*KITP “Beyond Standard Optical Lattices ---UC Santa Barbara --- 9 Dec 2010*

# Informal discussion: perspectives of cold atom physics

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[View of Pittsburgh---source: PittsburghSkyline.com]

# Acknowledgement


***Based on work done by Group members:***

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# *Future of cold atoms: Three directions (I find interesting)*

1. **Physics of higher orbital bands of optical lattice in various configurations.**
   - *Our "p-orbital" work:* PRA 2006; PRL 2006; PRL 2008a; PRL 2008b; PRL 2010; PRA 2010; arXiv:1005.4027; arXiv:1011.4301; Nature Phys (news & view, to appear)

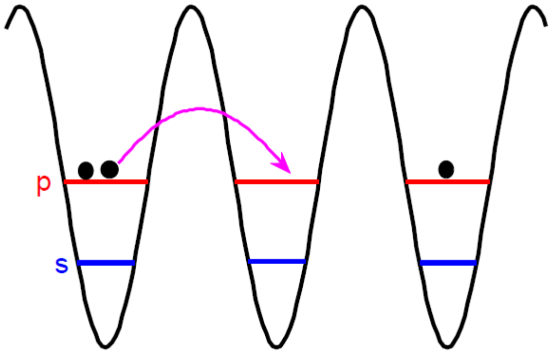
2. **Charge neutral bosons with momentum dependent interaction: dipolar interaction, cold polar molecues, Rydberg atoms, anisotropy as new feature**
   - *Our work:* arXiv:1005.4027

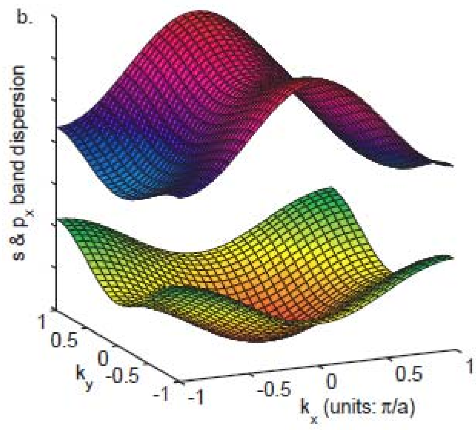
3. **Strongly interacting Fermi gases with unconventional conditions: a) spin imbalance; b) spinless p-wave**
   - Since the 1995 BEC realization, superfluidity has been a central subject of cold atom studies.
   - *Our work on (a):* PRL 2003a (**citations 178+**); PRL 2003b; PRA 2004; PRL 2005; Ann of Phys 2008; PRA 2008; PRL 2009; arXiv:1011.4967 *On (b):* WVL, PRA 2005.

Each topic: long list of studies by others – see KITP online talks quoted in next slides

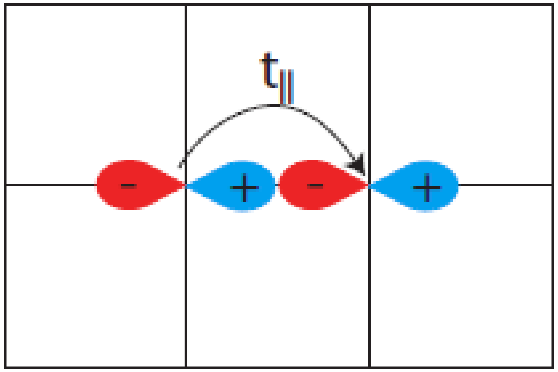
# Physics of higher orbital bands of optical lattice

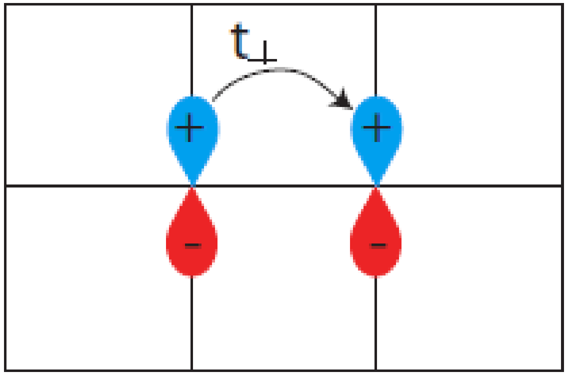
- Interested in some basics and perspectives: see, e.g., [M. Lewenstein and WVL, Nature Physics "News and View" ( to appear)]




- Reason: tunneling anisotropic and odd in parity.



