Unconventional Quantum Critical points

Cenke Xu

UCSB

Unconventional Quantum Critical points

Outline:

1, 2+1d O(3) nonlinear sigma model with conserved Skyrmion number, deconfined criticality.

2, stable critical phases, and CFT in 1d and 2d.

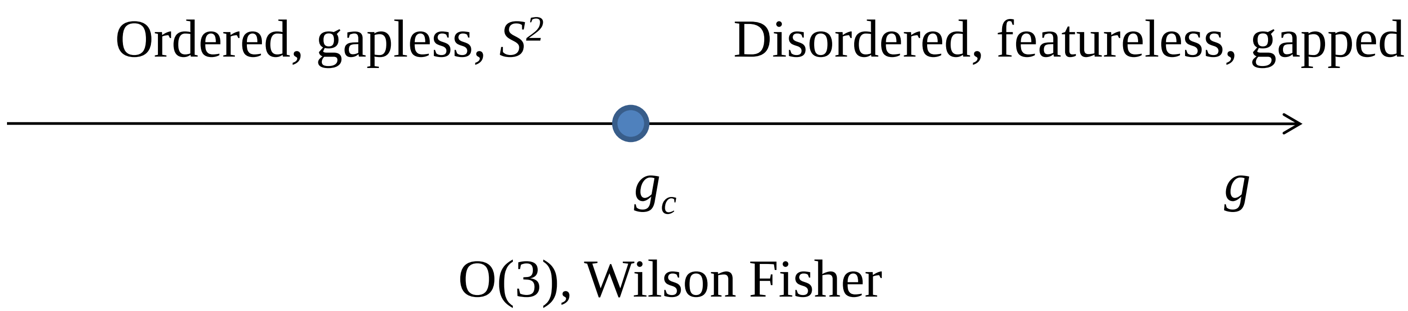
3, (optional) duality between spin and topological defects, phase transitions on the cubic lattice, and triangular lattice.

Unconventional Quantum Critical points

2+1d O(3) Nonlinear sigma model:

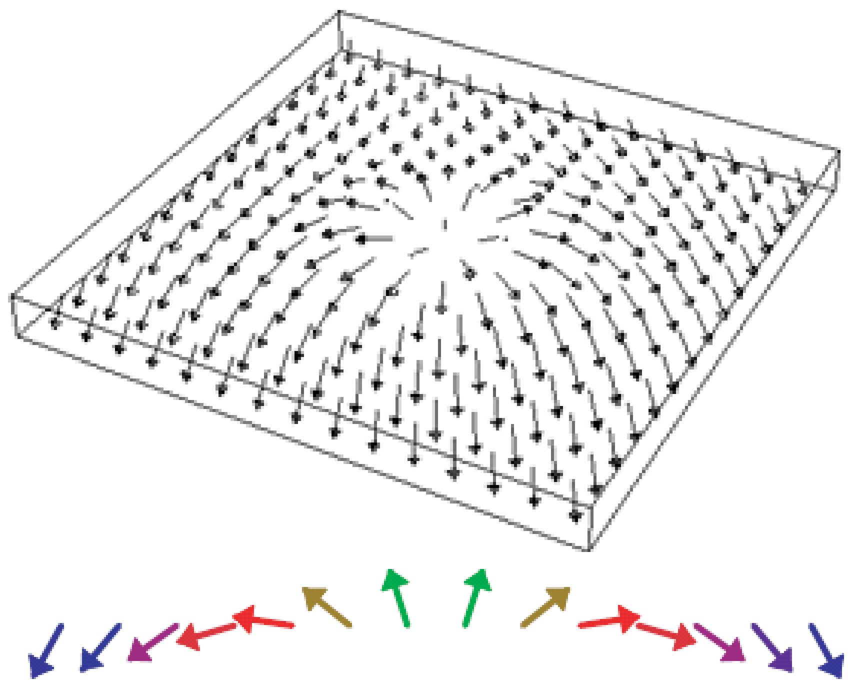
$$\mathcal{L} = \frac{1}{g}(\partial_\mu \vec{n})^2, \quad (\vec{n})^2 = 1$$

"Conventional" O(3) nonlinear sigma model:



Unconventional Quantum Critical points

"Conventional" O(3) nonlinear sigma model, means Skyrmion number is not conserved.

