

**The search for a perfect fluid: is string theory  
relevant for ultracold atoms ?**

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T. Schäfer and D. Teaney, Rep. Prog. Phys. (2009)

J. McGreevy, arXiv:0909.0518 [hep-th]

T. Enss, R. Haussmann, W. Zw., arXiv:1008.0007





# Wine Spectator



TENUTA  
**GREPPONE MAZZI**  
1999  
BRUNELLO DI MONTALCINO  
DENOMINAZIONE DI ORIGINE CONTROLLATA E GARANTITA

95 points

Ripe and opulent,  
with currant, mineral and  
fresh herbs.  
Layered and stylish

J.S. February, 2010



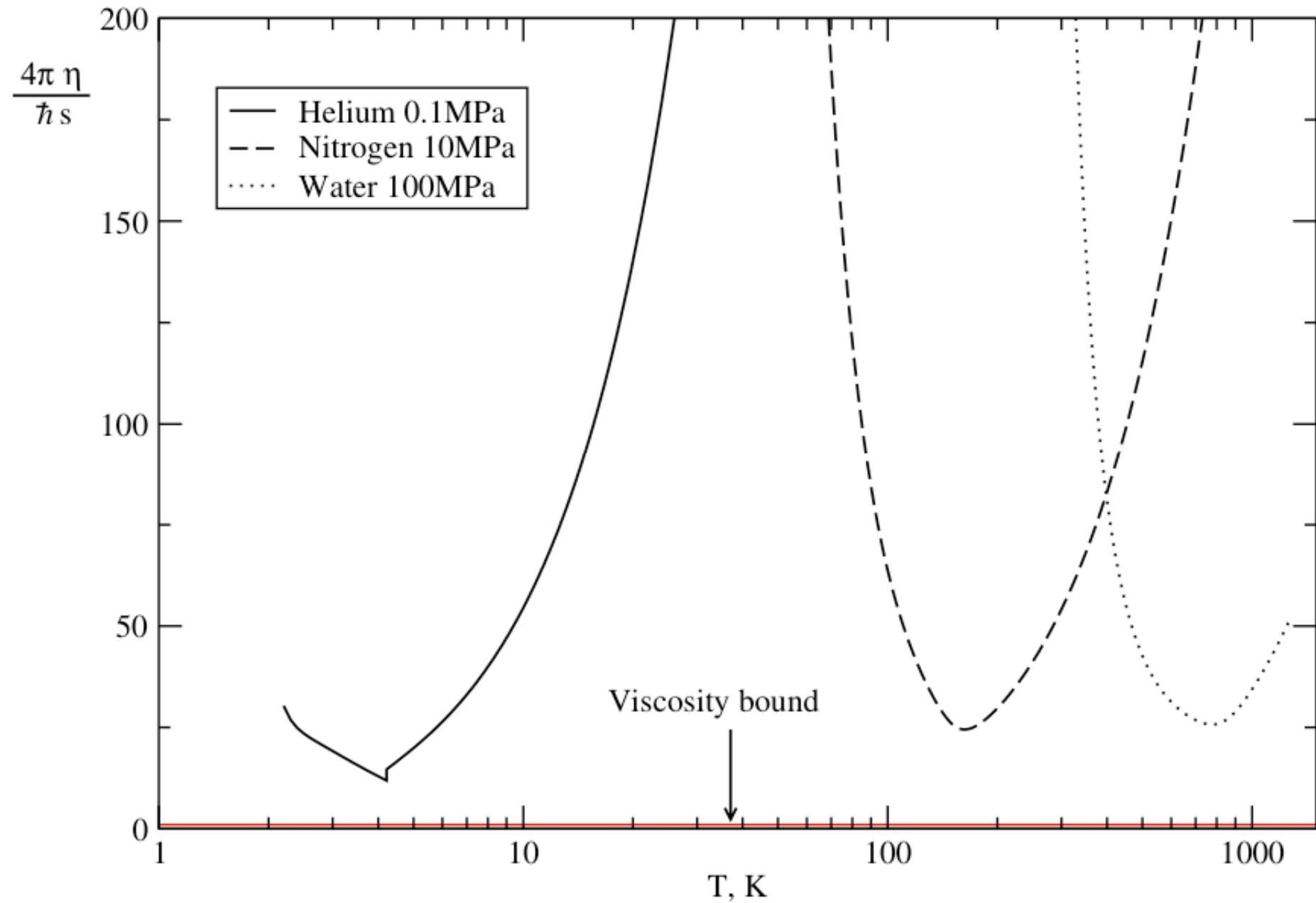
**Definition** A fluid is perfect if

$$\frac{\eta}{s} \equiv \frac{\hbar}{4\pi k_B}$$

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**Conjecture** Kovtun/Son/Starinets '05

**All** (relativistic, scale invariant) fluids have  $\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}$



## Outline

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Viscosity: ideal, viscous, super- and perfect fluids

