



# Learning by trial-and-error in the cerebellum

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# Learning and credit-assignment

Example: learning a complex movement  
(walking, riding a bike, skiing....)

A task generally composed of many  
coordinated sub-tasks but the error is often  
only global.

Many muscle involved, which muscle  
command should be modified to avoid  
falling?

A supplementary difficulty : the sign of the  
error is not known i.e. was one particular  
muscle too contracted or not enough?



## A possible solution : trial and error.

-During the task, perturb a few elements (cells, synapses,...)  
(...Doya and Sejnowski, 1995;...; Seung, 2004; Fiete et al, 2007;..)

-> Changes in the task performance can be attributed to the elements that were perturbed

An attractive framework that has been primarily considered in the context of song learning in birds but detailed implementation still to be worked out (how is the song evaluated, where are the sites of plasticity,...?)

Here : examine the question for learning complex coordinated movements at the level of the cerebellum.

# Learning by trial-and-error/stochastic gradient descent in the cerebellum?

The questions we would like to answer :

-During the task, a few cells should be perturbed ->

Question I : **what is the source of this perturbation?**

-Evaluation of the current performance needed to evaluate whether the perturbation has improved or deteriorated it ->

Question II : **where is current performance stored/how is the comparison made?**

Question III: **characteristics of the resulting learning algorithm (convergence, speed, ...)?**

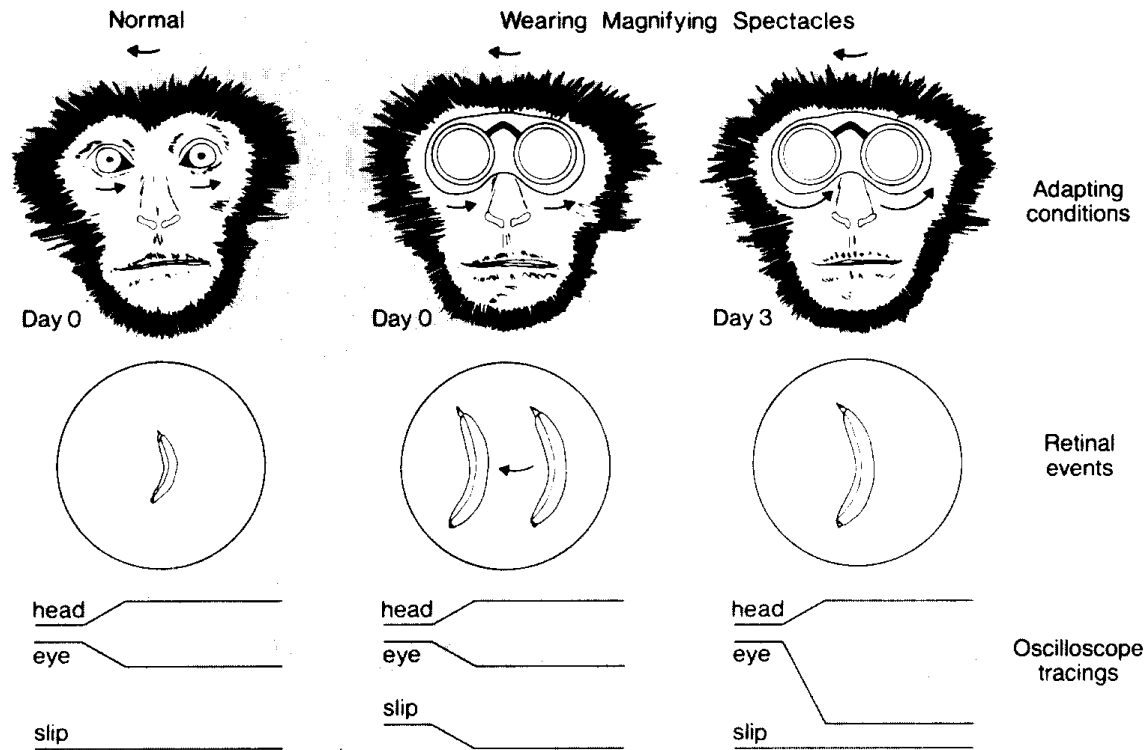
Question IV: **specific experimentally testable predictions?**

# Outline.

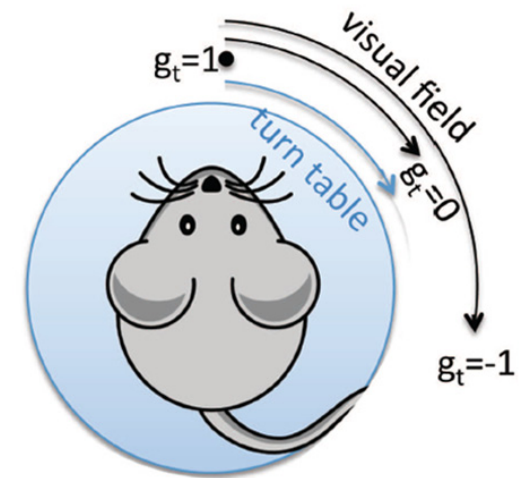
- Cerebellum and movement learning : cerebellar anatomy and the classical Marr-Albus theory
- A new proposal
- Some experimental results
- Simple mathematical implementations and analysis
- Further theoretical questions and experiments

# The cerebellum is a main site of motor learning

## Vestibulo-ocular reflex (VOR)



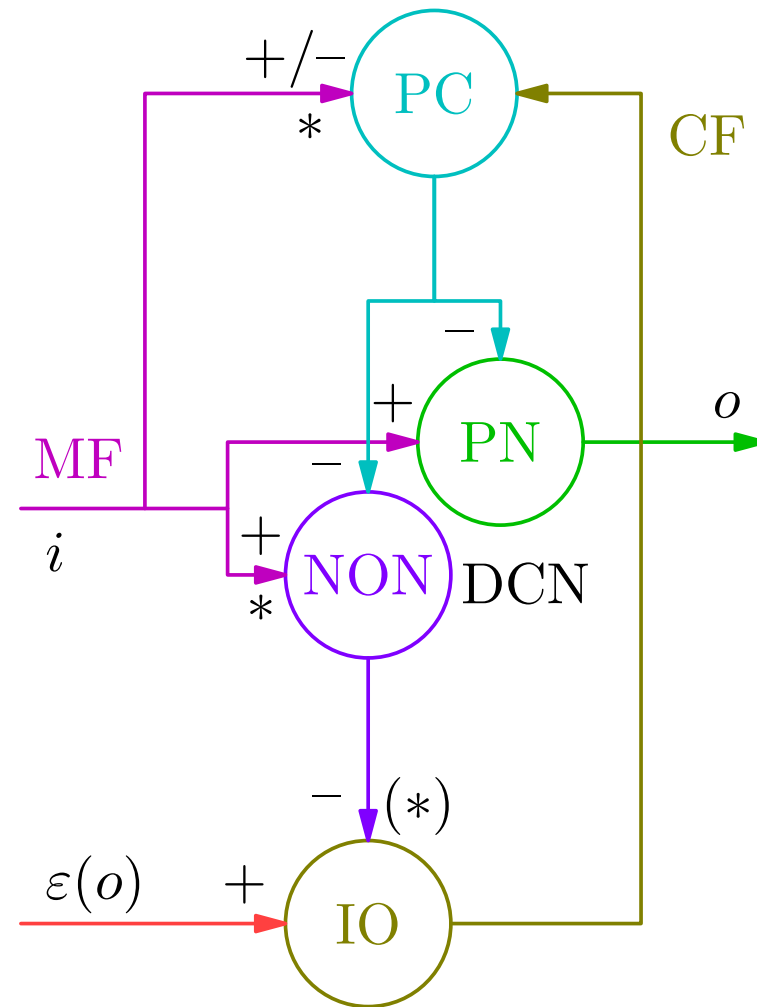
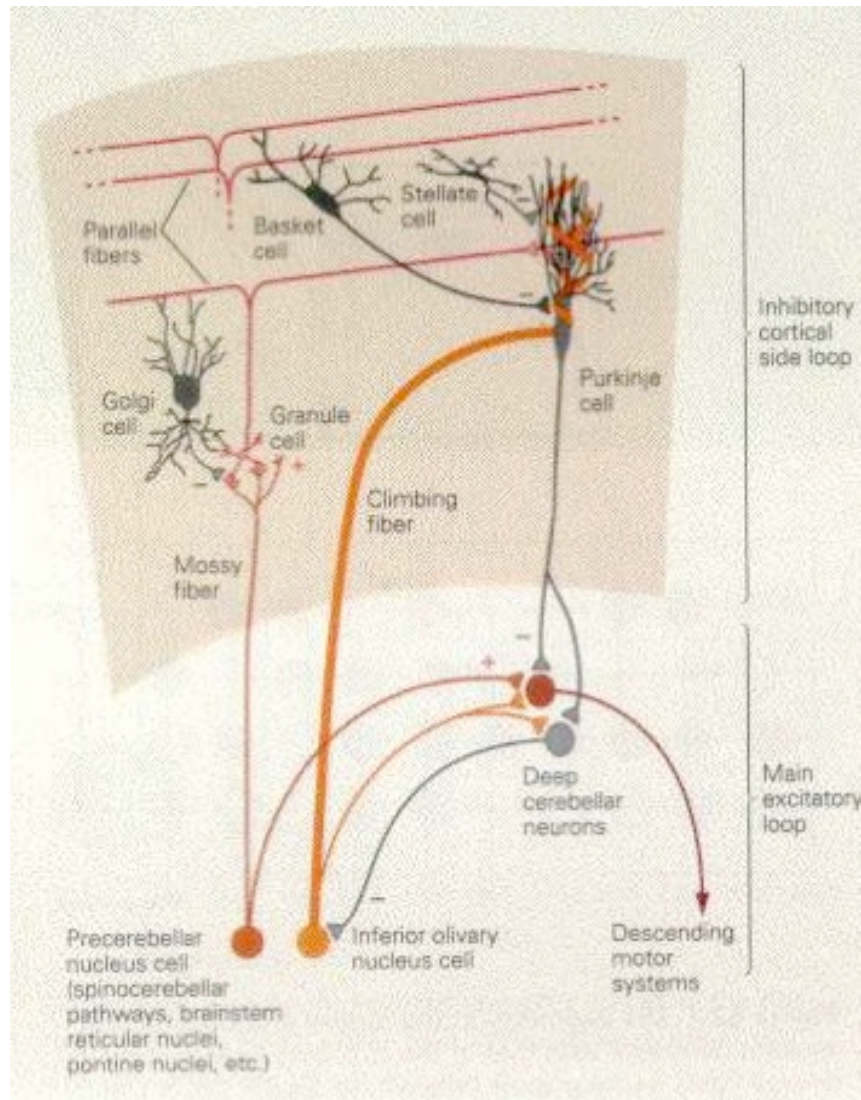
Lisberger, TINS 1988



