Information theory and the limits on color vision in natural scenes

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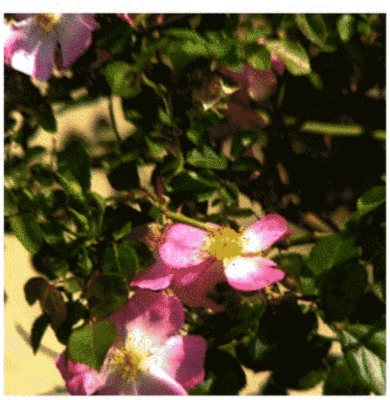
Local surface-color information



What limits out ability to identify surfaces despite changes in color of light on scene?

Three issues:

1. Number of different colors.
2. How surfaces coded neurally.
3. How well such codings label uniquely surfaces under different lights.

Theoretical limits, then experimental …

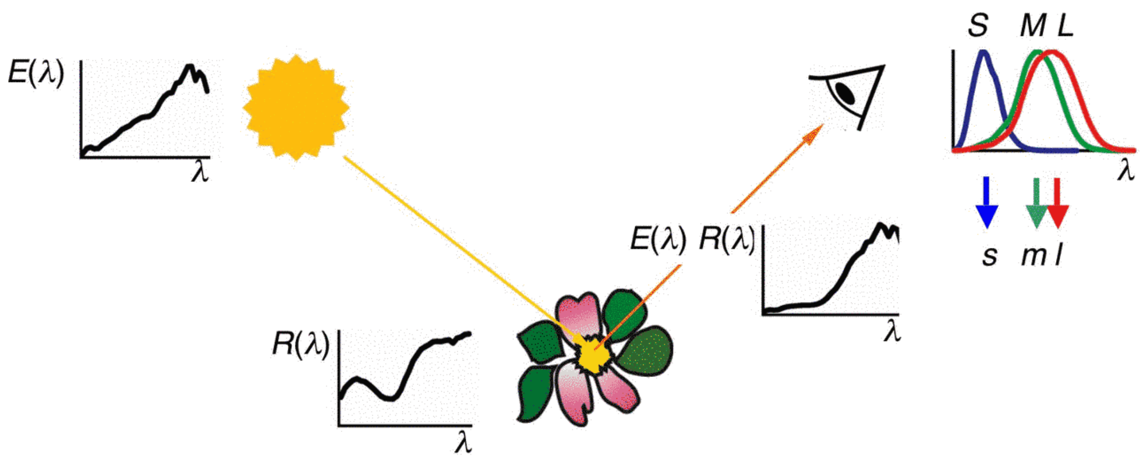
Local surface-color information

With just two surfaces, little confusion.

As number increases, risk of confusion increases:

1. Chance of similar reflectance spectra increases.
2. Codings not perfectly invariant under illuminant changes.
3. Physical, not perceptual or cognitive factors. …

Basic optics



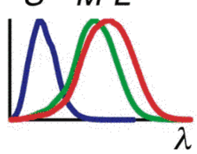
L-cone activity $l = \int E(\lambda)R(\lambda)L(\lambda)\,d\lambda$

M-cone activity $m = \int E(\lambda)R(\lambda)M(\lambda)\,d\lambda$

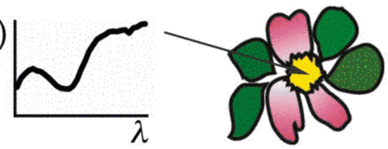
S-cone activity $s = \int E(\lambda)R(\lambda)S(\lambda)\,d\lambda$

Problem of recovery

From triple (l,m,s)



Want something like $R(\lambda)$



More precisely, want corresponding (l_0,m_0,s_0) values under some standard illuminant E_0 …

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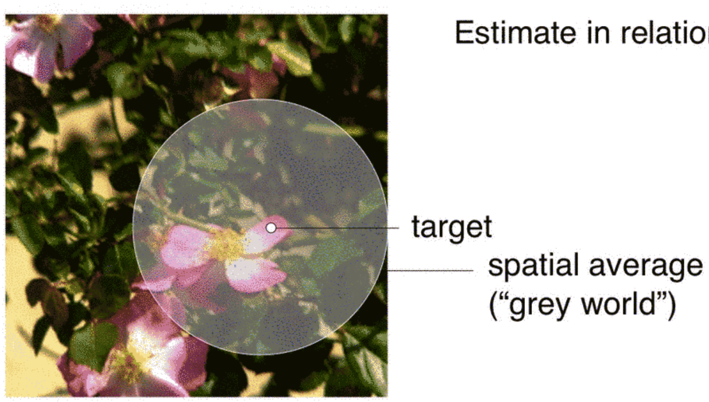
Illuminant-invariant codes



Estimate in relation to spatial average …

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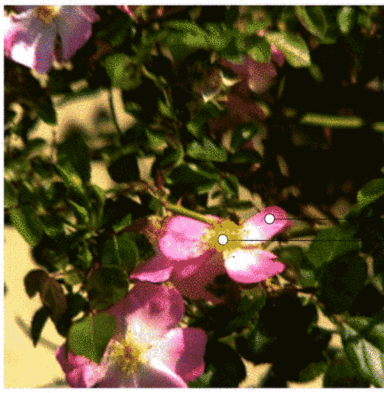
Illuminant-invariant codes



Take ratios for each cone class …

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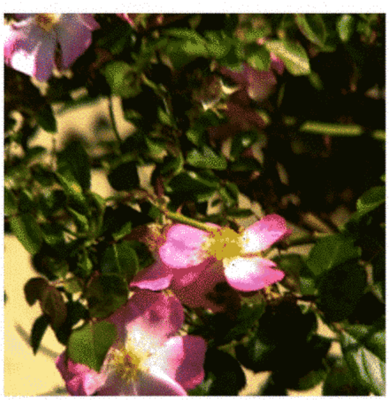
Scaling



Comparisons affected by:
- distance
- contrast
- gamut

Pairwise comparisons capture all illuminant-invariant operations.