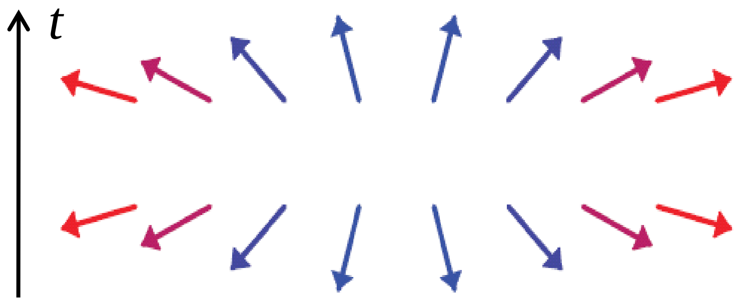


$$\pi_2[S^2] = \mathbb{Z} \qquad\qquad Q = \frac{1}{4\pi}\int d^2r\,\hat{n}\cdot\partial_x\hat{n}\times\partial_y\hat{n},$$

Unconventional Quantum Critical points

"Conventional" O(3) nonlinear sigma model, means Skyrmion number is not conserved.

Existence of spacetime hedgehog like monopole, Changes Skyrmion number by 1.

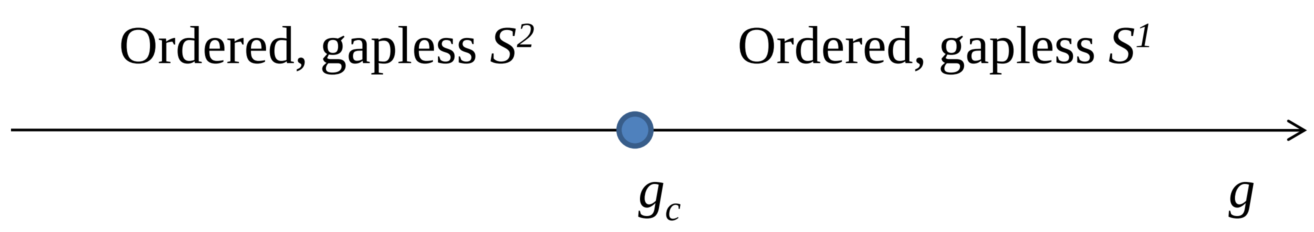


Unconventional Quantum Critical points

"Unconventional" O(3) nonlinear sigma model, means Skyrmion number is conserved i.e. monopole suppressed.

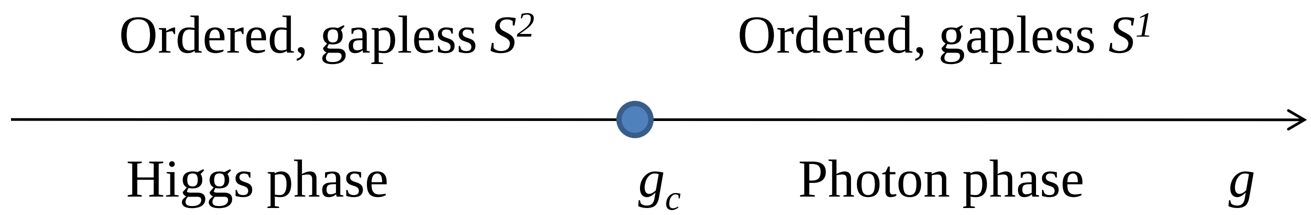
The symmetry of unconventional O(3) NSM is O(3) x U(1)

$$\mathcal{L} = \frac{1}{g}(\partial_\mu \vec{n})^2, \quad (\vec{n})^2 = 1$$



Universality class?

Unconventional Quantum Critical points



Universality class?

Map to CP(1) formalism: $\hat{n} = z^*_\alpha \sigma^a_{\alpha\beta} z_\beta$

$$\mathcal{L}_z = \sum_{a=1}^{2} |\left(\partial_\mu - i a_\mu\right) z_a|^2 + s|z|^2 + u\left(|z|^2\right)^2 + \kappa\left(\epsilon_{\mu\nu\kappa}\partial_\nu a_\kappa\right)^2$$

z_a condensed, gauge field Higgsed, gauge invariant operator is $\hat{n}$

z_a uncondensed, gauge field in photon phase, gapless photon

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Ordered, gapless S^2 Ordered, gapless S^1

Higgs phase g_c Photon phase g

Universality class?

$$\mathcal{L}_z = \sum_{a=1}^{2} \left| \left(\partial_\mu - i a_\mu\right) z_a \right|^2 + s|z|^2 + u\left(|z|^2\right)^2 + \kappa\left(\epsilon_{\mu\nu\kappa}\partial_\nu a_\kappa\right)^2$$

z_a uncondensed, gauge field in photon phase, gapless photon

2+1d photon is a “condensate” of its flux, dual to a superfluid:

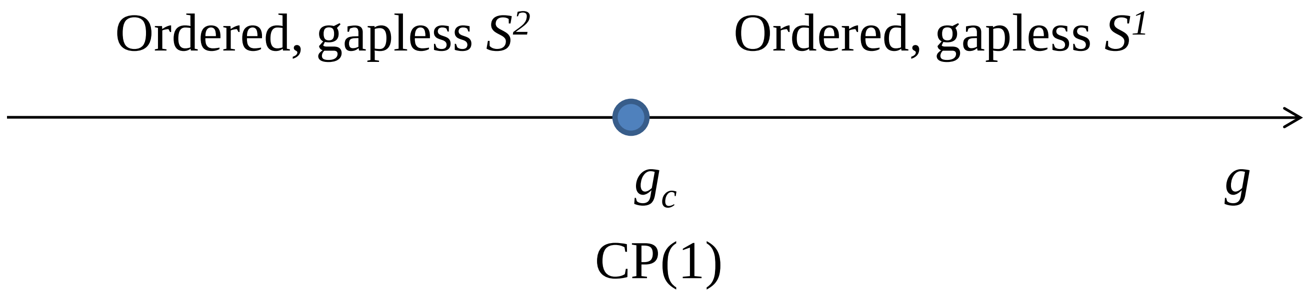
$$\text{flux} = \frac{1}{2\pi}\int d^2r(\partial_x a_y - \partial_y a_x) = \; Q = \frac{1}{4\pi}\int d^2r\, \hat{n}\cdot\partial_x\hat{n}\times\partial_y\hat{n},$$

Photon phase has ground state manifold S^1

Unconventional Quantum Critical points

Conclusion: When Skyrmion number is conserved, O(3) NSM is equivalent to Non-Compact CP(1) model.

$$\mathcal{L}_z = \sum_{a=1}^{2} \left| \left(\partial_\mu - i a_\mu\right) z_a \right|^2 + s|z|^2 + u\left(|z|^2\right)^2 + \kappa\left(\epsilon_{\mu\nu\kappa}\partial_\nu a_\kappa\right)^2$$



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Example 1, O(3) vector is Neel vector:

$$Q = \frac{1}{4\pi}\int d^2r\, \hat{n}\cdot\partial_x\hat{n}\times\partial_y\hat{n},$$

Carries lattice momentum,
Transform nontrivially on lattice.
Haldane, Read, Sachdev

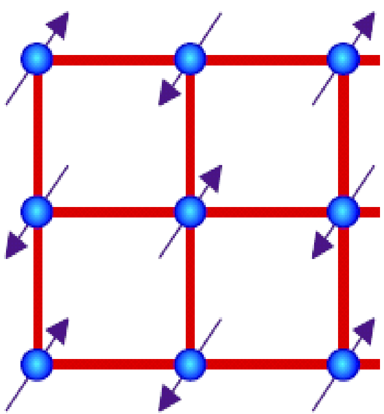
Ordered, Neel, gapless S^2 Ordered, gapless S^1

g_c

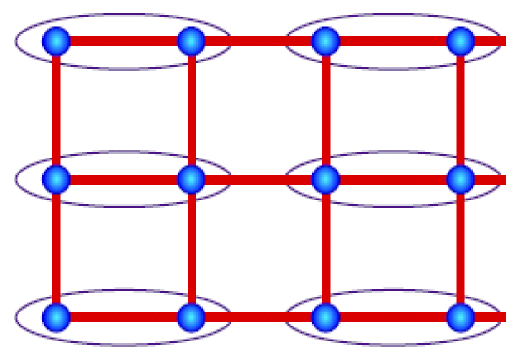
CP(1)

S^1 Further broken down to Z_4, VBS

g



Senthil, *et.al.*, 2004



Unconventional Quantum Critical points

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Ordered, Neel, gapless S^2 — Ordered, gapless S^1

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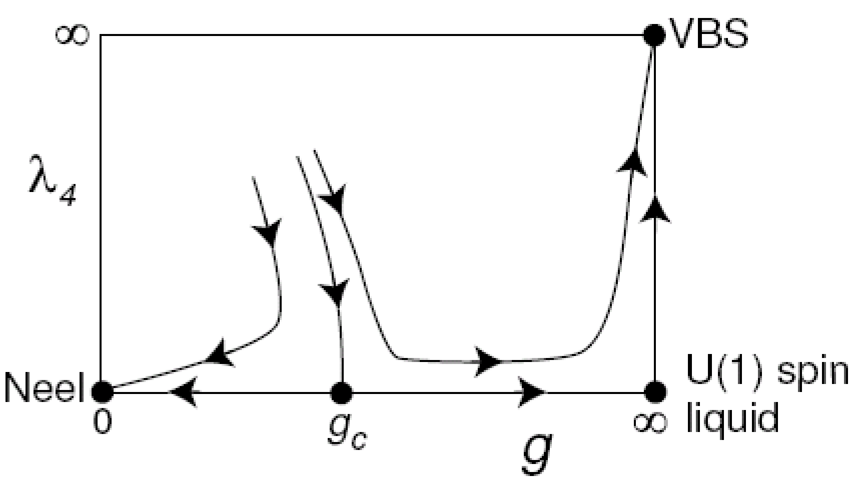