# More on higher orbitals and new quantum phases

*as seen from BOPTILATT Program Online Talks*

http://online.kitp.ucsb.edu/online/boptilatt10/

- 9/14, Andreas Hemmerich, “Orbital Optical Lattices”, Program seminar. http://online.kitp.ucsb.edu/online/boptilatt10/hemmerich/

- 10/12, Kai Sun, KITP “ultracold” conference, Tuesday talk: “Topological State of Matter from Orbital Degrees of Freedom”

- 10/14, Klaus Sengstock, KITP “ultracold” conference, Thursday talk: “Ultracold Quantum Gases in Triangular and Hexagonal Optical Lattices”

- 10/29, Congjun Wu, “Orbital Phases of Cold Atoms: Unconventional BEC, Ferromagnetism, and Fulde-Ferrell-Larkin-Ovchinnikov Pairing States”, Program seminar.

## Beyond short range contact: Momentum dependent interaction

- 10/11, KITP "ultracold" conference, supersession:
  - Peter Zoller, From Optical Lattices to Self-assembled Lattice Structures with Cold Atoms and Molecules
  - Sankar Das Sarma, Interesting Quantum Phases at the Frontiers of Cold Atom Physics: Perspectives of a Condensed Matter Theorist [had quantum phases on dipolar gases]

- 10/15, KITP "Ultracold" conference "Dipolar" and "hot topic" sessions
  - J Ye, "Polar Molecules in the Quantum Regime"
  - Guido Pupillo. "Solids and Supersolids in Dipolar Quantum Gases"
  - Tilman Pfau, "Coherent Control of Dense Rydberg Gases"
  - Misha Lukin, "Quantum Dynamics of Strongly Interacting Atoms and Photons in Dipolar Systems"
  - Hans Peter Büchler, "Quantum Critical Behavior in Driven and Strongly Interacting Rydberg Gases"
- 10/22, Gora Shlyapnikov (discussion leader), Program "Informal Discussion: Ultracold Dipolar Gases".

# Strongly interacting Fermi gas: spin imbalance

- 10/11, KITP “Ultracold” conference, “Fermi Gases” session
  - Randy Hulet, “Spin-Imbalance in a One-Dimensional Fermi Gas”
  - Martin Zwierlein, “Universal Spin Transport in Strongly Interacting Fermi Gases”
  - Christophe Salomon, “Thermodynamics of a Tunable Fermi Gas”
  - Sandro Stringari, “Spin Fluctuations and Magnetic Polarizability of a Nearly Ferromagnetic Fermi Gas”
  - 11/12, Nandini Trivedi, Simulations and Emulations of Fermions in Optical Lattices

- 10/12, ibid, “Optical lattices” session: Nandini Trivedi, “Simulations and Emulations of Fermions in Optical Lattices”

- 10/14, ibid, “low dimensional” session: Erhai Zhao, “Ultracold Fermi Gases in One and Quasi-one Dimension”

- 12/02, Program Informal Discussion: Adilet Imambekov, Pietro Massignan, Michael Forbes, Roman Lutchyn: “Exotic Fermi Superfluids with Ultracold Atoms”

# Spin imbalance: Pairing with mismatched Fermi surfaces

***---proposed possibilities beyond BCS***

FFLO

Larkin and Ovchinnikov; independently Fulde and Ferrell (1964)

Sarma / Breached Pairing (Sarma/BP)

Sarma phase [J. Phys. Chem. Solids (1963)]

WVL-Wilczek, PRL 2003
M. Forbes, E. Gobankova, WVL, F. Wilczek PRL 2005

⋮

# Interesting example of imbalanced Fermi gases

**(quantum) Superfluidity meets stripes (of soft condensed matter):**

**U(1)xU(1)/$Z_2$ Kosterlitz-Thouless Transition of the Larkin-Ovchinnikov phase in an anisotropic 2D Fermi gas**

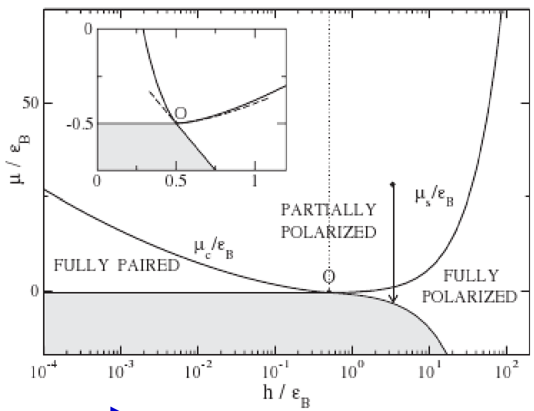
*[Chungwei Lin, Xiaopeng Li, WVL, arXiv:1011.4967]*

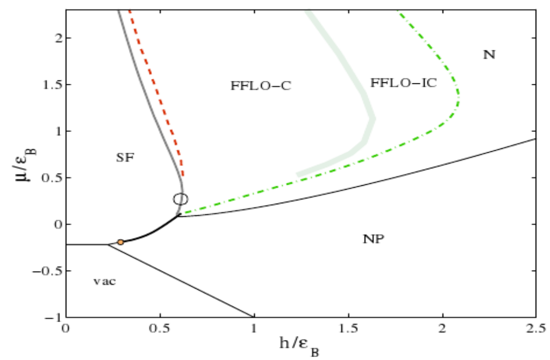
# Review: Searching for FFLO phase in cold atoms

