Universal Behavior in Quantum Chaotic Dynamics

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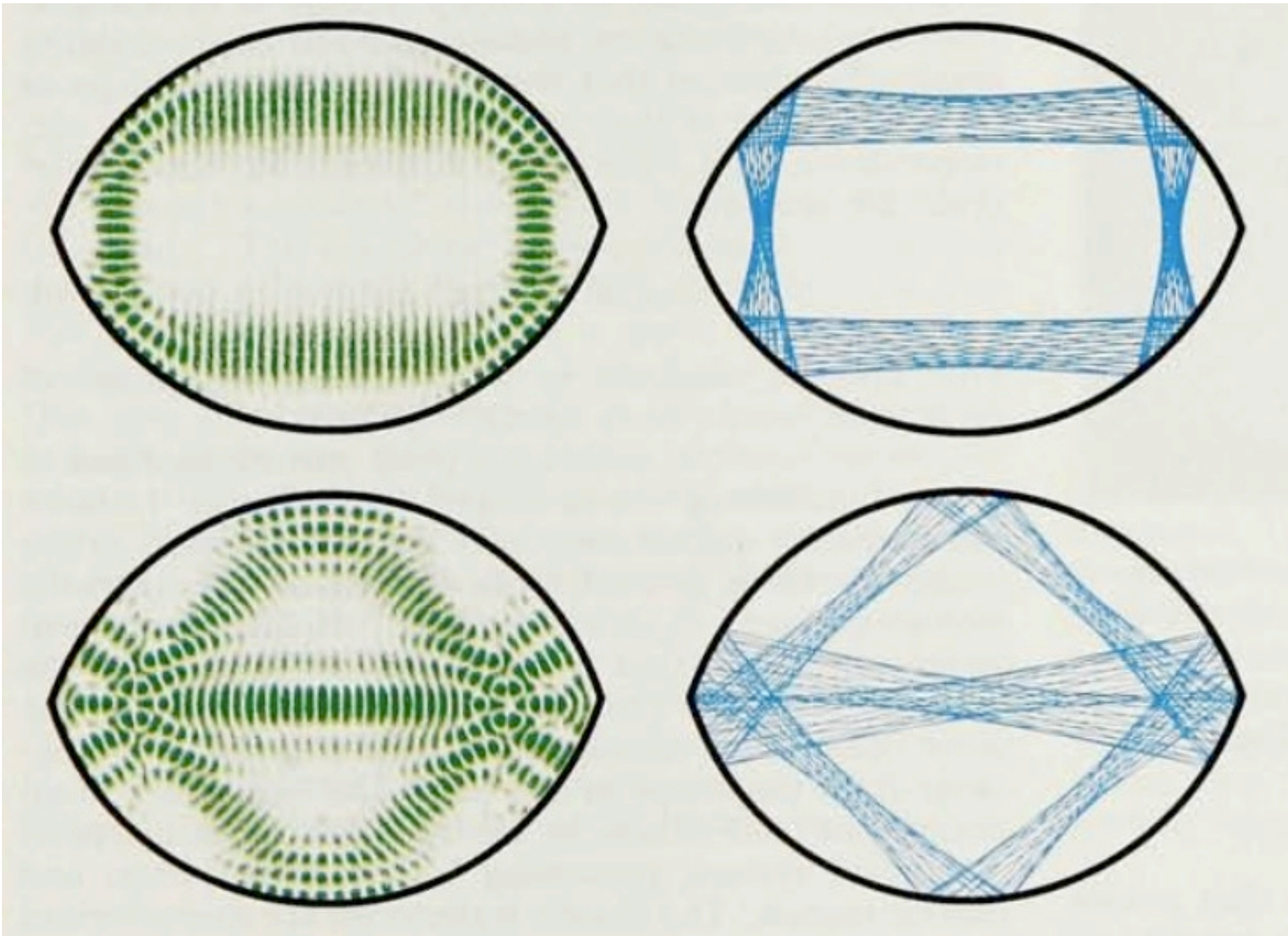
Reference: Xiong and Wu, arXiv:1007.2771

What is quantum chaos?

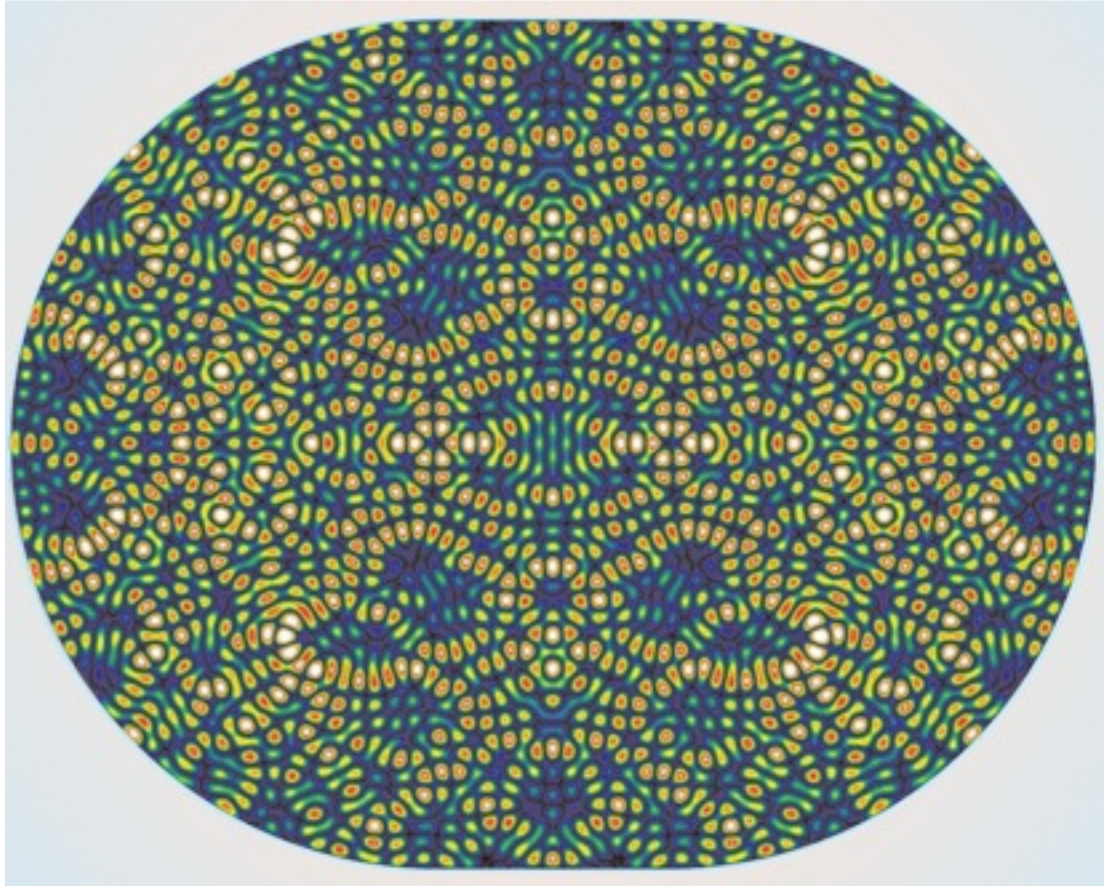
Quantum chaotic systems are quantum systems whose corresponding classical systems are chaotic.

Almost all quantum systems are quantum chaotic systems.

Billiards and their eigenstates



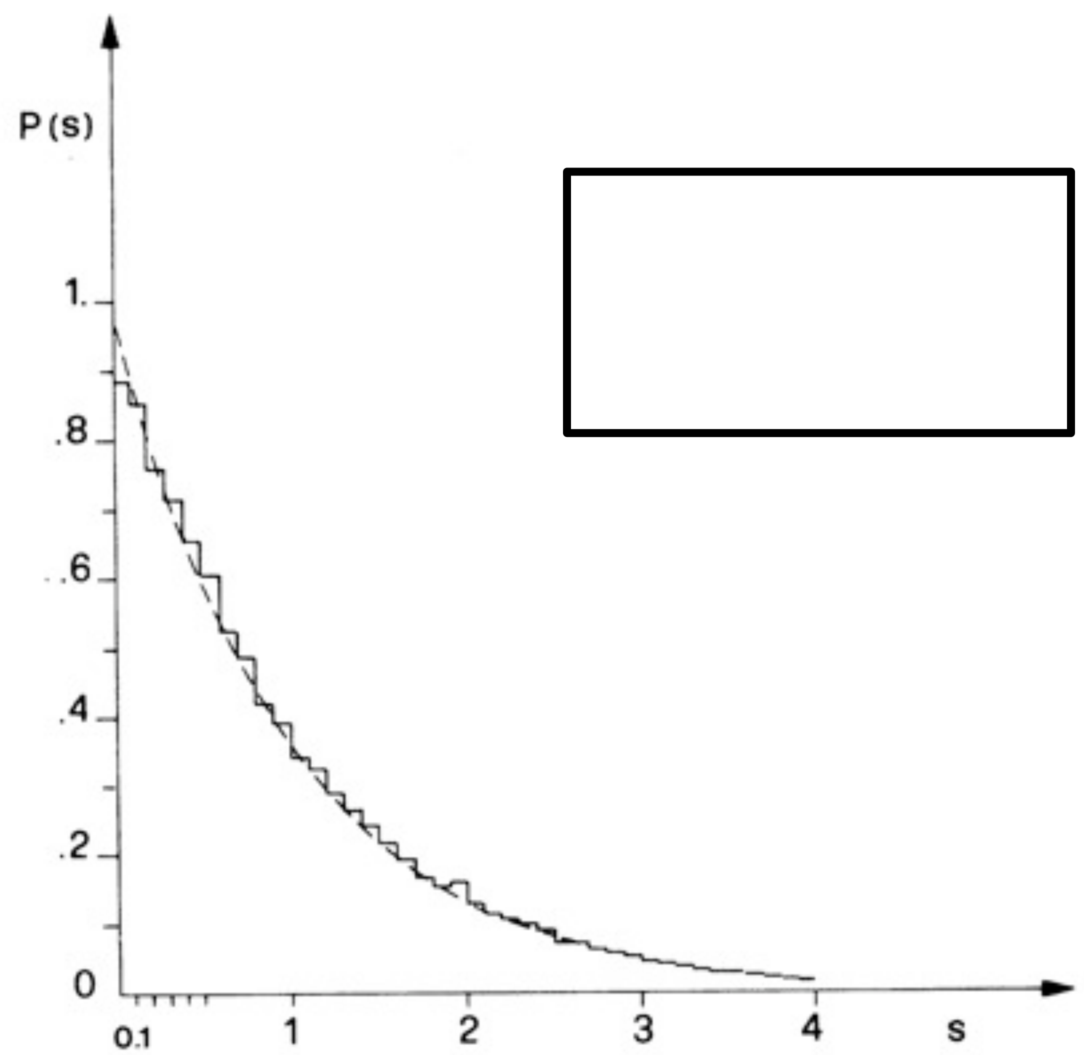
Heller and Tomsovic, **Physics Today**, 1993



Stone, **Physics Today**, 2005

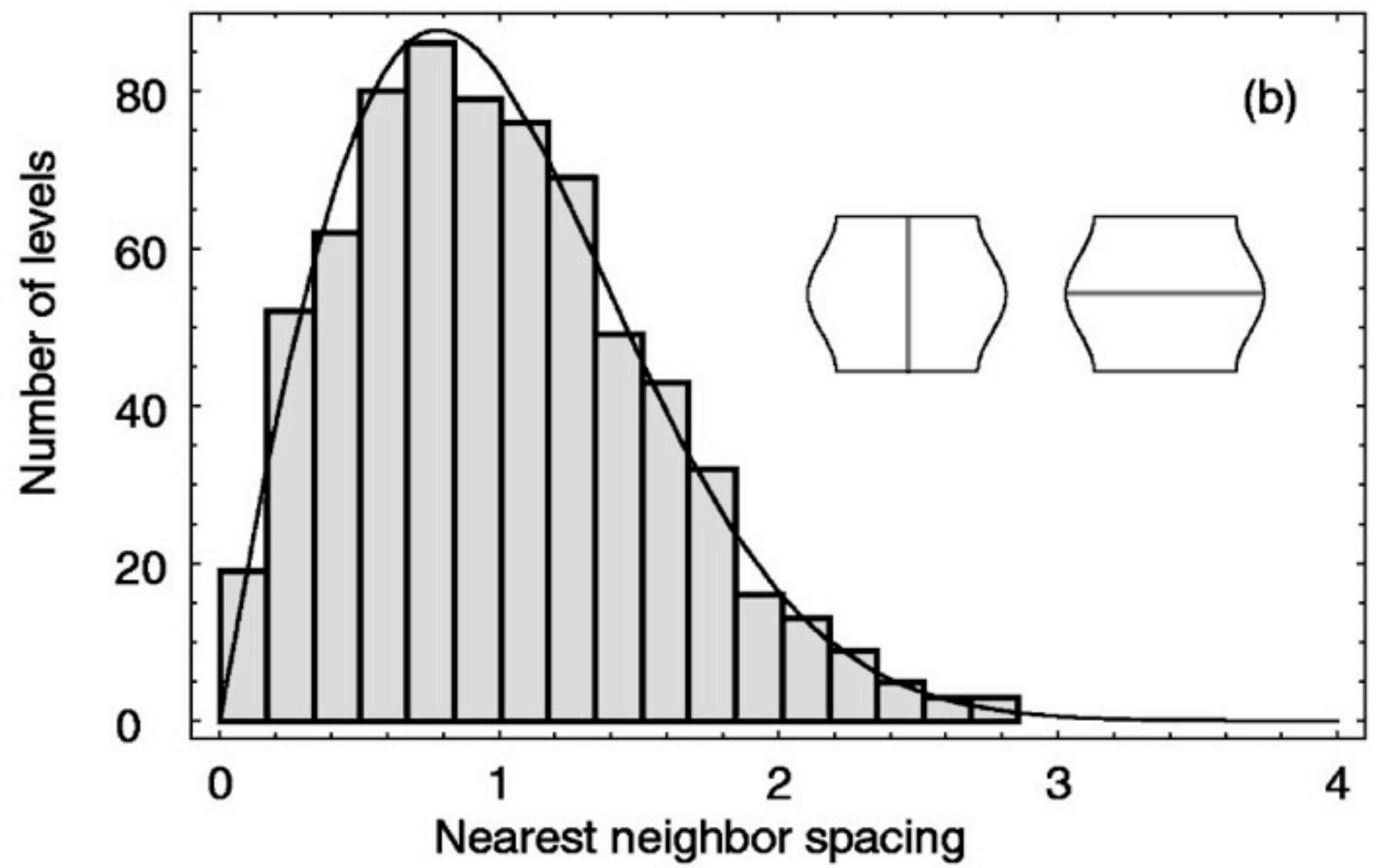
Eigen-energy level spacing

Poisson distribution



Casati *et al* (1985)

