Austrian Science Foundation

Institut für Theoretische Physik
Universität Innsbruck

Foundations and Applications of Quantum Science

Quantum Optics with ultracold gases

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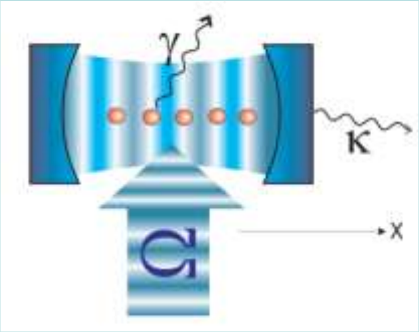
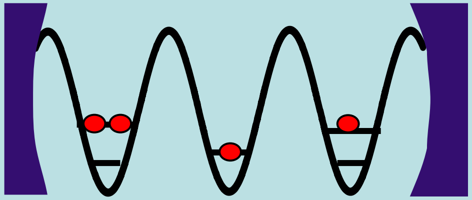
KITP-BOPTILAT
September 30, 2010

Outline

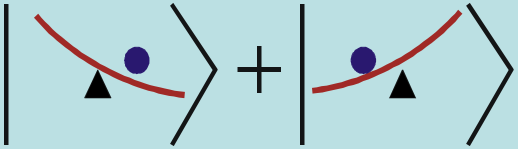
- *Light forces and Cavity QED*

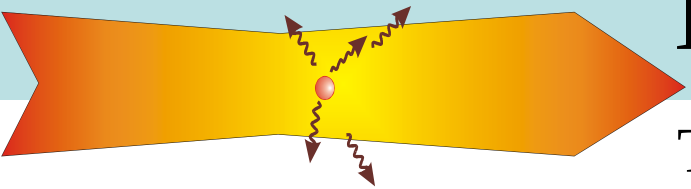


- *Atoms in a quantum optical lattice potential*



- *Quantum seesaw mechanism, selforganization + new "phases"*

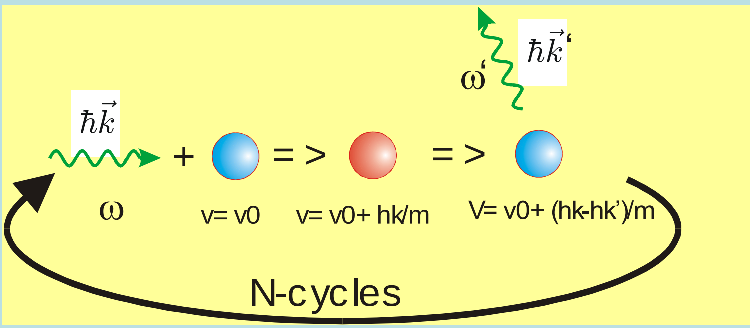




Laser light forces on atoms

Two classes of forces:

Radiation pressure
(absorption + spontaneous re-emission)



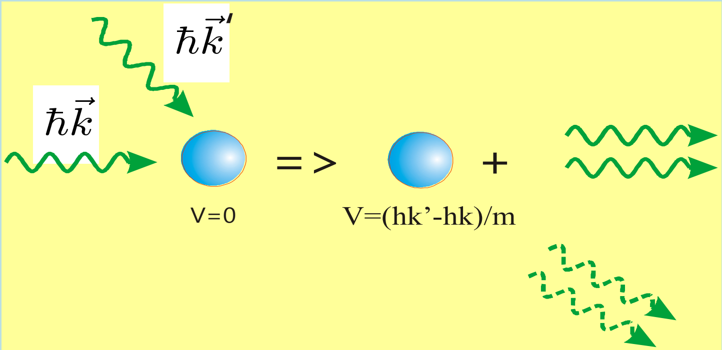
$$\vec{v} = \vec{v_0} + N\hbar\vec{k} - \sum_{i=1}^{N} \hbar\vec{k'} \approx N\hbar\vec{k}$$

emissions have random directions and cancel
N **depends on frequency and intensity**

$$\vec{F}_{rad} = \hbar\vec{k}\frac{\gamma}{2}\frac{\frac{1}{2}\Omega^2}{(\omega_l-\omega_a)^2+\Gamma^2/4+\frac{1}{2}\Omega^2}$$

- dissipative (random) force
- maximum force at resonance F=hk γ/2 (~10^5 g)
- **momentum diffusion** due to random emission

Dipole force
(absorption + stimulated emission)



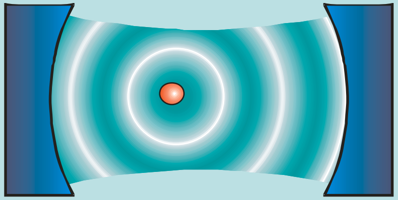
$$\vec{v} = \vec{v_0} + (\hbar\vec{k}' - \hbar\vec{k})/m = \vec{v_0} + \hbar\delta\vec{k}/m$$

Coherent transfer of momentum,
which **depends on relative phases** of fields

$$\vec{F}_{dip} = \hbar(\omega_l - \omega_a)\frac{\frac{1}{2}(\vec{\nabla}\Omega)^2}{(\omega_l-\omega_a)^2+\Gamma^2/4+\frac{1}{2}\Omega^2}$$

- conservative force with no upper limit => **optical potential** U(x) ~ I(x)/($\omega_l-\omega_a$)
- high field minima for red detuning

Cavity QED basics



Toy model of quantum electrodynamics:
Light enclosed in resonator with nonrelativistic atoms

field: small number M of quantized modes described by harmonic oscillators: $\{a_i, a_i^+\}$

$$H_f = \sum_{i=1}^{M} \hbar\omega_i a_i^\dagger a_i$$

$$[a_i, a_j^\dagger] = \delta_{ij}$$

atoms: finite number N of atoms with simple internal structure (e.g.: 2): $(\sigma_i^+, \sigma_i^-, \sigma_i^z)$

$$H_a = \sum_{i=1}^{M} \frac{1}{2}\hbar\omega_a\sigma_i^z$$

$\omega_a = (E_2 - E_1)/h$

interaction: near resonant dipole-coupling

$\omega_a \approx \omega_i$

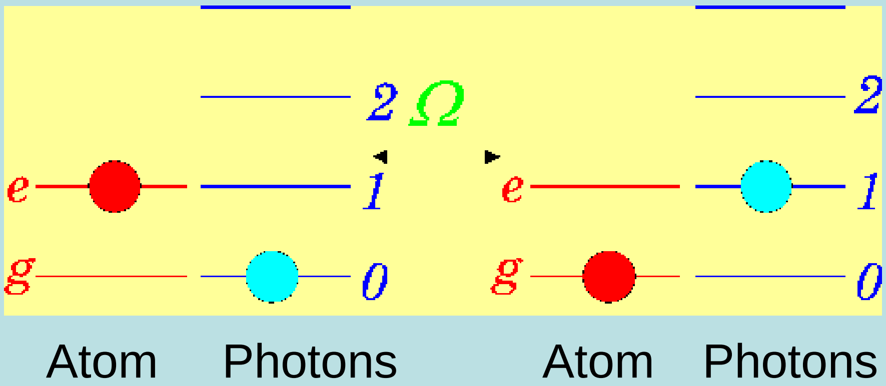
Simplest model: 2- level atom and one mode

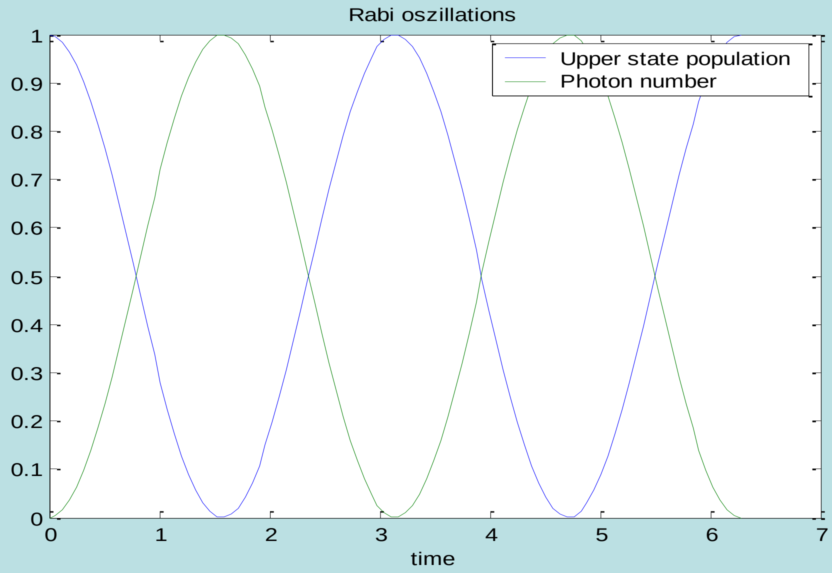
$$H = \hbar\omega_f a^\dagger a + \frac{1}{2}\hbar\omega_a\sigma^z + \hbar\mathbf{g}(a^\dagger\sigma^- + a\sigma^+)$$

Jaynes Cummings Model: 2- level atom and one mode

$$H = \hbar\omega_f a^\dagger a + \frac{1}{2}\hbar\omega_a \sigma^z + \hbar\mathbf{g}(a^\dagger\sigma^- + a\sigma^+)$$

Rabi oszillations:
periodic energy exchange
between
field and atom

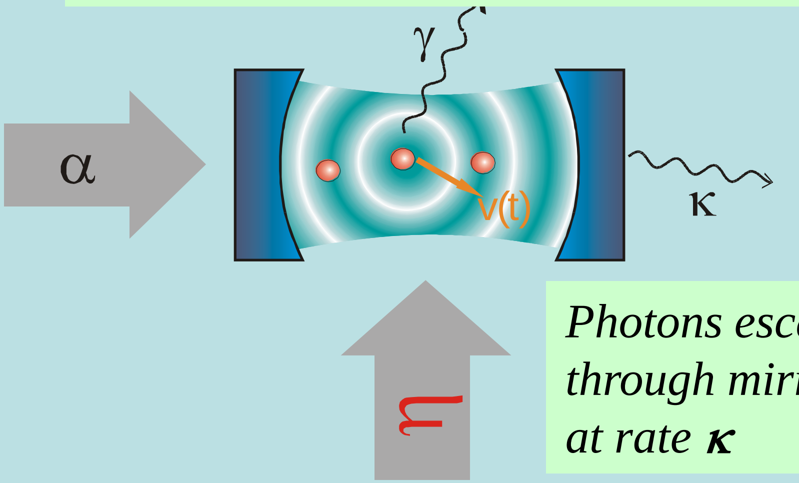




Nonlinear dynamics with
one atom + one photon
(saturation with one photon)

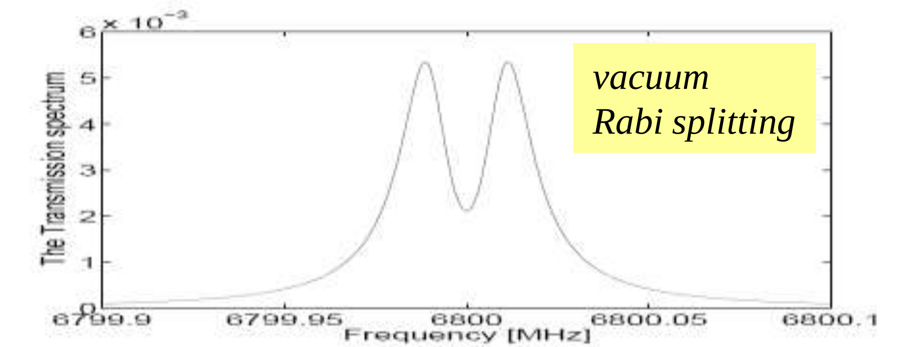
Real world: **dissipation** unavoidable and perturbs dynamics
But: input and output channels also provide information on system

Atoms decay spontaneously at rate γ



Photons escape through mirror at rate κ

Cavity QED limit achieved, if
$\omega_f \sim \omega_a >> g >> (\kappa,\gamma)$



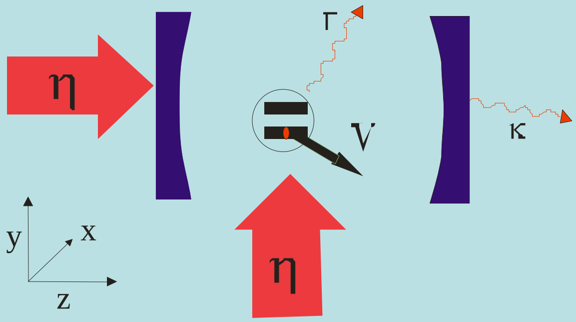
vacuum Rabi splitting

Experiment	
Micromaser	Optical cavity
ω ~ 10^10 Hz	**ω~ 10^14 Hz**
κ~10-100Hz	**κ ~ 10^6 Hz**
γ ~ Hz	**γ~10^7 Hz**
g ~10^4Hz	**g~10^8 Hz**

Consequences :
- Nonlinear atomic response to less to a single photon : n_0
- Single atom shifts cavity by more than a linewidth **N_0**

Many Gedankenexperiments of Quantum Mechanics realized:
e.g. Haroche, Walther, Kimble, Rempe,… + many more recently

Lightforces in optical resonators in dispersive regime



Atom(s) in field
of high Q resonator

$$\vec{E}(x,t) = E(t)\vec{f}(x),\ \vec{f}(x) \text{ Mode function}$$

lightforces of resonatorfield influence atomic motion

↓

interaction

↑

atoms influence resonator field dynamics

- **friction**
- **diffusion**
- **correlation and entanglement**

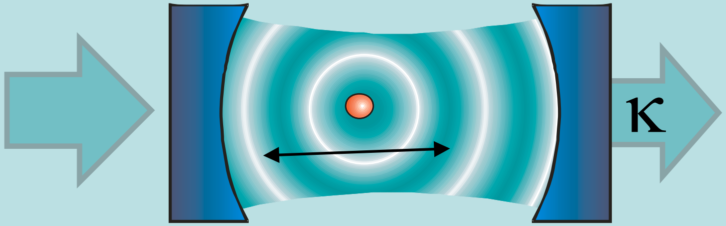
$$\dot{E} = [-\kappa - \boldsymbol{\gamma(x)} + i\Delta_c - i\boldsymbol{U(x)}]\,E - \alpha,$$
$$\dot{p} = -|E|^2 \frac{d}{dx}\boldsymbol{U(x)},$$
$$\dot{x} = p/m.$$

Detuning + loss of mode is position dependent !

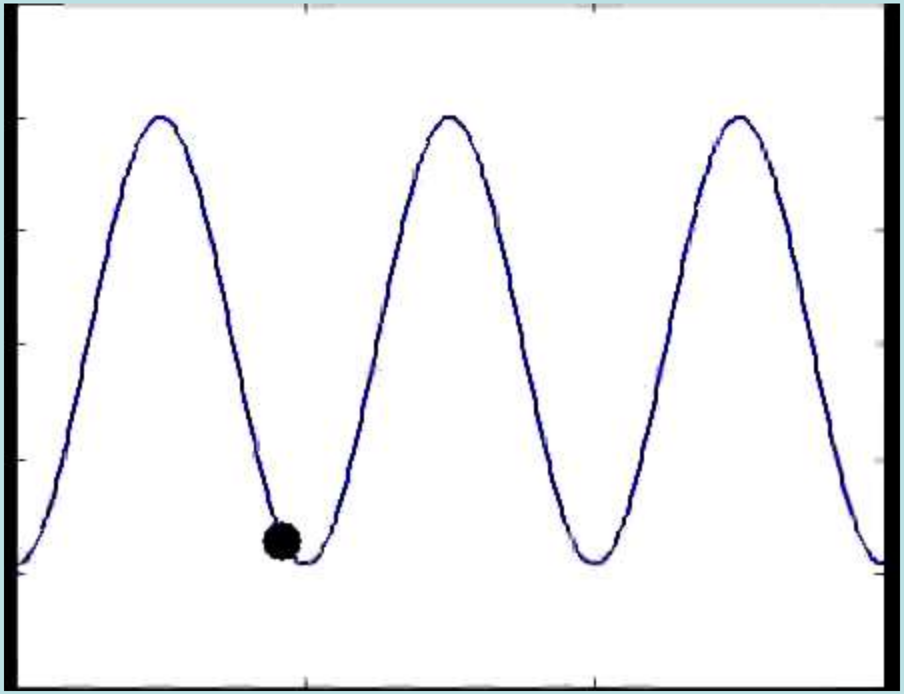
$U(x) = U_0 f(x)$ ... interaction energy = light shift/photon ~1/detuning
$\gamma(x)$ ... loss/photon ~ optical pumping rate ~ 1/detuning^2

Single 2-level atom dispersively coupled to single mode

atom moving along axis



• *red detuning: atoms drawn to field maxima*
• *field gets maximal for atom at antinode*



cavity cooling :

Horak, PRL 97, Vuletic, Chu PRL2000

$$k_B T = -\frac{\overline{D}}{\overline{F_1}} = \frac{\kappa}{2}$$

experiments + related work :

• ***field allows monitoring of position:*** ***Parkins, Kimble - Science 2000, Horak, PRL 2002***
• ***atom cavity cooling:*** ***Maunz, Rempe, Murr, Nature 2004, Nat. Phys. 2005***
Chapman 2007, Meschede 2009, Leibrandt 2009
• ***monitoring + feedback:*** *Vuletic 2004, Steck PRL 2004, Averbukh 2007*
Rempe 2009,
• ***many improvements recently ….***

Cavity cooling of polarizable particles/objects

Analytic solution for slow atoms for friction and diffusion

friction

$$\overline{\overline{F_1}} = -k^2 \frac{\eta^2 U_0^2}{4\kappa^4}$$

diffusion

$$\overline{D} = k^2 \kappa \frac{\eta^2 U_0^2}{8\kappa^4}$$

temperature

$$k_B T = -\frac{\overline{D}}{\overline{\overline{F_1}}} = \frac{\kappa}{2}$$

κ ... cavity linewidth

Experiment at MPQ Munich (Nature 2004)

Cavity cooling of a single atom

P. Maunz, T. Puppe, I. Schuster, N. Syassen, P. W. H. Pinkse & G. Rempe

Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Str. 1, D-85748 Garching, Germany

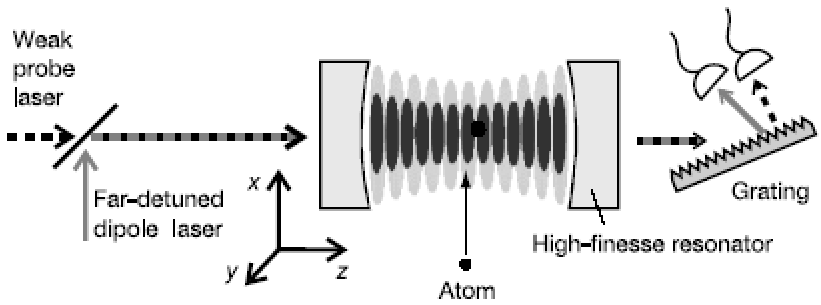


Figure 1 Experimental set-up. The high-finesse cavity ($F = 4.4 \times 10^5$) is excited by a weak near-resonant probe field and a strong far-red detuned dipole field. ^{85}Rb atoms are injected from below. Behind the cavity, the two light fields are separated by a grating. The probe light is further passed through a narrow-band interference filter before being directed onto a single-photon counting module. For this set-up, a quantum efficiency of 32% is achieved for the detection of probe light transmitted through the cavity while the dipole light is attenuated by more than 70 dB. The dipole light is also used to stabilize the cavity length with a radio frequency sideband technique. It is generated by a grating- and current-stabilized diode laser with a linewidth of 20 kHz r.m.s..

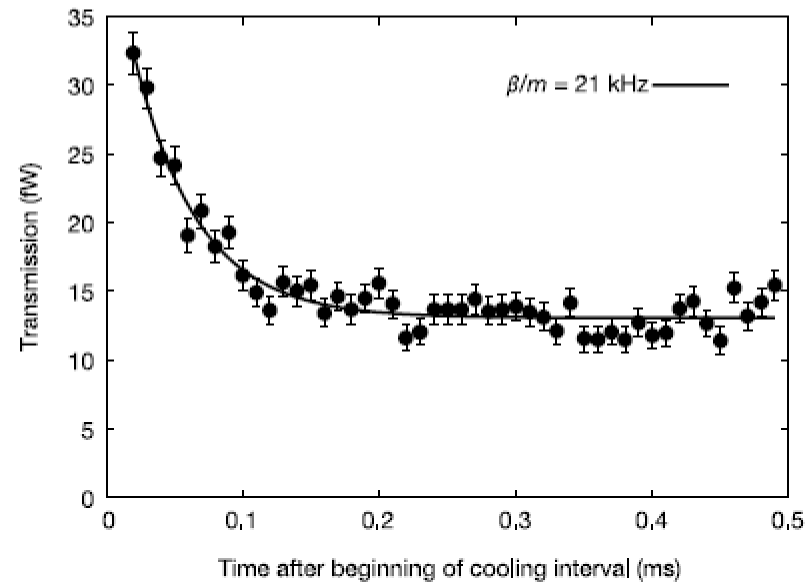
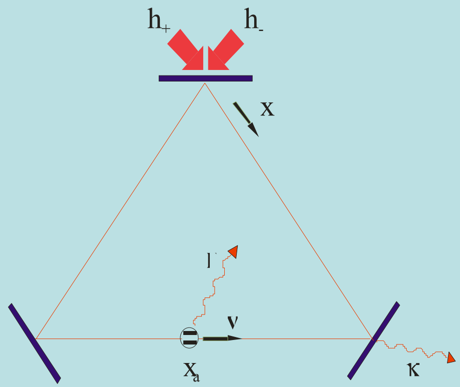
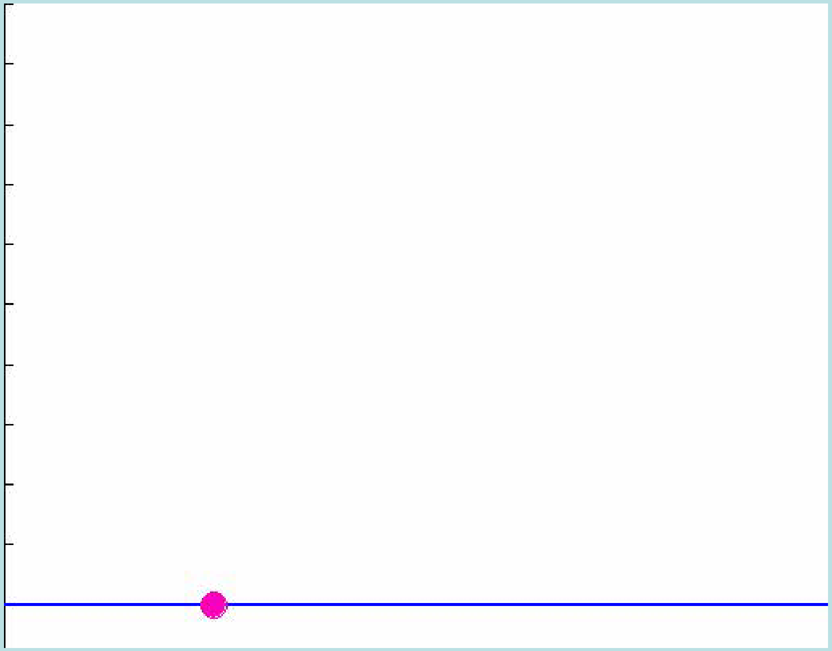


Figure 3 Cooling-induced localization. Average transmission during cooling intervals of length 0.5 ms after heating the atom for 0.1 ms. An incident power of $P_p = 2.25$ pW is chosen for a good signal to noise ratio. Without an atom, the cavity transmission on resonance is 300 fW. The atoms are cooled during the first 0.1 ms. This leads to a stronger coupling to the cavity mode and, hence, to a smaller transmission. A cooling rate of $\beta/m = 21$ kHz is estimated from an exponential fit. Radial heating occurs on a much longer timescale and is not visible here.