How invariant? ...

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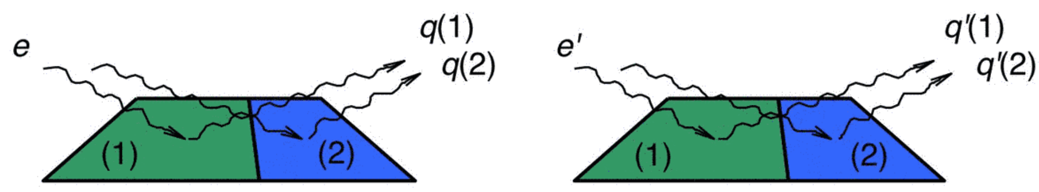
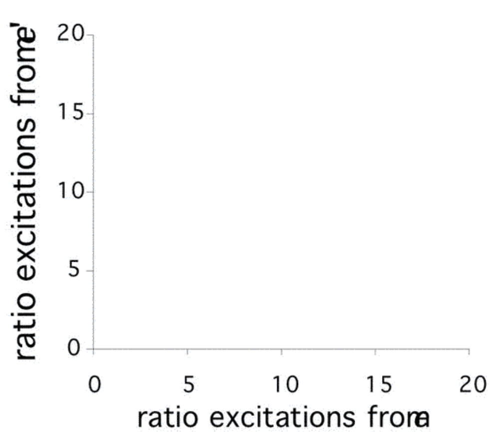
Cone-excitation ratios



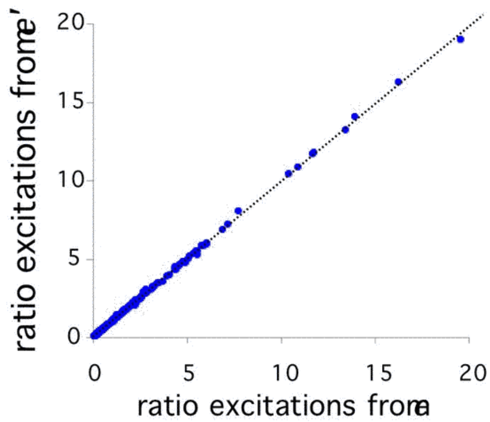
$r = q(1)\,/q(2)$ $r' = q'(1)\,/q'(2)$

Now plot r' against r …

Scatterplot ratios (S-cones)



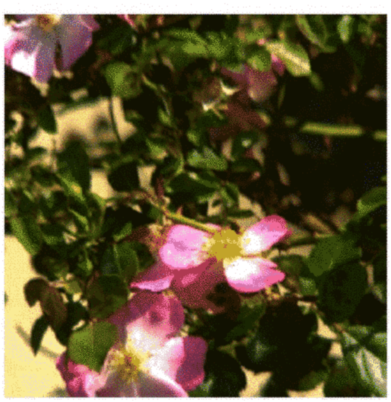
Scatterplot ratios (S-cones)



- Illuminants *e* and *e'* drawn at random from range 4300K to 25000K.
- Subset of 200 from 1000 surfaces drawn at random from natural scenes.
- Relative deviations for natural rural scenes:

  $|r - r'|/\min\{r, r'\} \sim 4\%$
- Ratios are stable, but the error is important …

Labelling

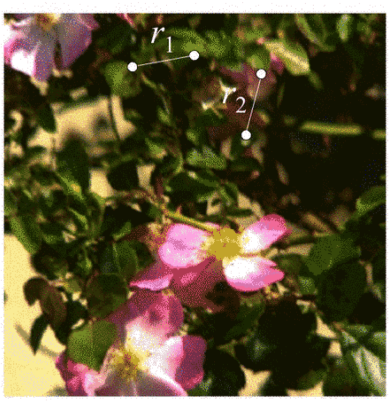


How much information is captured by color-labeling objects in a scene? …

Labelling by ratios

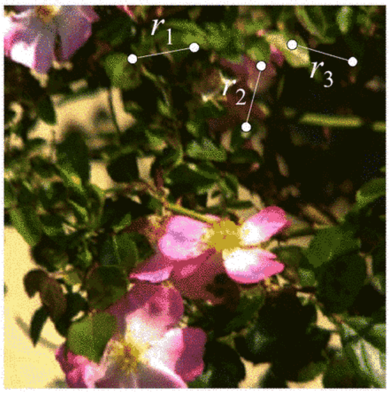
Represent surface colors as a collection of ratios r_1

Labelling by ratios



Represent surface colors as a collection of ratios r_1, r_2

Labelling by ratios



Represent surface colors as a collection of ratios r_1, r_2, r_3

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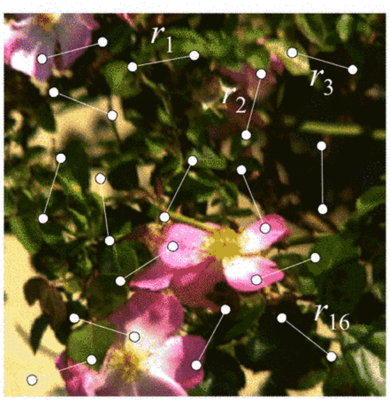
Labelling by ratios



Represent surface colors as a collection of ratios r_1, r_2, r_3, … , r_{16}.

