SELF SUSTAINED ACTIVITY AND FAILURE IN A SMALL WORLD NETWORK OF EXCITABLE NEURONS

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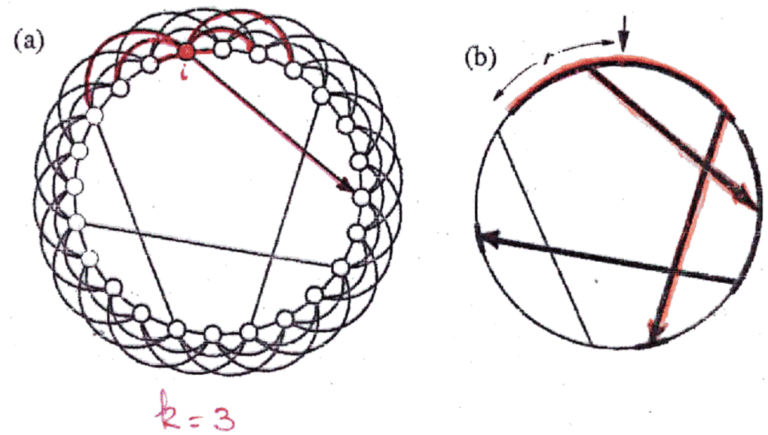
SMALL WORLD NETWORK

Local connectivity $W_{ij} = 1$ for $j = i \pm 1, i \pm 2, \ldots, i \pm k$

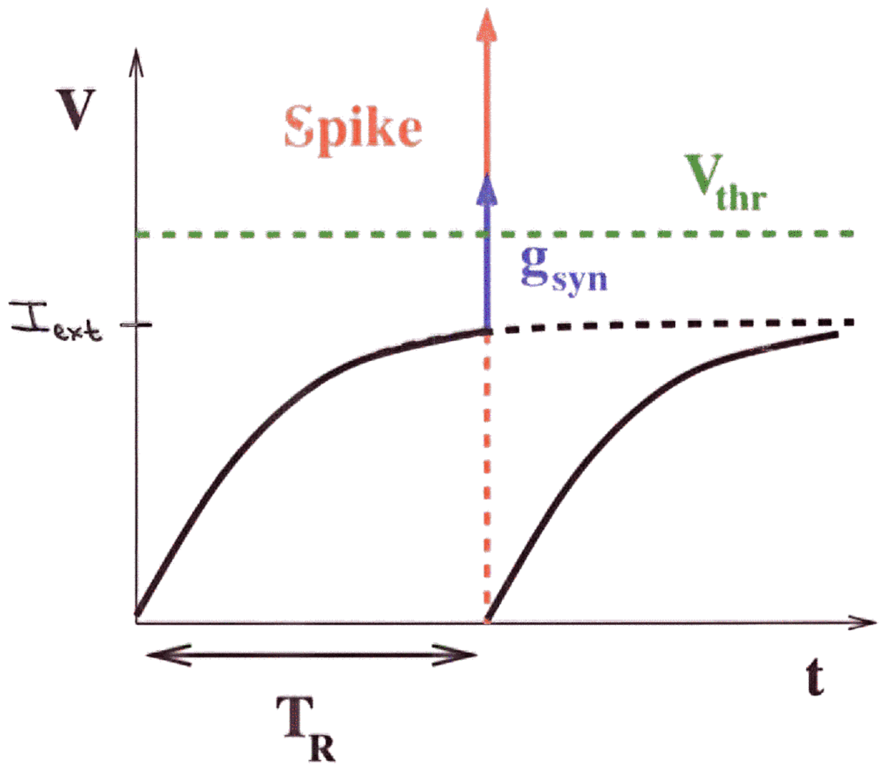
+ a fraction p of unidirectional couplings $W_{ij} = 1$
(an average number pN of shortcuts)



Shortcuts provide a means to transmit information efficiently around the network

Average shortest distance L between two nodes scales as $\ln N$

Integrate-and-Fire Neurons

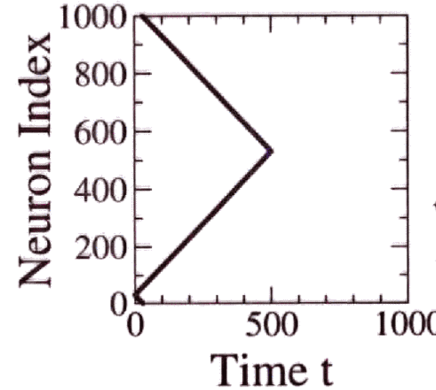


$$\tau_m \frac{dV_i}{dt} = -V_i + I_{ext} + g_{syn} \sum_{j\neq i} \sum_{m} w_{ij}\, \delta(t - t_j^{(m)} - \tau_D)$$

neuron j fires at time $t_j^{(m)}$ and provides input current to neuron i after a delay time τ_D