New data 2005: 17 s storage time

classical simulation for ring cavity: two modes + one atom or CMS of cloud





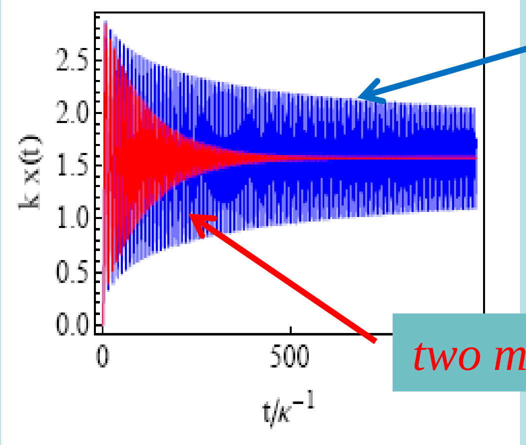
E(x,t) ~ a_c cos(kx)+a_s sinkx

$$\frac{da_c(t)}{dt} = \left[-\kappa + i\Delta_c(\hat{x})\right] a_c + i a_s U_{cs}(\hat{x}) + \eta + a_{in}^c$$

$$\frac{da_s(t)}{dt} = \left[-\kappa + i\Delta_s(\hat{x})\right] a_s + i a_c U_{cs}(\hat{x}) + a_{in}^s,$$

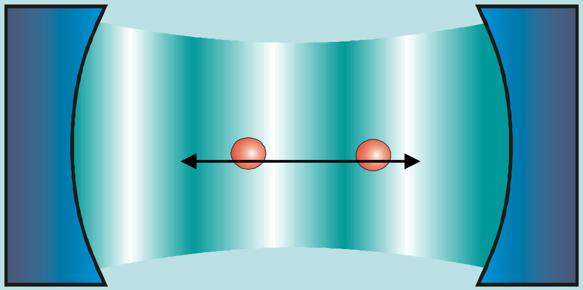
$$\frac{d\hat{x}(t)}{dt} = \frac{\hat{p}}{m},$$

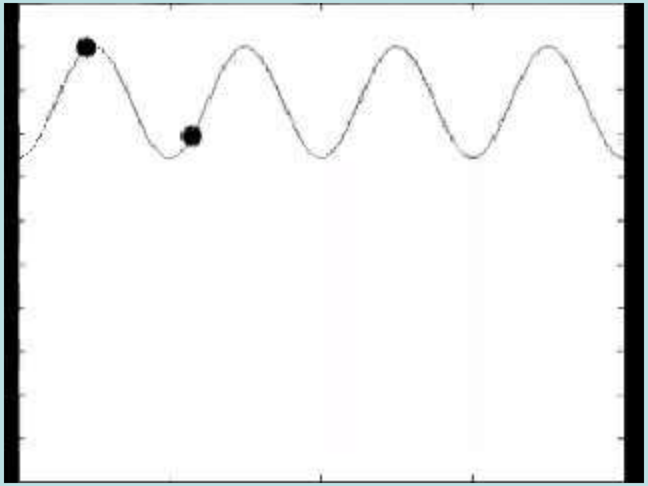
$$\frac{d\hat{p}(t)}{dt} = -\hbar \frac{dU(\hat{x})}{d\hat{x}}.$$



- ***atom drags node along its path to stay at intensity maximum = field minimum***
- ***atom phase locks field modes and changes intensity distribution over large distances***

Two 2-level atoms strongly coupled to a single mode





long (infinite) range interaction mediated by cavity field

- correlated motion and joint trapping
- collective nonlinear oscillations
- chaotic dynamics: results for up to 12 particles by Milburn/Holmes
- momentum space pairing in ring cavities

Theory: J. Asboth, PRA 2004
P. Domokos et. al. , JOSA B **20,** 2003
C.P. Meany, C.A. Holmes, … ICAP 2010 …

Experiment: Vuletic - group 2010

Many particles:
Cold atomic gas in a cavity generated optical potential

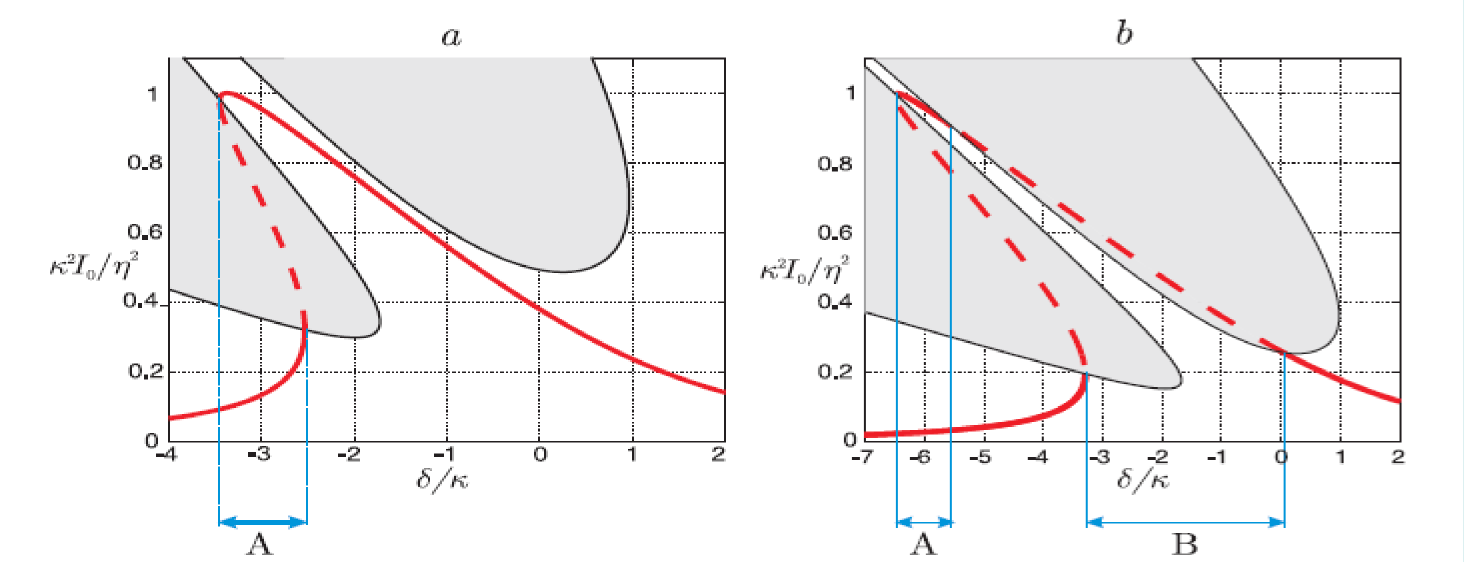
Vlasov-limit:
Continuous density approximation
for cold cloud of particles

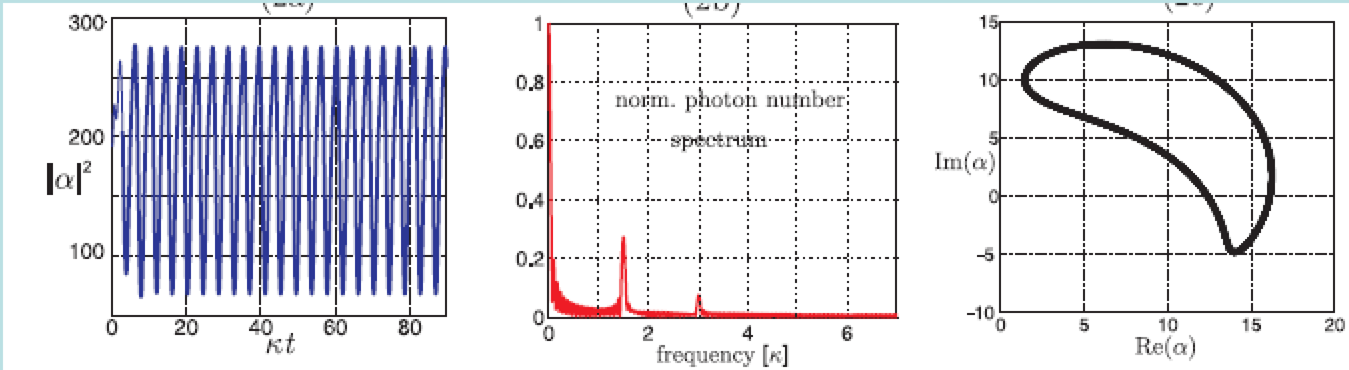
$$f(x,v,t) = \frac{m}{2N\pi\hbar}\int e^{-izmv/\hbar}\rho_{P,1}\left(x+\frac{z}{2}, x-\frac{z}{2}, t\right)dz$$

Kinetic limit-Vlasov equation

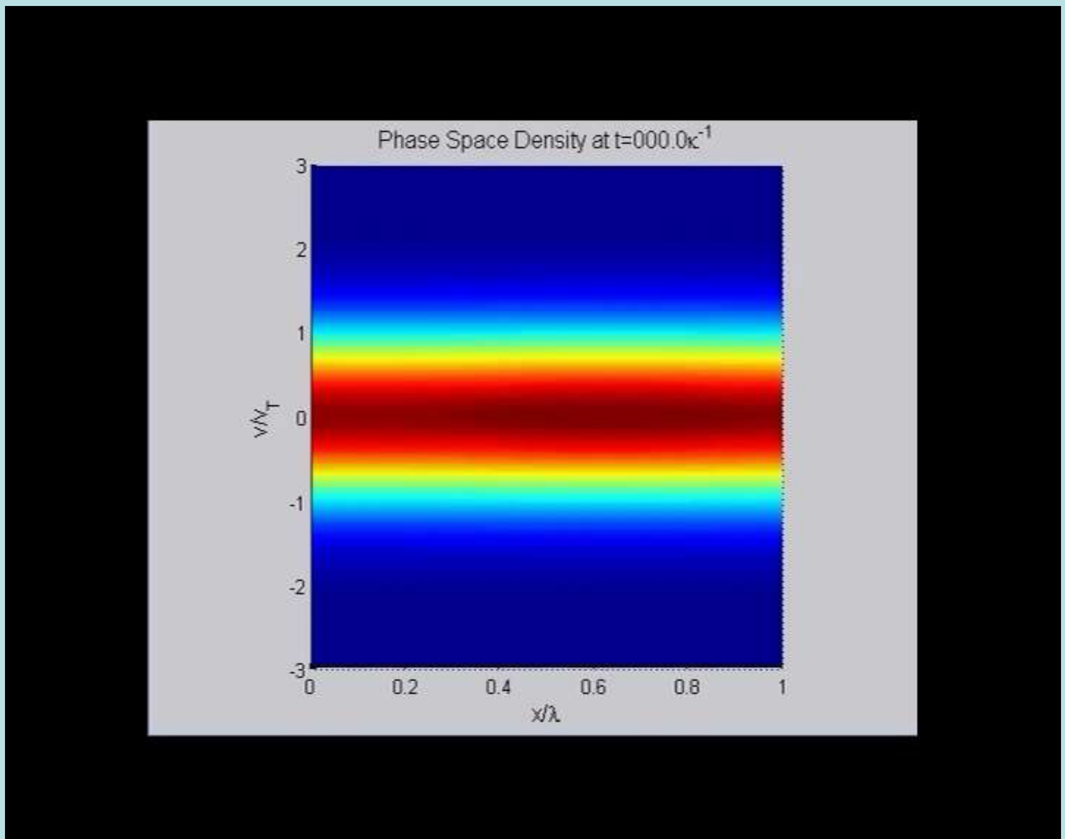
$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{U_0|\alpha|^2}{2}\sin(2kx)\Big(f(x,v+v_R) - f(x,v-v_R)\Big) = 0$$

$$\dot{\alpha} = [-\kappa + i(\Delta_c - NU_0/2)]\alpha - i\frac{NU_0}{2}\alpha\int_{-\infty}^{\infty}dv\int_0^{\lambda} f(x,v,t)\cos(2kx)dx + \eta$$





|α|²
κt
norm. photon number
spectrum
frequency [κ]
Im(α)
Re(α)

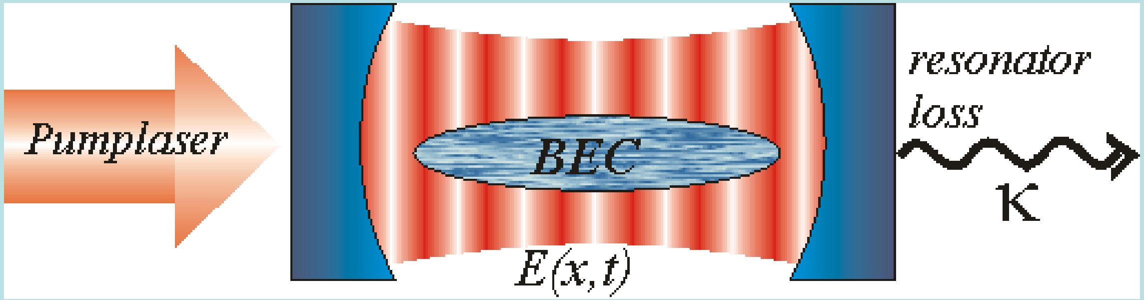


Phase Space Density at t=000.0κ⁻¹
v/v_T
x/λ

*Ultracold quantum gas in a **quantum** optical lattice potential (quantum description of many particles and field)*

* *Cavity field generates:* ***optical lattice with dynamic quantum properties***

* *Atoms /BEC: dynamic* ***refractive index depending on the quantum state***



mean field approximation for particles and field

$$\frac{d}{dt}\alpha(t)=[i\Delta_c - iN\langle U(\hat{x})\rangle - \kappa]\alpha(t)+\eta, \qquad (1a)$$

$$i\frac{d}{dt}\psi(x,t)=\left\{\frac{\hat{p}^2}{2m}+|\alpha(t)|^2U(x)+Ng_{coll}|\psi(x,t)|^2\right\}\psi(x,t).$$

coupled nonlinear and nonlocal equations with a wealth of dynamic effects

Refs:

Horak, Barnett, Zoller, Meystre, Liu, Bhattacherjee…

Experiments:

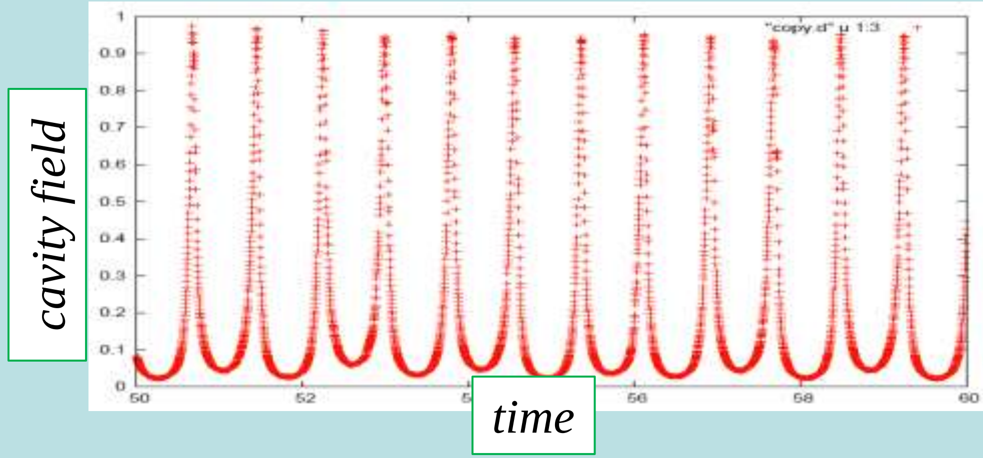
Esslinger, Reichel, Zimmermann, Hemmerich, Vuletic, Treutlein ...

simple effective theory :
two state expansion of BEC

$$\psi(x,t) = c_0(t) + c_2(t)\sqrt{2}\,\cos(2kx)$$

=> two x-X coupled oscillators
optomechanics – Hamiltonian
at T=0

operation in unstable regime
=> self-sustained oscillations at 4 ω_r

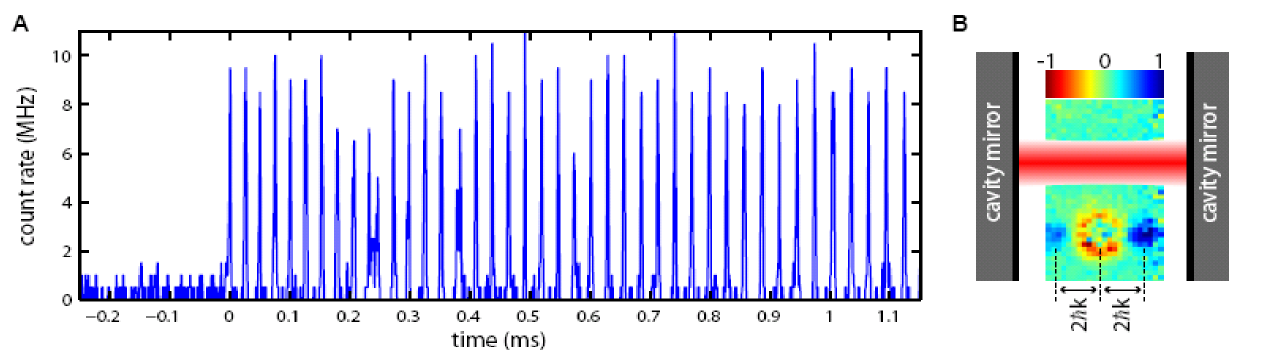


Experiment:
ETH Zürich

several
theoretical
descriptions

Dynamical Coupling between a Bose-Einstein Condensate and a Cavity Optical Lattice

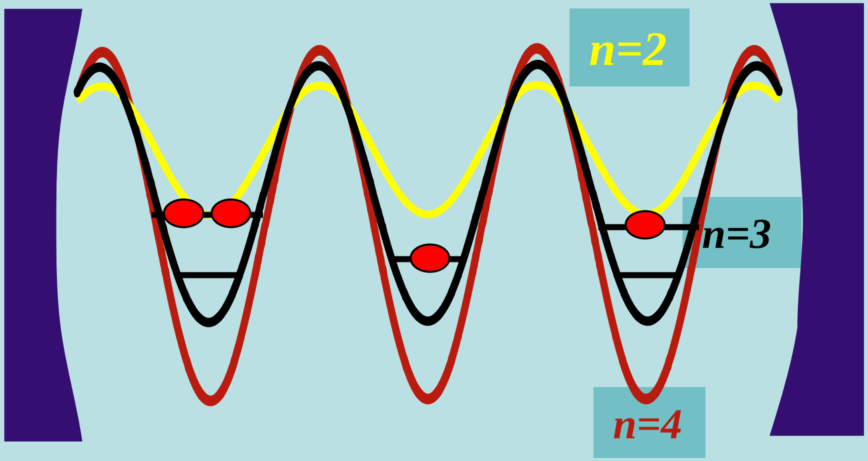
Stephan Ritter[1,2], Ferdinand Brennecke[1], Christine Guerlin[1],
Kristian Baumann[1], Tobias Donner[1,3], Tilman Esslinger[1]*
[1] *Institute for Quantum Electronics, ETH Zürich, CH-8093 Zürich, Switzerland*
[2] *Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany*
[3] *JILA, University of Colorado and National Institute of Standards and Technology, Boulder CO 80309, USA*
(Dated: November 24, 2008)



Beyond mean field

Atoms near T=0 in a ***quantum*** optical lattice potential

(quantum description of very cold particles and field)

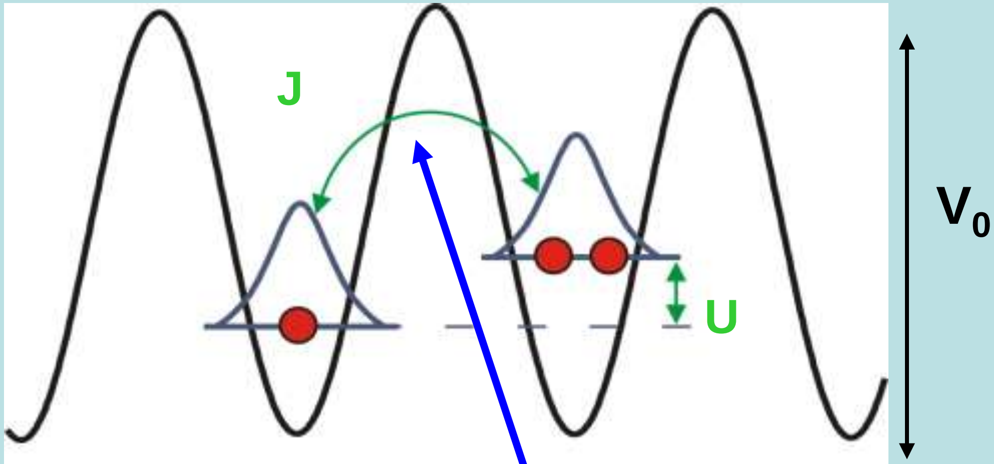


* *Cavity field generates* ***optical lattice with quantum properties***
* *Atoms act on the cavity field depending on the quantum state*

Central aim: Quantum description of atoms and field near T=0
⇔ Hubbard model in a quantized single mode field

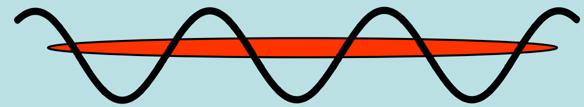
Beyond mean field

Reminder: Ultracold Atoms in far detuned optical lattices (Bose Hubbard model)



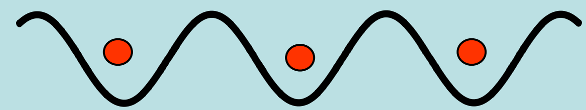
Tunneling J depends on laser power (V_0)
(or cavity photon number)

- **Superfluid Phase** J>>U



weakly interacting system; delocalized atoms

- **Mott-Insulator Phase:** J<<U



strongly interacting system: regular filling

Theory:

Fisher *et al*. (1989),
Jaksch *et al*. (1998)
Zwerger et. al.(2003)

Experiment:

Greiner et. al. (2002)
+ many more .

