Evidence for an Unfolded Border-collision Bifurcation in Paced Cardiac Tissue

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Joint work with
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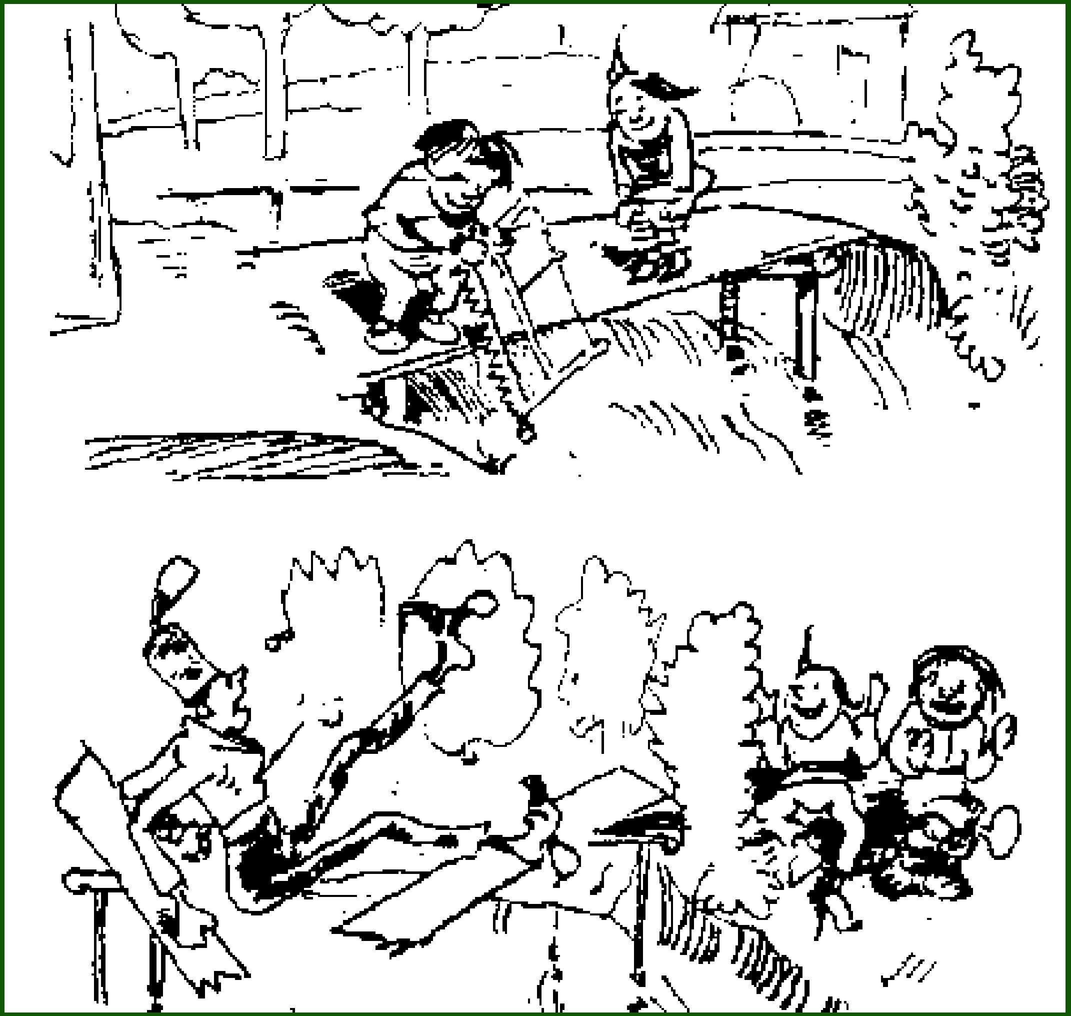
What Is Bifurcation?



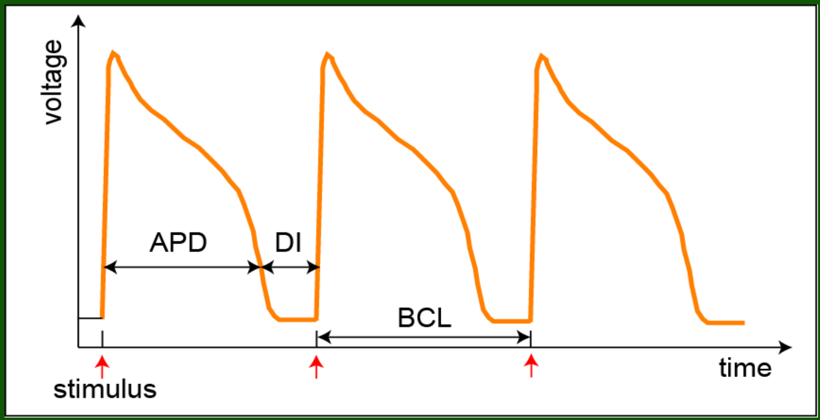
World of Bifurcation, R. Seydel

Bifurcation Theory

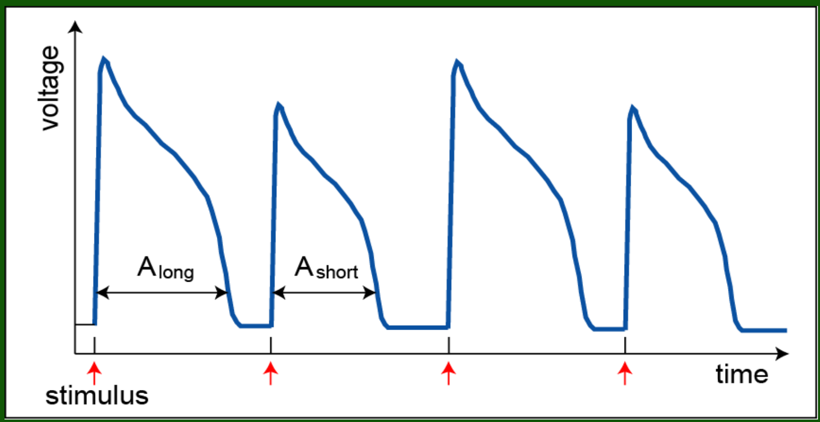
- Classical bifurcation
  - Context: smooth systems
  - Phenomena: period doubling, saddle node, …
- Border-collision bifurcation
  - Context: piecewise smooth systems
  - Typical Phenomena: period doubling, …,
  - Atypical Phenomena: instant chaos, …