De Zeeuw et al, Neuron (2010),  
...; Nat Rev Neur (2012)

Learning a gain change in the VOR is dependent on cerebellum

# Cerebellum circuitry

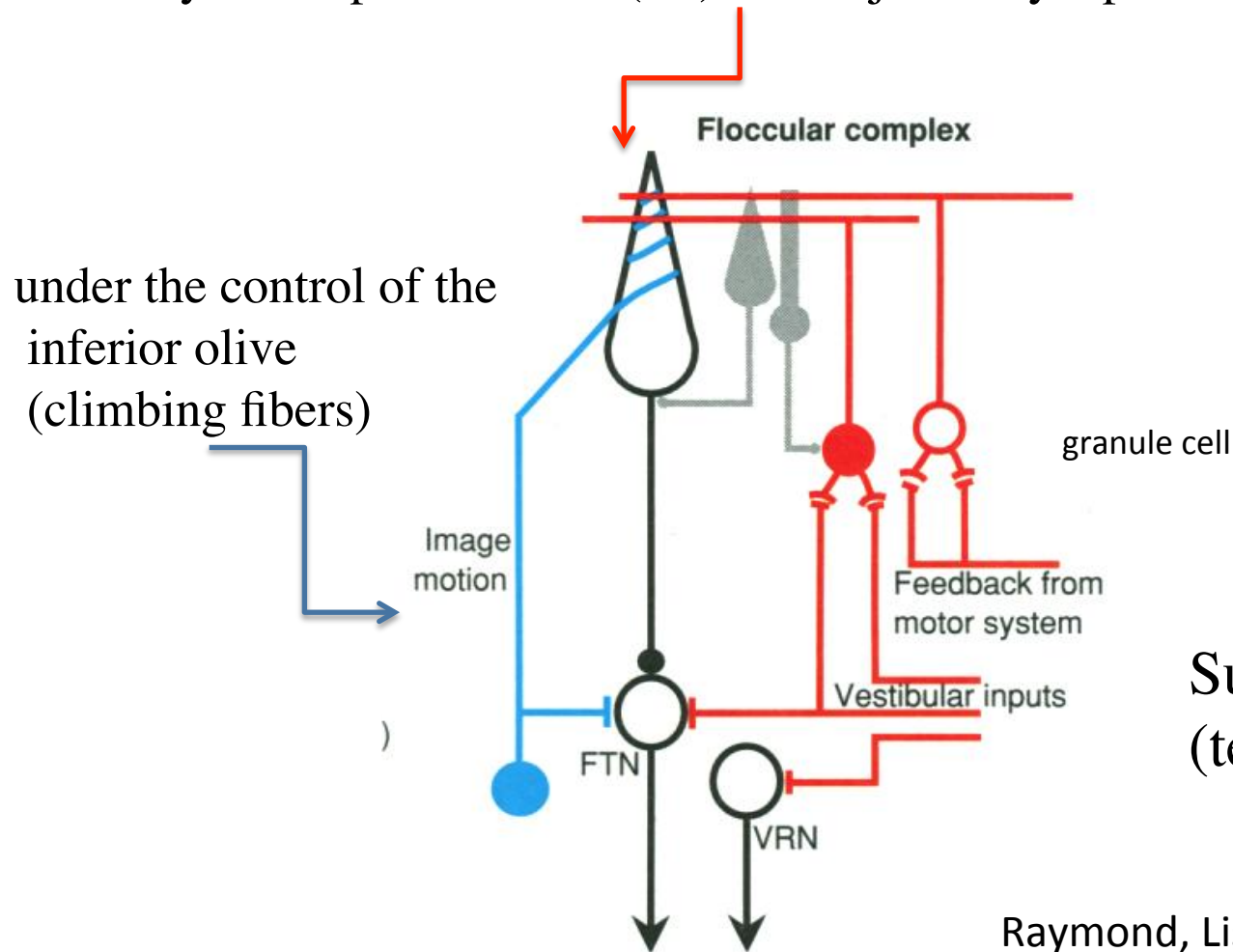


Two types of spikes in Purkinje cells (PC):

- **simple spikes** (SS) from granule cells/parallel fibers
- **complex spikes** (CS) from inferior olive/climbing fibers

# Classic theory of cerebellar learning (Marr-Albus-Ito):

Plasticity of the parallel fibre (PF)-Purkinje cell synapse



Supervised learning  
(teacher: climbing fiber)

Raymond, Lisberger, Mauk, Science (1996)



# Classic theory of cerebellar learning (Marr-Albus-Ito)

Climbing fibers : error signal -> depression of PF-PC synapse



Compensating potentiation : high frequency PF activity  
with no error signal

# Marr-Albus-Ito theory : some questions/difficulties

- In the theory of M-A-I, errors always lead to synaptic depression. Is it the case that errors always come from too strong synapses?
- the VOR is a simple task, sign of the error clear (given by retinal slip). For subtasks of complex movements less so, how can one determine if a synapse should be depressed or potentiated?
- For complex movements, how does the brain determine the precise source of error, the precise muscle that was not enough or too much contracted?

« Credit assignment problem » (Minsky, 1961)

The present proposal :

the cerebellum learns complex movements by performing stochastic gradient descent guided by a global estimate of the movement performance.

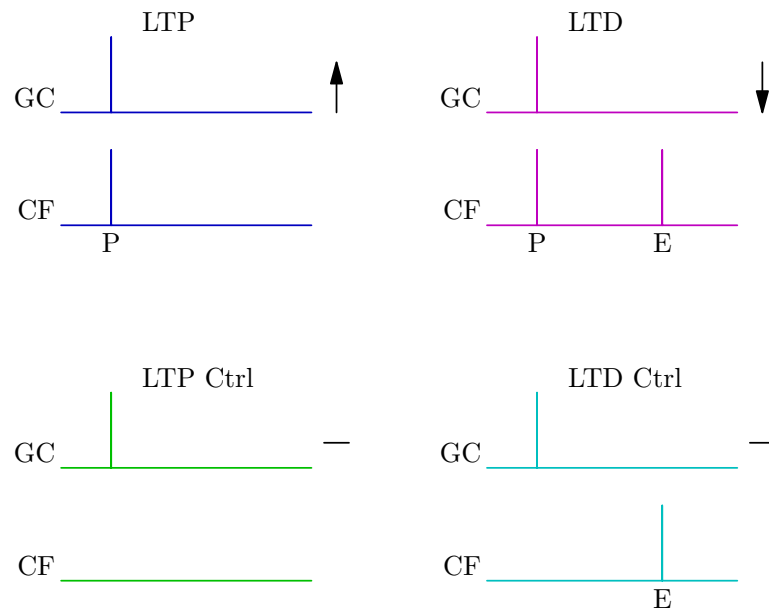
# Proposal I : perturbation source

Climbing fiber- induced **complex spikes** serve both as **perturbations** of movements and as a **signal errors** (as classically proposed)

