downarrow \neq m_\uparrow$
  - use bosonic impurities
- No decay to deeply bound molecular states

Pol→Mol decay

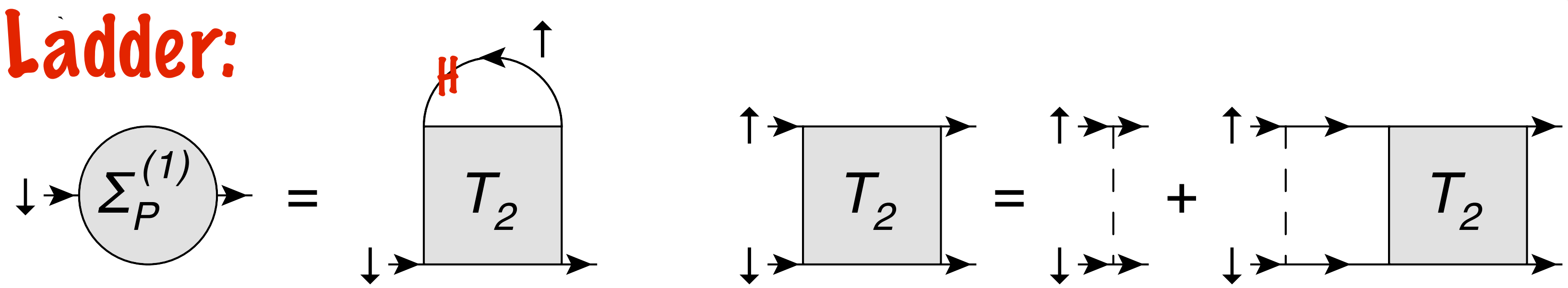
$$\Delta\omega = \omega_P - \omega_M > 0$$

Polaron: $G_\downarrow(\mathbf{p}, z)^{-1} = G^0_\downarrow(\mathbf{p}, z)^{-1} - \Sigma_P(\mathbf{p}, z)$

Decay rate: $\Gamma_P = -\mathrm{Im}\Sigma_P(p = 0, \omega_P)$

Hole expansion: $\Sigma_P(\mathbf{p}, z) = \Sigma_P^{(1)}(\mathbf{p}, z) + \Sigma_P^{(2)}(\mathbf{p}, z) + \ldots$

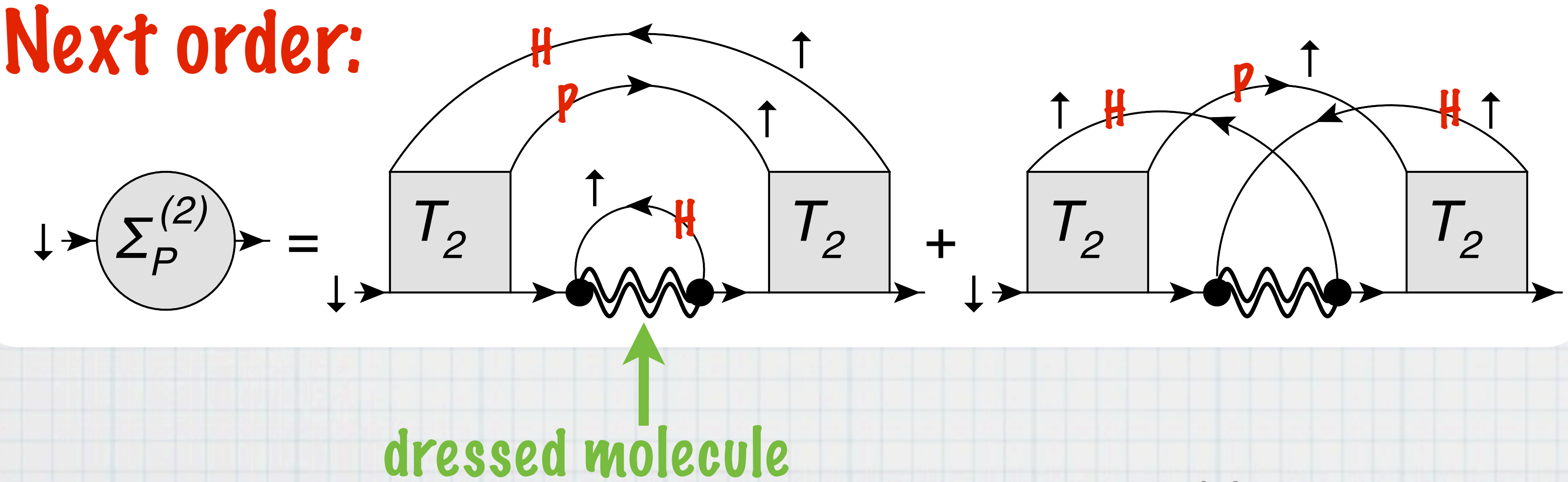
Ladder:

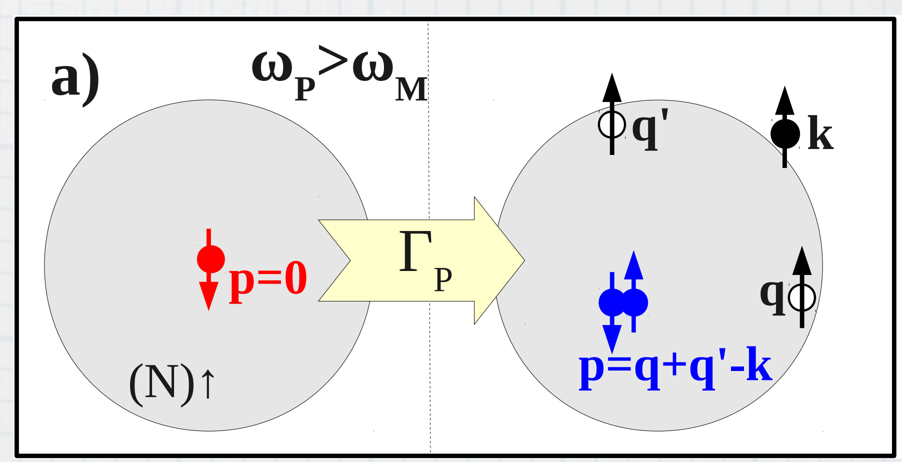


no damping
in the ladder approx.