Paced Cardiac Tissue

1:1 at slow pacing

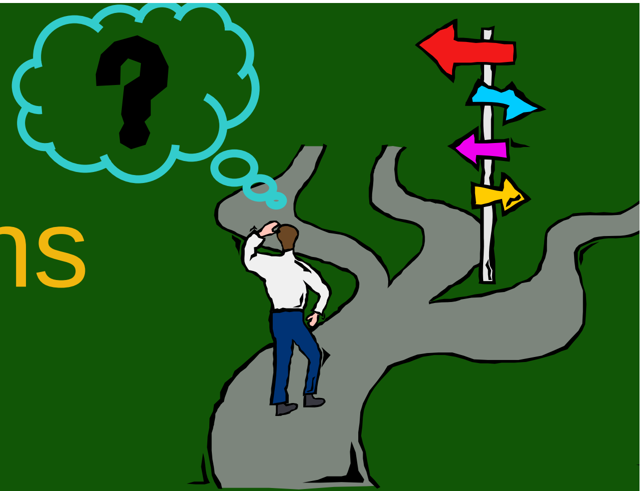


Alternans(2:2) at fast pacing



Bifurcation Diagram

Bifurcation to Alternans



- Classical period-doubling bifurcation
  - Nolasco & Dahlen (1968), Chialvo et al. (1990), Fox et al. (2002), …
- Border-collision period-doubling bifurcation
  - Sun et al. (1995)

Is alternans a classical bifurcation or a border-collision bifurcation?

A Model of Classical Bifurcation

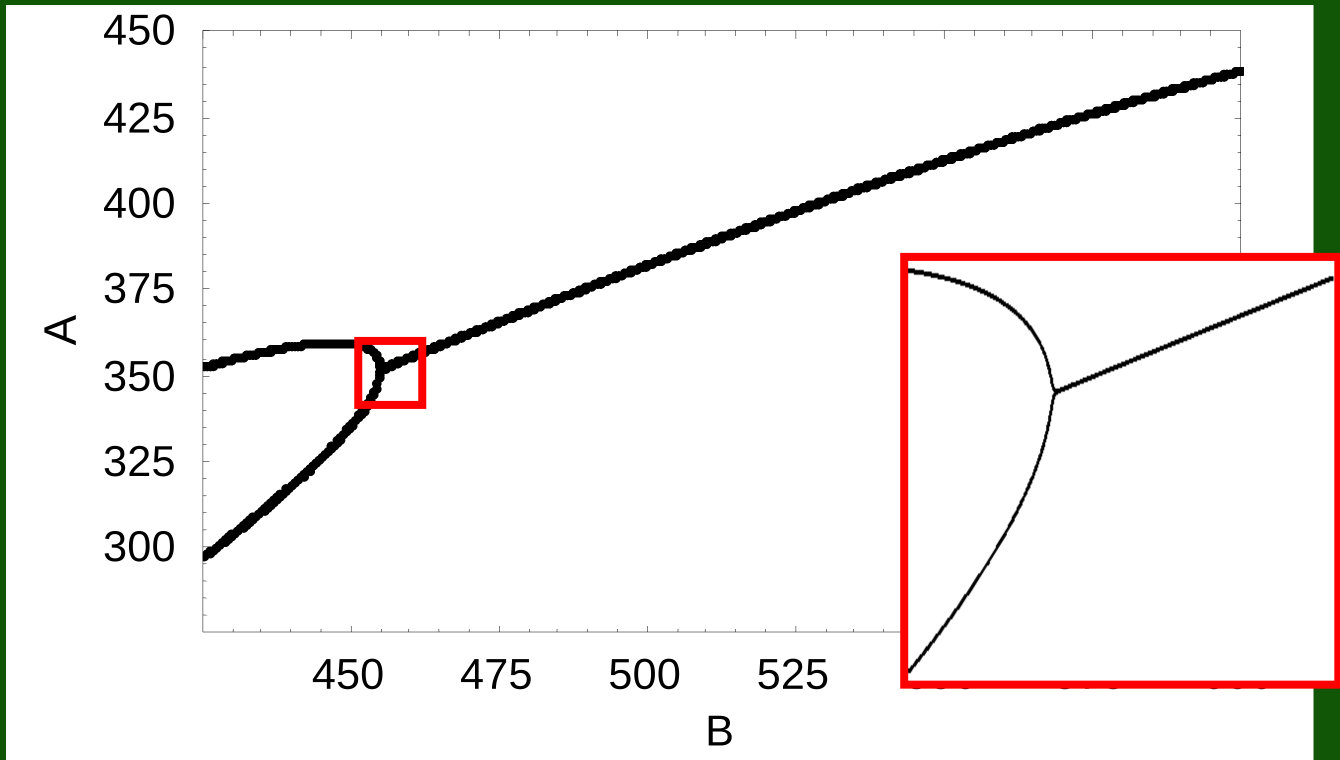
Chialvo et al., Circ. Res., 1990

$$\begin{pmatrix} A_{n+1} \\ M_{n+1} \end{pmatrix} = \begin{pmatrix} \left(1-M_{n+1}\right) A_{max} \left(1-\alpha e^{-\left(B-A_n\right)/\tau_1}\right) \\ \left(1-\left(1-M_n\right) e^{-A_n/\tau_2}\right) e^{-\left(B-A_n\right)/\tau_2} \end{pmatrix}$$

- Smooth map
- Bifurcation predictable from eigenvalues

A Model of Classical Bifurcation

Chialvo et al., Circ. Res., 1990



Note: the bifurcated branches are tangent.

A Model of Border-collision Bifurcation

Sun et al., J. Theor. Bio., 1995

$$A_{n+1}=\begin{cases} A_{min}+R_{n+1}+(500-3.0A_n)e^{-B/\tau_{rec}}, & \text{if } A_n \geq 130 \\ A_{min}+R_{n+1}+(201-0.7A_n)e^{-B/\tau_{rec}}, & \text{if } A_n \leq 130 \end{cases}$$

