Orbital Phases of Cold Atoms – Unconventional BEC, ferromagnetism, and f-wave pairing states

Congjun Wu

Department of Physics, UC San Diego

Bosons: C. Wu, Mod. Phys. Lett. 23, 1(2009);
C. Wu, W. V. Liu, J. Moore and S. Das Sarma, PRL 97, 190406 (2006).
W. V. Liu and C. Wu, PRA 74, 13607 (2006).
Fermions: S. Z. Zhang , H. H. Hung, and C. Wu, arXiv:0805.3031, to appear in PRA
W. C. Lee, C. Wu, S. Das Sarma, arXiv:0905.1146, to appear in PRA.
C. Wu, D. Bergman, L. Balents, and S. Das Sarma, PRL 99, 67004(2007).

Other work: http://www.physics.ucsd.edu/~wucj/Wu_Publication.html

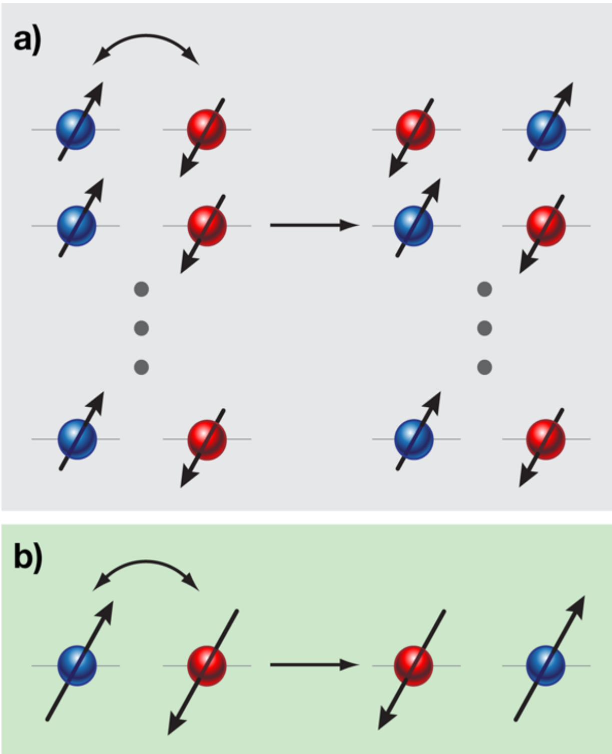
Oct 29, 2010, KITP

Collaborators:

W. C. Lee, H. H. Hung, Zi Cai	(UCSD)
S. Das Sarma	(UMD)
W. V. Liu	(Pittsburgh)
L. Balents	(UCSB)
J. Moore	(Berkeley)
Shi-zhong Zhang	(UIUC/OSU)

Thank I. Bloch, L. M. Duan, T. L. Ho for helpful discussions.

Large spin cold fermions: large S v.s. large N



a) Large spin electron systems: weak quantum fluctuation.

$$\Delta S_z = \pm 1$$

b) Large spin cold atom systems: strong quantum fluctuation .

$$\Delta F_z = \pm 1, \pm 2, \ldots \pm F$$

c) Spin-3/2 fermions: hidden **Sp(4) /SO(5)** symmetry without fine tuning.
Brief review: C. Wu, Mod Phys Lett B 20, 1707 (2006)

Online talk: **http://online.itp.ucsb.edu/online/coldatoms07/wu2/**

Recent exp: ^{87}Sr(I=9/2) (T. Killian), ^{173}Yb (I=5/2) (Y. Takahashi)

Theory: large N, Hermele, Gurarie, Rey et al.

Outline

- **Introduction: cold atoms in high orbital bands.**
  1. What is orbital physics in the condensed matter context?
  2. Why orbital physics with cold atoms is interesting? Some pioneering experiments.

- Orbital bosons: unconventional BEC beyond the “no-node” theorem.

- Orbital fermions:

Exotic band (quantum anomalous Hall) and Mott insulators (120 degree model): **http://online.itp.ucsb.edu/online/lowdim_c09/wu/**

Itinerant ferromagnetism (FM): flat band FM.

Unconventional Cooper pairing: f-wave.

FFLO states of fermions in the p-bands: 1D Fermi surface+2D phase coherence.

Research progress of cold atom physics

- Great success of cold atom physics in the past decade:

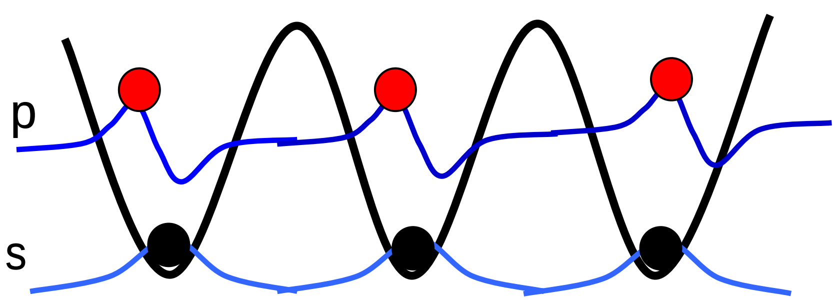
  BEC; superfluid-Mott insulator transition; multi-components bosons and fermions; BEC-BCS crossover ...

- **Orbital** Physics: new physics of bosons and fermions in high-orbital bands in optical lattices.

Here orbital refers to the different energy levels (e.g. s, p) of each optical site.

Good timing: pioneering experiments on orbital-bosons.

Square lattice (Mainz); double well lattice (NIST, Hamburg); polariton lattice (Stanford).



J. J. Sebby-Strabley, et al., PRA 73, 33605 (2006); T. Mueller et al., Phys. Rev. Lett. 99, 200405 (2007); C. W. Lai et al., Nature 450, 529 (2007).

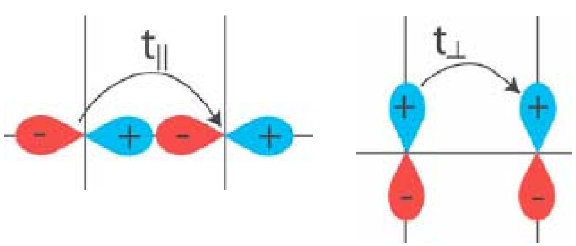
Orbital physics

- Orbital: a degree of freedom independent of charge and spin.

Tokura, et al., science 288, 462, (2000).

- Two fundamental features: orbital degeneracy and **spatial anisotropy.**

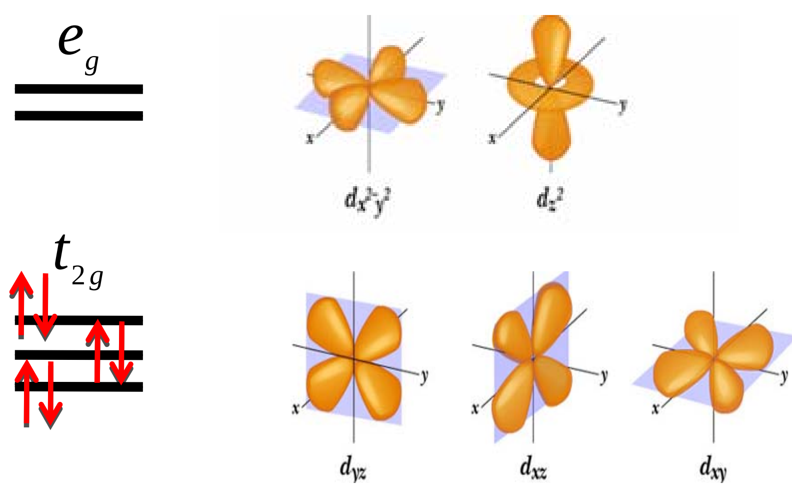
p-orbitals: p_x, p_y, p_z



σ-bond **π-bond**

$$t_{//} >> t_{\perp}$$

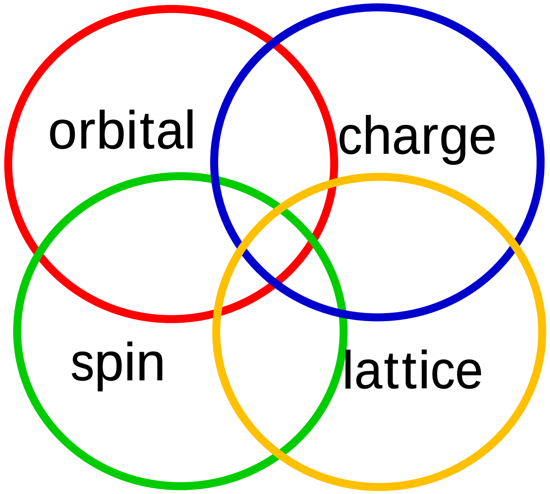
d-orbitals: $d_{x^2-y^2}, d_{r^2-3z^2}, d_{xy}, d_{yz}, d_{xz}$

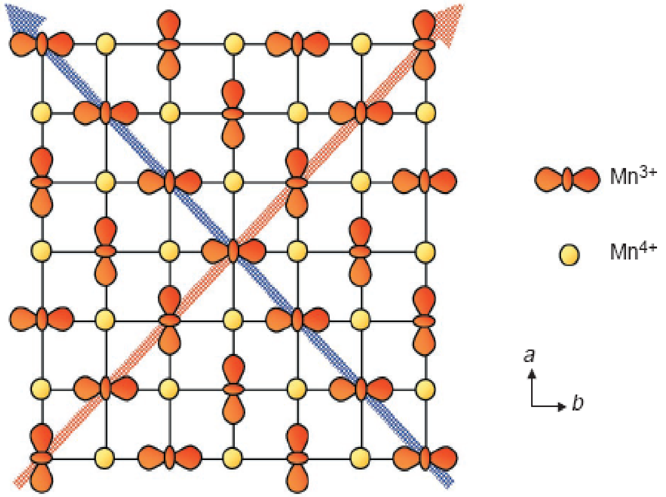


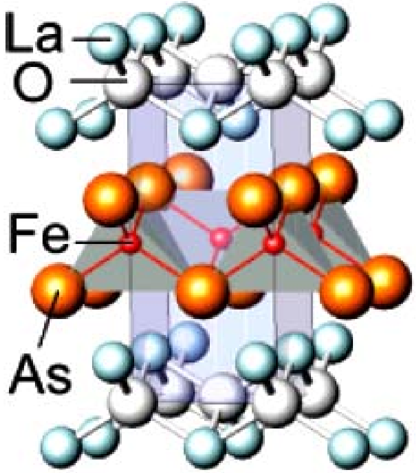
crystal field splitting

Transition metal oxide orbital systems

• Orbitals play an important role in magnetism, superconductivity, and transport properties.





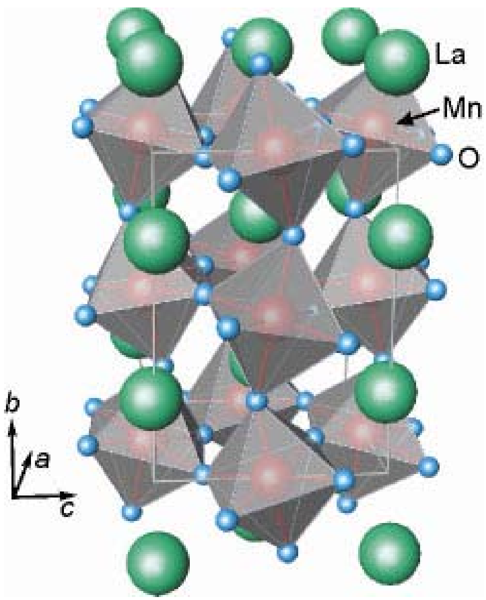


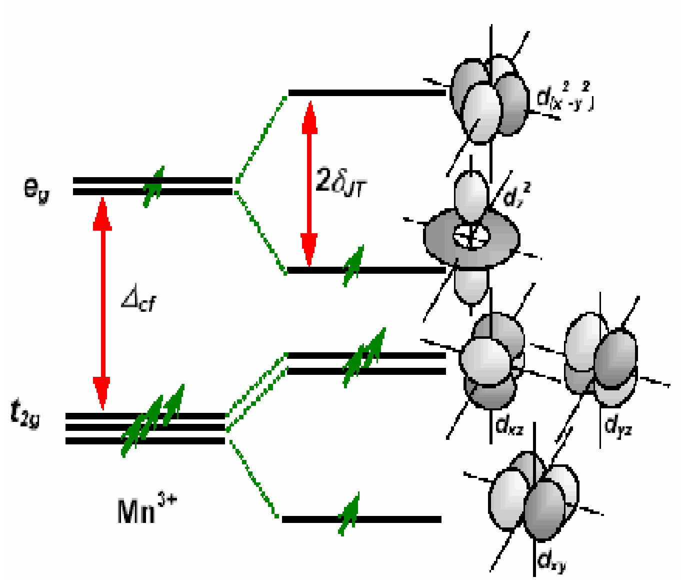
Manganite: $La_{1-x}Sr_{1+x}MnO_4$

Iron-pnictide: LaOFeAs

Cold atom orbital systems (I): no Jahn-Teller distortion

- Solid state orbital systems: lattice is not rigid. Jahn-Teller distortion lifts the orbital degeneracy and **quenches** the orbital degree of freedom.