Effective Hamiltonian

$$H = -J \sum_{\langle n,m \rangle} b_n^\dagger b_m + \frac{U}{2} \sum_n b_n^\dagger b_n \left(b_n^\dagger b_n - 1 \right) + \sum_i (\varepsilon_n - \mu) b_n^\dagger b_n$$

Hubbard model
for a single standing wave mode resonator

effective single atom Hamiltonian

$$H_{\text{eff}} = \frac{p^2}{2m} + \cos^2(kx)\left(\hbar U_0 a^\dagger a + V_{cl}\right) - \hbar\Delta_c a^\dagger a$$

quantized light potential

extra classical potential

many body generalization (M.L.)

$$H = \sum_{l=0,1} \hbar\omega_l a_l^\dagger a_l + \frac{1}{2}\frac{4\pi a_s \hbar^2}{m}\int d^3\mathbf{r}\,\Psi^\dagger(\mathbf{r})\Psi^\dagger(\mathbf{r})\Psi(\mathbf{r})\Psi(\mathbf{r}) + \int d^3\mathbf{r}\,\Psi^\dagger(\mathbf{r})H_0\Psi(\mathbf{r})$$

Dynamical lattice potential with effective parameters

Expand atomic field using 'estimated ' Wannier functions $\Psi(\mathbf{x}) = \sum_i b_i w(\mathbf{x} - \mathbf{x}_i)$

photon number dependent

$$E_{k,l} = \int \mathrm{d}^3x\, w(\mathbf{x} - \mathbf{x}_k)\left(-\frac{\hbar^2}{2m}\nabla^2\right) w(\mathbf{x} - \mathbf{x}_l)$$

$$J_{k,l} = \int \mathrm{d}^3x\, w(\mathbf{x} - \mathbf{x}_k)\cos^2(kx) w(\mathbf{x} - \mathbf{x}_l)$$

$$\tilde{J}_{k,l} = \int \mathrm{d}^3x\, w(\mathbf{x} - \mathbf{x}_k)\cos(kx) w(\mathbf{x} - \mathbf{x}_l)$$

keep only nearest neighbour terms ….

$$\begin{aligned} H &= E_0\hat{N} + E\hat{B} + \left(\hbar U_0 a^\dagger a + V_{\mathrm{cl}}\right)\left(J_0\hat{N} + J\hat{B}\right) \\ &- \hbar\Delta_c a^\dagger a - i\hbar\eta\left(a - a^\dagger\right) + \frac{U}{2}\hat{C}. \end{aligned}$$

$$\hat{N} = \sum_k \hat{n_k} = \sum_k b_k^\dagger b_k \qquad \hat{B} = \sum_k \left(b_{k+1}^\dagger b_k + h.c.\right)$$

Similar model as Bose Hubbard before, but
parameters for lattice dynamics are actually field operators

Field dynamically depends on atom number N and distribution **(See Nat. Phys. 2007)** *:*

Heisenberg equation for field amplitude ***a*** *:*

$$\dot{a} = \left\{ i \left[\Delta_c - U_0 \left(J_0 \hat{N} + J \hat{B} \right) \right] - \kappa \right\} a + \eta$$

$$\hat{N} = \sum_k \hat{n_k} = \sum_k b_k^\dagger b_k$$

atom number

$$\hat{B} = \sum_k \left(b_{k+1}^\dagger b_k + h.c. \right)$$

atom coherence

Formal approx. solution for field for fixed atom number N and for "bad cavity" $\kappa >> J$:

$$a = \frac{\eta}{\kappa - i\Delta_c'} \left[1 - \frac{iU_0}{\kappa - i\Delta_c'} J\hat{B} - \frac{U_0^2}{(\kappa - i\Delta_c')^2} J^2 \hat{B}^2 \right]$$

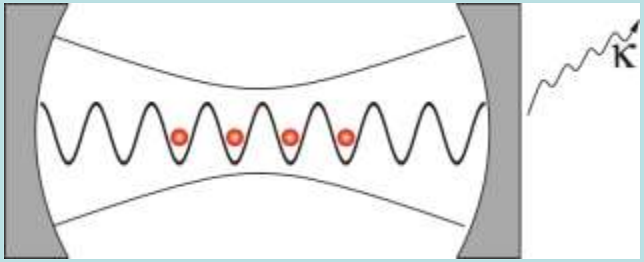
local atom-atom coherence

nonlocal pair correlations

Transmission spectrum of single Mode with quantum index in perturbative limit (V_cl >>U_o)

Only one mode: a_0

Standing wave cavity around partially filled lattice



$$a_0^\dagger a_0 = \frac{|\eta_0|^2}{(\Delta_p - \delta_0 \hat{D}_{00})^2 + \kappa^2}$$

detuning D_{00} is operator in particle space

$\Delta_p=\omega_{lp}-\omega_0$ ... probe to empty cavity detuning

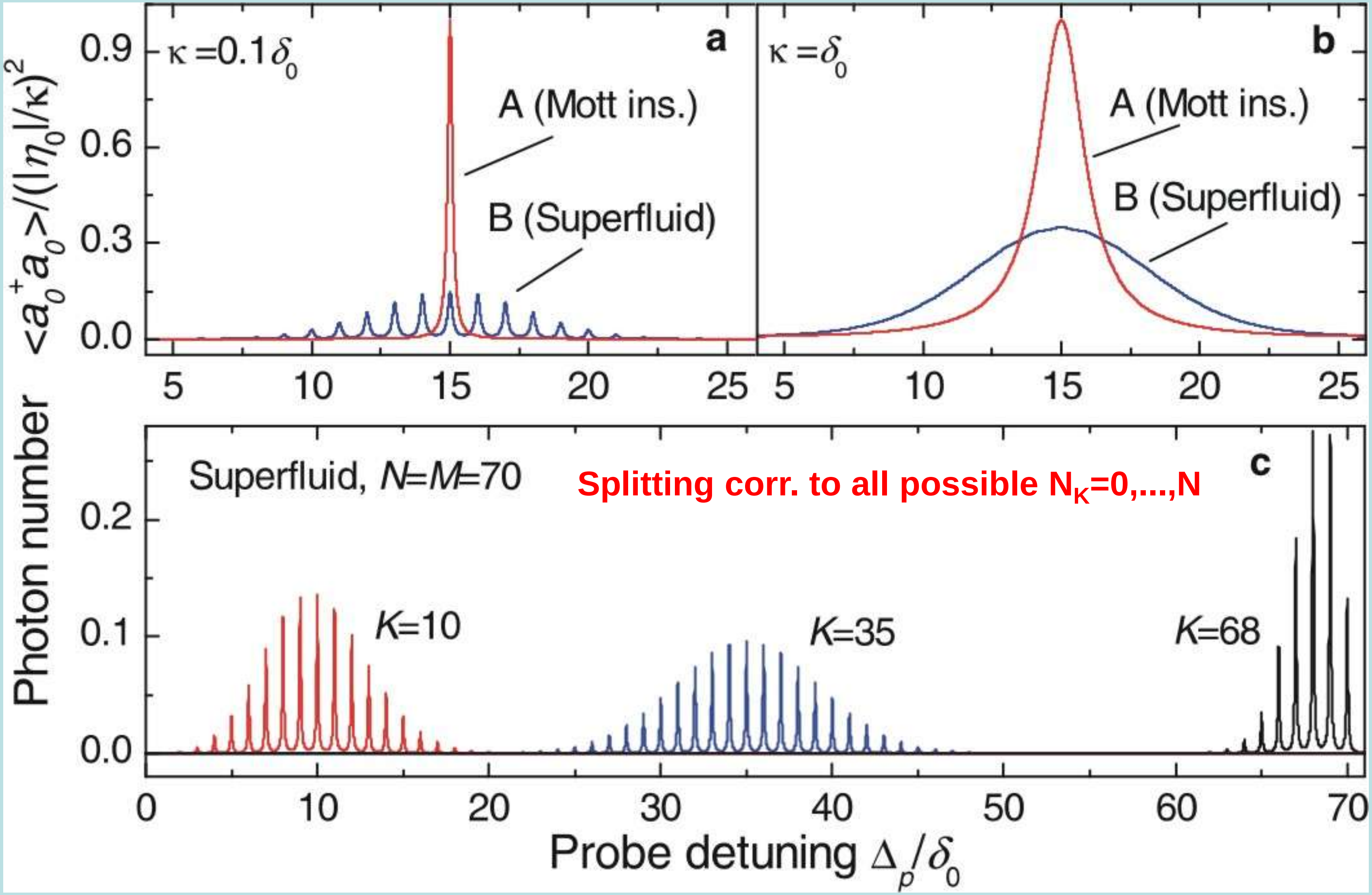
- Mott insulator: $\langle a_0^\dagger a_0 \rangle_{\mathrm{MI}}$ single lorentzian, with width κ and frequency shift $\delta_0 \langle \hat{D}_{00} \rangle_{\mathrm{MI}} = \delta_0 N_K$

  ➡ classical result

- Superfluid: $\langle a_0^\dagger a_0 \rangle_{\mathrm{SF}}$ different dispersion shifts, corresponding to all possible atom number distributions

  ➡ comb-like structure

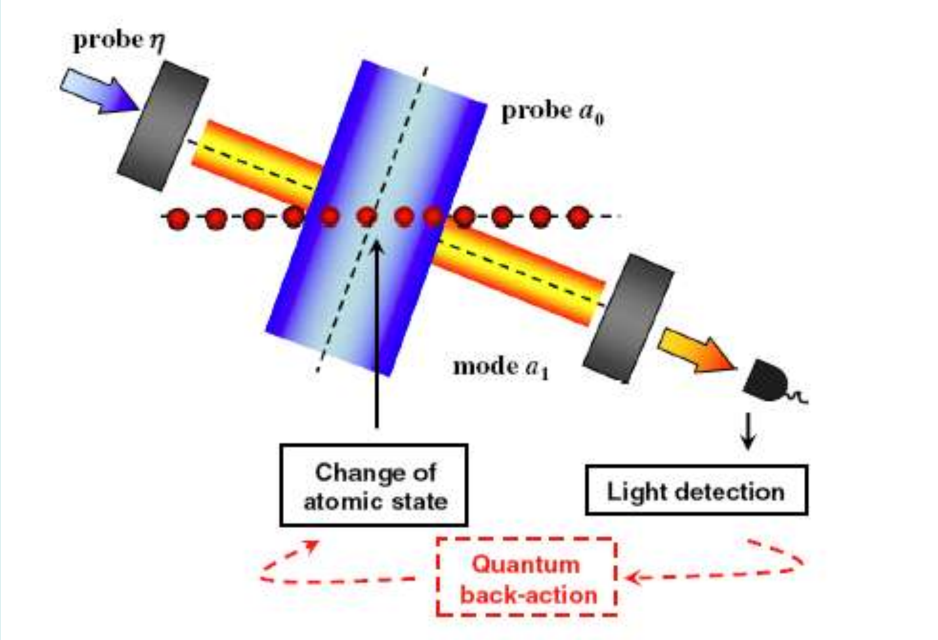
Numerical example

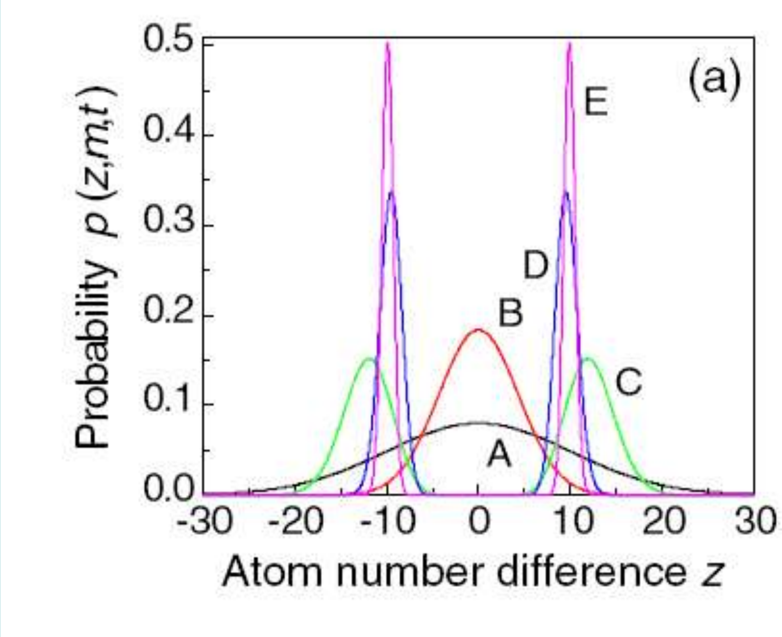


I.Mekhov, Nat. Physics 2007 and releated work by P. Meystre + al.

Quantum Measurement and Feedback:

(analogous to micromaser by S. Haroche and coworkers,
but exchange photons + atoms ....)





Monitoring cavity transmission of superfluid for weak probe =>
projection towards macroscopic superpositions

I. Mekhov, PRL 2009

Dynamics in a quantum potential:

bad cavity limit: effective Hamiltonian with adiabatically eliminated field

$$H = \left[E + J\left(V_{cl} - \hbar U_0 \eta^2 \frac{\kappa^2 - \Delta_c'^2}{\left(\kappa^2 + \Delta_c'^2\right)^2}\right)\right]\hat{B} \qquad (13)$$

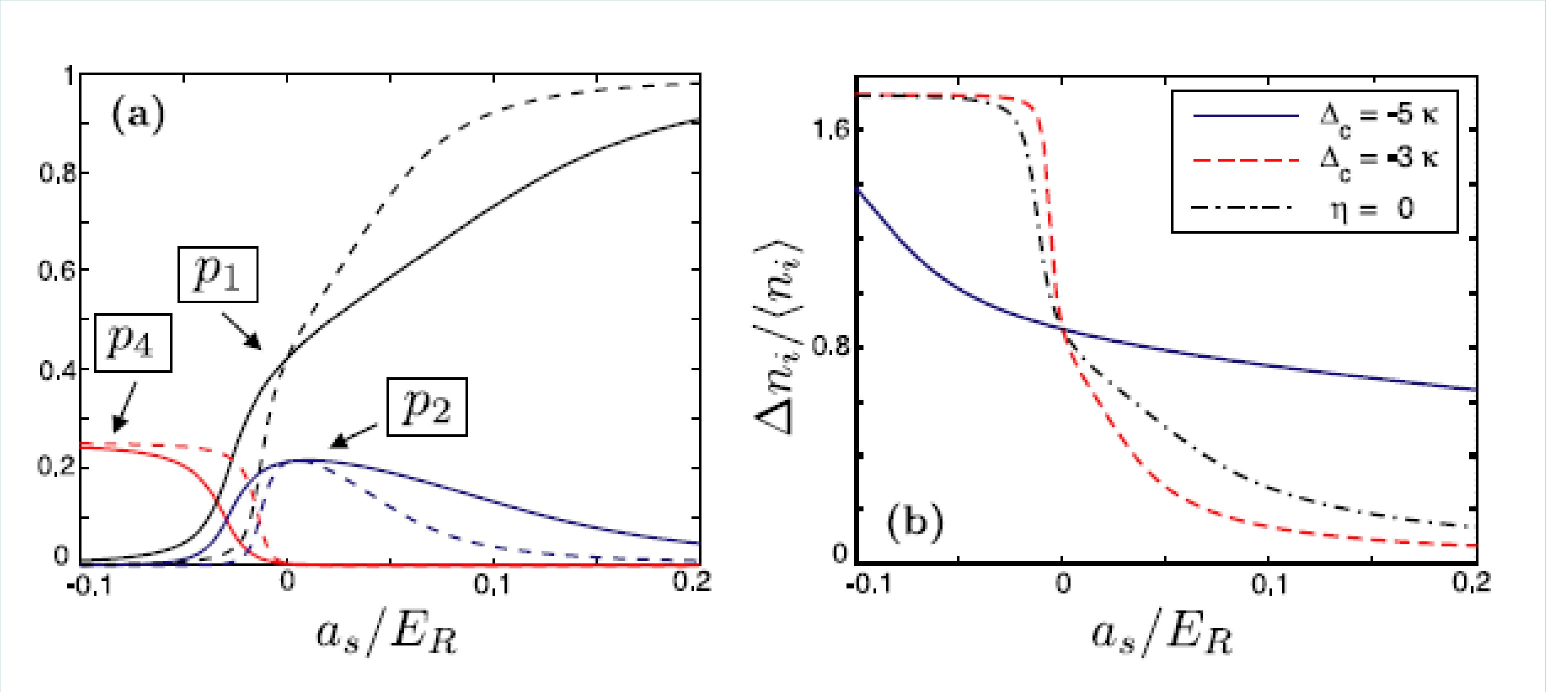
$$+ \; 3\hbar U_0^2 \eta^2 \Delta_c' \frac{3\kappa^2 - \Delta_c'^2}{\left(\kappa^2 + \Delta_c'^2\right)^4} J^2 \hat{B}^2 + \frac{U}{2} \sum_k \hat{n}_k \left(\hat{n}_k - 1\right)$$

rescaled hopping terms

Nonlocal atom-atom interaction
via nonlocal correlated hopping

Cavity parameters can be used to effectively tune size and type of interactions !

Find lowest energy state for four atoms in four wells (ring)

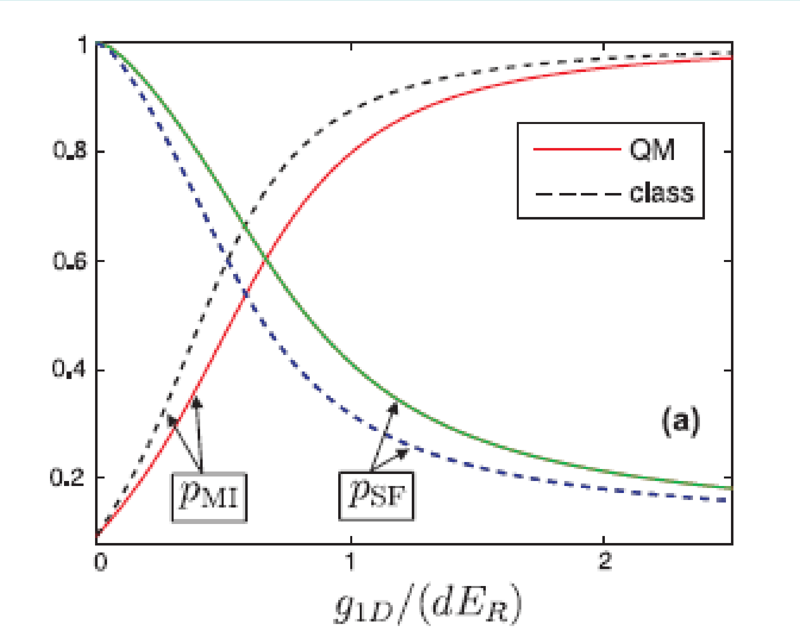


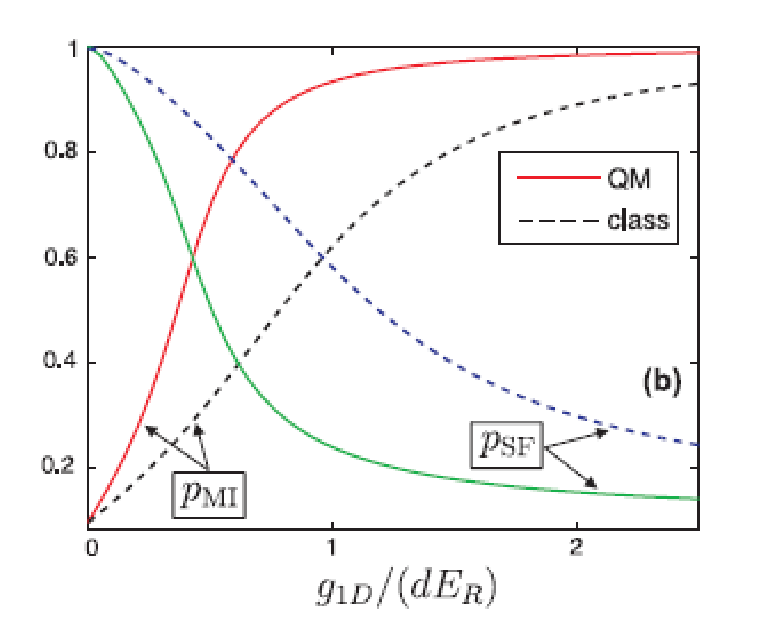
Occupation probability for single well

population fluctuations for single well

position and sharpness of ground state transition controlled by cavity parameters

Contribution of "Mott-insulator" and "superfluid" for 4 atoms in 4 wells





$$\Delta_c - U_0 J_0 N = \kappa.$$

blue detuning

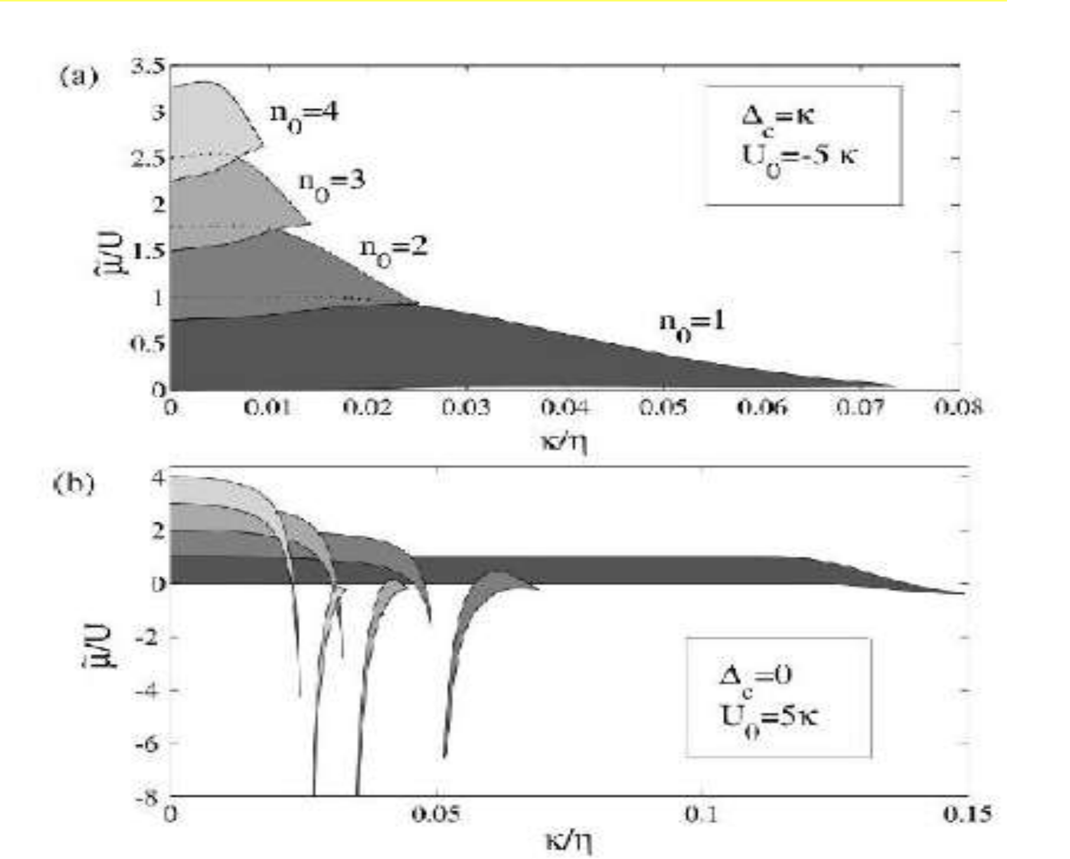
$$\Delta_c - U_0 J_0 N = -\kappa.$$

red detuning

Thermodynamic limit and phase transitions ?