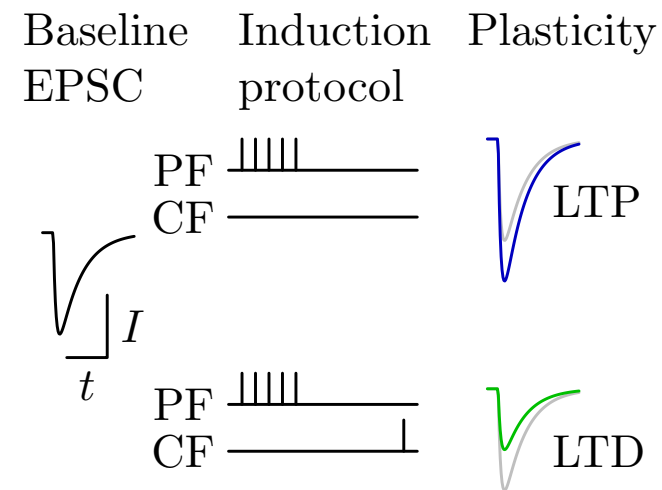
Olivary cells are spontaneously active at 1Hz

# Consequences for synaptic plasticity rules

## Predicted rules



## Classical theory/results



Change to standard protocols (to approach more physiological conditions) :

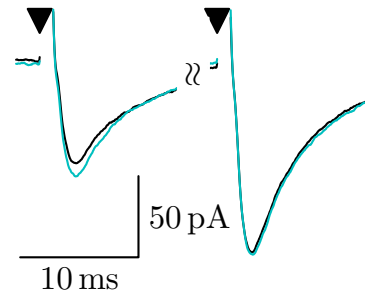
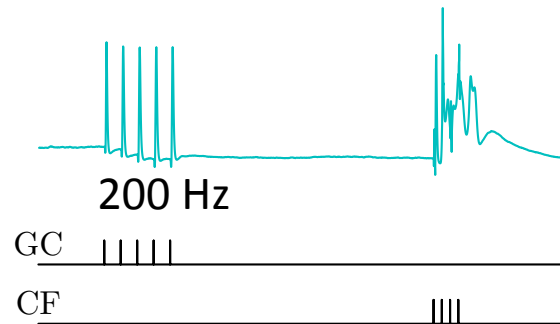
- lowered extracellular calcium (2mM->1.5mM)
- Weak stimulation of granule cell layer (instead of parallel fibers)
- inhibition not blocked

# Plasticity protocol

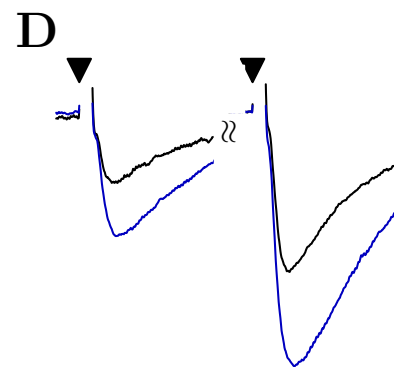
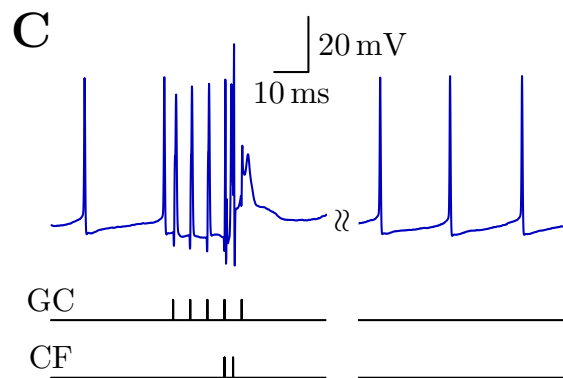
Test (pair of PF pulses before and after stimulation)

repeated every 2s between 10mn i.e. 300 times

No change



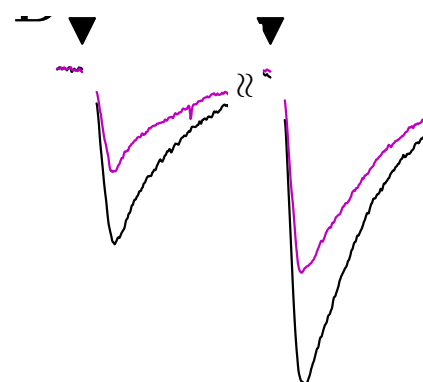
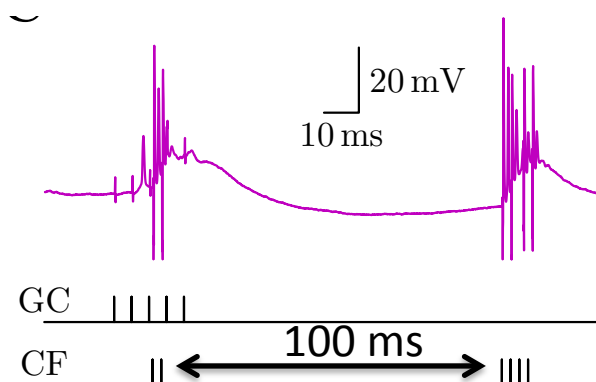
paired-pulse stimuli  
(50ms interval)



before

Potentiation

after



after

Depression

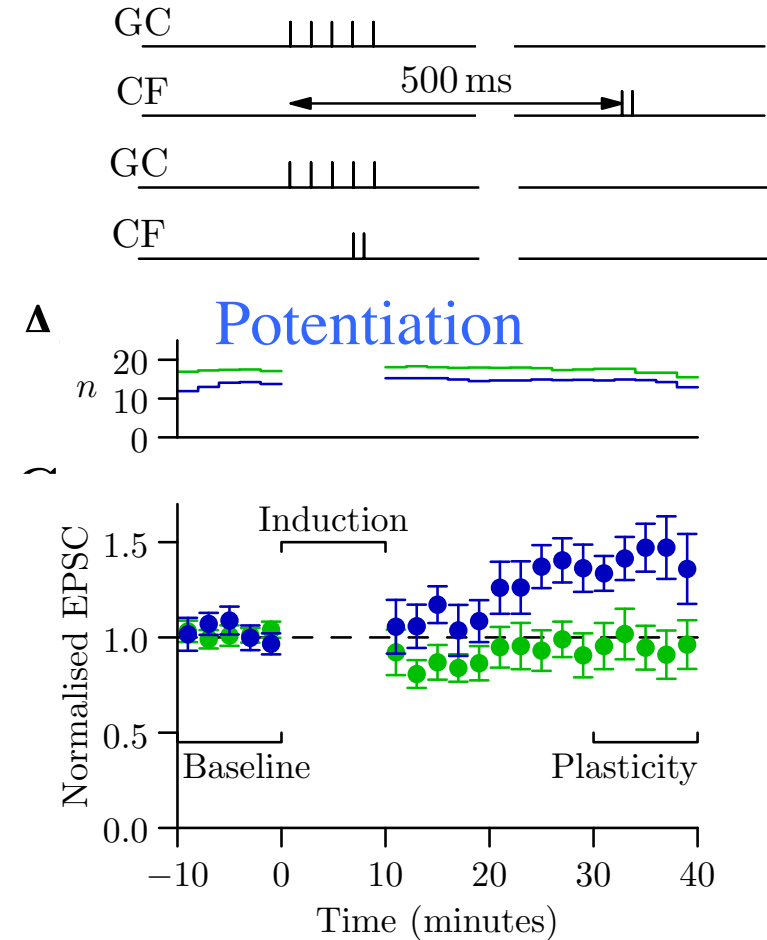
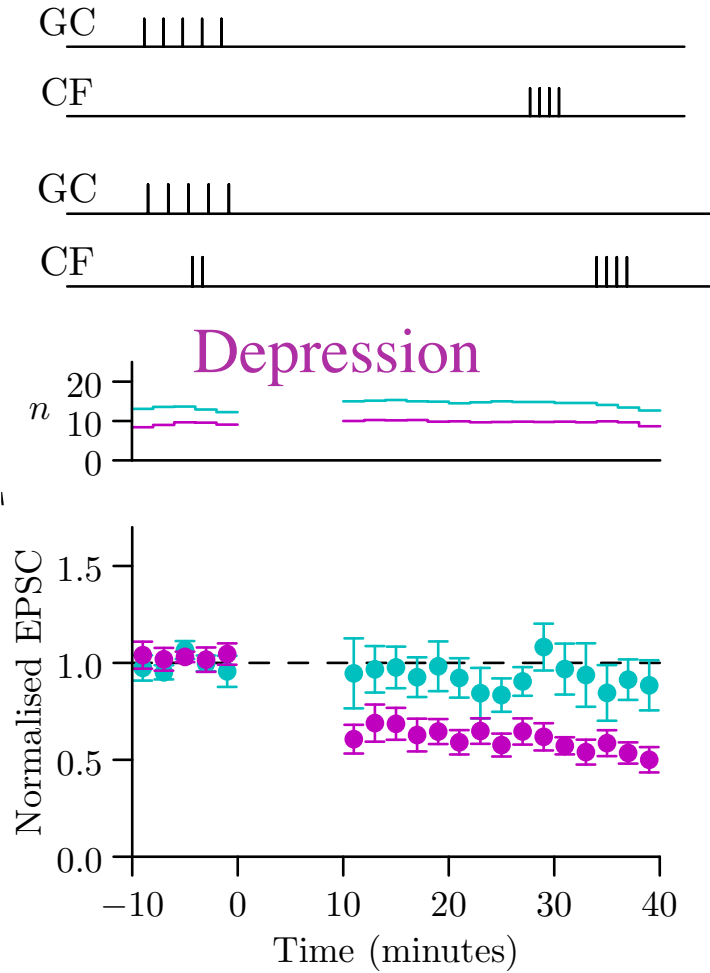
before