- Cold atom orbital systems: atoms in external optical lattices.

Rigid lattice **free of distortion**; orbital degeneracy is robust.

Cold atom orbital systems (II): orbital bosons

- Solid state orbital systems: orbital physics **only of fermions** .

- Cold atom orbital systems: both fermions and **bosons**.

The ordinary many-body ground states of bosons satisfy the "**no-node" theorem**, i.e., the wavefunction is positive-definite.

Orbital bosons: (meta-stable excited states of bosons).

New materials beyond the "**no-node**" theorem.
Unconventional BEC with spontaneous breaking of time-reversal symmetry.

Cold atom orbital systems (III): strongly correlated p-orbitals

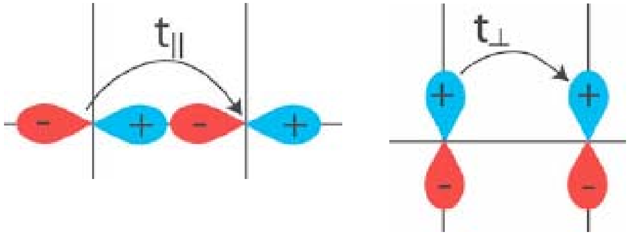
- Solid state: strongly correlated orbital systems are usually *d*-orbital transition metal oxides and *f*-orbital rare-earth compounds.

- Most *p*-orbital solid state materials are weakly correlated (e.g. semiconductors). **Not many *p*-orbital Mott-insulators**.

- Cold atom orbital systems:

New materials: strongly correlated *p*-orbital systems.

$t_{//} >> t_{\perp}$



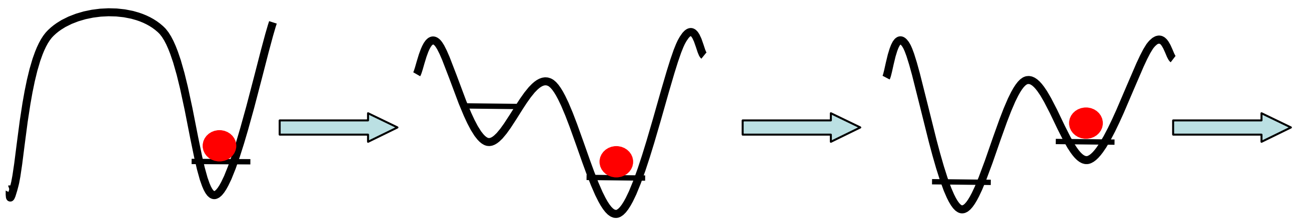
σ-bond π-bond

P-orbital has even **stronger anisotropy** than *d* and *f* ; combined with strong **correlation.**

Double-well lattice at NIST: transfer bosons to the excited band

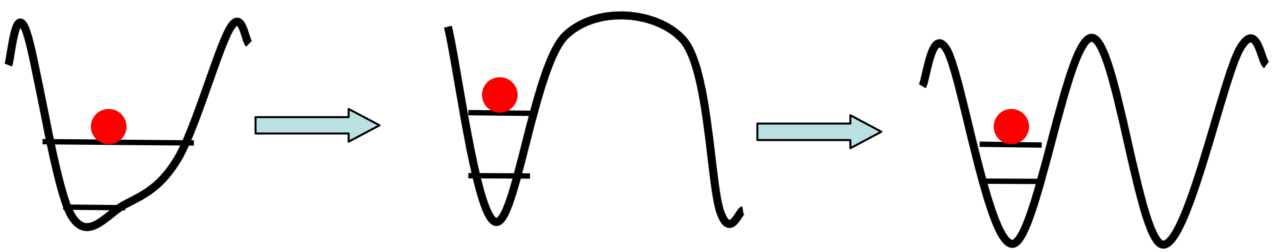
Grow the long period lattice

Avoid tunneling (diabatic)

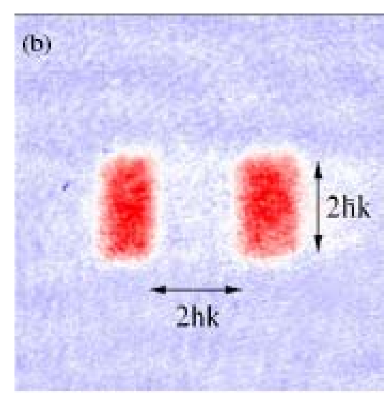


Create the excited state (adiabatic)

Create the short period lattice (diabatic)



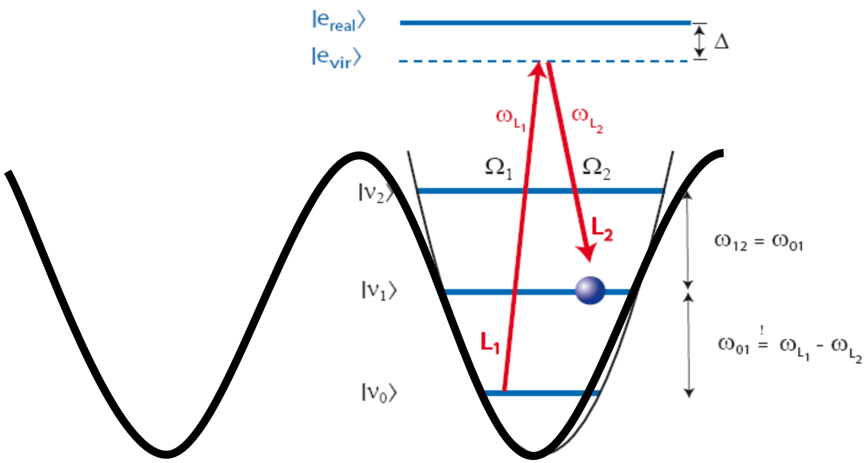
- Band mapping.
- Phase incoherence.

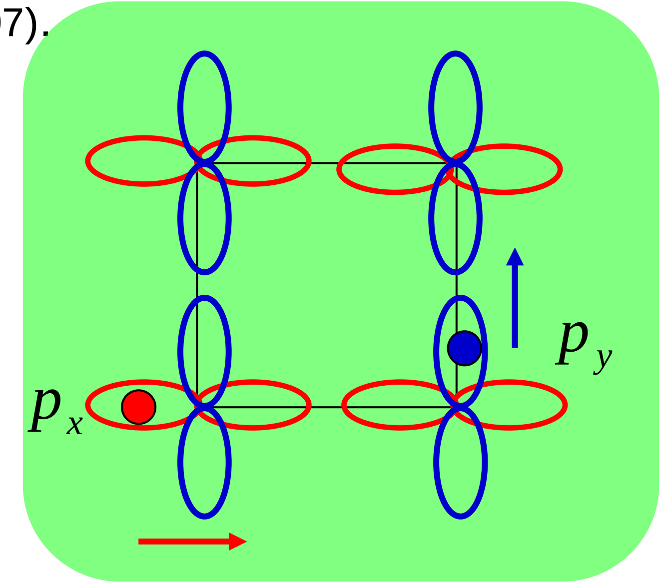


M. Anderlini, et al., J. Phys. B 39, S199 (2006).

Orbital bosons: pumping bosons by Raman transition

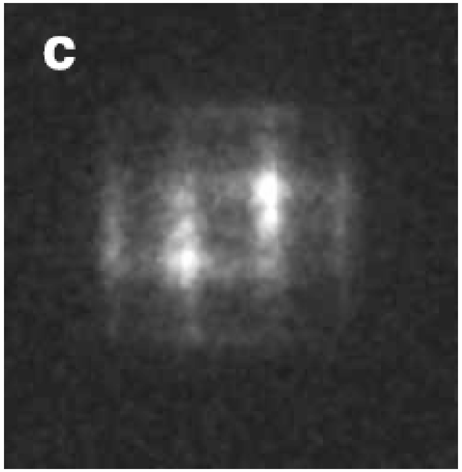
T. Mueller, I. Bloch et al., Phys. Rev. Lett. 99, 200405 (2007).



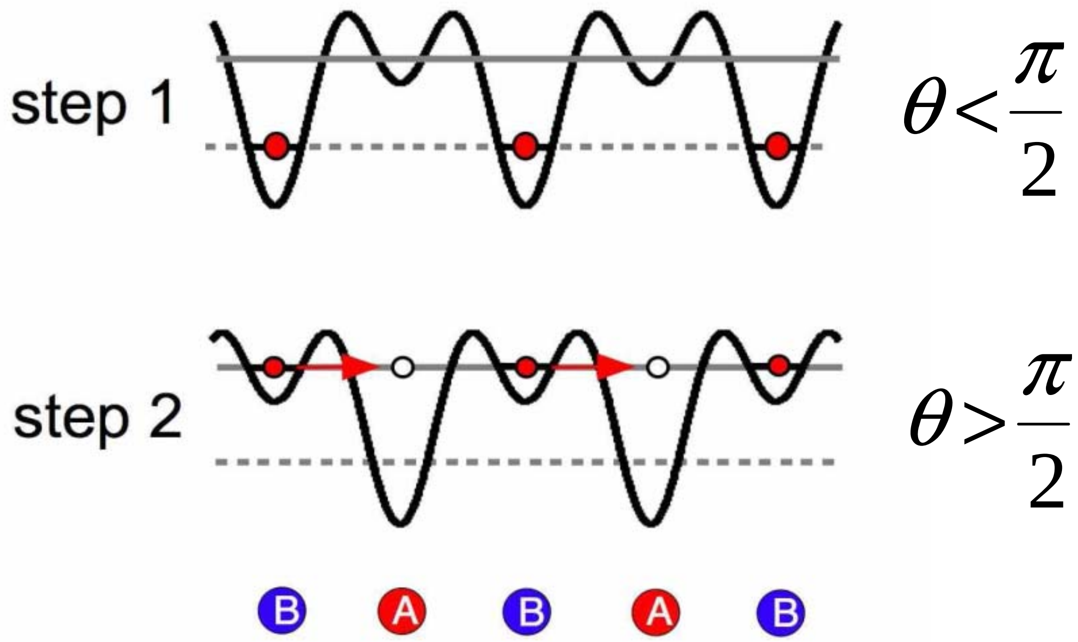


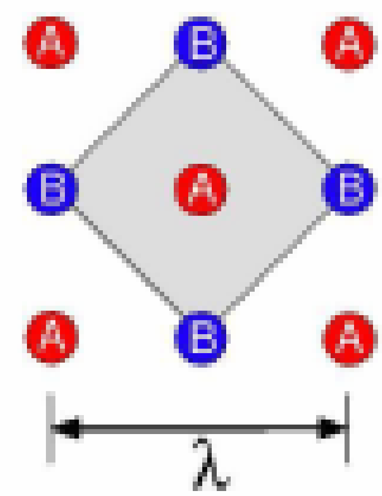
- Long life-time (~100 hopping time) to develop 1-d phase coherence.

- Quasi-1d band feature in the square lattice.



P-orbital BEC with non-zero center of mass momentum

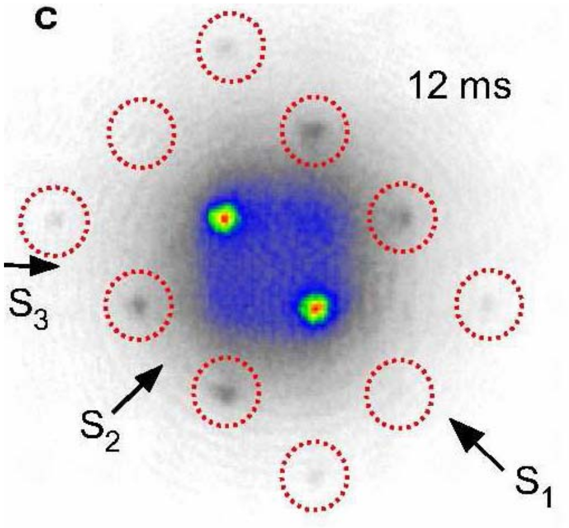


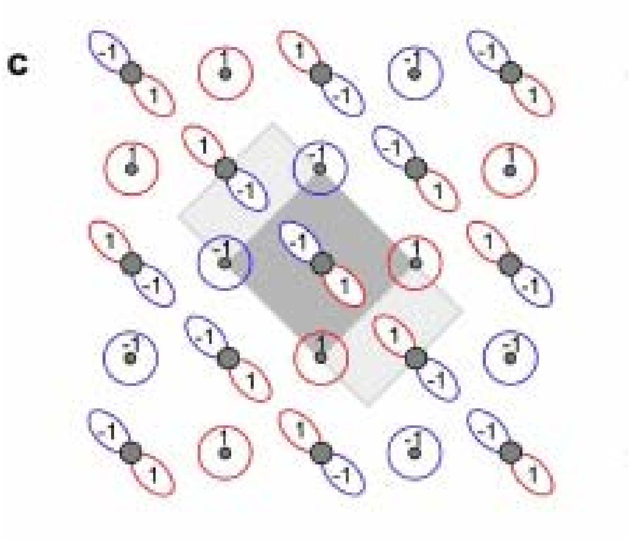


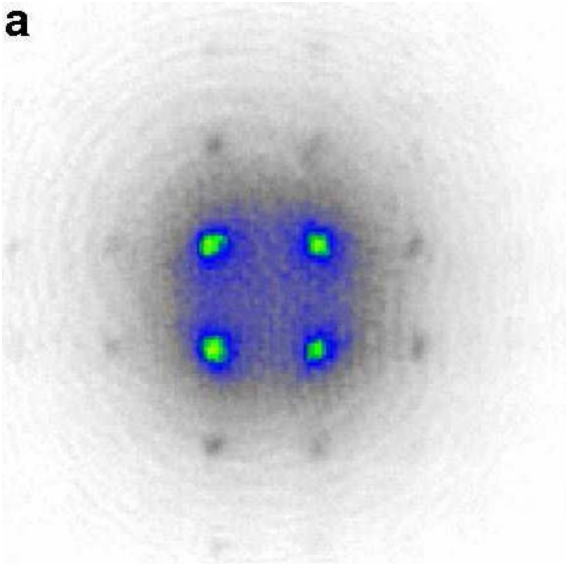
$$V(x,y) = -V_0\left[\sin^2(kx) + \sin^2(ky) + 2\cos(\theta)\sin(kx)\sin(ky)\right]$$