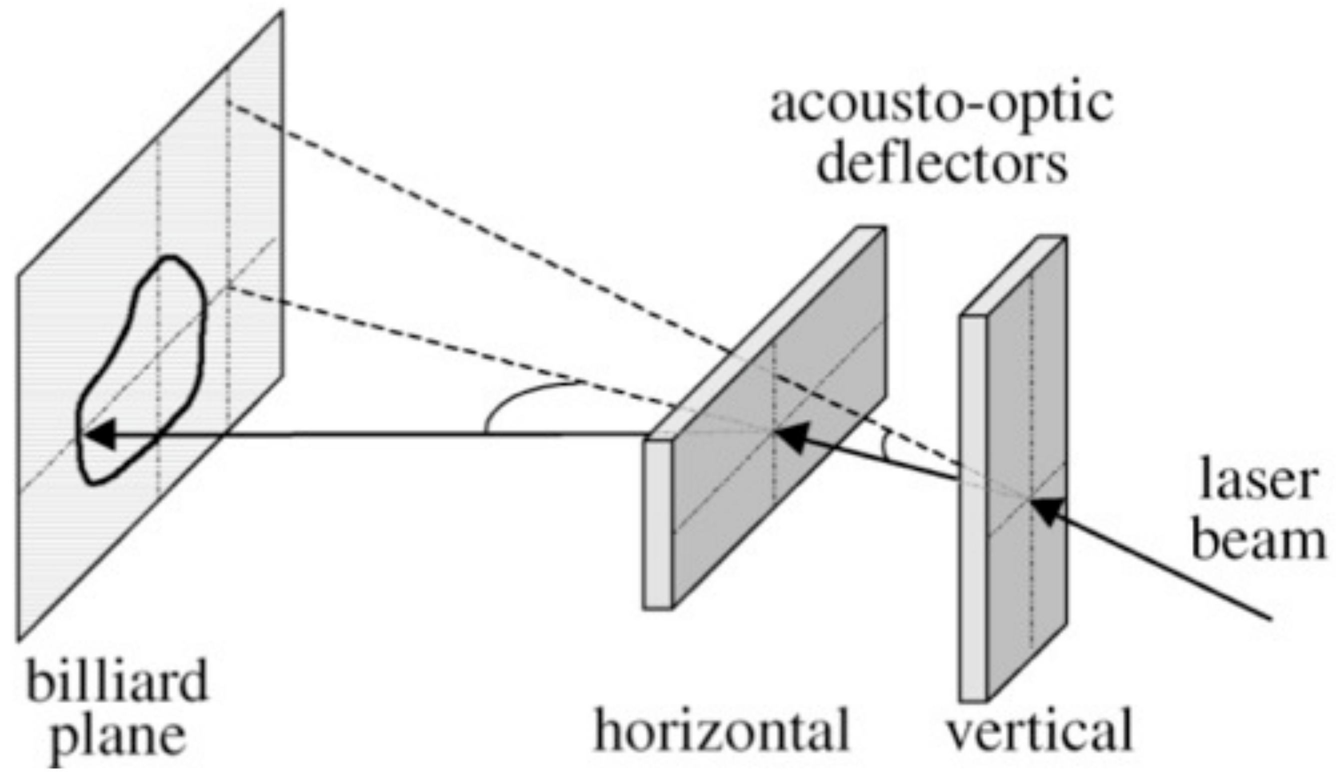
Wigner distribution

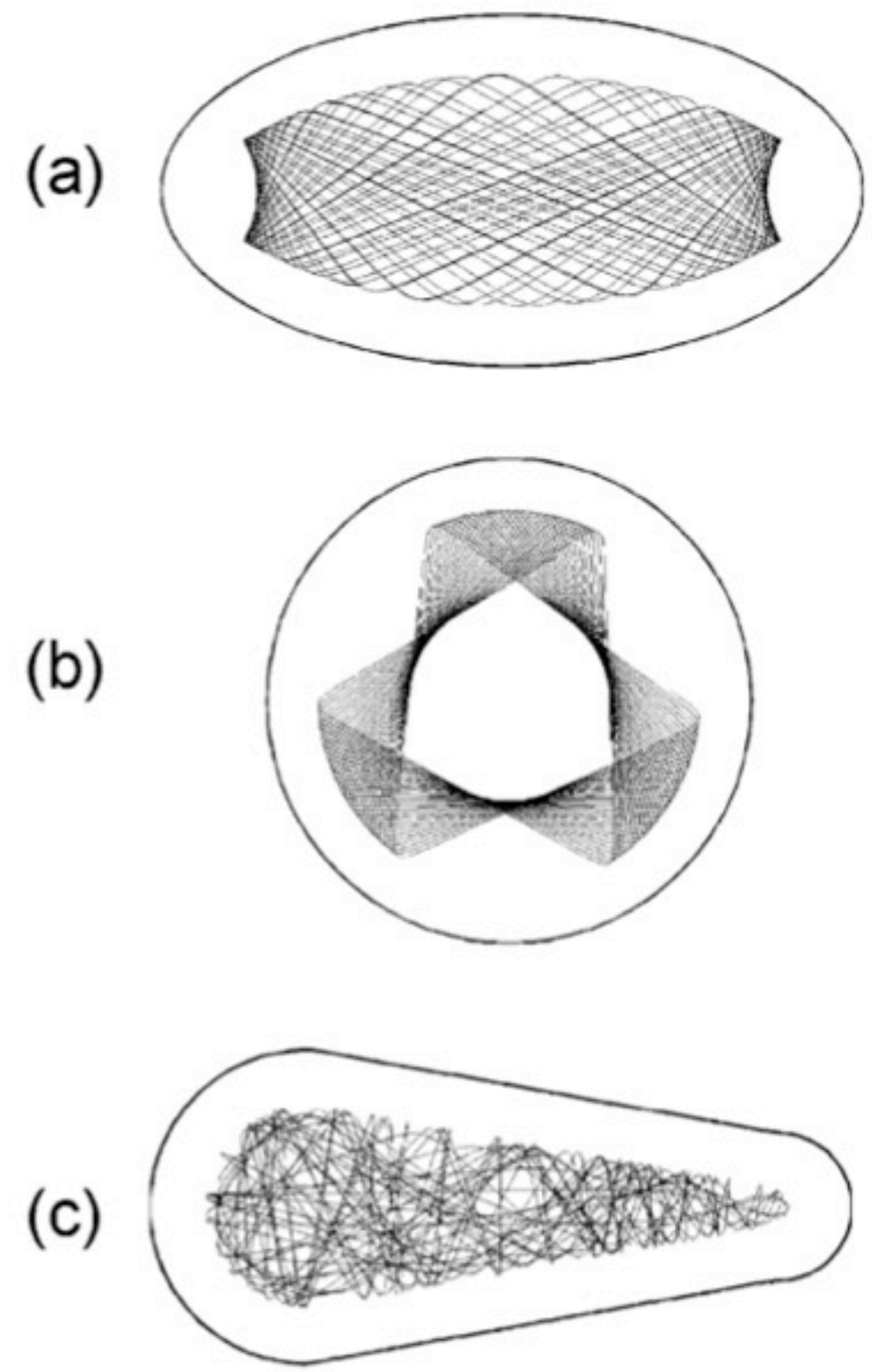


Li *et al* (2002)

Optical billiards with cold atoms

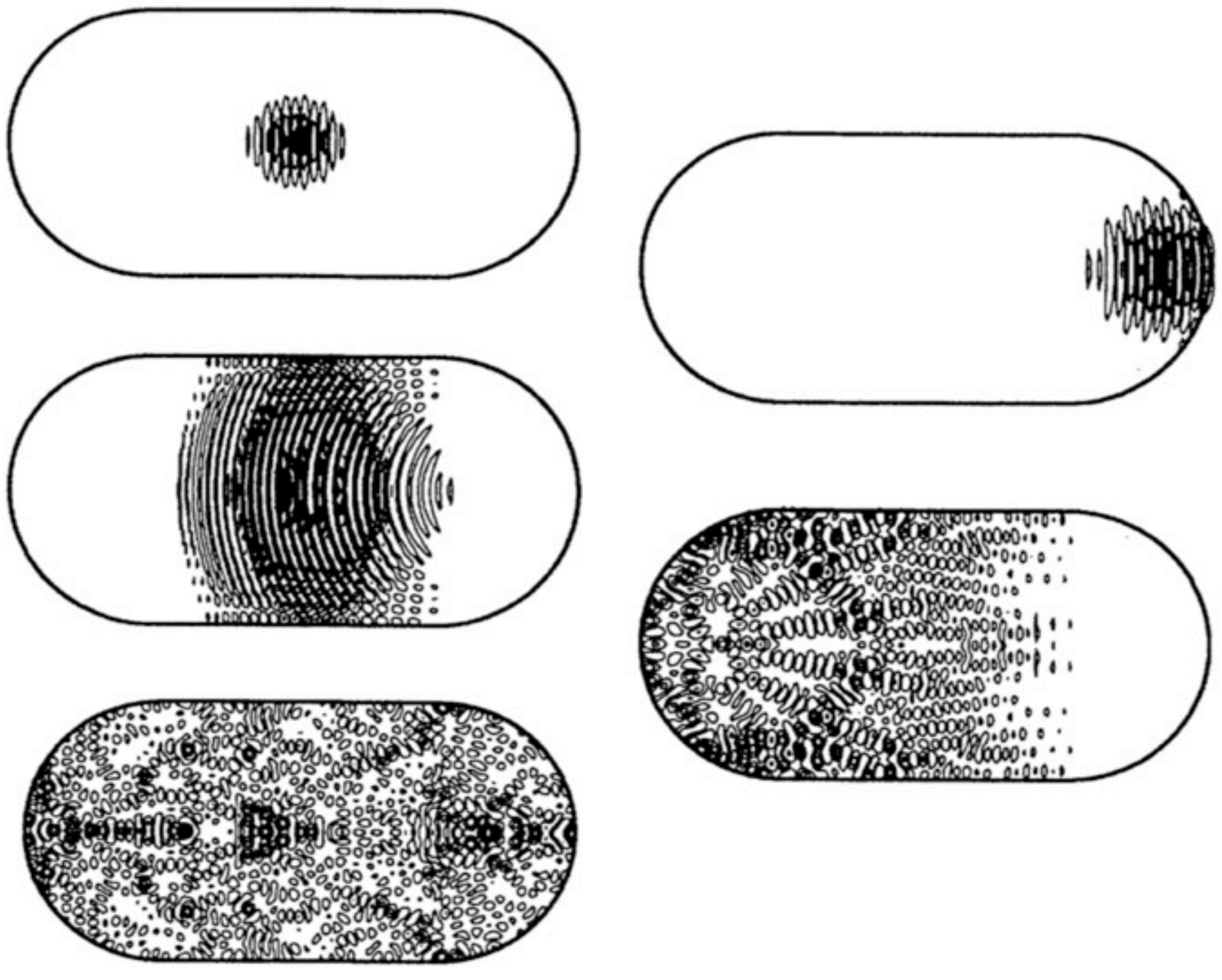
Davidson group and Raizen group (2001)





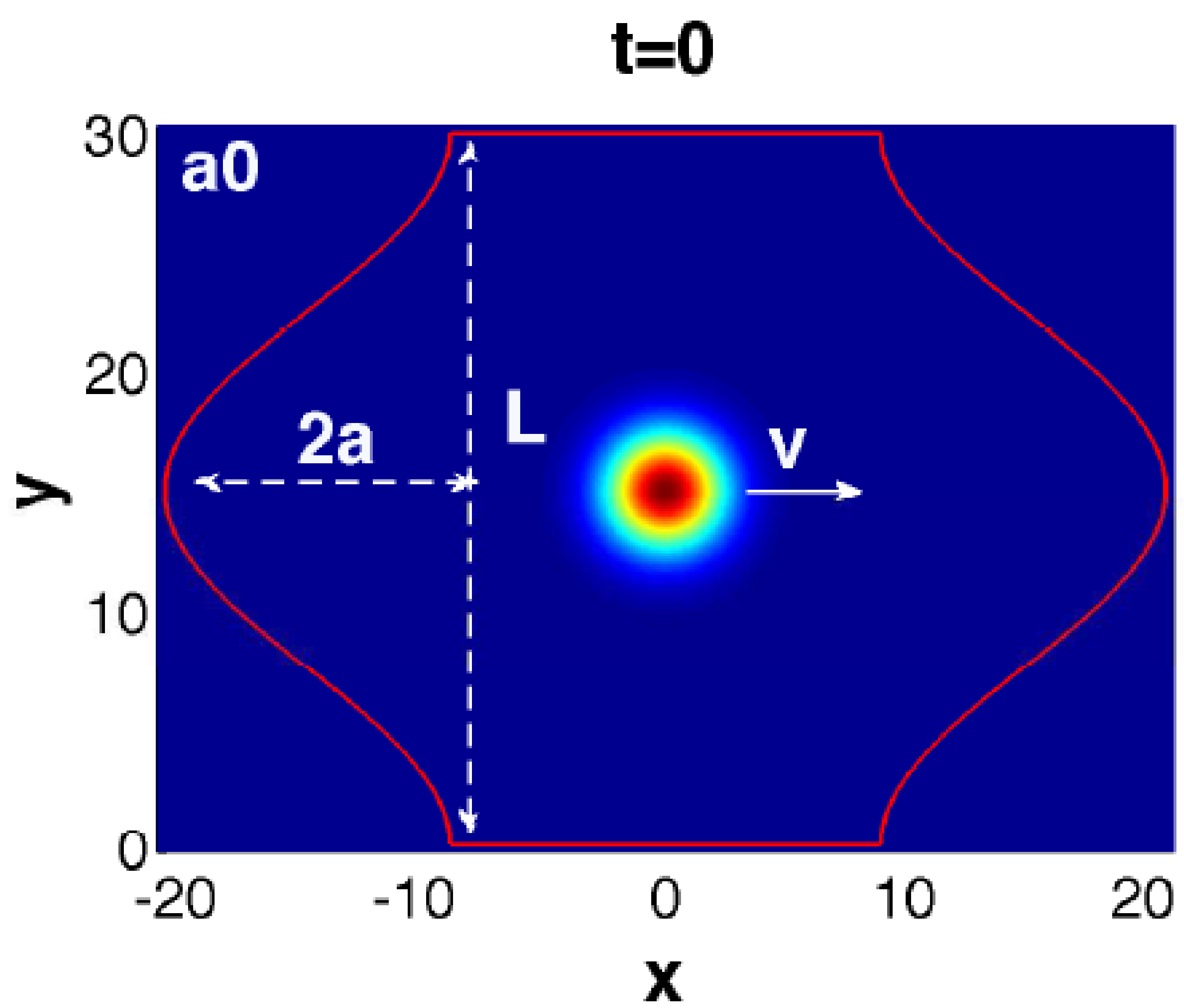
- cold atom are not condensed
- classical chaos

Wave Dynamics in Stadium Billiard



Tomsovic and Heller, **Phys. Rev. E**, 1993

Wave dynamics in ripple billard

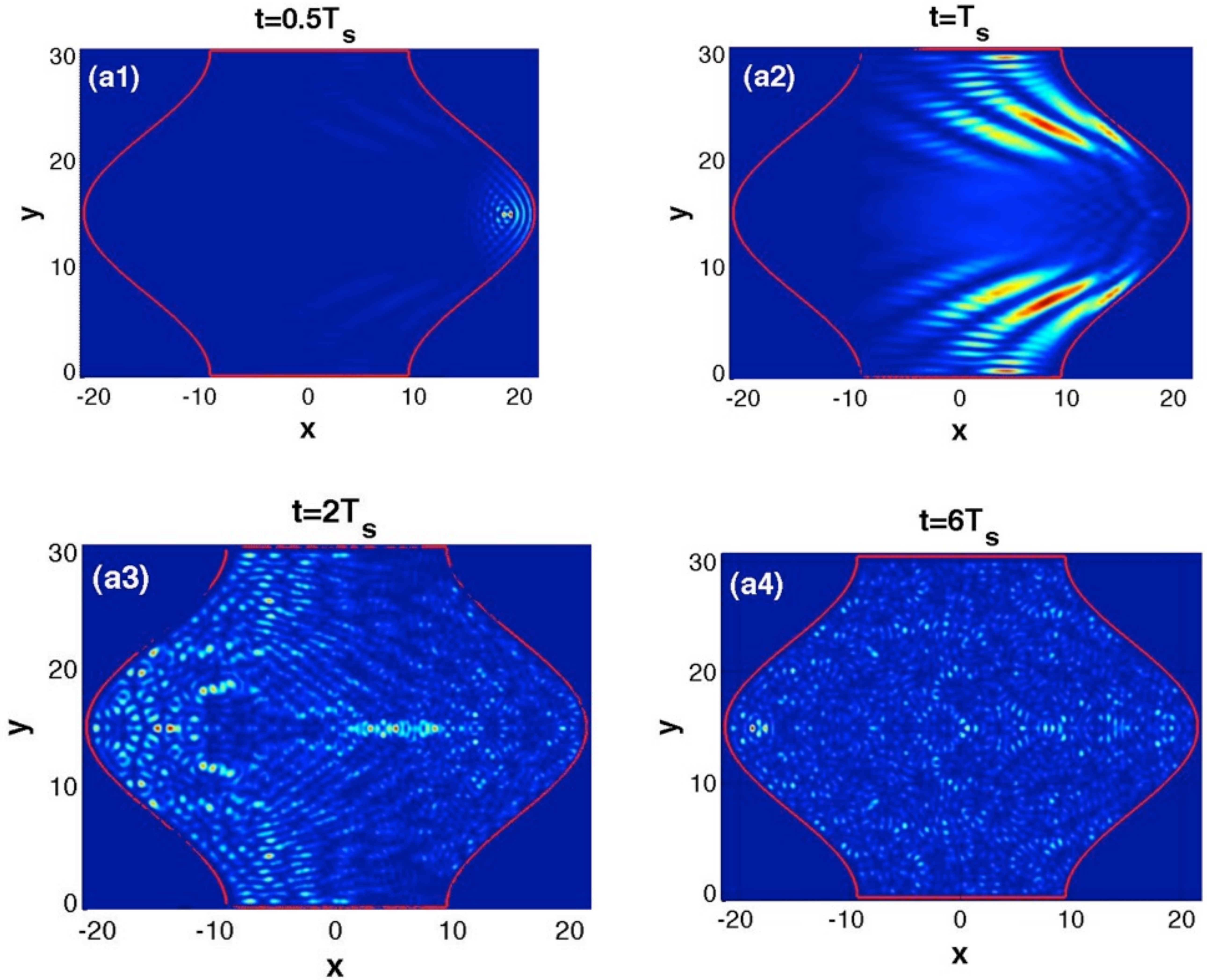


$$i\hbar\frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial \vec{r}^2}\Psi$$