Guy Bouvier

# Plasticity results summary

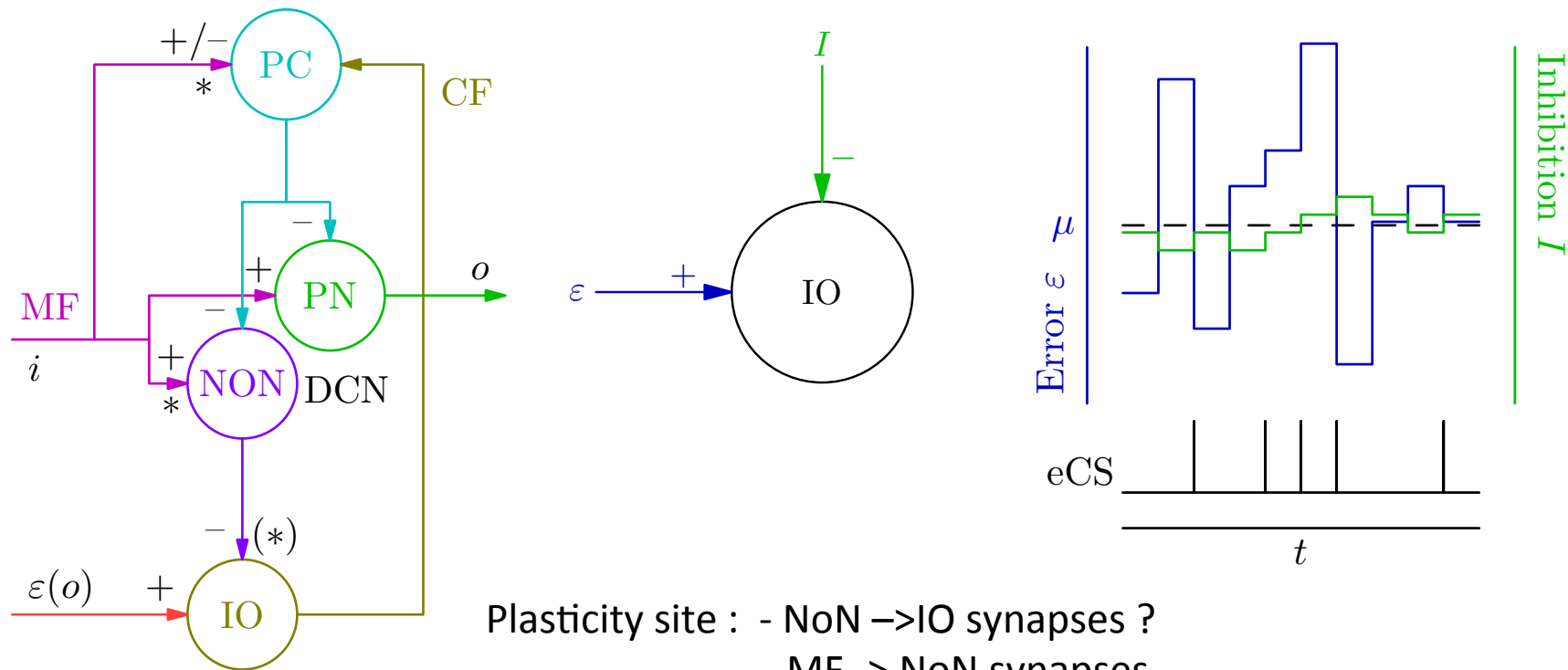
Guy Bouvier

Controls



# Proposal II : storage of current performance/ comparison with perturbed task

Proposal: the level of current performance is measured by the level of inhibition of olive neurones provided by NoN neurones



Plasticity site : - NoN  $\rightarrow$  IO synapses ?  
- MF  $\rightarrow$  NoN synapses  
(related plasticity observed at MF  $\rightarrow$  PN synapses)



# A toy model

N cells with firing rates  $P_1, \dots, P_N$   
Successful task firing rates  $T_1, \dots, T_N$

Mean error

$$E(t) = \frac{1}{N} \sum_i |P_i(t) - T_i|$$

Internal estimated value of the error : I

Learning task : bring the  $P_i$ 's to their target values  $T_i$ 's

# A toy model

Learning algorithm (I):

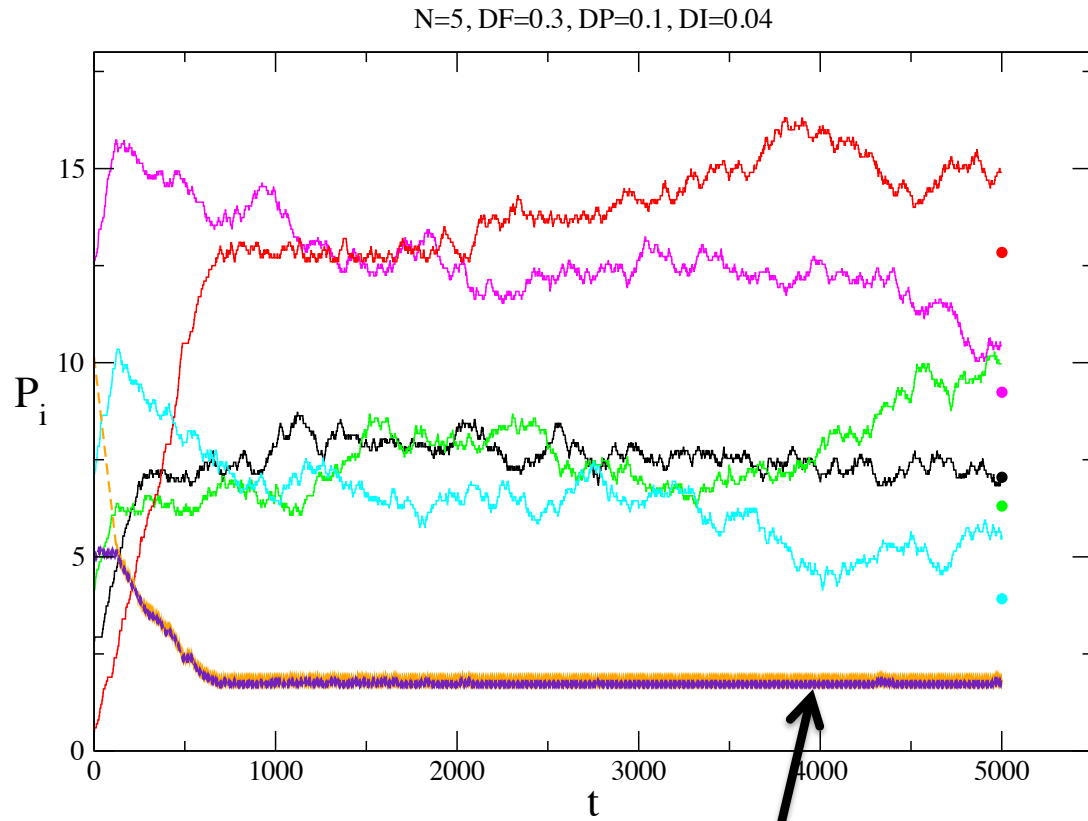
- choose randomly one of the rate  $i_c$  and perturb it by  $A > 0$ :  $P_{i_c} = P_{i_c} + A$
- Error with perturbation

$$E_p = \frac{1}{N} \sum_i |P_i(t) + \delta_{i,i_c} A - T_i|$$

-compare  $E_p$  estimated value of the error  $I$

If  $E_p < I$  good to increase the rate, modify  $P_{i_c} \rightarrow P_{i_c} + \Delta P$ ,  
update error estimation  $I \rightarrow I - \Delta I$

If  $E_p > I$  perturbation is bad, decrease  $P_{i_c} \rightarrow P_{i_c} - \Delta P$ ,  
 $I \rightarrow I + \Delta I$