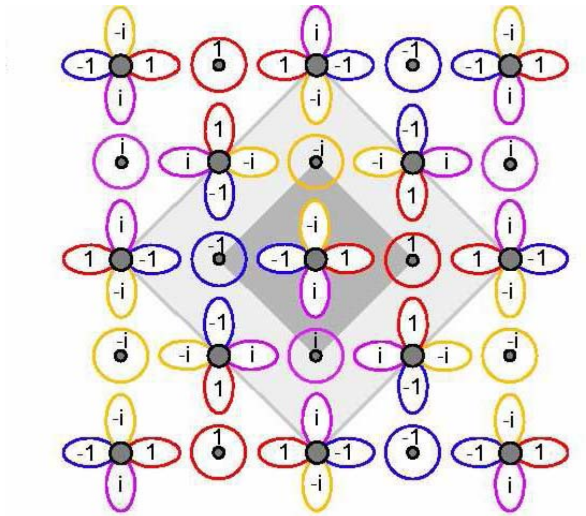
Wirth, Oelschlaeger, Hemmerich, arXiv:1006.0509.

Asymmetric (real, nodal) to symmetric (complex) p-BEC









Wirth, Oelschlaeger, Hemmerich, arXiv:1006.0509.

Outline

- Introduction: orbital physics, a new research direction of cold atoms.

- Orbital bosons: unconventional BEC with spontaneous time reversal symmetry breaking.

  C. Wu, Mod. Phys. Lett. 23, 1(2009) (brief review);
  V. M. Stojanovic, C. Wu, W. V. Liu and S. Das Sarma, PRL 125301(2008);
  C. Wu, W. V. Liu, J. Moore and S. Das Sarma, PRL 97, 190406 (2006);
  W. V. Liu and C. Wu, PRA 74, 13607 (2006).

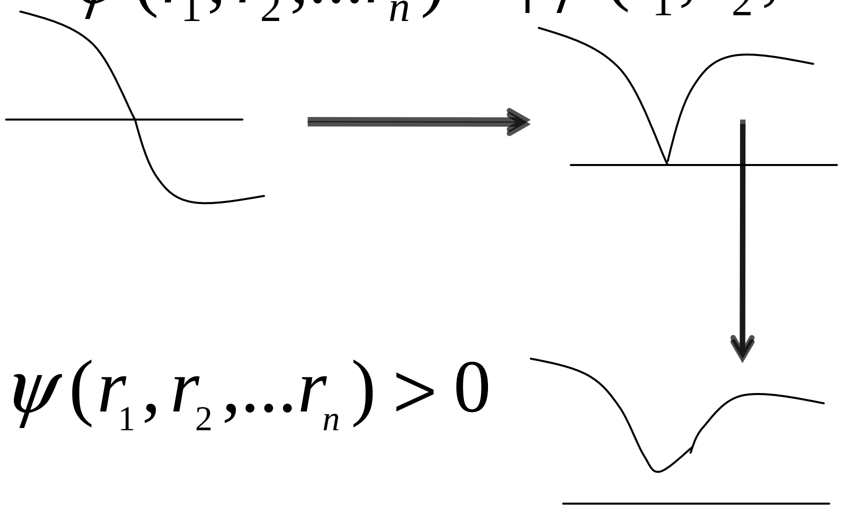
  Other group's related work: V. W. Scarola *et. al,* PRL, 2005; A. Isacsson *et. al.,* PRA 2005; A. B. Kuklov, PRL 97, 2006; C. Xu *et al.*, cond-mat/0611620 ....

- Orbital fermions in the hexagonal lattice.

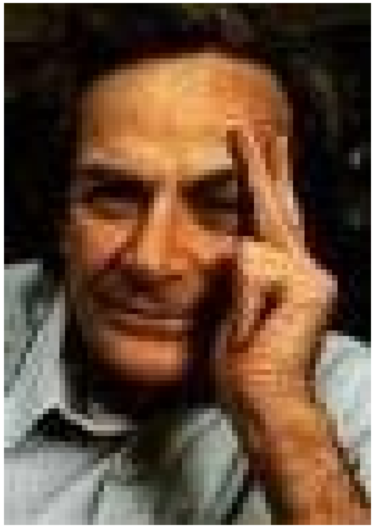
The “no-node” theorem (single component bosons)

- Many-body **ground state** wavefunctions of bosons in the coordinate-representation are positive-definite if without rotation.

$$\psi(r_1, r_2, \ldots r_n) \qquad |\psi(r_1, r_2, \ldots r_n)|$$



$$\psi(r_1, r_2, \ldots r_n) > 0$$



R. P. Feynman

$$\langle \psi | H | \psi \rangle = \int dr_1 \ldots dr_n \; \frac{\hbar^2}{2m} \sum_{i=1}^{n} \; |\nabla_i \psi(r_1, \ldots r_n)|^2 + |\psi(r_1, .. r_n)|^2 \sum_{i=1}^{n} U_{ex}(r_i)$$

$$+ |\psi(r_1, .. r_n)|^2 \sum_{i<j} V_{\text{int}}(r_i - r_j)$$

Go beyond the "no-node" paradigm

- It applies to all of the ground states of bosons, (e.g. superfluid, Mott-insulating, super-solid, density-wave states).

- Strong constraint!

Complex-valued wavefunction reduces to positive-definite → quantum Monte-Carlo is free of the sign problem.

Time-reversal (TR) symmetry cannot be spontaneously broken!

- Our goal: unconventional BEC spontaneously breaking TR.

Excited (meta-stable): bosons in high orbital bands ---- **orbital physics of bosons**.

Spin-orbit coupled BEC → spontaneous half-quantum vortex string.

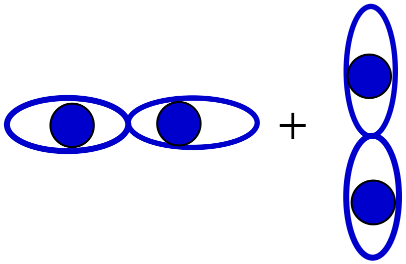
c.f. C. Wu and I. Mondragon-shem, arxiv:0809.3532; C. Wu, Mod. Phys. Lett. 23, 1 (2009).

Ferro-orbital interaction for spinless p-orbital bosons

- **A single site** problem: two orbitals p_x and p_y with two spinless bosons.

$$V(r_1-r_2)=g\delta(r_1-r_2) \qquad U=g\int dr\,|\phi_x(r)|^4=g\int dr\,|\phi_y(r)|^4$$

$$x^2+y^2 \quad L_z=0:\frac{1}{\sqrt{2}}(p_x^+p_x^+ + p_y^+p_y^+)\left|0\right\rangle$$



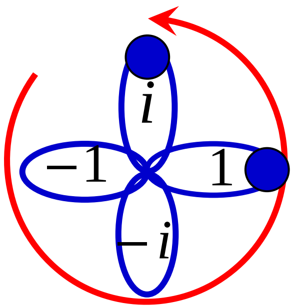
polar

$$E_0=\tfrac{4}{3}U$$

$$(x\pm iy)^2 \quad L_z=\pm 2:\ \{\frac{1}{\sqrt{2}}(p_x^+ \pm i p_y^+)\}^2\,|0\rangle$$

$$=\{\frac{1}{2}(p_x^+p_x^+ - p_y^+p_y^+)\pm\frac{i}{\sqrt{2}}p_x^+p_y^+\}\left|0\right\rangle$$

axial



$$E_{\pm 2}=\tfrac{2}{3}U$$

Axial states are spatially more extended!

$$E_{\pm 2} < E_0$$

Orbital Hund's rule for bosons

$$H_{\text{int}} = \frac{U}{2}\sum_r \{n_r^2 - \frac{1}{3}(L_r^z)^2\}$$

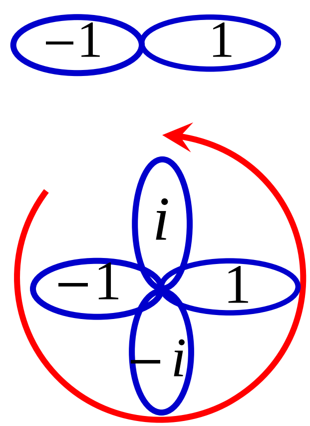
$$n = p_x^+ p_x + p_y^+ p_y, L_z = -i(p_x^+ p_y - p_y^+ p_x)$$

- If more than two bosons occupy in the same site, they aggregate into the same single-particle state.

- **Oribital angular momentum moments**: the same axial state (e.g. p+ip) instead of the polar state (e.g. p_x) to minimize repulsion.

- *cf*. Second Hund's rule for electrons.

- c.f. p+ip superconductors.

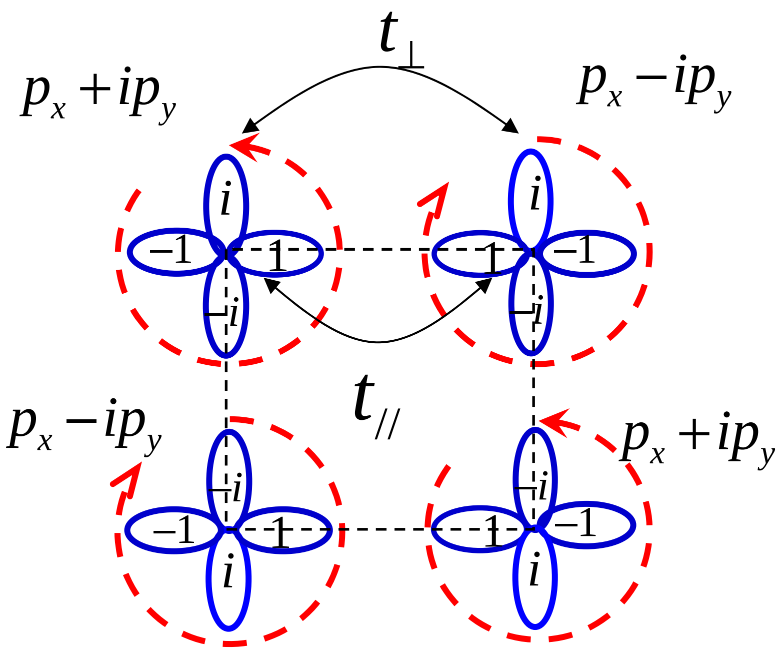


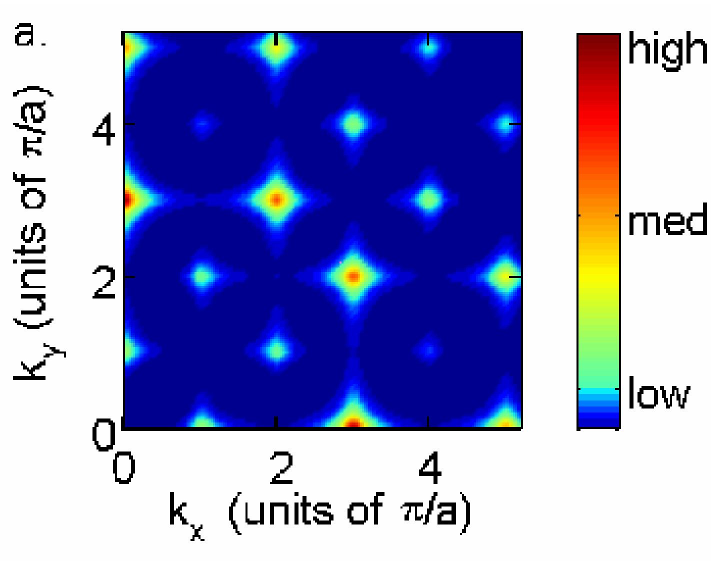
polar v.s.

axial

Unconventional BEC with TR symmetry breaking: square lattice

• Inter-site tunneling orders the onsite orbital angular momentum (OAM) moments. Staggered OAM ordering.