$$R_{n+1}=R_n e^{-(A_n+B)/\tau_{fat}}+\gamma e^{-B/\tau_{fat}}$$

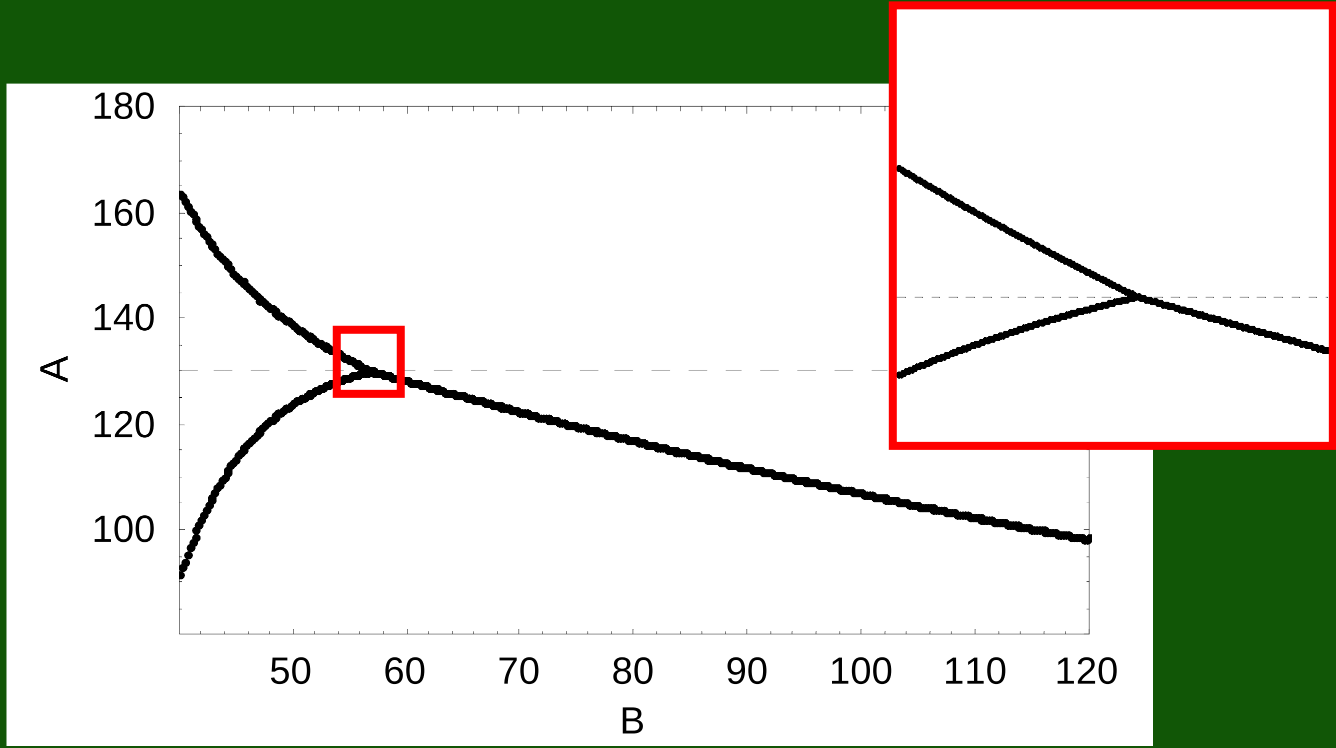
500-3.0*130=110

201-0.7*130=110

- Piecewise smooth map: continuous but derivatives jump
- Bifurcation **NOT** predictable from eigenvalues

A Model of Border-collision Bifurcation

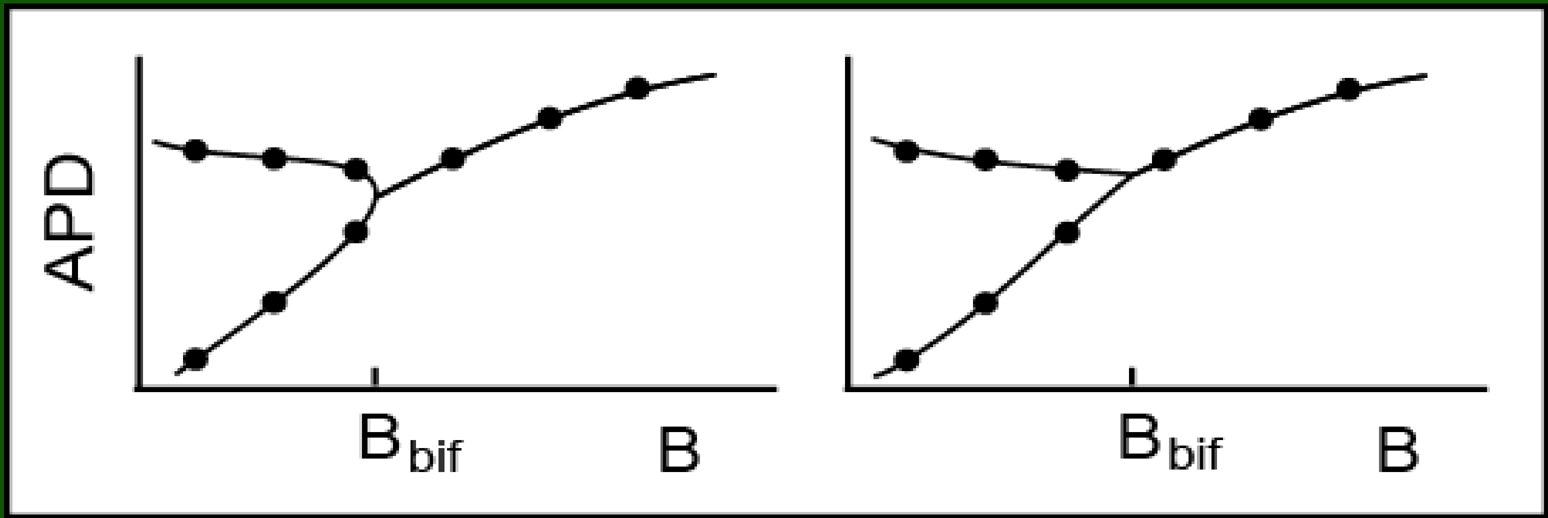
Sun et al., J. Theor. Bio., 1995



Note: the bifurcated branches are **NOT** tangent.

Differentiation

Who Is Who

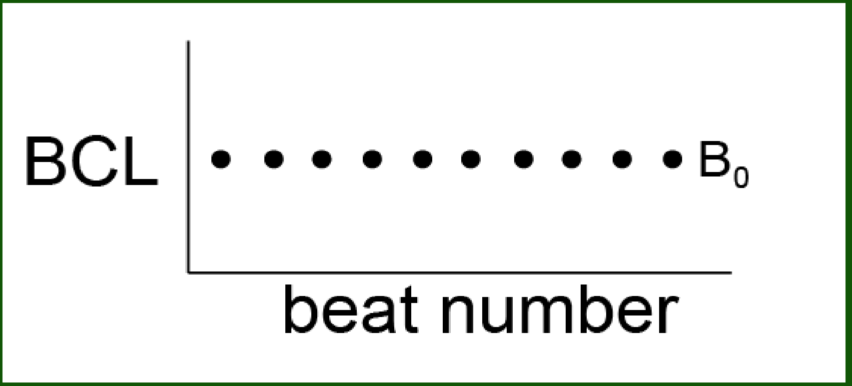


- Experimental bifurcation diagrams difficult to interpret
- New technique based on **Alternate Pacing**