3-body process

Next order:

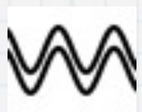




molecule w.f. in vacuum:

$$\phi_q = \frac{\sqrt{8\pi a^3}}{1+q^2a^2} \qquad \text{or} \qquad \phi_r \propto \frac{\mathrm{e}^{-r/a}}{r}$$

dressed molecule:



$$D(\mathbf{p},\omega) \simeq \frac{Z_M}{\omega - \omega_M - p^2/2m_M^*}.$$

atom-molecule coupling:

$$\bullet = \frac{1}{g(\mathbf{p},z)} = \int \frac{d^3q}{(2\pi)^3}\phi_q \frac{1-f(\xi_{\mathbf{p}-\mathbf{q}\uparrow})}{z-\xi_{\mathbf{p}-\mathbf{q}\uparrow}-\xi_{q\downarrow}} \sim -\sqrt{\frac{m_r^2 a}{2\pi}}$$

(Bruun&Pethick, PRL 2004)

$$\Gamma_P = \frac{g^2 Z_M}{2}\int \frac{d^3k\ d^3q\ d^3q'}{(2\pi)^9}\left[F(\mathbf{q},\mathbf{k},\omega_P) - F(\mathbf{q}',\mathbf{k},\omega_P)\right]^2 \delta\left(\Delta\omega + \xi_{q\uparrow} + \xi_{q'\uparrow} - \xi_{k\uparrow} - \frac{(\mathbf{q}+\mathbf{q}'-\mathbf{k})^2}{2m_M^*}\right)$$

$$q, q' < k_F \ , \ k > k_F$$

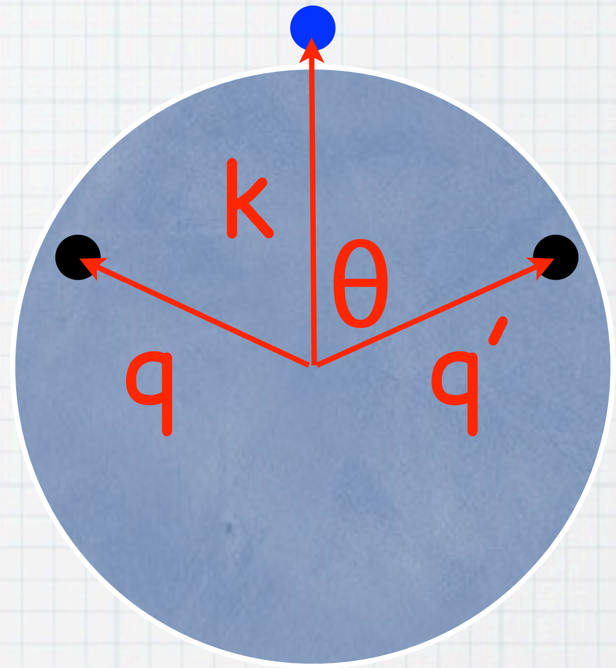
$$F(\mathbf{q},\mathbf{k},\omega) = T_2(\mathbf{q},\omega+\xi_{q\uparrow})G^0_\downarrow(\mathbf{q}-\mathbf{k},\omega+\xi_{q\uparrow}-\xi_{k\uparrow})$$

$\Delta\omega \ll \epsilon_F$

$q \simeq k \simeq k' \simeq k_F$

In the neighborhood of the P-M crossing,

$$\int \frac{d^3k\ d^3q\ d^3q'}{(2\pi)^9}\delta(\ldots) \sim (m_M^*)^{3/2}(\Delta\omega)^{7/2}$$



The P+H+H form an equilateral triangle, since $q + q' - k \sim 0$

At the crossing, Fermi antisymmetry yields a vanishing of the matrix element:

$$F(\mathbf{q}, \mathbf{k}, \omega_P) - F(\mathbf{q}', \mathbf{k}, \omega_P)$$

the angular dependence of F is only on θ

Expand the difference to get an extra factor of Δω:

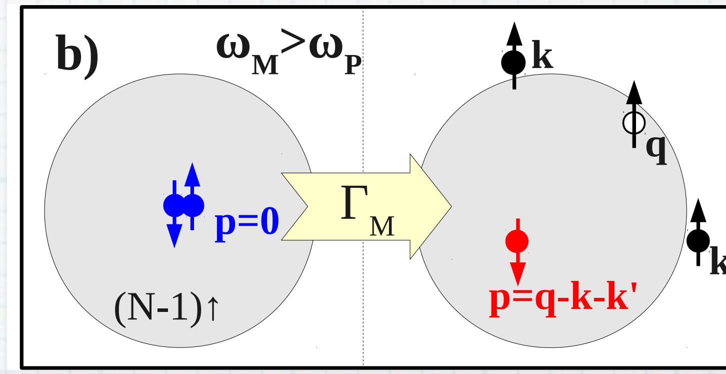
$$\boxed{\Gamma_P \sim Z_M(k_F a)\,(m_M^*)^{3/2}\,(\Delta\omega)^{9/2}}$$

1st order transition between the P&M states (no coupling at the crossing)

Mol→Pol decay

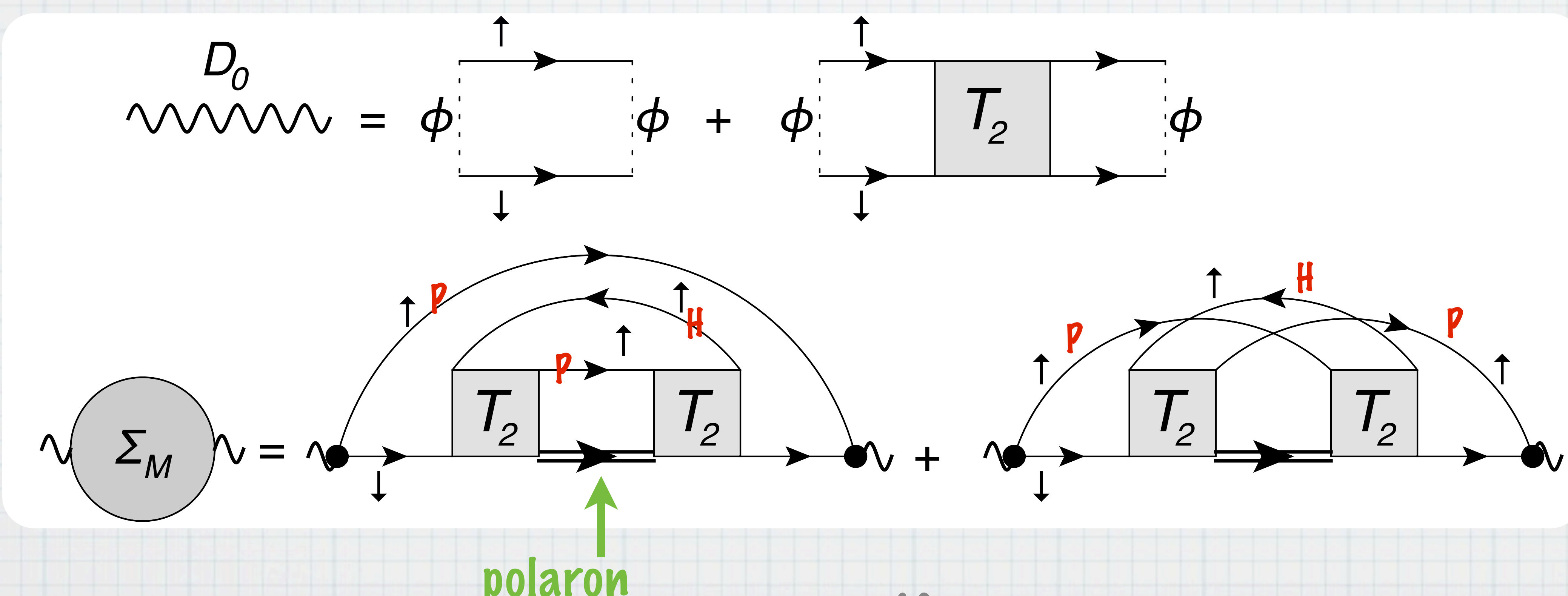
$$\Delta\omega = \omega_P - \omega_M < 0$$