• Time of flight (zero temperature): Bragg peaks located at fractional values of reciprocal lattice vectors.

$$((m+\tfrac{1}{2})\frac{\pi}{a},0) \quad (0,(n+\tfrac{1}{2})\frac{\pi}{a})$$

W. V. Liu and C. Wu, PRA 74, 13607 (2006).

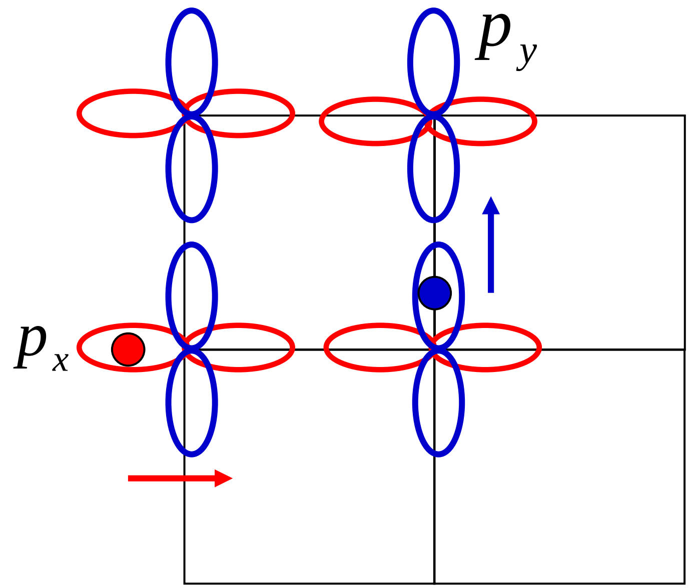
Quasi-1D behavior at finite temperatures

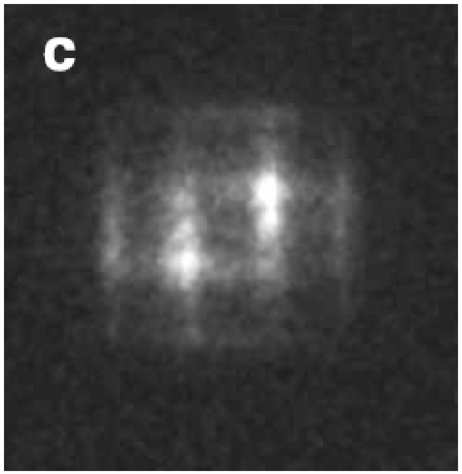
- Because $t_{\perp} << t_{//}$, p_x-particles can maintain phase coherence within the same row, but loose inter-row phase coherence at finite temperatures.

- Similar behavior also occurs for p_y-particles.

- The system effectively becomes 1D-like as shown in the time of flight experiment.

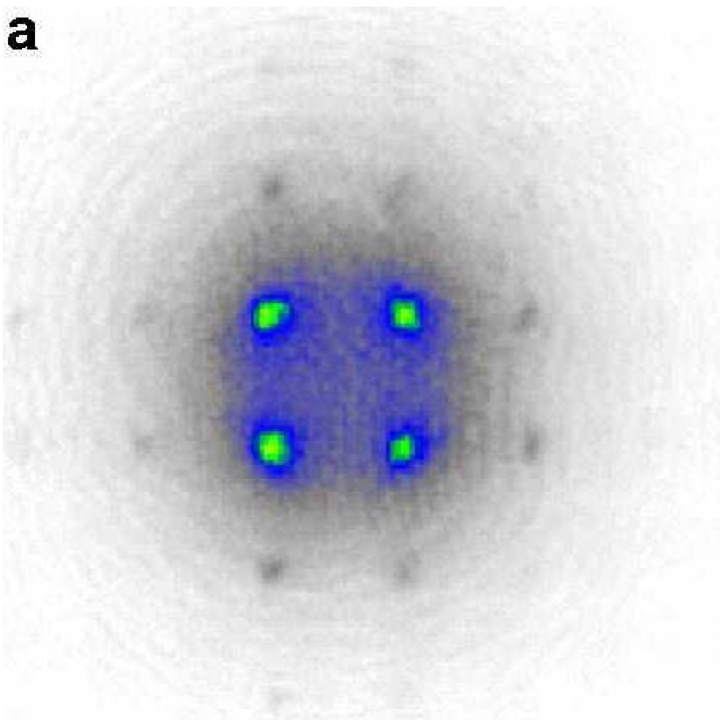
A. Isacsson et. al., PRA 72, 53604 (2005).

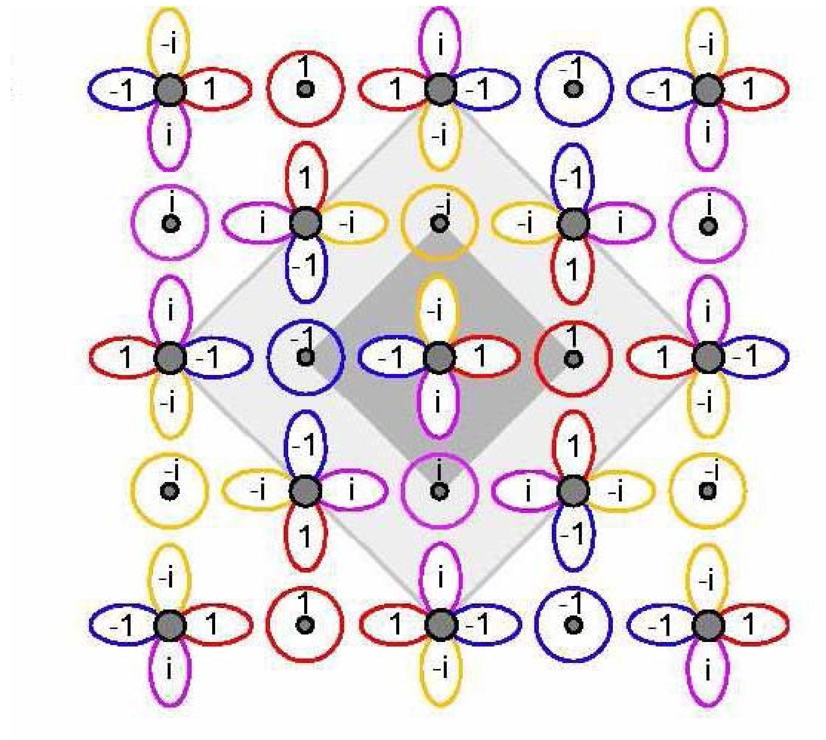




T. Mueller, I. Bloch et al.

Complex BEC beyond the “no-node” theorem

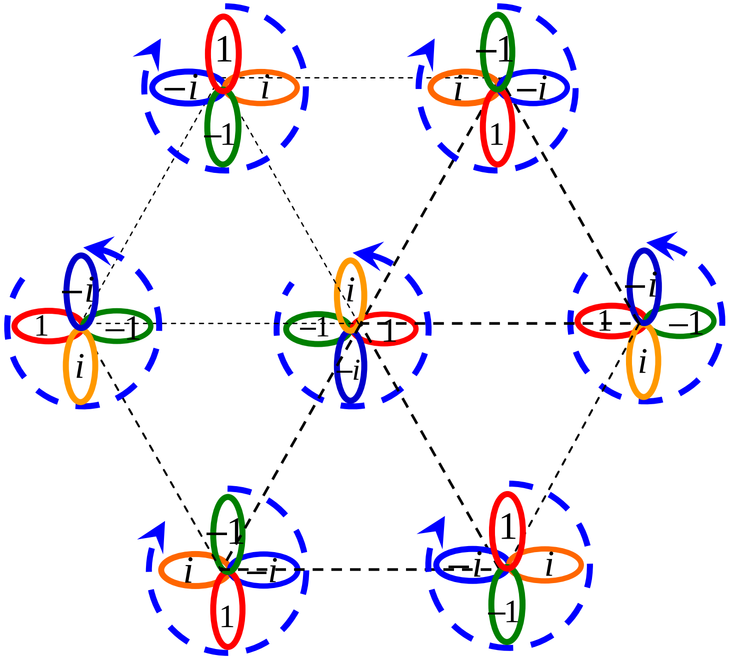


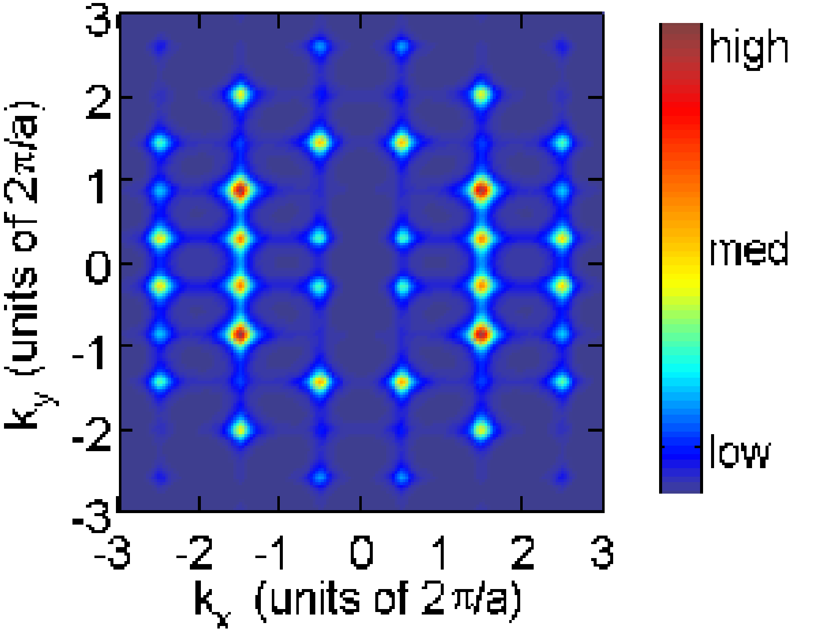


Wirth, Oelschlaeger, Hemmerich, arXiv:1006.0509.

Unconventional BEC with TR symmetry breaking: triangular lattice

- Stripe ordering of orbital angular momentum moment in the triangular lattice.

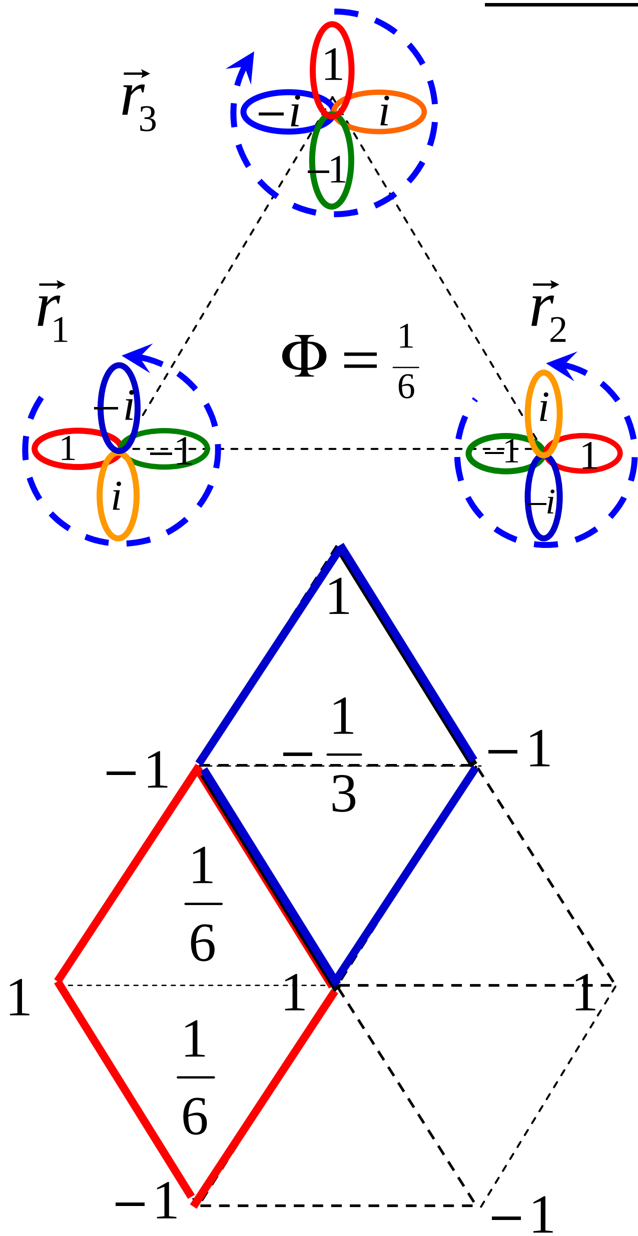




- Each site behaves like a vortex with long range interaction in the superfluid state. Stripe ordering to minimize the global vorticity.