M. Lewenstein, G. Morigi + coworkers:
Phase diagram in thermodynamic limit (PRL 2007, 2008)

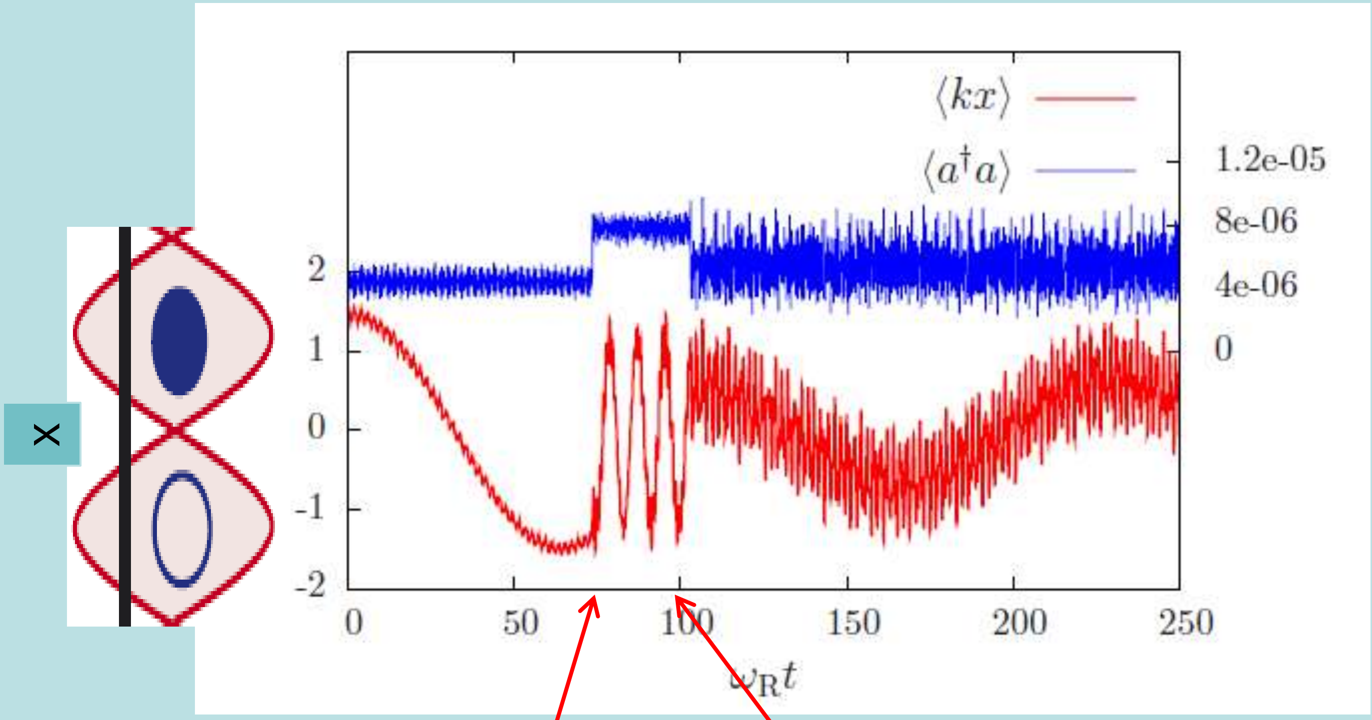
Cavity creates extra effective attraction or repulsion :
bistable phases (~ optical bistability)
phase superpositions ??



Generalization to fermions: Morigi PRA 2008

Simulated single atom dynamics for two wells : photon-assisted or photon blocked tunneling

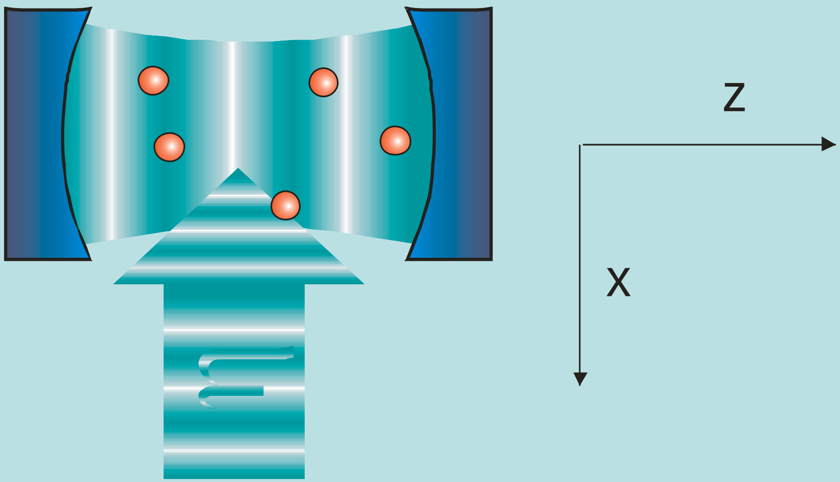
at t=0 atom prepared at right well:



- jumps in photon number + atomic state
- effective model contains weighted average of tunnel amplitudes

Selforganisation of large ensembles through superradiant light scattering

***New-geometry:* transverse pump: direct excitation of atoms from side !**



phase of excitation light depends on position x

$$\dot{\sigma}_i = (i\Delta_A - \gamma)\sigma_i - g(z_i)a + \eta_x + \xi_A$$

$$\dot{a} = (i\Delta_C - \kappa)a + \sum_{i=1}^{N} g^*(z_i)\sigma_i + \xi_i \,.$$

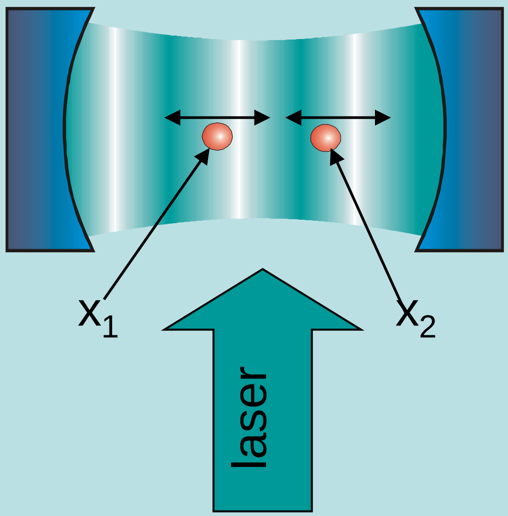
collective pumpstrength ***R***

Field in cavity generated only by atoms

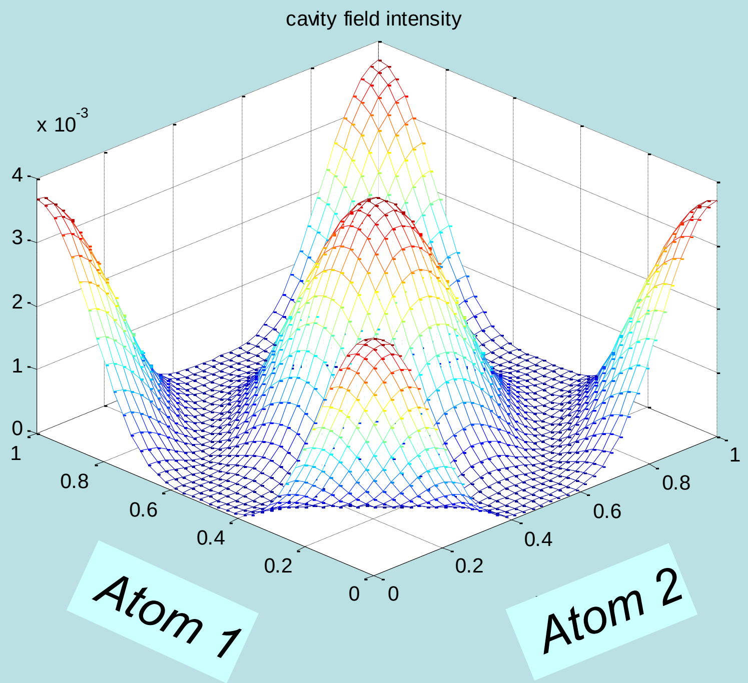
R *= 0 for random atomic distribution*

R ~ Ng *for regular lattice (Bragg)*

Two Atoms at fixed positions



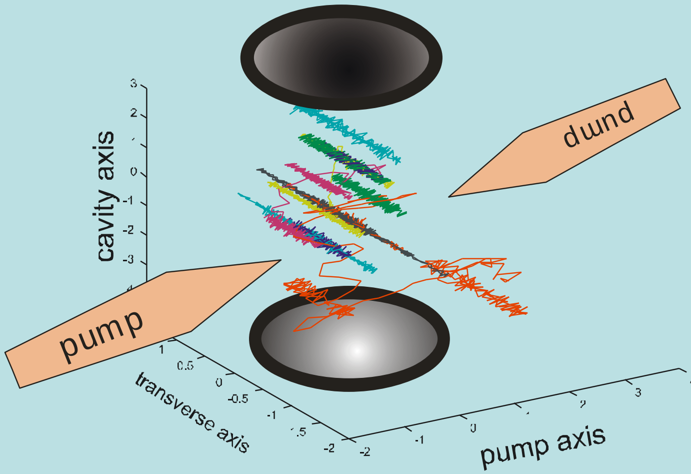
Cavity field as a function of positions for two atoms

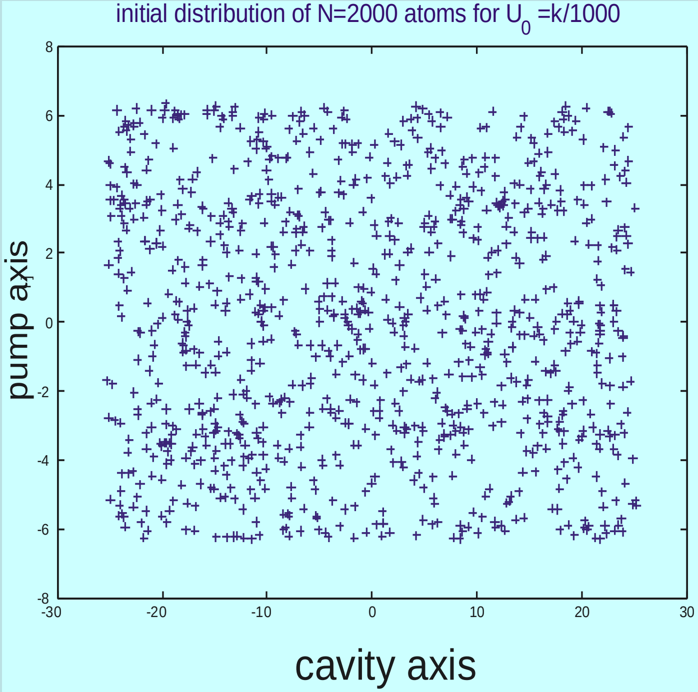


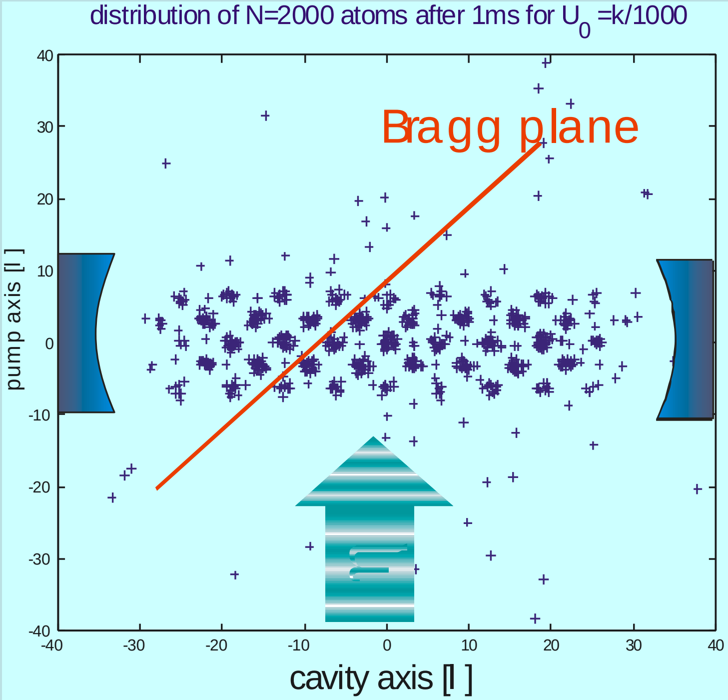
Maximum photon number for 0 and l distance
Minimum photon number for l/2 distance
(~ two pinhole interference)
=> for high field seekers ordering is favourable

3D simulation of Selforganisation

Simulation:
coherent light emission
in connection with cooling



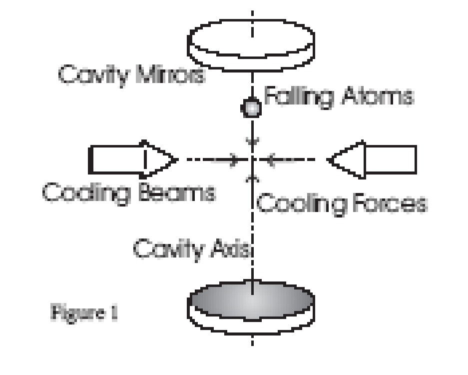




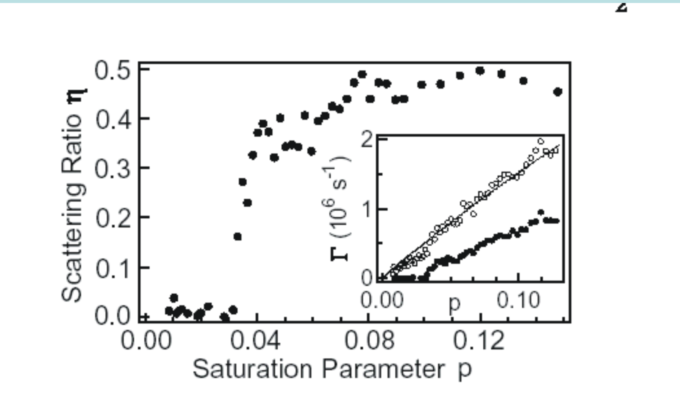
Atoms form Bragg planes which optimizes scattering into cavity and maximizes their trapping potential

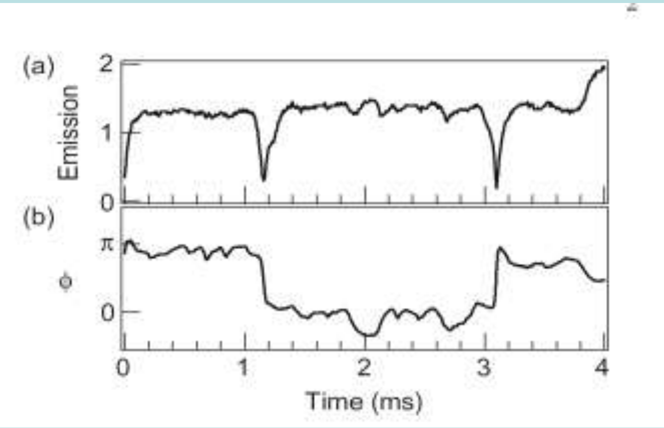
Experiment with atoms:

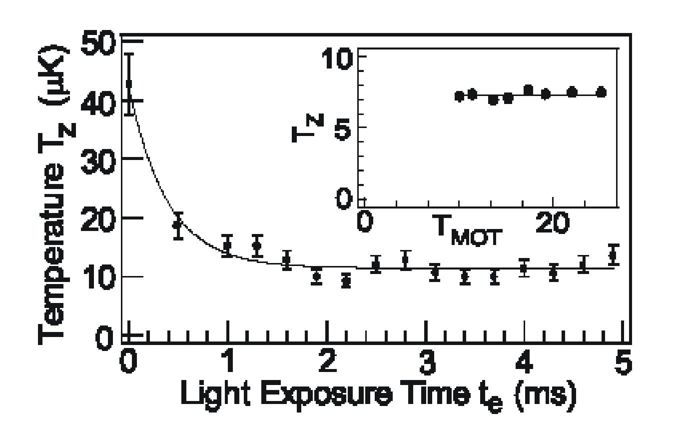
Vladan Vuletic: Stanford University (=>MIT)



10^6 Caesium atoms in resonator with transverse coherent pump field







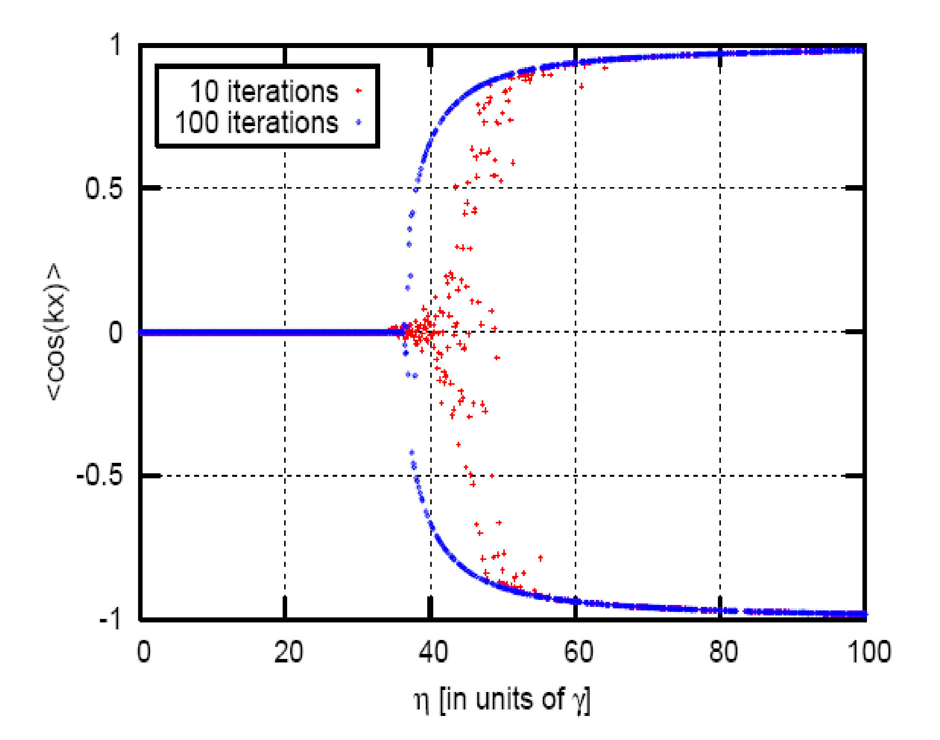
Phase stability of coherent emission with Pi-jumps (bistable pattern)

** >10^6 Atoms trapped and cooled to ~mK with simultaneous coherent light emission*

Experiment works better than predictions and even close to cavity resonance
New experiment with accelerations of >10^6 g at very low saturation

Selforganisation analogous to a phase transition

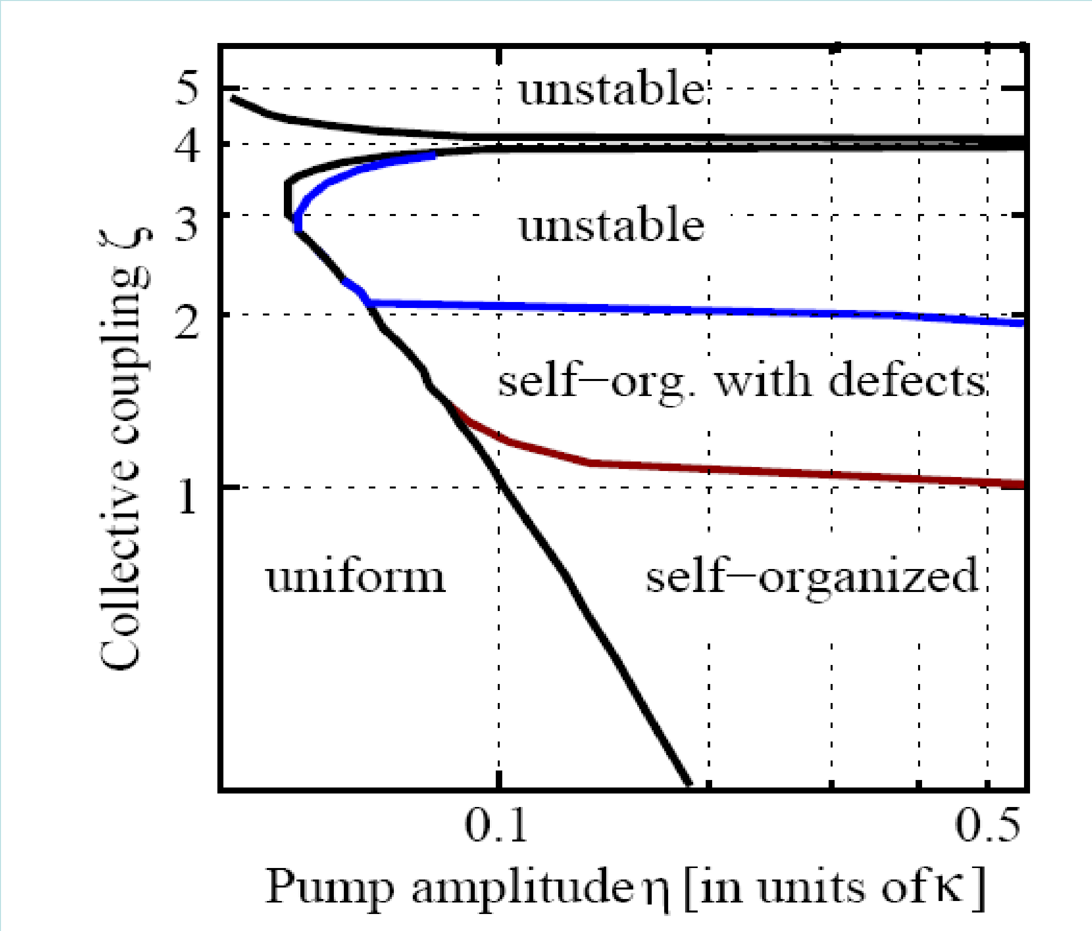
Stable steady state solution in a continuous density approximation



Critical Point:

$$\eta > \sqrt{\frac{k_B T}{\hbar\kappa}\frac{\sqrt{2}\kappa\,|\Delta_A|}{\sqrt{N}g}}.$$

Numerically obtained phase diagram
(transversely driven ring cavity geometry)



Quantum description of selforganization for a BEC in a cavity in mean field approximation

P. Domokos et. al, EPJD 48, 2008

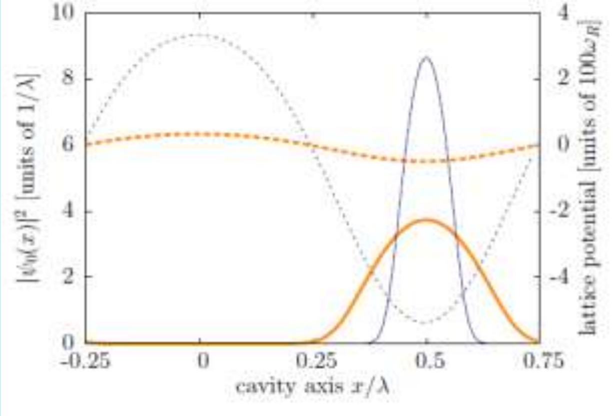
$$i\frac{\partial}{\partial t}\alpha = \left[-\Delta_C + N\langle U(x)\rangle - i\kappa\right]\alpha + N\langle \eta_t(x)\rangle,$$

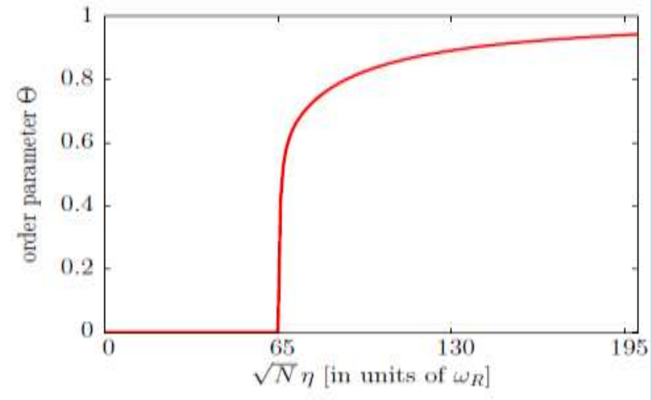
$$i\frac{\partial}{\partial t}\psi(x,t) = \left\{\frac{p^2}{2\hbar m} + |\alpha(t)|^2 U(x) + 2\mathrm{Re}\{\alpha(t)\}\eta_t(x) + N g_c |\psi(x,t)|^2\right\}\psi(x,t).$$

$$\sqrt{N}\eta_c = \sqrt{\frac{(\Delta_C - NU_0/2)^2 + \kappa^2}{(NU_0 - 2\Delta_C)}}\sqrt{\omega_R + 2Ng_c}$$

threshold even for T=0, g=0 !