What Is Alternate Pacing

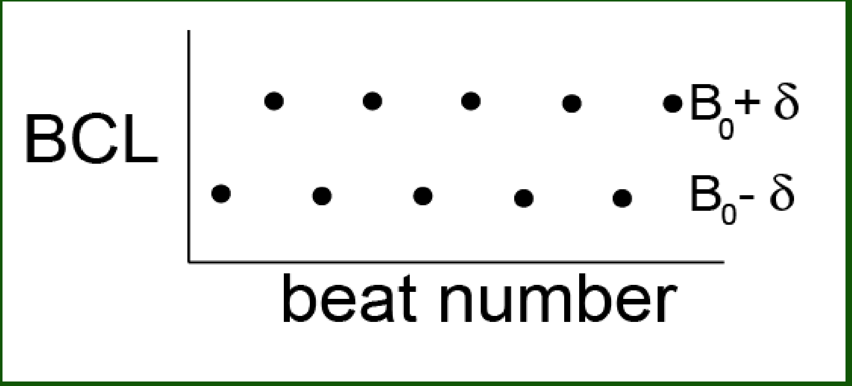
Constant Pacing:

$$B_n = B_0$$



Alternate Pacing:

$$B_n = B_0 + \left(-1\right)^n \delta$$



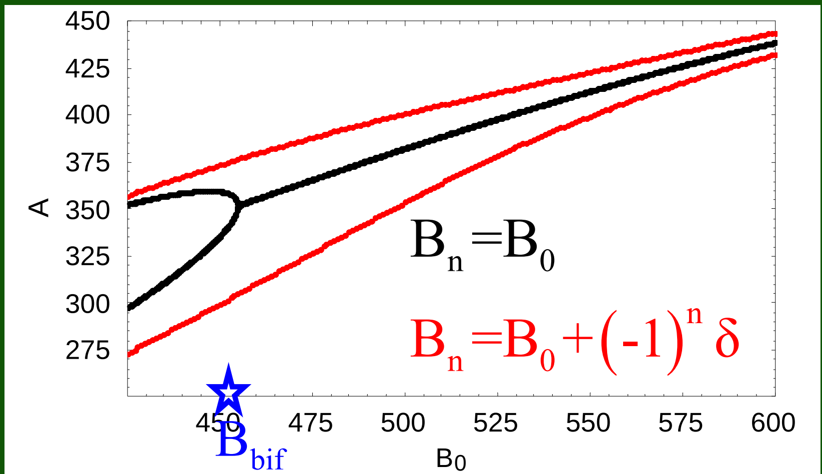
Alternate Pacing

Who Did What

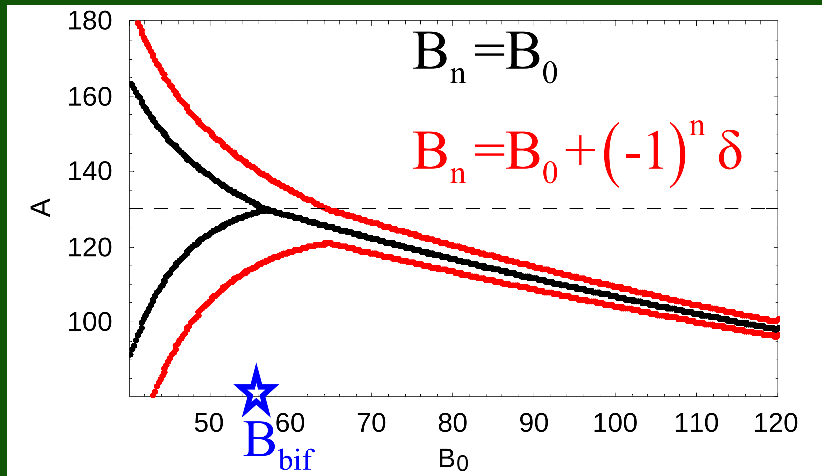
- First studied by Heldstab et al. (1983)
- Numerous theoretical and experimental work with application in physics
- Clinical use to detect cardiac alternans suggested by Karma and Shiferaw (2004)
- Analytical solution of the response in classical & border-collision bifurcations, Zhao et al. (2006)