The KSS-bound

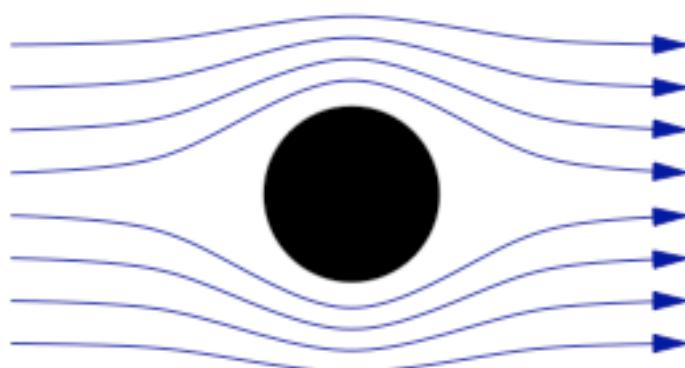
Viscosity of the unitary Fermi gas

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shear force per area

$$T_{xy} = \eta \cdot \partial_y v_x$$



Reynolds number

$$\text{Re} = \frac{vL}{\nu} \quad \nu = \frac{\eta}{\rho} \equiv D_{\perp}$$

**momentum balance**

$$\partial_t(\rho v_i) + \partial_j \Pi_{ij} = 0$$

$$\Pi_{ij} = p\delta_{ij} + \rho v_i v_j - \eta \left( \partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \cdot \partial_k v_k \right) - \zeta \delta_{ij} \cdot \partial_k v_k$$

positivity:  $\eta \geq 0$  and  $\zeta \geq 0$  due to  $dS/dt \geq 0$

fluids are (approx.) **ideal**  $\eta_{\text{id}} \equiv 0$  if  $\eta |\nabla_{\perp} v| \ll p \rightarrow L \gg \ell$

**shear viscosity: liquids versus gases**

liquids: thermally activated  $\rightarrow \eta(T)$  grows as  $T \downarrow$

gases  $\eta = \frac{1}{3} m n \langle v \rangle \ell \simeq \sqrt{m k_B T} / \sigma(T)$  grows as  $T \uparrow$

## A lower bound on the viscosity ?

mean free path  $\ell \gtrsim n^{-1/3}$     average velocity  $\langle v \rangle \gtrsim \frac{\hbar}{m} n^{1/3}$

gives  $\eta \geq \alpha_\eta \cdot \hbar n$     ( $\alpha_\eta \simeq 0.2$  for  ${}^4\text{He}$  at 2K)

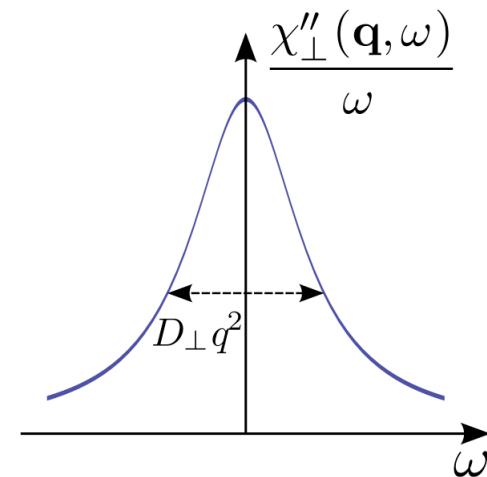
**superfluids** have  $\eta_{SF} \equiv 0$  but at any finite  $T$  there is

a normal comp.  $\rho_n(T) \neq 0$ ; relaxation of shear due to

phonon-phonon collisions  $\eta \sim T^{-5}$  Landau/Khal. '49

**shear diffusion** transverse currents relax diffusively

$$\frac{\chi''_{\perp}(\mathbf{q}, \omega)}{\omega} \rightarrow \frac{\eta q^2}{\omega^2 + (D_{\perp} q^2)^2}$$



sum rule  $\int \frac{\chi''_{\perp}(\mathbf{q}, \omega)}{\omega} = \chi_{\perp}(\mathbf{q}) \rightarrow \rho_n$  normal fluid density

**Einstein relation**  $\boxed{\eta \equiv D_{\perp} \cdot \rho_n}$  Hohenberg/Martin '65

$\eta = \alpha_{\eta} \cdot \hbar n$  implies  $D_{\perp} = \alpha_{\eta} \cdot \frac{\hbar}{m}$  (exp. Zwierlein '10)