Molecule: $D(\mathbf{p}, z)^{-1} = D_0(\mathbf{p}, z)^{-1} - \Sigma_M(\mathbf{p}, z)$



Decay rate: $\Gamma_M = -\mathrm{Im}\Sigma_M(p = 0, \omega_M)$

Vacuum: $$D_0(\mathbf{p}, z) = \int d^3\check{q}\phi_q^2 \frac{1 - f(\xi_{\mathbf{p}-\mathbf{q}\uparrow})}{z - \xi_{\mathbf{p}-\mathbf{q}\uparrow} - \xi_{q\downarrow}} + \frac{T_2(\mathbf{p}, z)}{g(\mathbf{p}, z)^2}$$



3-body process

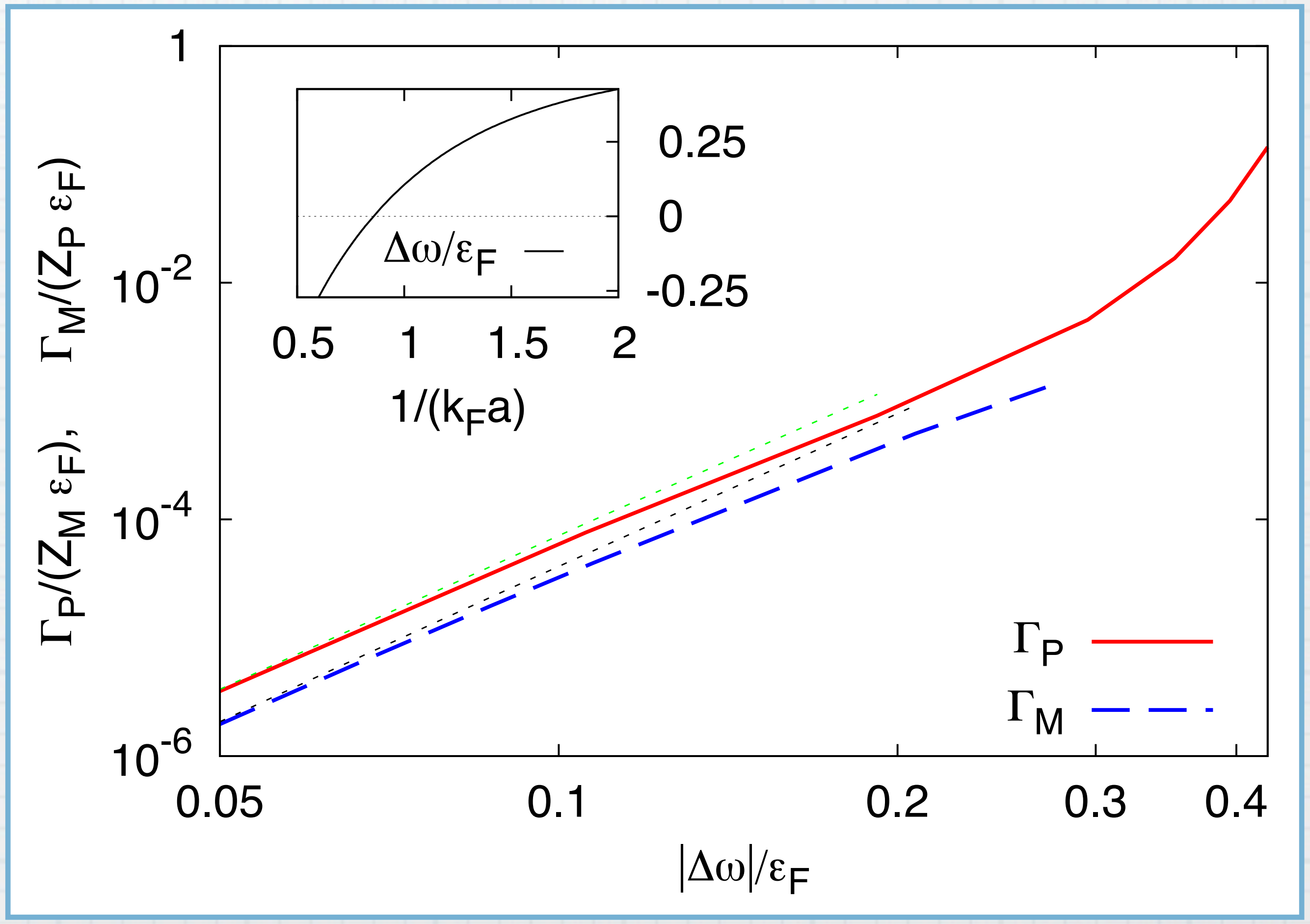
$$\Gamma_M = \frac{g^2 Z_P}{2} \int \frac{d^3k\ d^3k'\ d^3q}{(2\pi)^9} \left[C(\mathbf{q},\mathbf{k},\omega_M) - C(\mathbf{q},\mathbf{k}',\omega_M)\right]^2 \delta\left(|\Delta\omega| + \xi_{q\uparrow} - \xi_{k\uparrow} - \xi_{k'\uparrow} - \frac{(\mathbf{q}-\mathbf{k}-\mathbf{k}')^2}{2m_P^*}\right)$$

In the neighborhood of the M-P crossing, $\Gamma_M \sim Z_P(k_F a)\,(m_P^*)^{3/2}\,(-\Delta\omega)^{9/2}$

For both decay processes,
very **long lifetimes** are ensured by:

- limited phase-space
- Fermi antisymmetry

much longer than usual Fermi liquids



In the numerics:

$$\omega_M = -\frac{\hbar^2}{2m_r a^2} - \epsilon_F + g_3 n_\uparrow$$

$$a_3 = 1.18a$$

$$T_2(\mathbf{p},\omega) = \frac{2\pi a/m_r}{1 - \sqrt{2m_r a^2\left(\frac{p^2}{2m_M} - \omega - \epsilon_F + g_3 n_\uparrow\right)}}$$