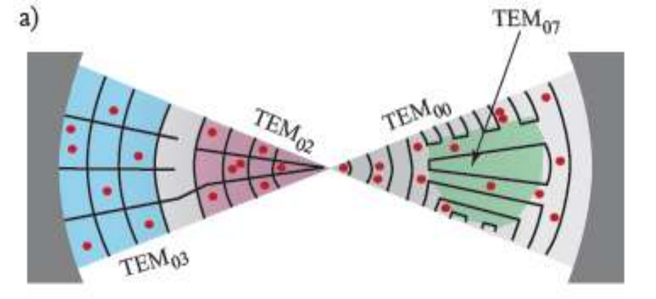
kT => ω_rec



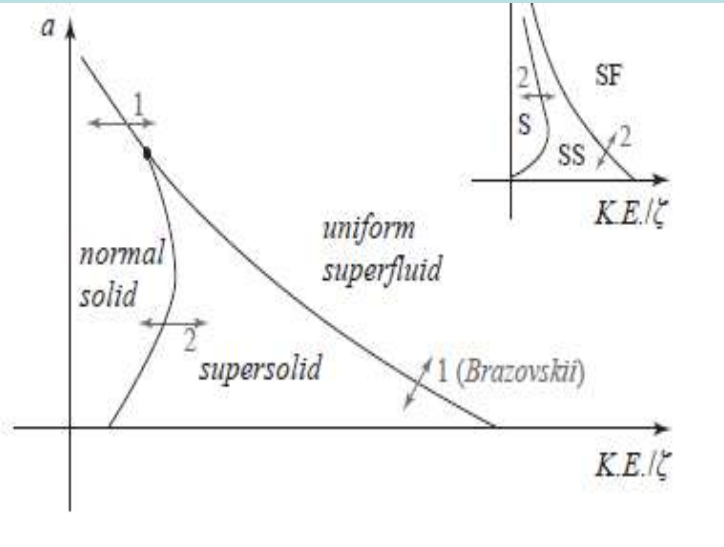


Generalization to multimode confocal cavity :

S. Gopalakrishnan, B. L. Lev, and P. M. Goldbart, Nat.VPhys. 5, 845 (2009).



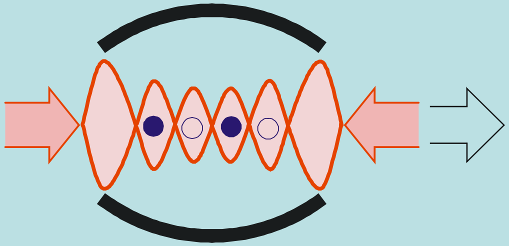
$$\frac{\Omega_{th} - \Omega_{th}^{mf}}{\Omega_{th}^{mf}} \simeq 2.5 \left[\frac{\alpha U \sqrt{\hbar^2 K_0^2 / 2M}}{(\hbar \zeta N \chi)^{3/2}} \right]$$



Quantum Brazovskii transition

Beyond mean field

multiparticle quantum description of selforganization in a lattice



- Classical standing pump wave creates optical lattice
- Atoms prepared in lowest band of lattice
- field in the cavity by scattering of lattice light

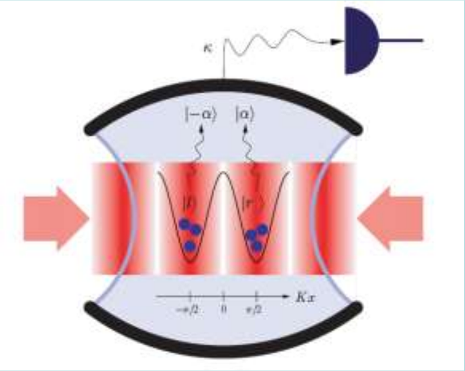
Effective Hamiltonian (large detuning):

$$H = \sum_{k,l} E_{k,l} b_k^\dagger b_l + \hbar U_0 \eta' g \sum_{k,l} J_{k,l} b_k^\dagger b_l + \hbar \eta' \left(a + a^\dagger\right) \sum_{k,l} \tilde{J}_{k,l} b_k^\dagger b_l - \hbar \left(\Delta_c - U_0\right) a^\dagger a$$

pump amplitude determined by atomic distribution operator

How and when will selforganization happen here ?

Lowest energy states for atoms at only two sites …



$$a = -i\frac{\eta'}{\kappa - i(\Delta_c - U_0)}\tilde{J}_0\left(b_1^\dagger b_1 - b_2^\dagger b_2\right)$$

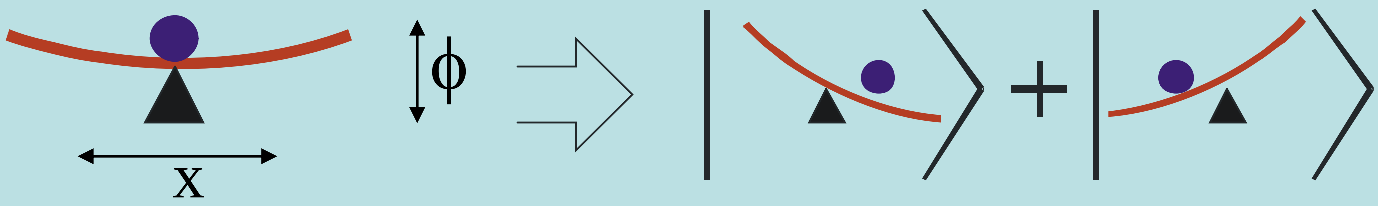
$$a^\dagger a \sim \left(b_1^\dagger b_1 - b_2^\dagger b_2\right)^2$$

$$\frac{1}{\sqrt{2}}\left(|\text{left}\rangle\,|\alpha\rangle \pm |\text{right}\rangle\,|-\alpha\rangle\right)$$

… shows atom field entanglement

- Note: strongly entangled state
- Symmetry leads to zero field but nonzero intensity (photons)
- How does entanglement and intensity grow ?

Very simple toy model:
"decay of a quantum seesaw "

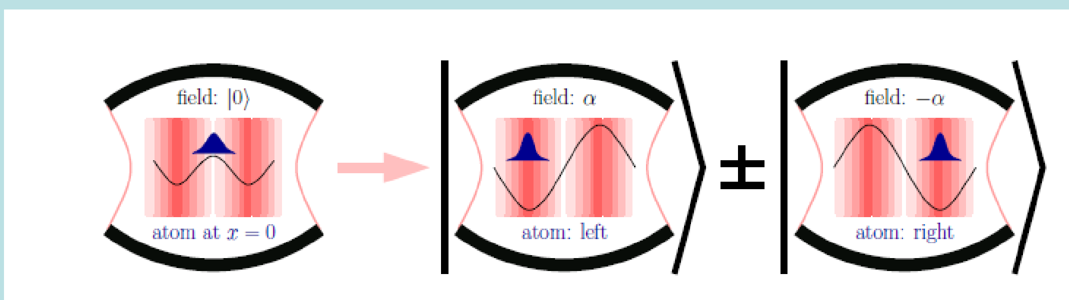


classical

quantum

Two degrees of freedom: tilt angle ϕ and particle position x

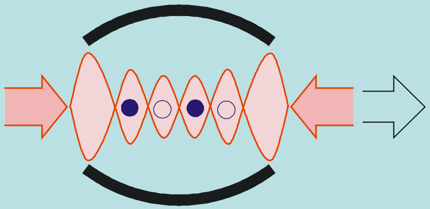
Note: classical equilibrium point at x=ϕ=0
but
product state of oscillator ground states is not stationary



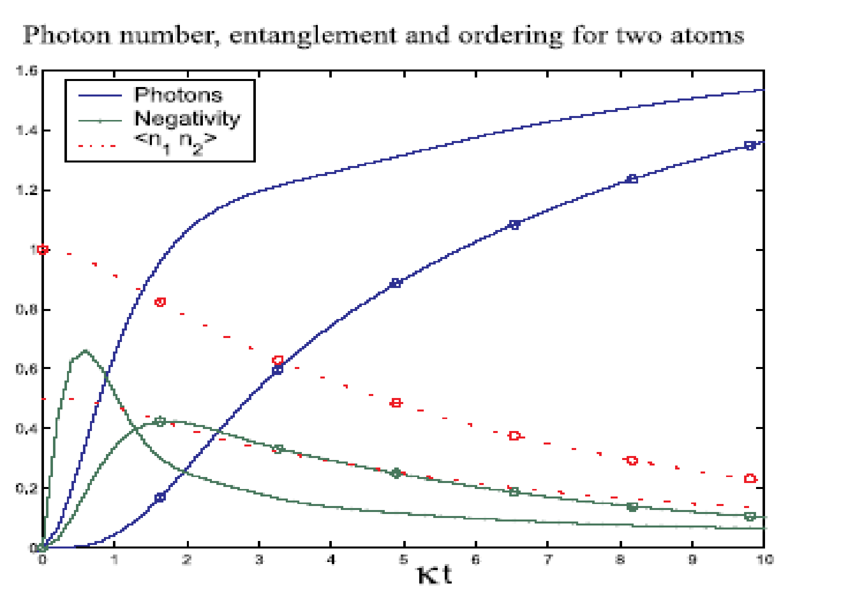
field phase replaces tilt angle <> occupation difference replaces position

Selforganization of atoms in a lattice as seesaw

$$H = \sum_{k,l} E_{k,l} b_k^\dagger b_l + \hbar U_0 \eta' g \sum_{k,l} J_{k,l} b_k^\dagger b_l + \hbar \eta' \left(a + a^\dagger\right) \sum_{k,l} \tilde{J}_{k,l} b_k^\dagger b_l - \hbar \left(\Delta_c - U_0\right) a^\dagger a$$



$$a = -i \frac{\eta'}{\kappa - i(\Delta_c - U_0)} \tilde{J}_0 \left(b_1^\dagger b_1 - b_2^\dagger b_2\right)$$



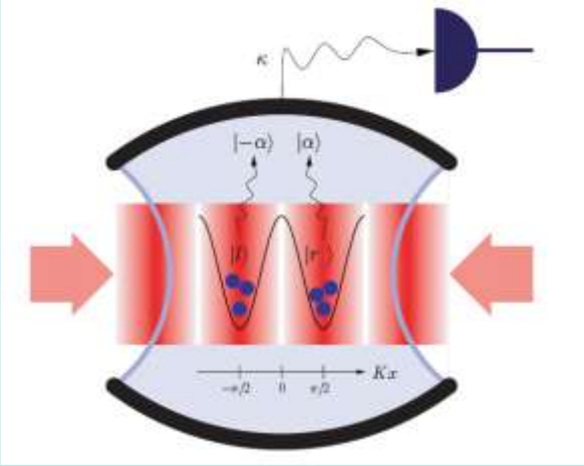
atom + field evolve fast towards entangled cat state !

$$1/\sqrt{2} \left(|\text{left}\rangle\, |\alpha\rangle \pm |\text{right}\rangle\, |-\alpha\rangle\right)$$

„superfluid selforganizes much faster than Mott insulator „

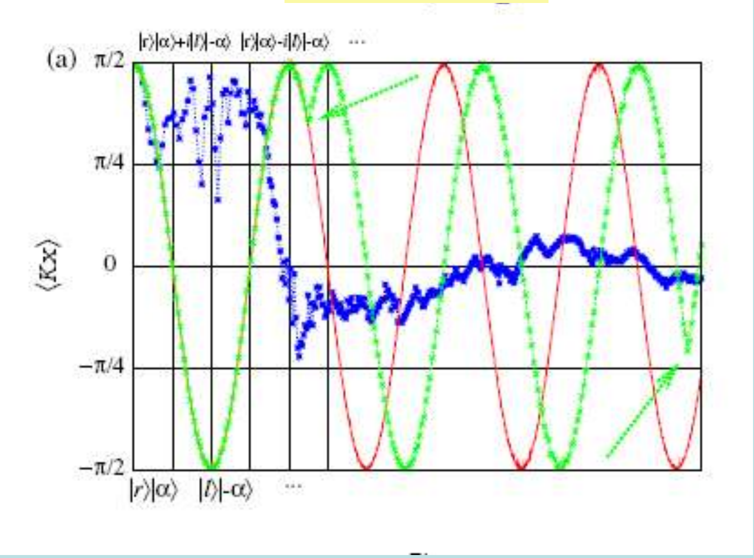
Quantum Model for field and atoms

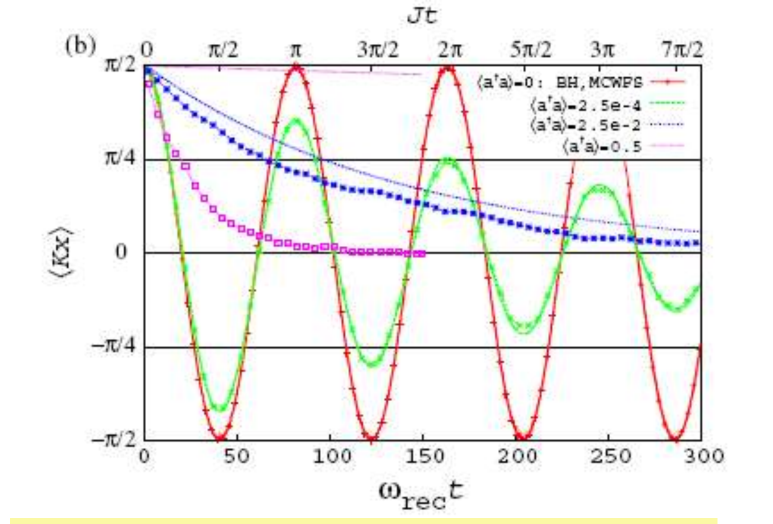
Microscopic dynamics of selforganization



How do the atoms evolve into an ordered state at T=0 ?

single atom





fast decay towards entangled state

Two atoms

$\langle n_1 n_r \rangle$ measures ordering:

- 1 „Mott"-insulator (1,1)
- 0 Ordered states {(2,0) +/-(0,2)}

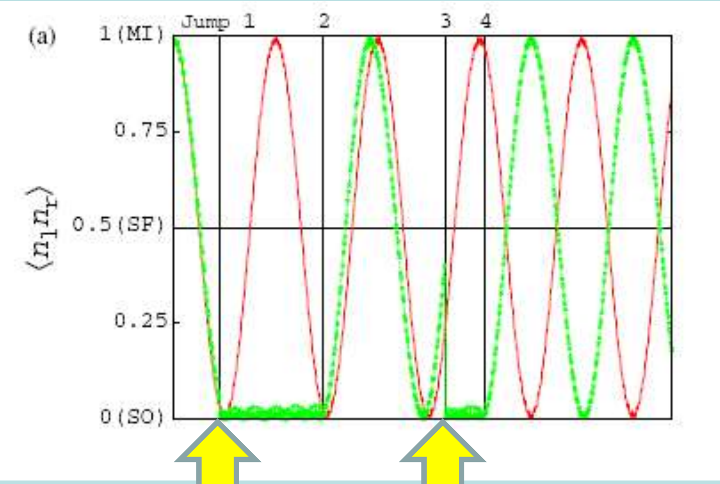
„spontaneous „ ordering
via photon scattering by sign change

$$|\Psi(t)\rangle = (|-,-2\alpha\rangle + |+,2\alpha\rangle)$$

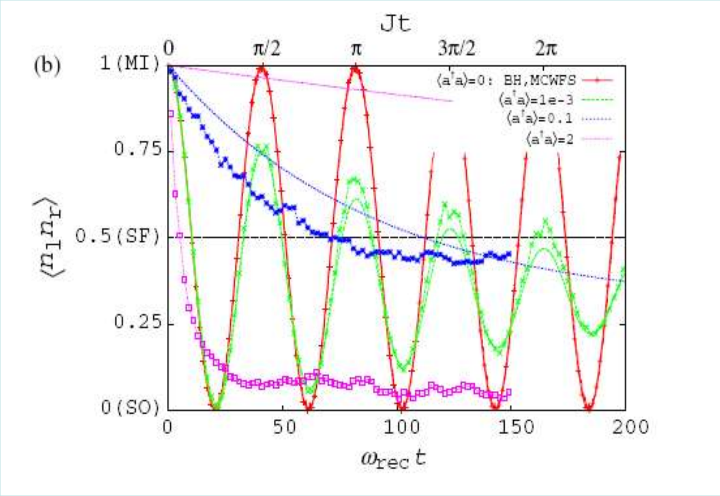
$$|\Psi'(t)\rangle \propto a\,|\Psi(t)\rangle \propto |-,-2\alpha\rangle - |+,2\alpha\rangle$$

|-> =[(2,0)-(0,2)] state does not tunnel !

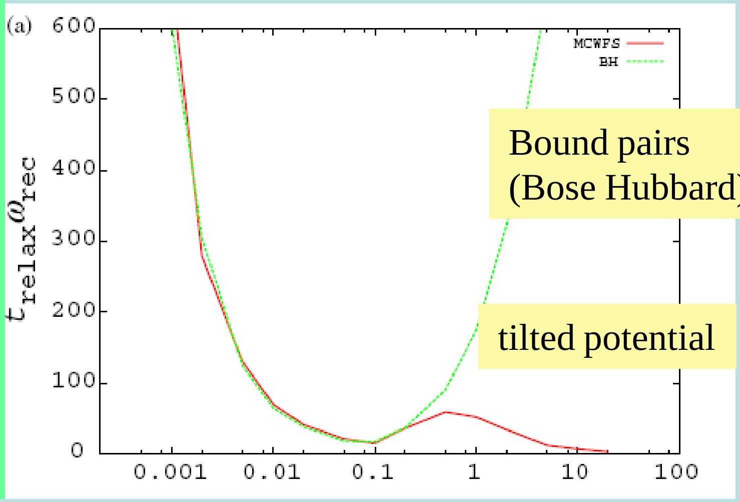
Single trajectory



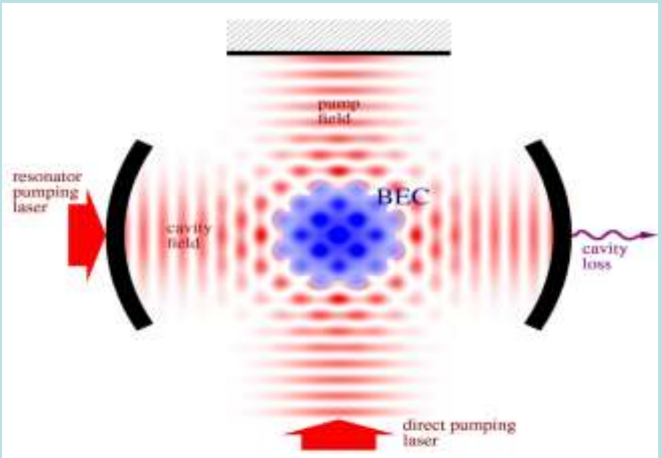
ensemble average



Time scale of ordering

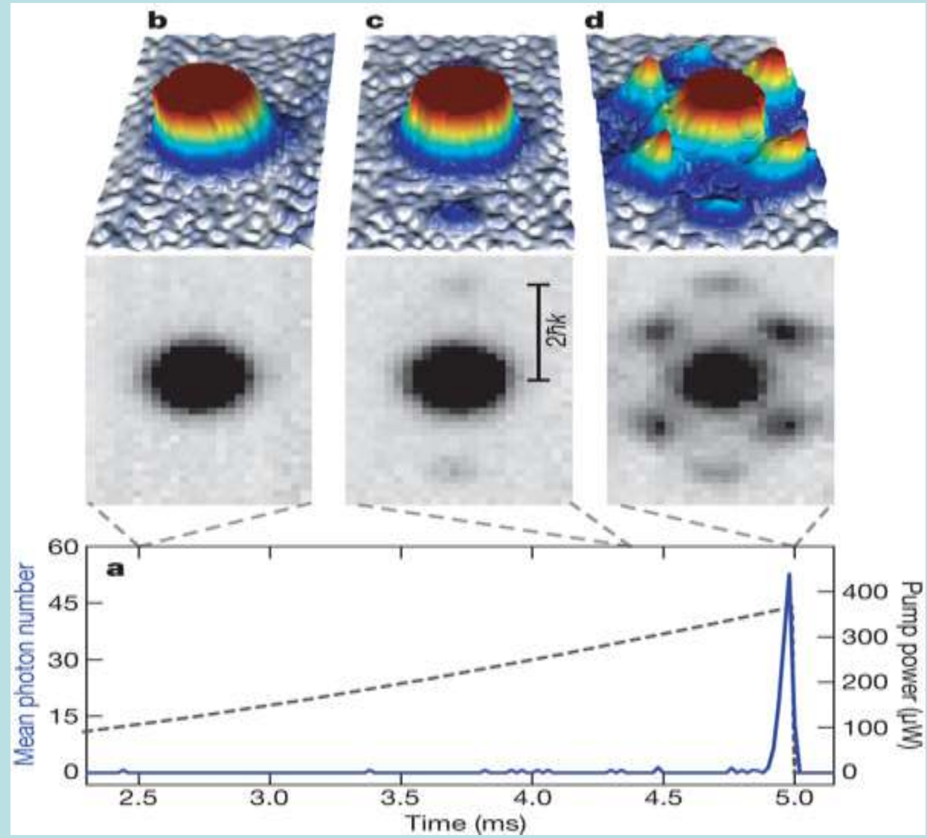


Observation of the phase transition to new phase with coherence + ordering present



$$\Psi(x) = \frac{1}{\sqrt{L}} c_0 + \sqrt{\frac{2}{L}} c_1 \cos kx$$

$$H = -\delta_C \, a^\dagger a + \omega_R \hat{S}_z + iy(a^\dagger - a)\hat{S}_x/\sqrt{N} + u a^\dagger a \left(\frac{1}{2} + \hat{S}_z/N\right)$$



