Interaction induced novel condensates in a double-well lattice

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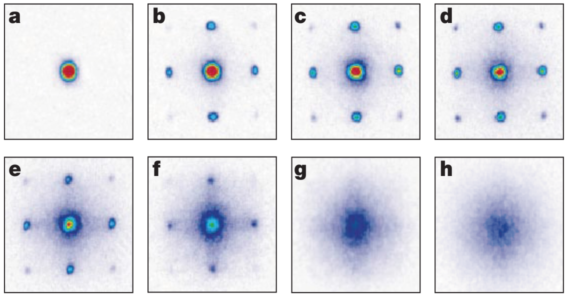
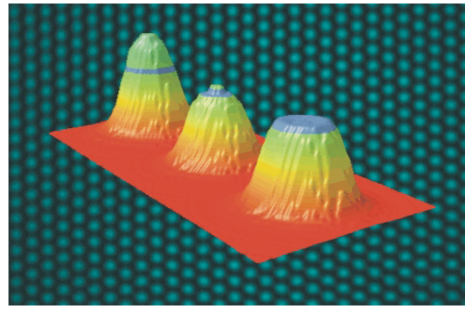
KITP, Beyond Standard Optical Lattices, 2010

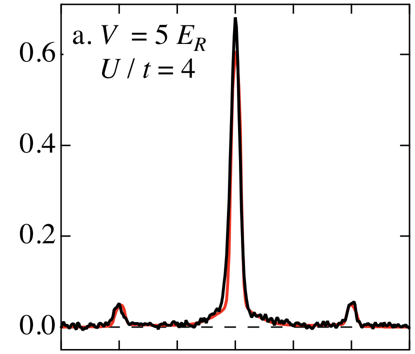
Last decade has seen many exciting development in optical lattices



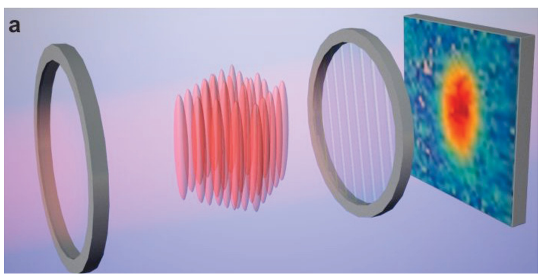
I. Bloch(2002)



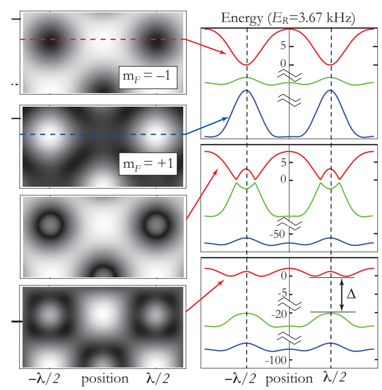
C. Cheng(2009)



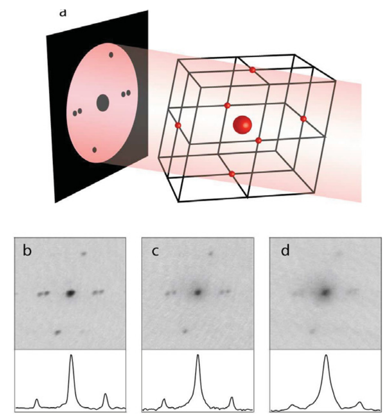
I. B. Spielman (2008)



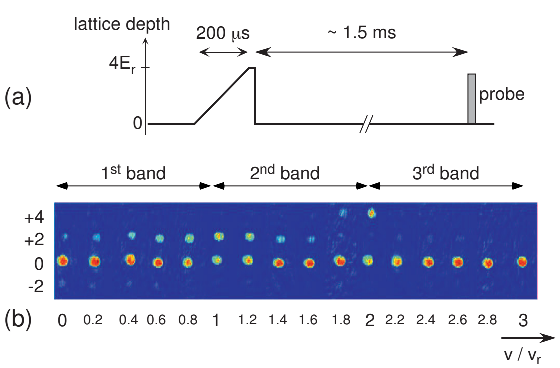
R. G. Hulet (2009)



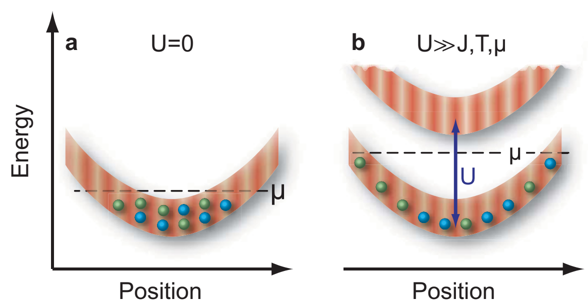
J. V. Porto (2008)



W. Ketterle (2006)



W.D. Phillips (2008)



T. Esslinger (2008)

and many more

。。。

A few obstacles:

- Cold atoms are not cold enough in optical lattices

$$t \sim nK \qquad J_{ex} = t^2/U \sim 0.01nK$$

- Many physical quantities are not accessible so far

Superfluid density, entropy density, temperature in lattices…

- No systematical way to overcome the finite size effect in an inhomogeneous trap

$$N \sim 10^5 - 10^7 \qquad D \sim 100d \qquad V(\mathbf{r}) = \frac{1}{2}M\omega^2\mathbf{r}^2$$

Theoretical proposals:

T.L Ho, QZ, PRL, 99, 120404 (2007)

T.L.Ho, QZ, PNAS, 106, 6916 (2009)

QZ, et al, PRL, 103, 085701 (2009)

T.L. Ho, QZ, Nat Phys 6, 131 (2010)

QZ, T.L. Ho, arXiv:1006.1174 (2010)

Experimental efforts:

Ketterle' group, arXiv:1006.4674(2010)

Ingusico' group, PRL 103, 140401 (2009)

Chin's group, Nature 460, 995 (2009)

Salomon's group, Nature 463, 1057 (2010)

Chin's group, arXiv:1009.0016 (2010)

New Physics in Optical lattices?

Optical lattices

- Atoms with spin
- Higher bands
- Mixtures: BB, BF, FF
- Alkali earth atoms
- …

E. Demler 2001, F. Zhou 2002, 2003,
M. Lewenstein 2005, 2006, A. Vishwanath, 2007,
C. Wu 2006, 2007, 2008, 2009, 2010,
C. Xu 2006, 2008, S. Powell 2005, 2009,
S. Girvin 2006, B. Svistunov 2002, 2003,
R. Lutchyn 2008, J. Cirac 2006,
S. Das Sarma 2006, 2007, 2008, 2009, 2010,
M. Troyer 2006, L. Santos 2003,
A. Gorshkov 2009, A. M. Rey 2010
and many more

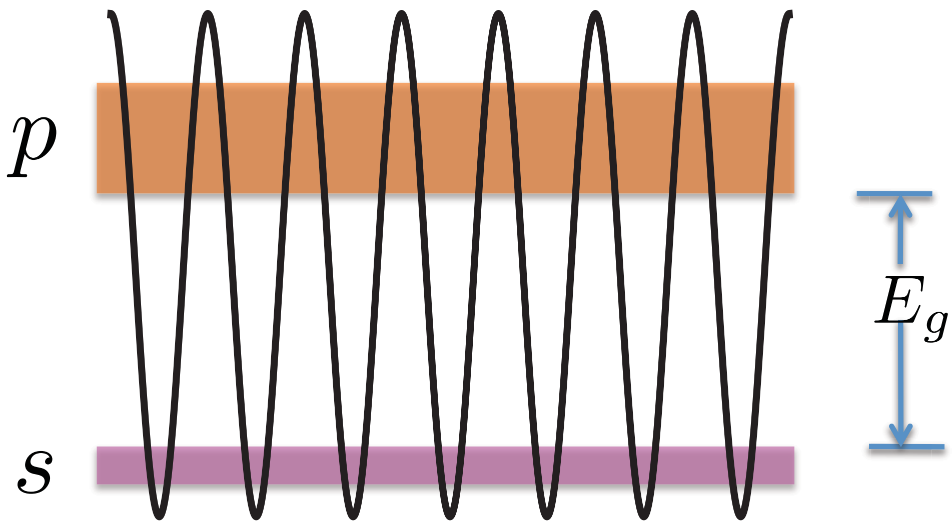
A class of problem that has been less discussed

Multiple band effects and inter-band coupling

R. Diener 2005, H. Zhai 2007, H. Zhai 2008, J. Larson 2009, W. Vincent Liu 2010

Single band models are usually good for standard optical lattices

Satisfied in standard lattices



$$V = V_0 \sum_{i=1,2,3} \sin^2(2\pi x_i/\lambda)$$

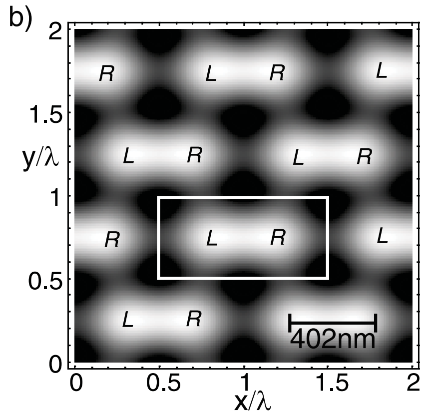
V_o/E_R	5	10	15	20
$E_g\ (nK)$	294	678	956	1171
$U\ (nK)$	24.2	44.6	63.7	82.0
$t\ (nK)$	10.4	3.01	1.03	0.39

$$E_g \gg t, U$$

$$\boxed{H = -t \sum_{\langle i,j \rangle} (b_i^\dagger b_j + c.c) + \frac{U}{2} \sum_i n_i(n_i - 1) - \mu \sum_i n_i}$$