C. Wu, W. V. Liu, J. Moore and S. Das Sarma, Phy. Rev. Lett. 97, 190406 (2006).

Strong coupling analysis



- Ising variable for vortex vorticity:

$$\sigma = \pm 1$$

- The minimum of the effective flux per plaquette is $\pm 1/6$.

$$\Phi_i = \frac{1}{2\pi} \sum_{\langle r,r' \rangle} A_{r,r'} = \frac{1}{6}(\sigma_{r1} + \sigma_{r2} + \sigma_{r3})$$

- The stripe pattern minimizes the ground state vorticity.

- cf. The same analysis also applies to p+ip Josephson junction array.

Outline

• Introduction: orbital physics is an interesting research direction of cold atoms.

• Orbital bosons: unconventional BEC beyond the “no-node” theorem.

• Orbital fermions in the honeycomb lattice. **P-orbital Fermions are stable due to Pauli’s exclusion principle!**

Ferromagnetism from band flatness.

C. Wu, and S. Das Sarma, PRB 77, 235107(2008);
C. Wu et al, PRL 99, 67004(2007).
Shi-zhong zhang, Hsiang-hsuan Hung, and C. Wu, arXiv:0805.3031, to appear in PRA.

A new mechanism for the **unconventional pairing: the f-wave**.

The early age of ferromagnetism

World's first compass
(司南 South-pointer)

↑
south

Thales says that a stone (lodestone) has a soul because it causes movement to iron.
----*De Anima*, Aristotle (384-322 BC)

The magnetic stone attracts iron.
慈 (ci) 石(shi) 召(zhao) 铁(tie)

---- *Guiguzi (鬼谷子),* **(4th century BC)**

慈 (loving, kind): the original Chinese character for magnetism

↓

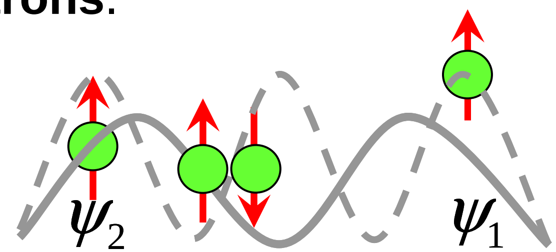
磁: magnetism, magnetic

http://en.wikipedia.org/wiki/Compass#China

Itinerant ferromagnetism (FM): one of the central problems in condensed matter physics

E. C. Stoner

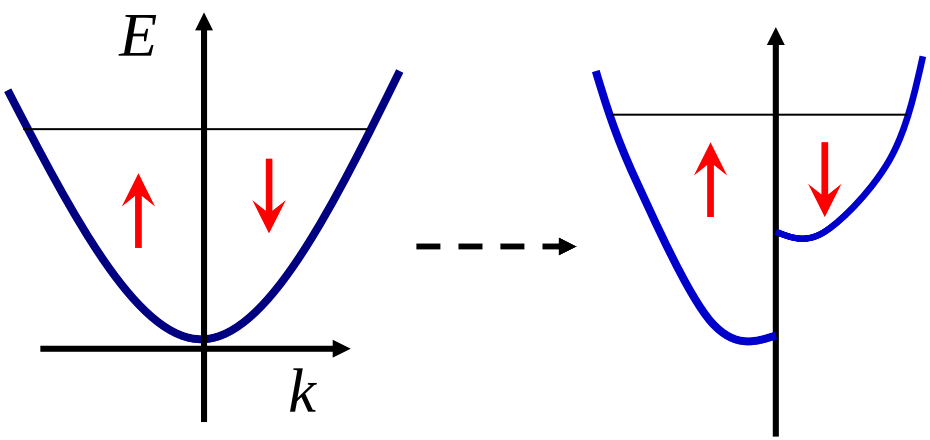
• Driving force: direct **exchange interaction among electrons**.



$$E_{\uparrow\uparrow} < E_{\uparrow\downarrow}$$

• Stoner criterion:

$$UN_0 > 1$$



U – average interaction strength; N_0 – density of states at the Fermi level

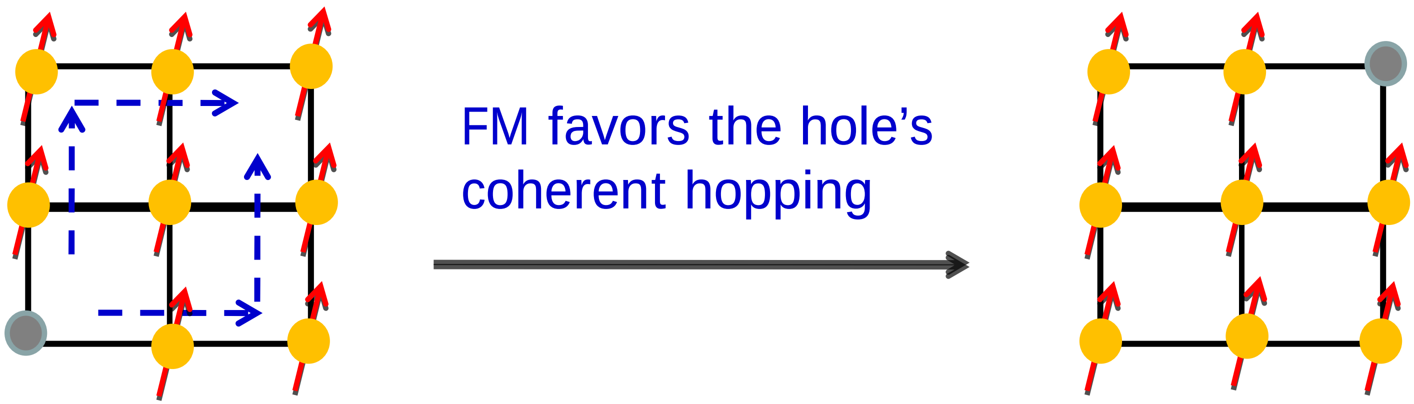
Fe	Co	Ni

A major challenge of condensed matter physics

• FM is essentially strong coupling physics. **NO** reliable weak coupling picture. In comparison, for superconductivity, we still have weak coupling BCS theory.

Few Exact results:

1. Nagaoka FM in Hubbard models ($D \geq 2$) with a single hole and $U \to +\infty$

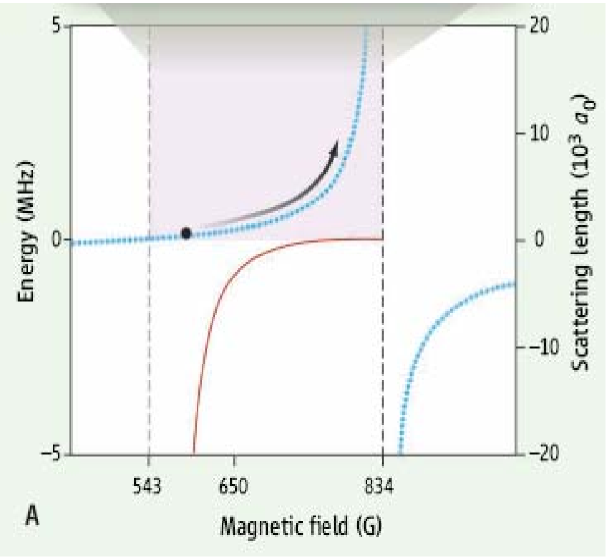


2. Flat band FM: divergence of density of states stabilize ferromagnetism.

A. Mielke and H. Tasaki, Comm. Math. Phys 158, 341 (1993).

Evidence of FM with cold atoms?

• Two-component fermions through Feshbach resonances from the side of **positive** scattering lengths.



When $k_f a_s > 1.9$, the increase of kinetic energy and the decrease of three-body loss are observed.

G. B. Jo, et al., Science 325, 1521(2009).

• Criticism: Magnetic domains are not observed.

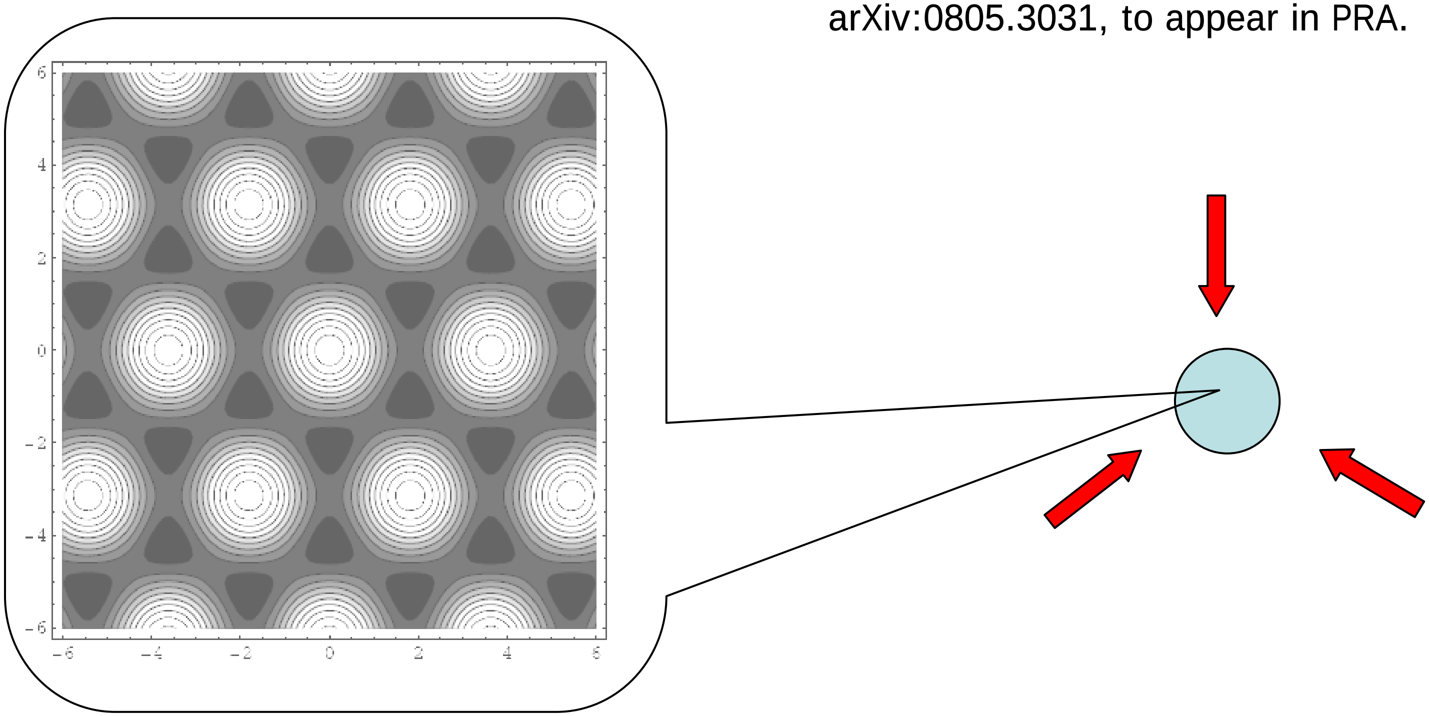
Hui Zhai, arxiv:0909.4917.

• Drawback: unstable to the dimer formation. To avoid this, fast ramping is needed, thus adiabaticity is not be ensured.

The p-orbital honeycomb lattice

- Our approach: flat band FM to avoid the stability issue.

S. Z. Zhang, H. H. Hung, and C. Wu, arXiv:0805.3031, to appear in PRA.

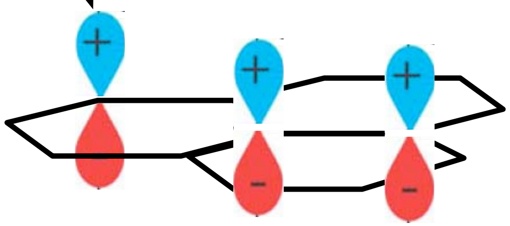


- Three coherent laser beams polarizing in the z-direction.

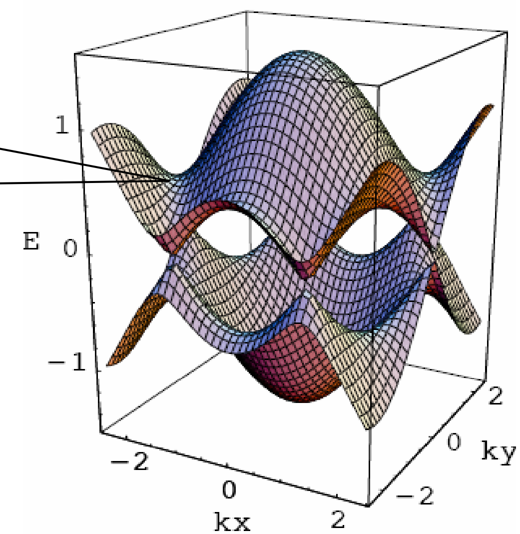
G. Grynberg et al., Phys. Rev. Lett. **70**, 2249 (1993).
also K. Sengstock's recent work.

p-orbital fermions in the honeycomb lattice

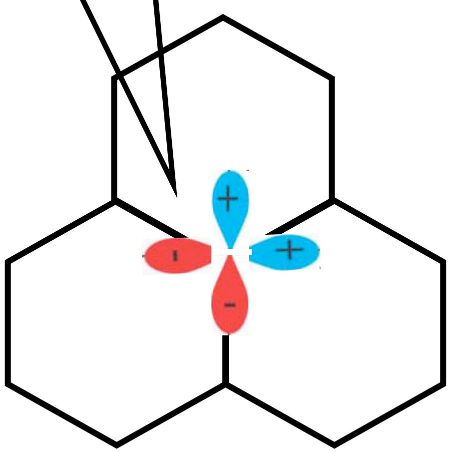
Not our interest



cf. graphene: p_z-orbital; Dirac cones.