Or, if local grey-world normalization …

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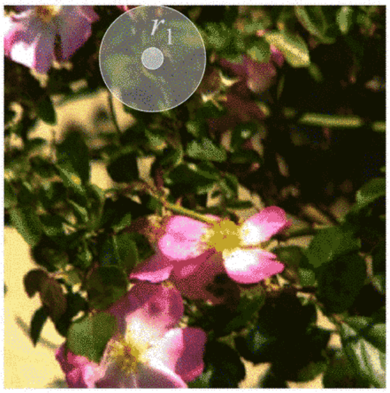
Labelling by grey-world normalization



Represent surface colors as a collection of ratios r_1

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Labelling by grey-world normalization

Represent surface colors as a collection of ratios r_1, r_2

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Labelling by grey-world normalization

Represent surface colors as a collection of ratios r_1, r_2, r_3

Labelling by grey-world normalization

Represent surface colors as a collection of ratios r_1, r_2, r_3, … , r_{16}.

Can think of these 16 points carrying 4 bits of information.

If n points, then $\log_2 n$ bits ...

Now change illuminant

Suppose illuminant changed, is information preserved by eye?

Change to cone-ratio pairs as easier to see …

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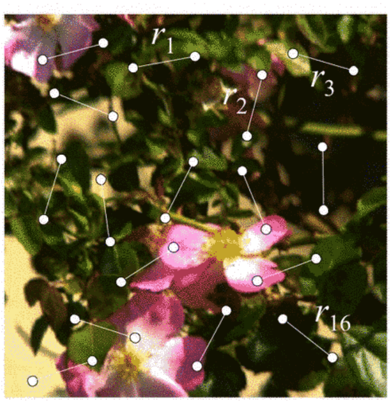
Preserving information?



Under warm light 4300K.

Change to cold light 25000K …

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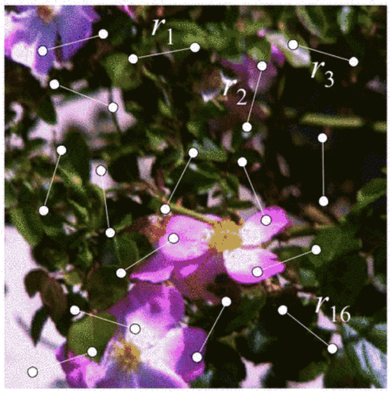
Preserving information?



Under cold light 25000K.

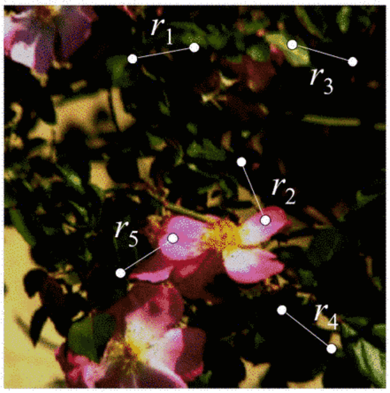
How well do ratios to identify original surfaces? …

Preserving information?

Suppose just 5 pairs under old illuminant.

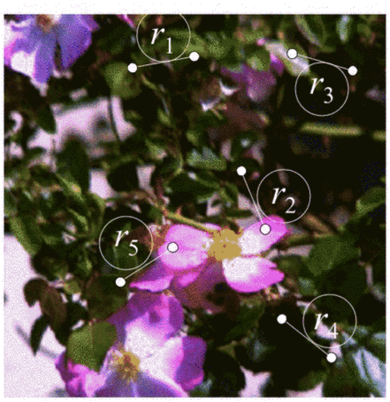
Suppose perfect recovery under new illuminant …

Preserving information?



old illuminant:

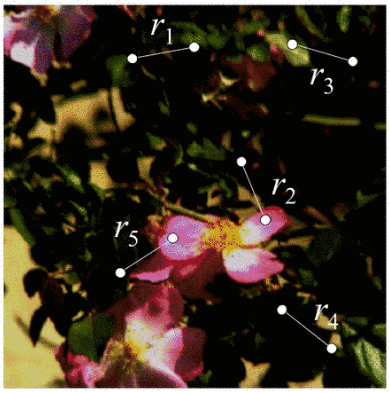
Perfect recovery



old illuminant:	1	2	3	4	5
	↓	↓	↓	↓	↓
new illuminant:	1	2	3	4	5

No information lost.

Imperfect recovery



old illuminant:

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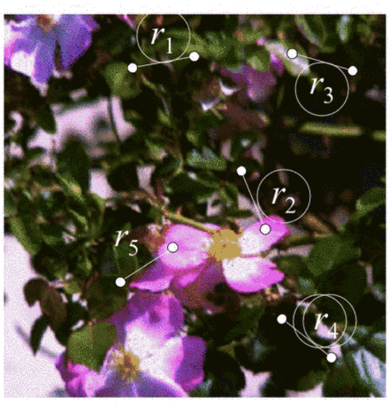
Imperfect recovery



old illuminant: 1 2 3 4 5

new illuminant: 1 2 3 4

Now information lost: from (4) cannot determine original reliably …

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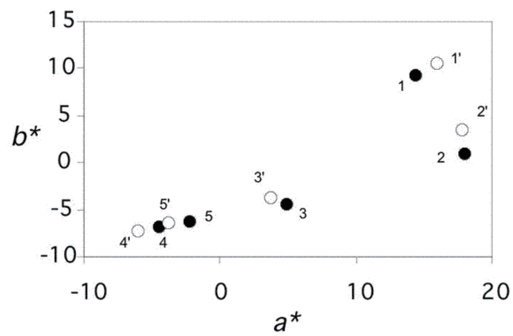
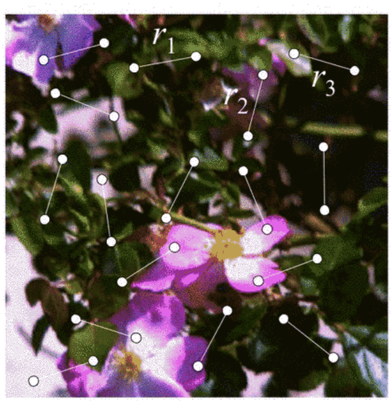
Imperfect recovery

In a suitable color space:



In general, need to describe identifications by probabilities ...