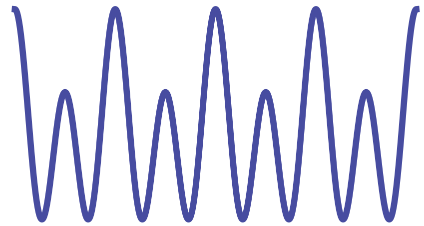
$$\boxed{H = -t \sum_{\langle i,j \rangle \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + c.c) + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i}$$

A new system: double-well optical lattices

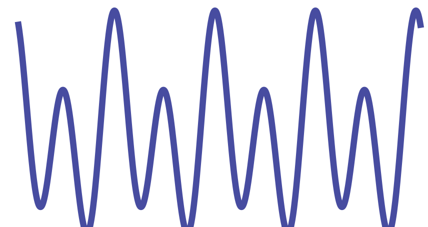


Trey Porto, JQI (2006, 2008)
See previous talk

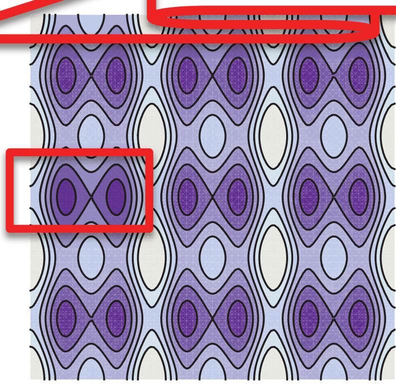
Symmetrical case $V(x) = V(-x)$



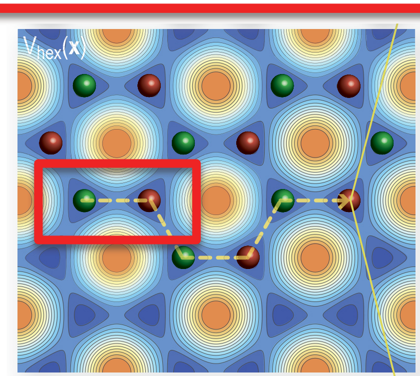
Asymmetrical case $V(x) \neq V(-x)$



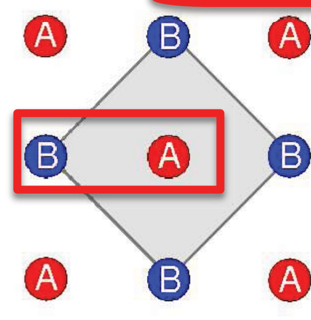
Beyond standard optical lattices



I. Bloch (2007)



K. Sengstock (2010)

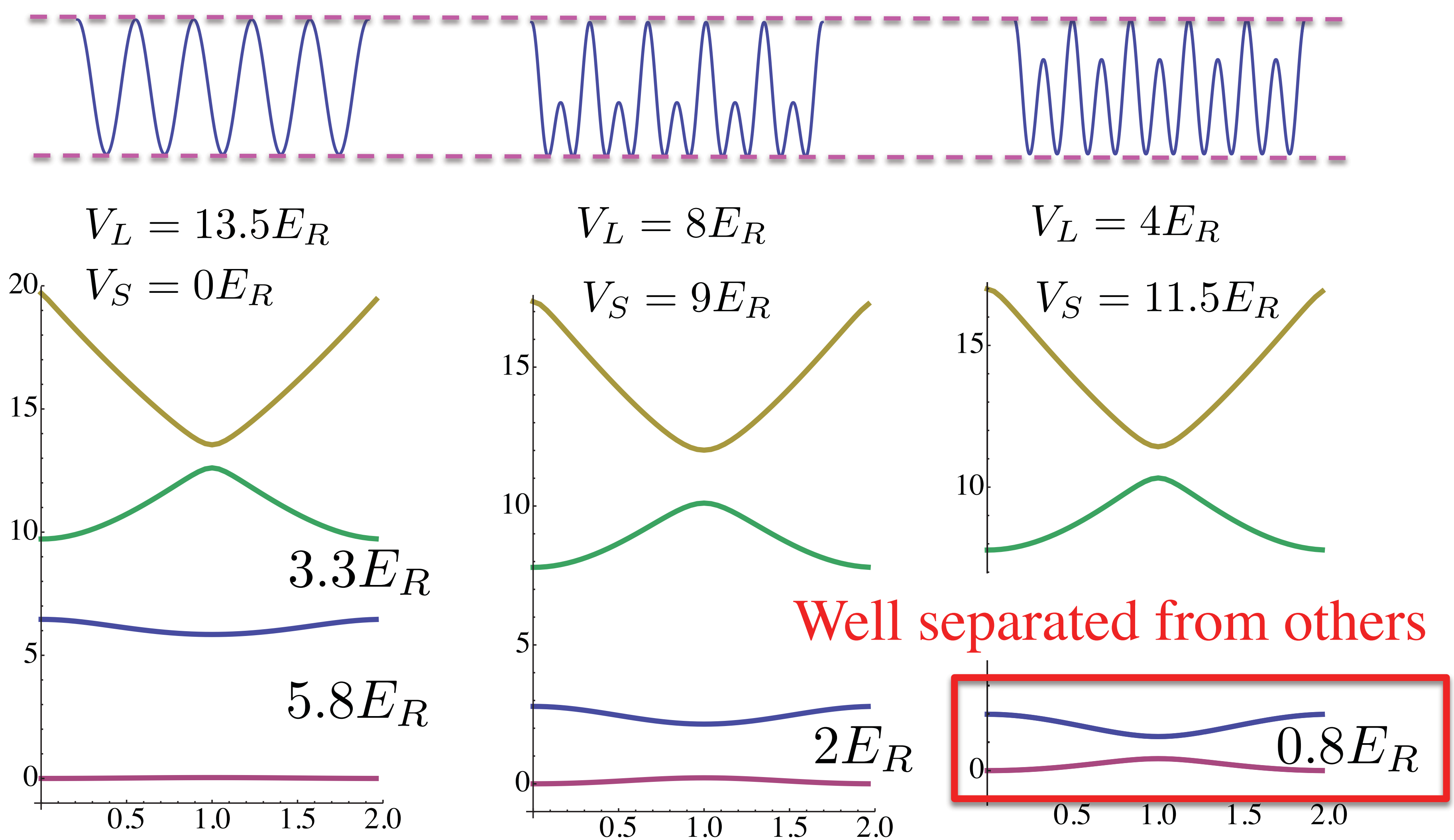


A. Hemmerich (2010)

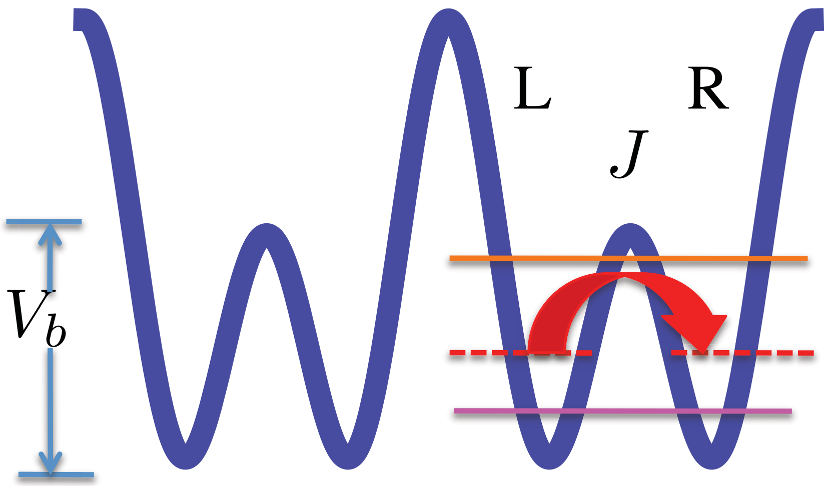
A unique advantage of double-well lattices

Tunable band gap between the s and p band at a fixed lattice depth

$$V(x) = V_L \sin^2\left(\frac{\pi}{d}x\right) - V_S \sin^2\left(\frac{2\pi}{d}x\right)$$



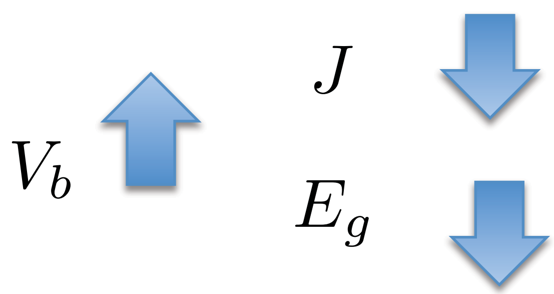
A simple way to understand the reduced band gap