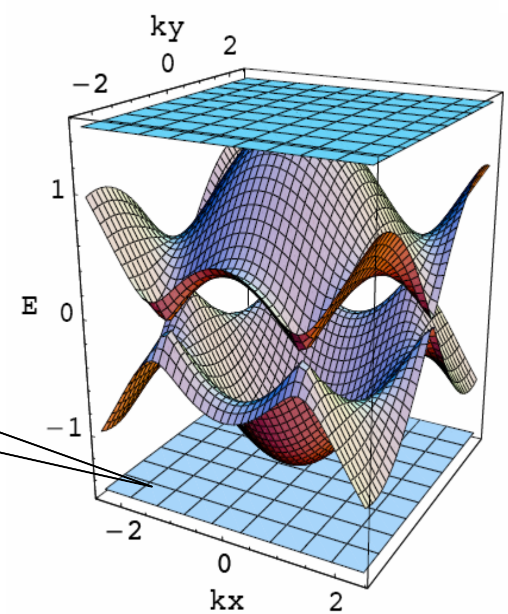


Our interest



p_x;p_y-orbitals: flat bands; interaction effects dominate→ ferromagnetism.

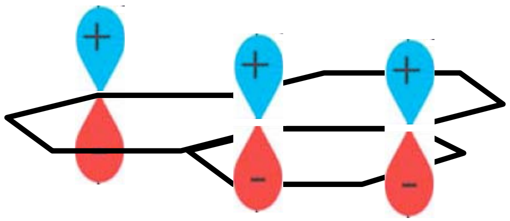
C. Wu, D. Bergman, L. Balents, and S. Das Sarma, PRL 99, 70401 (2007).



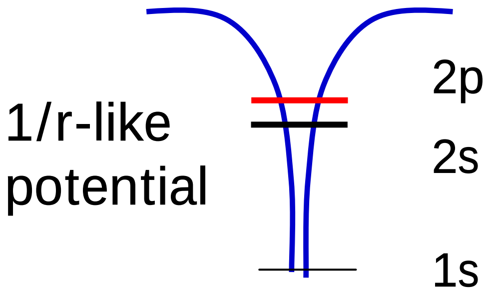
p_x, p_y-physics: get rid of the hybridization with s

- p_z-orbital band is not a good system for orbital physics.

  isotropic within 2D; non-degenerate.

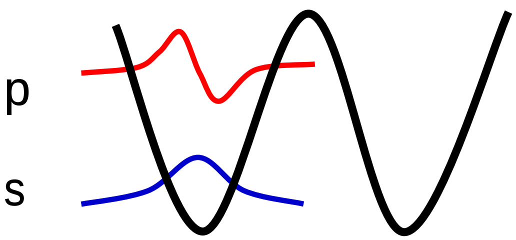


- Interesting orbital physics in the p_x, p_y-orbital bands.



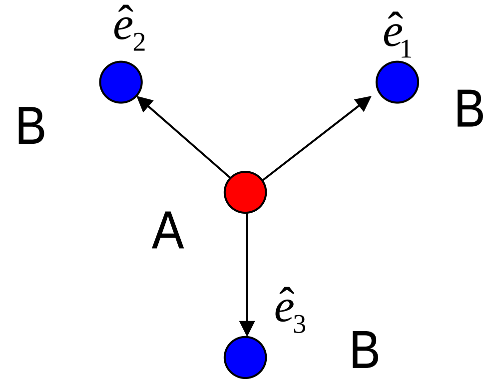
- However, in graphene, $2p_x$ and $2p_y$ are close to $2s$, thus strong hybridization occurs.

- In optical lattices, p_x and p_y-orbital bands are well separated from s.

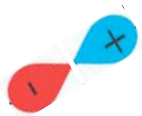


p-orbital honeycomb optical lattice

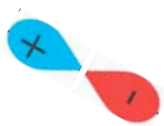
- Band Hamiltonian (σ-bonding) for spin-polarized fermions.



$$H_t = t_{//}\Big\{ \sum_{\vec{r}\in A} [p_1^+(\vec{r})p_1(\vec{r}+\hat{e}_1)+h.c.]$$
$$+[p_2^+(\vec{r})p_1(\vec{r}+\hat{e}_2)+h.c]$$
$$+[p_3^+(\vec{r})p_3(\vec{r}+\hat{e}_3)+h.c] \Big\}$$

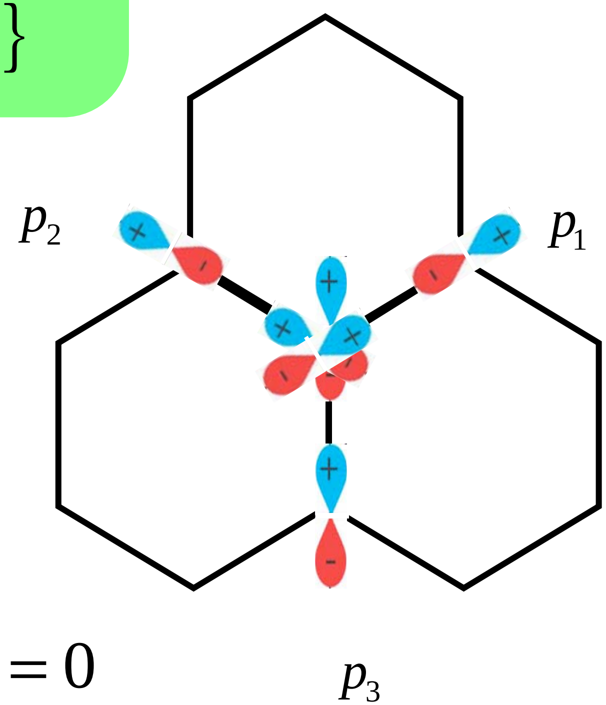


$$p_1 = \frac{\sqrt{3}}{2}p_x + \frac{1}{2}p_y$$



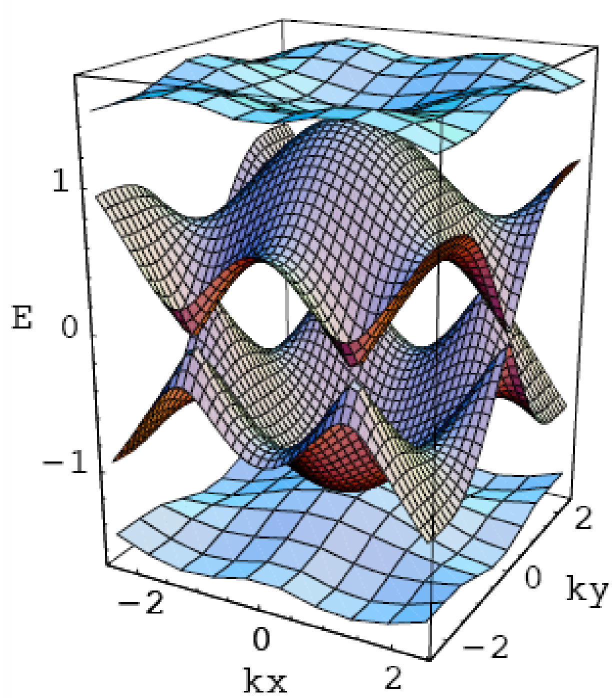
$$p_2 = -\frac{\sqrt{3}}{2}p_x + \frac{1}{2}p_y$$

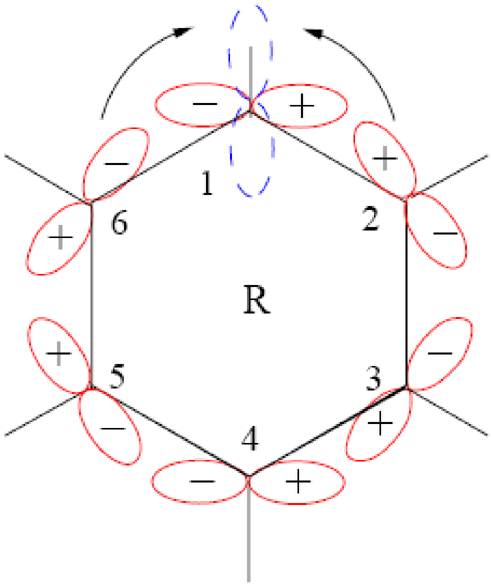
$$p_3 = -p_y$$



$$p_1 + p_2 + p_3 = 0$$

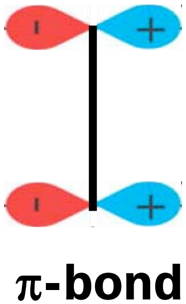
Flat bands from **localized** eigenstates





- Flat band + Dirac cone.
- localized eigenstates.

- If π-bonding is included, the flat bands acquire small width at the order of $t_{\perp}$. Realistic band structures show $t_{\perp} / t_{//} \rightarrow 1\%$



$t_{//} >> t_{\perp}$

Two-component fermions: six two-particle onsite states

- Multi-orbital Hubbard model: U, Hund's rule J, pair hopping Δ.

$$H_{\text{int}} = U\sum_{\vec{r}}[n_{p_x,\uparrow}(\vec{r})n_{p_x,\downarrow}(\vec{r}) + n_{p_y,\uparrow}(\vec{r})n_{p_y,\downarrow}(\vec{r})] - J\sum_{r}[\vec{S}_{p_x}(\vec{r})\cdot\vec{S}_{p_y}(\vec{r}) - \frac{1}{4}n_{p_x}(\vec{r})n_{p_y}(\vec{r})]$$

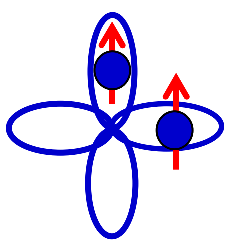
$$+\Delta\sum_{r}[p^{+}_{x,\uparrow}(\vec{r})p^{+}_{x,\downarrow}(\vec{r})p_{y,\downarrow}(\vec{r})p_{y,\downarrow}(\vec{r}) + h.c.]$$

- A single site problem: the onsite triplet states do not cost energy within the s-wave scattering approximation.

$p^{+}_{x\uparrow}p^{+}_{y\uparrow}$

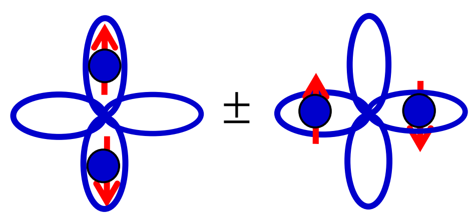
$p^{+}_{x\downarrow}p^{+}_{y\downarrow}$

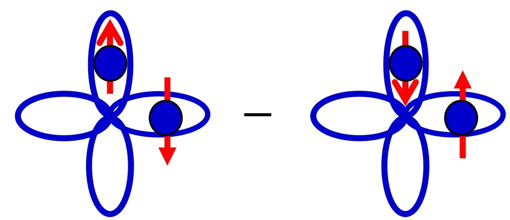
$p^{+}_{x\uparrow}p^{+}_{y\downarrow} + p^{+}_{x\downarrow}p^{+}_{y\uparrow}$



$E=0$

3-fold triplet states





$p^{+}_{x\uparrow}p^{+}_{x\downarrow} \pm p^{+}_{y\uparrow}p^{+}_{y\downarrow},$

$E = U \pm \Delta > 0$

$p^{+}_{x\uparrow}p^{+}_{y\downarrow} - p^{+}_{x\downarrow}p^{+}_{y\uparrow},$

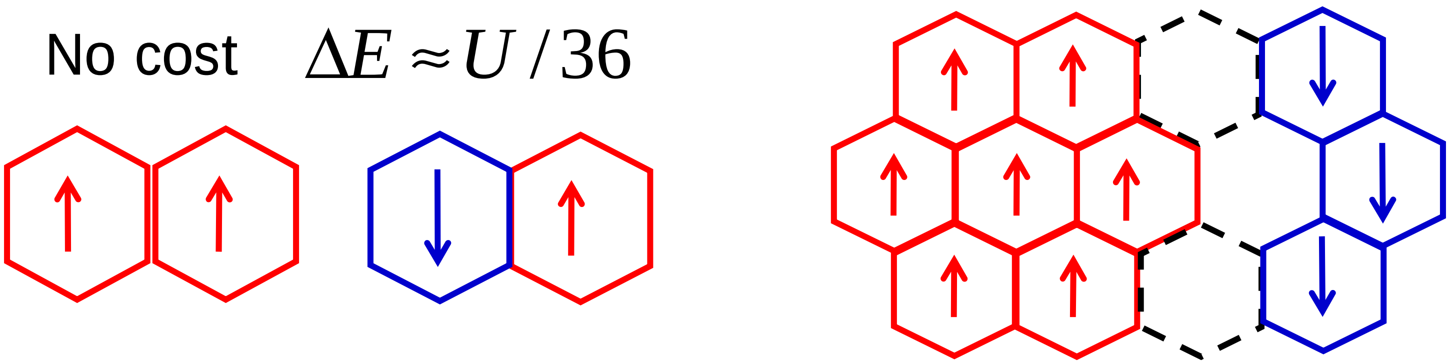
$E = J > 0$

3 different singlet states

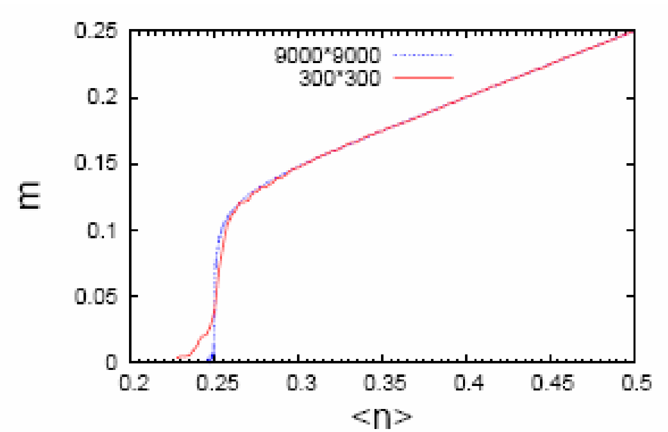
Two-component fermion: interaction and percolation

- When localized eigenstates touch, their spins are aligned by the direct exchanges → formation of FM clusters.
- Spins in disconnected clusters are uncorrelated.