**relativistic fluids**: replace  $\rho$  by  $sT/c^2 \rightarrow D_{\perp}^{\text{rel.}} = \frac{\eta c^2}{sT}$

KSS-bound

$$\frac{D_{\perp}}{c^2} = \tau_{\perp} \geq \frac{\hbar}{4\pi k_B T}$$

applies to QFT's

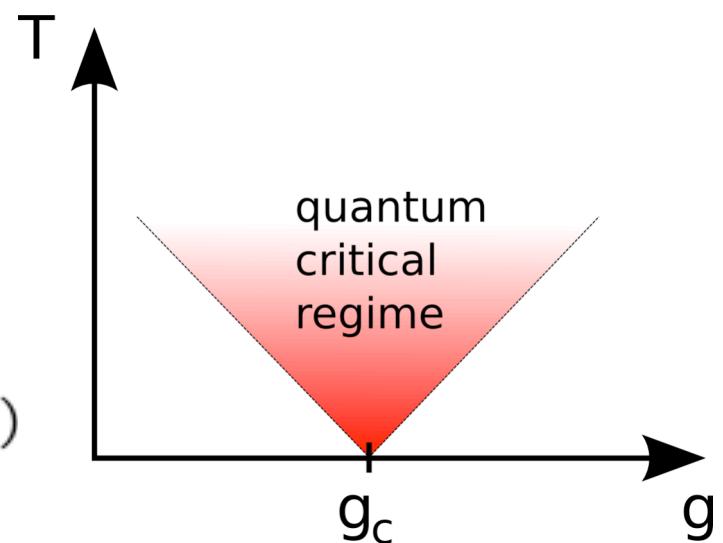
with no well defined quasi-particles ( $\hbar/\tau \ll \epsilon_{qp} \simeq k_B T$ )

**quantum critical regime**

above a QPT

$$\tau_{\Psi} = C \frac{\hbar}{k_B T}$$

$C$  is a universal number (Sachdev)



The KSS bound

extend Yang-Mills Theory

$\mathcal{L}_{YM} = -\frac{1}{4g^2} F_{\mu\nu}^a F^{\mu\nu a}$  to a  $\mathcal{N}=4$  supersymmetric one

$\beta(g) \equiv 0 \rightarrow$  no confinement or asymptotic freedom!

entropy/viscosity

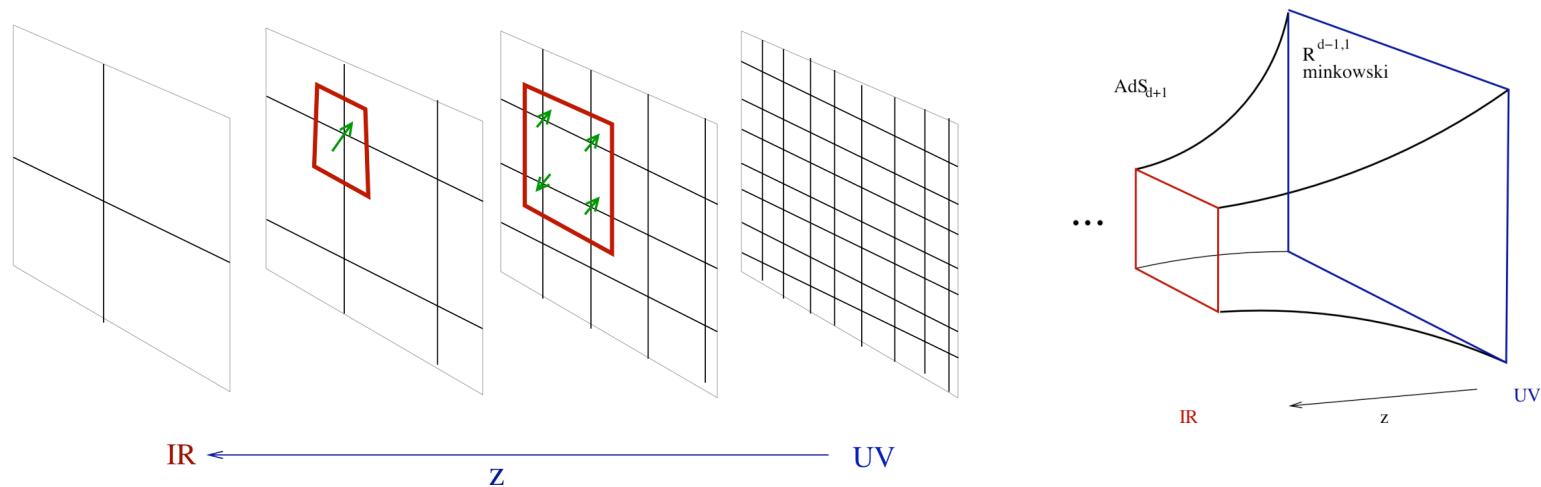
gas of massless bosons with

interactions  $\mathcal{O}(1)$  gives  $s(T) \simeq k_B (k_B T / \hbar c)^3$  and

$\eta(T) \simeq \hbar (k_B T / \hbar c)^3$  with  $\eta/s = \mathcal{O}(\hbar/k_B)$  at all  $T$ !