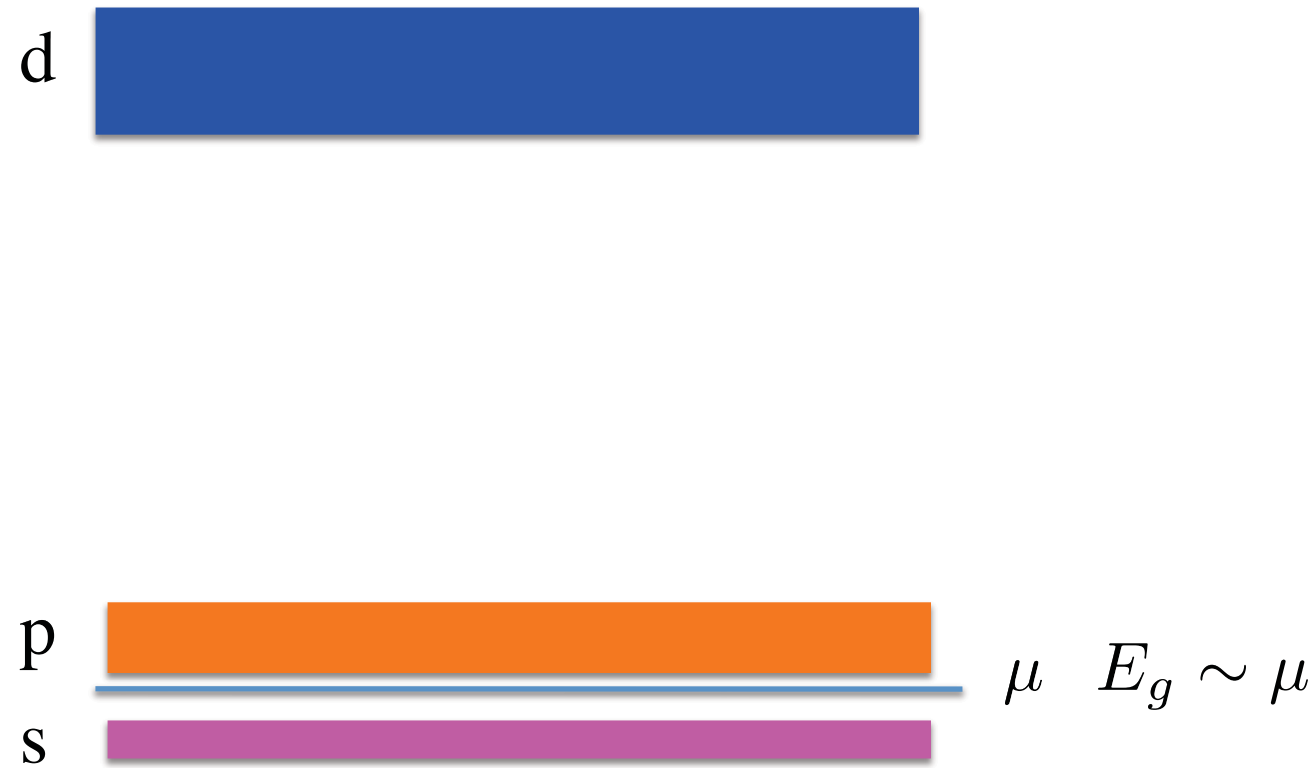
$p \quad \frac{1}{\sqrt{2}}(|L\rangle + |R\rangle)$

$s \quad \frac{1}{\sqrt{2}}(|L\rangle - |R\rangle)$

$$E_g \sim J$$



See also: J. Larson, A. Collin, J.-P. Martikainen, Phys. Rev. A 79, 033603 (2009)



Reduced band gap + Interaction effects

Novel many body phenomena in two coupled bands

Single component of bosons in a double-well lattice

Novel topology of the phase diagram

Novel inter-band condensates

A simple way to generate higher band condensate

Can be generalized to optical superlattice

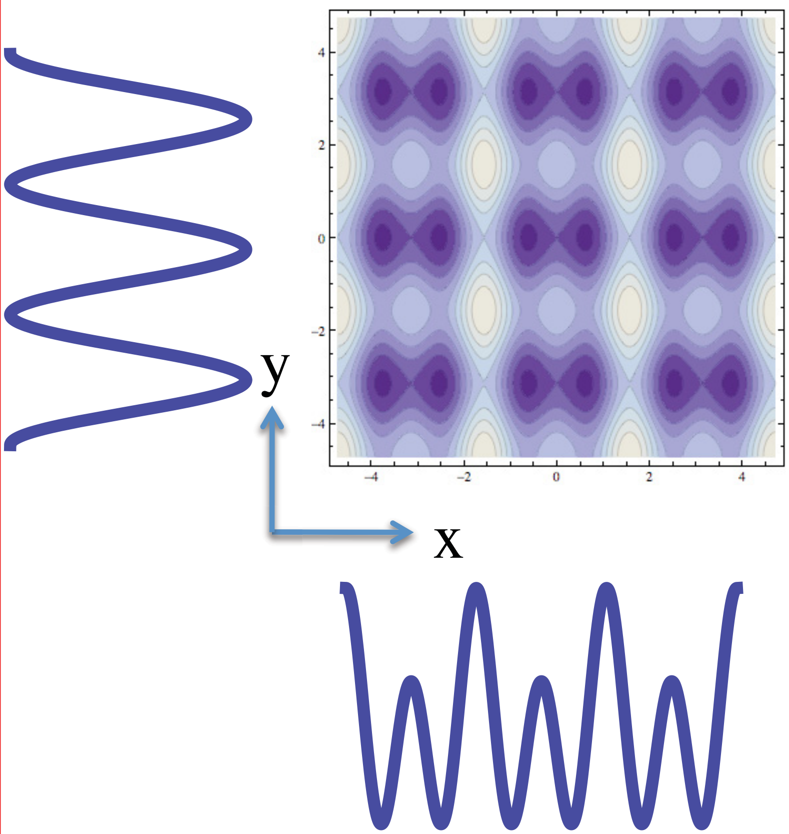
Qi Zhou, J.V. Porto and S. Das Sarma, arXiv:1010.1534 (2010)

Qi Zhou, J.V. Porto and S. Das Sarma, to appear soon

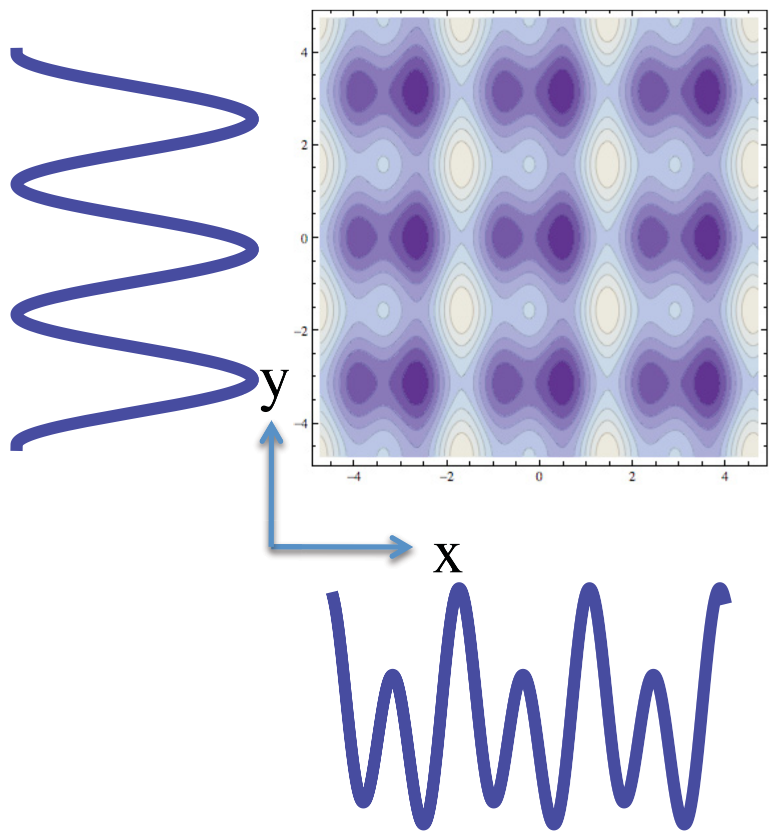
Lattice configuration considered here

$$V = V_L \sin^2(\frac{\pi}{d}x) - V_S \sin^2(\frac{2\pi}{d}x + \frac{\eta}{2}) + V_y \sin^2(\frac{\pi}{d}y) + V_z \sin^2(\frac{\pi}{d}z)$$