Information available vs number



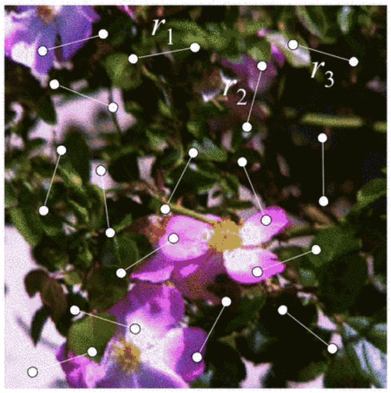
If $p(i, j)$ is probability of identifying point j under illuminant E_2 with point i under illuminant E_1, then information preserved:

$$I(1;2) = \sum_i \sum_j p(i, j) \log p(i, j)/(p(i)p(j)),$$

where $p(i) = \sum_j p(i, j)$ and $p(j) = \sum_i p(i, j)$.

Evaluate for $i, j = 1, .. n$, by numerical simulation …

Information available vs number



- How does *I* vary with scene?
- How does *I* vary with coding?
- Need reflectance data at each point in scene …

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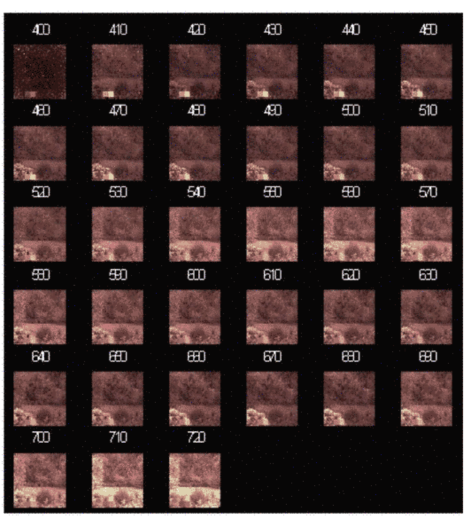
Hyperspectral imaging



- Spectral samples at 10-nm intervals over 400–720 nm.
- Avoids temporal variations in illuminant or position.
- Spatial resolution similar to eye (≤1 arcmin × ≤1 arcmin) …

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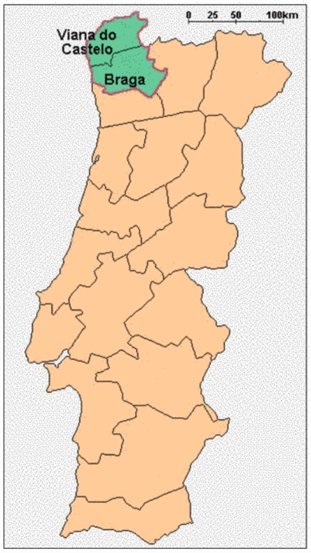
On location



- Recorded during summers of 2002 and 2003.
- Almost always under clear sky.
- Particular care taken to avoid scenes containing movement ...