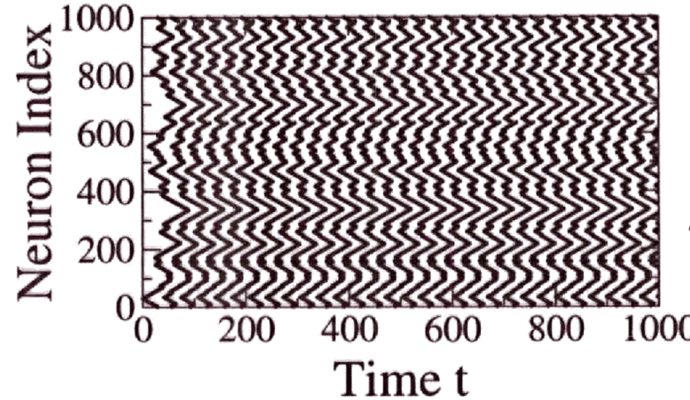
$I_{ext} < V_{thr}$ ⇒ excitable neuron

Fast Waves : $\tau_D = 1.0$



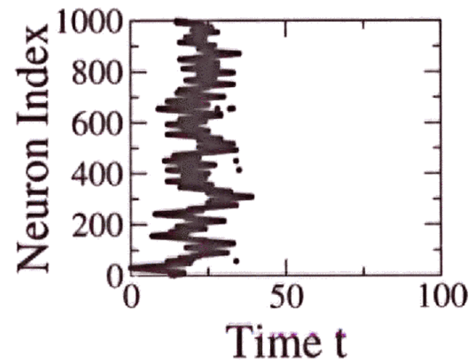
$p = 0$

Mutual annihilation leads to extinction



$p = 0.05$

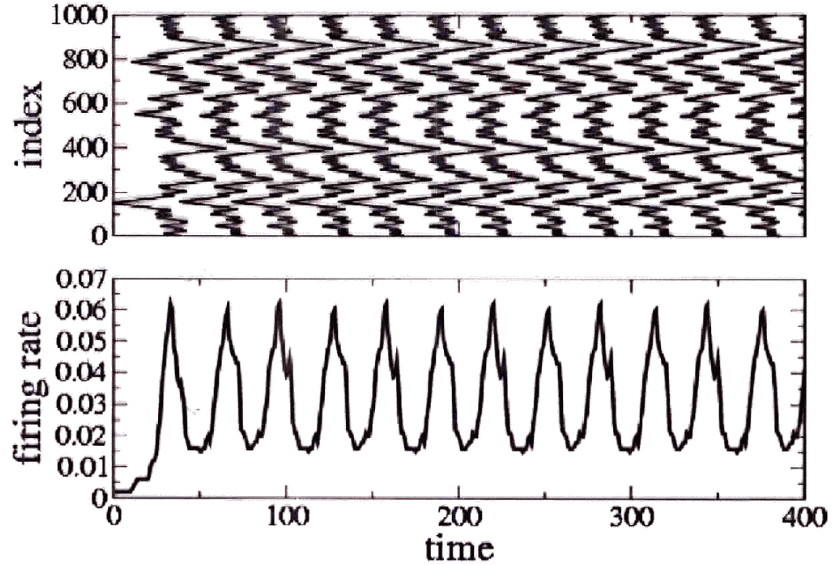
persistent activity



$p = 0.10$

failure

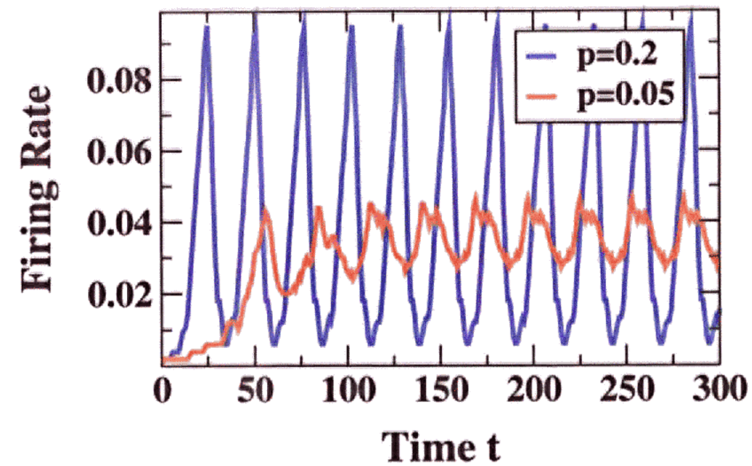
Persistent Activity

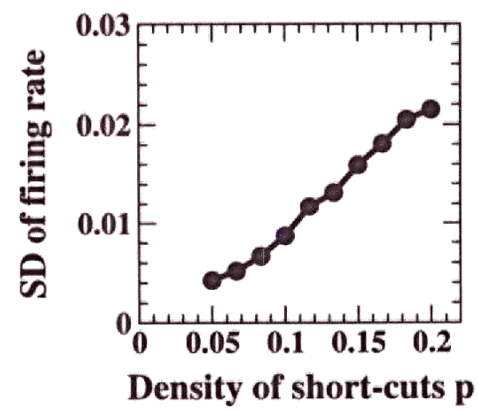


$N = 1000$, $p = 0.10$
$I_{ext} = 0.85$, $g_{syn} = 0.2$
$\tau_m = 10$, $\tau_D = 1$