$\eta = 0, 2\pi$ Symmetrical case



$\eta \neq 0, 2\pi$ Asymmetrical case



The lowest two bands (s and px) are very close to each other

Many-body Hamiltonian for symmetrical case

Hubbard model for each single band $\sigma = g, e$ at site $\mathbf{m}$

$$\hat{H} = \sum_{\sigma \mathbf{m} \vec{r}} t_{\sigma,\vec{r}} (\hat{b}^{\dagger}_{\sigma\mathbf{m}} \hat{b}_{\sigma\mathbf{m}+\vec{r}} + \text{c.c}) - \sum_{\sigma\mathbf{m}} \mu_\sigma \hat{n}_{\sigma\mathbf{m}} + \frac{U_\sigma}{2} \sum_{\sigma\mathbf{m}} \hat{n}_{\sigma\mathbf{m}} (\hat{n}_{\sigma\mathbf{m}} - 1)$$

$$+U_{ge} \sum_{\mathbf{m}} \hat{n}_{g\mathbf{m}} \hat{n}_{e\mathbf{m}} + \boxed{\sum_{\mathbf{m}} \left(W \hat{b}^{\dagger}_{e\mathbf{m}} \hat{b}^{\dagger}_{e\mathbf{m}} \hat{b}_{g\mathbf{m}} \hat{b}_{g\mathbf{m}} + \text{c.c} \right)}$$

$$\mu_g = \mu$$

$$\mu_e = \mu - \Delta_g$$

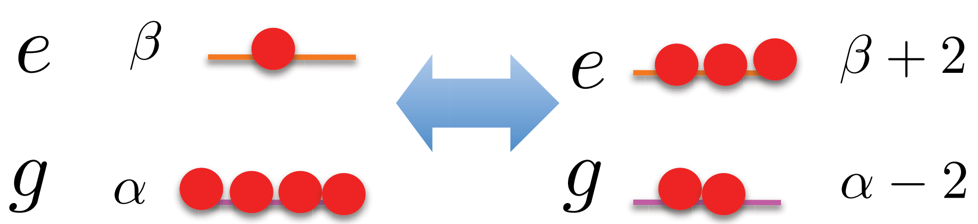
Inter-band density-density interaction

Inter-band paring coupling

$$\psi_g(x) = \psi_g(-x)$$

$$\psi_e(x) = -\psi_e(-x)$$

$$W(\mathbf{r}) = \frac{2\pi\hbar^2 a_s}{M} \int d\mathbf{r} \psi_g^2(\mathbf{r}) \psi_e^2(\mathbf{r})$$



$$\int d\mathbf{r} \psi_g(\mathbf{r}) \psi_g(\mathbf{r}) \psi_g(\mathbf{r}) \psi_e(\mathbf{r}) = 0$$

Gutzwiller Mean-field solution

$$|G\rangle = \prod_{\mathbf{m}} \sum_{\alpha,\beta} c_{\alpha\beta} |\alpha, \beta\rangle_{\mathbf{m}}$$

Particle number in band g e

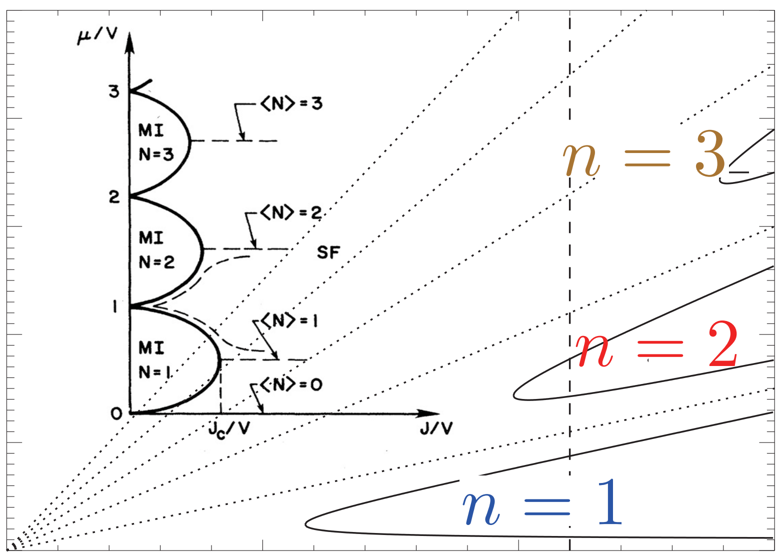
Self-consistently

$$\Psi_g = \langle G|\hat{b}_g|G\rangle \quad = 0$$

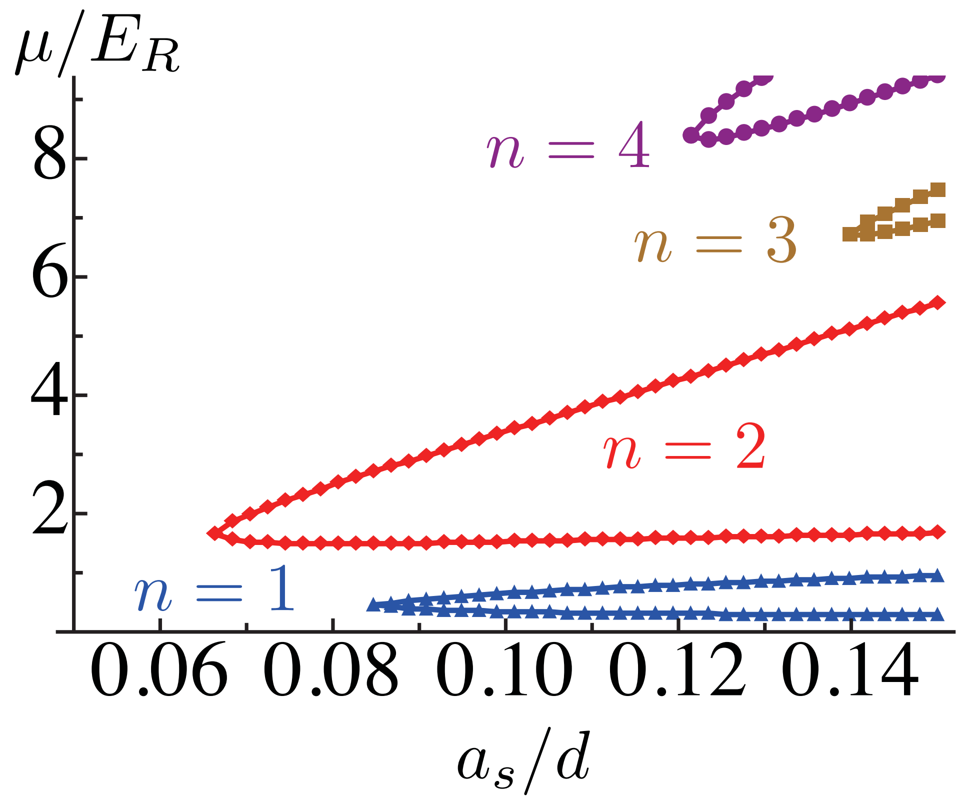
$$\Psi_e = \langle G|\hat{b}_e|G\rangle \quad = 0$$

Phase boundary

Usual result for the single-band model



Even-odd effect

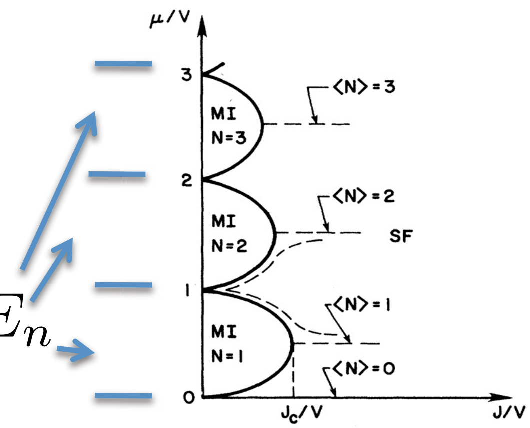


Understand the odd-even effect

Why Mott lobes monotonically decreases
in a single band model?