## ***dimensional dependence***

Resonance regime

Chemical potential



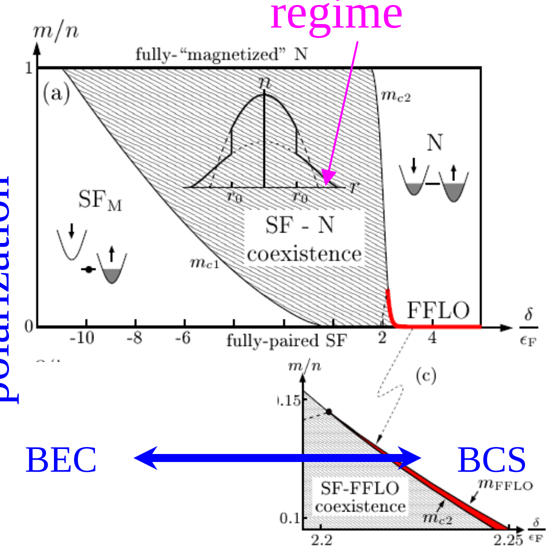




Zeeman splitting - "magnetic" h

Pure 1D
Orso, PRL (2006)

Quasi 1D (3D)
Parish et al, PRL (2007)

polarization




BEC BCS

3D (BCS side)

Sheehy & Radzihovsky, PRL (2006)

**Remarks:**

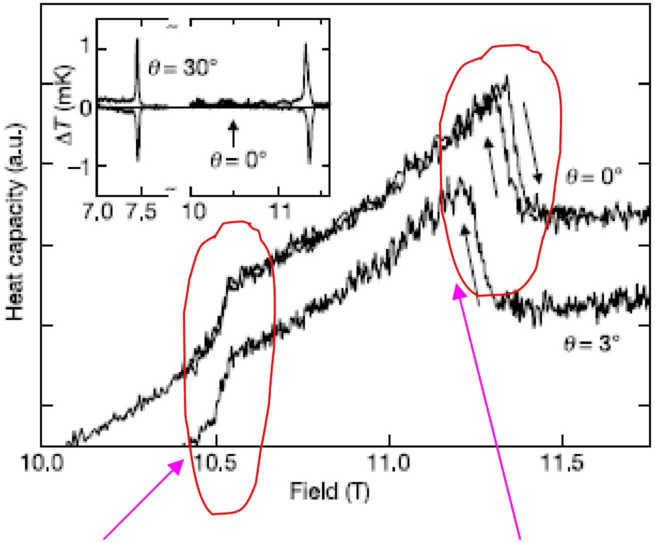
- From **3D mean field theory**, FFLO only occupies a tiny region
  - 3D FFLO can be complex crystals (FCC, )[Bowers and Rajagopal, PRD 2002]
- 1D has no true long range order.
- Quasi-1D appears to be the most promising candidate [Parish, Baur, Mueller, Huse, PRL 2007]

Note: FFLO=Fulde-Ferrell-Larkin-Ovchinnikov,1964

# Some experimental evidence in heavy fermion (inconclusive yet!)

❖ Heavy fermion $CeCoIn_5$:
quasi-2D with in-plane or tilted magnetic field

Radovan et al, Nature (2003)



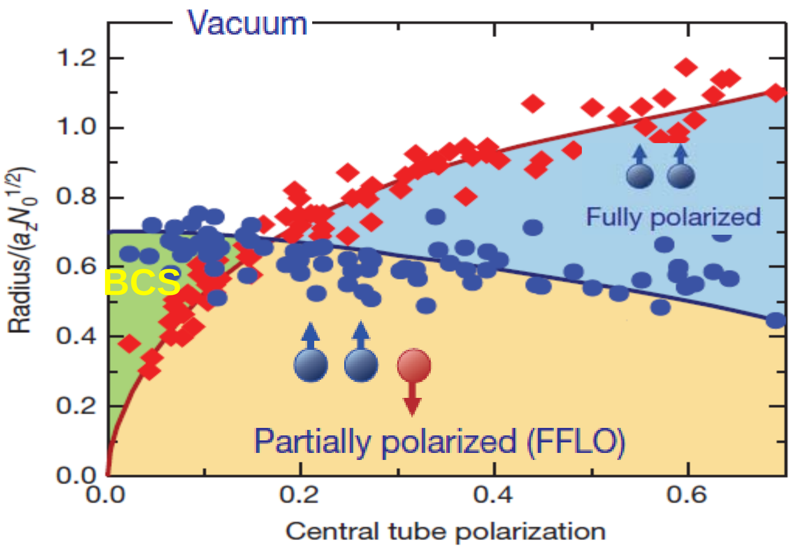

Uniform d-wave superconductor to FFLO transition. Continuous?