## AdS/CFT $\mathcal{N}=4$ SSYM-Theory in the t'Hooft limit

$\lambda = g^2 N \rightarrow \infty$  is equiv. to a classical theory of gravity



$$ds^2 = \frac{L^2}{z^2} (-dt^2 + d\mathbf{x}^2 + dz^2)$$

$$\frac{L}{\ell_P} = \lambda^{1/4} \rightarrow \infty$$

radial coord.  $z$  is effectively an RG-scale McGreevy '09

## The unitary Fermi gas

two-component Fermigas with zero-range interactions

$$\mathcal{L}_E = \sum_{\sigma=\uparrow,\downarrow} \Psi_\sigma^\dagger \left( \hbar \partial_\tau - \frac{\hbar^2}{2m} \nabla^2 \right) \Psi_\sigma + \frac{g(\Lambda)}{2} \Psi_\sigma^\dagger \Psi_{-\sigma}^\dagger \Psi_{-\sigma} \Psi_\sigma$$

renormalized coupling  $g(\Lambda) \rightarrow g = 4\pi\hbar^2 a/m$

Hubbard-Stratonovich transformation

$$\mathcal{L}[\Psi] \rightarrow \mathcal{L}[\Psi, \Phi] = \mathcal{L}_0 + (\Psi_\uparrow \Psi_\downarrow \Phi + \text{h.c.}) - \frac{1}{g} \Phi^2$$

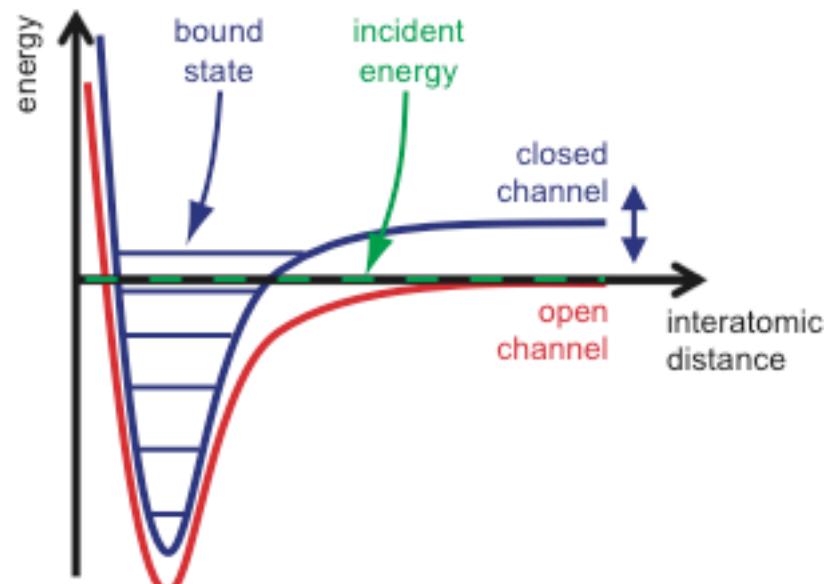
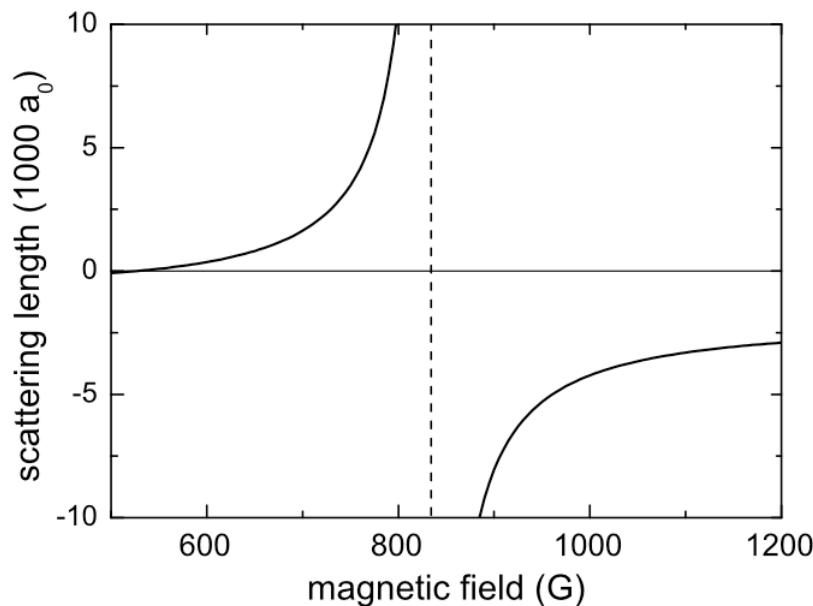
$\Phi$  is massless at infinite coupling  $g = \infty$

## Feshbach-resonances

closed channel bound state

couples resonantly

$$a_s = a_{bg} \left( 1 - \frac{\Delta B}{B - B_0} \right)$$



scattering length in  ${}^6\text{Li}$

(two lowest hyperfine states)

## Scale invariance at infinite scattering length

$x \rightarrow \lambda x$  gives  $H \rightarrow H/\lambda^2 \rightarrow \partial_\nu D^\nu = 0 \rightarrow \text{Tr } T = 0$