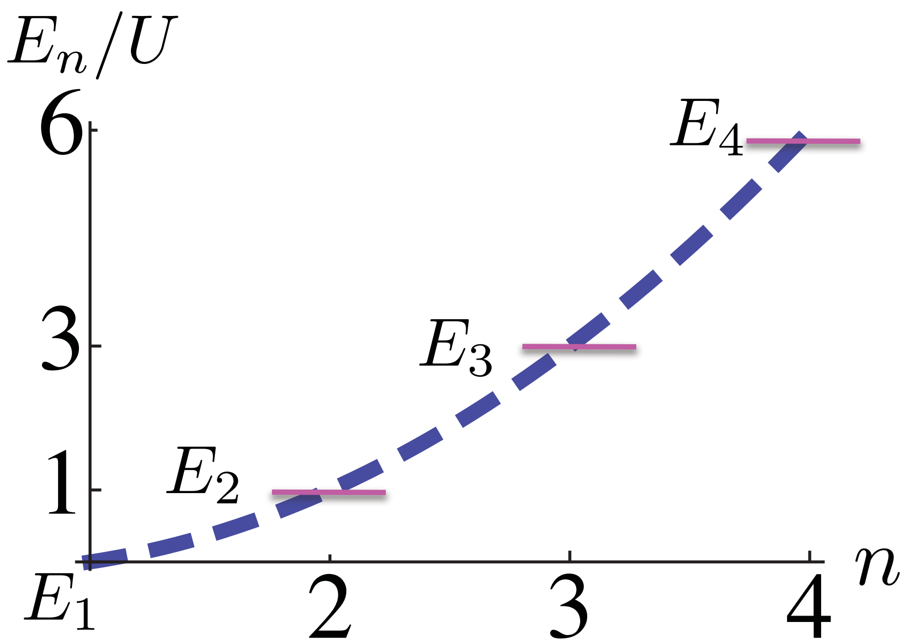
$$E_n = Un(n-1)/2$$

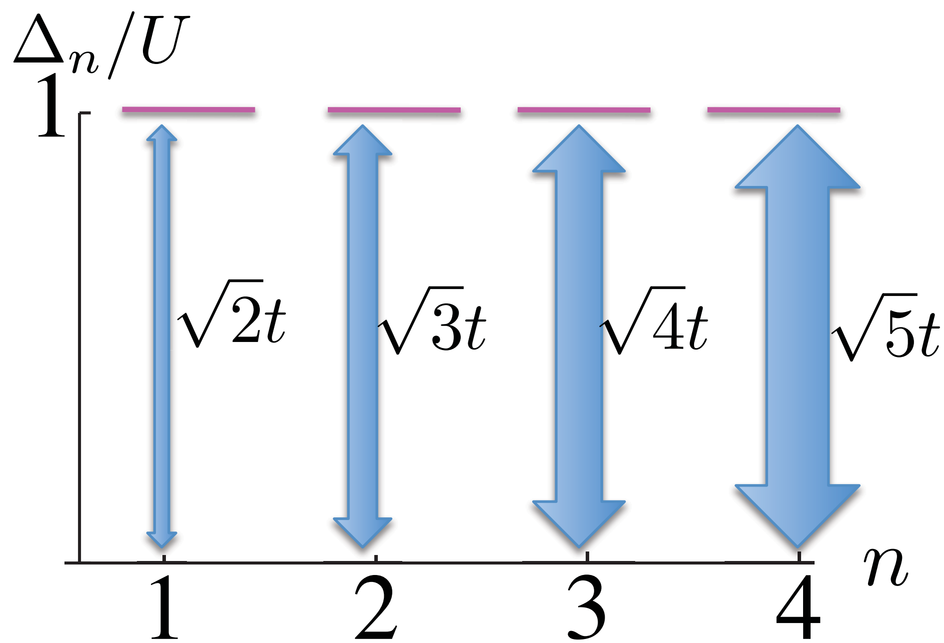
Excitation gap $\quad \Delta_n = E_{n+1} + E_{n-1} - 2E_n$



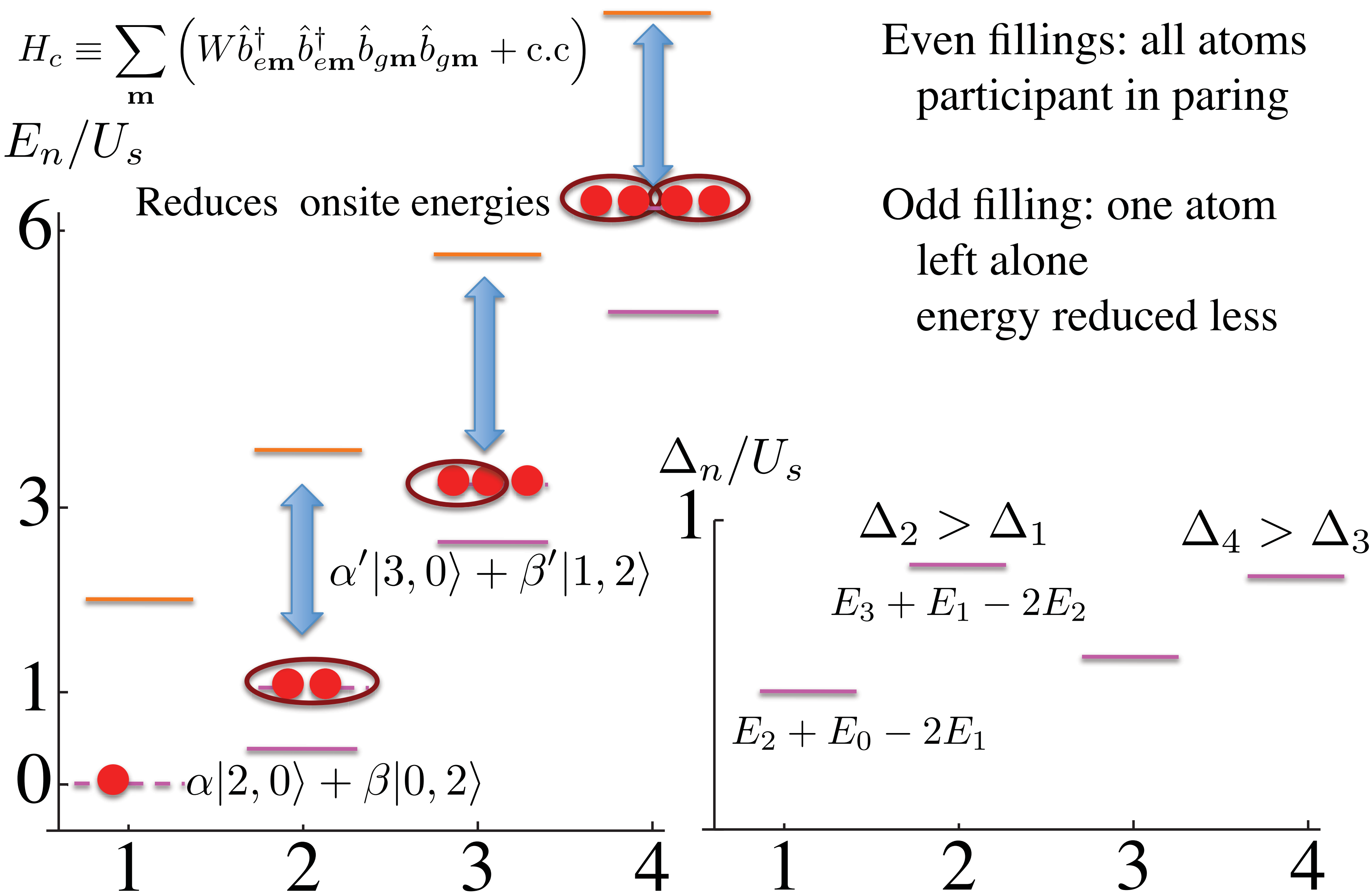
$\Delta_n = U$ independent on n!

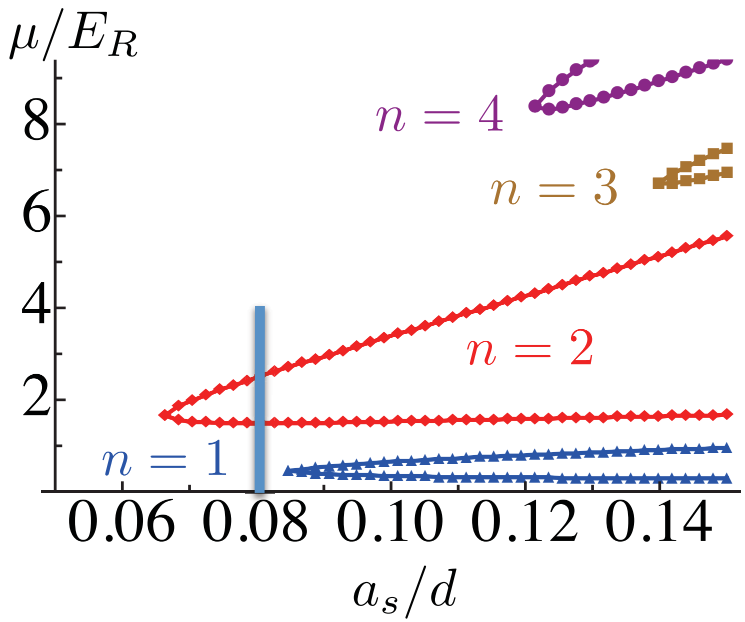
Tunneling $\quad -t\hat{b}_i^{\dagger} b_j \sim -t\sqrt{(n+1)n}$ Bosonic enhancement





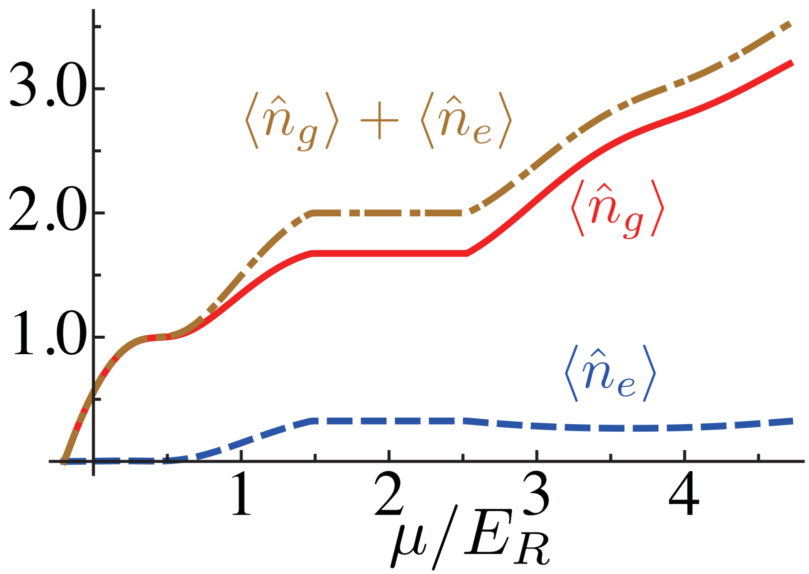
In a double-well lattice: excited band approaches from above