Persistent activity due to reinjection of activity via a short cut into a previously active domain that has had time to recover. Rapid establishment of collective oscillations

Modulation of Firing Rates

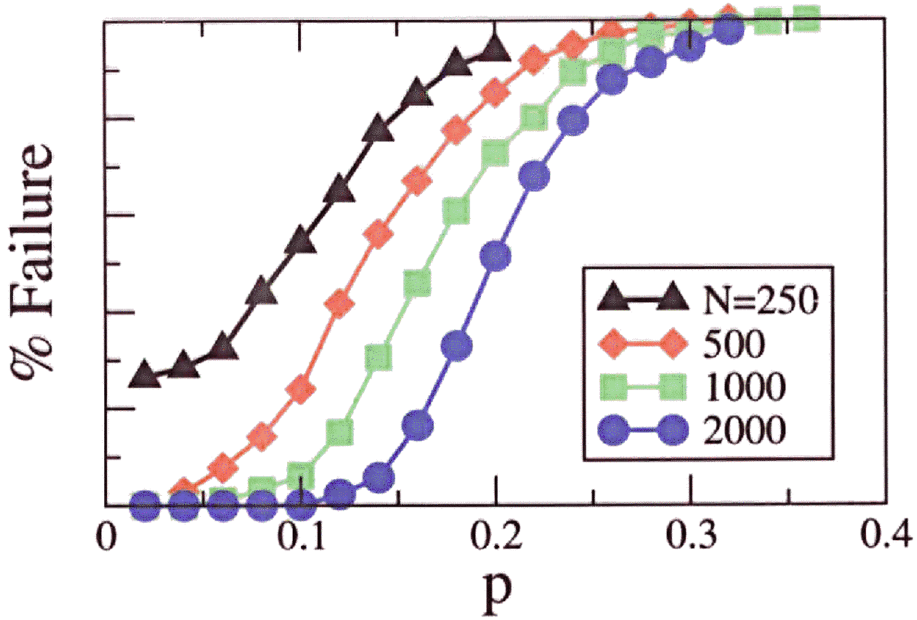




As p increases: faster establishment of oscillations, increasing oscillation amplitude, increasing recruitment of neurons into collective oscillation

BUT: increased probability of failure!

Failure Transition



Probability of failure is a monotonically increasing function of p; it crosses over from 0 to 1 with increasing steepness as the size N of the network increases.