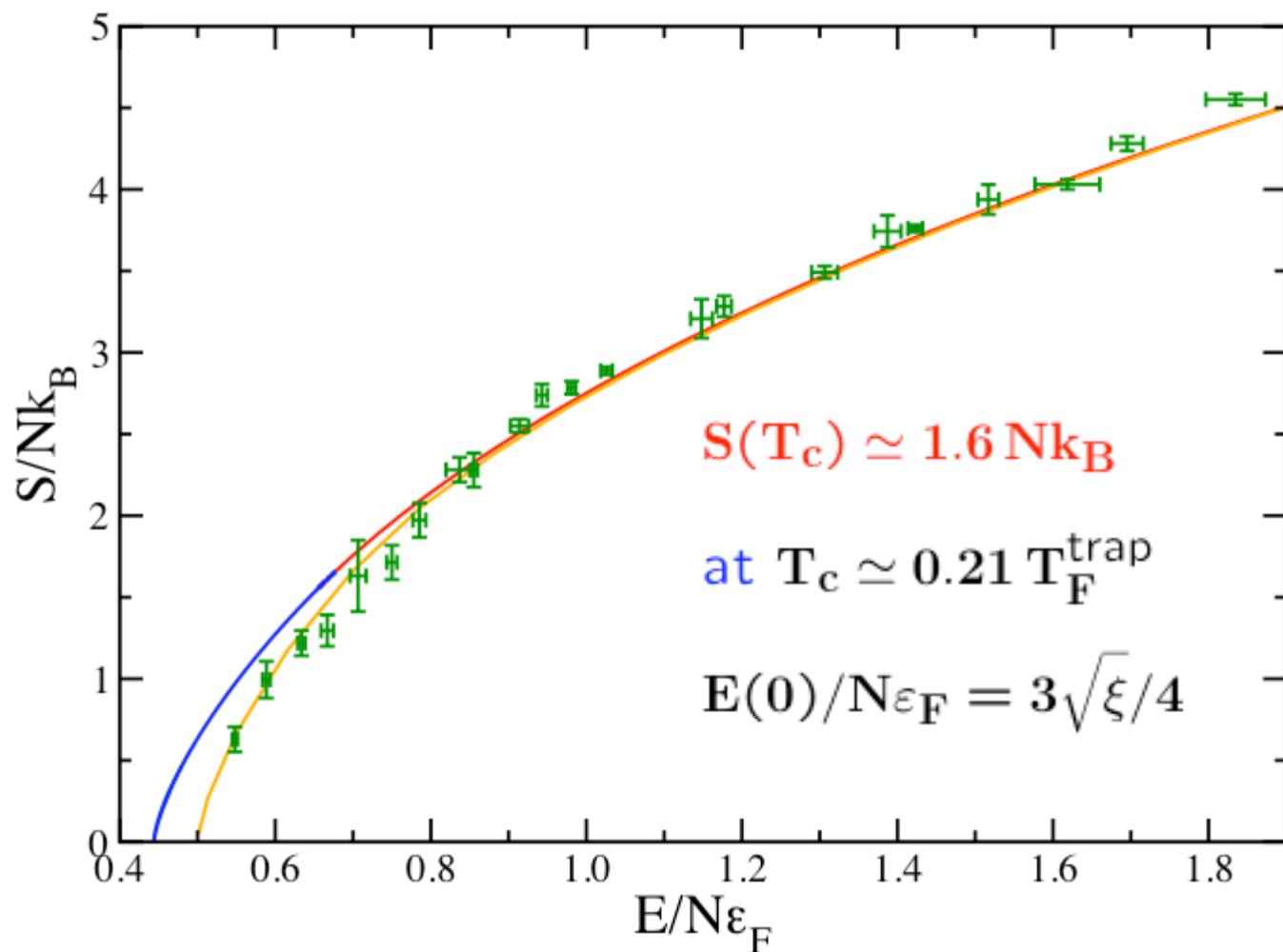
pressure  $p = 2u/3$  Ho '04 bulk viscosity  $\zeta = 0$  Son '07

virial theorem  $\langle \hat{H}_{\text{tot}} \rangle = 2 \int U_{\text{trap}}(x) n(x)$  Thomas '05

$p(\infty) = \xi \cdot p_F^{(0)}$  **Bertsch-parameter**  $\xi \simeq 0.4$

determines cloud size in a trap  $R_{TF} = R_{TF}^{(0)} \cdot \xi^{1/4}$

## Unitary gas entropy in a trap Thomas '07+'09



## viscosity of the unitary gas

Boltzmann-limit  $\eta(T \gg T_F) = 2.8\hbar n(T/T_F)^{3/2} \sim \frac{\hbar}{\lambda_T^3}$  (!)

