# Cardiac re-entry dynamics in MRI- and micro-CT based models of the heart 

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in collaboration with

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${ }^{6}$ Karlsruhe Institute of Technology, Germany; ${ }^{7}$ The University Heart Center, Freiburg, Germany.


## Dissipative vortices


in liquid crystals (S.Residori, Institut Non Linéair de Nice)


Ca waves in oocytes (D.E.Clapham, Harvard)

re-entry of excitation in human atrium


Galaxy M5 (Hubble telescope).

spreading depression in chicken retina (M.A.Dahlem, Magdeburg)

in lasers (numerics: J.Simonotto, Herriot-Watt; J.Simonotto, Herriot-Wat
experiment: Someone at Lebedev Inst. Moscow?)


Catalytic CO oxidation on Pt surface (Y.Kevrekidis, Princeton)


Map plotting the track and intensity of the storm, according to the Saffir -Simpson scale


Spinning combustion wave (numerics: A.Merzhanov+T.Ivleva, Chernogolovka)


1 cm
Belousov-Zhabotinsky (BZ) reaction medium (Stefan C.Muller + Theo Plesser)

in slime moulds (C.Weijer, Dundee)




Hurricane Maria near peak intensity NW of Dominica Sept 192017.
https://en.wikipedia.org/wiki/Hurricane Maria

## Unperturbed Dynamics

- Reaction-diffusion system for $\ell$ components

$$
\partial_{t} \mathbf{u}=\mathbf{f}(\mathbf{u})+\mathbf{D} \nabla^{2} \mathbf{u}+\epsilon \mathbf{h}, \quad \mathbf{u}, \mathbf{f}, \mathbf{h} \in \mathbb{R}^{\ell}, \mathbf{D} \in \mathbb{R}^{\ell \times \ell}, \ell \geq 2
$$

- Steadily rotating spiral wave solutions $(\epsilon=0)$ :

$$
\mathbf{u}(\vec{r}, t)=\mathbf{U}(\rho(\vec{r}-\vec{R}), \vartheta(\vec{r}-\vec{R})+\omega t-\Phi)
$$

## 1D: Front existence (initiation)__\&__dissipation

E.g. V.N. Biktashev and R. Suckley, Phys. Rev. Lett, 93(16):168103, 2004
I.V. Biktasheva, et. al, FIMH 2005, LNCS 3504: 293-303, 2005.


## 3D: Filament tension (negative)

$\checkmark$ Scroll turbulence
V.N. Biktashev, et.al, PTRSA, 347: 611-630, 1994 V.N. Biktashev, IJBC, 8(4): 677-684, 1998

Buckling
H. Dierckx, et.al, PRL, 109(17):174102, 2012 >>


## Perturbation theory: the problem

- Reaction-diffusion system for $\ell$ components

$$
\partial_{t} \mathbf{u}=\mathbf{f}(\mathbf{u})+\mathbf{D} \nabla^{2} \mathbf{u}+\epsilon \mathbf{h}, \quad \mathbf{u}, \mathbf{f}, \mathbf{h} \in \mathbb{R}^{\ell}, \mathbf{D} \in \mathbb{R}^{\ell \times \ell}, \ell \geq 2 .
$$

- Steadily rotating spiral wave solutions $(\epsilon=0)$ :

$$
\begin{gathered}
\mathbf{u}(\vec{r}, t)=\mathbf{U}(\rho(\vec{r}-\vec{R}), \vartheta(\vec{r}-\vec{R})+\omega t-\Phi) . \\
(\vec{r}=(x, y), \vec{R}=(X, Y)=\text { const, } \Phi=\text { const, } \omega \text { is an eigenvalue }) .
\end{gathered}
$$

- For $\epsilon \neq 0$, the spiral drifts: solution remains approximately as above, but $\mathrm{d} \vec{R} / \mathrm{d} t=\mathcal{O}(\epsilon), \mathrm{d} \Phi / \mathrm{d} t=\mathcal{O}(\epsilon)$.