Even fillings are more stable than odd fillings

Directly observable in experiments measuring the density profile



Mott plateau with 2 particle per site emerge first when increasing interaction

Similar phenomena in the metal-insulator transition

More than one orbital + Hund's coupling

Condensate phases

$(\mathcal{C}1)$ $\quad \langle \hat{b}_g \rangle \neq 0 \quad \langle \hat{b}_e \rangle \neq 0$

A superposition of single-particle condensates of both bands

$(\mathcal{C}2)$

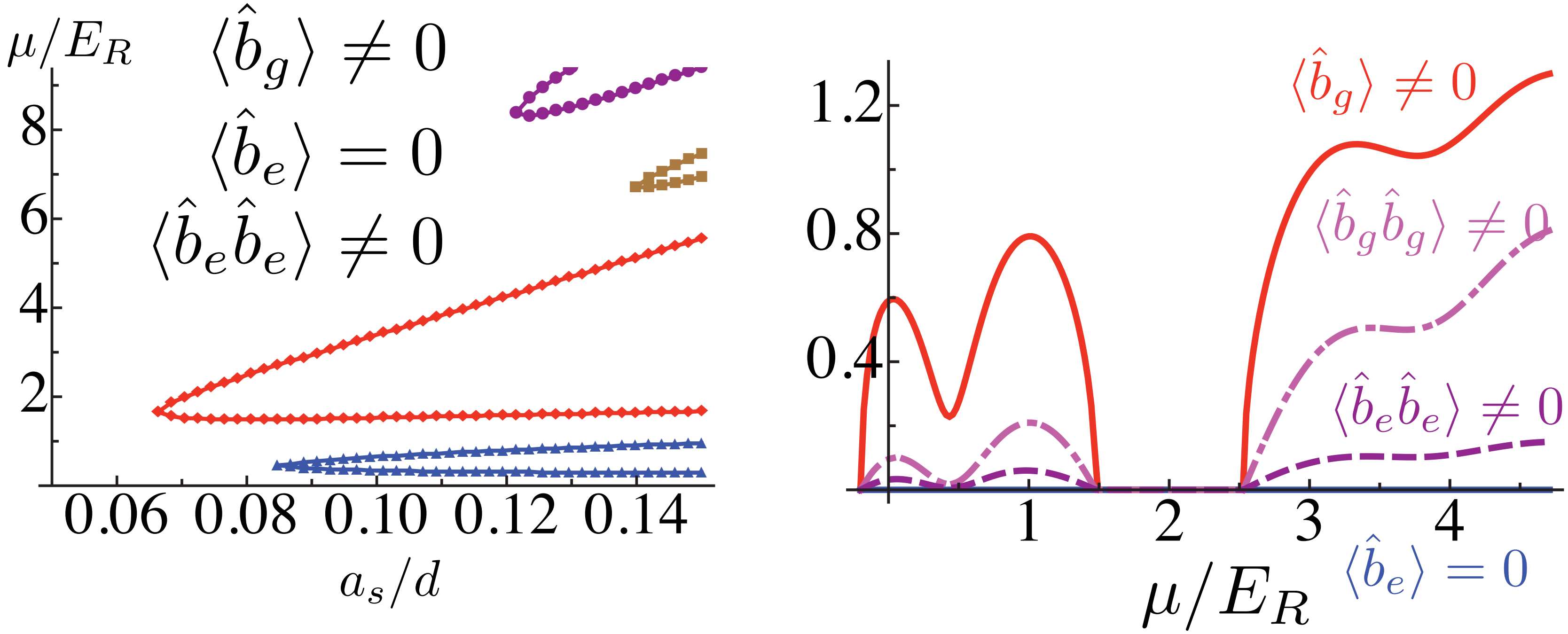
$$\langle \hat{b}_g \rangle \neq 0 \quad \langle \hat{b}_e \rangle = 0 \quad \langle \hat{b}_e \hat{b}_e \rangle \neq 0$$

$$\langle \hat{b}_e \rangle \neq 0 \quad \langle \hat{b}_g \rangle = 0 \quad \langle \hat{b}_g \hat{b}_g \rangle \neq 0$$

A superposition of a single-particle condensate of one band and a pair-condensate in another band

$$H_c \equiv \sum_{\mathbf{m}} \left(W \hat{b}^\dagger_{e\mathbf{m}} \hat{b}^\dagger_{e\mathbf{m}} \hat{b}_{g\mathbf{m}} \hat{b}_{g\mathbf{m}} + \text{c.c} \right) \qquad \langle \hat{b}^\dagger_{g\mathbf{m}} \rangle \langle \hat{b}^\dagger_{g\mathbf{m}} \rangle \langle \hat{b}_{e\mathbf{m}} \hat{b}_{e\mathbf{m}} \rangle$$

A linear driving term for $\langle \hat{b}_{e\mathbf{m}} \hat{b}_{e\mathbf{m}} \rangle$



μ/E_R
$\langle\hat{b}_g\rangle \neq 0$
$\langle\hat{b}_e\rangle = 0$
$\langle\hat{b}_e\hat{b}_e\rangle \neq 0$
8
6
4
2
0.06
0.08
0.10
0.12
0.14
a_s/d
1.2
0.8
0.4
1
2
3
4
μ/E_R
$\langle\hat{b}_g\rangle \neq 0$
$\langle\hat{b}_g\hat{b}_g\rangle \neq 0$
$\langle\hat{b}_e\hat{b}_e\rangle \neq 0$
$\langle\hat{b}_e\rangle = 0$

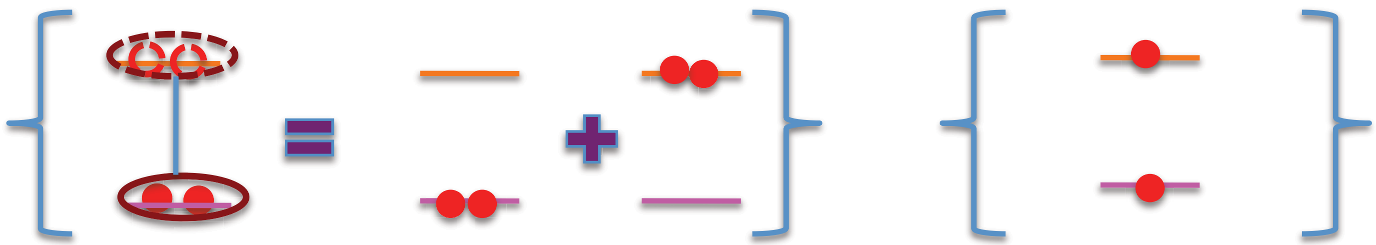
An inter-band condensate

$$\boxed{\langle \hat{b}_e \rangle = 0 \qquad \langle \hat{b}_g \rangle = 0 \qquad \langle \hat{b}_g^\dagger \hat{b}_e \rangle = \langle \hat{b}_e^\dagger \hat{b}_g \rangle \neq 0}$$

On-site Hilbert space for n particles per site divides into two subspace

$$\{|n-2l, 2l\rangle_{\mathbf{m}}\} \qquad \{|n-2l-1, 2l+1\rangle_{\mathbf{m}}\}$$

$$H_c \equiv \sum_{\mathbf{m}} \left(W \hat{b}^\dagger_{e\mathbf{m}} \hat{b}^\dagger_{e\mathbf{m}} \hat{b}_{g\mathbf{m}} \hat{b}_{g\mathbf{m}} + \mathrm{c.c} \right)$$