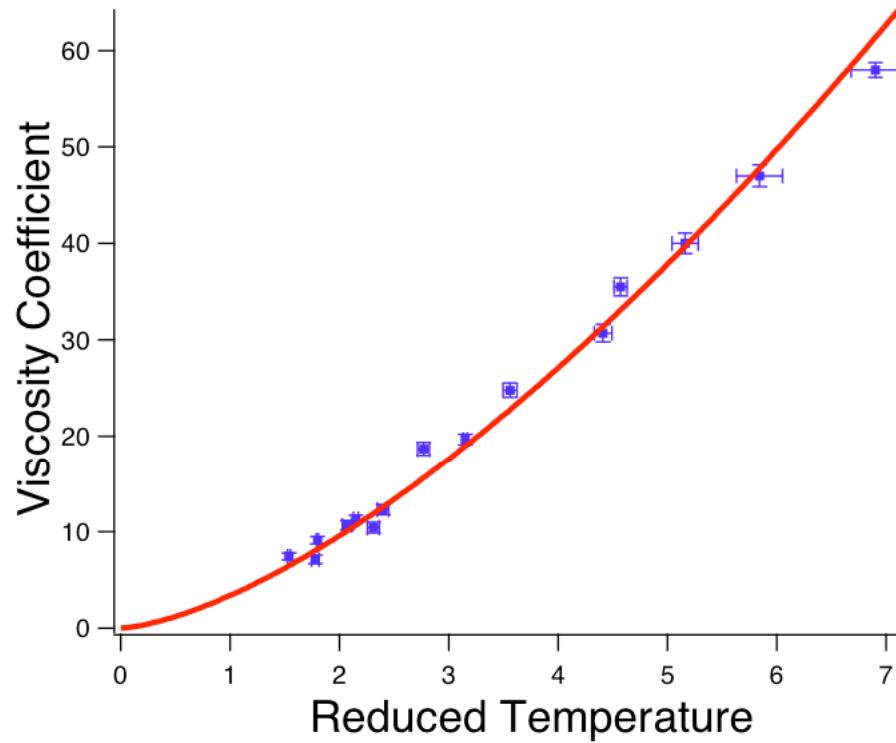
relies on well defined quasiparticles  $\hbar/\tau_\eta \ll k_B T$

Kubo formula  $Re \eta(\omega) = \frac{Im \chi_{xy}^{ret}(\omega)}{\omega}$  lin. response

$$\hat{H}' = h_\ell(t) \cdot \hat{\Pi}_\ell \quad (\ell = 0, 2 \rightarrow \text{bulk, shear})$$

$$\text{euclidean time } \tau \rightarrow \chi_\ell(\tau) = \int d^3x \langle \tilde{T} \hat{\Pi}_\ell(\mathbf{x}, \tau) \hat{\Pi}_\ell(\mathbf{0}, 0) \rangle$$

from  $\chi_\ell(\tau) = -\frac{\delta^2 \Omega}{\delta h_\ell(\tau) \delta h_\ell(0)}|_{h=0}$



shear viscosity  
for trapped atoms | [exp: Cao et al. 2010]

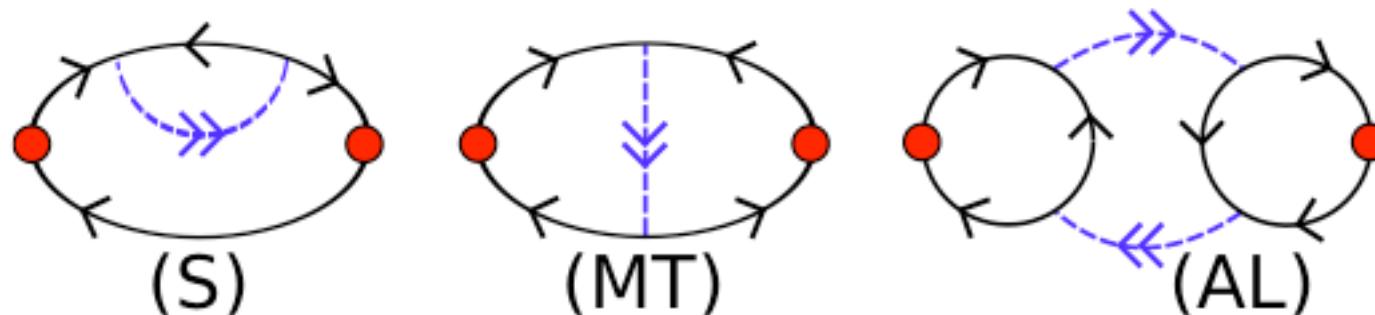
## Luttinger-Ward functional $\Omega[G]$ in T-Matrix approx.