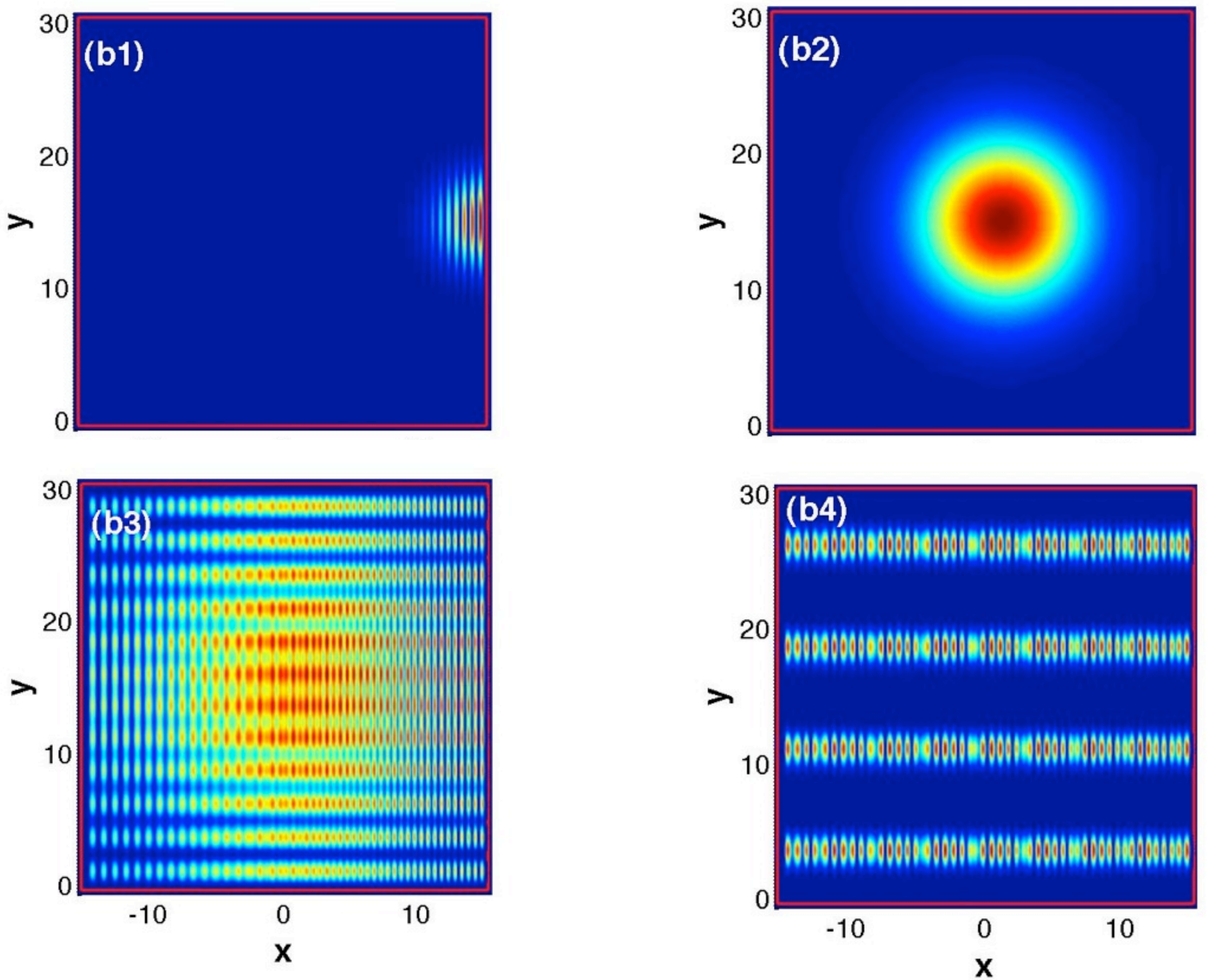
ripple boundary: $x = b - a\cos(2\pi y/L)$