Failure to Reinject

Failure to sustain activity due to the dynamics of neuronal recovery. After the emission of a spike, $V(t=t_{spike}^{+})=0$. Time is needed for the neuron to recover from the reset potential to a new potential such that $V+g_{syn} \geq V_{thr}=1$. A single synaptic input will be able to reinject activity and elicit a spike only if the elapsed time exceeds

$$T_R^{(1)} = \tau_m \ln \left[\frac{I_{ext}}{I_{ext}+g_{syn}-1} \right]$$

For $\tau_m=10$, $I_{ext}=0.85$, $g_{syn}=0.20$,

$$T_R^{(1)} = 28.3$$

Recurrence Time

What is the time needed for activity to spread accross the whole network, $T_A(p)$?

$$T_A(p) = \tau_D \, L_A(p)$$

Where $L_A(p)$ is the largest distance accross the network. This distance has been computed for bi directional shortcuts using a mean-field approach (Newman, Moore, and Watts, PRL 14, 3201 (2000)).

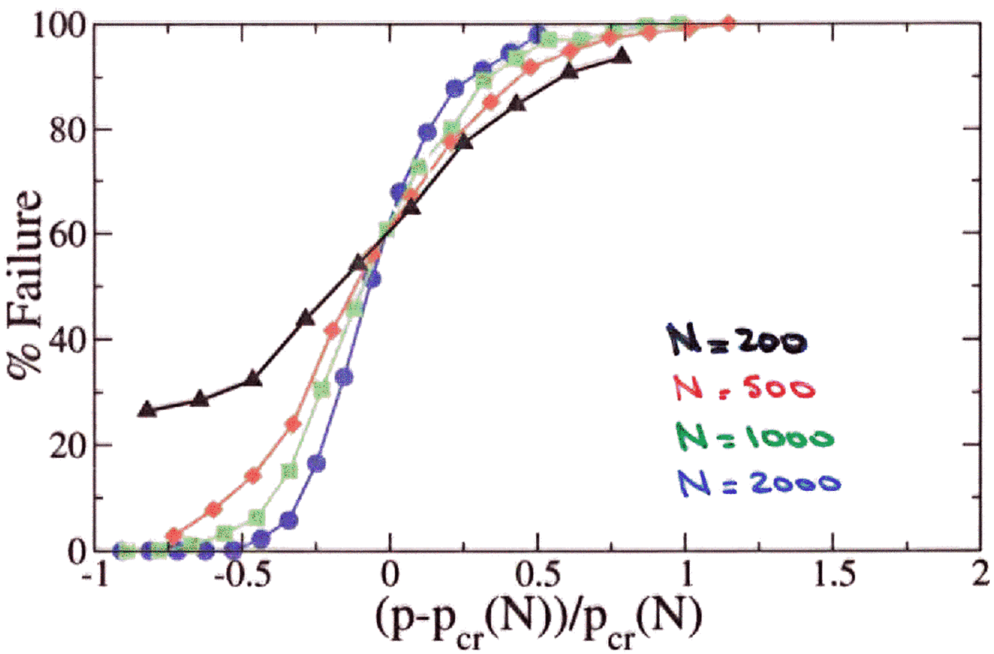
The mean-field theory can be extended to uni directional short cuts to obtain:

$$\left(1+\frac{4}{pN}\right)^{1/2} \tanh\left[\left(1+\frac{4}{pN}\right)^{1/2} \frac{p\,T_A(p)}{2\tau_D}\right] = 1$$

Then, $T_A(p) = T_R^{(1)}$

determines a critical concentration $p_{cr}(N)$ for failure to sustain activity by reinjection