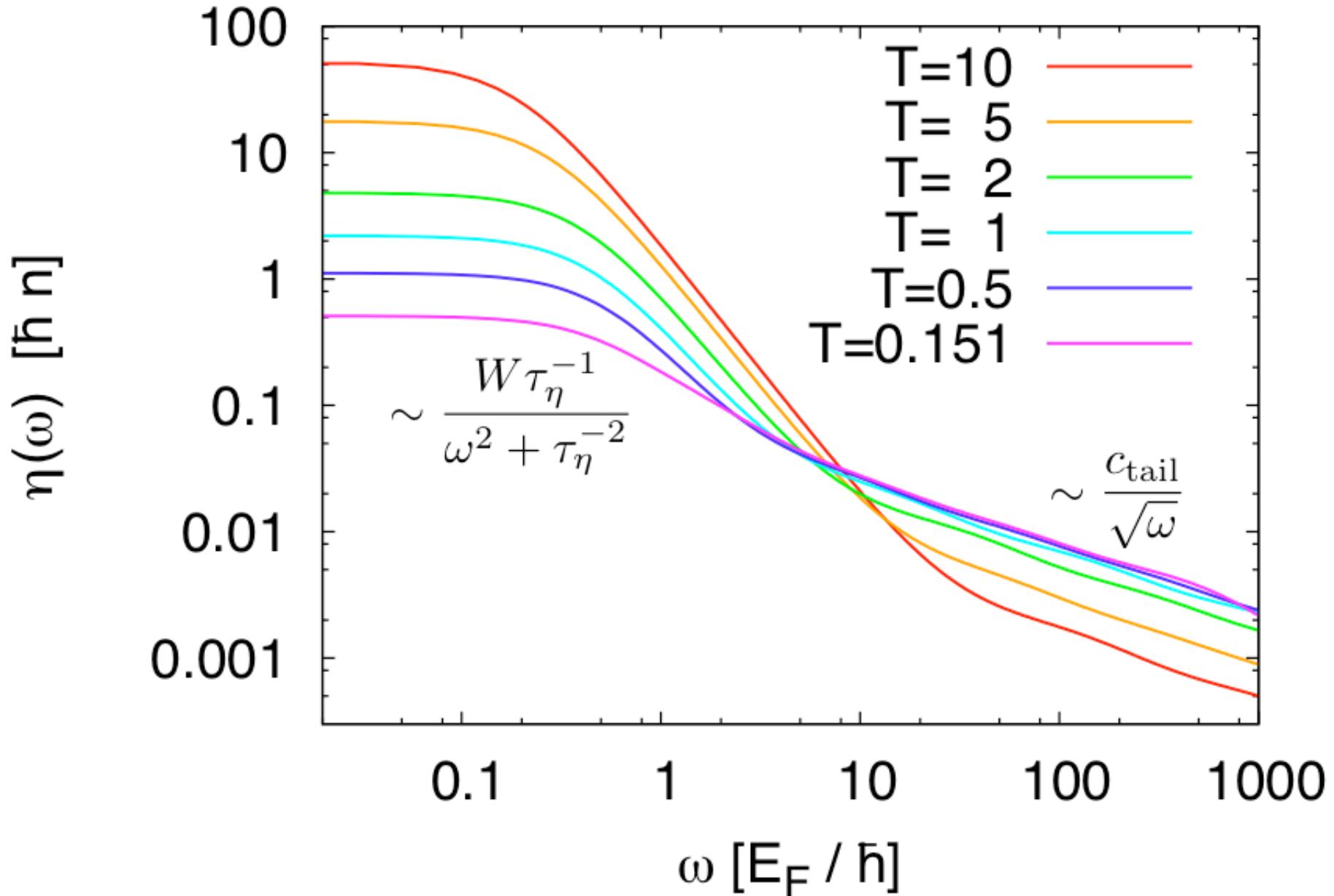
a) symm. + conserv. laws  $\zeta(\omega) \equiv 0$

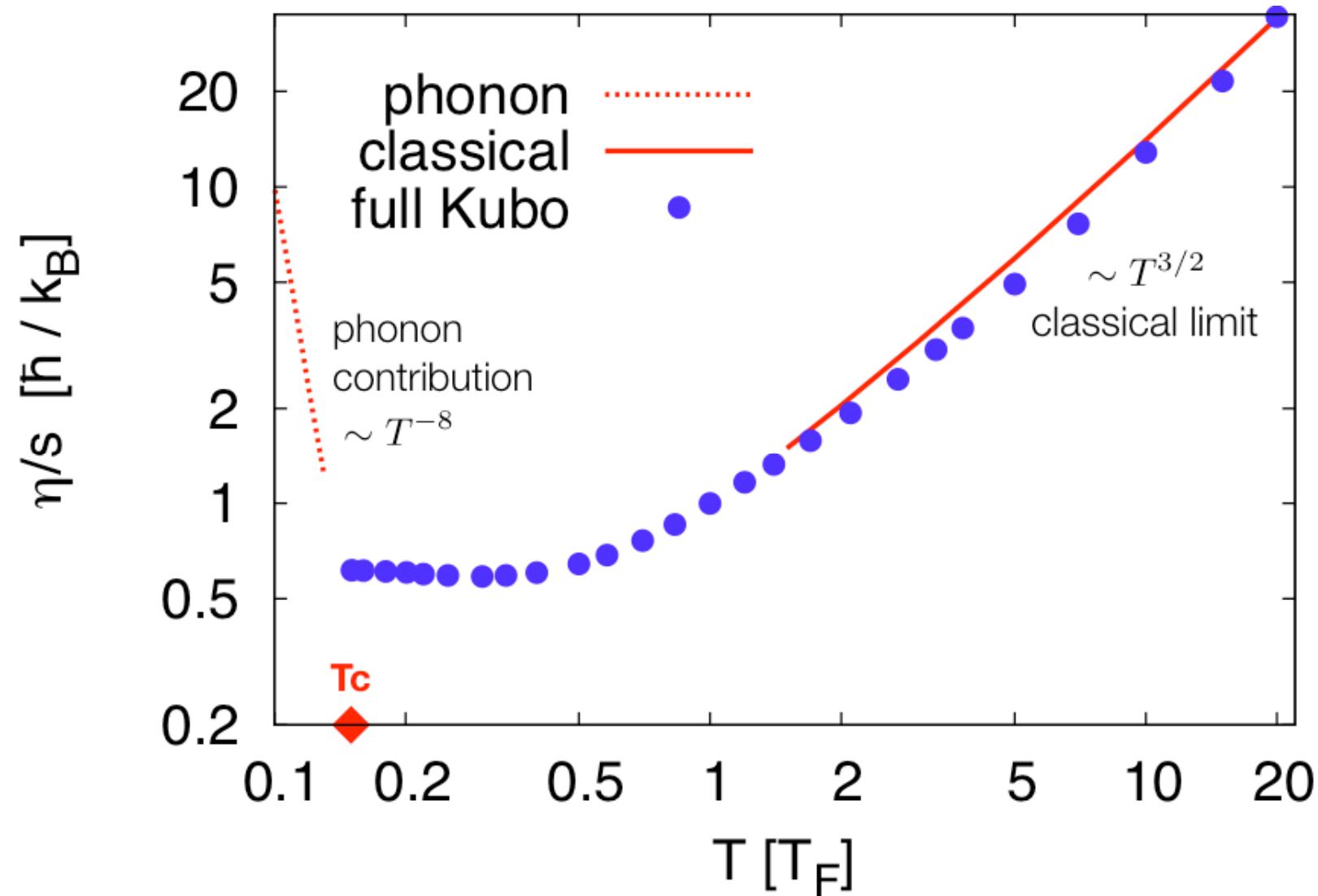
b) sum rule  $\frac{2}{\pi} \int_0^\infty d\omega \left[ \text{Re } \eta(\omega) - \frac{\hbar^{3/2} C_\eta}{\sqrt{m\omega}} \right] \equiv p$