FFLO to Normal. 1st order?

See also: Bianchi et al. (2003); and review by Matsuda-Shimahara (2007)

❖ $^6Li$ gas in 1D optical trap:

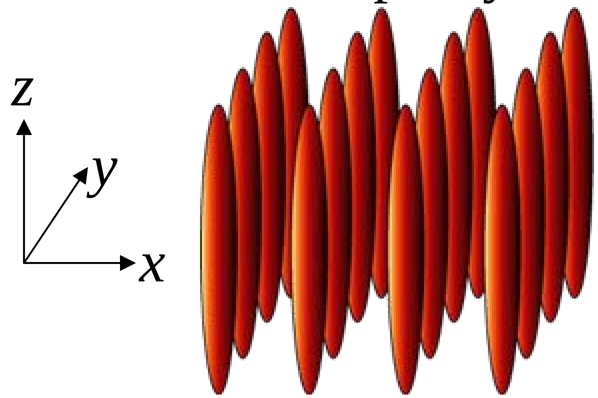
Rice experiment: Liao et al, Nature (2010)



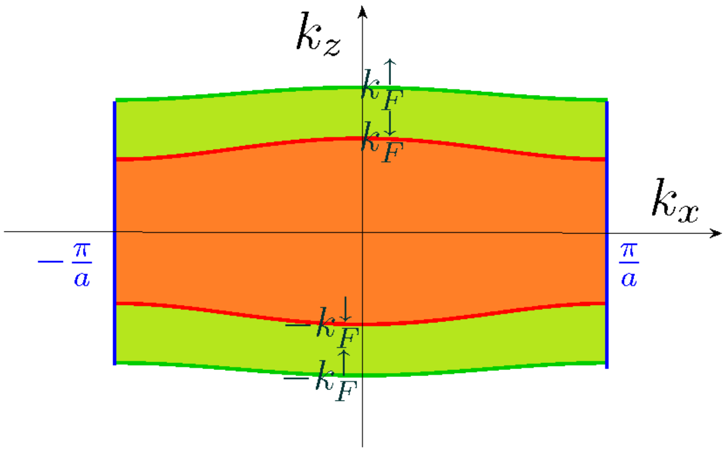

- Experiments confirm Bethe ansatz 1D phase diagram
- See R. Hulet opening talk.

# What phase do we expect in the Rice experiment?

Anisotropic system

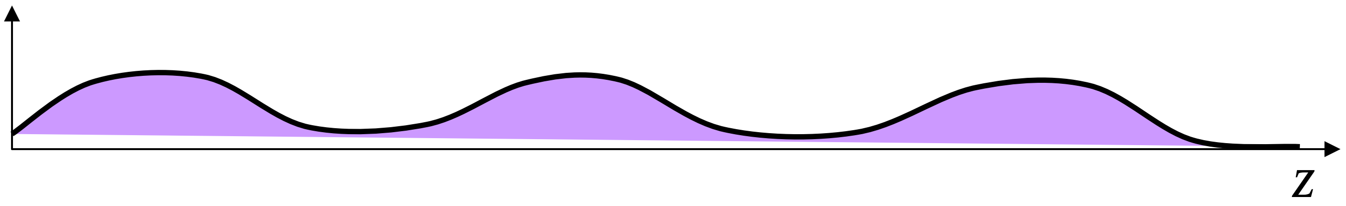



Sketch of Fermi surface




**Larkin-Ovchinnikov (LO) phase:** Pairing order parameter oscillates in space

Order parameter:




$$\Delta(\mathbf{r}) = \Delta_0 \cos(Qz)\,, \quad \mathbf{Q} \parallel \hat{z}$$

$$\mathbf{Q} = (0,0,Q), \quad \mathbf{r} = (x,y,z)\,, \quad Q \sim \delta k_F = k_{zF\uparrow} - k_{zF\downarrow}$$

LO = Striped Superfluid!

# What unique properties of LO ?

## *compared with conventional uniform superfluid*

1. **Crystalline order breaks more symmetries (translational in addition to U(1) of phase transformation)**