Response to Alternate Pacing

Chialvo model



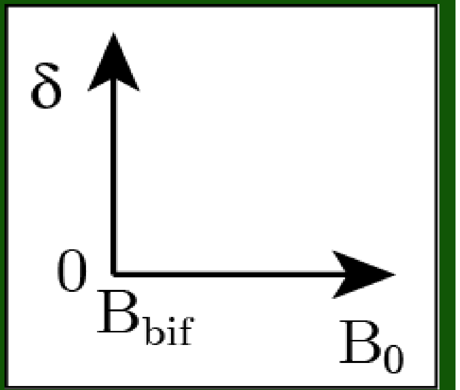
Sun model



Quantify the results by a gain

$$\Gamma(B_0, \delta) = \frac{A_{long} - A_{short}}{2\delta}$$

Prebifurcation Gain

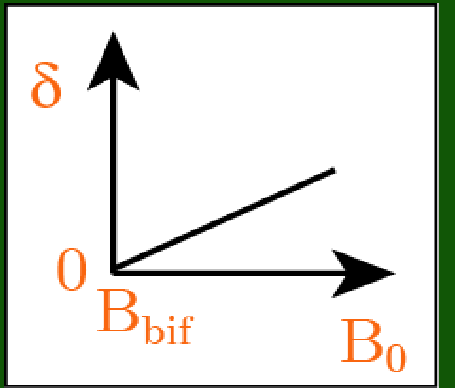


Smooth Bifurcation

$$\delta^2\Gamma^3+\left(B_0-B_{bif}\right)\Gamma-1=0$$

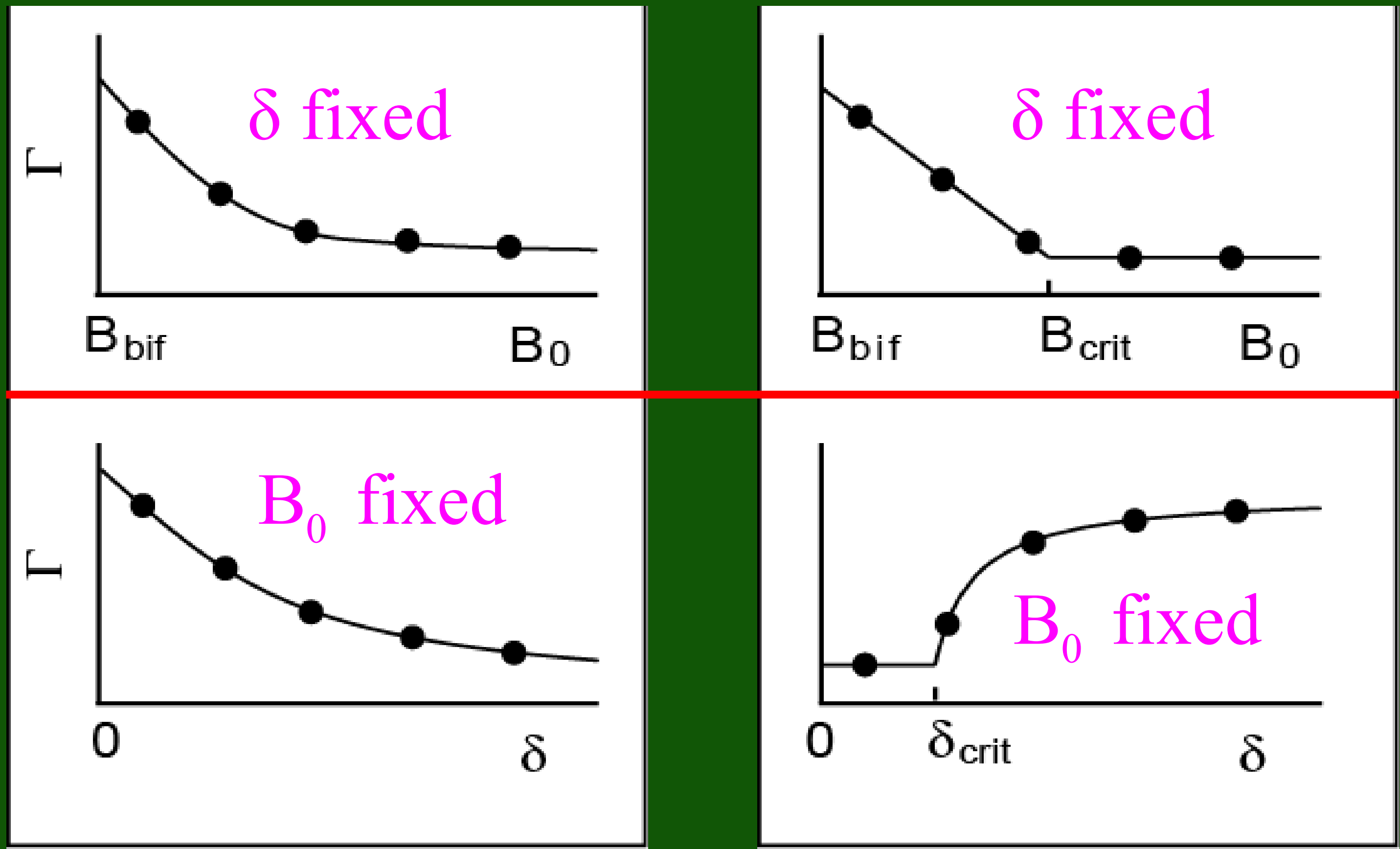
Nonsmooth Bifurcation

$$\Gamma=\begin{cases}\Gamma_{const}+\gamma\left(\rho-\dfrac{B_0-B_{bif}}{\delta}\right), & \text{if } \delta\geq\left(B_0-B_{bif}\right)/\rho\\ \Gamma_{const}, & \text{if } \delta\leq\left(B_0-B_{bif}\right)/\rho\end{cases}$$

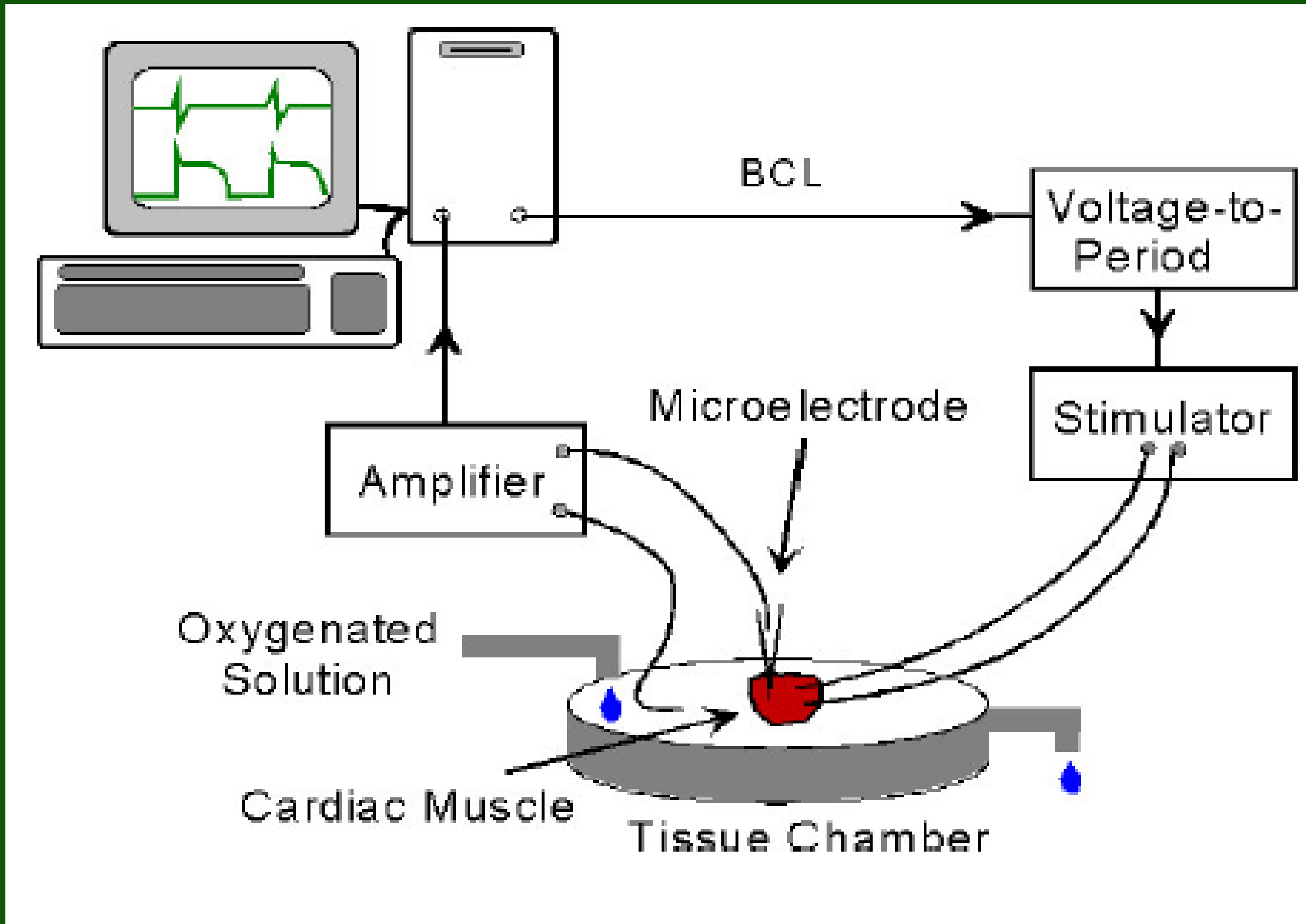


Prebifurcation Gain

Smooth Bifurcation | Nonsmooth Bifurcation



Experimental Setup



Experimental Protocol

1. Constant pacing:

   $B_n = B_0$ (2 minutes)