## E.g.

- alternanses
E. Koch, Pflügers Arch., 181: 106, 1920
A. Karma, "Electrical alternans and spiral wave breakup in cardiac tissue", Chaos, 4(3), pp 461-472, 1994.
- inhomogeneity
V.N. Biktashev, D. Barkley and I.V. Biktasheva, "Orbital motion of spiral waves in excitable media", PRL, 104(5): 058302, 2010 >>

- anisotropy
F. Fenton, A. Karma, "Fiber-rotation-induced vortex turbulence in thick myocardium", PRL, 81(2), pp 481-484, 1998
O. Berenfeld, M. Wellner, J. Jalife, A.M. Pertsov, "Shaping of a scroll wave filament by cardiac fibers", PRE, 63(6): 061901,2001
- curvature of the domain
H. Dierckx, E. Brisard, H. Verschelde, A.V. Panfilov, "Drift laws for spiral waves on curved anisotropic surfaces", PRE, 88(1): 012908 , 2013


## Drift caused by inhomogeneity



- Half of the medium is slightly different than the other.
V.N. Biktashev \& A.V. Holden
"Resonant drift of autowave vortices in 2D and the effects of boundaries and inhomogeneities', Chaos, Solitons and Fractals 5(3,4): 575-622, 1995
I.V. Biktasheva, "Drift of spiral waves in the Complex Ginzburg-Landau Equation due to media inhomogeneities", Phys. Rev. E, 62(6):8800-8803, 2000

- Orbital motion around localized inhomogeneity
- Here, the spiral is kept at a stable distance, which depends on the RFs (i.e. medium parameters) not inhomogeneity strength!
V.N. Biktashev, D. Barkley and I.V. Biktasheva " Orbital motion of spiral waves in excitable media" Phys. Rev. Lett., 104(5): 058302, 2010


## Scroll turbulence at negative filament tension



- Filament equation of motion:

$$
(\vec{N}+\mathrm{i} \vec{B}) \cdot \vec{R}=\left(b_{2}+\mathrm{i} c_{3}\right) \kappa
$$

where $b_{2}$ is filament tension.
Total length:
$\frac{\mathrm{d}}{\mathrm{dt}} \int \mathrm{d} s=-\int b_{2} \kappa^{2} \mathrm{~d} s+$ bound.eff.
V.N. Biktashev, A.V. Holden \& H. Zhang, ``Tension of Organizing Filaments of Scroll Waves" Phil. Trans. Roy. Soc. London, ser A 347: 611-630, 1994
V.N. Biktashev " A Three-Dimensional Autowave

Turbulence" Int. J. Bifurcation \& Chaos, 8(4):
677-684, 1998

## Macroscopic Dissipative Wave-Particle Duality


> Localization of the adjoint symmetry modes, aka the Response Functions (RFs), means that spiral waves behave like point objects, and scroll waves behave like string objects.
> The asymptotic theory based on the RFs

- is (within its limits) in good quantitative agreement with direct simulations.
- predicted new qualitative phenomena:
- orbital motion,
- pinning to repelling inhomogeneity,
- interaction with small variation of
thickness in a layer, etc.

in liquid crystals
(S.Residori, Institut Non Linéair de Nice)


Ca waves in oocytes (D.E.Clapham, Harvard)

re-entry of excitation in human atrium


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Spinning combustion wave (numerics: A.Merzhano v+T.Ivleva, Chernogolovka)

rusting of steel surface in nitric acid (O.Steinbock, Florida)


Catalytic CO oxidation on Pt surface (Y.Kevrekidis, Princeton)


Belousov-Zhabotinsky (BZ) reaction medium (Stefan C.Muller + Theo Plesser

in slime moulds (C.Weijer, Dundee)


Hurricane Maria near peak intensity NW of Dominica, Sept 192017.