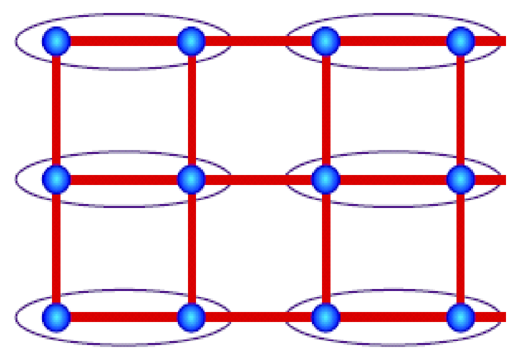

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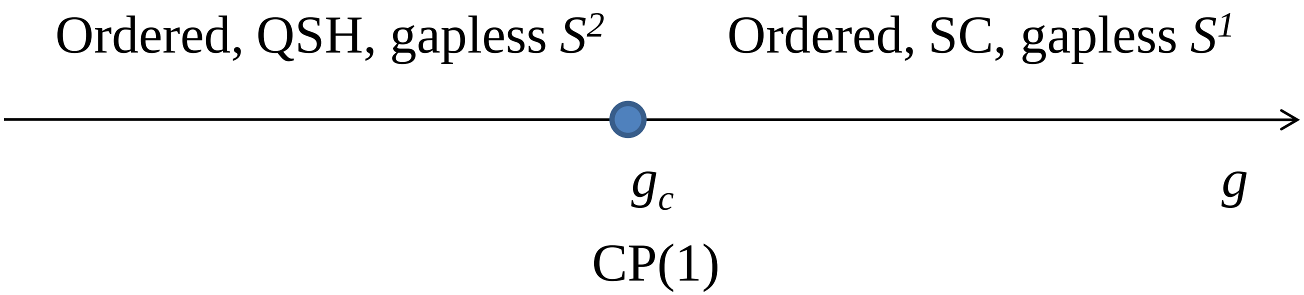
Example 2, O(3) vector is QSH vector

$$S = \int d^3x\, \overline{\psi}(-i\gamma_\mu \partial_\mu + im\vec{\sigma}.\hat{N})\psi$$

$$Q = \frac{1}{4\pi}\int d^2r\, \hat{n}\cdot\partial_x\hat{n}\times\partial_y\hat{n} \;= 2e$$



Grover, Senthil, 2008

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More quantitative about CP(N-1) model:

$$\mathcal{L}_z = \sum_{a=1}^{N} \left| \left(\partial_\mu - i a_\mu \right) z_a \right|^2 + s|z|^2 + u\left(|z|^2\right)^2 + \kappa \left(\epsilon_{\mu\nu\kappa} \partial_\nu a_\kappa \right)^2$$

$$\xi \sim s^{-\nu} \qquad\qquad \nu = 1 - \frac{48}{\pi^2 N}$$

$$\eta_{\hat{n}} = 2\left(\Delta[z^\dagger \vec{\sigma} z] - (D-2) \right) = 1 - \frac{32}{\pi^2 N}$$

Large anomalous dimension, “confirmed” by numerics:

Sandvik, 2007,
Melko, Kaul, 2008,
Mortrunich, Vishwanath, 2008

$$\eta \sim 0.2 - 0.4$$

Unconventional Quantum Critical points

$$\xi \sim s^{-\nu} \qquad\qquad \nu = 1 - \frac{48}{\pi^2 N}$$

$$\eta_{\hat{n}} = 2\left(\Delta[z^\dagger \vec{\sigma} z] - (D-2)\right) = 1 - \frac{32}{\pi^2 N}$$

Compare CP(N) and O(N)

O(N) $N = \infty, \nu = 1, \eta = 0$

CP(N) $N = \infty,\ \nu = 1,\ \eta = 1$

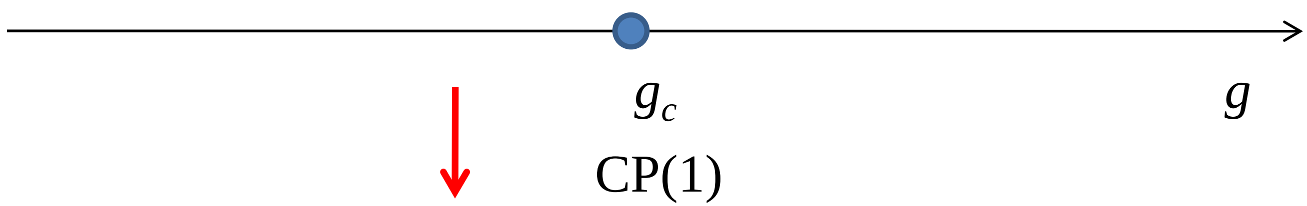
N		ν	η
1	FD exp [13]	0.6303(8)	0.0335(25)
	ϵ exp [13]	0.6305(25)	0.0365(50)
	HT exp [14]	0.63012(16)	0.03639(15)
	MC [15]	0.6297(5)	0.0362(8)
2	FD exp [13]	0.6703(15)	0.0354(25)
	ϵ exp [13]	0.6680(35)	0.0380(50)
	HT exp [16]	0.67155(27)	0.0380(4)
	MC [16]	0.6716(5)	0.0380(5)
3	FD exp [13]	0.7073(35)	0.0355(25)
	ϵ exp [13]	0.7045(55)	0.0375(45)
	HT exp [17]	0.7112(5)	0.0375(5)
	MC [17]	0.7113(11)	0.0378(6)