Failure Transition

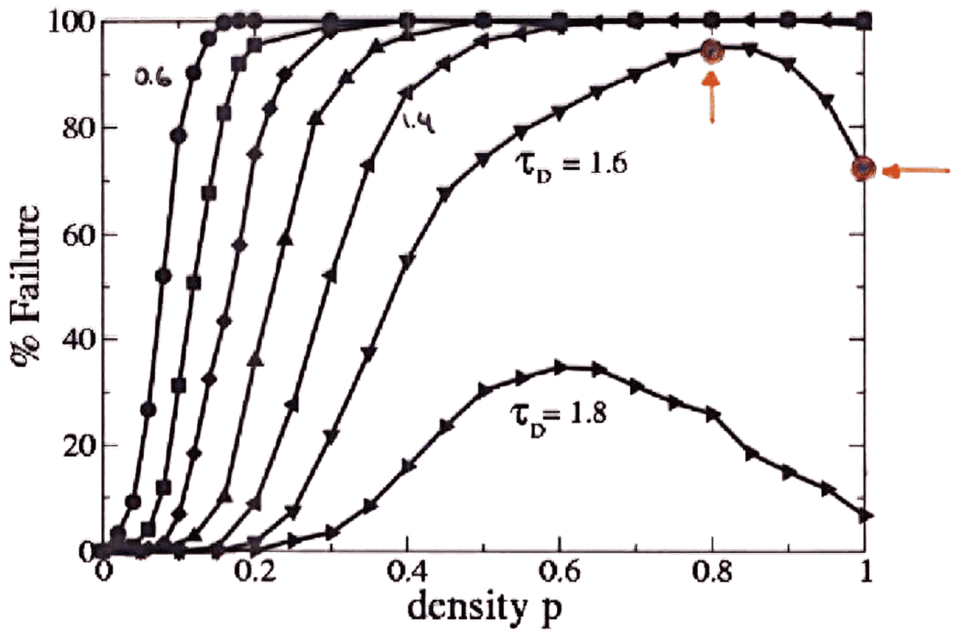


A well defined transition to failure as $N \to \infty$.

Note that $p_{cr}(N) \sim \ln N$.

Only for sufficiently fast waves, small τ_D

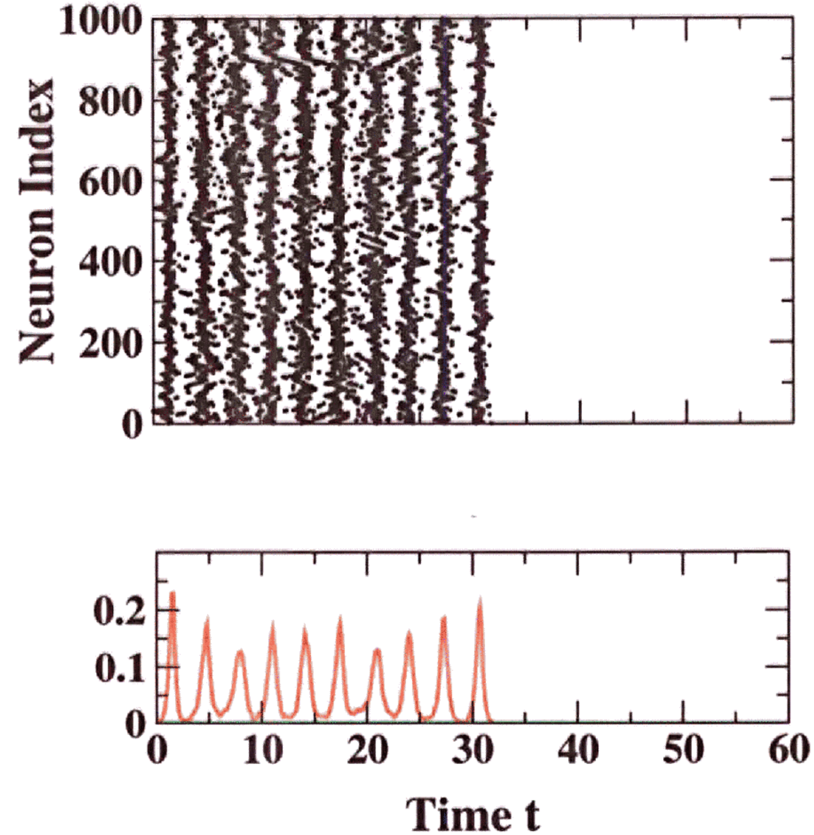
Slow Waves



N=1000. As τ_D increases, more complex network dynamics.

Reentrant phenomena: a nonmonotonic probability of failure with increasing p.