With these moves only, conservation of:

$$C = I(t) \Delta P + \Delta I \sum_i P_i(t) \quad \text{cannot converge!}$$

## Learning algorithm (II)

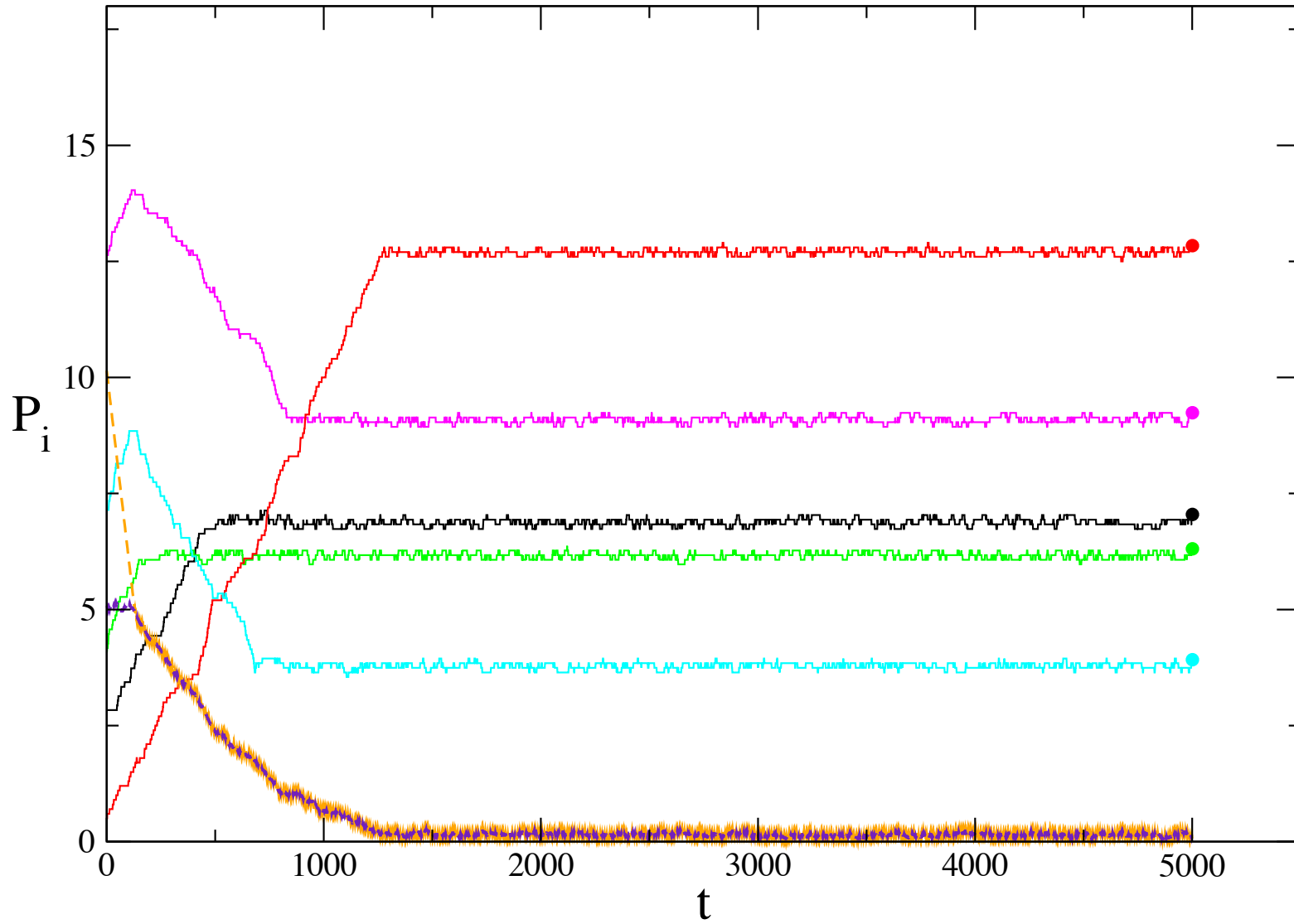
Remedy :

- make the previous moves with perturbation (« type A » ) with probability  $\rho$
- Make moves with no perturbation and just error estimation adjustment (« Type B » ) with probability  $1-\rho$

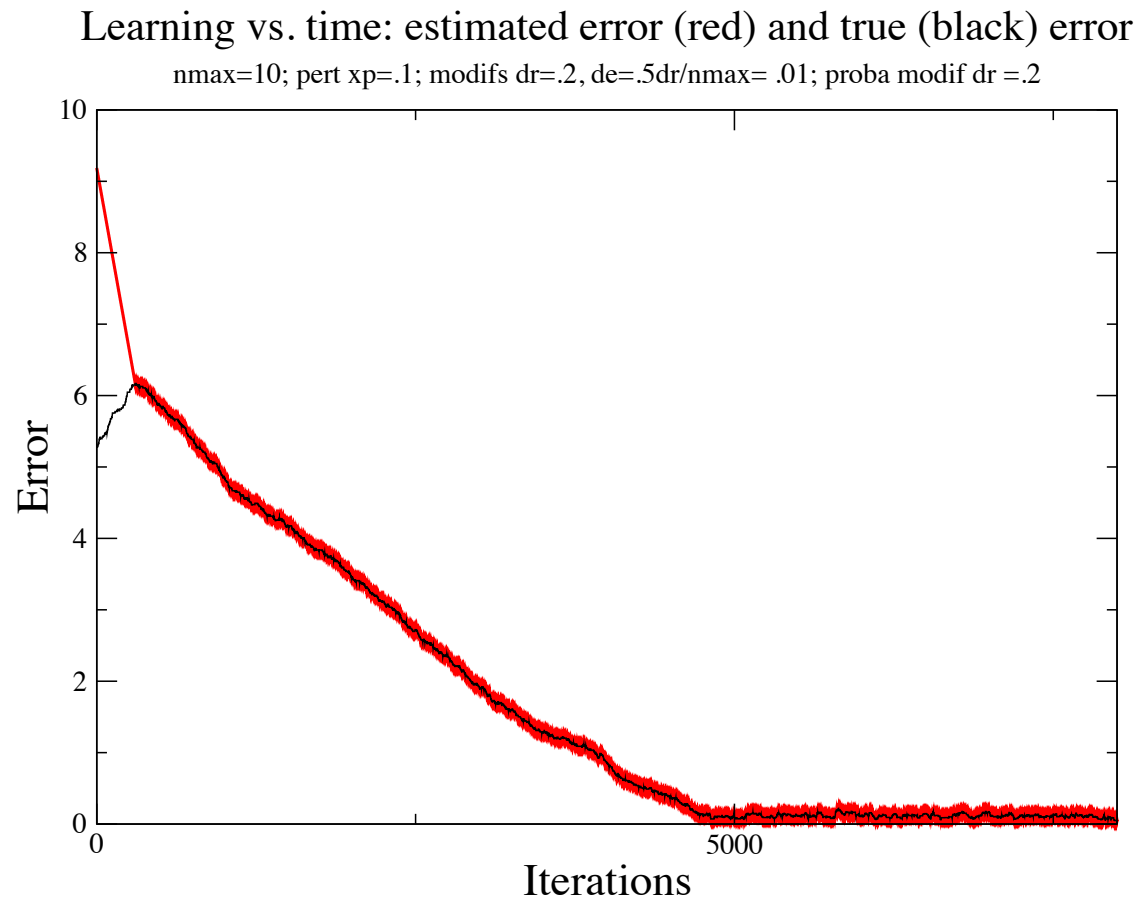
« Type B » moves:

If  $E < I$  then  $I \rightarrow I - \Delta I$   
If  $E > I$  then  $I \rightarrow I + \Delta I$

$N=5, DF=0.3, DP=0.1, DI=0.04$



# Toy model : convergence



Three successive phases in the learning dynamics

Role of the different parameters? convergence speed?