Calabrese *et.al.*
Condmat/0306273

Unconventional Quantum Critical points

Conjecture: NCCP(1) model is equivalent to the O(5) NSM with WZW term (Senthil, Fisher, 2006)

Ordered, gapless S^2 Ordered, gapless S^1



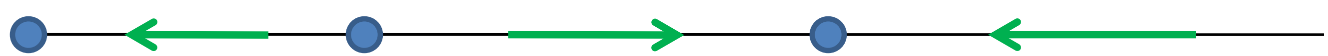
g_c g

CP(1)

$$S = \int d^3x \frac{1}{2G} (\partial_i \phi_a)^2 + 2\pi i \Gamma \left[\vec{\phi}\right]$$

$$\Gamma = \frac{\epsilon_{abcde}}{\mathrm{Area}(S^4)} \int_0^1 du \int d^3 x \phi_a \partial_x \phi_b \partial_y \phi_c \partial_\tau \phi_d \partial_u \phi_e$$

0 $G_1{}^*$ $G_2{}^*$



G

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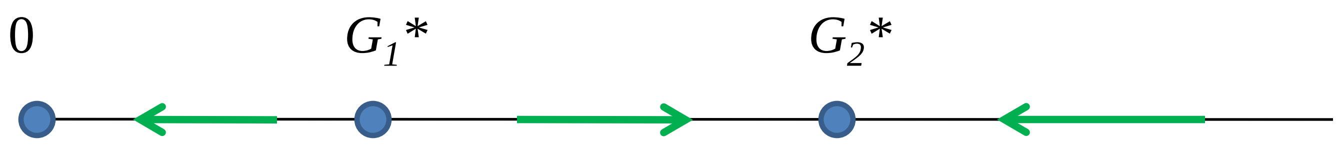
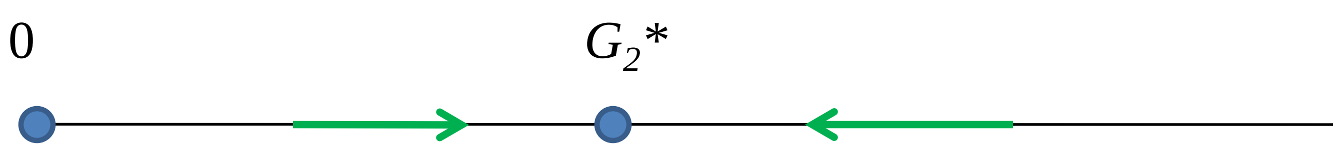
$$S = \int d^3x \frac{1}{2G}(\partial_i \phi_a)^2 + 2\pi i \Gamma\left[\vec{\phi}\right]$$



For 1+1d O(4) nonlinear sigma model with WZW term:

$$S = \int d^2x \frac{1}{2G}(\partial_i \phi_a)^2 + 2\pi i \Gamma[\vec{\phi}]$$



$SU(2)_1$ Spin-1/2 chain

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Stable critical phase

Stable, gapless, fixed point, described by 2+1d CFT.

$$\vec{S}_i = f_{i,\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} f_{i,\beta} \qquad\qquad \sum_a f_{i,a}^{\dagger} f_{i,a} = C$$

Spinon fills a band structure, some with Dirac fermion dispersion.

Low energy theory, Dirac fermions + gauge field fluctuation, gauge field can be SU(2), U(1), Z2, SU(n)……

We should only count gauge invariant operators as physical quantities.

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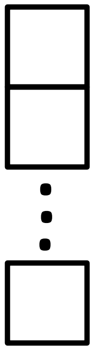
Example 1: honeycomb lattice SU(N) magnets, or square lattice π-flux state:



$N/2$

$$\sum_a f^\dagger_{i,a} f_{i,a} = \frac{N}{2}$$

$$\mathcal{L} = \sum_{a=1}^{2N} \bar{\psi}_a \gamma_\mu (\partial_\mu - i a_\mu) \psi_a + \frac{1}{e^2} f^2_{\mu\nu} + O(\psi^4)$$

$N > N_c$, it is a CFT, with $e^{2*} \sim \frac{1}{N}$, emergent SU(2N)

$N < N_c$, (maybe) chiral symmetry breaking mass generation, break SU(2N) to SU(N)