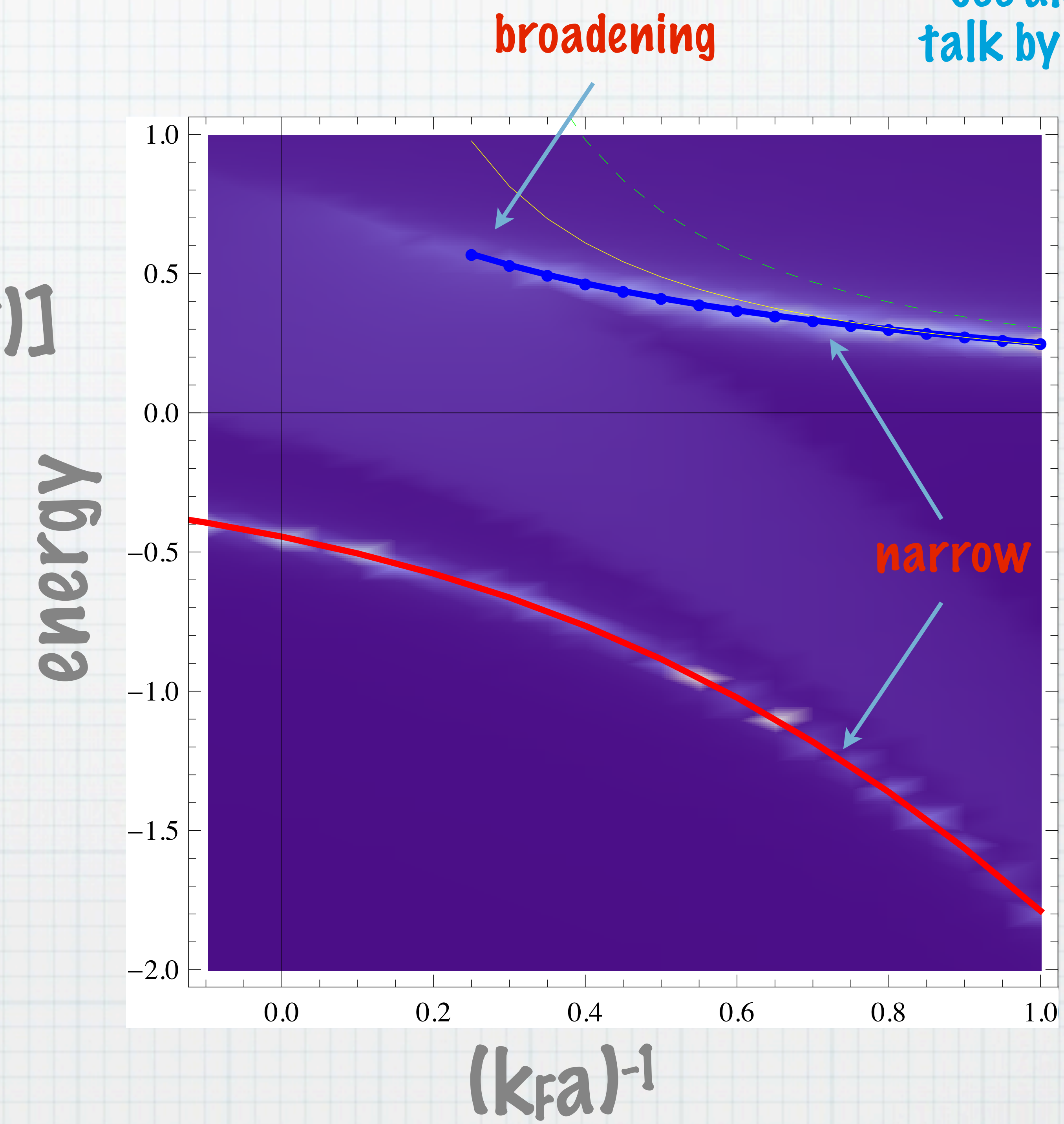
...is there more?

Yes..