Map plotting the track and intensity of the storm, according to the Saffir-Simpson scale
https://en.wikipedia.org/wiki/Hurricane Maria

## Perturbation theory: result

- Drift velocity due to perturbation:

$$
\dot{R}=\epsilon \int_{\phi-\pi}^{\phi+\pi} e^{-i \xi}\langle\mathbf{W}, \tilde{\mathbf{h}}(\mathbf{U} ; \rho, \theta, \xi)\rangle \frac{\mathrm{d} \xi}{2 \pi}+\mathcal{O}\left(\epsilon^{2}\right),
$$



Beeler-Reuter (heart ventricles)
where $(\rho, \theta)$ are corotating polar coordinates and $\phi$ and $\xi$ measure the rotation phase, $\phi=\omega t-\Phi(t)$, and inner product

$$
\langle\mathbf{w}, \mathbf{v}\rangle=\int_{\mathbb{R}^{2}} \mathbf{w}^{+}(\vec{r}) \mathbf{v}(\vec{r}) \mathrm{d}^{2} \vec{r}=\oint \int_{0}^{\infty} \mathbf{w}^{+}(\rho, \theta) \mathbf{v}(\rho, \theta) \rho \mathrm{d} \rho \mathrm{~d} \theta .
$$

- (Translational) response function $\mathbf{W}(\rho, \theta) \in \mathbb{C}$ : eigenfunction of the adjoint linearized operator, corresponding to eigenvalue $\mathrm{i} \omega$.

[^0]
## Superposition principle

## Superposition of various perturbation

$$
\epsilon \mathbf{h}=\sum_{j} \epsilon_{j} \mathbf{h}_{j},
$$



Beeler-Reuter (heart ventricles)
has summative effect on the drift velocity

$$
\dot{R} \approx \sum_{j} \epsilon_{j} \gamma_{j}
$$

where "specific forces" are

$$
\gamma_{j}=\oint e^{-i \xi}\left\langle\mathbf{W}, \tilde{\mathbf{h}}_{j}\right\rangle \frac{\mathrm{d} \xi}{2 \pi} .
$$

[^1]
## Spiral waves generated near recovering ischaemic boundary: cell culture experiment


V.N. Biktashev, A Arutunyan and N.A. Sarvazyan
"Generation and escape of local waves from the boundary of uncoupled cardiac tissue" Biophys. J., 94: 3726-3738,
2008


## Ischaemic boundary specific forces



3D boundary movement


Spiral wave escape

$\gamma_{D}$ - force of gradient of diffusivity
( = connectivity)
$\gamma_{\alpha}$ - force of gradient of excitability $\alpha$
$F$ - force caused by a localized inhomogeneity of $\alpha$
$\gamma_{\kappa}$ - force due to filament curvature $K$

$$
\gamma_{\kappa}=b_{2}+i c_{3}
$$

where
$b_{2}$ - filament tension
$c_{3}$ - binormal drift coefficient of the filament

NB. $\quad \gamma_{D}=-\gamma_{\kappa}$

[^2] of ischaemic border zone" PLoS ONE, 6(9):e24388, 2011

## Drift due to variation of thickness of the layer.

Epicardial View


- Ridge --- the CT and PM (attached to wall) ridge structures

- Ditch --- in between PM (attached to wall) ridge structures
Up >>

Down >>


Simulation





(a)

(b)
S.R. Kharche, I.V. Biktasheva, G. Seeman, H.G. Zhang, and V.N. Biktashev, Biomed Research International, 2015: 731386, 2015.
I.V. Biktasheva, H.Dierckx, and V.N.Biktashev, PRL, 114, 068302, 2015

## specific forces



3D boundary movement


$\checkmark$ Inhomogeneity
$\gamma_{D}$ - gradient of diffusivity ( = coupling )
$\gamma_{\alpha}$ - gradient of excitability $\alpha$
$F$ - localized inhomogeneity of $\alpha$
$\checkmark \gamma_{\kappa}$ - Filament curvature $\kappa$

$$
\begin{aligned}
& \quad \gamma_{\kappa}=b_{2}+i c_{3} \\
& \text { where } \quad b_{2} \text { - filament tension } \\
& \\
& c_{3} \text { - binormal drift velocity }
\end{aligned}
$$