2. Alternate pacing:

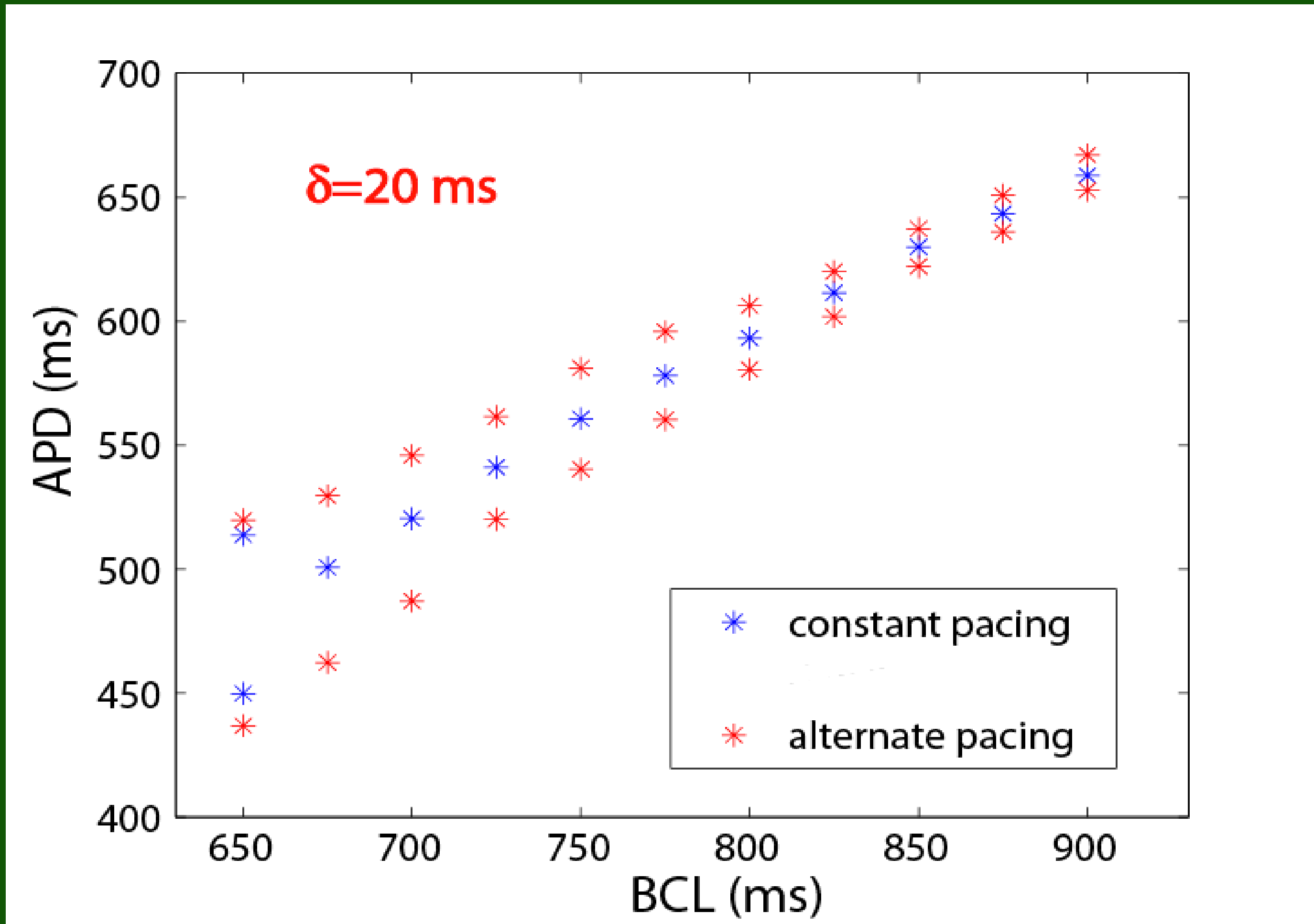
   $B_n = B_0 + (-1)^n \delta$ with δ=20ms (20 seconds)

3. Repeat step 2 with δ=15ms, 10ms, and 5ms

4. Constant pacing: $B_n = B_0$ (20 seconds)

Reduce B_0 by 25ms and repeat steps 1-4.