Unconventional Quantum Critical points

This formalism seems crazy, but it works perfectly well in 1+1d. Slave fermion formalism gives us SU(N)$_1$ WZW CFT.

$$\mathcal{L} = \sum_{a=1}^{2N} \bar{\psi}_a \gamma_\mu (\partial_\mu - i a_\mu) \psi_a + \frac{1}{e^2} f_{\mu\nu}^2 + O(\psi^4)$$

Solve this model in 1+1d, or 1/N expansion, obtain the scaling dimension of magnetic order parameters:

$$\Delta[\bar{\psi} T^a \psi] = 1 - \frac{1}{N}$$

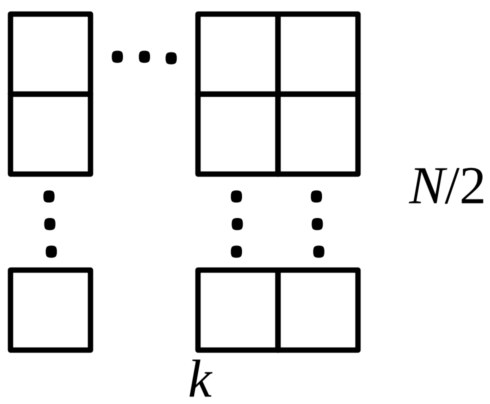
Same answer as SU(N)$_1$ WZW model.

N = 2, back to the spin-1/2 Heisenberg chain.

Kim, Lee, 1998

Unconventional Quantum Critical points

Example 2: honeycomb lattice *n*-orbital SU(*N*) magnets, or square lattice *π*-flux state:



$$\sum_{\beta=1}^{k}\sum_{a=1}^{N} f^{\dagger}_{i,a,\alpha} f_{i,a,\beta} = \frac{kN}{2}$$

+ SU(k) singlet on every site

$$\mathcal{L} = \sum_{a=1}^{2N}\sum_{\alpha\beta} \bar{\psi}_{a,\alpha}\gamma_{\mu}(\partial_{\mu}\delta_{\alpha\beta} - i a_{\mu}\delta_{\alpha\beta} - i\sum_{l=1}^{k^2-1} A^{l}_{\mu}T^{l}_{\alpha\beta})\psi_{a,\beta} + \cdots$$

Unconventional Quantum Critical points

Example 2: honeycomb lattice *n*-orbital SU(*N*) magnets, or square lattice *π*-flux state:

$$\mathcal{L} = \sum_{a=1}^{2N} \sum_{\alpha\beta} \bar{\psi}_{a,\alpha} \gamma_\mu (\partial_\mu \delta_{\alpha\beta} - i a_\mu \delta_{\alpha\beta} - i \sum_{l=1}^{k^2-1} A_\mu^l T_{\alpha\beta}^l) \psi_{a,\beta} + \cdots$$

Still, 2+1d, $N > N_c$ stable CFT, not sure what happens when *N* is small;

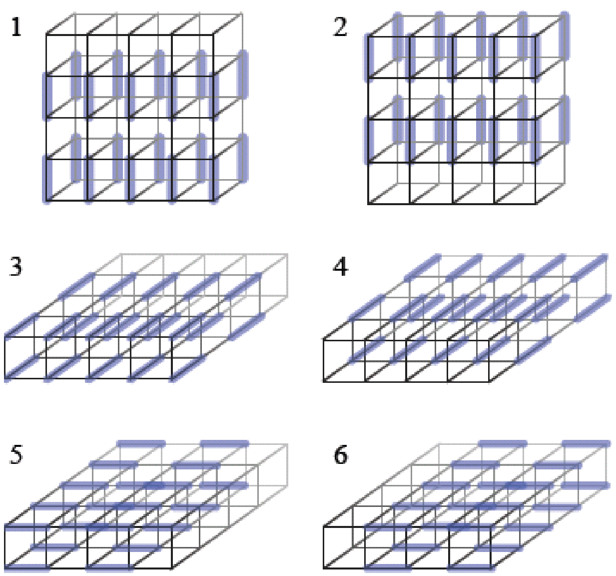
1+1d, equivalent to $SU(N)_k$ WZW CFT.

With alkaline earths atoms, might be realizable.

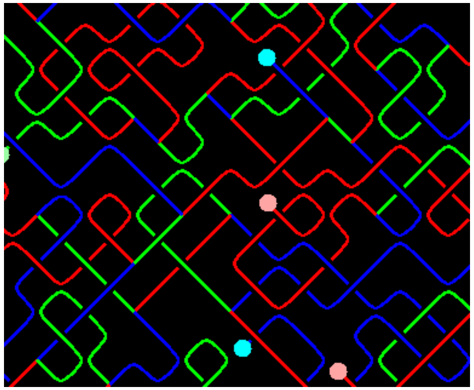
Unconventional Quantum Critical points

3d quantum dimer model:

$$H = -t\left(|=\rangle\langle||\,| + h.c.\right) + v\left(|=\rangle\langle=| + |\,||\,\rangle\langle||\,|\right)$$



Photon phase



Unconventional Quantum Critical points

3+1d CP(1) model, 3d quantum dimer model:

$$H = -t\left(|\text{=}\rangle\langle \text{II}| + h.c.\right) + v\left(|\text{=}\rangle\langle \text{=}| + |\text{II}\rangle\langle \text{II}|\right)$$

VBS,
condensate of monopole

Photon phase
U(1) gauge field

v/t

Confinement transition,
Condensation of monopoles.