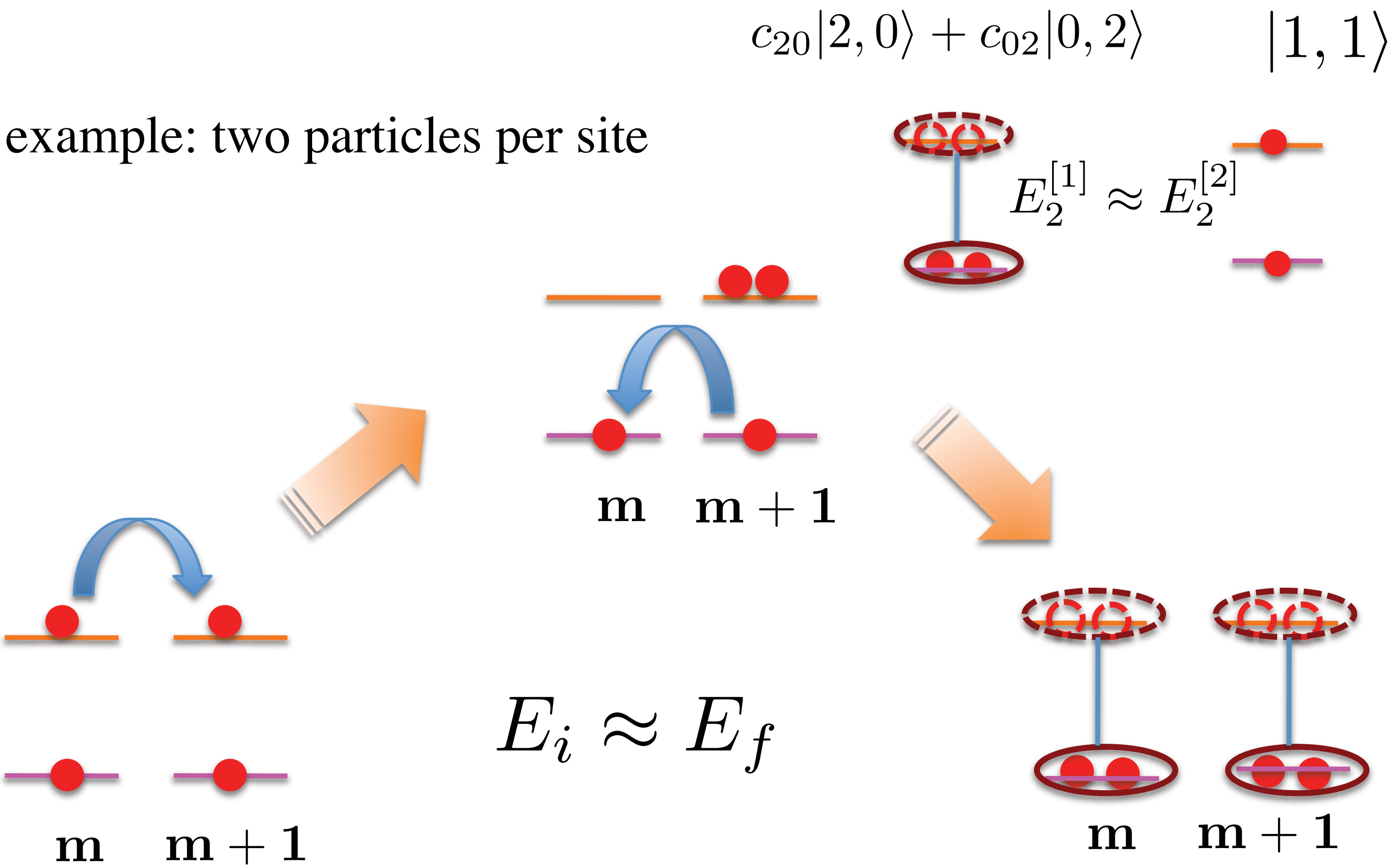
$c_{20}|2,0\rangle + c_{02}|0,2\rangle$ $\qquad$ $|1,1\rangle$

Ground state energy of each subspace $\quad E_n^{[1]} \approx E_n^{[2]}$

Exotic inter-band condensate

See also A. Kuklov, N. Prokof'ev, B. Svistunov, Phys. Rev. Lett. 92, 050402 (2004): a different system with two component bosons

An example: two particles per site



$$u\left(c_{20}|2,0\rangle + c_{02}|0,2\rangle\right) + v|1,1\rangle$$

$$\boxed{\begin{array}{l} \langle \hat{b}_e \rangle = 0 \qquad \langle \hat{b}_g \rangle = 0 \\ \langle \hat{b}_g^\dagger \hat{b}_e \rangle = \langle \hat{b}_e^\dagger \hat{b}_g \rangle \neq 0 \end{array}}$$

Effective spin model

Strong interaction quenches the "charge" degree of freedom, but not the "pseudospin"

$|\uparrow\rangle$ $|\downarrow\rangle$

$$H_{spin} = \overbrace{\sum_{\mathbf{m},\vec{r}} J_{\vec{r}}(S^{\dagger}_{\mathbf{m}} S_{\mathbf{m}+\vec{r}} + c.c) + \sum_{\mathbf{m},\vec{r}} J^{z}_{\vec{r}} S^{z}_{\mathbf{m}} S^{z}_{\mathbf{m}+\vec{r}} + h\sum_{\mathbf{m}} S^{z}_{\mathbf{m}}}^{\text{Usual XXZ model}}$$

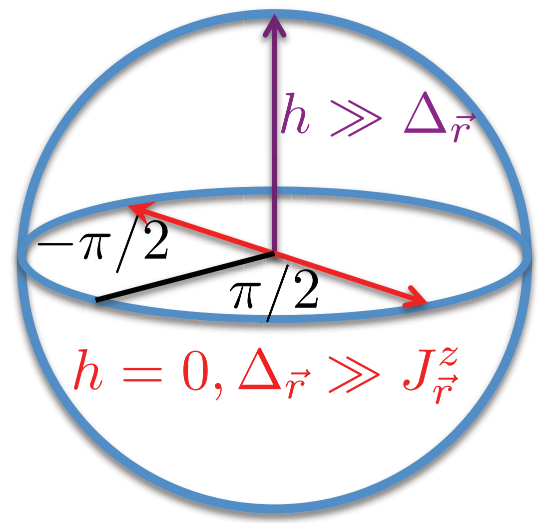
$$+ \sum_{\mathbf{m},\vec{r}} \Delta_{\vec{r}}(S^{\dagger}_{\mathbf{m}} S^{\dagger}_{\mathbf{m}+\vec{r}} + c.c)$$

$$h \approx E^{[2]}_{n} - E^{[1]}_{n}$$

$\sum S^{z}_{\mathbf{m}}$ does not need to be conserved

conserved the particle number in the original bosonic model

Z_2 symmetry



In one dimension $$\hat{c}_m = \left(\prod_{n<m} S_n^z\right) S_m^\dagger \qquad \hat{c}_m^\dagger = \left(\prod_{n<m} S_n^z\right) S_m^-$$

$$H_F = \sum_{\langle mn\rangle} J_{d\hat{x}}(\hat{c}_m^\dagger \hat{c}_n + c.c) + 4J_{d\hat{x}}^z \sum_{\langle mn\rangle} \hat{c}_m^\dagger \hat{c}_m \hat{c}_n^\dagger \hat{c}_n - (2h + 4J_{d\hat{x}}^z) \sum_m \hat{c}_m^\dagger \hat{c}_m$$

$$+ \sum_{\langle mn\rangle} \Delta_{mn}(\hat{c}_m^\dagger \hat{c}_n^\dagger + c.c) \qquad \Delta_{mn} = -\Delta_{mn}$$

1d p-wave superconductor: A. Kitaev, Physics Uspekhi, 44, 131 (2001)

$$H_{is} = \sum_{\langle mn\rangle} (J_x S_m^x S_n^x + J_y S_m^y S_n^y) + h \sum_m S_m^z$$

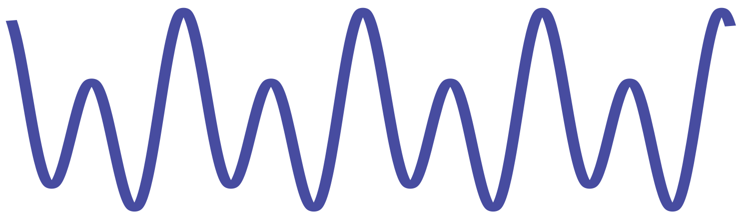
Ising model in a transverse field:
anisotropy in Jx and Jy $\Rightarrow$ Z2

Here Jx=Jy

$$\sum_{\mathbf{m},\vec{r}} \Delta_{\vec{r}}(S_{\mathbf{m}}^\dagger S_{\mathbf{m}+\vec{r}}^\dagger + c.c) \Rightarrow \text{Z2}$$

Asymmetrical case

$$V(x) \neq V(-x)$$



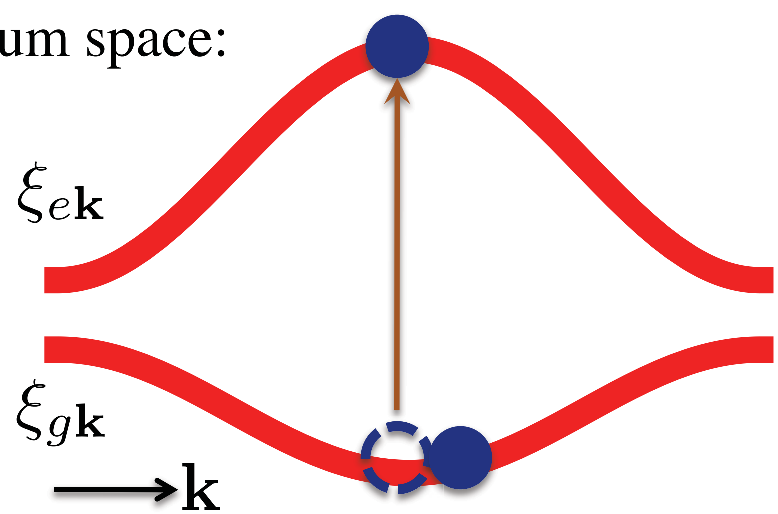
Inversion symmetry is broken

$$\hat{H} \longrightarrow \hat{H} + \hat{H}'$$

$$\hat{H}' = G\hat{b}_g^\dagger \hat{b}_g^\dagger \hat{b}_g \hat{b}_e + G'\hat{b}_e^\dagger \hat{b}_e^\dagger \hat{b}_e \hat{b}_g + c.c$$