NB. $\gamma_{D}=-\gamma_{\kappa}$
$\checkmark$ Anatomy

- Anisotropy
- Thickness gradients \& fluctuations
- Curvature of the Domain


## Anatomy Models

Human Atrium


The model derived from the Visible Female dataset: the transversal cryosections with a resolution of 0.33 mm , and an algorithm for fiber orientation .
G. Seemann, C. Höper, F. B. Sachse, O. Dössel, A. V. Holden, and H. Zhang. Heterogeneous three-dimensional anatomical and electrophysiological model of human atria. In Phil. Trans. Roy. Soc. A, vol. 364(1843) , pp. 1465-1481, 2006

## Human Foetal Heart



Fast low- angle shot and diffusion tensor magnetic resonance imaging (DT-MRI) of 139 days of gestation age (DGA) foetal heart, with the voxel resolution of 0.1 mm .

E Pervolaraki, RA Anderson, AP Benson, B Hays-Gill, AV Holden, BJR Moore, MN Paley, HG Zhang, "Antenatal architecture and activity of the human heart", INTERFACE FOCUS, v. 3(2), 20120065, 2013.

## Rat Pulmonary Vein Wall



Micro_CT resolution 3.5 microns in each plane.
[G.S. Ramlugun, B.H. Smaill, unpublished.]
G.S. Ramlugun, B. Thomas, V.N. Biktashev, I.J. LeGrice, B.H. Smaill, J.C. Zhao, and I.V.
Biktasheva, "Comparative in-silico study of cardiac re-entry dynamics in micro-CT based models of mammalian hearts", submitted to Frontiers Physiology, 2018.

## BeatBox Virtual Heart.

$\checkmark$ software package following the modular philosophy, with built-in extendable repository of anatomical and cell models.
$\checkmark$ built in script interpreter to allow combinatorial flexibility to change from one model to another, without re-compilation of the whole system.


## $\checkmark$ High Performance Computing:

"BeatBox - HPC Environment for
Biophysically and Anatomically Realistic Cardiac Simulations"

[^3] for biophysically and anatomically realistic cardiac electrophysiology". PLoS ONE 12(5): e0172292, 2017.

## A sample BeatBox script

```
<fhn.par>
state geometry=ffr.bbg
def real T; def real begin; def real out; def real end; // real vars control works of "devices"
// The computation
diff v0=[u] v1=[i] Dpar=D Dtrans=D/4 hx=hx; // anisotropic diffusion
k_func when=force pgm={u[i]=u[i]+force;;
euler v0=[u] v1=[v] ht=ht ode=fhncubpar={eps=eps bet=@[b] gam=gam lu=@[i]};
// Output
ppmout when=out file="[0]/%04d.ppm" mode="w" // every "out" timesteps
record when=end file=[0].rec when=end v0=0 v1=1; // ascii dump in the end of run
stop when=end;
end;
```


## Excitation Re-entry in Human Atrium*



- Regular Cartesian coordinates (cryosections + algorithm fibers):
box of $237 \times 271 \times 300$ points, $\Delta x=0.33 \mathrm{~mm}$
- Explicit Euler in time
- Second order approximation of Laplacian
- Neumann boundary conditions: $1<$ approximation order $\leq 2$ **
- Spiral wave initiation by phase distribution
- MPI HPC
-G. Seemann, C. Höper, F. B. Sachse, O. Dössel, A. V. Holden, and H. Zhang. Heterogeneous three-dimensional anatomical and electrophysiological model of human atria. In Phil. Trans. Roy. Soc. A, vol. 364(1843), pp. 1465-1481, 2006
** M. Antonioletti, V.N. Biktashev, A. Jackson, S.R. Kharche, T. Stary, I.V. Biktasheva, "BeatBox-HPC simulation environment for biophysically and anatomically realistic cardiac electrophysiology". PLoS ONE 12(5): e0172292, 2017.