see also next talk by Hui Zhai

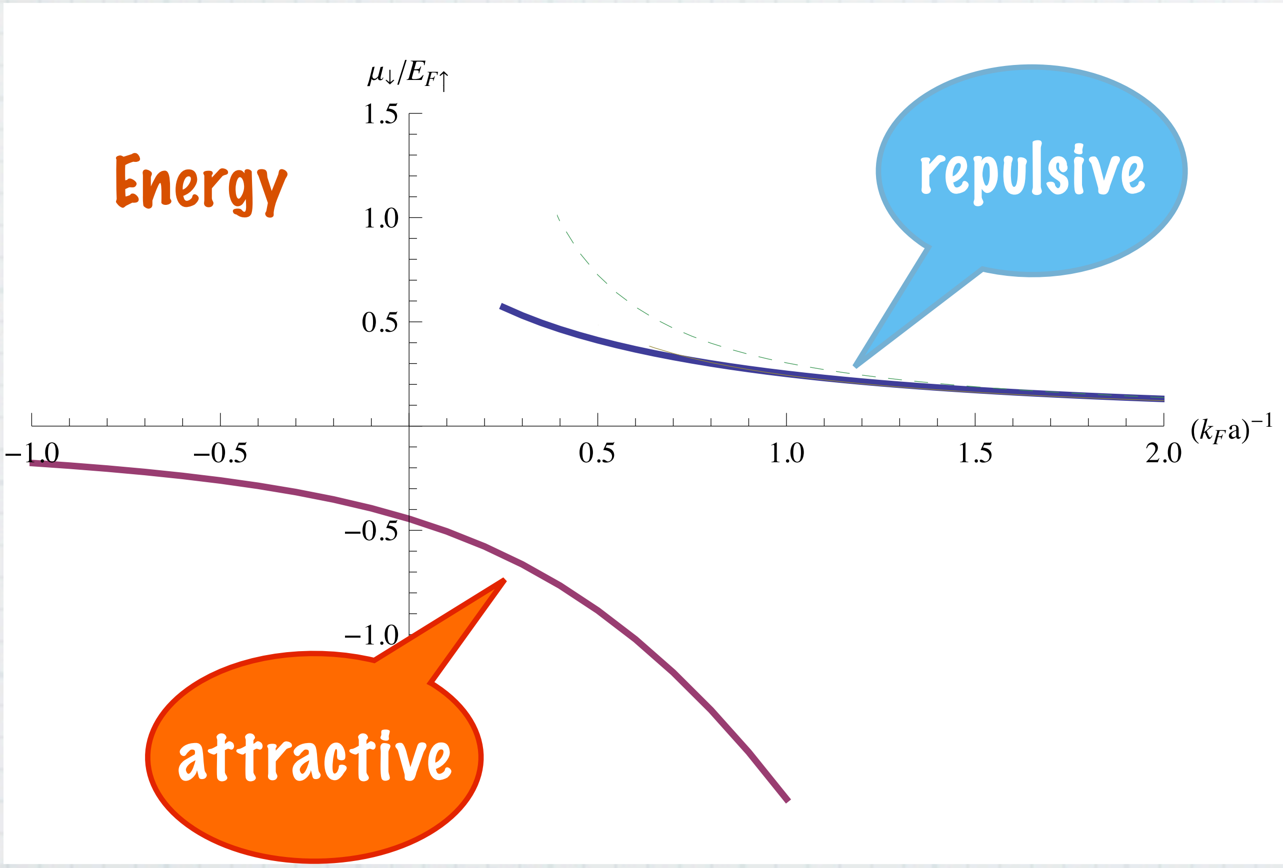
spectral function

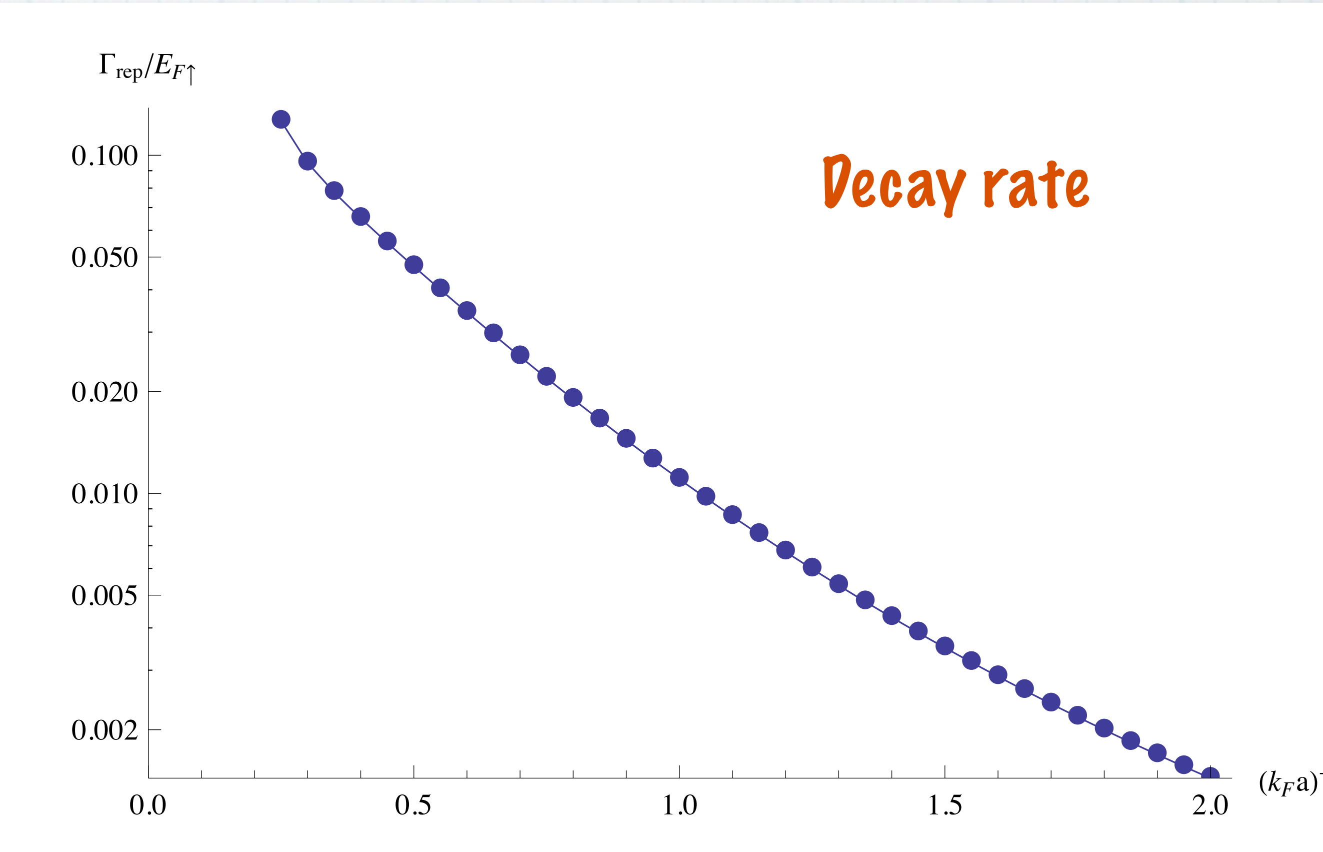
$A_{\downarrow}(\omega)=-\mathrm{Im}[G_{\downarrow}(k=0,\omega+i0^+)]$