# The one cell case

One pattern, cell fires with **rate P**, desired firing rate T.  
Current **inhibition** on olive **I**

Two types of trials :

-**Perturbation** with probability  $\rho$ ,  $P \rightarrow P + \Delta$ ,  $E = |P + \Delta - T|$

If  $E > I$ , error CS,  $P \rightarrow P - \Delta P$ ,  $I \rightarrow I + \Delta I$

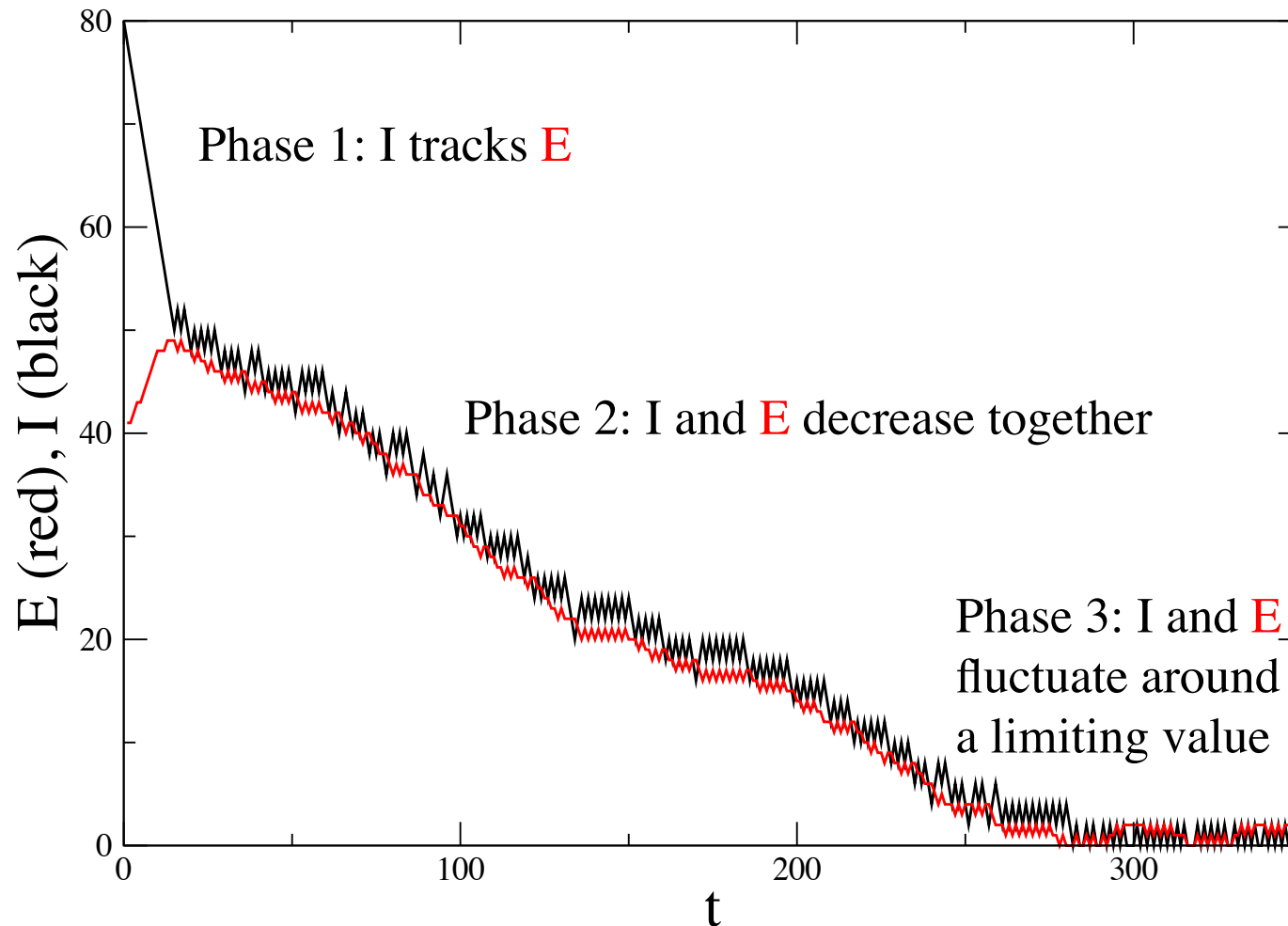
$E < I$ , no error CS,  $P \rightarrow P + \Delta P$ ,  $I \rightarrow I - \Delta I$

-**No perturbation** with probability  $1 - \rho$ ,  $E = |P - T|$

If  $E > I$ , error CS,  $I \rightarrow I + \Delta I$  (P unchanged)

$E < I$ , no error CS,  $I \rightarrow I + \Delta I$  (P unchanged)

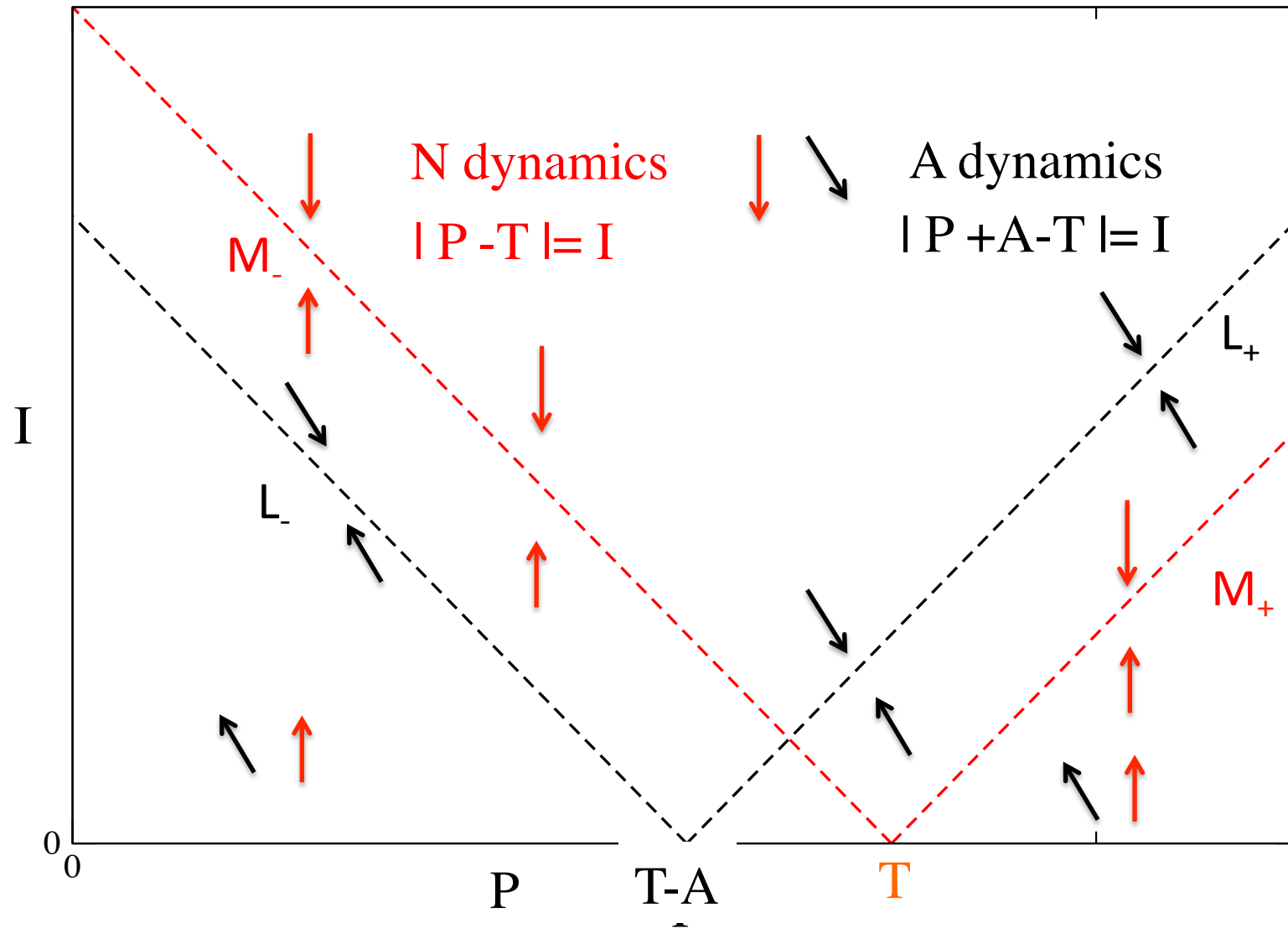
# Convergence in 3 phases



- Origin of the 3 successive phases?
- Role of the different parameters?
- convergence speed?



# One cell (P, I) phase-plane dynamics



- Two convergence « corridors »

# Different parameter regimes

$A, \rho, \Delta P, \Delta I$

(large perturbation  $A \gg \Delta P, \Delta I$ )