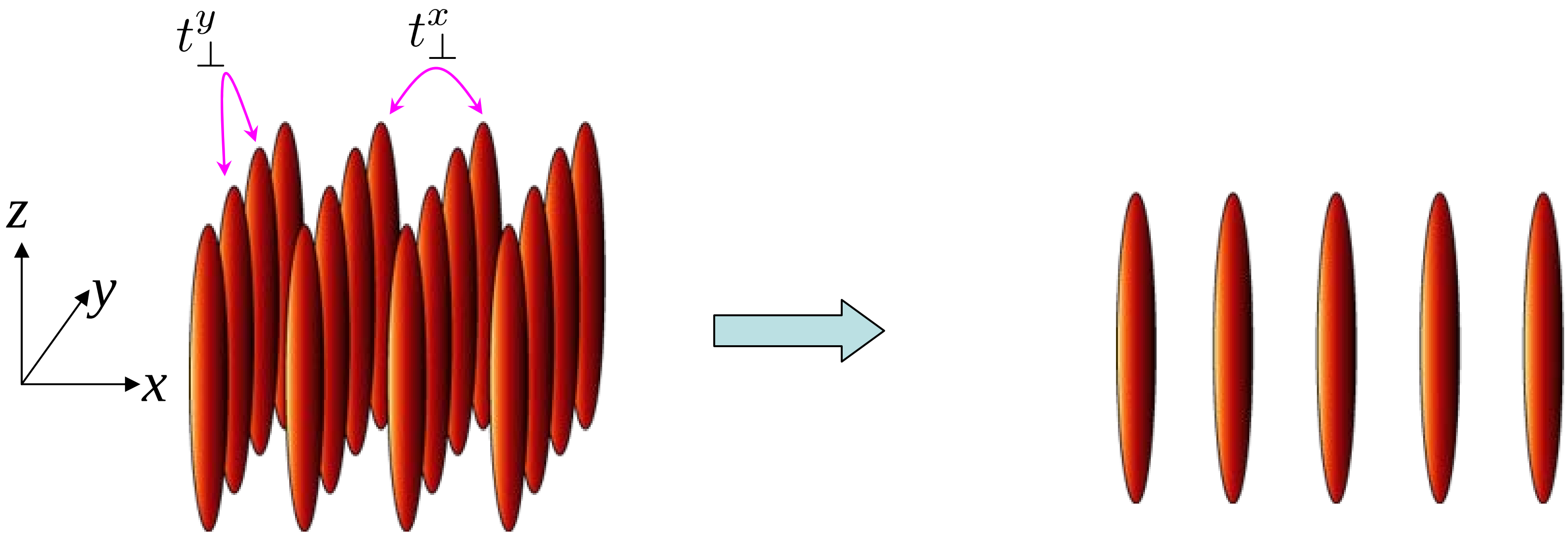
2. **More Goldstone modes?**
   - Superfluid sound/phase mode
   - fluctuation of the nodal lines of the order parameter (stripe vibration).

3. **Different kinds of topological defects**
   - Vortices, edge dislocations of the super-crystal, fractional defects.

4. **Significance of understanding the defects**
   - Key to the topological nature of Kosterlitz-Thouless transition in 2D

5. **Focus of this talk:  2D KT transition of LO phase.**

# Anisotropic 2D system




Assume a deeper lattice potential in y-direction

$$\frac{(\hbar k_F^z)^2}{2m} \geq t_\perp^x \gg t_\perp^y, \qquad \text{i.e.} \qquad t_\perp^y \to 0$$

# Low energy excitations above mean field ground state

❖ LO state, the order parameter is

$$\Delta(x,z) \propto \langle c_\downarrow(\vec{r}) c_\uparrow(\vec{r})\rangle \propto \Delta_0 f(z) \qquad f(z+L/2) = -f(z)\,, \quad f(z+L) = f(z)$$

**Broken symmetries:**

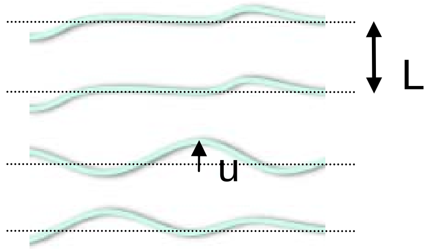
- Phase U(1) completely broken.
- Translational symmetry: Continuous group down to lattice translation (subgroup): $T_z^{\text{Cont}}/T_z^{\text{Lat}} \sim U(1)$

❖ Two Goldstone modes: "elastic" scalar fields

$$\Delta(x,z) = \Delta_0 f(z) \quad \Rightarrow \quad \Delta(x,z) = \Delta_0 e^{i\theta(x,z)} f(z+u(x,z))$$

$u$ : displacement of stripe

$\theta$ : phase field




Some features:

1. $0 \le \theta < 2\pi, \quad 0 \le u < L$
2. Quasi-1D → no rotational symmetry; x and z are not equivalent.

**Unbroken $Z_2$ symmetry**

$$\theta \to \theta + \pi$$
$$u \to u + L/2$$

# Effective theory

Effective theory: two **anisotropic XY models**

$$F = \int dxdz \left[ \frac{A}{2}(Q\partial_x u)^2 + \frac{B}{2}(Q\partial_z u)^2 + \frac{C}{2}(\partial_x \theta)^2 + \frac{D}{2}(\partial_z \theta)^2 \right] \qquad Q = \frac{2\pi}{L}$$

Related to the model theory of:

• stripe-ordered superconductors (e.g, Nd doped LSCO cuprate). [E. Berg. E. Fradkin, S Kivelson. Nat Phys. 2009]

• Spinor BEC (with Zeeman effect, easy axis) [F. Zhou, Phys. Rev. Lett. (2001); S. Mukerjee, C. Xu, and J. Moore, Phys. Rev. Lett. (2006); A. Lamacraft, KITP Program talk (online) on Sep 23, 2010 and references therein.]