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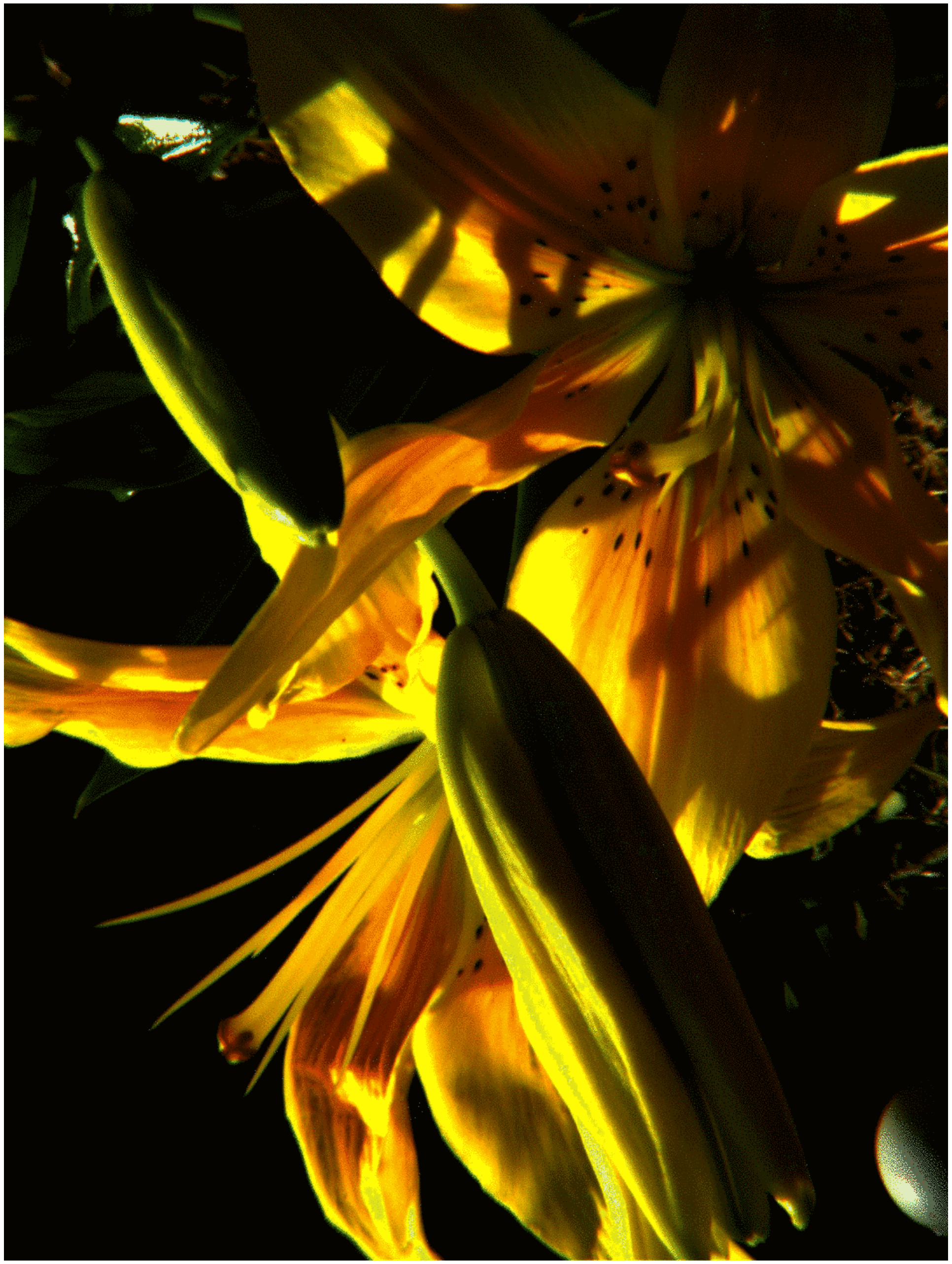
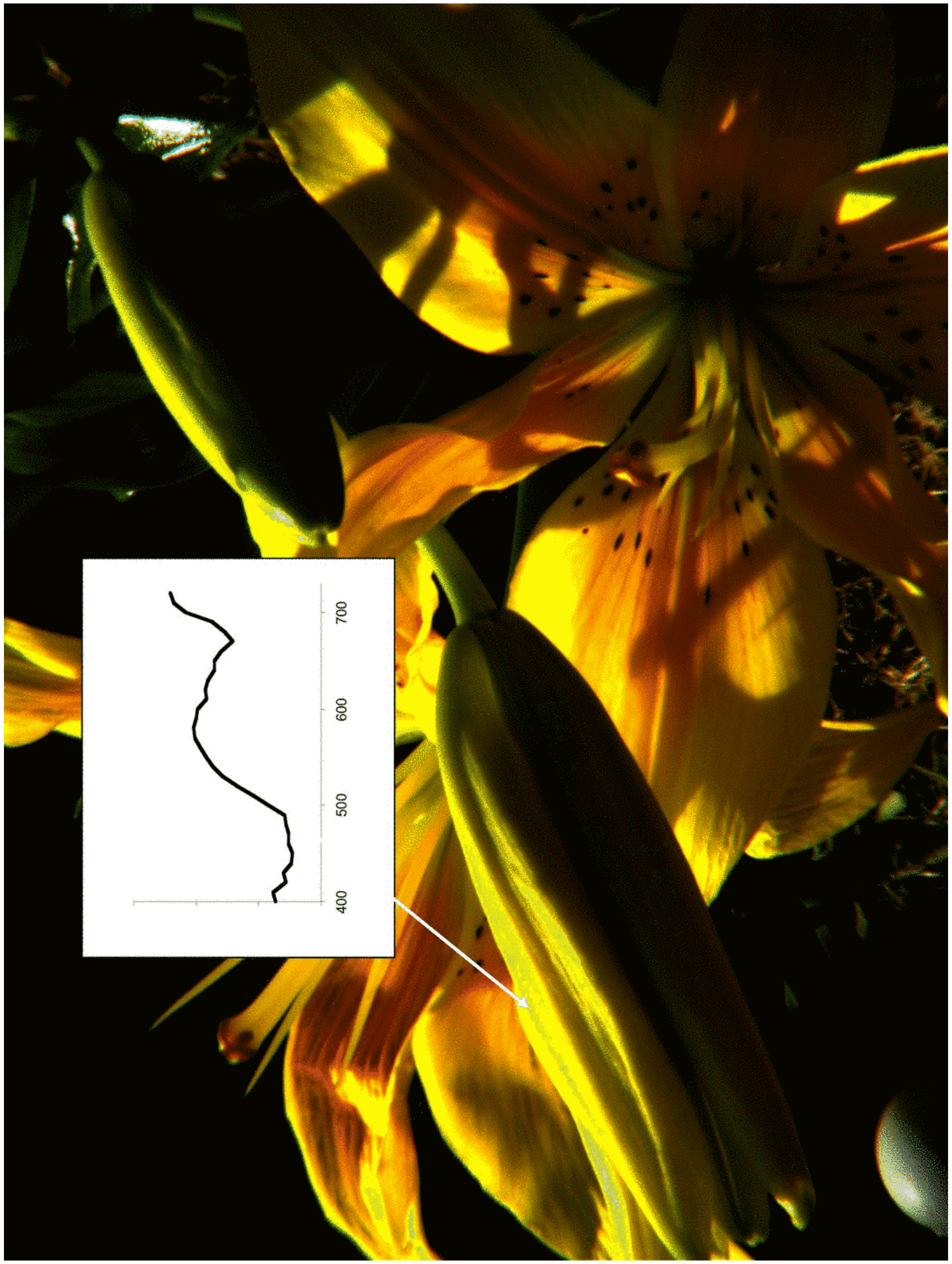
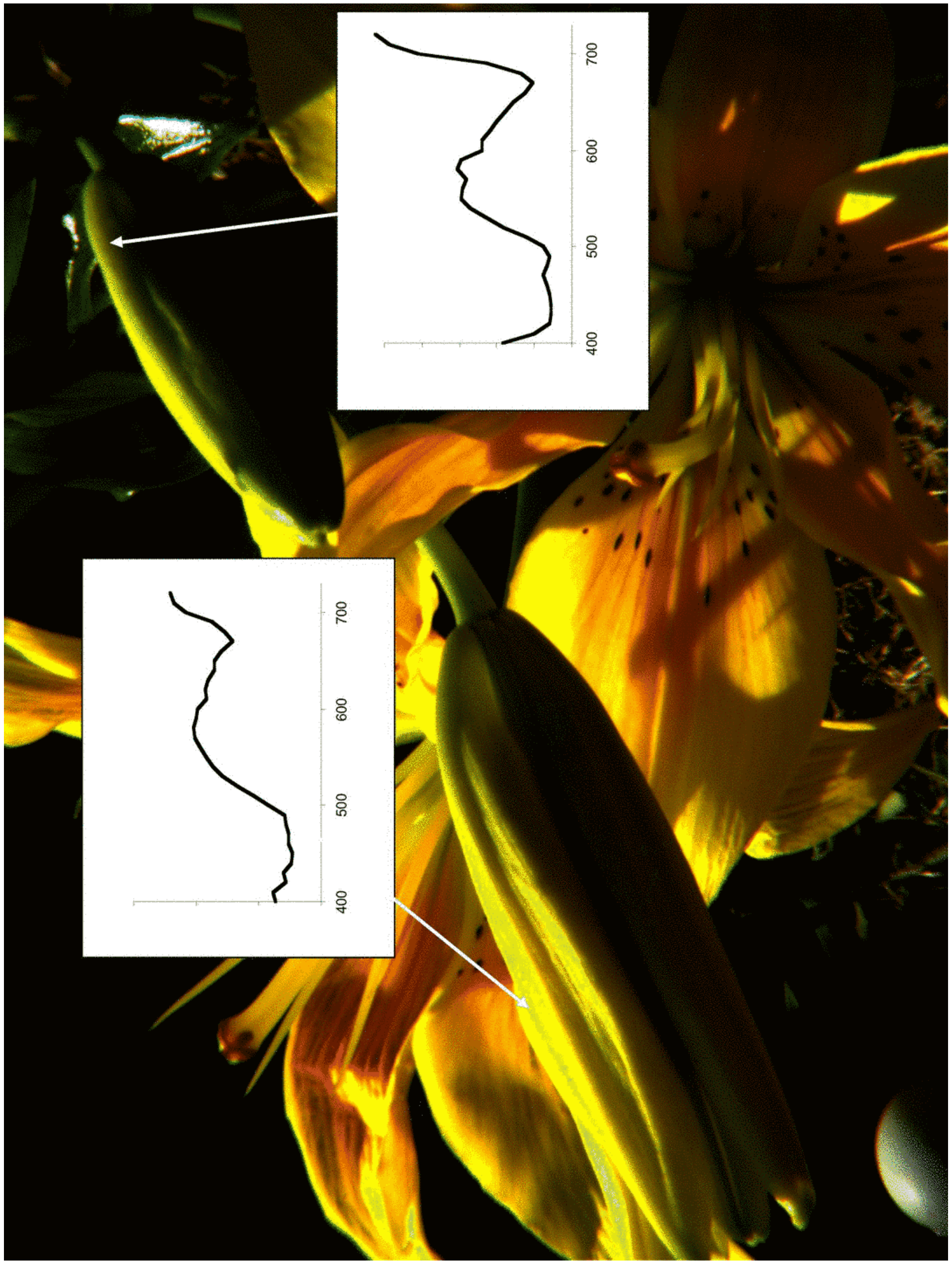