## Atrium



Wu T et al. Circulation Research. 1998;83:448-462
G. Seemann, C. Höper, F. B. Sachse, O. Dössel, A. V. Holden, and H. Zhang. Heterogeneous three-dimensional anatomical and electrophysiological model of human atria. In Phil. Trans. Roy. Soc. A, vol. 364(1843) , pp. 1465-1481, 2006

## Anatomy induced drift in Human Atrium



Epicardial View


- Ridge --- the CT and PM (attached to wall) ridge structures


## Anatomy induced drift in Human Atrium



Isotropic, $\mathrm{z}_{0}=0.75 \times \mathrm{z}_{\text {max }}, \mathrm{y}_{0}=0.8 \times \mathrm{y}_{\text {max }}$
Isotropic, $\mathrm{z}_{0}=0.6 \times \mathrm{z}_{\text {max }}, \mathrm{y}_{0}=0.8 \times \mathrm{y}_{\text {max }}$


Isotropic, $\mathrm{z}_{0}=0.6 \times \mathrm{z}_{\text {max }}, \mathrm{y}_{0}=0.8 \times \mathrm{y}_{\text {max }}$


Anisotropic, $\mathrm{z}_{0}=0.6 \times \mathrm{z}_{\text {max }} \mathrm{y}_{0}=0.8 \times \mathrm{y}_{\text {max }}$ .. work in progress...


Anisotropic, $\mathrm{z}_{0}=0.6 \times \mathrm{z}_{\text {max }} \mathrm{y}_{0}=0.8 \times \mathrm{y}_{\text {max }}$
I.V. Biktasheva, V.N. Biktashev, S. R. Kharche, G. Seemann, unpublished

## Excitation Re-entry in Foetal Heart *



- Regular Cartesian coordinates (DT-MRI):
box of $128 \times 128 \times 128$ points, $\Delta x=0.1 \mathrm{~mm}$;
- Explicit Euler in time
- Second order approximation of Laplacian
- Neumann boundary conditions: $1<$ approximation order $\leq 2$ **
- FHN model: $\varepsilon=0.3 ; \beta=0.71 ; \gamma=0.5$ (rigid rotation, positive filament tension)
- Spiral wave initiation by phase distribution
- MPI HPC
- E Pervolaraki, RA Anderson, AP Benson, B Hays-Gill, AV Holden, BJR Moore, MN Paley, HG Zhang, "Antenatal architecture and activity of the human heart", INTERFACE FOCUS, v. 3(2), 20120065, 2013.
** M. Antonioletti, V.N. Biktashev, A. Jackson, S.R. Kharche, T. Stary, I.V. Biktasheva, "BeatBox-HPC simulation environment for biophysically and anatomically realistic cardiac electrophysiology". PLoS ONE 12(5): e0172292, 2017.


## Role of anisotropy in cardiac re-entry dynamics \& self-termination.

(b) fibre helix angle


Formed laminar intramural fibres vs chaotic outer-wall regions*

conduction: 2D - pinning;



3D - persistent re-entry;


3D - self-termination;

## Role of anisotropy in cardiac re-entry dynamics \& self-termination.


I.V. Biktasheva, R.A. Anderson, A.V. Holden, E. Pervolaraki, and F.C. Wen, "Cardiac re-entry dynamics \& self-termination in DT-MRI based model of Human Foetal Heart", Frontiers Phys. 6:15, 2018

## Role of anisotropy in cardiac re-entry dynamics \& self-termination.


I.V. Biktasheva, R.A. Anderson, A.V. Holden, E. Pervolaraki, and F.C. Wen, "Cardiac re-entry dynamics \& self-termination in DT-MRI based model of Human Foetal Heart", Frontiers Phys. 6:15, 2018

## Excitation Re-entry in Rat Pulmonary Vein Wall*



- Regular Cartesian coordinates (micro-CT):

box of $255 \times 920 \times 542$ points, $\Delta x=3,5 \mu \mathrm{~m}$
- Explicit Euler in time
- Second order approximation of Laplacian
- Neumann boundary conditions: $1<$ approximation order $\leq 2$ **
- FHN model: $\varepsilon=0.3 ; \beta=0.71 ; \gamma=0.5$ (rigid rotation, positive filament tension)
- Spiral wave initiation by phase distribution
- MPI HPC