**Some features:**

——— related to 2D nature ———

1. low temperature
   quasi-long range order, no true long-range order!

$$\langle O(\mathbf{r}) O(0) \rangle \sim r^{-\eta}, \qquad \mathbf{r} \equiv (z, x)$$

2. proliferation of topological defects leads to Kosterlitz-Thouless transition

3. two fields, fractional defects and possibly two-stage transitions in temperature

# Topological Defects - General

$$\Delta(x,z) = \Delta_0 e^{i\theta} f(z+u)$$

$$f(z+L) = f(z)$$

**Boundary condition:**

(circulation around a defect)

$$\oint \vec{\nabla}\theta \cdot d\vec{l} = 2\pi n_v$$

polar coordinate:

$$x, z \to \rho, \phi$$

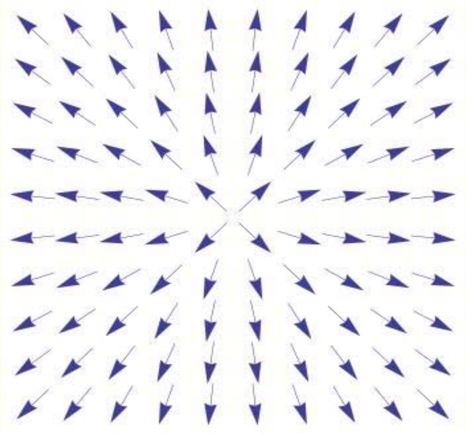
$$\oint \vec{\nabla} u \cdot d\vec{l} = L n_d$$

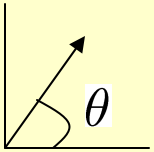
$(n_v, n_d)$--- topological charge or strength of the defect