400
500
600
700

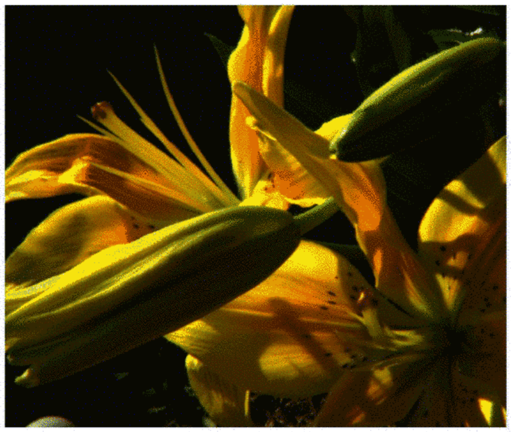

700
600
500
400
700
600
500
400


700
600
500
400


700
600
500
400
700
600
500
400

Hyperspectral reconstruction

ylilly1b



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Hyperspectral reconstruction

yrose1b

42

Hyperspectral reconstruction

brufe1b

43

Hyperspectral reconstruction

braga1b

44

Hyperspectral reconstruction

crown 3bb

45

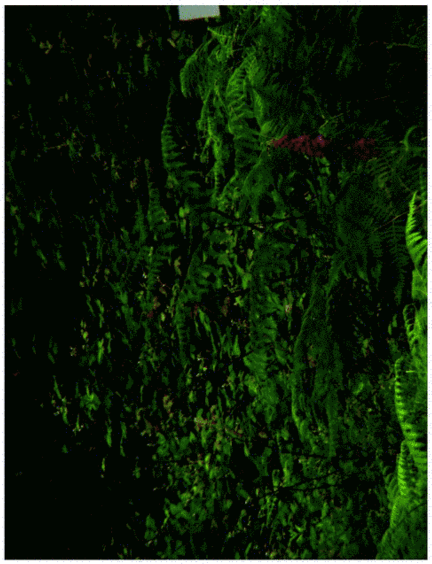
Hyperspectral reconstruction

crown4bbbb

46

Hyperspectral reconstruction

ruivaes1b



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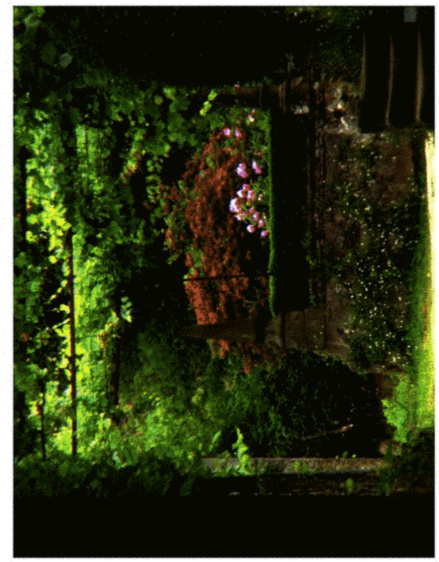
Hyperspectral reconstruction