W. Li, L. E. Reichl, and B. Wu
Phys. Rev. E 65, 056220 (2002)

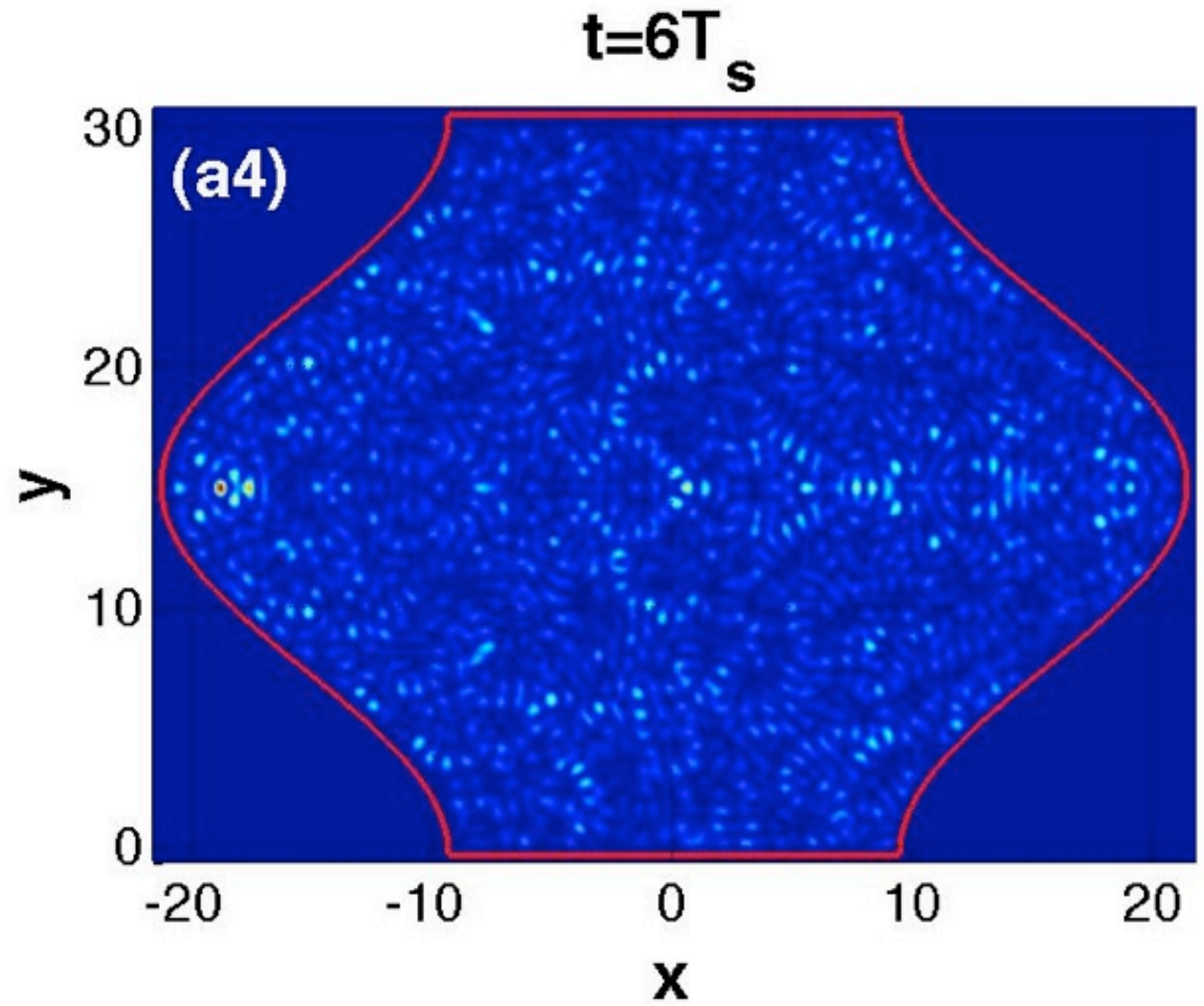
Wave dynamics in ripple billard



Wave dynamics in square billiard

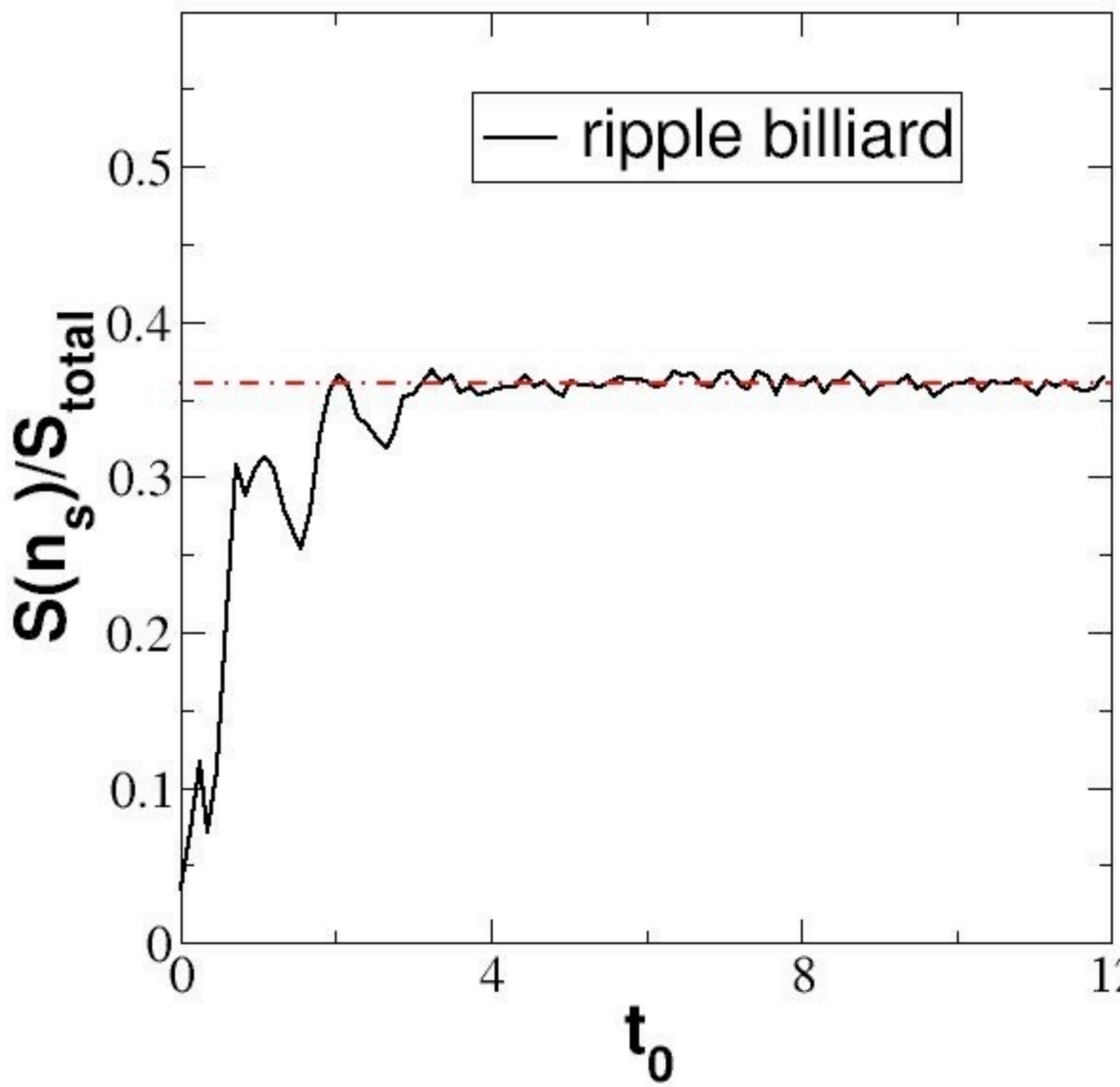


Statistics of random-looking wave

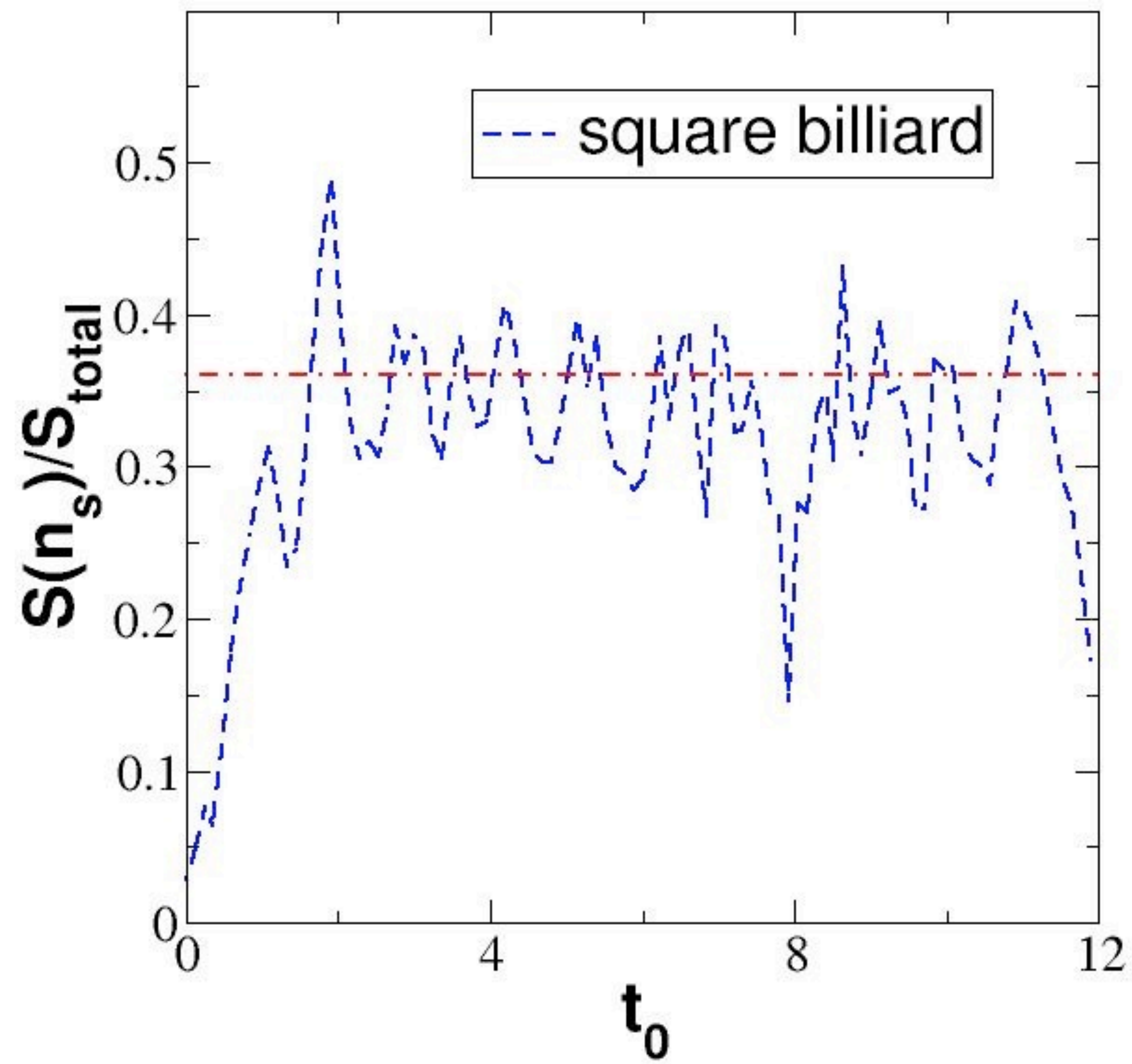


$S\left(n,t\right)$: area of the regions, where the density is larger than *n*

Probability above average density



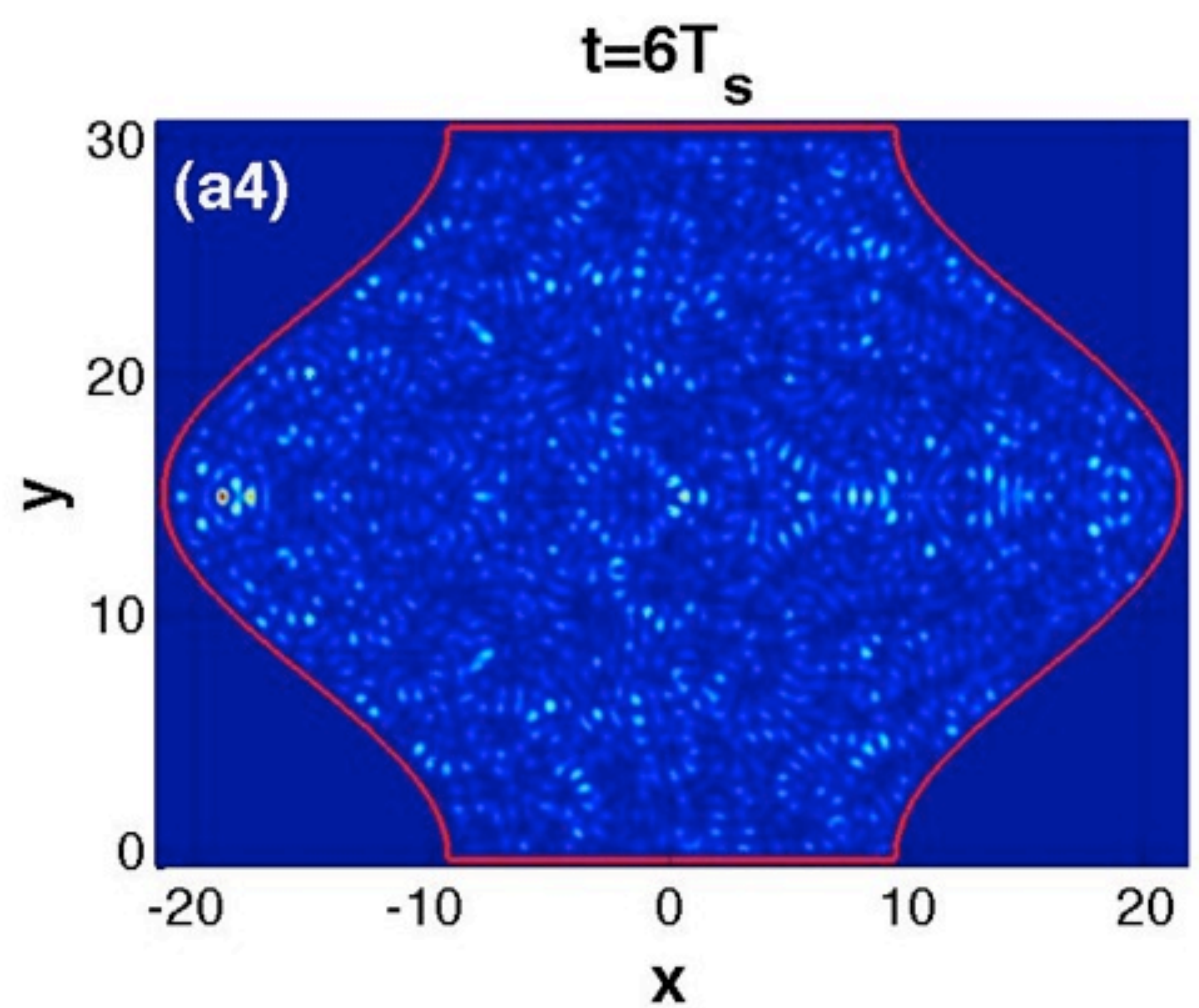
1/e



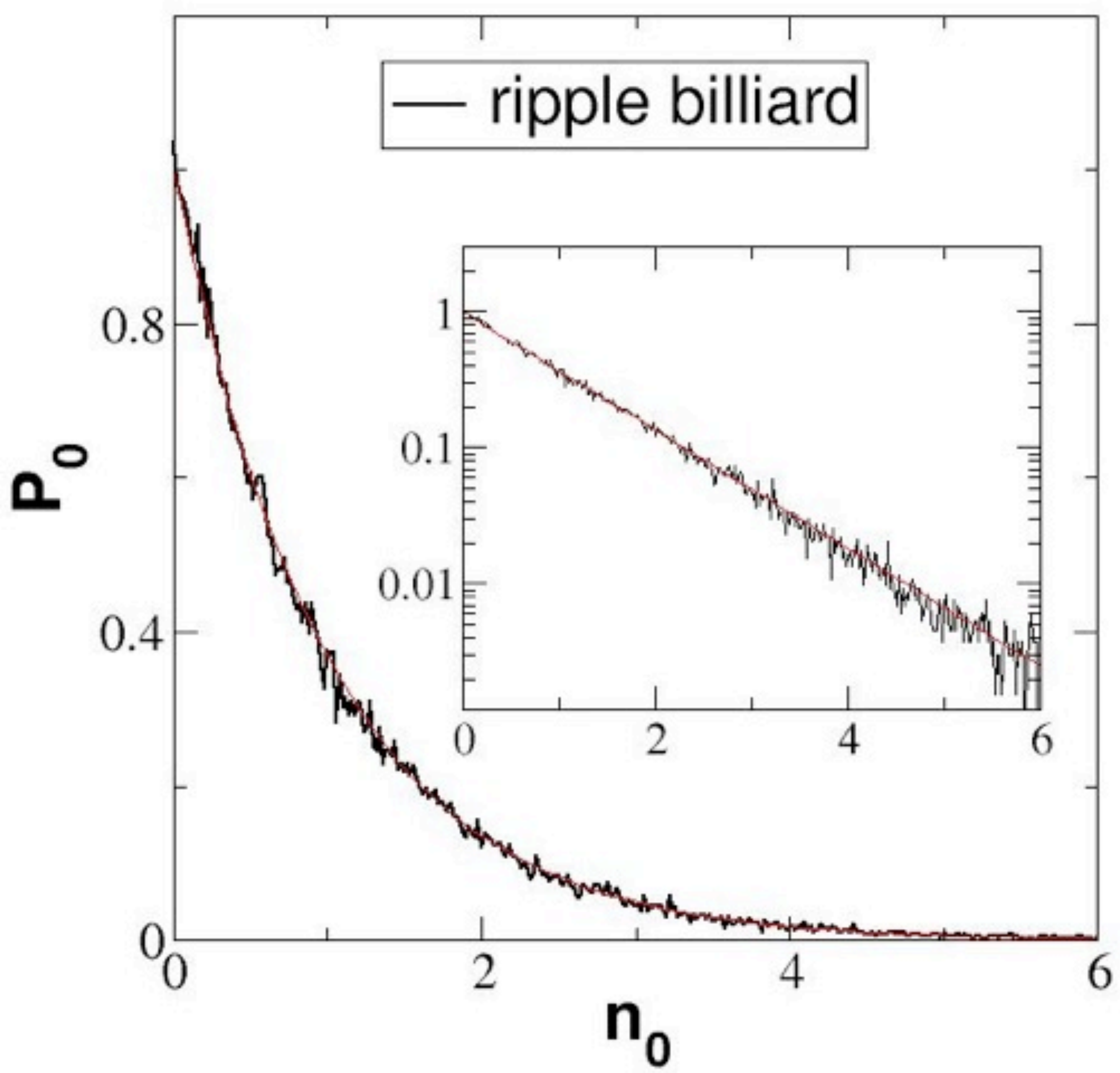
More statistics

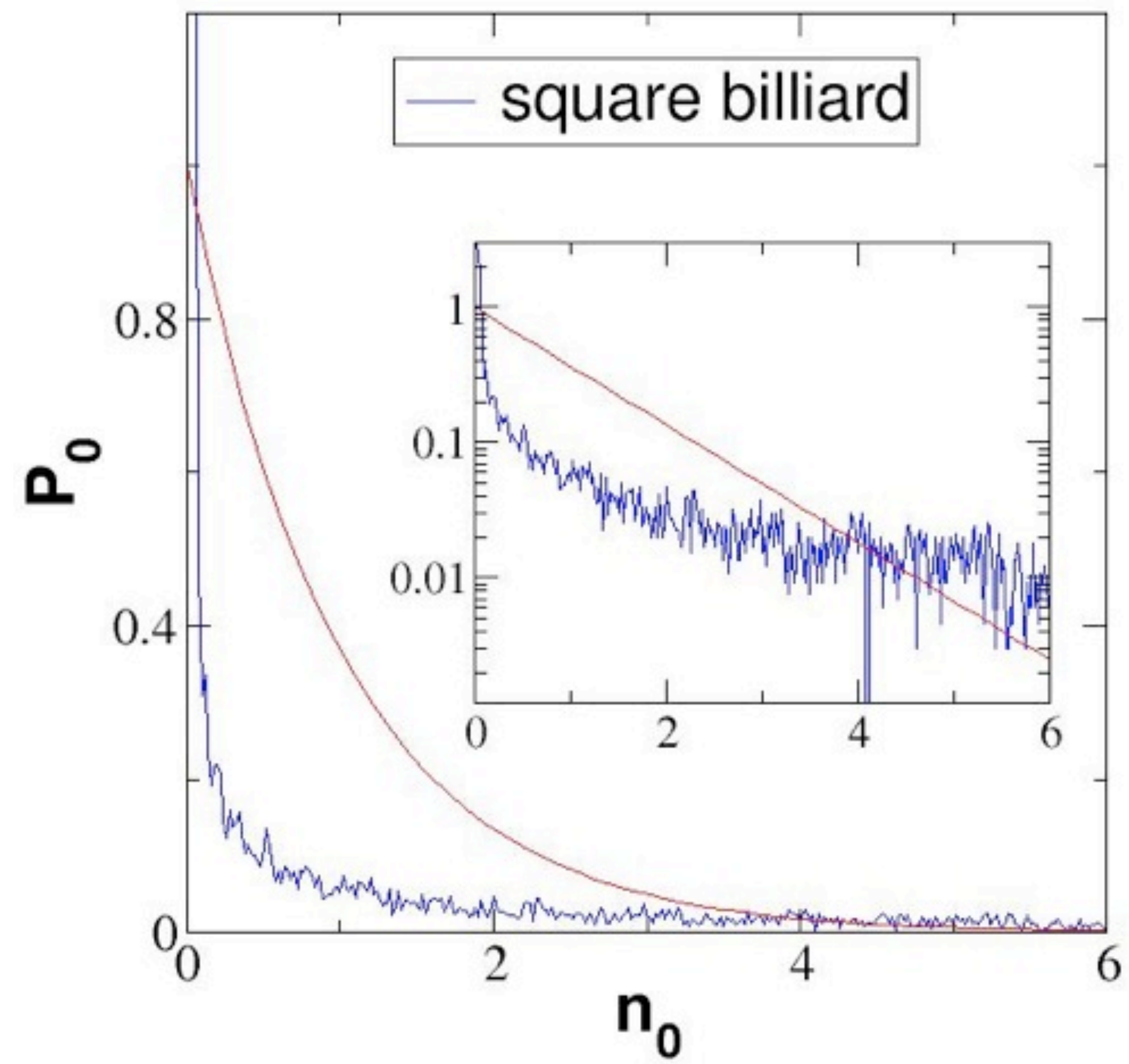
Probability at a given density *n*:

$$P(n,t) = \frac{S(n - \delta n/2, t) - S(n + \delta n/2, t)}{\delta n \cdot S_{\mathrm{total}}}$$