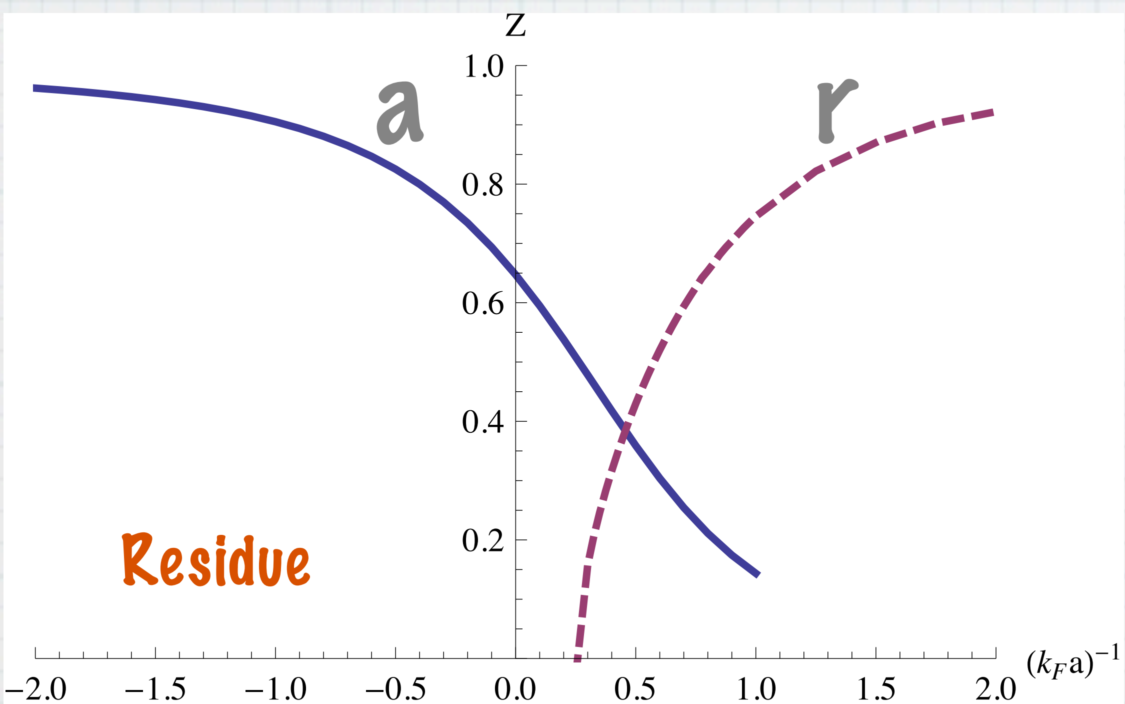


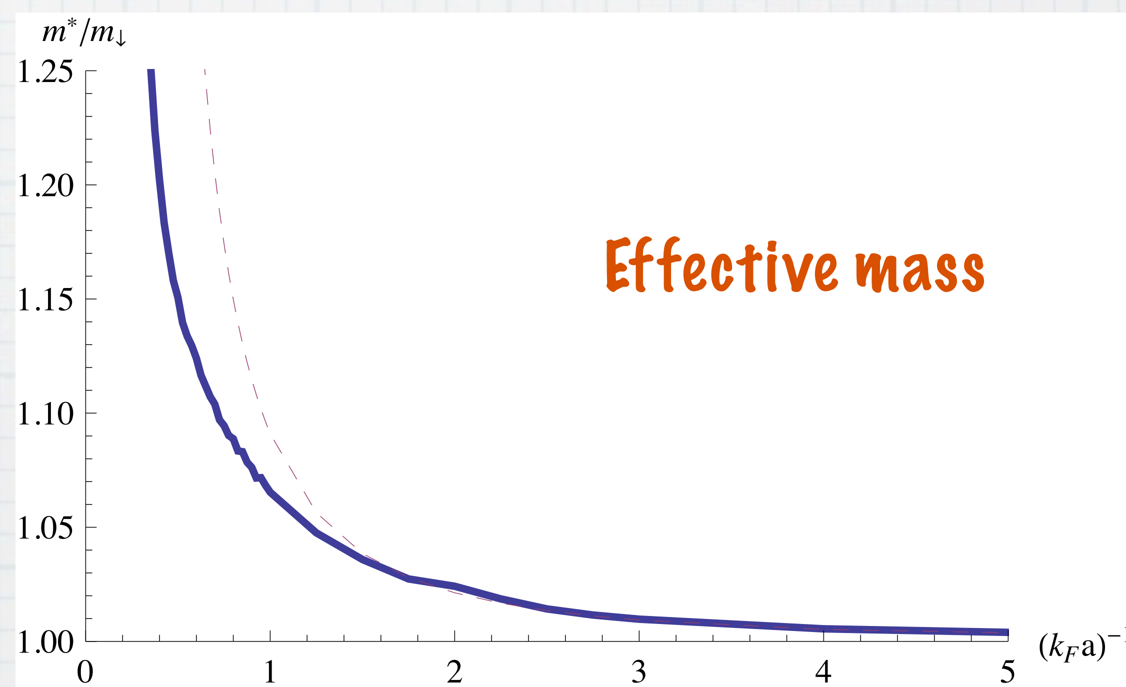
^{40}K impurity in a Fermi sea of ^{6}Li

The repulsive polaron revealed





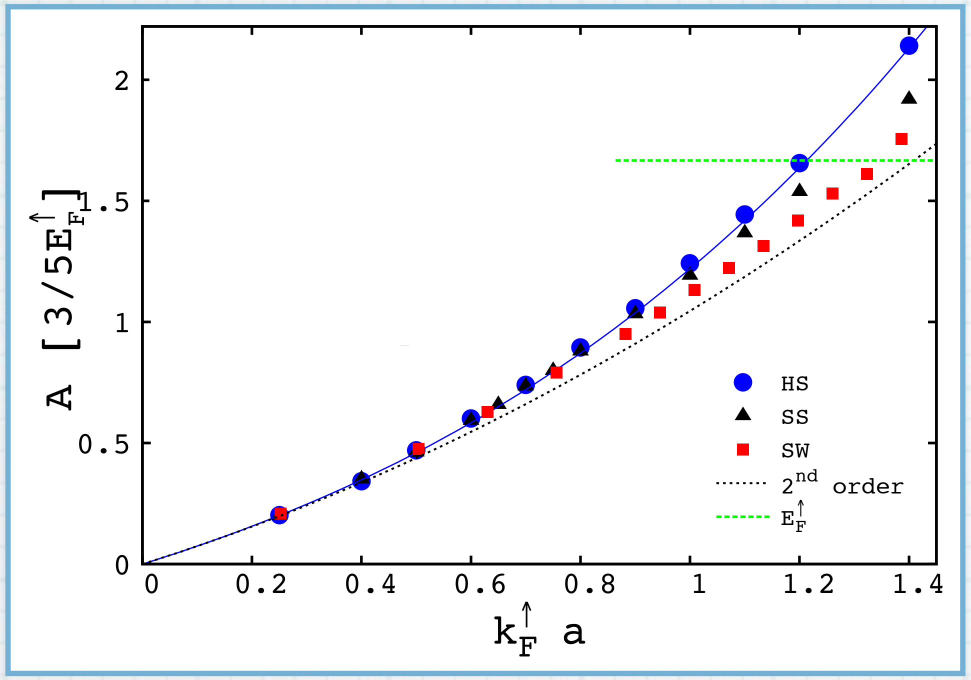




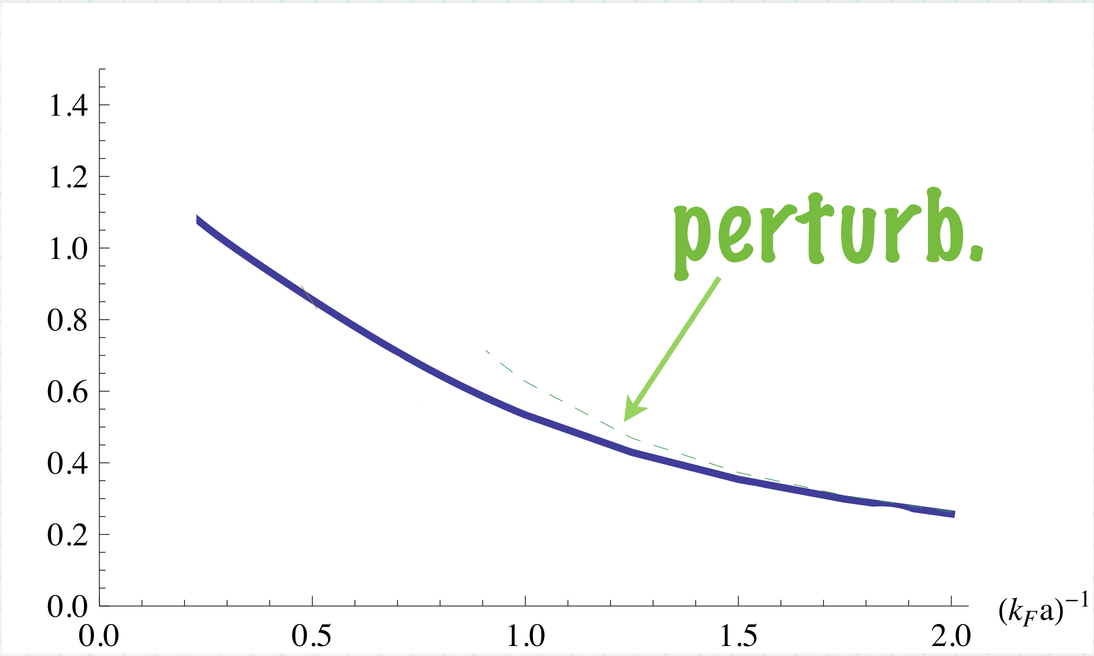
^{40}K impurity in a Fermi sea of ^{6}Li