mosteiro4b



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Hyperspectral reconstruction

ribeira1bb

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Hyperspectral reconstruction

farme1a

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Hyperspectral reconstruction

ruivaes1b

Plot mutual information $I(1;2)$ against number n of sample surfaces …

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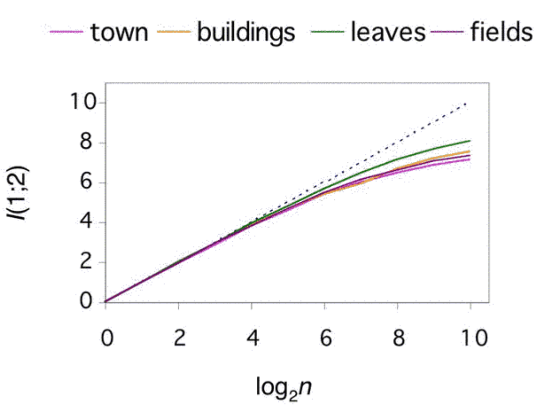
Mutual information $I(1;2)$ vs sample n



$I(1;2) = \sum_i \sum_j p(i, j) \log p(i, j)/(p(i)p(j))$,

with $i, j = 1, .. n$,.

- Estimated $I(1;2)$ increases monotonically.
- Similar values for four types of scenes.
- Each tends towards asymptote.
- Unreliable estimates with small $p(i, j)$ and large n.
- Fortunately, when n so large that distribution of code values continuous, can approximate by additive Gaussian channel …

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Additive Gaussian channel

Suppose each point coded as triple of cone responses (l_1, m_1, s_1) under illuminant E_1

Under illuminant E_2, triple becomes $(l_1+\Delta l, m_1+\Delta m, s_1+\Delta s)$.

Let K_1 be covariance of (l_1, m_1, s_1) and K_Δ covariance of $(\Delta l, \Delta m, \Delta s)$.

Then max $I(1; 2) = C$, where

$$C = \_ \log\{\det(K_1 + K_\Delta)/\det(K_\Delta)\}.$$

If noise not Gaussian, then still max if nearest-neighbor identification (Lapidoth, 1996).

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Information capacity

illuminant change		grey-world	ratios
25000K – 6500K	mean C	11.43	11.02
	S.d. C	**0.76**	**0.69**
4000K – 6500K	mean C	11.78	11.37
	S.d. C	**0.72**	**0.69**

data based on 25 rural and urban scenes, 1344 × 1024 pixels.

- Despite variety of scenes, C remarkably stable.
- Suggests performance in using color information can be optimized independent of location for any particular range of illuminants …

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Implications

- Fundamental limit on use of surface color to label objects in scenes.
- Can estimate ideal match for any given surface by minimizing variance in cone-excitation ratios.
- Are human observers' matches as good as these limiting values?
- Traditionally, use computer-controlled images of geometric displays or more recently of natural scenes, but gamut problem …

Gamut problem

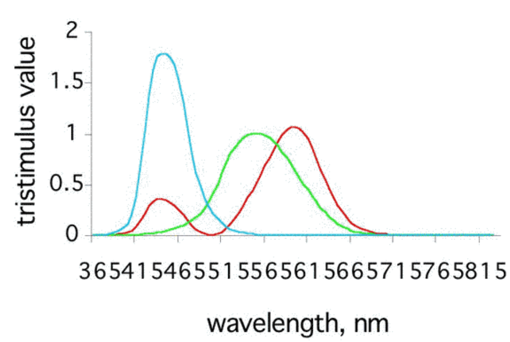
To specify colors, use CIE chromaticity diagram.

Need XYZ tristimulus values for arbitrary spectral distribution f (_):

$$X = \int f(\lambda)\,\bar{x}(\lambda)\,d\lambda$$

$$Y = \int f(\lambda)\,\bar{y}(\lambda)\,d\lambda$$

$$Z = \int f(\lambda)\,\bar{z}(\lambda)\,d\lambda$$



Gamut problem

Normalize to get chromaticity coordinates:

$x = X/(X + Y + Z)$ ("red")

$y = Y/(X + Y + Z)$ ("green")

$z = 1 - x + y$ ("blue")

Now plot phosphor gamut …

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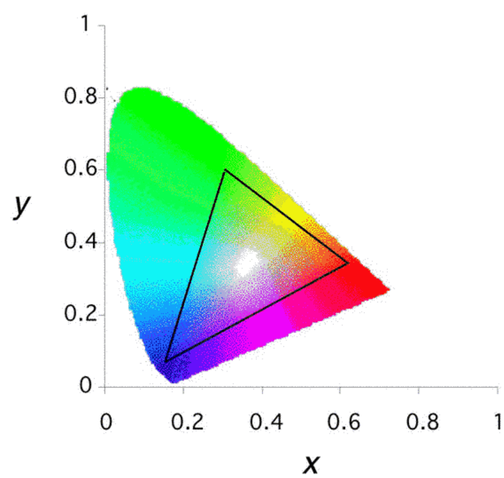
Gamut problem



Displayable monitor colors contained within triangle of three phosphors.