Monopole has two minima in its BZ,

$$\mathcal{L} = \sum_a |(\partial_\mu - i a_\mu)\varphi_a|^2 + r|\varphi_a|^2 + g(\sum_a |\varphi_a|^2)^2 + \cdots$$

Unconventional Quantum Critical points

Electric-magnetic duality:

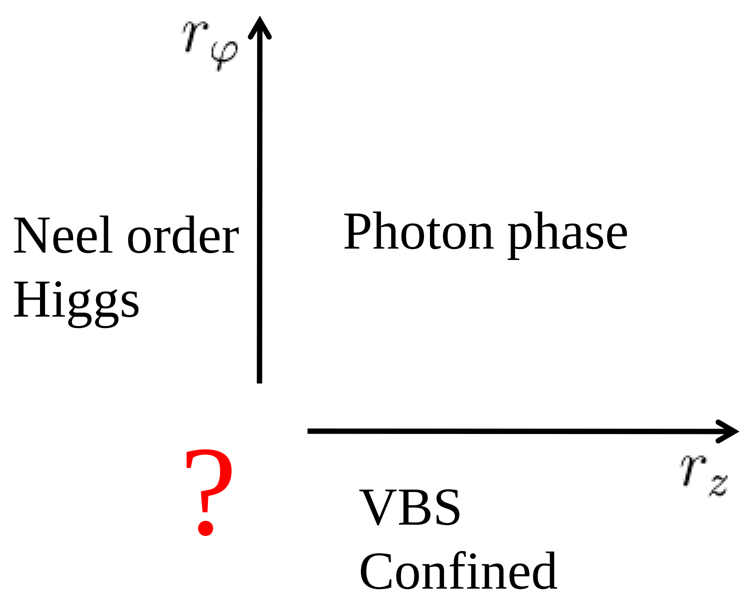
$$\mathcal{L} = \sum_a |(\partial_\mu - iA_\mu) z_a|^2 + r_z |z_a|^2 + g(\sum_a |z_a|^2)^2 + \cdots \qquad z^\dagger \vec{\sigma} z, \quad \text{Neel}$$

$$\mathcal{L} = \sum_a |(\partial_\mu - i a_\mu) \varphi_a|^2 + r_\varphi |\varphi_a|^2 + g(\sum_a |\varphi_a|^2)^2 + \cdots \qquad \varphi^\dagger \vec{\sigma} \varphi, \quad \text{VBS}$$

Spinon condense, Neel order,
Monopole condense, VBS order,
Both gapped, photon phase

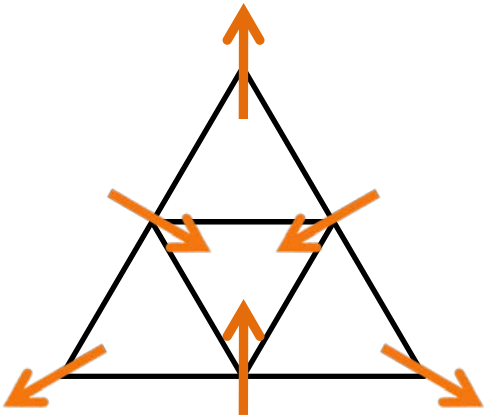
Both condense?
Direct transition?

Motrunich, Senthil 2004



Unconventional Quantum Critical points

Phase transition starting with spiral order



Standard spin order on the triangular lattice: 120 degree state.

The GSM of 120 degree state is SO(3) $\simeq S^3/Z_2$ with enlarged O(4) symmetry,

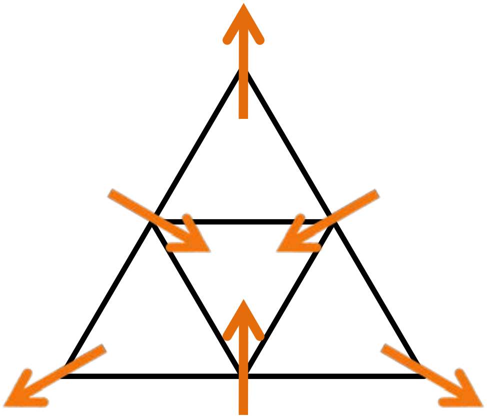
$$\vec{S} = \vec{n}_1 \cos(2\vec{Q}\cdot\vec{r}) + \vec{n}_2 \sin(2\vec{Q}\cdot\vec{r})$$