Energy: comparison with QMC

(equal masses case)



QMC by Pilati et al., PRL 2010



Massignan & Bruun
(in preparation)

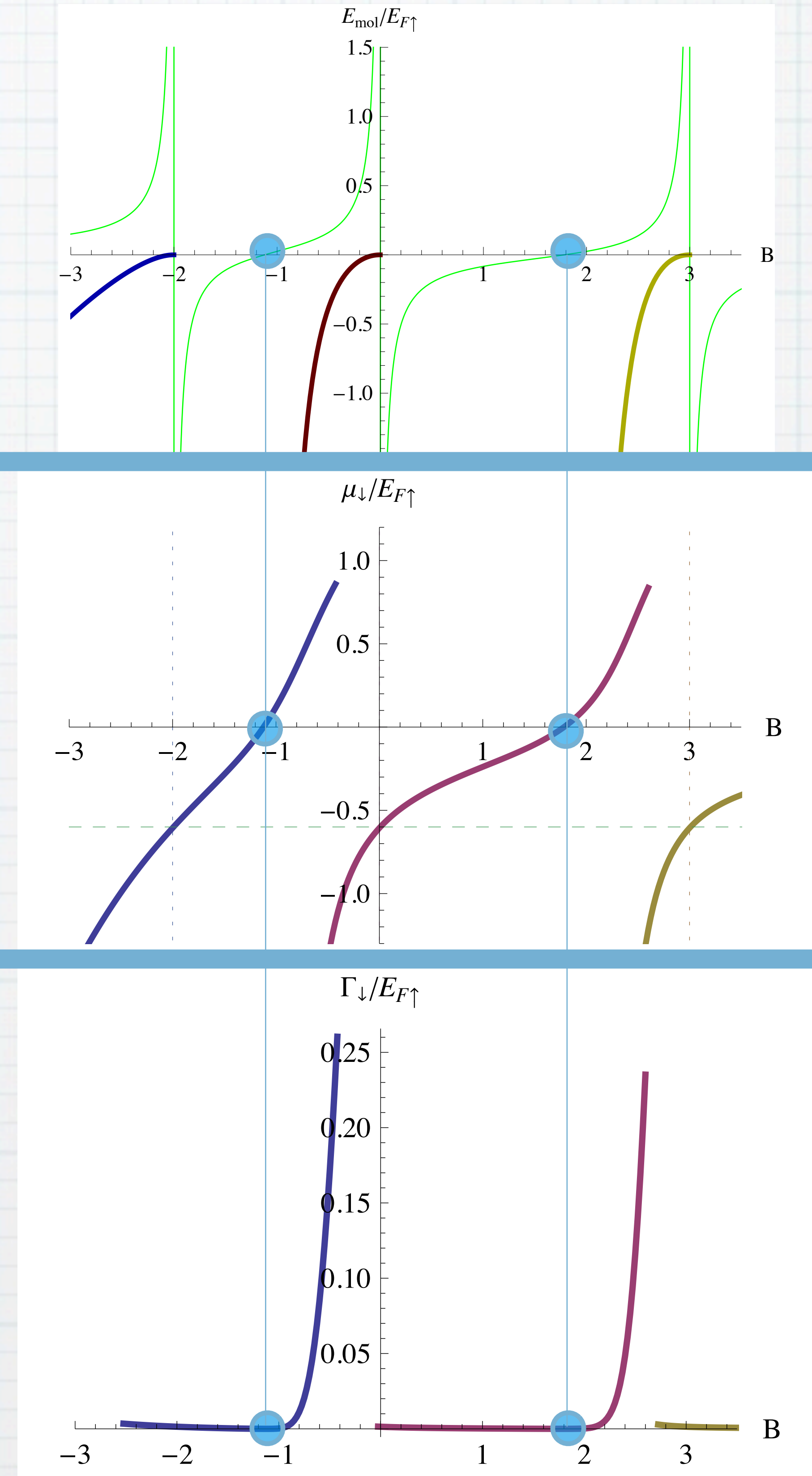
weak coupling: $$\frac{\mu_\downarrow}{E_{F\uparrow}} = \frac{4}{3\pi}(k_F a) + \frac{2}{\pi^2}(k_F a)^2 + \ldots$$

a toy model with 3 FR

2-body bound states:

Polaronic states:

Decay rates:



- a=0

weak coupling:

$E \propto a$

$\Gamma \propto \Theta(a)$

Conclusions

- At small momenta, the process coupling molecules and polarons requires at least 3-bodies
- Strongly suppressed P-M decay due to a combination of small final density of states and Fermi statistics
- Expected lifetimes ~ 10-100ms
- Complete characterization of the repulsive branch