Interaction induced inter-band hybridization

Momentum space:



A significant result:

$\langle \hat{b}_e \rangle \neq 0$ $\quad\Rightarrow\quad$ $\langle \hat{b}_g \rangle \neq 0$

$\langle \hat{b}_g \rangle \neq 0$ $\quad\Rightarrow\quad$ $\langle \hat{b}_e \rangle \neq 0$

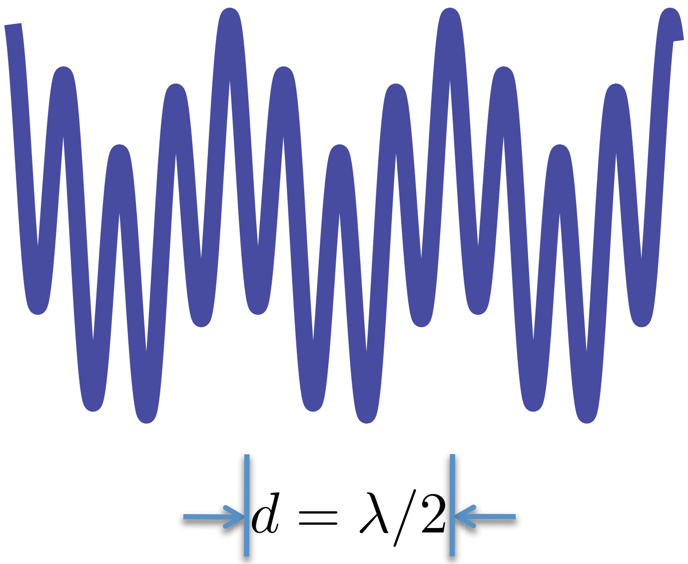
A simple way to generate a condensate in an excited band

Experimental observation has been reported by Sengstock in KITP conference Oct 2010

Generalization to an optical superlattice

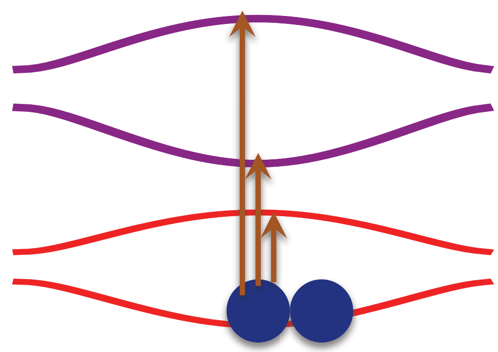
$$V(x) = V_L \sin^2(\frac{\pi}{d}x) - V_S \sin^2(\frac{n\pi}{d}x + \frac{\eta}{2})$$

Separate the lowest a few bands
from higher ones



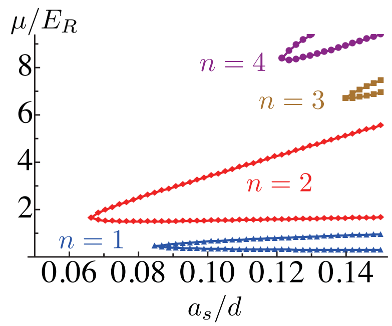
λ

λ/n

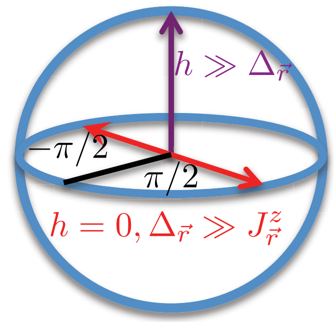


Induced condensate in the lowest a few bands

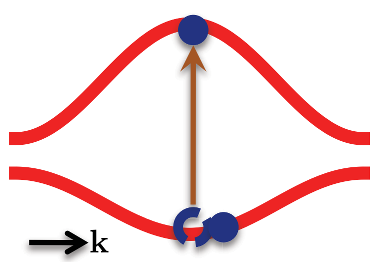
Novel topology of the phase diagram



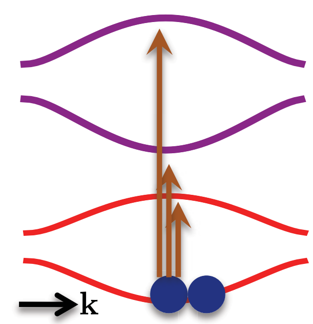
Novel condensates



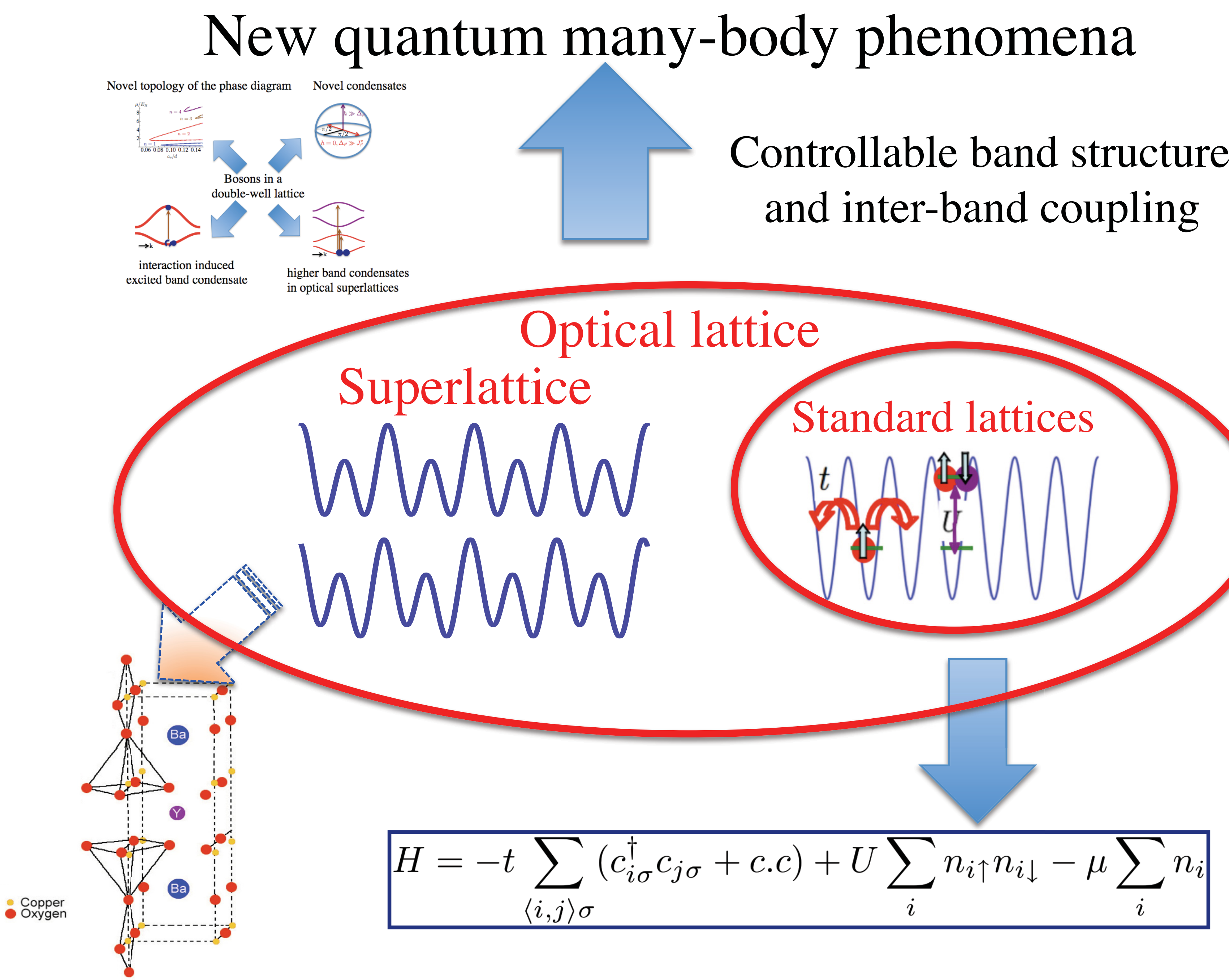
Bosons in a
double-well lattice



interaction induced
excited band condensate



higher band condensates
in optical superlattices


New quantum many-body phenomena
Novel topology of the phase diagram
Novel condensates
Bosons in a double-well lattice
interaction induced excited band condensate
higher band condensates in optical superlattices
Controllable band structure and inter-band coupling
Optical lattice
Superlattice
Standard lattices
t
U
Ba
Y
Ba
Copper
Oxygen
$H = -t \sum_{\langle i,j \rangle \sigma} (c^\dagger_{i\sigma} c_{j\sigma} + c.c) + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i$

Thank you for your attention