Exponential distribution

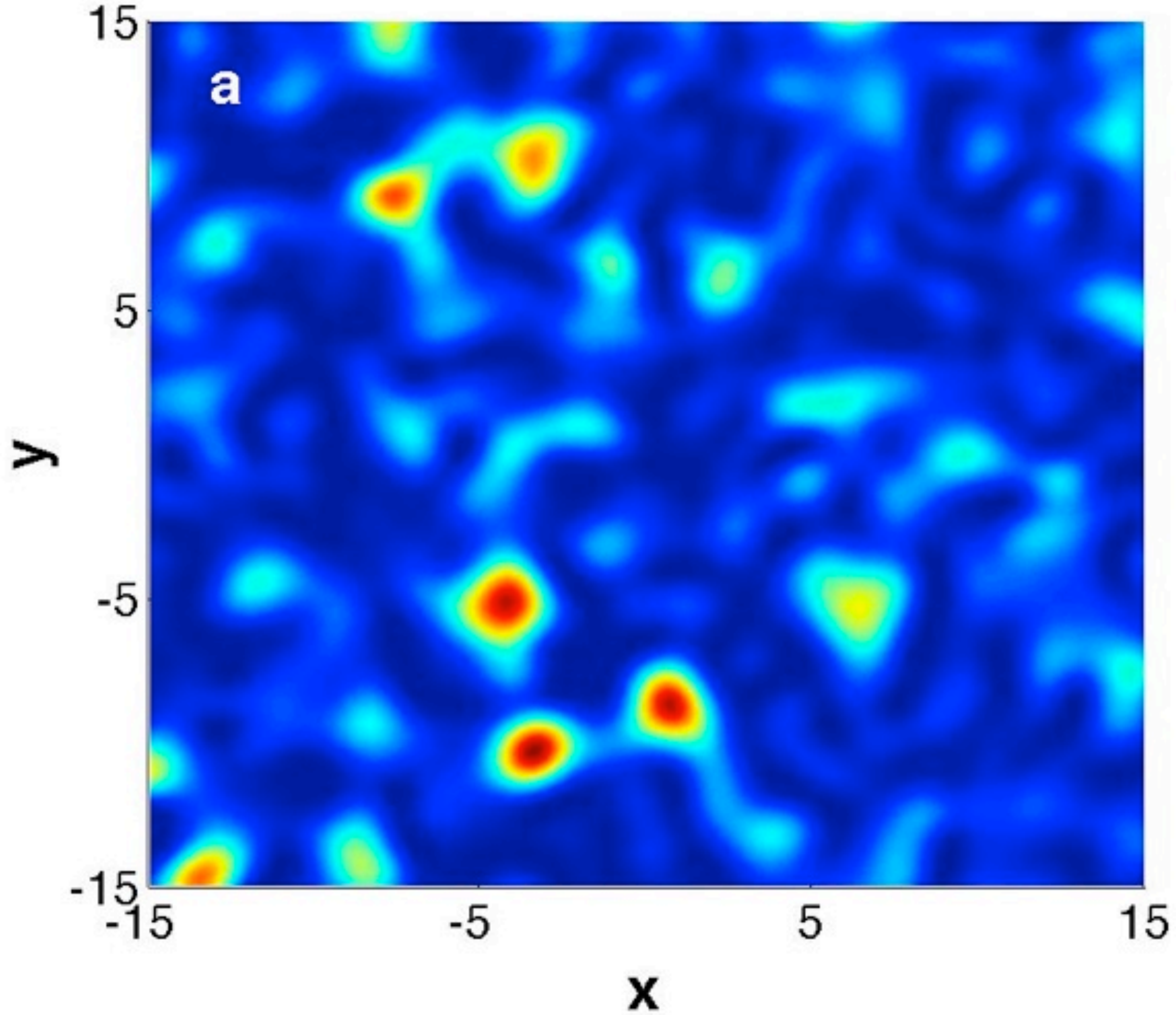


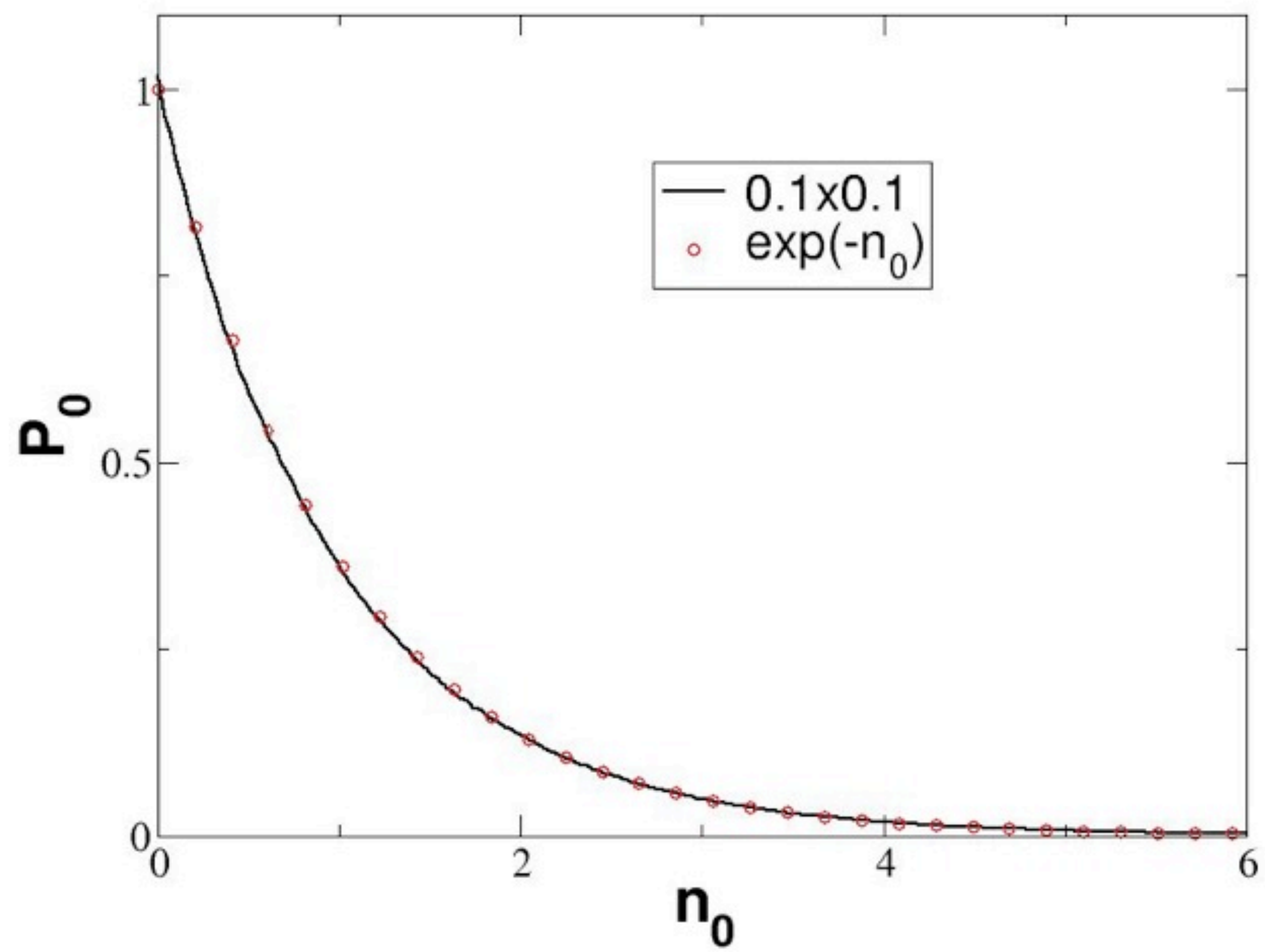


$$P_0^{\mathrm{eq}}\left(n_0\right) = e^{-n_0}$$

Random superposition of plane waves

$$\Psi(x,y) \propto \sum_{\vec{k}} c_{\vec{k}} \exp(i\vec{k}\cdot\vec{x})$$



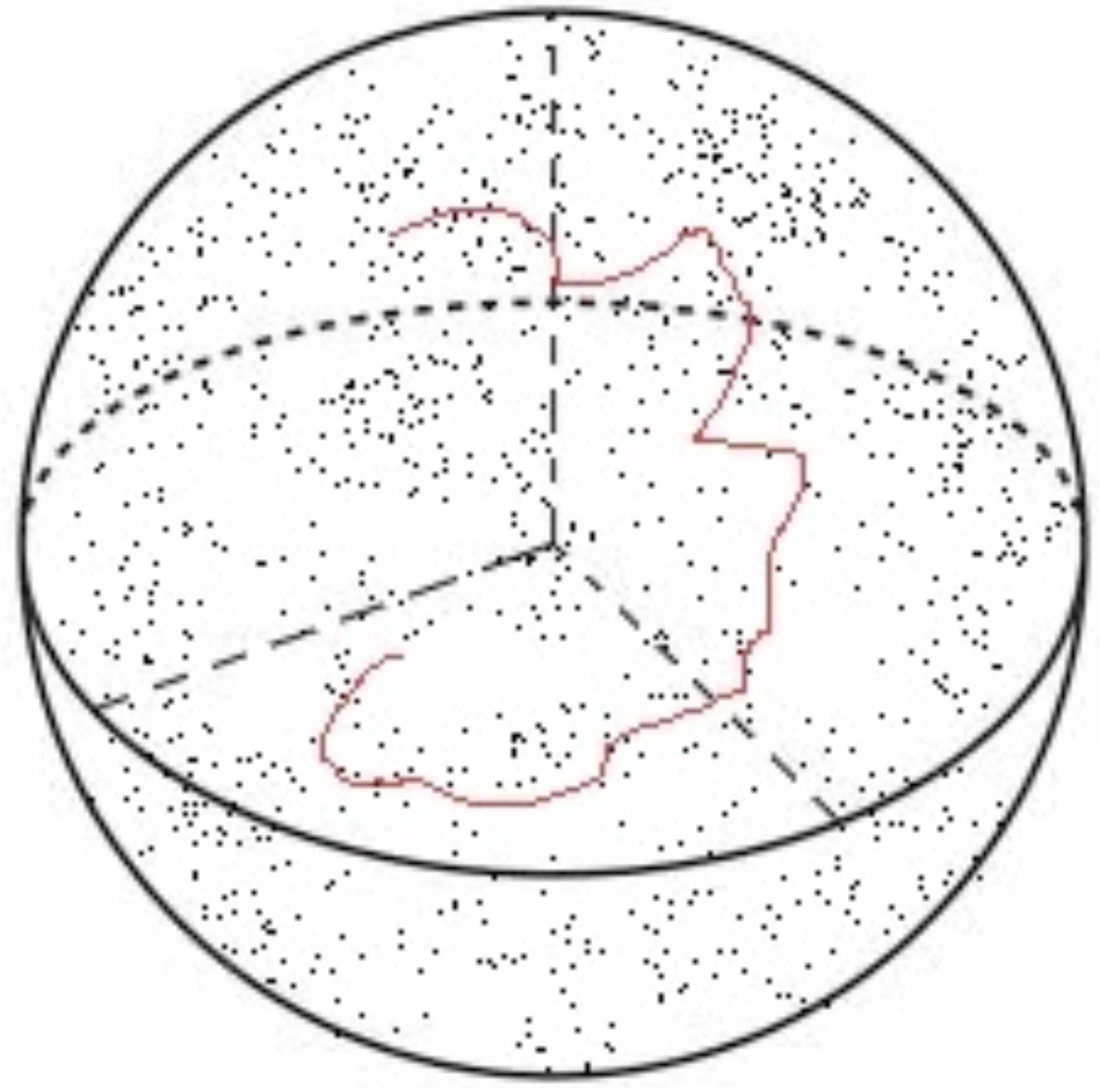


Rigorous proof

complex number

$$|\Psi(t)\rangle \approx \sum_{j=1}^{N} \alpha_j(t)\,|x_j, y_j\rangle$$

$$\sum_{j=1}^{N} |\alpha_j|^2 = 1$$



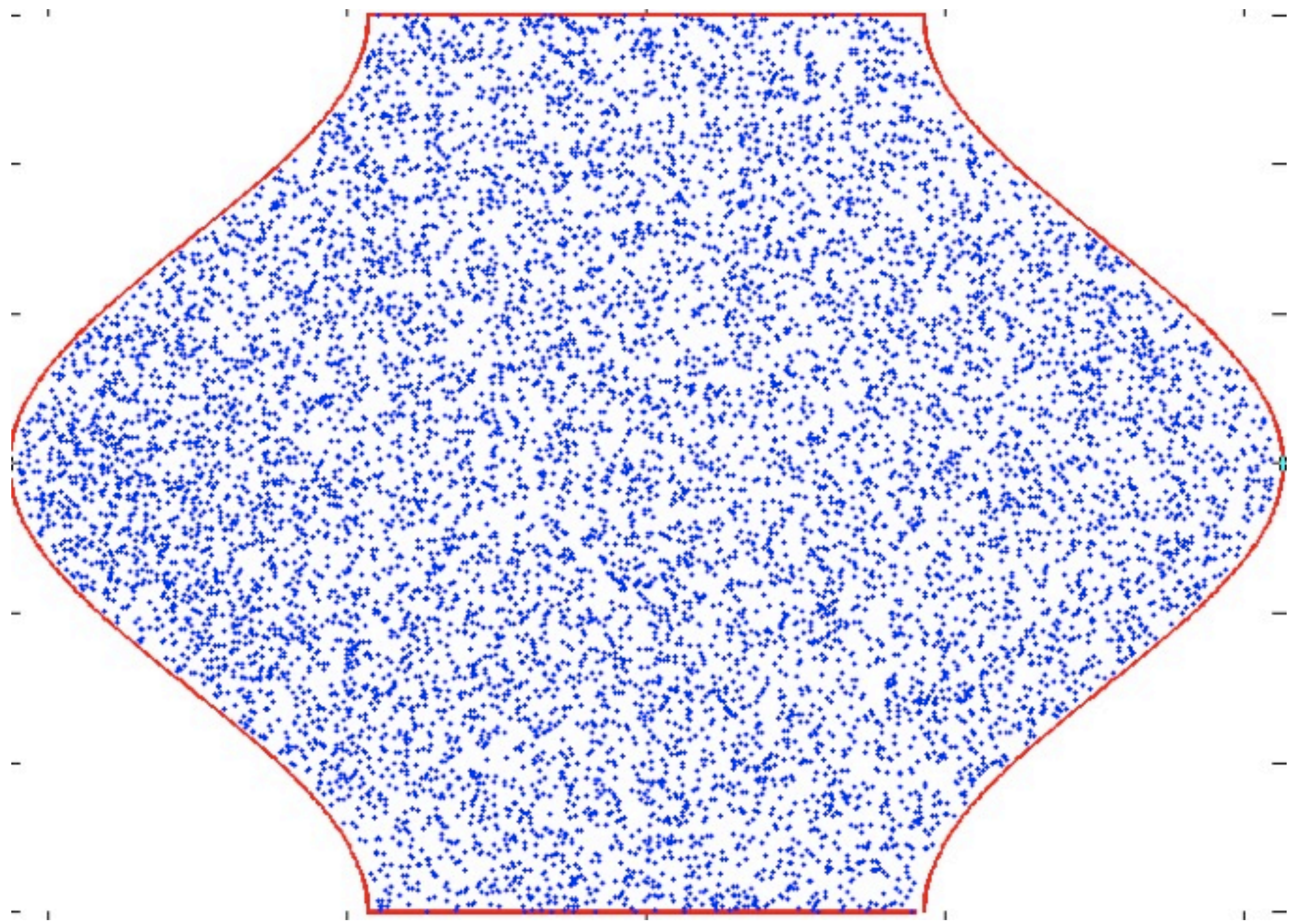
the probability of $|\alpha_j|^2$ being between γ_j and $\gamma_j + d\gamma_j$

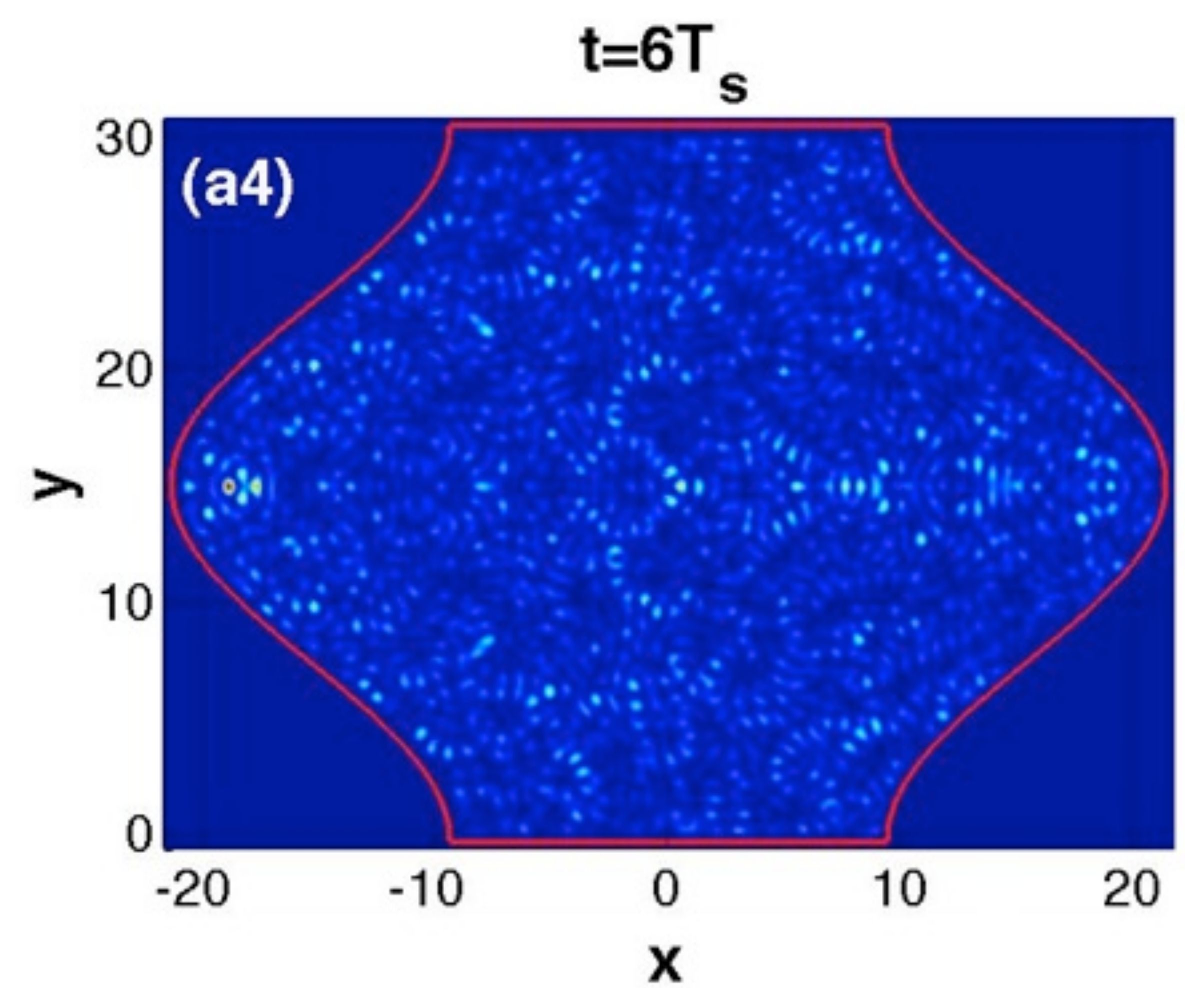
$$P(\gamma_j)\,d\gamma_j = \frac{\int d^2\alpha_1 \cdots d^2\alpha_N \delta\left(\gamma_j - |\alpha_j|^2\right)\delta\left(1 - \sum_{i=1}^{N} |\alpha_i|^2\right)}{\int d^2\alpha_1 \cdots d^2\alpha_N \delta\left(1 - \sum_{i=1}^{N} |\alpha_i|^2\right)} d\gamma_j$$