K Baumann *et al.* *Nature* 464, 1301-1306 (2010) doi:10.1038/nature09009

nature

Summary and Outlook

- ***light forces are strongly modified in optical resonators:***

  **** self trapping and cooling => single photon single atom trap***
  **** ground state cooling with suppressed spontaneous emission***

- ***atom atom interaction via field:***

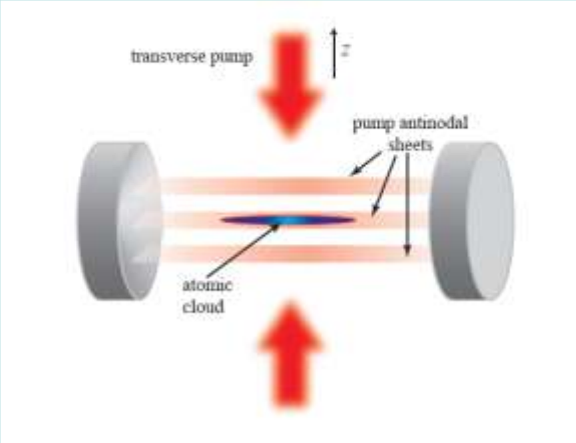
  **** selforganisation and superradiance enhance cooling***
- **** quantum field description entangles distant atoms and fields***

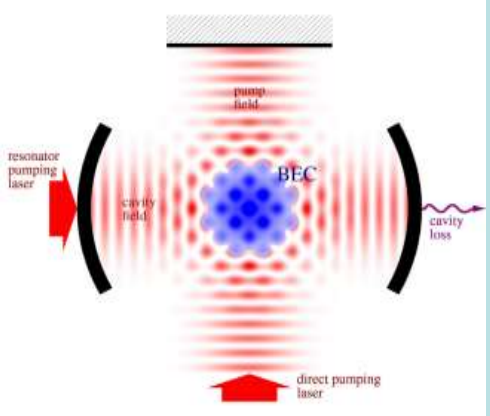
- ***"Quantum potentials"***

  **** superposition of different atomic distributions***
  **** entangled atom field states***
  **** new decay channels for "phase transitions"***

Refs: *P. Horak et. al., PRL* ***79****, 1997*
T. Salzburger and H. R., PRA, 75, 2007
A. Vukics, P. Domokos and H.R. al, PRL ***92****, 2004*
S. Zippilli, G. Morigi and H.R., PRL ***93****, 2004*
C. Maschler and H.R., PRL ***95****, 2005*
I. Mekhov,C. Maschler and H.R., PRL ***98****, 2007, Nat. Phys.3, 2007*

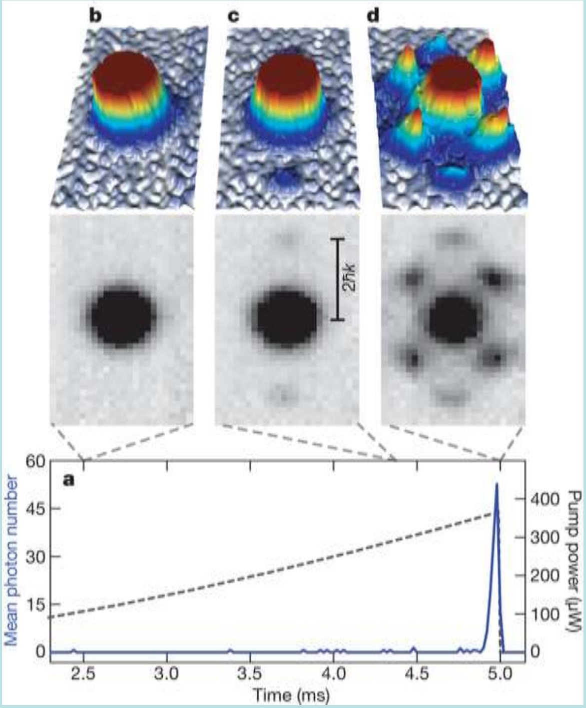
Experiment Esslinger 2010:





$$
\begin{aligned}
H = &-\delta_c\, a^\dagger a + i\,(\eta\, a^\dagger - \text{h.c.}) \\
&+ \int d^3r\, \Psi^\dagger(\mathbf{r}) \Big\{ -\frac{\Delta}{2m} + u\, a^\dagger a\, f(k\mathbf{r}) \\
&\qquad + i\,(\eta'(k\mathbf{r})\, a^\dagger - \text{h.c.}) \Big\}\, \Psi(\mathbf{r}),
\end{aligned}
$$

Observation of the phase transition.

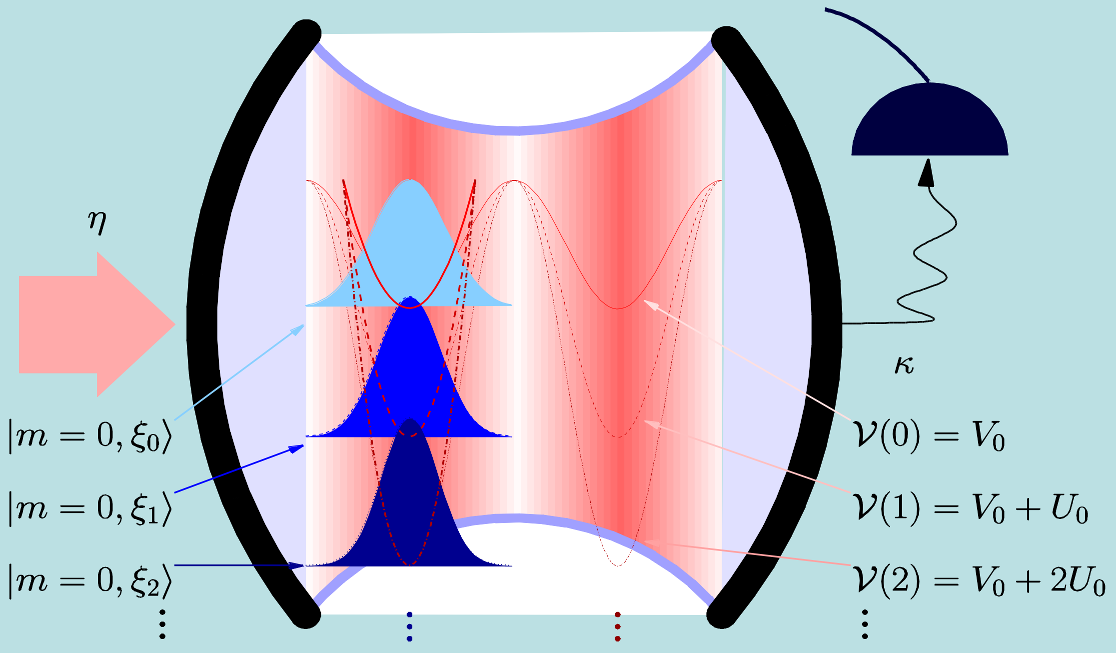


K Baumann *et al. Nature* **464**, 1301-1306 (2010) doi:10.1038/nature09009

nature

Cavity non-linear optics with ultracold quantum particles

*Cold atoms in **deep** optical potential of far detuned cavity field*



Atomic eigenstate wavefunction depends on field intensity = photon number refractive index (phase shift) depends on width of eigenstate => atoms create effective nonlinearity although only weakly excited

classical limiting case: Gangl PRA 2000, Stamper-Kurn PRL 2007 heuristic derivation

Hamiltonian for a single quantum particle in deep well:

$$H = \frac{\hat{P}^2}{2m} + m\omega_R U_0 a^\dagger a \hat{X}^2 + i\eta(a - a^\dagger)$$

adiabatic elinination of atomic motion => effective nonlinear medium

atom in n-th eigenstate:

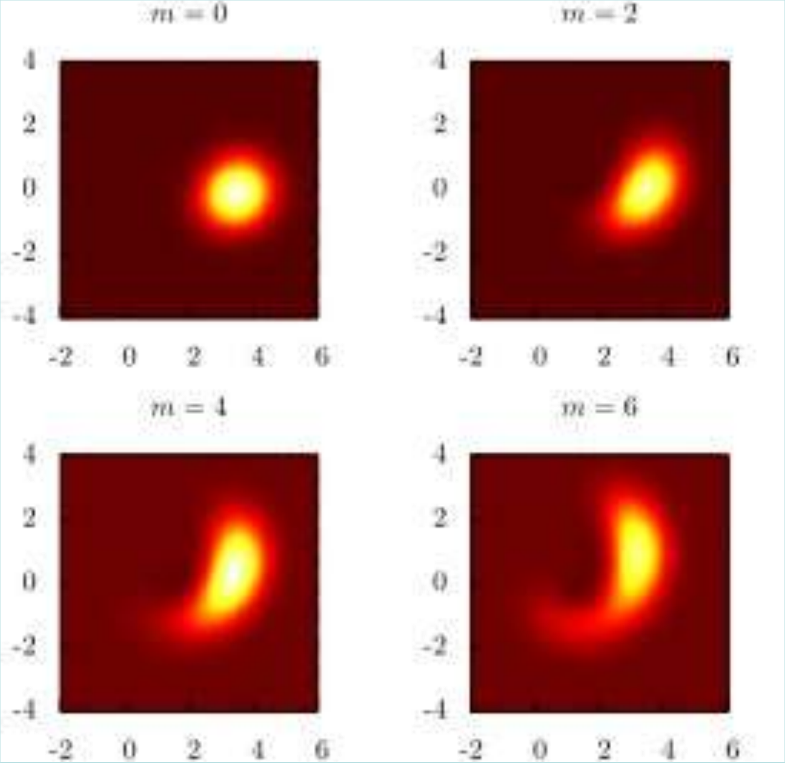
$$< X^2 > \approx \frac{n}{\sqrt{\omega_R U_0 a^\dagger a}}$$

effective field Hamiltonian

$$H_{eff} = \frac{\hat{P}^2}{2m} + (2n+1)\sqrt{\omega_R U_0 a^\dagger a} + i\eta(a - a^\dagger)$$

effektive SQRT nonlinearity => nonclassical light (G. Milburn 19??)
large difference between 0,1,2,3 …photons

Numerical calculated stationary field Wignerfunction when the atom sits in different vibrational states starting with coherent state



Atoms in different vibrational levels cause different nonlinearties

Different photon numbers create different optical potentials

Tailorable nonlinear system at low photon and low atom numbers

Summary and Outlook

- *light forces are strongly modified in optical resonators:*

  *** self trapping and loading of micro traps: single photon trap***
  *** tailored dissipation for cooling with suppressed spontaneous emission***

- *atom atom interaction via field:*

  *** selforganisation and superradiance enhance cooling***
- *** quantum field description entangles distant atoms and fields***

- *"Quantum potentials"*

  *** superposition of different atomic distributions***
  *** entangled atom field states***
  *** new decay channels for phase transitions***

Refs: *P. Horak et. al., PRL* ***79****, 1997*
T. Salzburger and H. R., PRA, 75, 2007
A. Vukics, P. Domokos and H.R. al, PRL ***92****, 2004*
S. Zippilli, G. Morigi and H.R., PRL ***93****, 2004*
C. Maschler and H.R., PRL ***95****, 2005*
I. Mekhov,C. Maschler and H.R., PRL ***98****, 2007, Nat. Phys.3, 2007*


Innsbruck University – visitors welcome !

Many particles:
Cold atomic gas in a cavity generated optical potential

Vlasov-limit:
Continuous density approximation for cold cloud of particles

$$f_K(x,v,t) := \frac{1}{N}\sum_{j=1}^{N} \delta(x - x_j(t))\delta(v - v_j(t))$$

Kinetic limit-Vlasov equation

$$\frac{\partial f_K}{\partial t} + r_1 v \frac{\partial f_K}{\partial x} - r_2 \partial_x \phi(x, \alpha_\pm) \frac{\partial f_K}{\partial v} = 0$$

$$\dot{\alpha}_\pm = (-1 + i\delta)\alpha_\pm - iNu_0\alpha_\mp\sigma_\pm - iN\eta\theta_\pm + \eta_\pm$$

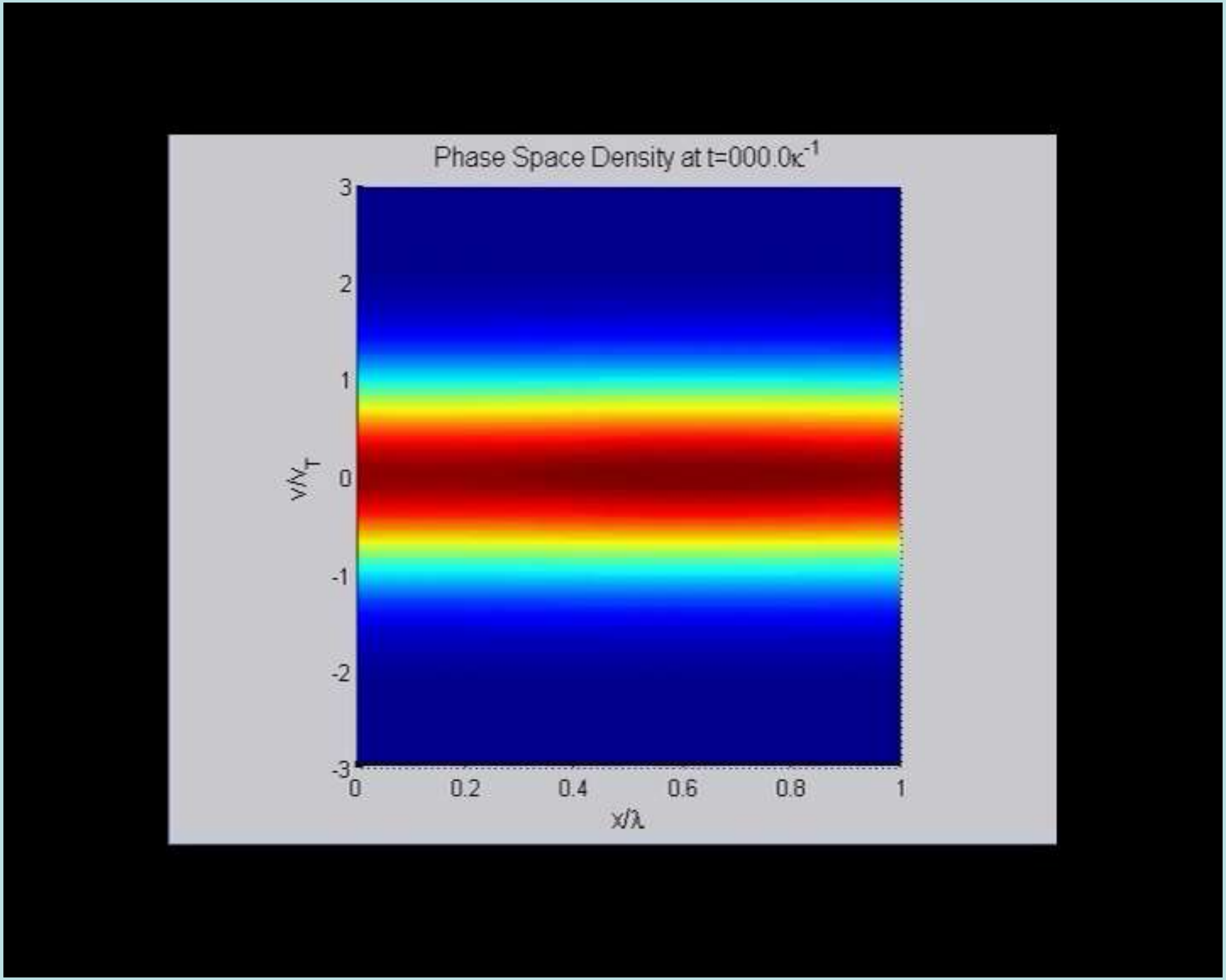
$$\sigma_\pm := \int_{-\infty}^{\infty}\int_0^{2\pi} f_K(x,v,t) e^{\mp 2ix} dx dv$$

$$\theta_\pm := \int_{-\infty}^{\infty}\int_0^{2\pi} f_K(x,v,t) e^{\mp ix} dx dv$$

Stability-limit for a homogenous distribution:

$$\eta_c^2 = \frac{mv_0^2}{2\hbar\kappa}\frac{1+\delta^2}{2Nu_0^2|I|}\left(-\frac{\delta}{2} + \sqrt{\frac{\delta^2}{4} + \frac{I}{\pi F''(0)}\frac{2kv_0}{\kappa}}\right)$$

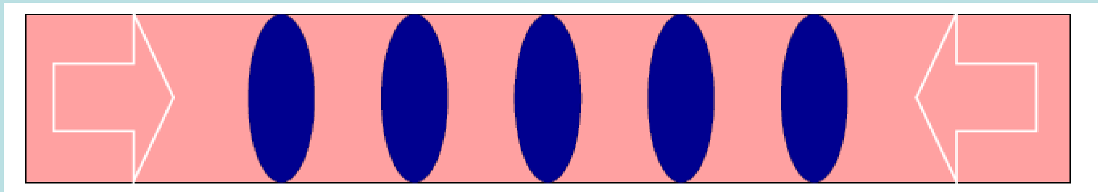
Numerical simulation:



Experiment with thermal gas in cavity at IFCO (M. Cristinani, J. Eschner)

Optomechanical approach to an optical lattice for atoms

1D
optical lattice



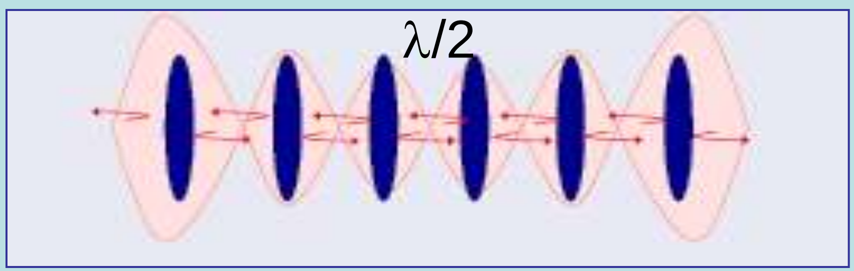
λ/2

3D

Dipole force
traps atoms
near field maxima

Radiation pressure
from both sides cancels

trapped clouds assume pancake shape
and form
small partly reflecting mirrors
(=> Bragg reflector)



Paradox

"Light seems to create a structure into which it cannot propagate"

Photonic band gaps in optical lattices

I. H. Deutsch, R. J. C. Spreeuw,* S. L. Rolston, and W. D. Phillips
National Institute of Standards and Technology, PHYS A167, Gaithersburg, Maryland 20899

model Hamiltonian for a single particle in one deep well:
a…photon excitation, b…atomic excitation

$$H = \frac{p^2}{2m} + (|V_{\text{cl}}| + \hbar |U_0| a^\dagger a) k^2 x^2 - -\hbar(\Delta_c - U_0) a^\dagger a - i\hbar\eta(a - a^\dagger) - |V_{\text{cl}}|$$

harmonic approximation for optical potential

$$k^4 \hat{x}_0^4 = \frac{E_R}{|V_{\text{cl}}| + \hbar |U_0| a^\dagger a}$$

$$H = \Omega\left(a^\dagger a\right)\left(b^\dagger b + \frac{1}{2}\right) + H^{(\text{free})} + H^{(\text{pump})},$$

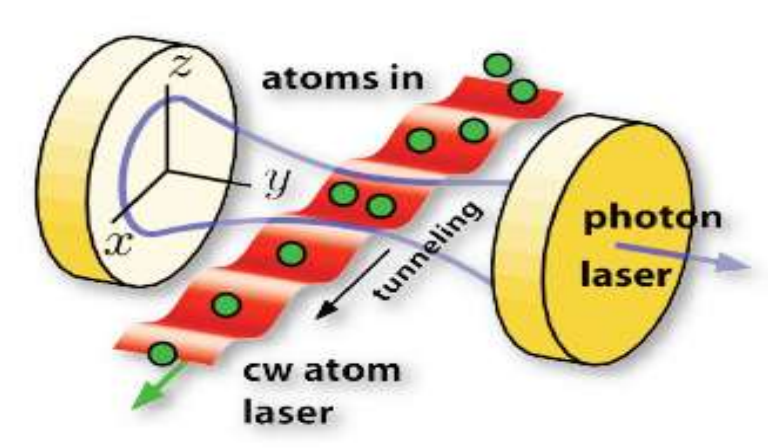
adiabatic elinination of atomic motion: atom follows light field => effective nonlinear medium

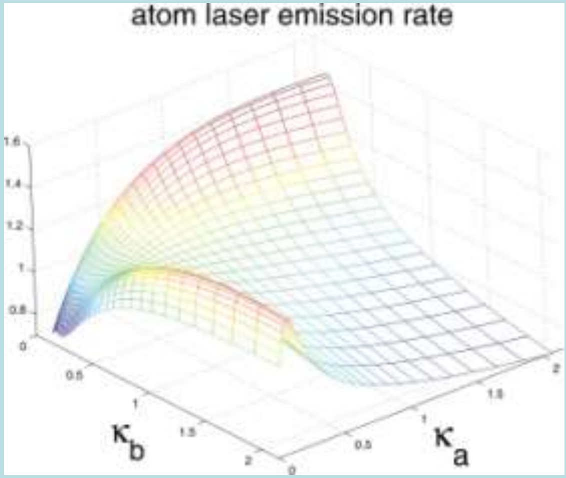
$$H_m \equiv \sqrt{\omega_{\text{rec}}\left(|V_0| + |U_0|\, a^\dagger a\right)}\,(2m+1) + H^{(\text{free})} + H^{(\text{pump})}$$

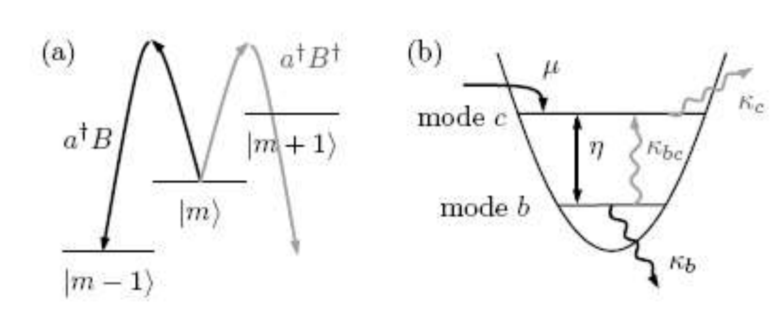
coherent input field evolves into nonclassical light