Instead, as a more realistic test, use real surfaces with optical projection device …

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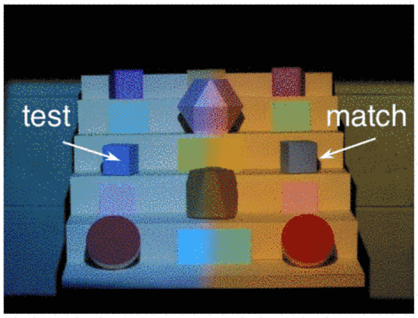
Real 3-dimensional physical surfaces



de Almeida, Fiadeiro, Nascimento (2004)

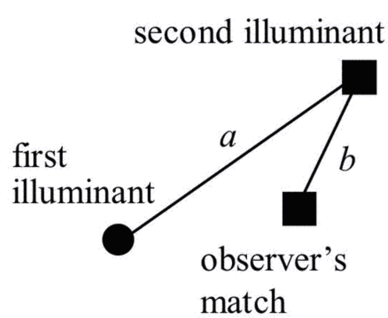
Left-side illuminant 25000 K; right-side, 4000 K.

Right-side cube is a virtual 3D image.

Subject adjusts apparent color until surface matches surface of left-side cube (measure of “color constancy”)
...

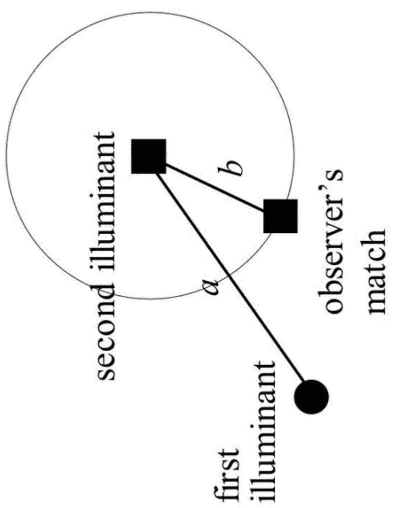
Constancy index

In a suitable color space (CIE 1976 $L^*u^*v^*$ space)



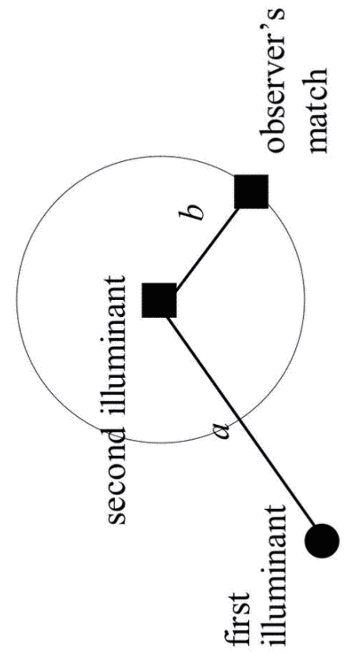
$$\mathrm{CI} = 1 - b/a$$

Constancy index



CI = 1 − b/a

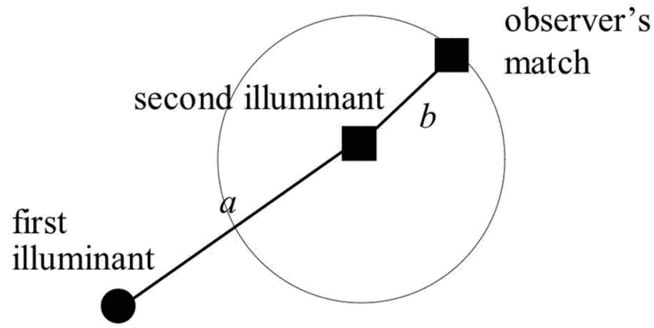
Constancy index



CI = 1 − b/a

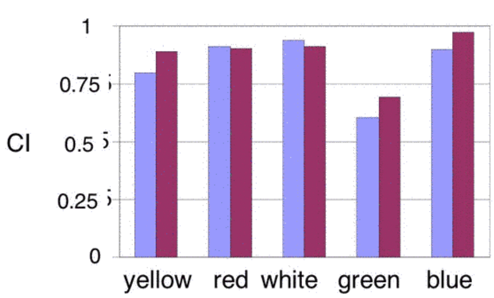
Constancy index

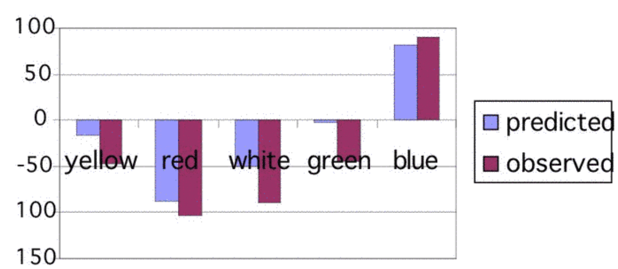
$$CI = 1 - b/a$$



Measure both CI and angular error …

Constancy index and angular error





Nascimento, de Almeida, Fiadeiro, and Foster (2004)

- Typical color-constancy indices 0.4-0.8.
- Residual bias towards color of illuminant of fixed comparison surface, suggesting light-based matching.
- No parameters in predicted fit …

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Summary

- Despite variation in scenes, information capacity remarkably stable.
- Reveals physical limit on observers' surface-color matching.
- Previous models of surface-color matching involve multiple empirically determined parameters.
- Minimum-variance principle requires only chromatic specification of stimuli: no free parameters.
- Variation in constancy and directional biases generally well reproduced °ı