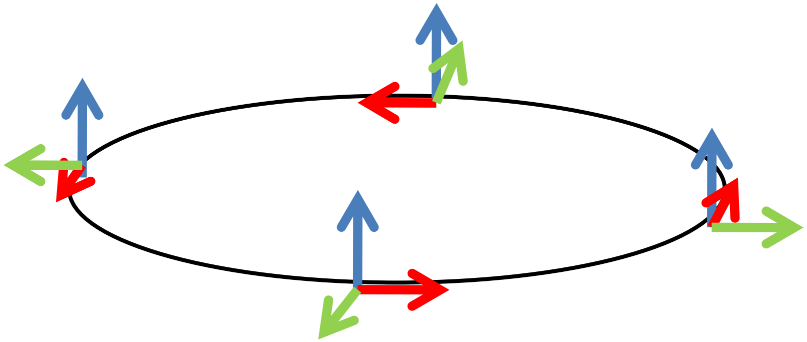
$$\vec{n}_3 = \vec{n}_1 \times \vec{n}_2$$

Unconventional Quantum Critical points

Phase transition starting with spiral order

The GSM of 120 degree state is SO(3) $\cong S^3/Z_2$ with enlarged O(4) symmetry, Supports stable half-vortex *i.e.* vison,





$$\vec{n}_1 = \mathrm{Re}[z^t i\sigma^y \vec{\sigma} z],$$

$$\vec{n}_2 = \mathrm{Im}[z^t i\sigma^y \vec{\sigma} z],$$

$$\vec{n}_3 = z^\dagger \vec{\sigma} z.$$

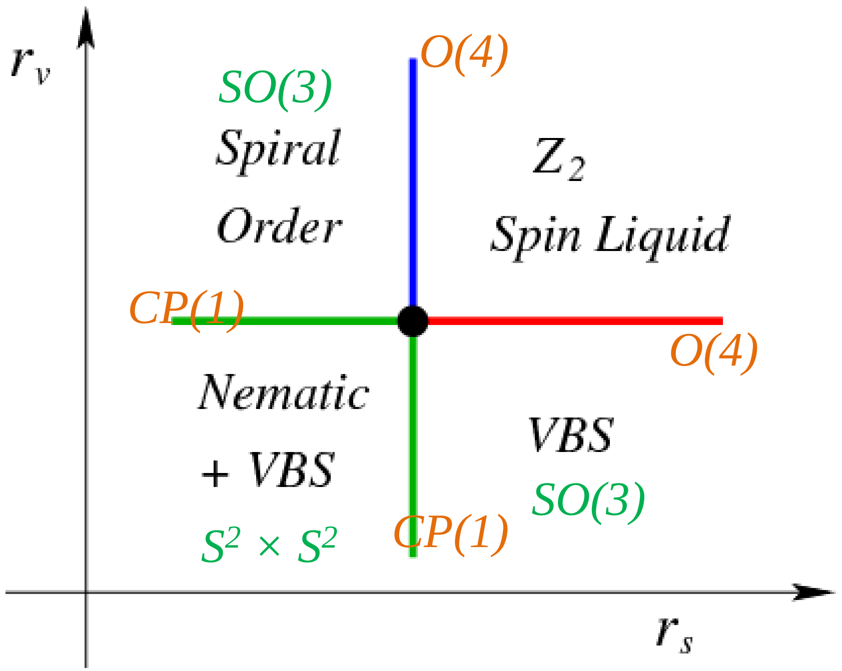
Vison also has multiple minima in the BZ, in the end also becomes two complex bosons.

Vison and spinon have mutual statistics

Unconventional Quantum Critical points

2d triangular lattice, spinon and vison also dual to each other:

$$L = \sum_{\alpha=1}^{2} |(\partial_\mu - i a_\mu) z_\alpha|^2 + |(\partial_\mu - i b_\mu) v_\alpha|^2 + \frac{ik}{4\pi}\epsilon_{\mu\nu\rho} a_\mu \partial_\nu b_\rho + r_s |z_\alpha|^2 + r_v |v_\alpha|^2 + \cdots$$



$\mathrm{Re}[z^t i\sigma^y \vec{\sigma} z], \quad \mathrm{Im}[z^t i\sigma^y \vec{\sigma} z], \quad z^\dagger \vec{\sigma} z.$

Spiral order parameters,

$\mathrm{Re}[v^t i\sigma^y \vec{\sigma} v], \quad \mathrm{Im}[v^t i\sigma^y \vec{\sigma} v], \quad v^\dagger \vec{\sigma} v.$

VBS parameters,

Xu, Sachdev, 2008

Unconventional Quantum Critical points

Hand over to Leon next time……