Further research topics in cavity QED with cold atoms

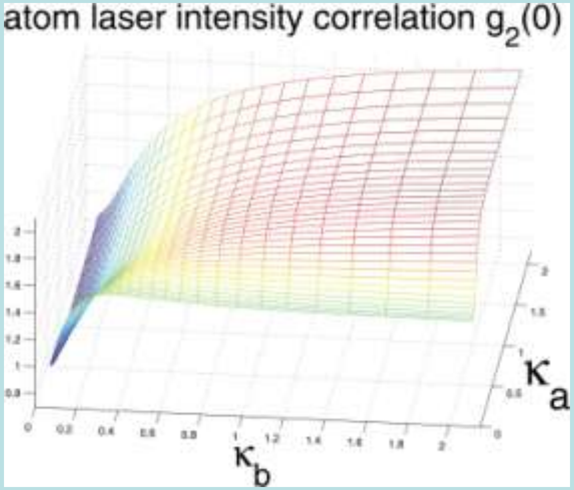
Atom-photon pair laser







$$\mathcal{H}_{\text{int}} = \eta\left(a^\dagger b^\dagger c + a\, b\, c^\dagger\right)$$



Stimulated amplification of light and atomic groundstate via blue Raman sideband

Excitons and polaritons as resonant excitaions of Mott insulator in a cavity

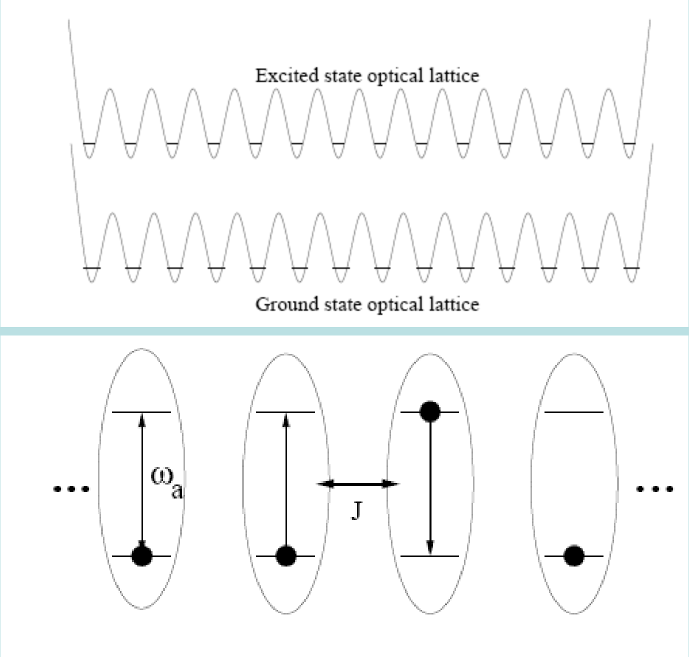
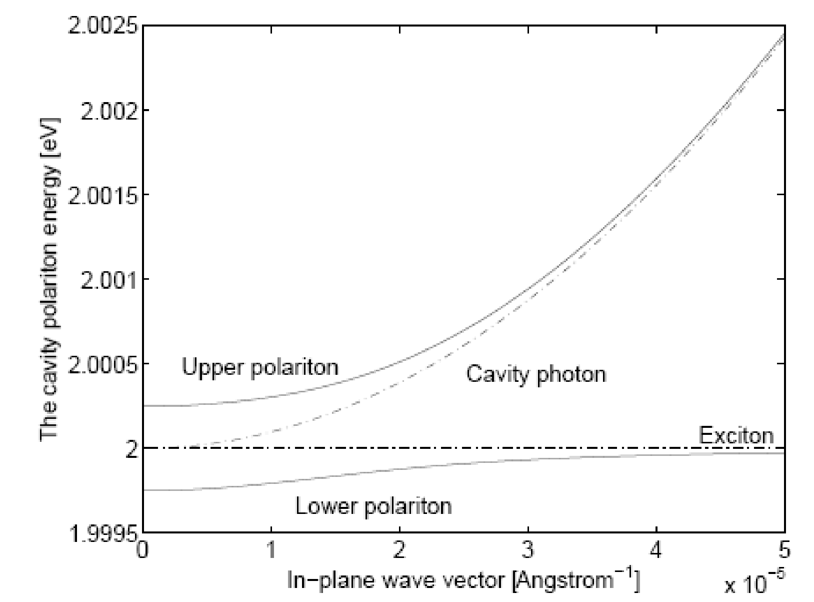
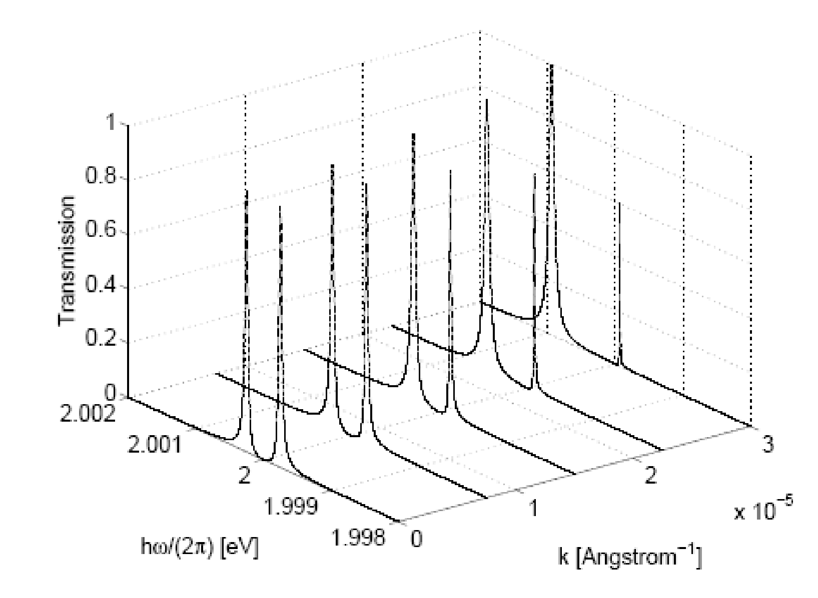


FIG. 2: Energy transfer in 1D atoms chain.

$$H_a = \sum_i \hbar\omega_a \; S_i^\dagger S_i + \sum_{i,j} \hbar J_{ij} \; S_i^\dagger S_j,$$



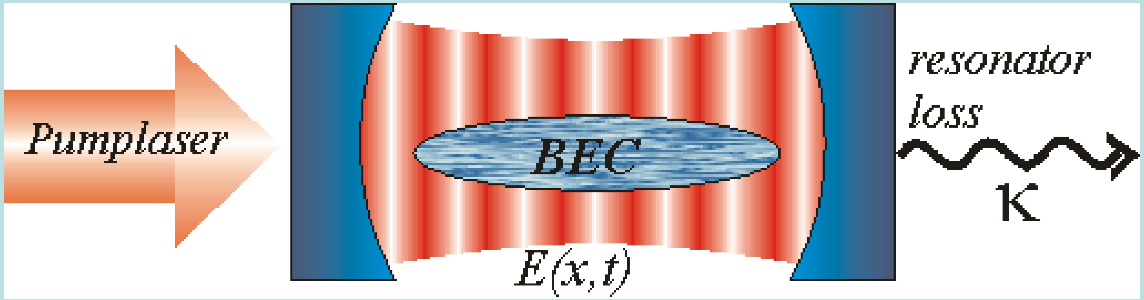


Applications: Lattice clock shifts, lattice defect detection,

*III) Ultacold quantum gas in a **quantum** optical lattice potential (quantum description of many particles and field)*

* *Cavity field generates:* ***optical lattice with dynamic quantum properties***

* *Atoms /BEC: dynamic* ***refractive index depending on the quantum state***



mean field approximation for particles and field

$$\frac{d}{dt}\alpha(t) = [i\Delta_c - iN\langle U(\hat{x})\rangle - \kappa]\alpha(t) + \eta, \qquad (1a)$$

$$i\frac{d}{dt}\psi(x,t) = \left\{ \frac{\hat{p}^2}{2m} + |\alpha(t)|^2 U(x) + N g_{coll}|\psi(x,t)|^2 \right\} \psi(x,t).$$

coupled nonlinear and nonlocal equations with a wealth of dynamic effects

Refs:

Horak, Barnett, Zoller, Meystre, Liu, Bhattacherjee…

Experiments:

Esslinger, Reichel, Zimmermann, Hemmerich, Vuletic, Treutlein ...

Generalized Bose-Hubbard model in multimode cavity generated fields

$$H = \sum_{l=0,1} \hbar\omega_l a_l^\dagger a_l + \frac{1}{2}\frac{4\pi a_s \hbar^2}{m} \int d^3\mathbf{r}\Psi^\dagger(\mathbf{r})\Psi^\dagger(\mathbf{r})\Psi(\mathbf{r})\Psi(\mathbf{r}) + \int d^3\mathbf{r}\Psi^\dagger(\mathbf{r})H_0\Psi(\mathbf{r})$$

quantized light modes

Nonlinear atom-atom interaction

Adiabatically eliminated single-particle Hamiltonian

$$H_0 = \frac{\mathbf{p}^2}{2m_a} + V_{\mathrm{cl}}(\mathbf{r}) + \hbar g^2 \sum_{l,m=0,1} \frac{u_l^*(\mathbf{r})u_m(\mathbf{r})a_l^\dagger a_m}{\Delta_{ma}}$$

One-dimensional optical lattice: $\mathbf{r}_m = x_m \mathbf{e}_x = md\mathbf{e}_x \quad \text{for } m = 1, 2, \ldots, M$

Travelling wave cavities: $u_{0,1}(\mathbf{r}_m) = \exp[i(mk_{0,1}d + \phi)]$

Standing wave cavities: $u_{0,1}(\mathbf{r}_m) = \cos(mk_{0,1}d + \phi)$

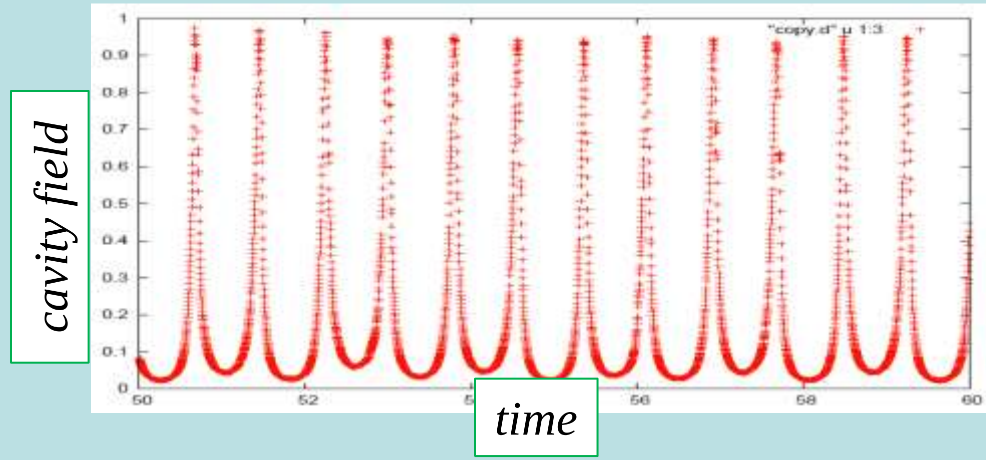
$k_{0,1} = \mathbf{k}_{0,1} \cos\theta_{0,1}$

simple effective theory :
two state expansion of BEC

$$\psi(x,t) = c_0(t) + c_2(t)\sqrt{2}\,\cos(2kx)$$

=> two x-X coupled oscillators
optomechanics – Hamiltonian

operation in unstable regime
=> self-sustained oscillations at 4 ω_r

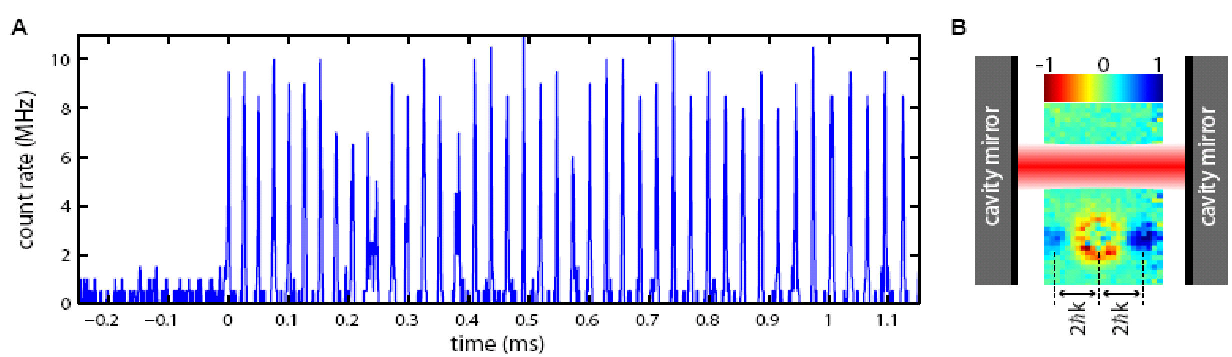


Experiment:
ETH Zürich

several theoretical descriptions

Dynamical Coupling between a Bose-Einstein Condensate and a Cavity Optical Lattice

Stephan Ritter[1,2], Ferdinand Brennecke[1], Christine Guerlin[1],
Kristian Baumann[1], Tobias Donner[1,3], Tilman Esslinger[1]*
[1] *Institute for Quantum Electronics, ETH Zürich, CH-8093 Zürich, Switzerland*
[2] *Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany*
[3] *JILA, University of Colorado and National Institute of Standards and Technology, Boulder CO 80309, USA*
(Dated: November 24, 2008)



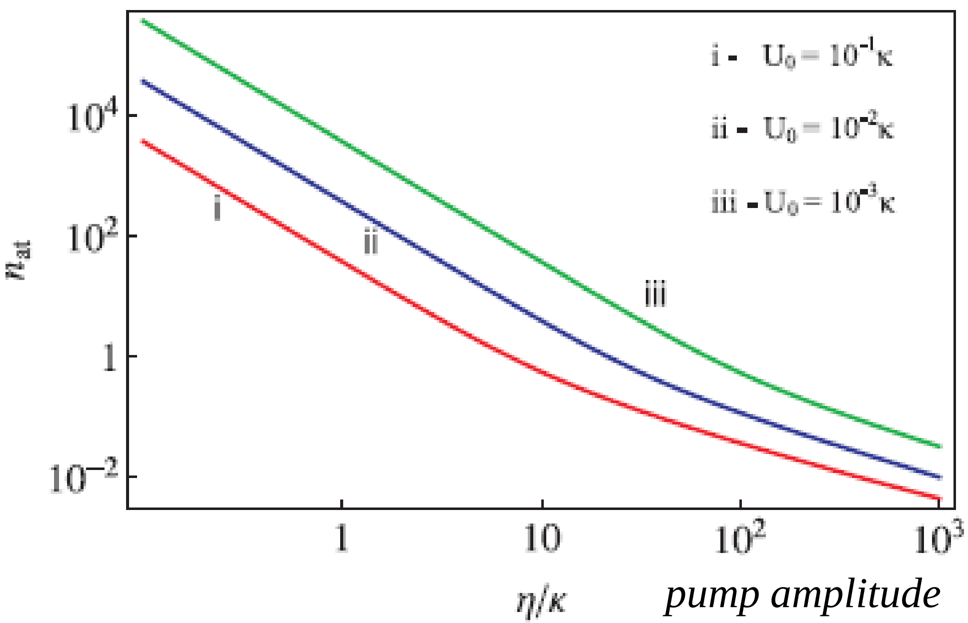
Quantum model is analytically solvable in the linear regime:

$$\langle a^\dagger a\rangle = -\frac{U_0^2(\Delta^2+\kappa^2)}{4\omega_m\Delta(\kappa^2+\Delta^2)+8\bar{U}_0^2\Delta^2},$$

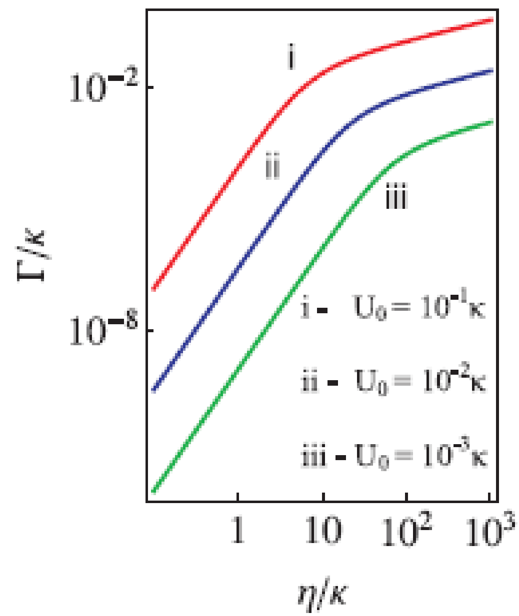
$$\langle Q^2\rangle = -\frac{(\kappa^2+\omega_m^2+\Delta^2)(\kappa^2+\Delta^2)+2\bar{U}_0^2\omega_m\Delta}{4\omega_m\Delta(\kappa^2+\Delta^2)+8\bar{U}_0^2\Delta^2} \qquad (10)$$

$$\langle P^2\rangle = -\frac{(\kappa^2+\omega_m^2+\Delta^2+2\bar{U}_0^2\Delta/\omega_m)(\kappa^2+\Delta^2)+2\bar{U}_0^2\omega_m\Delta}{4\omega_m\Delta(\kappa^2+\Delta^2)+8\bar{U}_0^2\Delta^2}$$

final occupation number



cooling rate

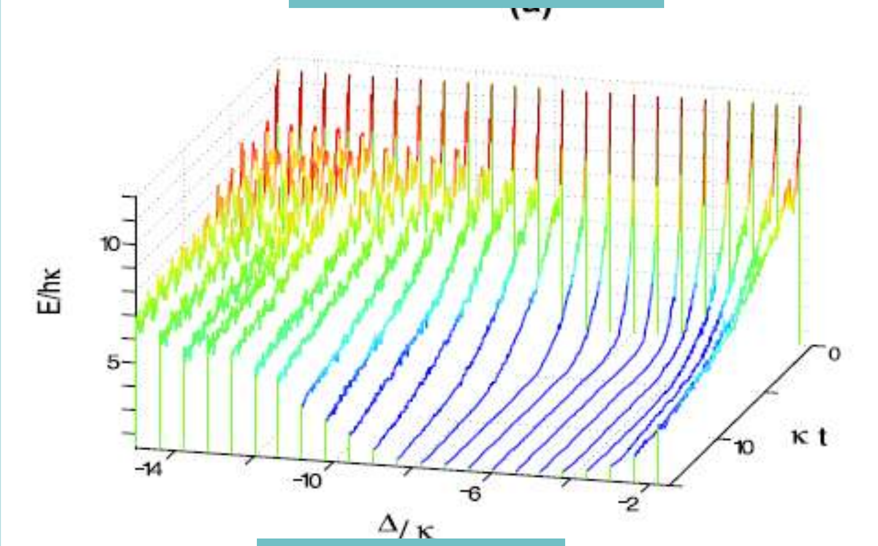


* Ground-state cooling possible
* faster cooling by increasing power

Numerical Monte Carlo wave function simulations :**C++QED**

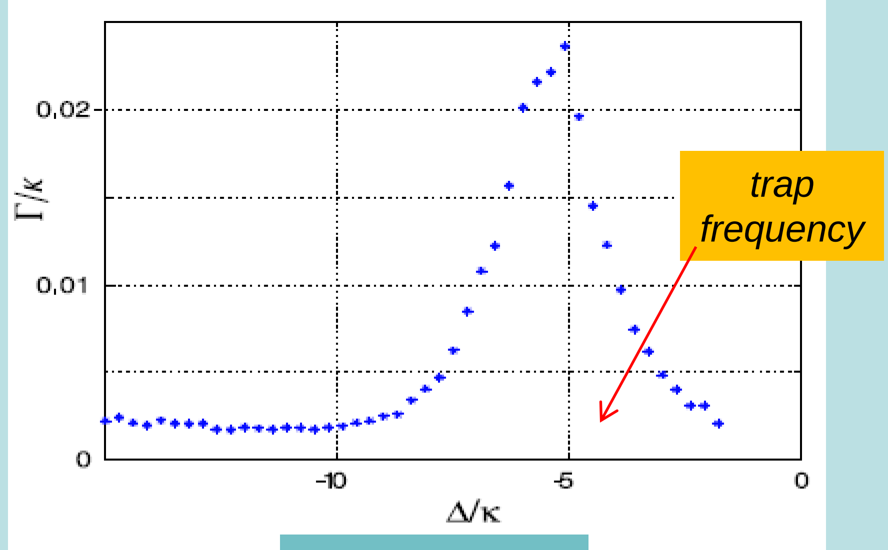
Single quantum particle in a ring cavity with dispersive interaction:
frequency dependence of cooling

Particle energy



pump frequency

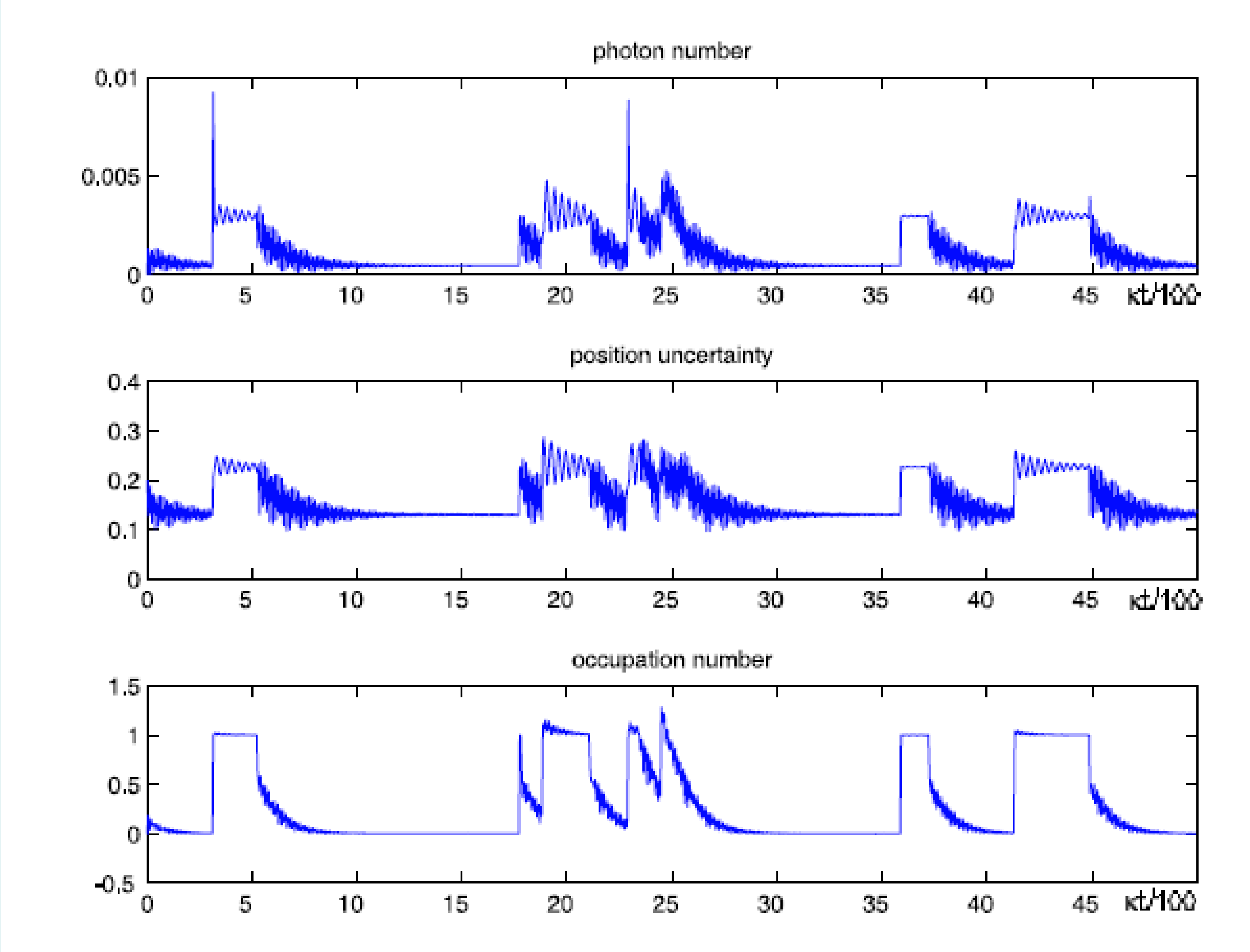
cooling rate



pump frequency

ground state cooling when mode is tuned the antistokes line : $\Delta \sim \omega >> \kappa$