Slow Waves: Failure

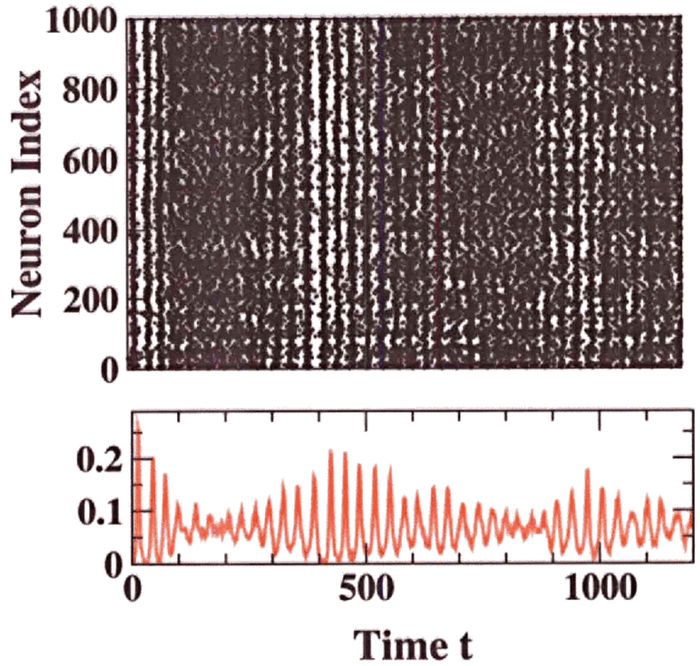


$\tau_D = 1.6$, $p = 0.8$

Regular activity, collective oscillations

Failure after a few cycles

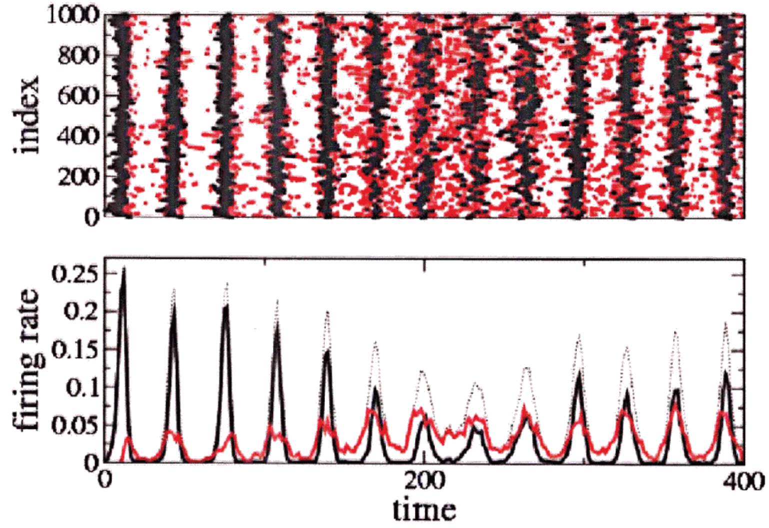
Slow Waves: Persistence



$\tau_D = 1.6$, $p = 1.0$

Irregular firing patterns, long transients

Quasiquiescent Epochs

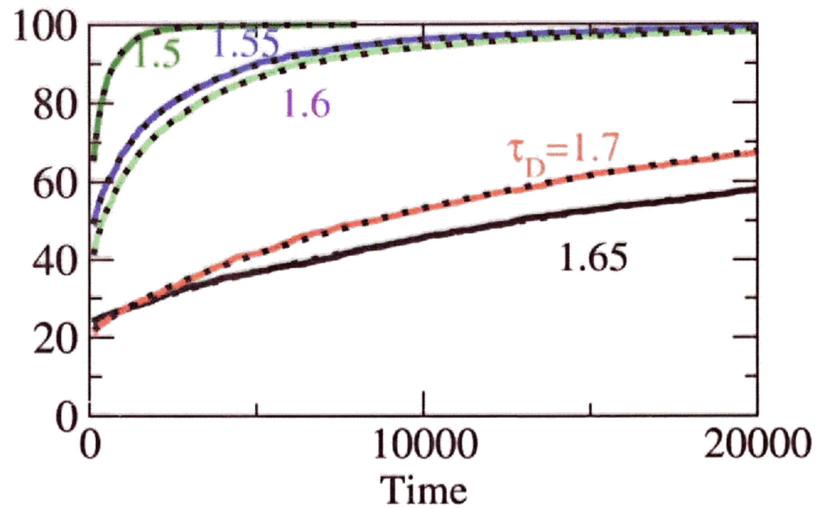


Neurons that receive n excitatory inputs during one cycle of network activity can have short recovery times:

$$T_R^{(n)} = \tau_m \ln \left[\frac{I_{ext}}{I_{ext} + n g_{syn} - 1} \right]$$

These neurons carry network activity across silent epochs of neurons that need $T_R^{(1)}$