No cost $\Delta E \approx U/36$



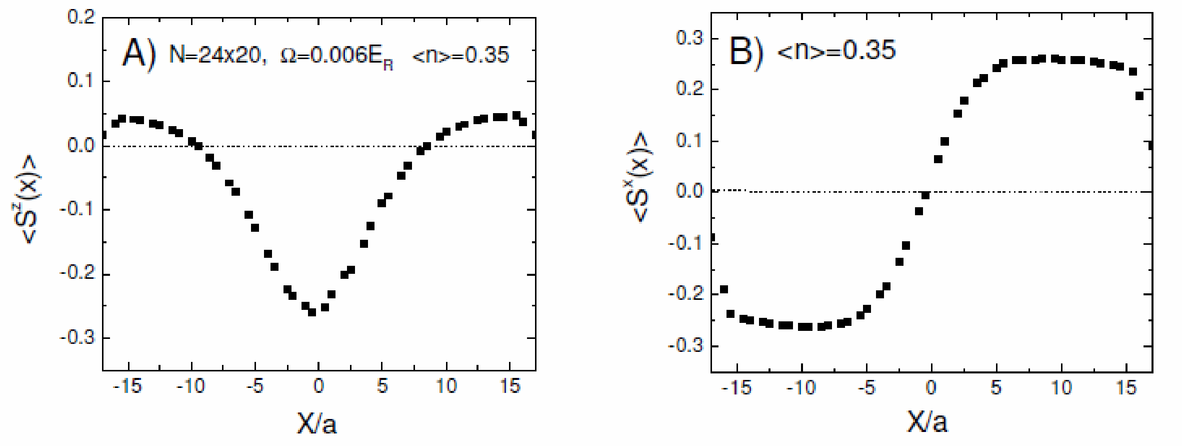
- **Exact** result in homogenous systems: FM appears above the percolation threshold while filling remains in the flat band: $0.25 < n < 0.5$



Flat-band ferromagnetism in the p-orbitals

• Realistic system with a soft harmonic trap. Particle numbers of spin up and down are separately conserved.

$$\Omega = 0.005 E_R,\ l = 4.5a,$$
$$N_\uparrow = N_\downarrow = 255,$$
$$t_{//} = U = 0.24 E_R$$



• Self-consistent B-de G calculation. Skrymion spin texture. It is reliable for soft inhomogeneity because of its exact starting point.

• Advantage: FM stabilized by weak or intermediate repulsions. The instability towards to the dimer molecule states is avoided.

• Entropy is large!

Outline

• Introduction: orbital physics is an interesting research direction of cold atoms.

• Orbital bosons: unconventional BEC beyond the “no-node” theorem.

• Orbital fermions in the hexagonal lattice.

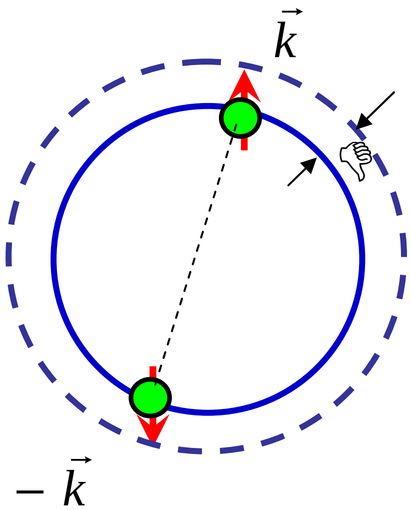
Ferromagnetism from band flatness.

The unconventional f-wave Cooper pairing of the spinless fermions.

W. C. Lee, C. Wu, S. Das Sarma, arXiv:0905.1146, to appear in PRA.

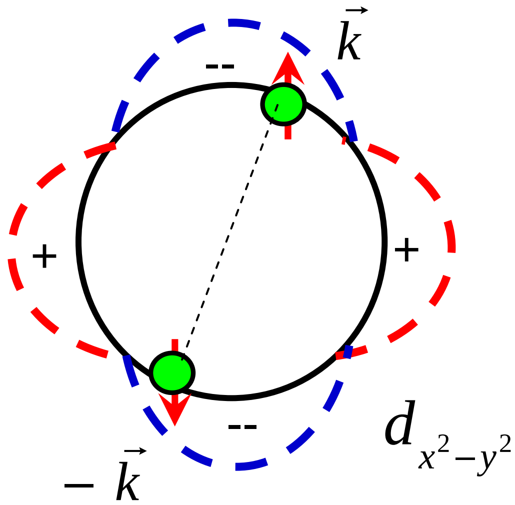
Conventional v.s. unconventional Cooper pairings

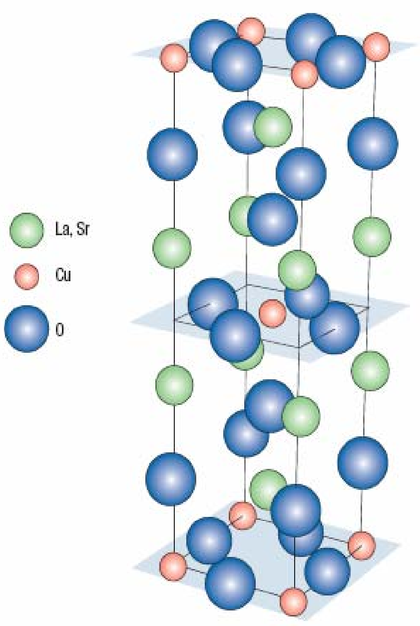
- Conventional superconductivity:



s-wave: pairing amplitude does not change over the Fermi surface.

- *d*-wave (high T_c cuprates). Pairing amplitude changes sign on the Fermi surface.





Unconventional Cooper pairing

- Most of unconventional pairing states arise from strong correlation effects. Predictions and analysis are difficult.

p-wave: superfluid ^{3}He-A and B; Sr_2RuO_4;

d-wave: high T_c cuprates;

Extended s-wave: iron-pnictide superconductors (?);

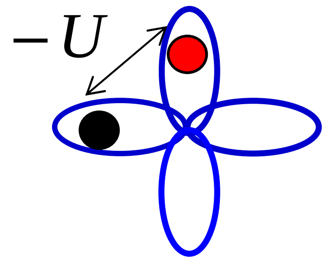
- Can we arrive at unconventional pairing in a simpler way, say, from **nontrivial band structures** but with **conventional** interactions?

C. W. Zhang et al., Phys. Rev. Lett. 101, 160401 (2008).

- No strong correlation effects. Analysis is controllable.
- **f-wave** pairing with spinless fermions in the p-orbital hexagonal optical lattice.

Onsite attraction for **SPINLESS** *p*-orbital fermions

$$H_{\text{int}} = -U\sum_{\vec{r}} n_{p_x}(\vec{r})n_{p_y}(\vec{r})$$
$$= -U\sum_{\vec{r}} n_{p_x+ip_y}(\vec{r})n_{p_x-ip_y}(\vec{r})$$

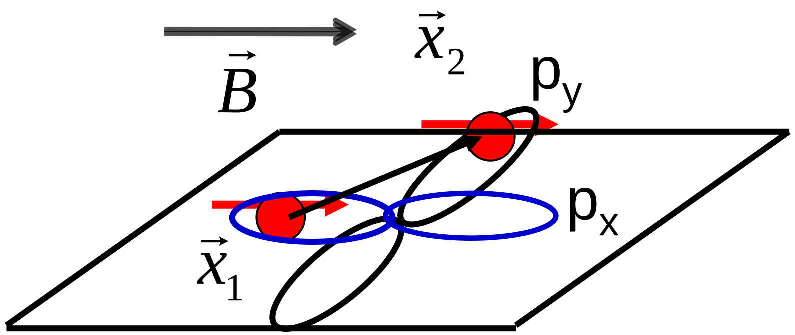


- Problem: contact interaction vanishes for spinless fermions.

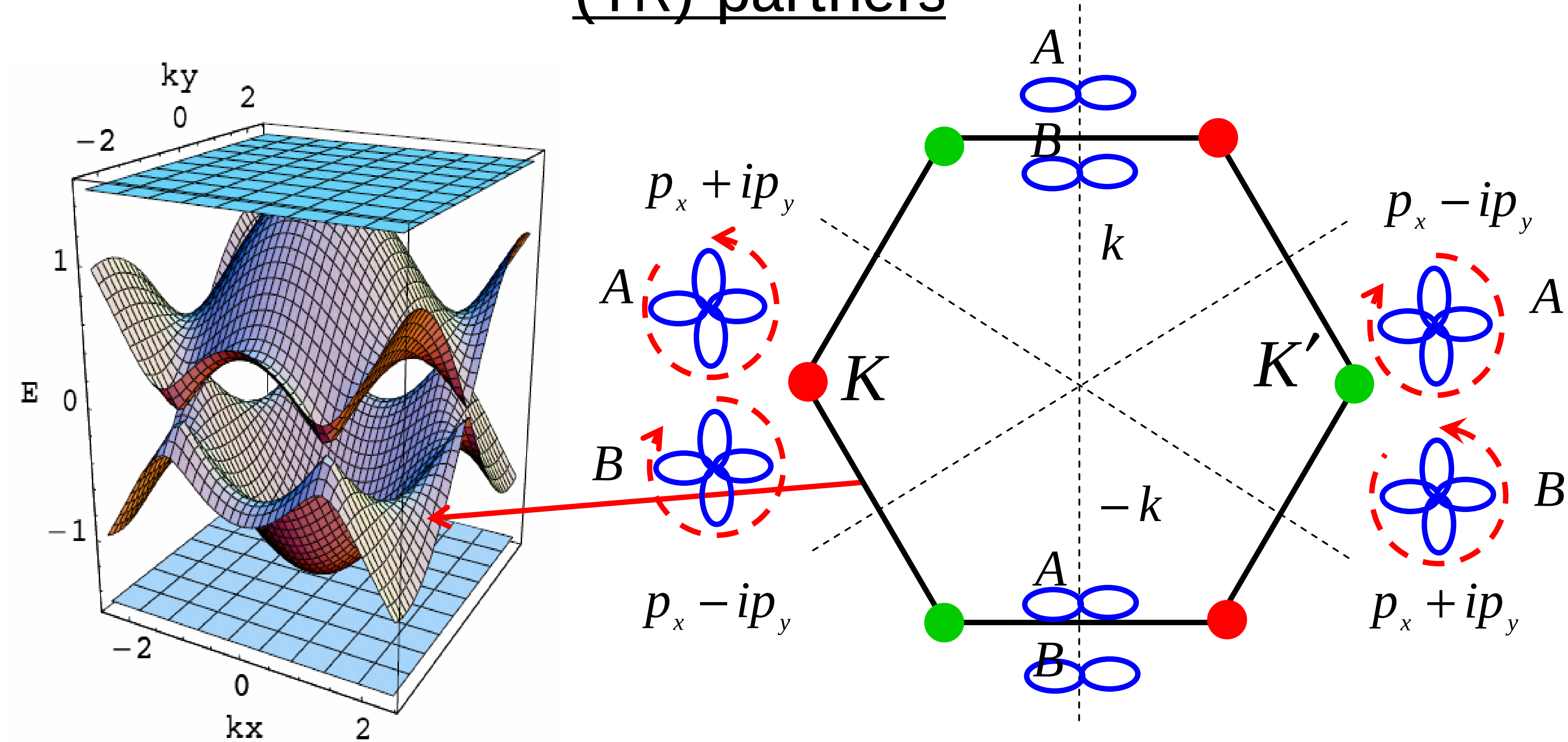
- Use fermions with large magnetic moments whose laser cooling has been performed.

$$^{167}\text{Er}\ (7\mu_B)$$

- Under strong 2D confinement, U is attractive and can reach the order of 100nK.