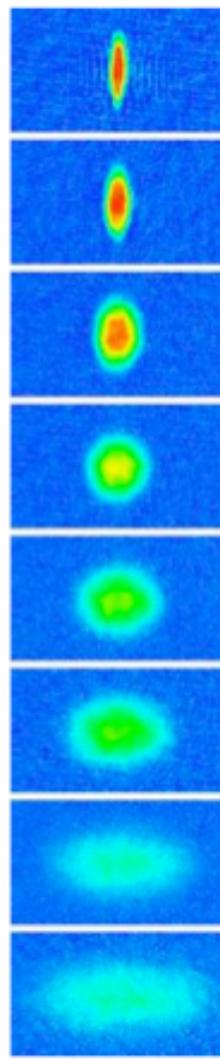
$C_\eta = C/15\pi$  det. by the Tan contact density  $C$

c) Boltzmann equ. at high temp. (exact to order  $z^2$ )



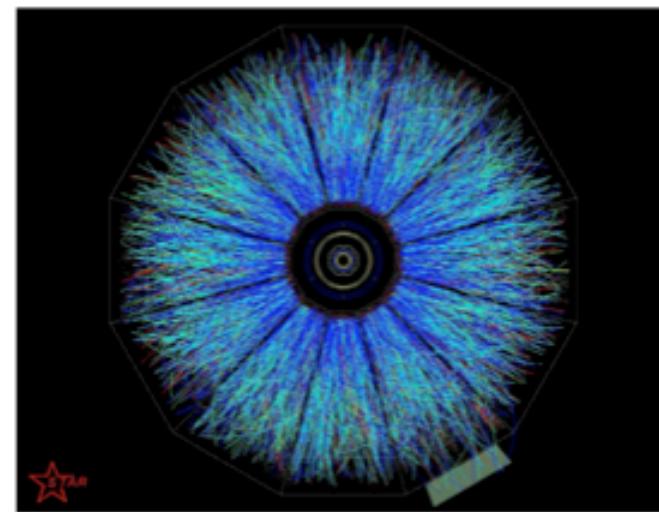






Trapped Atoms

$$\eta = 1.7 \cdot 10^{-15} Pa \cdot s$$



$$QGP \quad \eta = 5 \cdot 10^{11} Pa \cdot s$$



Liquid Helium

$$\eta = 1.7 \cdot 10^{-6} Pa \cdot s$$

Consider ratios

$$\frac{\eta}{s} \gtrsim 0.4; 0.5; 1 \text{ for QGP, } {}^6\text{Li, } {}^4\text{He}$$

## Conclusions

- 1) All known fluids obey the KSS bound on  $\eta/s$ . The quark-gluon-plasma and the unitary Fermi gas come closest to saturating it.
- 2) Ideas from string theory provide motivation for theory and experiments in the area of ultracold atoms.



## Scale invariant many-body problems

pseudopotential  $g_2\delta(x)$  in 2d Pitaevskii/Rosch '97

unitary gas ( $a = \infty$ ) in 3d Son/Wingate '06

gases in mixed dimensions Nishida/Tan '09

electrons with  $1/r$ -interaction in Graphene Son '07