$N \to \infty$ ⇒

$$P(n_j) = e^{-n_j}$$

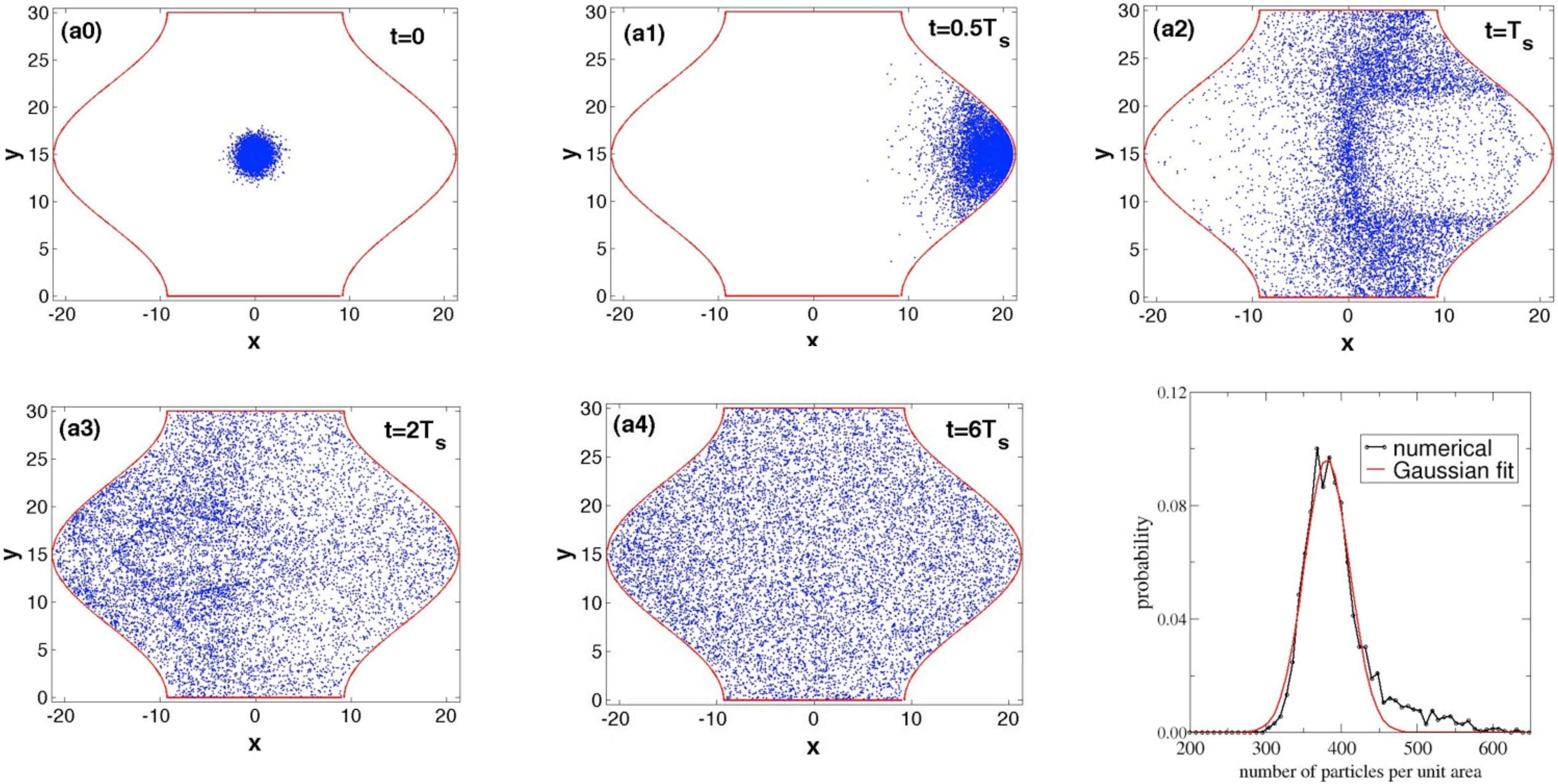
Exponential distribution is quantum



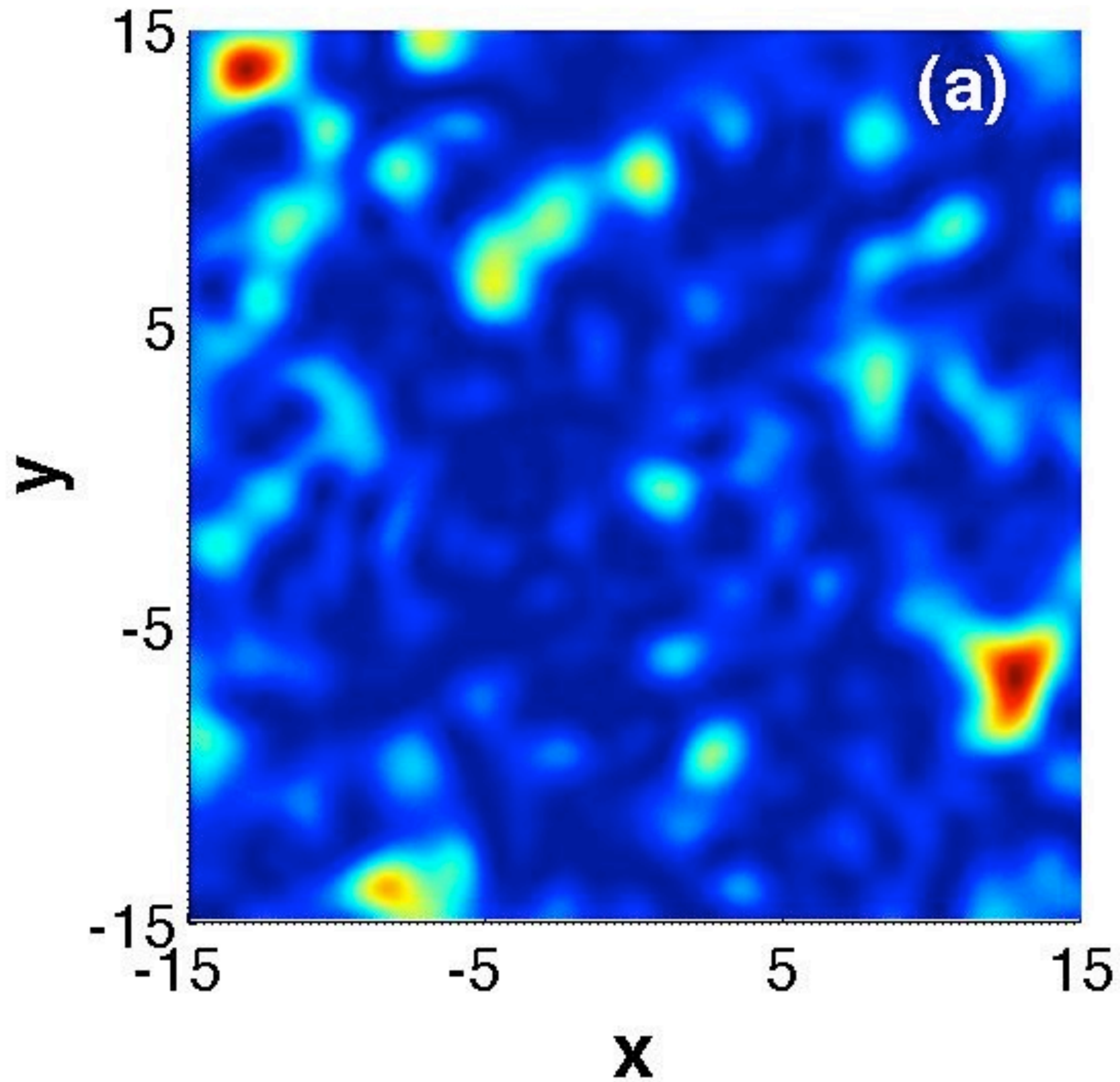


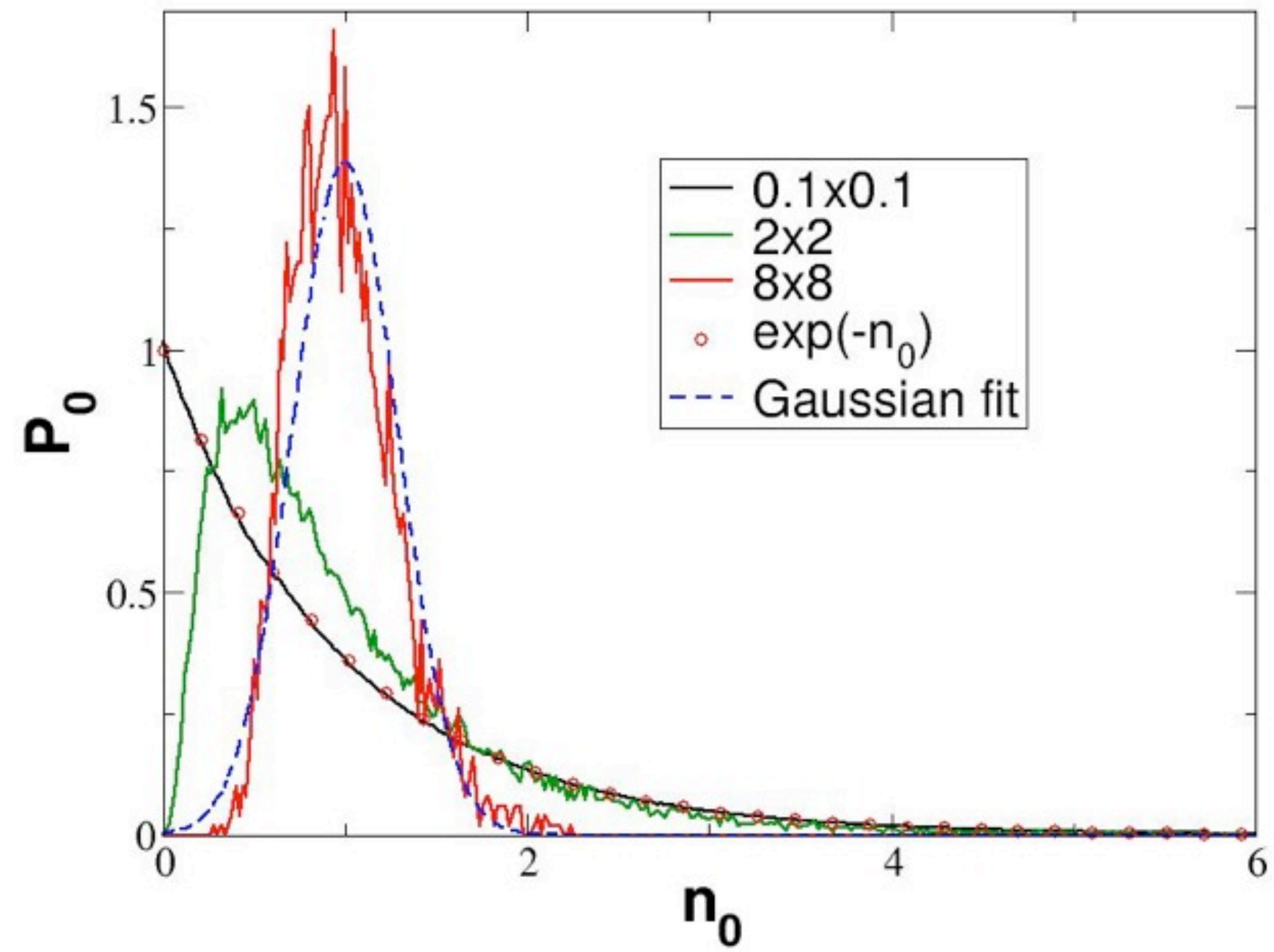
For a classical (or thermal) gas, it should be Gaussian distribution.

Evolution of a classical cloud



Coarse graining: quantum->classical





Properties of exponential distribution

For the exponential distribution

$$P_0^{\text{eq}}\left(n_0\right) = e^{-n_0}$$

- the number fluctuation is $\delta n^2 = n^2$
- the entropy is the largest

$$H(P) = -\int_0^\infty P(n)\ln P(n)dn$$

Previous works

* Review: Backer, Eur. Phys. J. Special Topics **145**, 161(2007)

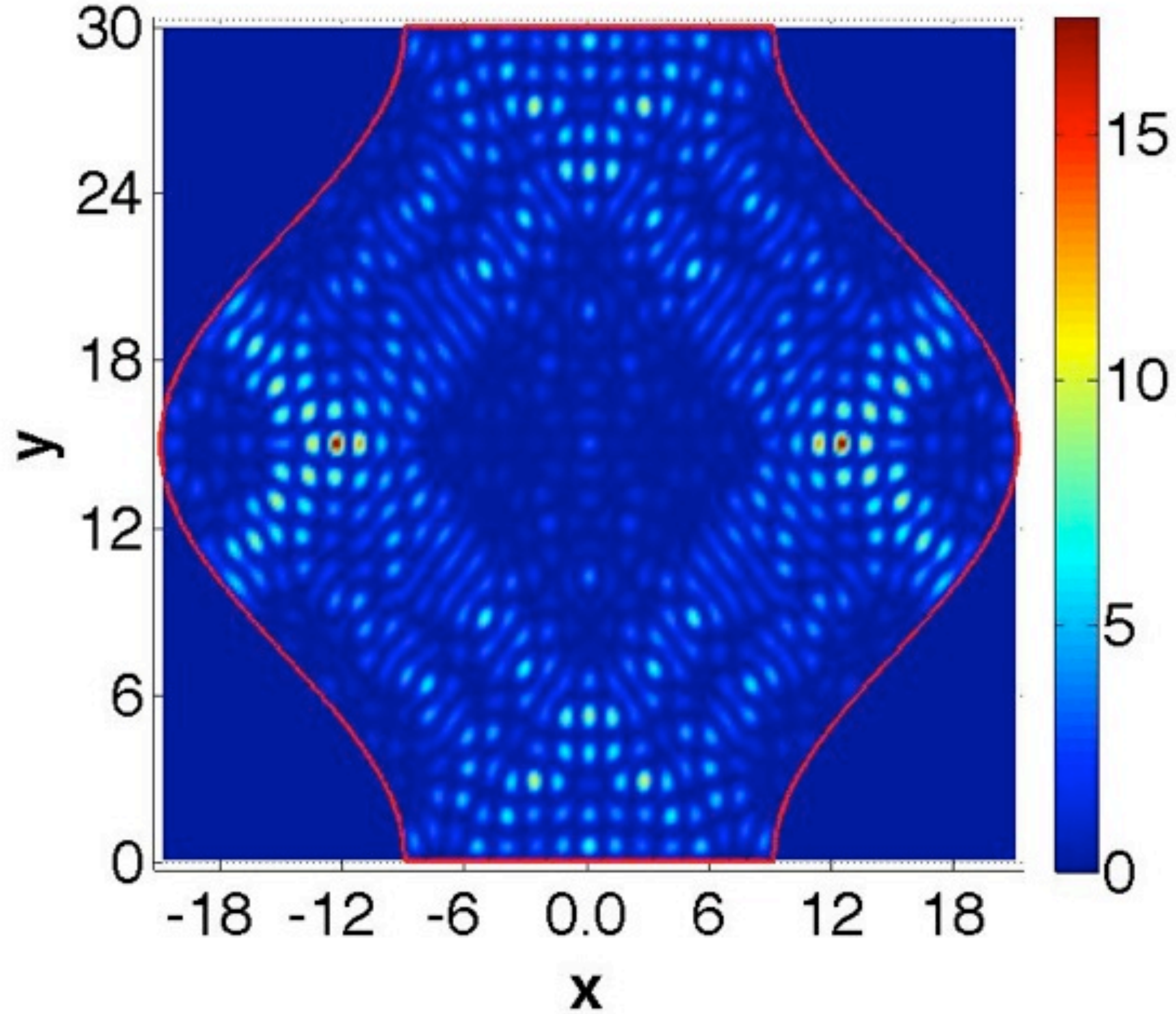
- amplitude distribution of eigenstates
- random superposition of plane waves with real coefficients