Nontrivial orbital configurations between time reversal (TR) partners

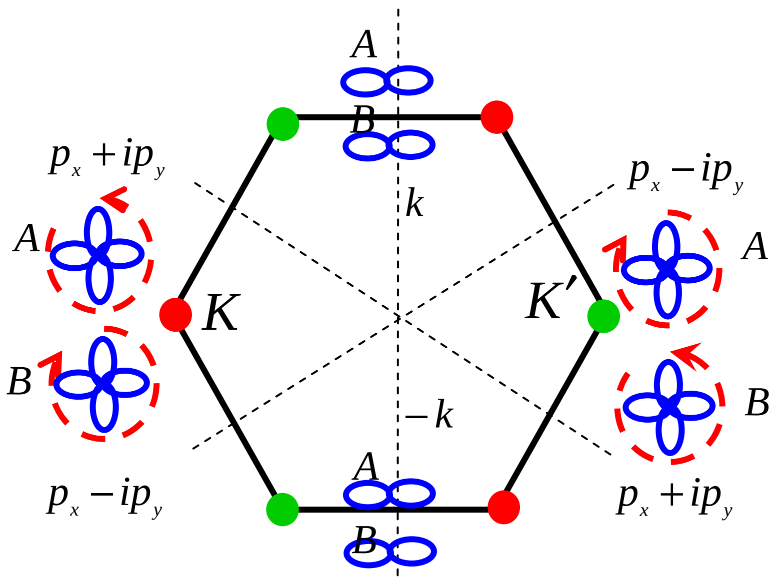


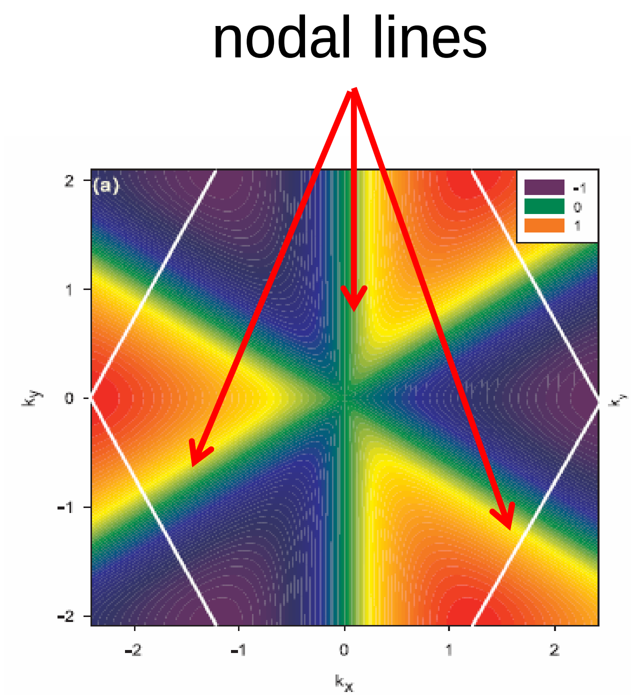
- Along the three middle lines of Brillouin zone, eigen-orbitals are real.

- At K and K’, eigen-orbitals are complex and orthognoal.

The f-wave structure because of the symmetry reason

- Along middle lines, TR pairs cannot be paired → nodal lines.
- The TR pair at K and K' has the largest pairing.
- Odd parity.

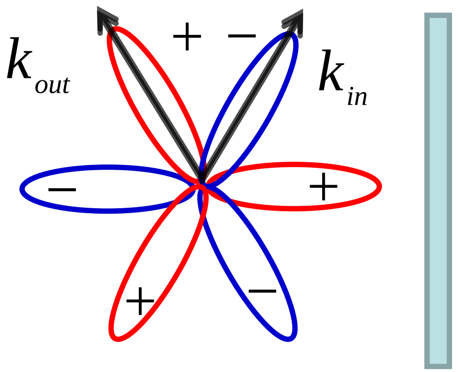




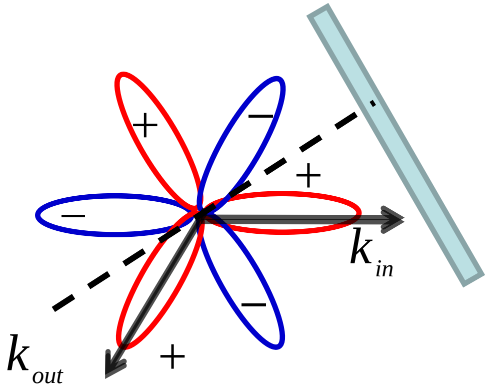
- The mean-field gap value can reach 10nK; and the 2D Kosterlitz-Thouless temperature can reach 1nK.

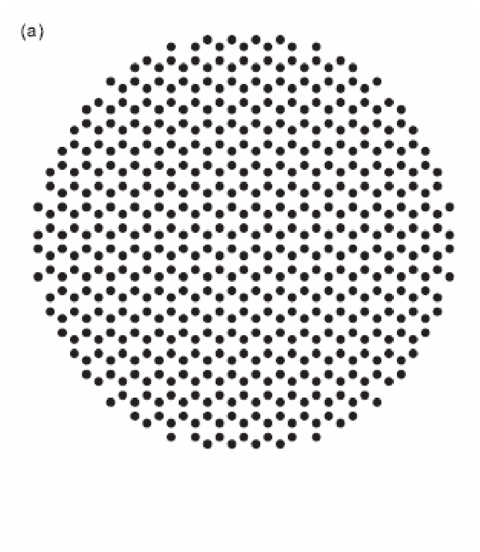
Phase sensitive detection: zero energy Andreev bound states

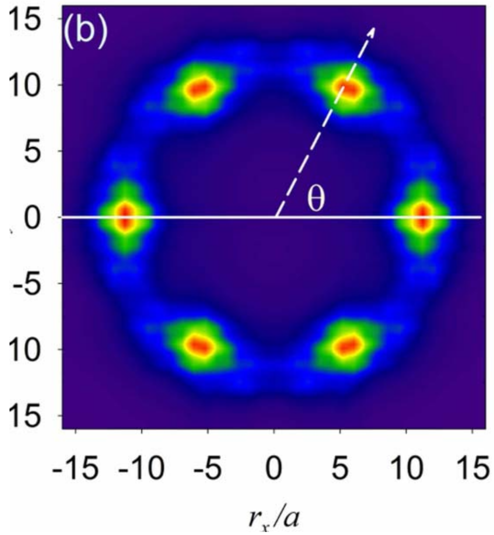
With zero energy Andreev Bound States



No Andreev Bound States

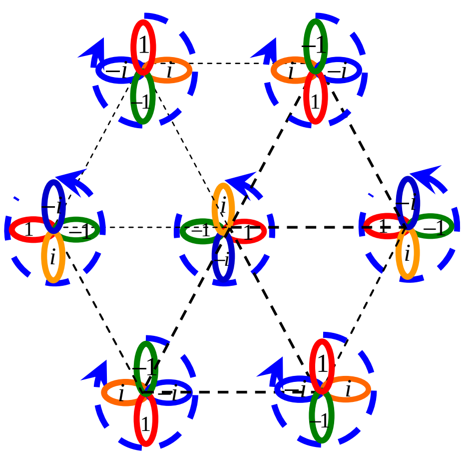


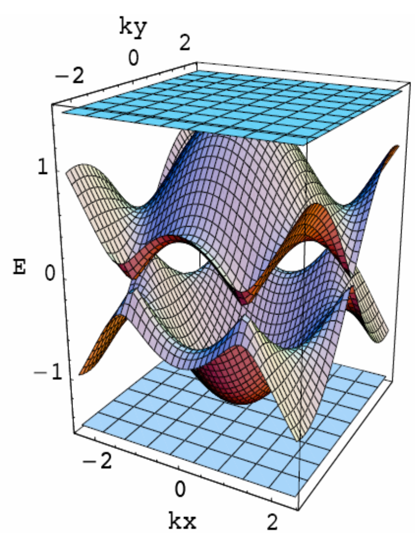


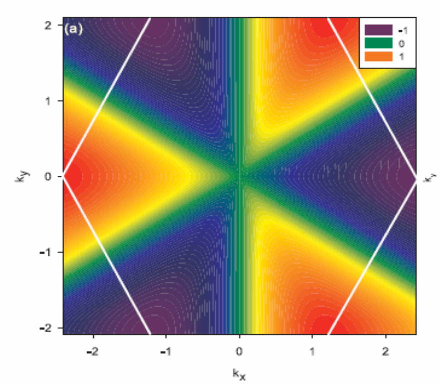


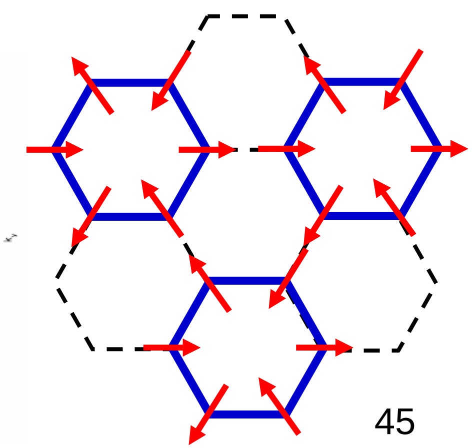
Summary: orbital physics with cold atoms

- Unconventional BEC beyond the "no-node" theorem.
- Exact flat-band ferromagnetism.
- Novel mechanism for f-wave Cooper pairing;
- Mott-insulator: a new type of frustrated magnet-like model.
- Band insulator (topological): quantum anomalous Hall effect.



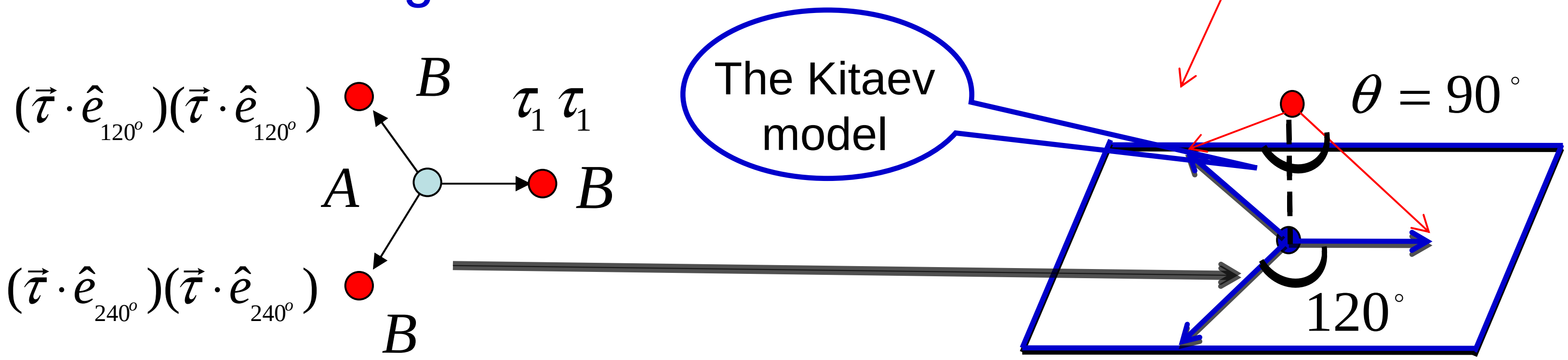






Other selected work

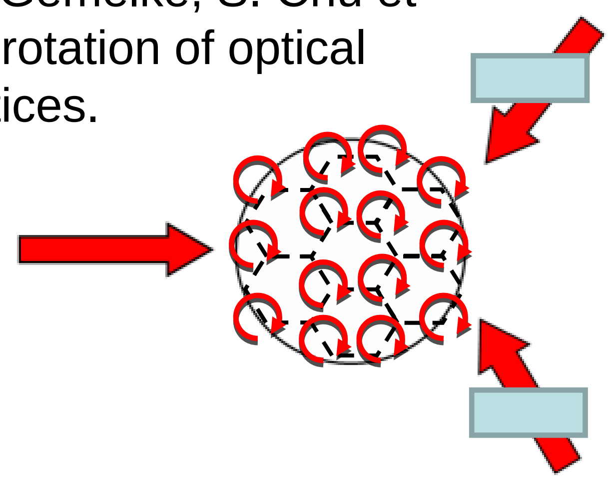
- Mott-insulator: quantum 120° model in the honeycomb lattice. A new frustrated magnet-like model.



C. Wu, PRL 100, 200406 (2008).

- Band insulator (topological): quantum anomalous Hall effect.

N. Gemelke, S. Chu et al. rotation of optical lattices.



C. Wu, PRL 101, 168807 (2008).