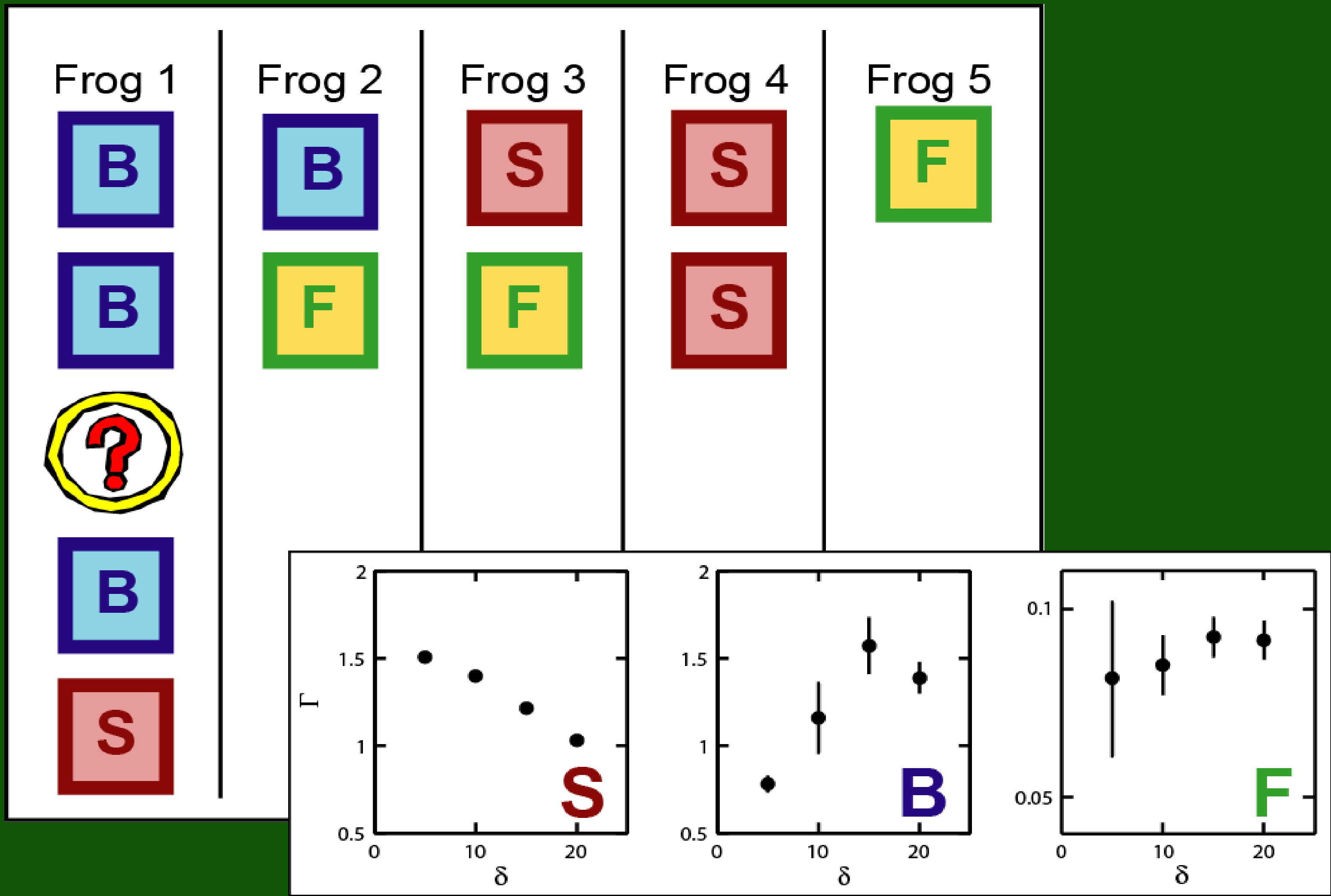
One Example Trial



Categorization of Data

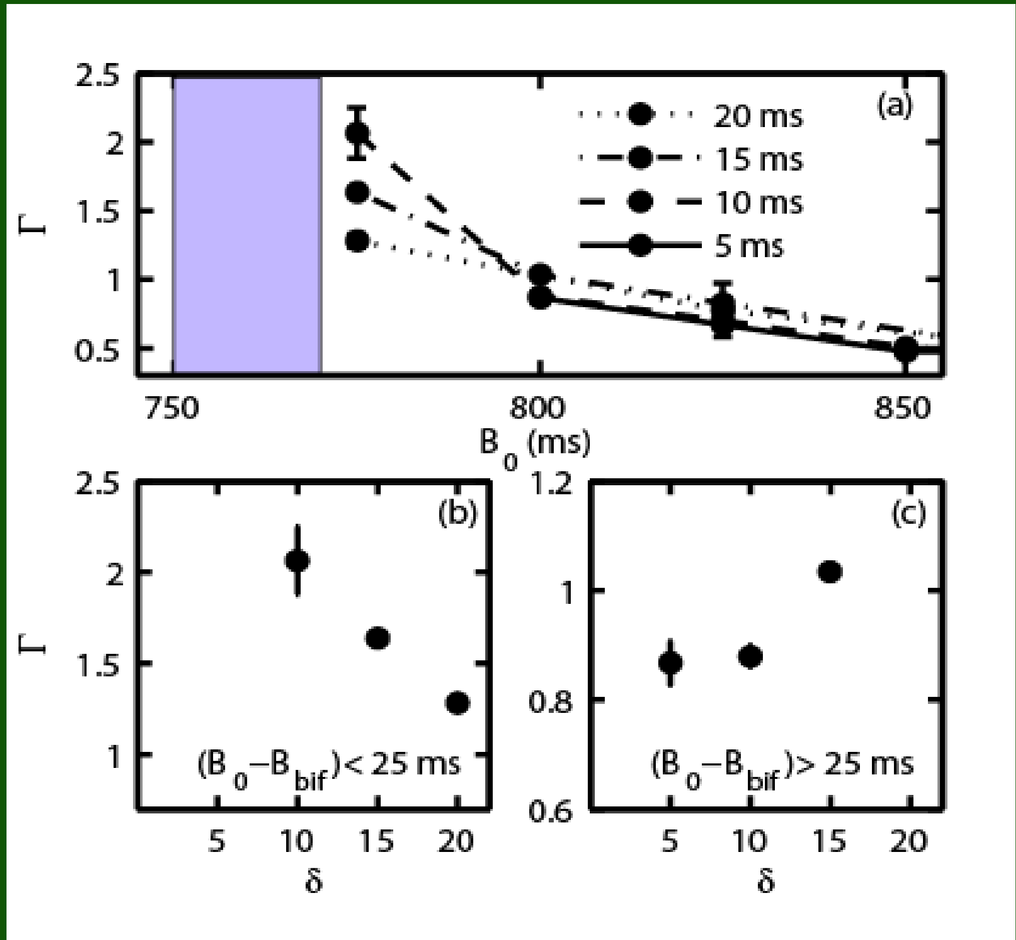
12 trials:

- 4 S
- 4 B
- 3 F
- 1 ?



Surprise: Remaining Trial

- Alternans seen at B_0=750ms
- Smooth-like behavior at B_0=775ms
- BCB-like behavior at B_0=800ms



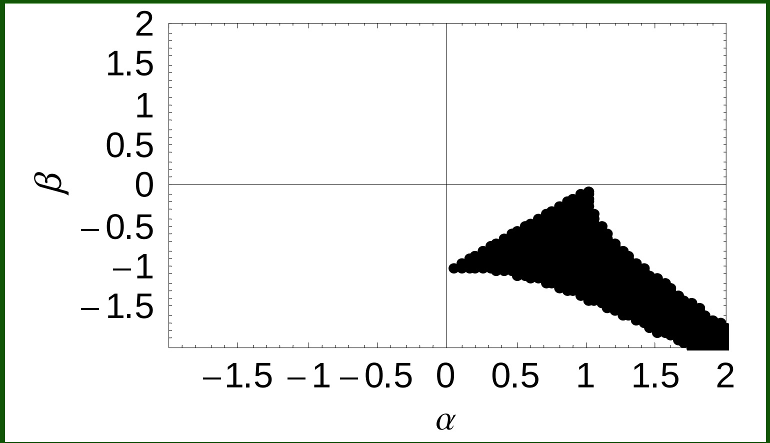
Unfolded Border-collision Bifurcation (BCB)

Skewed tent map:

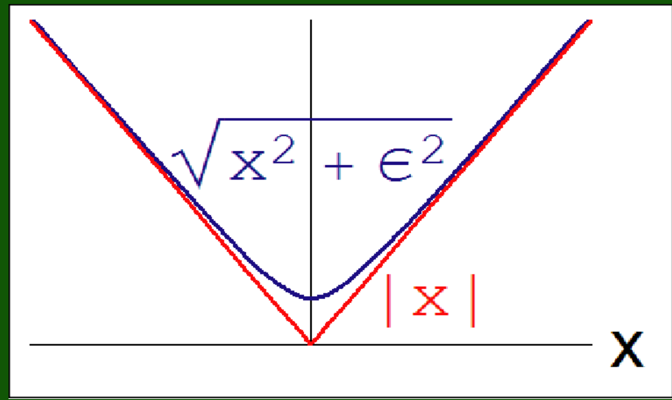
$$x_{n+1} = \mu - \alpha x_n - \beta \, | x_n |$$

Border-collision period-doubling bifurcation occurs when

$$-1<\alpha+\beta<1<\alpha-\beta \text{ and } -1<\alpha^2 - \beta^2 < 1$$



Unfolded Border-col Bifurcation (BCB)



Skewed tent map:

$$x_{n+1} = \mu - \alpha x_n - \beta \, | \, x_n \, |$$

Border-collision period-doubling bifurcation occurs when

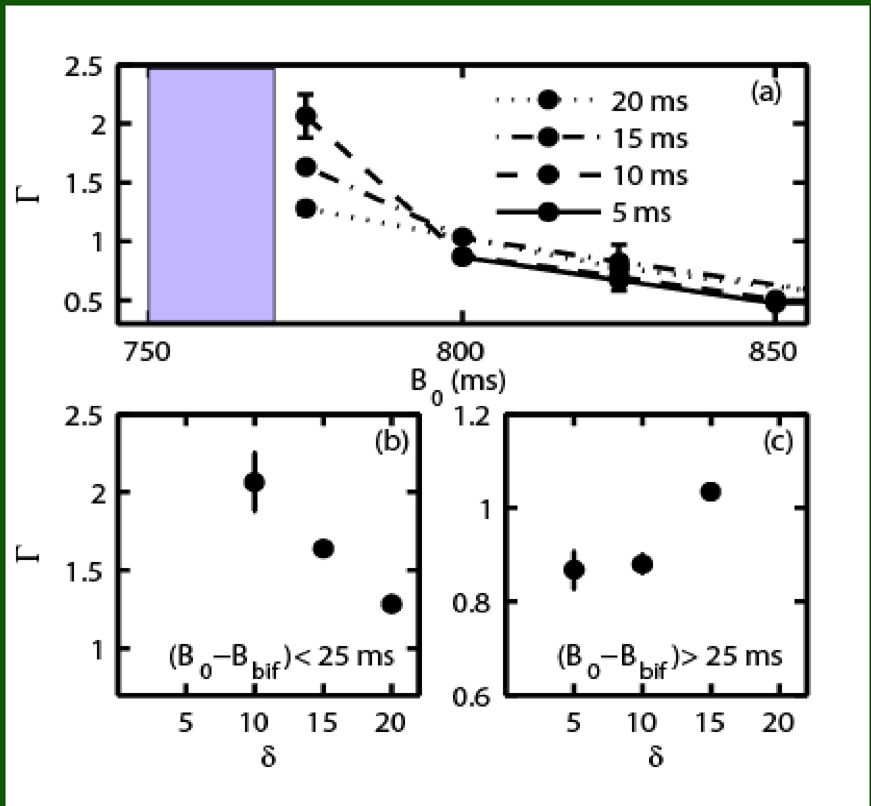
$$-1<\alpha+\beta<1<\alpha-\beta \text{ and } -1<\alpha^2-\beta^2<1$$

Unfolded map:

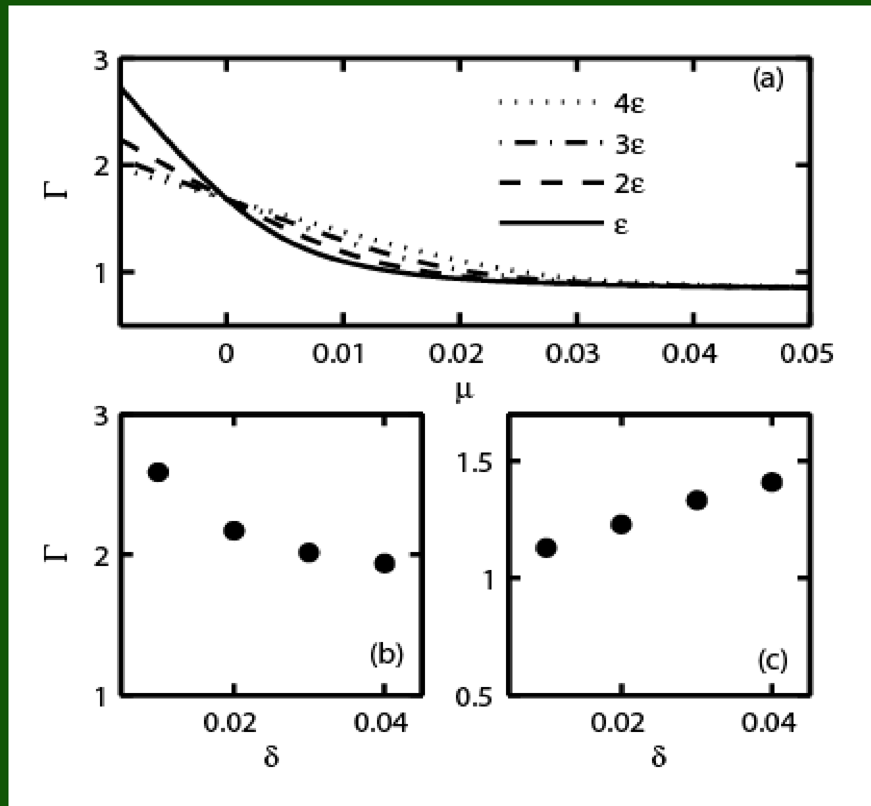
$$x_{n+1} = \mu - \alpha x_n - \beta \sqrt{x_n{}^2 + \varepsilon^2}$$

Unfolded Bifurcation

Experiment



Model



Summary

- Alternate pacing protocol can differentiate between the classical and border-collision period-doubling bifurcations
- Alternans in paced frog tissue exhibits hybrid behavior, which is captured by a model of unfolded border-collision bifurcation

References

1. X. Zhao, D.G. Schaeffer, C.M. Berger, and D.J. Gauthier, "Small signal amplification of period-doubling bifurcations in smooth iterated maps," to appear in Nonlinear Dynamics
2. X. Zhao and D.G. Schaeffer, "Alternate pacing of border-collision period-doubling bifurcations," submitted to Nonlinear Dynamics
3. C.M. Berger, X. Zhao, D.G. Schaeffer, W. Krassowska, H. Dobrovolny, and D.J. Gauthier, "Evidence for an unfolded border-collision bifurcation in paced cardiac tissue," submitted to Physical Review Letters

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THANK YOU!