FAILURE TIMES



Cumulative distribution at $p=1.0$

$1.5 \le \tau_D \le 1.7$

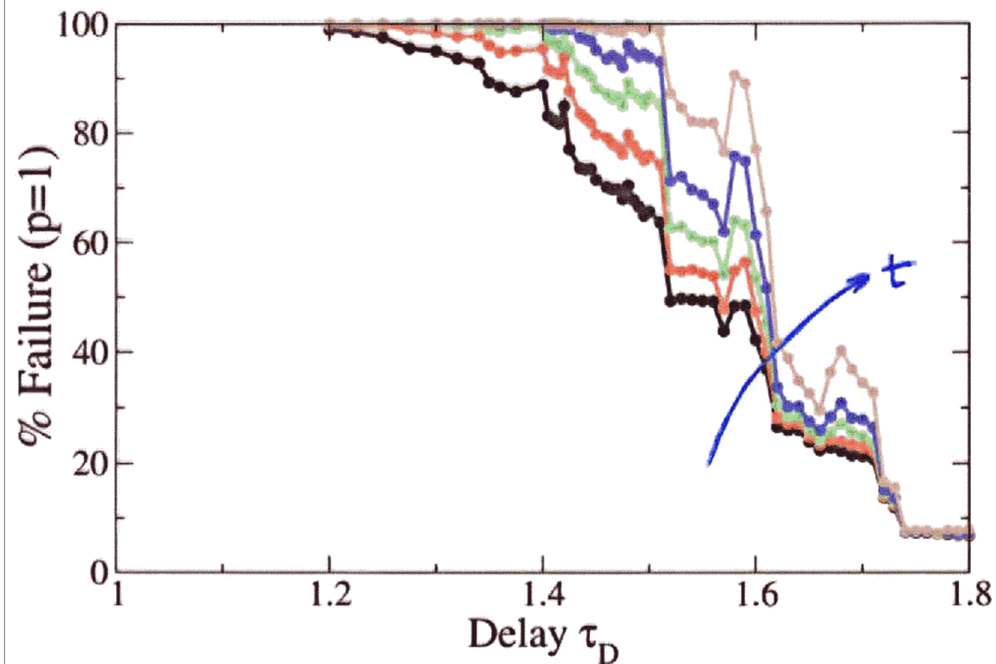
Fits a stretched exponential:

$$F(t) = f_\infty(\tau_D) - C e^{-\alpha t^\beta} , \quad \beta \approx 0.4$$

For $\tau_D = 1.65$, runs up to $t = 300000$ provide

$0.97 \le f_\infty(\tau_D = 1.65) \le 1.0$

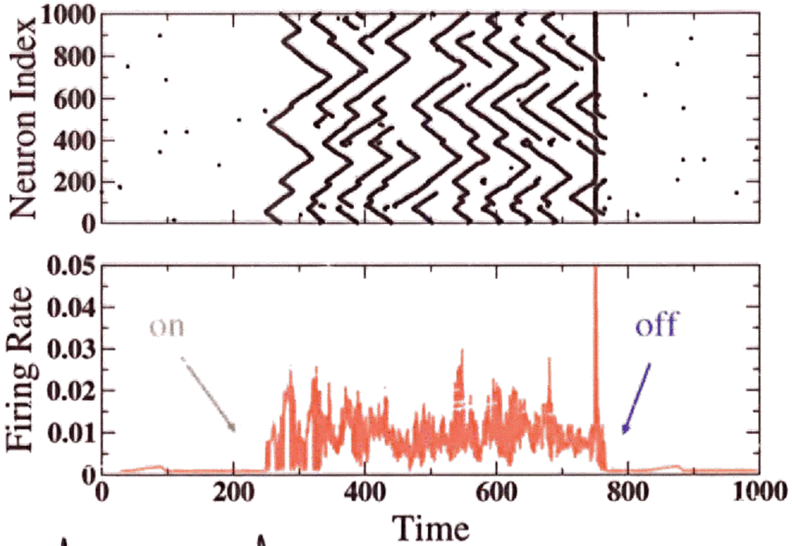
FAILURE RATES



$t = (5, 10, 20, 40, 100) \times T_R^{(1)}$

Failure rates $F(t, \tau_D)$ at fixed t exhibit complex structure as function of τ_D, suggestive of "resonances"

Bistability



$p = 0.10$, $k = 5$

The network is bistable between an "on" state of persisent, self-sustained activity, and an "off" quiet state. The simultaneous activation of a small number of neurons suffices to switch between them.

Summary

- A small-world network of integrate-and-fire neurons can sustain persistent activity.
- A low density of shortcuts provides a mechanism for reinjection of activity.
- Propagating pulses of activity are sustained by branching and reinjection.
- Network bistability provides a neural substrate for short-term "working" memory.

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