**Field configuration:**

$$\theta(\rho,\phi) = n_v \phi$$

$$u(\rho,\phi) = n_d L \frac{\phi}{2\pi}$$

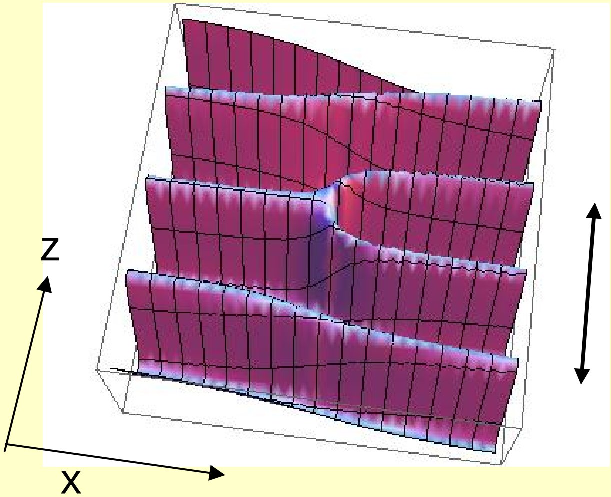





Vortex $(n_v, n_d) = (1,0)$

Arrow = phase angle

$f\left(z + u(x,z)\right)$




dislocation $(n_v, n_d) = (0,1)$

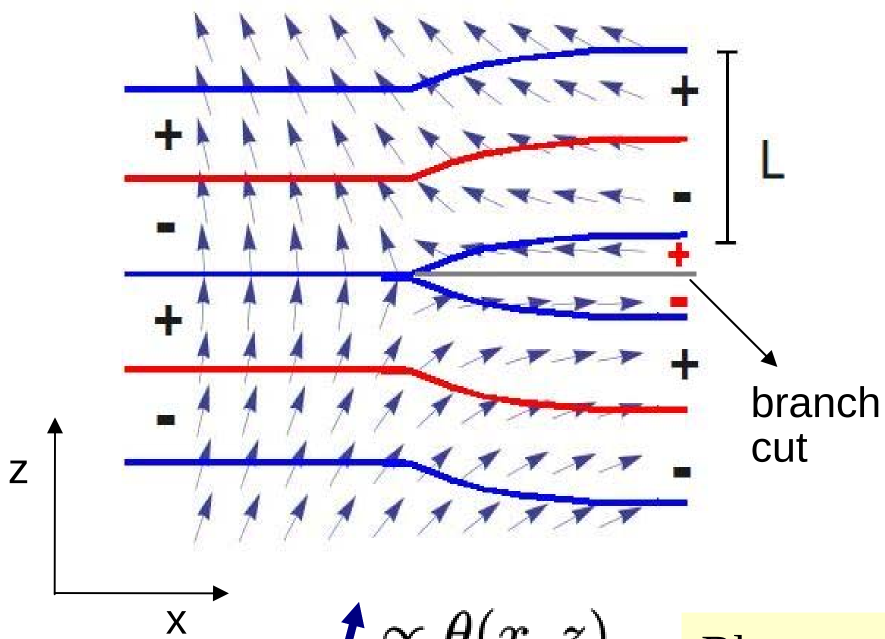
# Topological defect - Fractional

$$\Delta(x,z) = \Delta_0 e^{i\theta} f(z+u)$$

*half-vortex half-dislocation

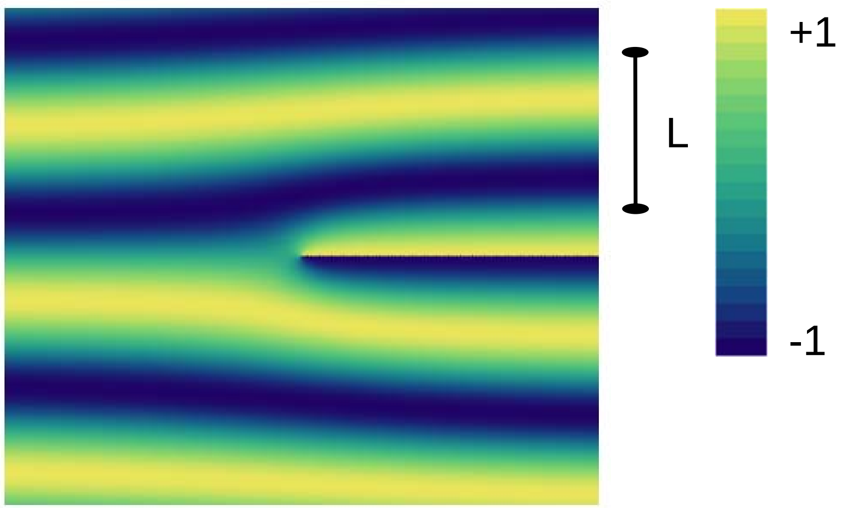
$$\oint \vec{\nabla} u \cdot d\vec{l} = L n_d \quad \oint \vec{\nabla}\theta \cdot d\vec{l} = 2\pi n_v \quad (n_d, n_v) = (\pm\frac{1}{2}, \pm\frac{1}{2})$$

destroy both SF and crystalline order
restore both translational and U(1) (phase) symmetry




$\uparrow \propto \theta(x,z)$

plot of $f(z+u)$




Blue or red solid lines= zeros of gap function

# Three competing algebraic orders (low T)

**Correlation functions from our effective field theory:** (low T)

$$K_u(r) = \langle e^{iQu(r)} e^{-iQu(0)} \rangle \sim r^{-\eta_u} \qquad \eta_u = 2\pi T/\sqrt{AB}$$

$$K_\theta(r) = \langle e^{i\theta(r)} e^{-i\theta(0)} \rangle \sim r^{-\eta_\theta} \qquad \eta_\theta = 2\pi T/\sqrt{CD}$$

$$\mathbf{r} \equiv (z, x)$$

$$r = \sqrt{\tfrac{B}{A} z^2 + \tfrac{A}{B} x^2}$$

$$r = \sqrt{\tfrac{D}{C} z^2 + \tfrac{C}{D} x^2}$$

**Order parameter**

$$\Delta(x,z) = \Delta_Q e^{iQz} + \Delta_{-Q} e^{-iQz} \quad \text{with} \quad \Delta_Q = \frac{\Delta_0}{2} e^{i(\theta + Qu)} \quad \Delta_{-Q} = \frac{\Delta_0}{2} e^{i(\theta - Qu)}$$

---

**quasi long range orders:**

1. **LO order** (charge 2, wave vector Q)

   $$\langle \Delta_Q(r) \Delta_Q(0)^* \rangle \sim K_u(r) K_\theta(r)$$

2. **Charge 4 SF** (no spatial order): $K_u(r)$ becomes exponential decay

   $$\Delta_4 \equiv \Delta_Q \Delta_{-Q} \propto e^{i2\theta} \qquad \langle \Delta_4(r) \Delta_4(0)^* \rangle \sim K_\theta(r)^4 \sim r^{-4\eta_\theta}$$

3. 2Q **Charge Density Wave (CDW)**: $K_\theta(r)$ becomes exponential decay

   $$\rho_{2Q} \equiv \Delta_Q \Delta_{-Q}^* \propto e^{i2Qu} \qquad \langle \rho_{2Q}(r) \rho_{2Q}(0)^* \rangle \sim K_u(r)^4 \sim r^{-4\eta_u}$$