[^4]
## Anatomy Initiated Re-entry in the Rat PV Wall


G.S. Ramlugun, B. Thomas, V.N. Biktashev, I.J. LeGrice, B.H. Smaill, J.C. Zhao, and I.V. Biktasheva, "Comparative in-silico study of cardiac re-entry dynamics in micro-CT based models of mammalian hearts", under review, 2018.

## Excitation Re-entry in Rat Pulmonary Vein Wall


G.S. Ramlugun, B. Thomas, V.N. Biktashev, I.J. LeGrice, B.H. Smaill, J.C. Zhao, and I.V. Biktasheva, "Comparative in-silico study of cardiac re-entry dynamics in micro-CT based models of mammalian hearts", under review, 2018.

## Conclusion


> DT-MRI/micro-CT models allow in-silico testing of the effects of individual heart anatomy and anisotropy on cardiac re-entry dynamics.
$>$ Major Limitation: only "tissue"|"not tissue" points differentiation taken into account. Currently, tissue segmentation is acquired from DT-MRI data sets via image post-processing. In the future, multichannel computer tomography might offer automatic tissue segmentation.
$>$ Minor Limitation: challenging visualization - further research necessary.
$>$ The DT-MRI/micro-CT based comparative study suggests cardiac anatomy and anisotropy functional effect on cardiac re-entry: normally self-termination due to the structured shape and anisotropy of the heart; occasionally transition to anatomical re-entry as well as pinning to fine anatomical features.
> In particular, the anatomically realistic cardiac re-entry simulations concur with propensity to pin to PM-CT junction reported in experiment:
T. J. Wu, et al., Circulation Research 83, 448 (1998). M. Yamazaki, et al., Cardiovascular Research 94, 48 (2012).
$>$ Anatomy defined initiation of re-entry.
> Possibility of different endo- vs epi cardial manifestation of trans mural PV wall re-entry.
> Important implications for cardiac re-entry control, e.g. ablation, ICDs, etc.

## Acknowledgements:

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Software
- BeatBox http://empslocal.ex.ac.uk/people/staff/vnb262/software/BeatBox/
- EZView
(Barkley and Dowle's EZScroll descendant for visualization http://homepages.warwick.ac.uk/~masax/ )


[^0]:    V.N. Biktashev and A.V. Holden, Chaos, Solitons and Fractals 5(3,4): 575-622, 1995
    V.N. Biktashev, I.V. Biktasheva, and N.A. Sarvazyan, PLoS ONE, 6(9):e24388, 2011

[^1]:    V.N. Biktashev, I.V. Biktasheva and N.A. Sarvazyan, "'Evolution of spiral and scroll waves of excitation in a mathematical model of ischaemic border zone" PLoS ONE, 6(9):e24388, 2011

[^2]:    V.N. Biktashev, I.V. Biktasheva and N.A. Sarvazyan, ' ${ }^{\prime}$ Evolution of spiral and scroll waves of excitation in a mathematical model

[^3]:    M. Antonioletti, V.N. Biktashev, A. Jackson, S.R. Kharche, T. Stary, I.V. Biktasheva, "BeatBox-HPC simulation environment

[^4]:    *G.S. Ramlugun, B. Thomas, V.N. Biktashev, I.J. LeGrice, B.H. Smaill, J.C. Zhao, and I.V. Biktasheva, "Comparative in-silico study of cardiac re-entry dynamics in micro-CT based models of mammalian hearts", under review, 2018.
    ** M. Antonioletti, V.N. Biktashev, A. Jackson, S.R. Kharche, T. Stary, I.V. Biktasheva, "BeatBox-HPC simulation environment for biophysically and anatomically realistic cardiac electrophysiology". PLoS ONE 12(5): e0172292, 2017.