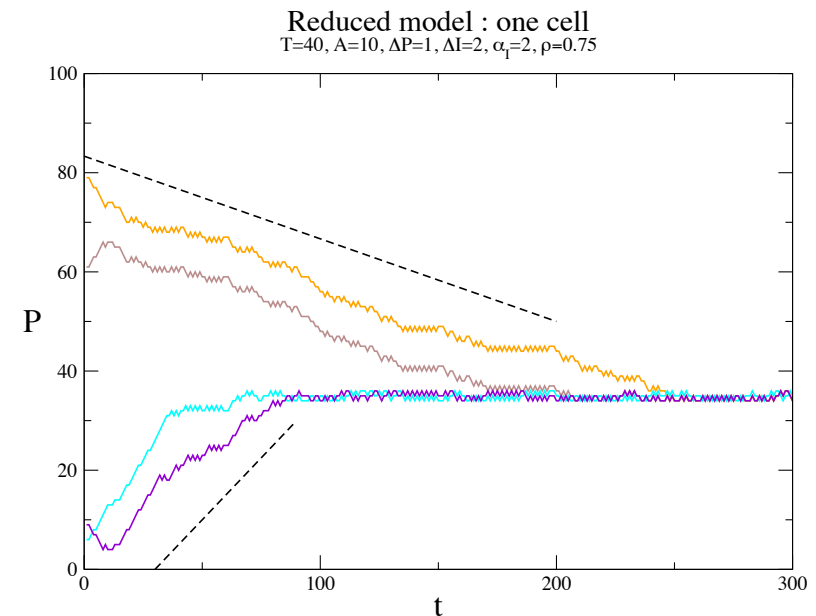
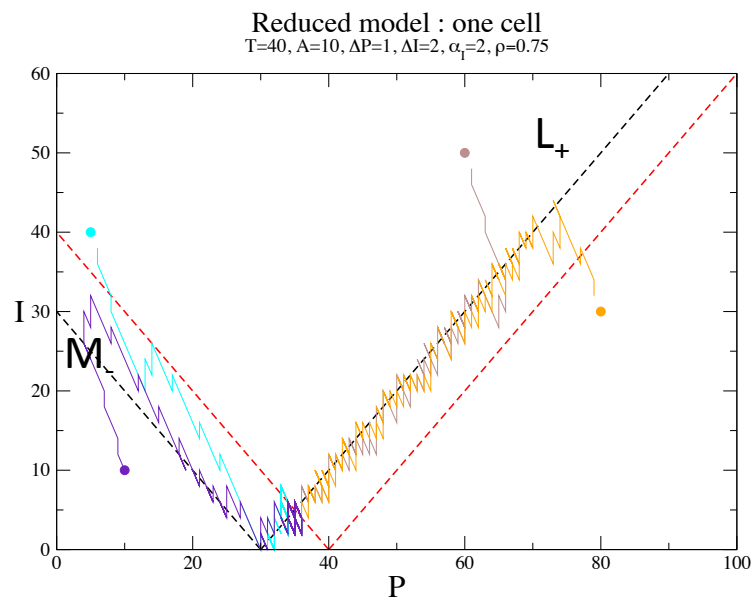
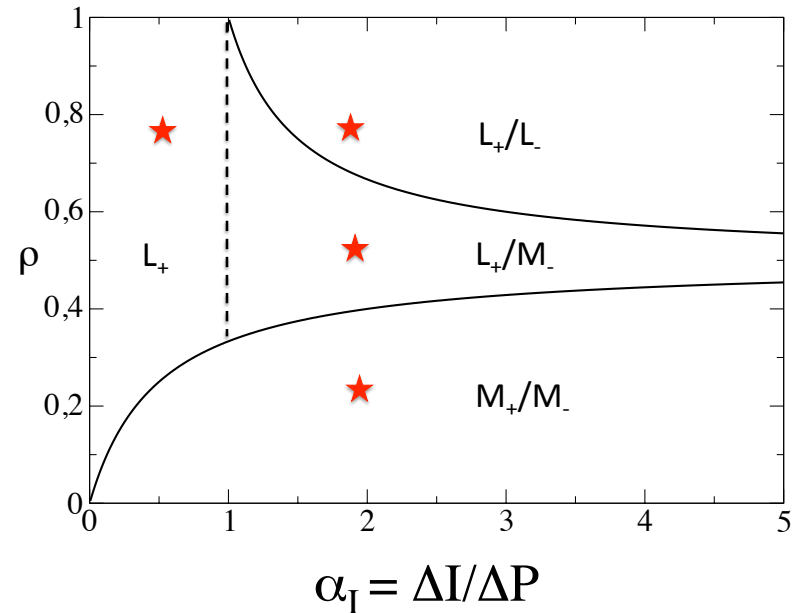
Convergence speed

$$L_+ : v_c = -\frac{1 - \rho}{1 + \alpha_I} \Delta I$$

Dominated by error estimation

$$M_- : v_c = \rho \Delta P$$

Dominated by rate update



# Simple mathematical description

- Purkinje cell as an analog perceptron

$g_i^\mu, i=1, \dots, N_g$  inputs 0,1 with prob.  $f$

$\mu=1, \dots, N_p$  patterns, target rates  $T^\mu$

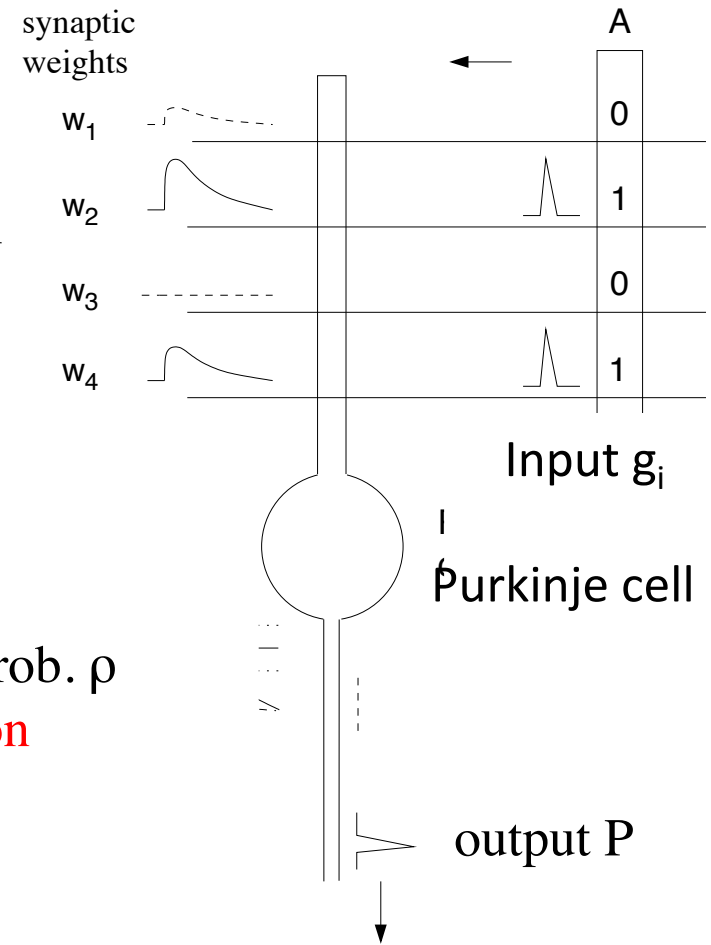
$$P = \left[ \frac{1}{\sqrt{N_g}} \left( \sum_i w_i g_i^\mu - \theta N_g \right) \right]_+ + \eta A$$

$\eta=0,1$  with prob.  $\rho$   
**A perturbation**

$$E = |P - T^\mu| \quad \text{Error for pattern } \mu$$

- Input to DCN

$$D = E - I, \quad \text{with } I = \left[ \frac{1}{\sqrt{N_g}} \left( \sum_i q_i g_i^\mu - \theta N_g \right) \right]_+$$



## Perceptron plasticity rules

$$P = \left[ \frac{1}{\sqrt{N_g}} \left( \sum_i w_i g_i^\mu - \theta N_g \right) \right]_+ + \eta A \quad \eta=0,1 \text{ with prob. } \rho \text{ perturbation } A$$

$$D = E - I, \quad \text{with } I = \left[ \frac{1}{\sqrt{N_g}} \left( \sum_i q_i g_i^\mu - \theta N_g \right) \right]_+$$

$$E = |P - T^\mu| \quad \text{Error for pattern } \mu$$

### Synaptic weight changes :

$$c = \text{sign}(E - I) \quad \text{climbing fiber discharge/presence or absence of perturbation CS}$$