For “stripe” high Tc cuprates, discussed by Berg, et al Nat. Phys. 2009

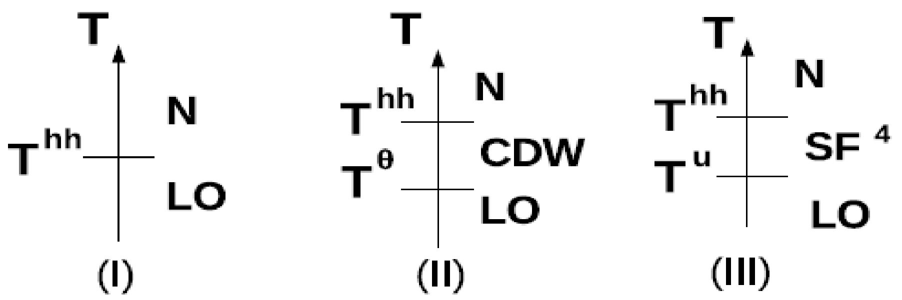
# Three topological defects. Three KT transitions

**properties of defects:**

| *Type* | *Energy cost* | *KT critical Temp* | *Restored symmetries* |
|---|---|---|---|
| dislocation | $\pi\sqrt{AB}$ | $T^u = \frac{\pi}{2}\sqrt{AB}$ | translation |
| vortex | $\pi\sqrt{CD}$ | $T^\theta = \frac{\pi}{2}\sqrt{CD}$ | U(1) (particle conservation) |
| half-half | $\pi(\sqrt{AB}+\sqrt{CD})/4$ | $T^{hh} = \left(T^u + T^\theta\right)/4$ | Both U(1) and translation |

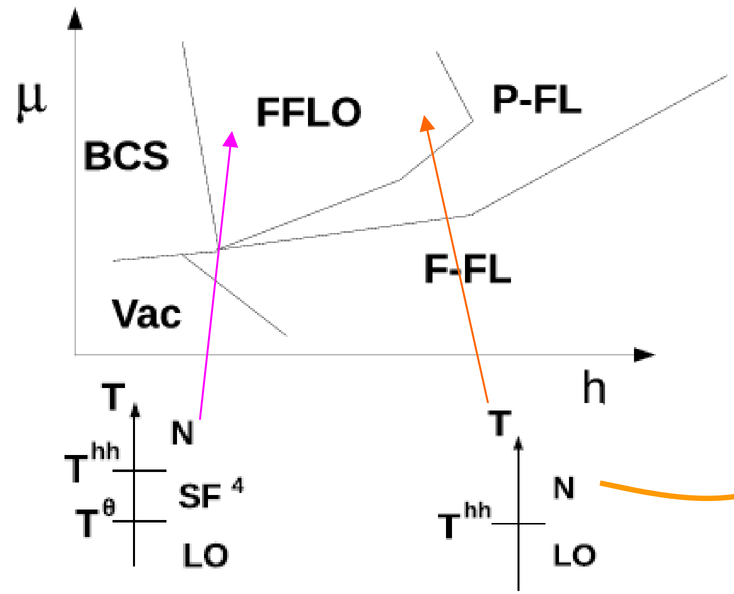
$T^{hh}$ cannot be the highest!!

**Three possible phase diagrams of melting order:**

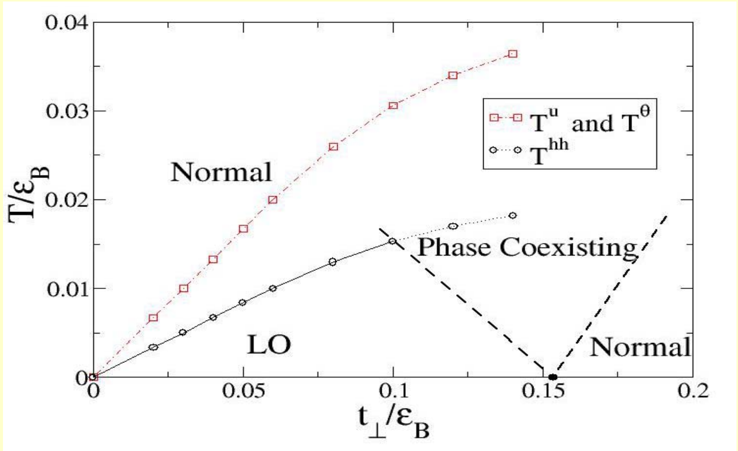


# **Summary -** *for a FFLO state in an anisotropic 2D system*

**Conjectured μ–h phase diagram**




**KT transitions due to half vortex half dislocation**



# Conclusion
# for the informal discussion on Future of Cold Atom Physics

**See the next KITP "cold atoms" program, to be held in 201??**

**Coordinators: YOU**
**Scientific advisors: to be picked by the Advisory Board**

**Thank the KITP!! and**
**Thank you – the past and present participants!!**