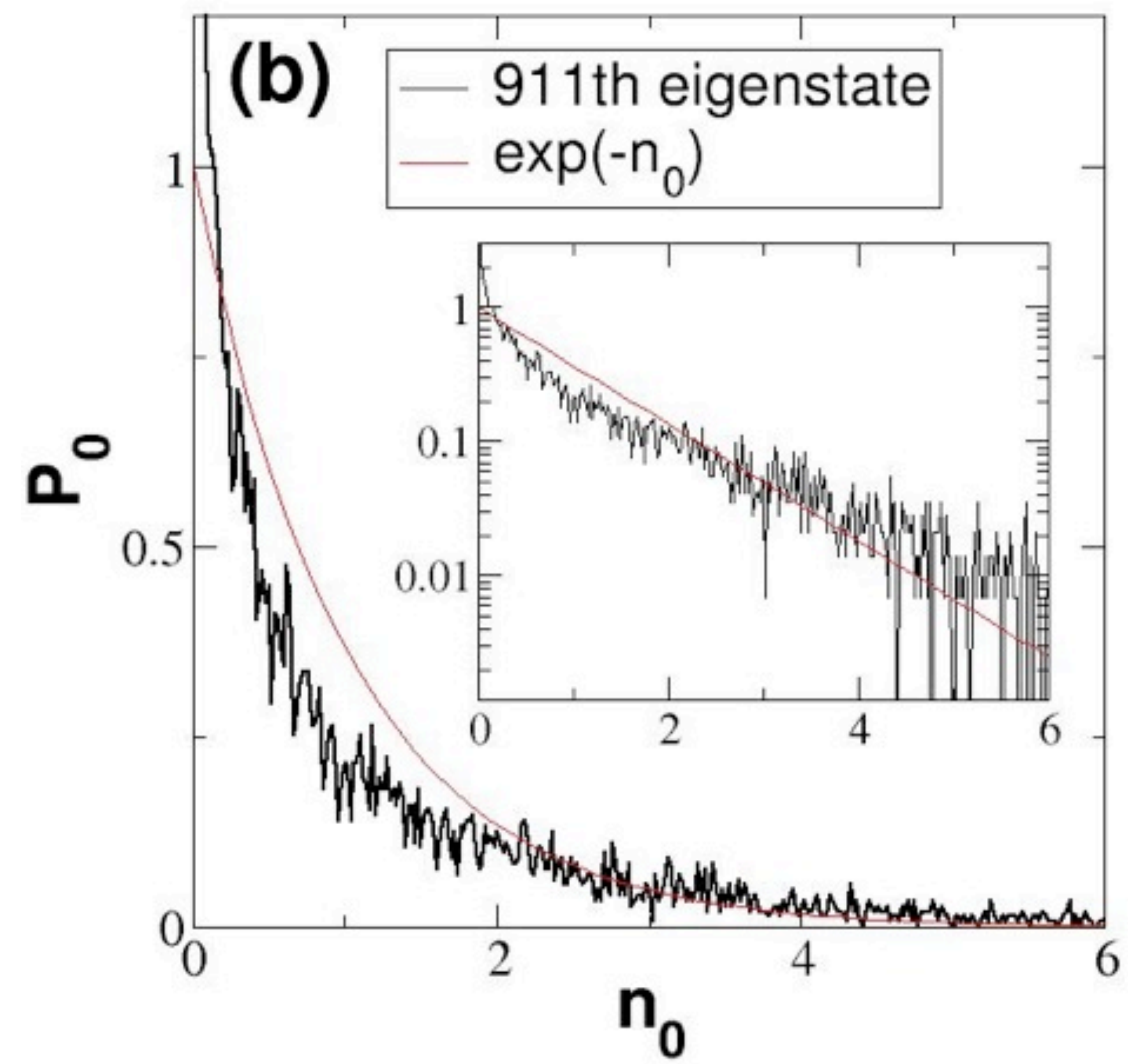
* *Statistical optics* by Goodman *and Introduction to Wave Scattering, Localization, and Mesoscopic Phenomena* by Ping Sheng

- Intensity distribution of a "thermal" light

$$D(I) = \frac{1}{\langle I \rangle_c} \exp\left(- \frac{I}{\langle I \rangle_c} \right)$$

Eigenstates and ETH





For eigenstates, the exponential distribution does not hold.

Counter example to ETH (eigenstate thermalization hypothesis)?

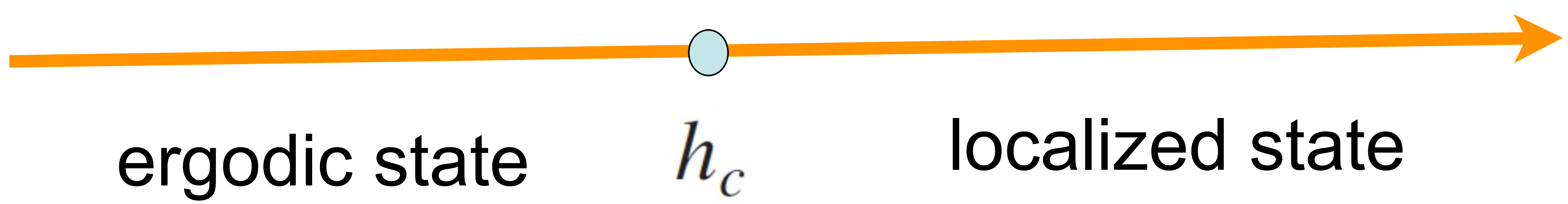
Deutsch(1991),Srednicki(1994),Rigol et al(2008)

Dynamical phase transition

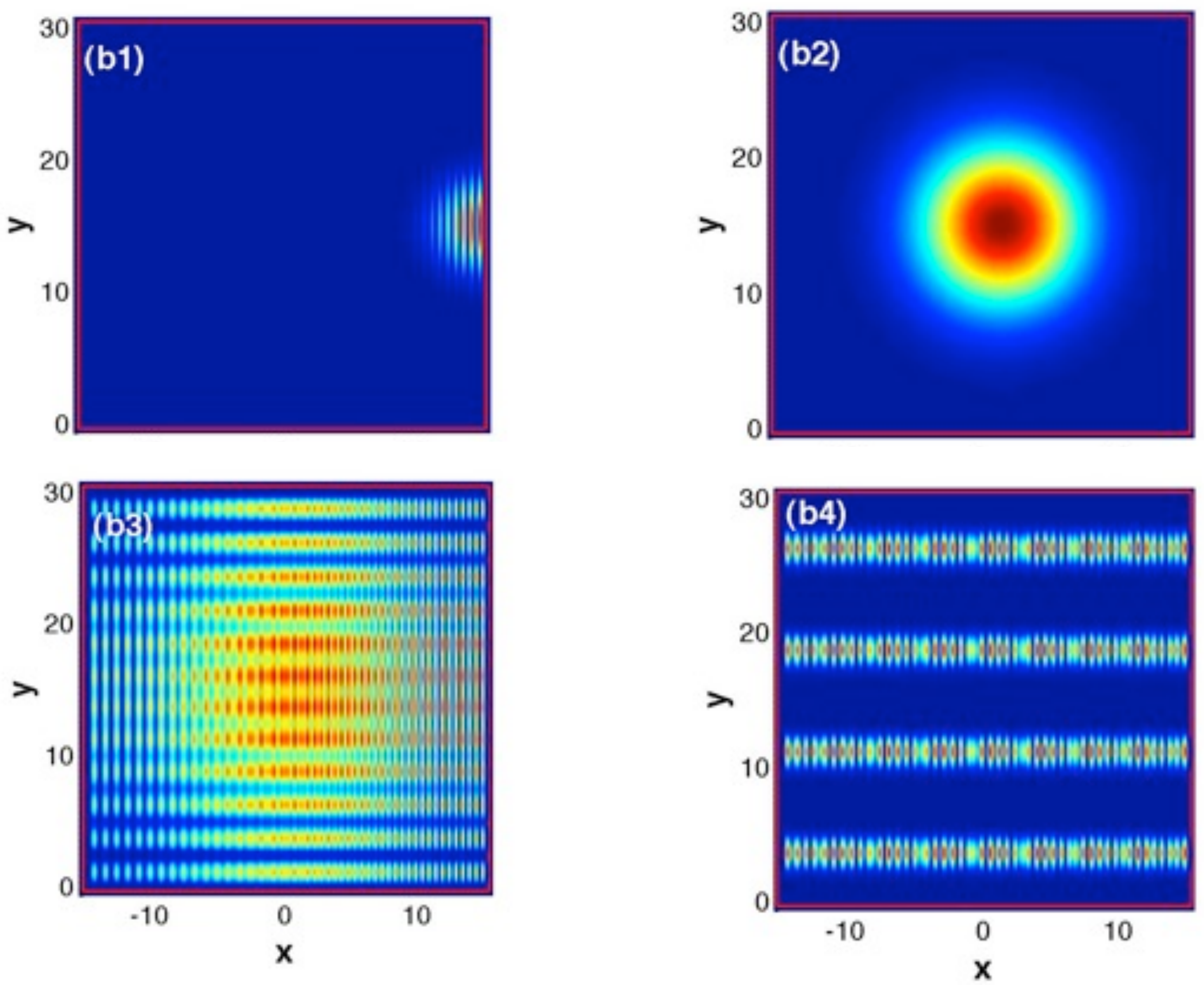
Pal and Huse, PRB(2010)

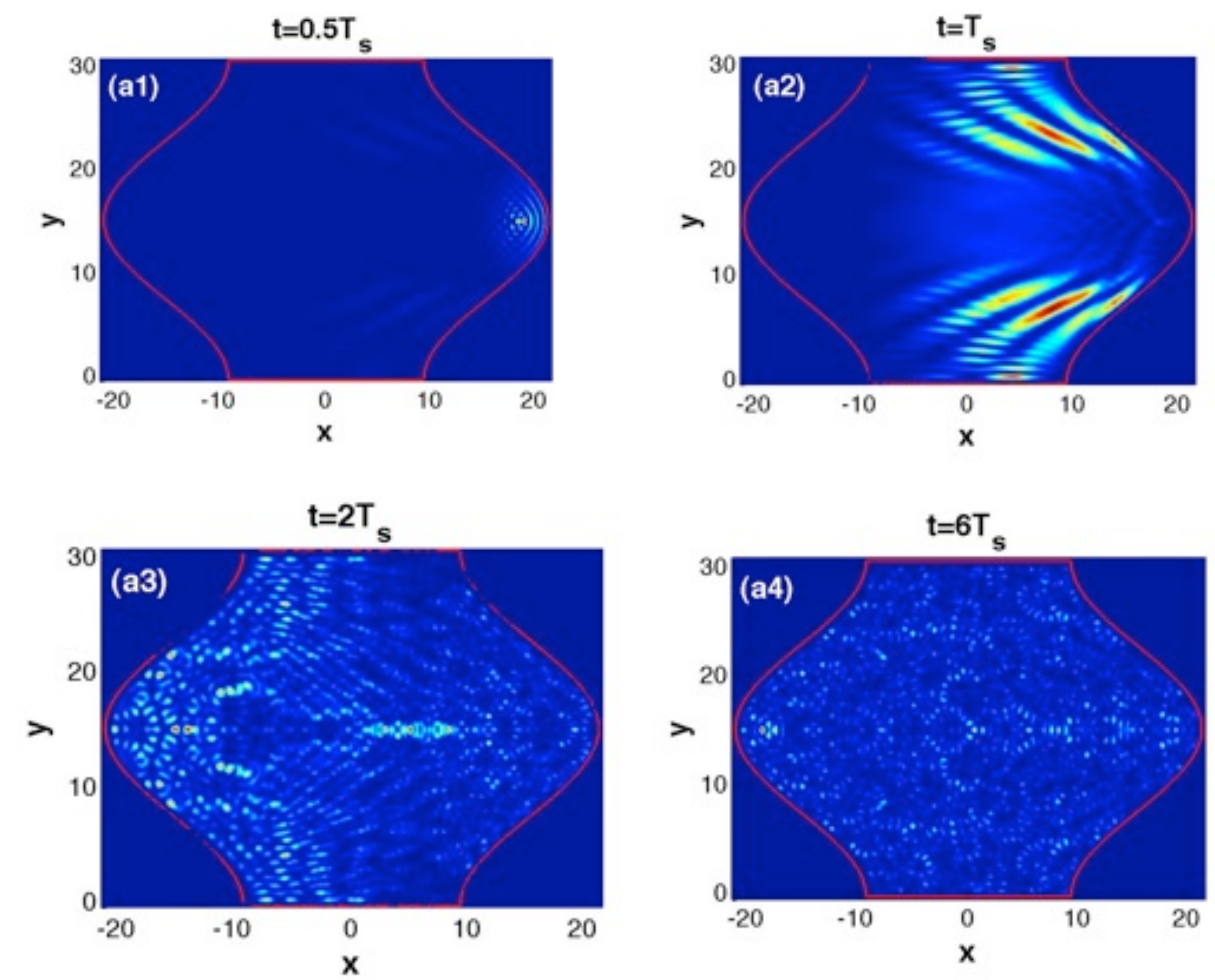
$$H = \sum_{i=1}^{L} [h_i \hat{S}_i^z + J \hat{\vec{S}}_i \cdot \hat{\vec{S}}_{i+1}],$$

h_i a random field between $[-h, h]$



Dynamic phase transition?





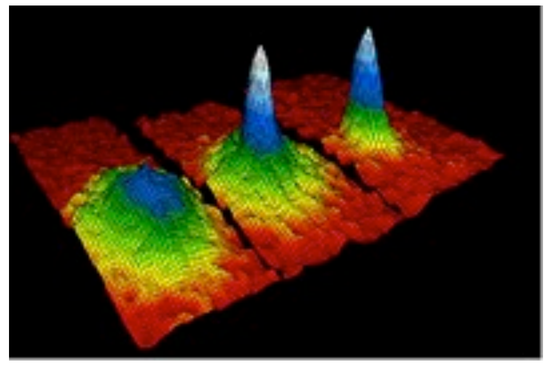
$$x = b - a\cos(2\pi y/L)$$

a_c

Application in cold atoms

CCD imaging is a primary experimental tool in cold atom physics.