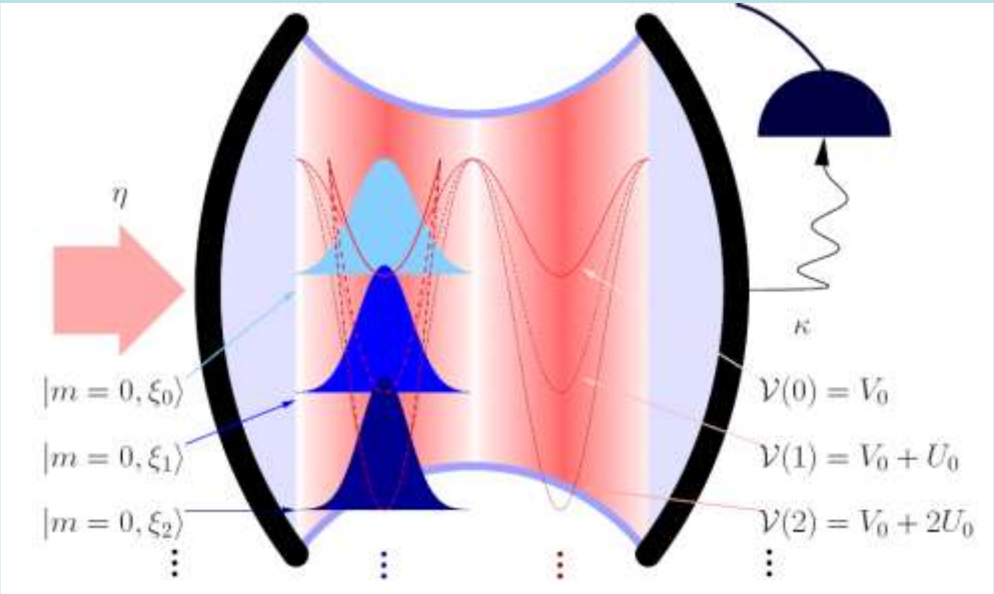
Single trajectory analysis:
Quantum jumps of molecules near ground state :



Particle jumps between two lowest states (parity change !)
first excited state in deep trap is "metastable"

Cavity non-linear optics with trapped quantum particles: x^2 term

Deep optical potential in quadratic approximantion



- ***photon number determines atomic ground state width***
- ***ground state width determines refractive index = detuning***

=> atoms create optical nonlinearity at single or few photon level

classical limiting case: Gangl PRA 2000, Stamper-Kurn PRL 2007 , Grießer 2010 heuristic derivation

Hamiltonian for a single quantum particle in deep well:

$$H = \frac{\hat{P}^2}{2m} + m\omega_R U_0 a^\dagger a \hat{X}^2 + i\eta(a - a^\dagger)$$

adiabatic elinination of atomic motion => effective nonlinear medium

atom in n-th eigenstate:

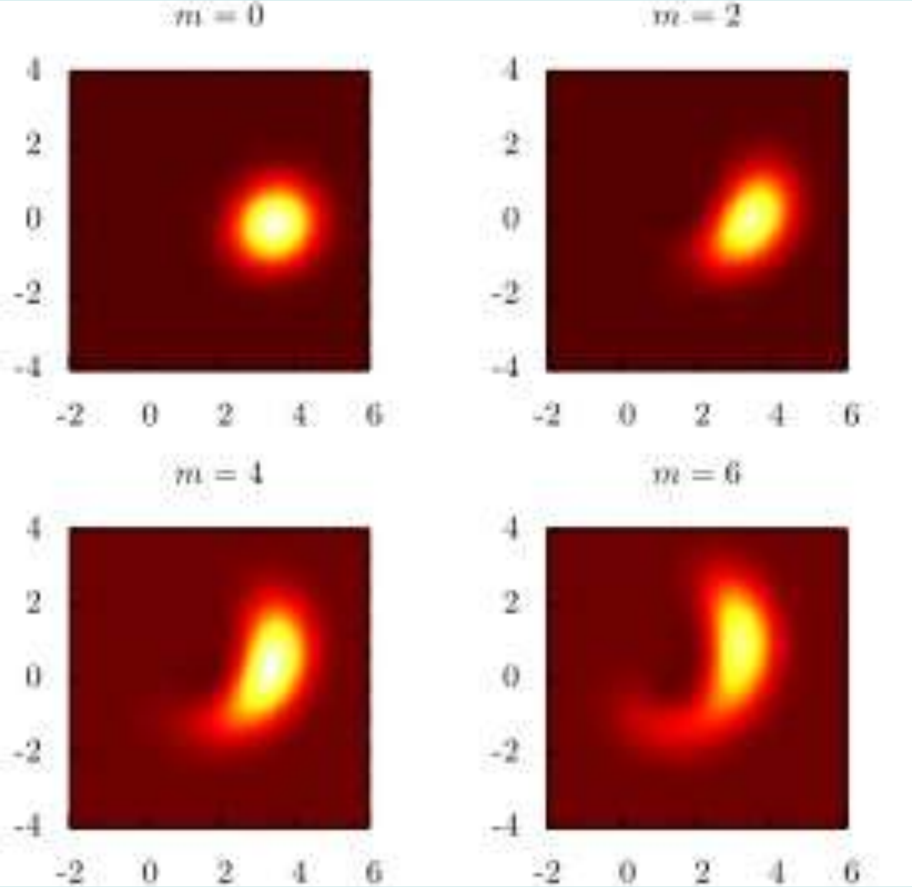
$$< X^2 > \approx \frac{n}{\sqrt{\omega_R U_0 a^\dagger a}}$$

effective field Hamiltonian

$$H_{eff} = \frac{\hat{P}^2}{2m} + (2n+1)\sqrt{\omega_R U_0 a^\dagger a} + i\eta(a - a^\dagger)$$

effektive SQRT nonlinearity => nonclassical light (G. Milburn 19??)
large difference between 0,1,2,3 …photons

Numerical calculated stationary field Wignerfunction when the atom sits in different vibrational states



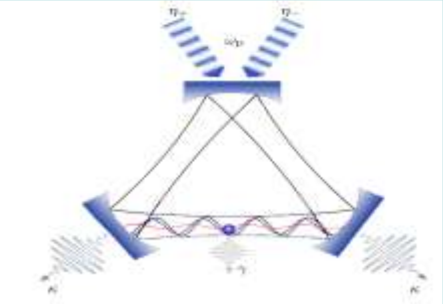
Atoms in different vibrational levels cause different nonlinearity

Tailorable nonlinear system at low photon and low atom numbers

A. Vukics, W. Niedenzu, PRA 79(2009)

Single quantum particle in a ring cavity

cooling to quantum regime
=> atom-field Hamiltonian for quantized motion:



$$H = \frac{\hat{p}^2}{2m} - \hbar\Delta\left(a_c^\dagger a_c + a_s^\dagger a_s\right) - \hbar U(\hat{x}) + i\hbar\left(\eta a_c^\dagger - \eta^* a_c\right)$$

$$U(\hat{x}) = a_c^\dagger a_c U_c(\hat{x}) + a_s^\dagger a_s U_s(\hat{x}) + \left(a_c^\dagger a_s + a_c a_s^\dagger\right) U_{cs}(\hat{x})$$

E(x,t) ~ a_c cos(kx)+a_s sinkx

generic setup:
strong pump of cosine mode:
=> deep trap for particle
=> mode in coherent state

$$\omega_m^2 = 4\omega_R U_0 \alpha^2$$

field amplitude

$$H = \left[\frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega_m^2 \hat{x}^2\right] - \hbar\Delta a^\dagger a - \hbar U_0'(a + a^\dagger)\hat{x}$$

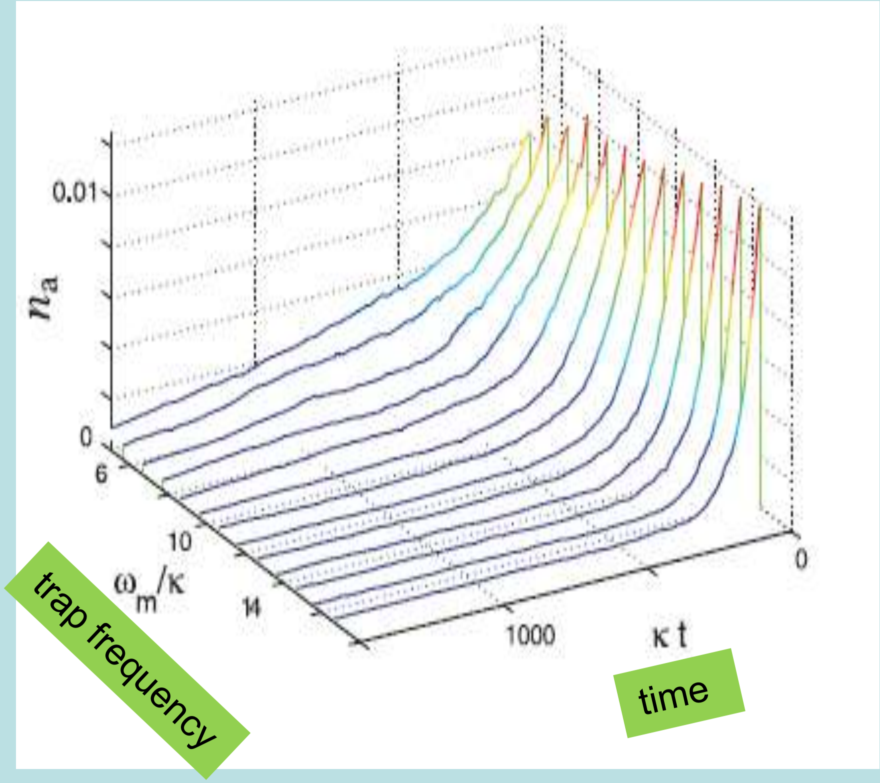
linear coupling 'x'
=> "optomechanical cooling"

quadratic coupling 'x^2' => harmonic trap

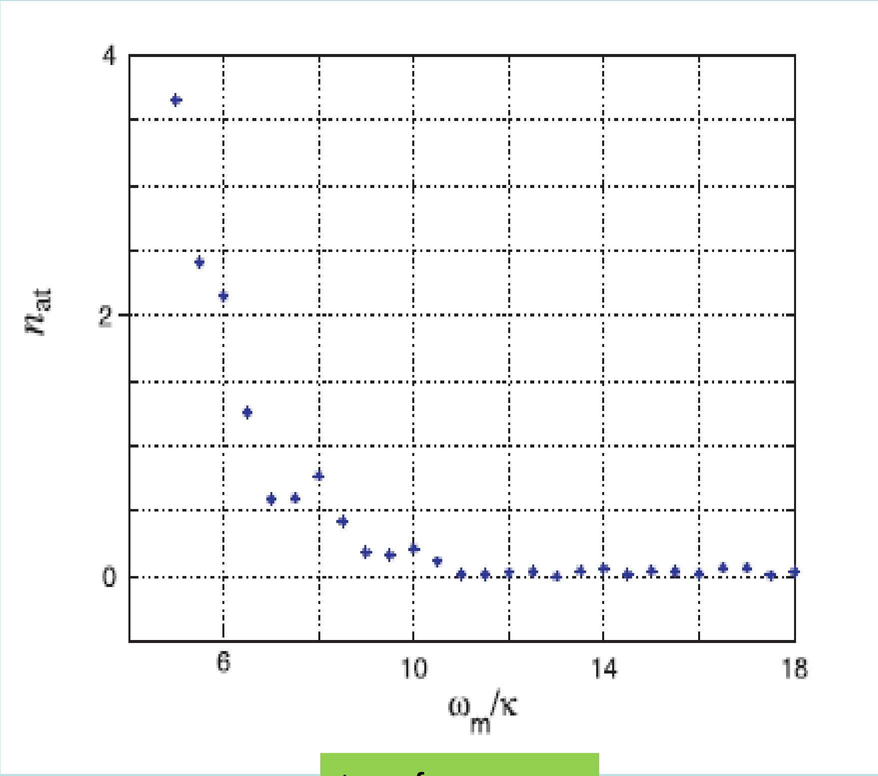
Optomechanics: all you need is power !

$$\omega_m^2 = 4\omega_R U_0 \alpha^2$$

Time evolution of quantum number



final quantum number

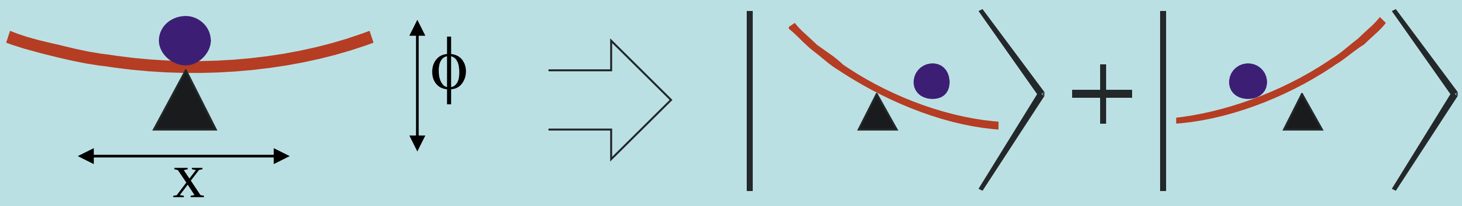


trap frequency

any polarizable nonabsorptive particle can be cooled !

Interlude

Very simple toy model:
"decay of a quantum seesaw "



classical

quantum

Two degrees of freedom: tilt angle ϕ and particle position x
=> simple model Hamiltonian:

$$V(x,\varphi) = \omega_x^2 x^2 + \omega_\varphi^2 \varphi^2 - 2J\sin(\varphi)x.$$

$$H = \frac{1}{2}(P_x^2 + P_\varphi^2 + V(x,\varphi))$$

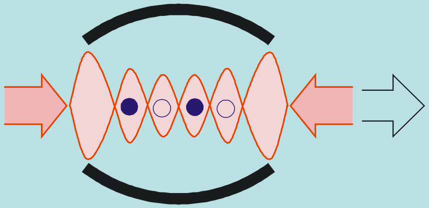
linear approx. in φ :

$$\hbar\omega_x a_x^\dagger a_x + \hbar\omega_\varphi a_\varphi^\dagger a_\varphi - \frac{J}{4}(a_\varphi^\dagger + a_\varphi)(a_x^\dagger + a_x)$$

Note: classical equilibrium point at x=ϕ=0
but
product state of oscillator ground states is not stationary

Selforganization of atoms in a lattice as seesaw

$$H=\sum_{k,l}E_{k,l}b_k^\dagger b_l+\hbar U_0\eta' g\sum_{k,l}J_{k,l}b_k^\dagger b_l+\hbar\eta'\left(a+a^\dagger\right)\sum_{k,l}\tilde{J}_{k,l}b_k^\dagger b_l-\hbar\left(\Delta_c-U_0\right)a^\dagger a$$



$$a=-i\frac{\eta'}{\kappa-i(\Delta_c-U_0)}\tilde{J}_0\left(b_1^\dagger b_1-b_2^\dagger b_2\right)$$

field replaces tilt angle <>occupation difference replaces position

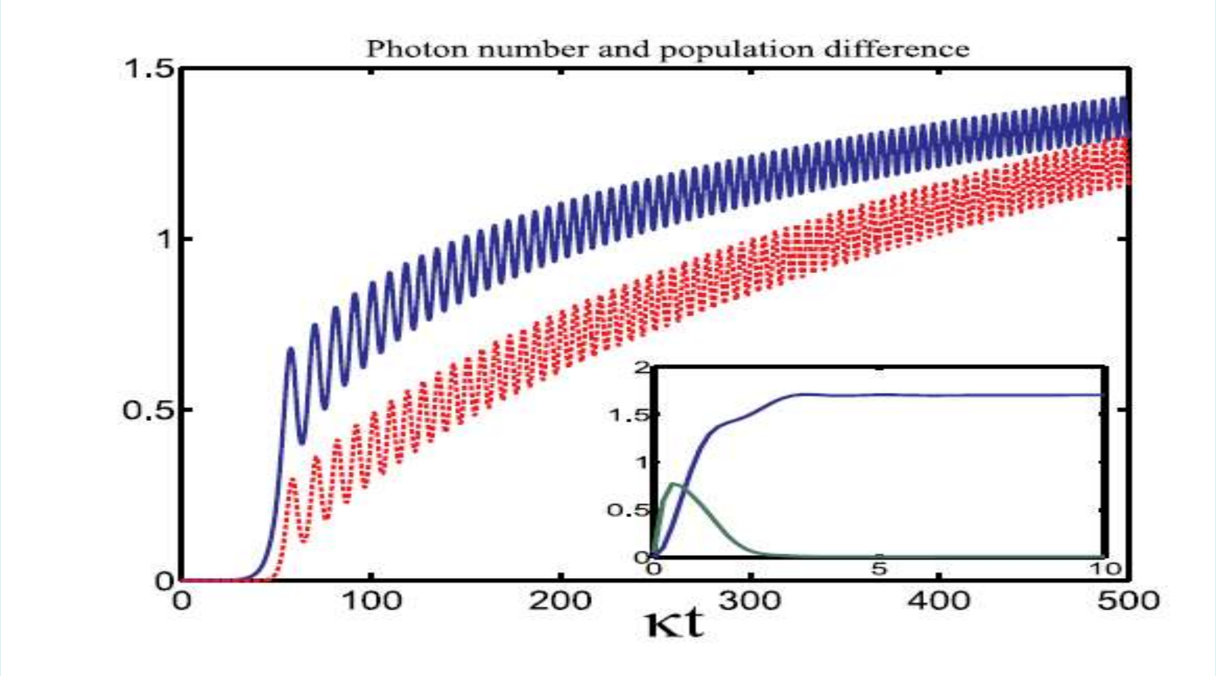
,Semiclassical‘ approximation of lattice field (n>>1) :

$$\dot{\alpha}(t)=\left[i\left(\Delta_c-U_0N\right)-\kappa\right]\alpha(t)-i\tilde{J}\left\langle b_l^\dagger b_l-b_r^\dagger b_r\right\rangle$$

$$H=J\left(b_l^\dagger b_r+b_r^\dagger b_l\right)+\hbar\tilde{J}\left(b_l^\dagger b_l-b_r^\dagger b_r\right)2\mathrm{Re}\left\{\alpha(t)\right\}$$

For symmetric initial condition (e.g. Superfluid, Mott-insulator)
no fields is created =>
symmetric initial state is stationary !

Numerical solution within classical field approximation for equal atoms number at two sites starting at almost equal population at right and left site

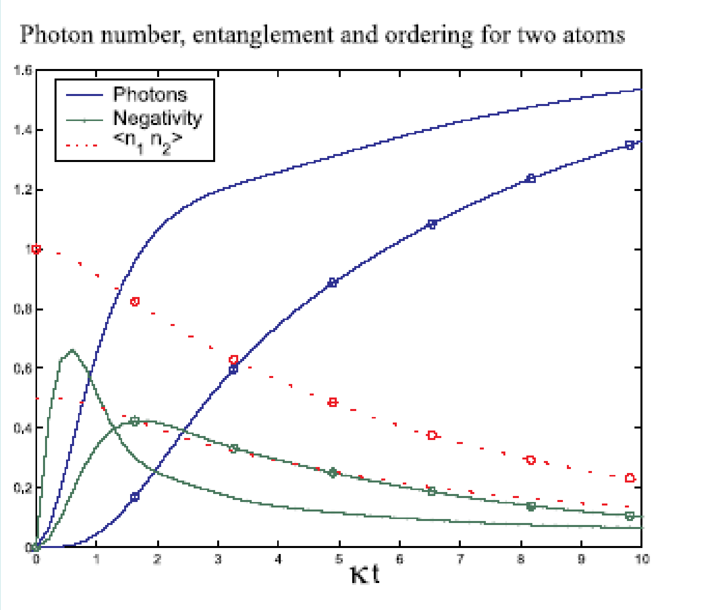


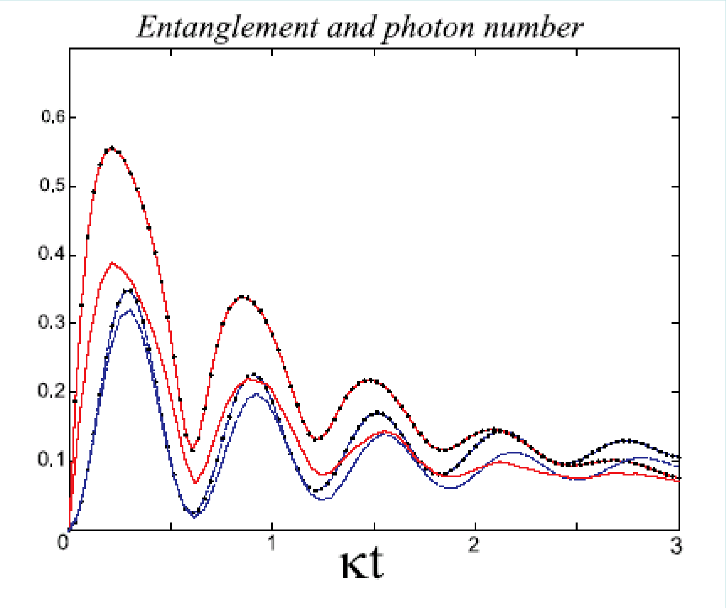
Population is stable for long time and only eventually organizes to ordered state

"Field is a measurement apparatus"

Selforganization , entanglement and statistics

Numerical solution for two atoms at two sites in a quantum potential starting at equal population at right and left site





atom + field evolve in short time towards entangled cat state !

$$1/\sqrt{2}\,(|\mathrm{left}\rangle\,|\alpha\rangle \pm |\mathrm{right}\rangle\,|-\alpha\rangle)$$

Note: superfluid selforganizes much faster than Mott insulator !!