$$w_i \rightarrow [w_i - \alpha_w c \eta g_i^\mu]_+$$

$$q_i \rightarrow [q_i + \alpha_q c g_i^\mu]_+$$

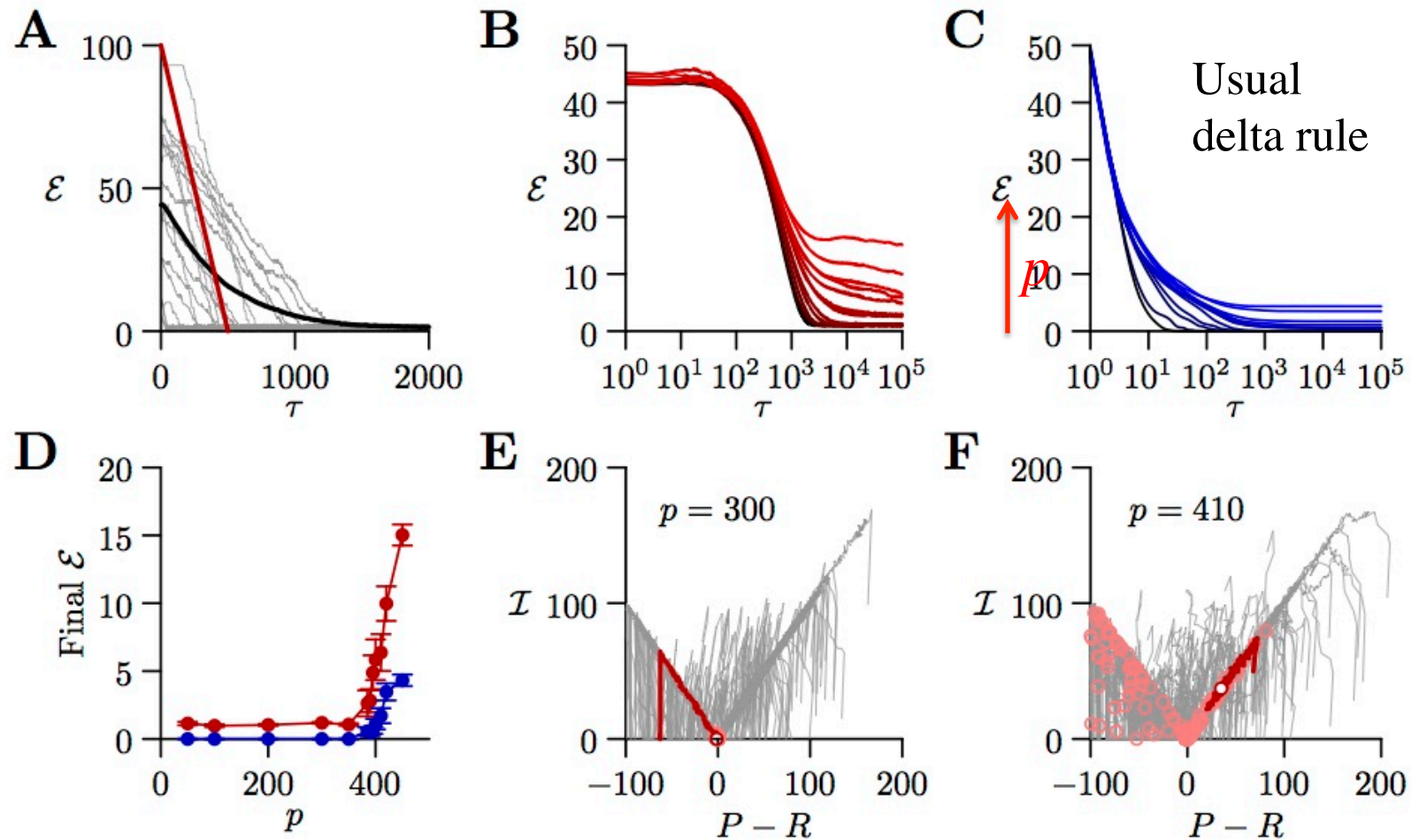
Relation to toy model

$$\Delta P = \alpha_w f \sqrt{N_g}$$

$$\Delta I = \alpha_q f \sqrt{N_g}$$

# Perceptron : learning $p$ patterns with estimated error

$N=1000$ , coding fraction  $f=0.2$

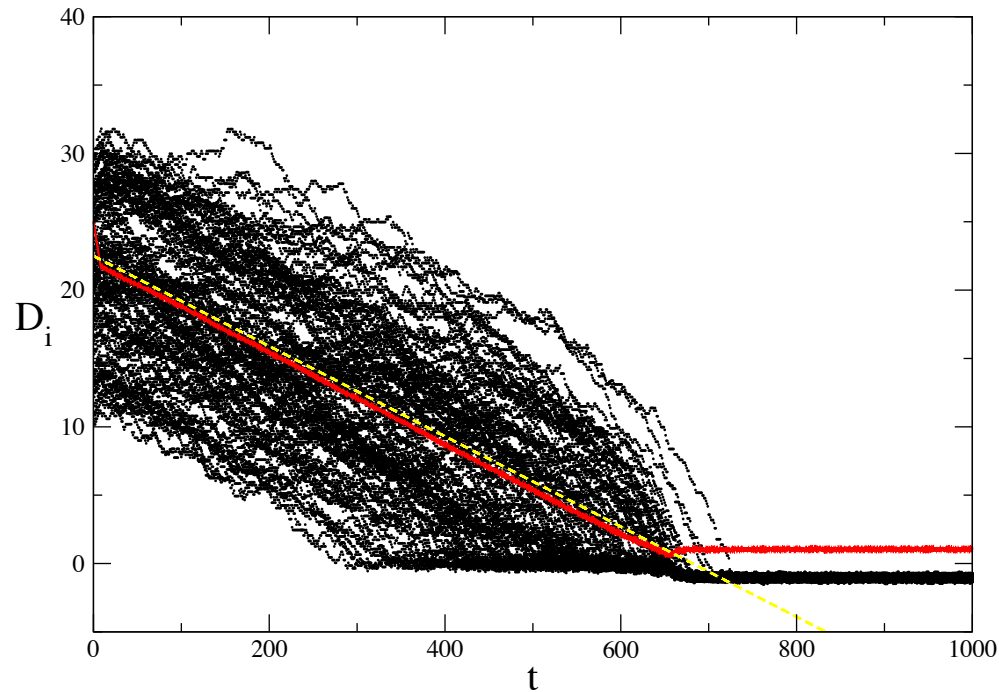


N cells, estimated **global** error

# Many cells

Reduced model : multiple cells

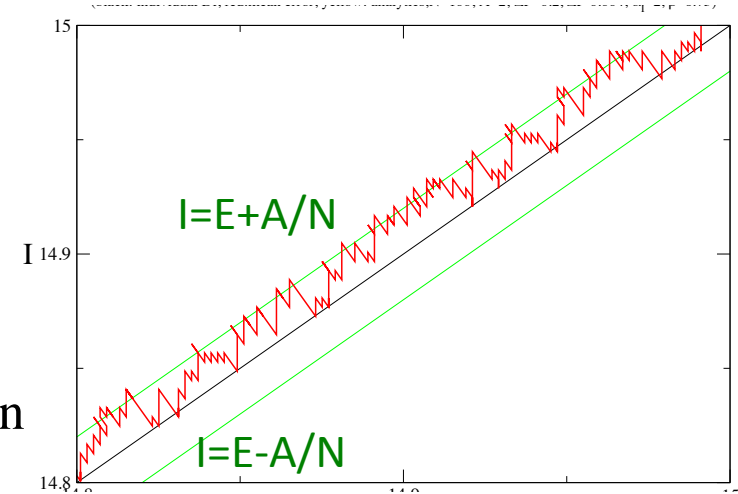
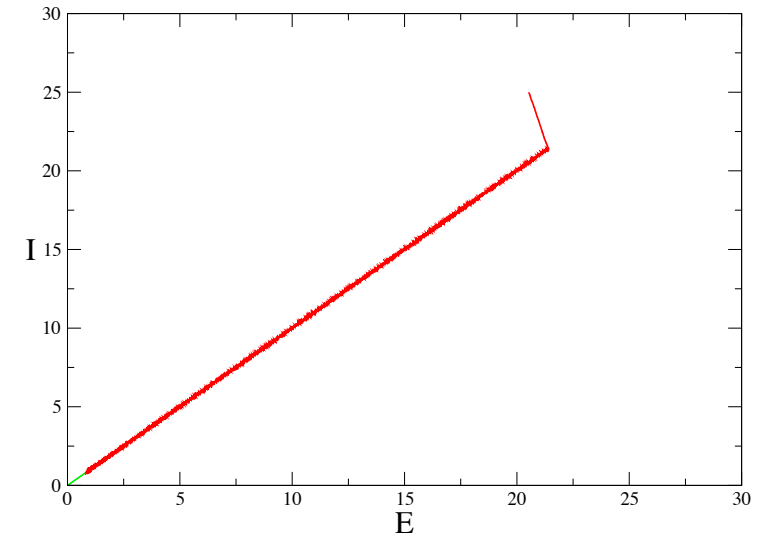
(black: individual  $D_i$ , red: mean error, yellow: analytics;  $N=100$ ,  $A=2$ ,  $\Delta P=0.2$ ,  $\Delta I=0.004$ ,  $\alpha_i=2$ ,  $\rho=0.75$ )



Generalized analysis possible, cells successively reach their limiting rates convergence curve depend on initial condition

Reduced model : multiple cells

(black: individual  $D_i$ , red: mean error, yellow: analytics;  $N=100$ ,  $A=2$ ,  $\Delta P=0.2$ ,  $\Delta I=0.004$ ,  $\alpha_i=2$ ,  $\rho=0.75$ )



What's next ?

**Theory :**

- convergence dynamics with 'interfering' patterns
- different forms of global error

**Experiment :** in vivo data

- evidence from perturbation during movement
- extraction of plasticity rules from data

**Both :** - Other structures where a similar type of learning may apply? (e.g. basal ganglia; dopamine release signals reward but also promotes movement initiation, plasticity rules involving this two successive releases?).



**Thank you!**

Nicolas Brunel

Claudia Clopath

Jean-Pierre Nadal

Yonatan Aljadeff

Jonas Ranft



**Boris Barbour**



**Guy Bouvier**

Celian Bimbard