- Time of flight



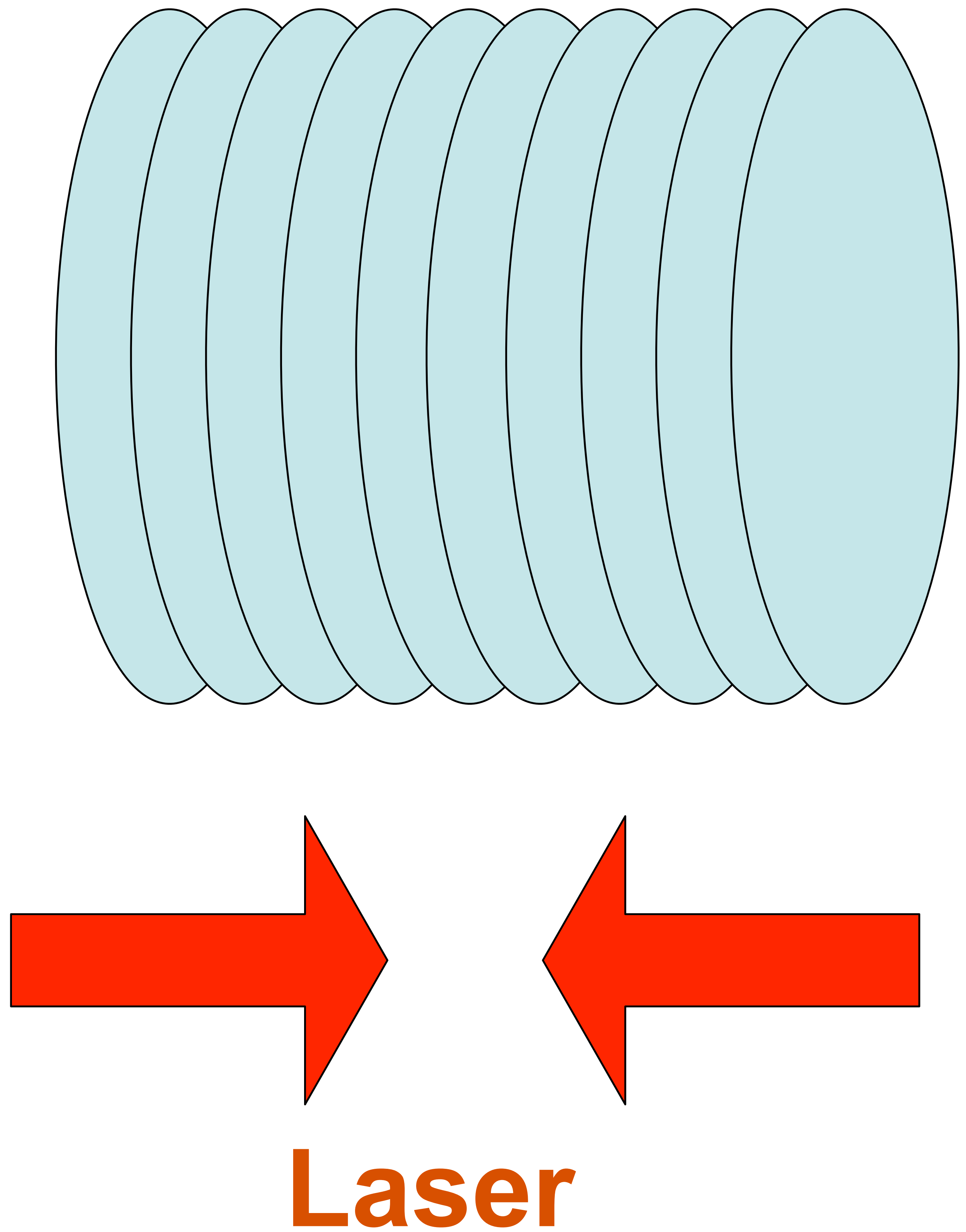
- Noise correlation (Altman et al (2004))

- Homogeneous properties from a trapped gas (Ho and Zhou, 2010)

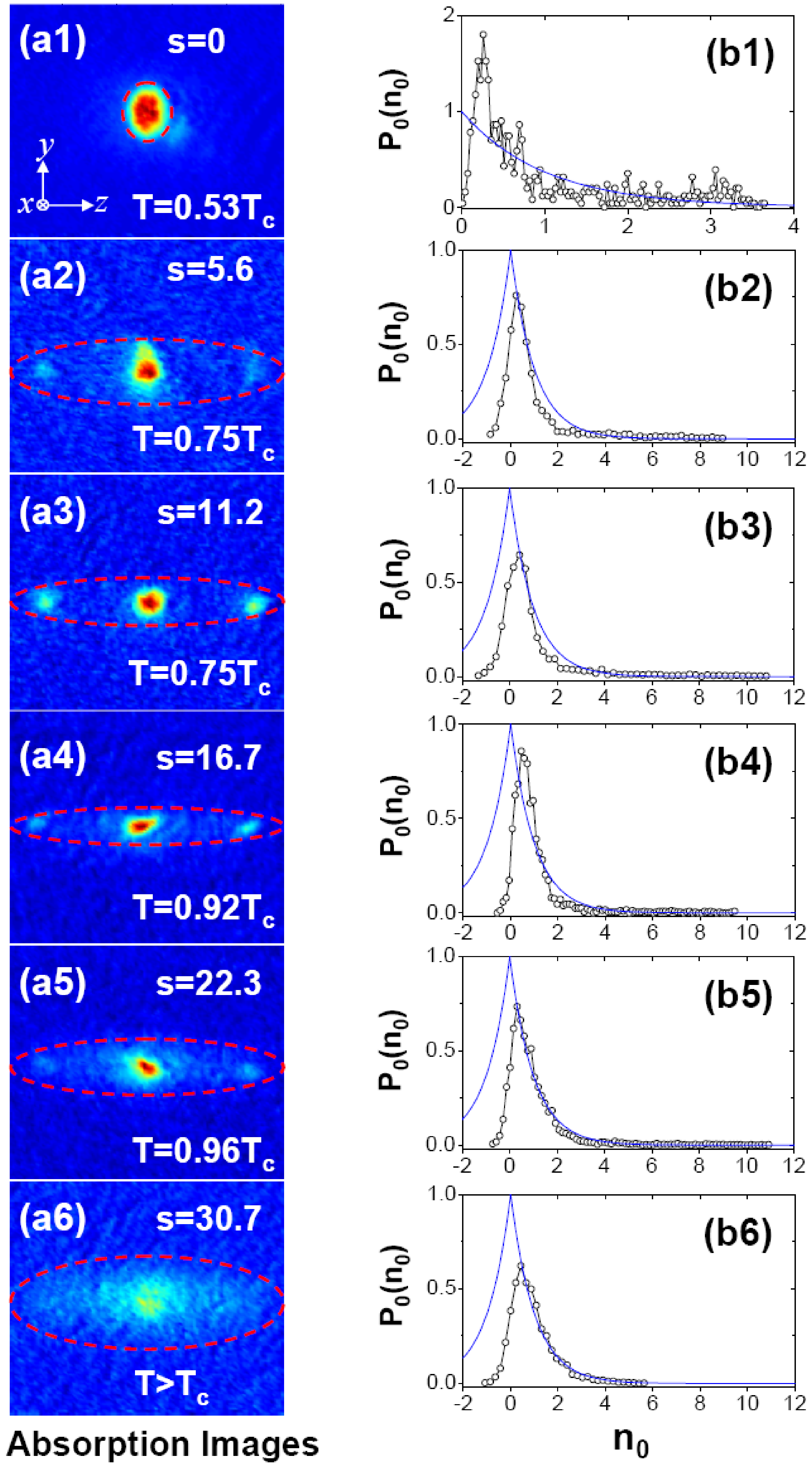
....

- The exponential distribution may be a useful tool

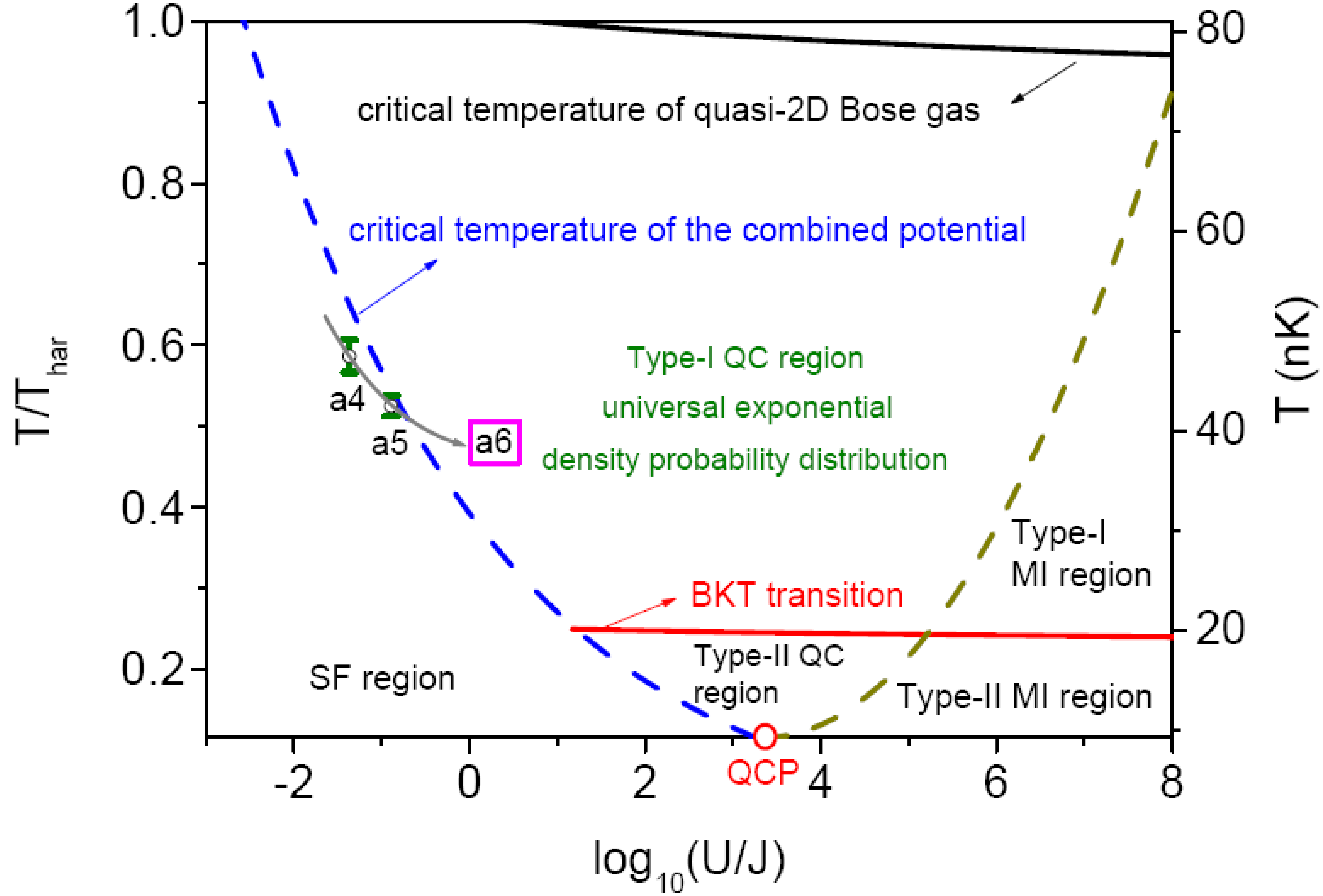
BEC in 1D Optical Lattice



Experiment: BEC in 1D lattice

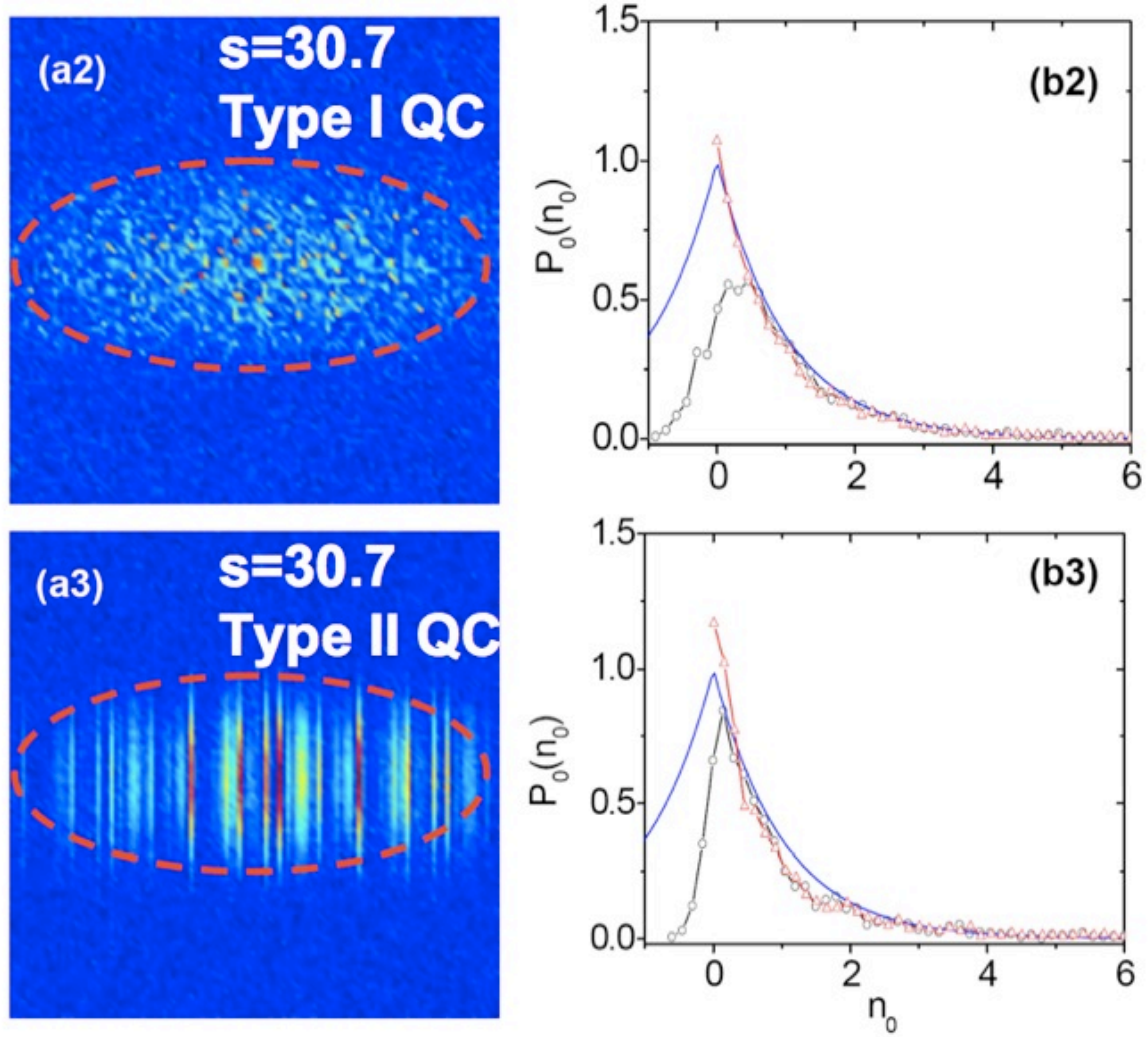


Hongwei Xiong, Xinzhou Tan, Bing Wang, Lijuan Cao, and Baolong Lü
arXiv:1007.4877 (2010)



Theoretical simulation of QC

$$n_{2D}(y,z,t) = \int dx \left| \sum_k \sqrt{N_k} e^{i(\phi_{k\perp}(x,y,t)+\phi_k^s(z,t))} \varphi_{k\perp}(x,y,t)\, \varphi_{kz}(z,t) \right|^2 + n_{2D}^{opt}(x,y)$$



Summary

- Quantum chaos can drive a quantum state to an "steady" or "equilibrium" state, whose density distribution is exponential.

quantum "random" gas	classical "random" gas (thermal gas)
exponential density distribution	Gaussian density distribution

- application to quantum critical gas

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Thank you for your attention.