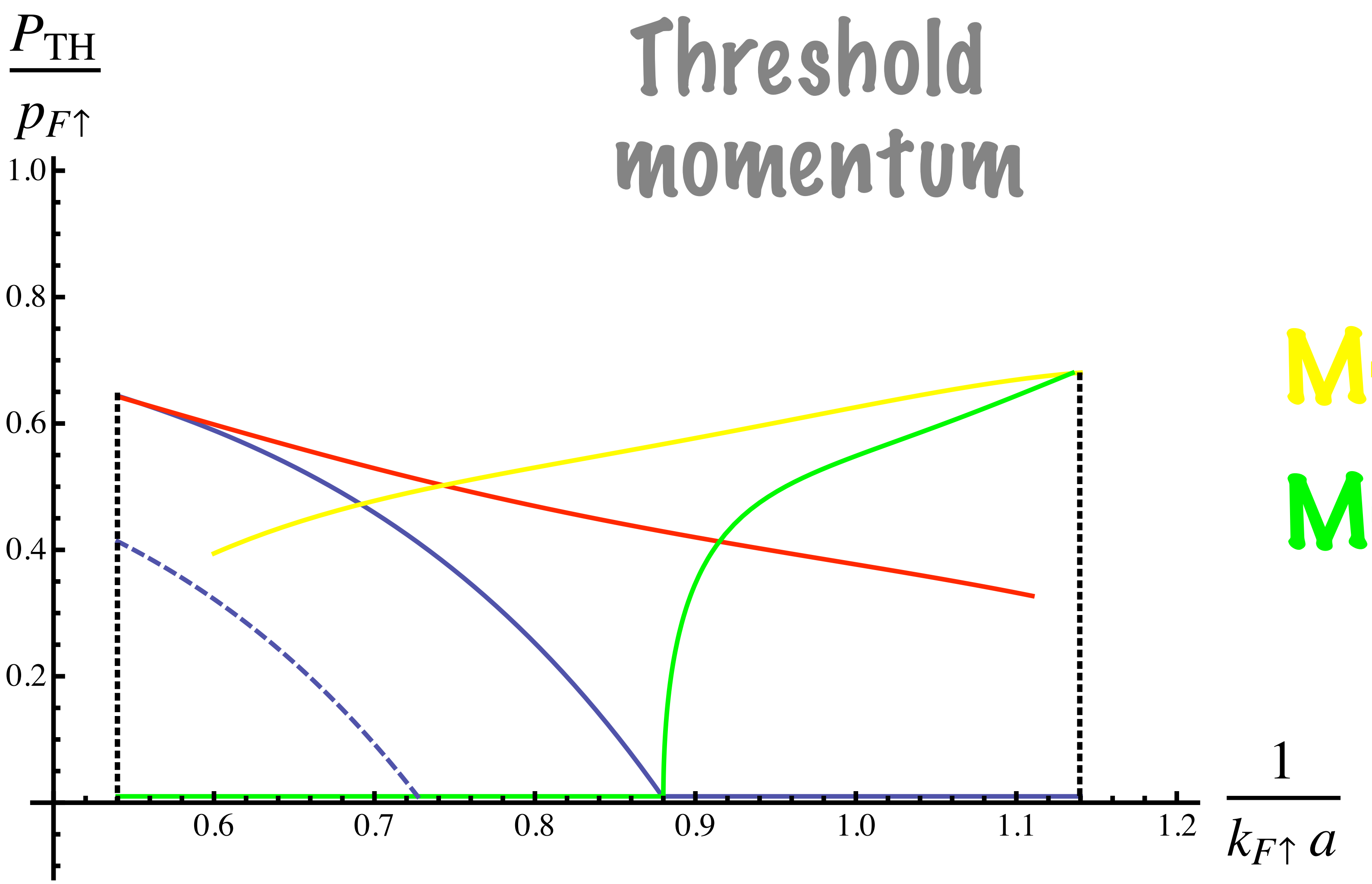
G. Bruun and P. Massignan, Phys. Rev. Lett. 105, 020403 (2010)
P. Massignan and G. Bruun, coming soon

Decay of p≠0 QP

Threshold momentum

P⇢M+h

P⇢M+2h+p

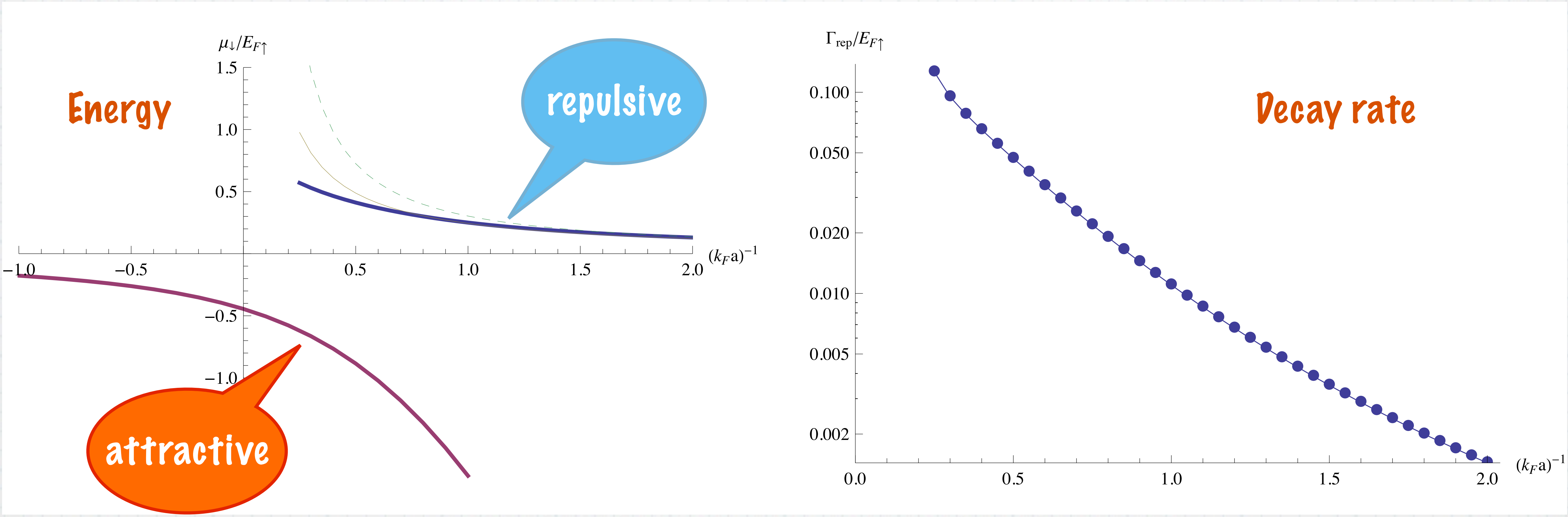


M⇢P+p

M⇢P+2p+h

(preliminary)

Repulsive polaron



A ^{40}K impurity in a Fermi sea of ^{6}Li

atom-molecule coupling:

$$\bullet = \frac{1}{g(\mathbf{p}, z)} = \int \frac{d^3q}{(2\pi)^3} \phi_q \frac{1 - f(\xi_{\mathbf{p}-\mathbf{q}\uparrow})}{z - \xi_{\mathbf{p}-\mathbf{q}\uparrow} - \xi_{q\downarrow}} \sim -\sqrt{\frac{m_r^2 a}{2\pi}}$$

Vacuum:

$$D_0(\mathbf{p}, z) = \int d^3\check{q}\,\phi_q^2 \frac{1 - f(\xi_{\mathbf{p}-\mathbf{q}\uparrow})}{z - \xi_{\mathbf{p}-\mathbf{q}\uparrow} - \xi_{q\downarrow}} + \frac{T_2(\mathbf{p}, z)}{g(\mathbf{p}, z)^2}$$

Notice

The many-body physics discussed here can be

defined

calculated

and measured!