



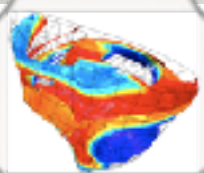
Kavli Institute for
Theoretical Physics
University of California, Santa Barbara



CARDIAC ELECTRO-MECHANICS & MEF: THE STRESS-ASSISTED DIFFUSION 'SAD' APPROACH

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NONLINEAR PHYSICS AND MATHEMATICAL MODELING LAB
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ACKNOWLEDGMENTS



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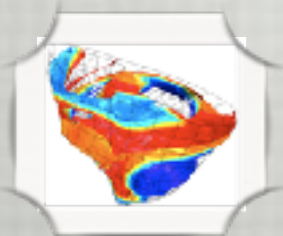
FLAVIO H. FENTON



DAISUKE SATO

YOHANNES SHIFERAW

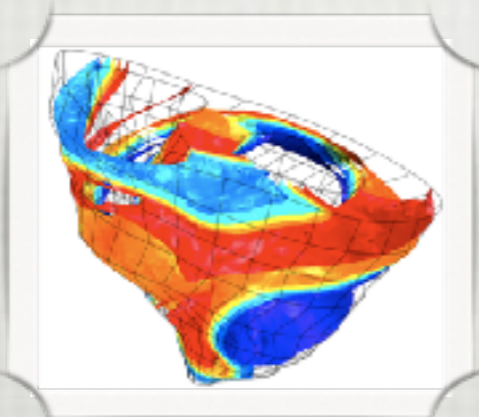
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GEORGIA TECH



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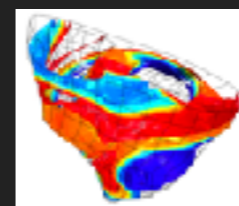
SIMONETTA FILIPPI (HEAD)
CHRISTIAN CHERUBINI (ASSOCIATE PROF.)
LETIZIA CHIDO (ASSISTANT PROF.)
ALESSIO GIZZI (ASSISTANT PROF.)
ALESSANDRO LOPPINI (ASSISTANT PROF.)
MARTINA NICOLETTI (PHD)

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INTERNATIONAL CENTER FOR RELATIVISTIC ASTROPHYSICS NETWORK

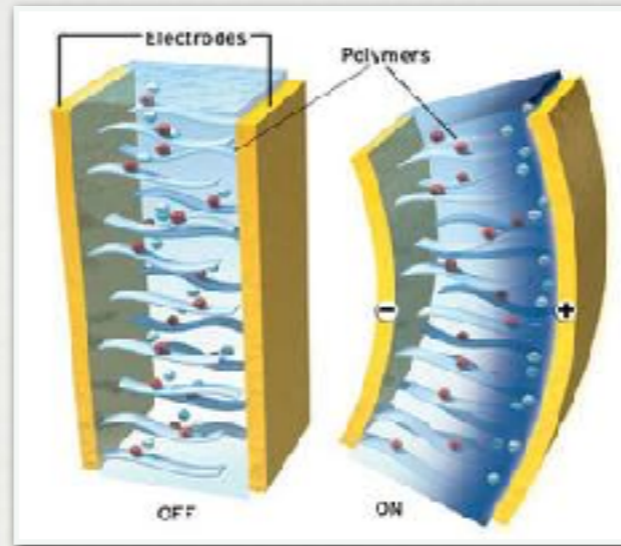


ACTIVE DEFORMABLE MEDIA

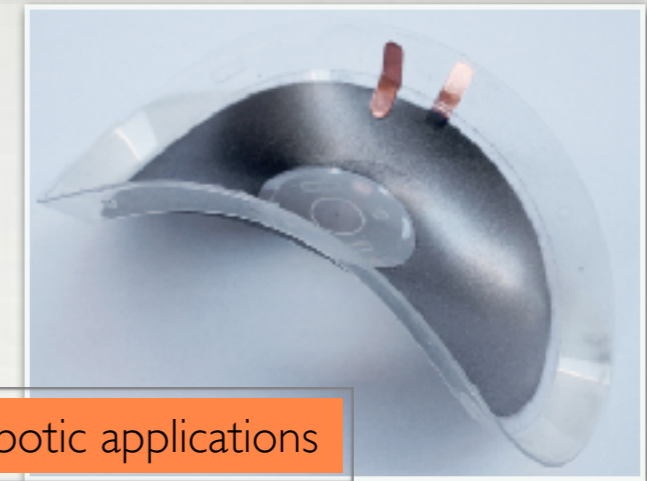
ELECTRO-ACTIVE POLYMERS (EAP)



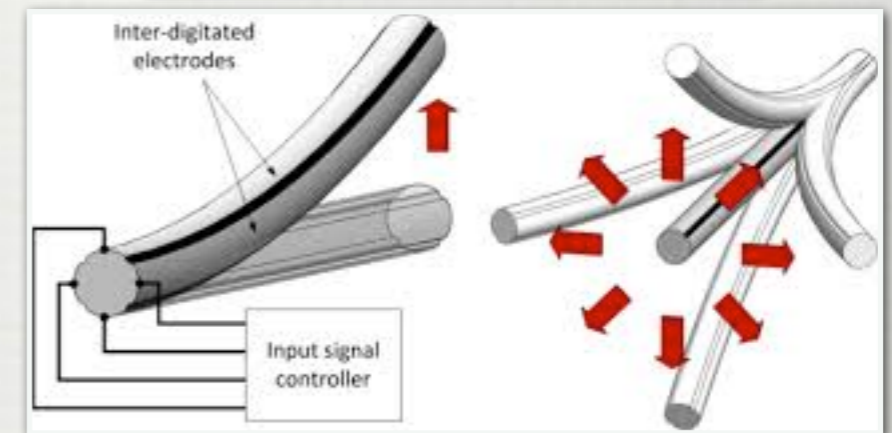
Ion exchange resins



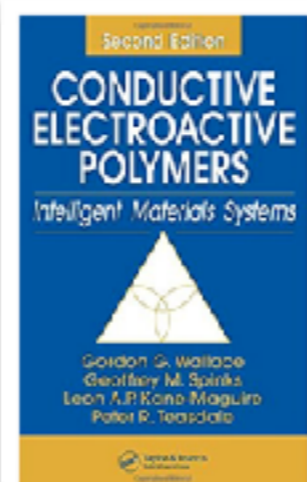
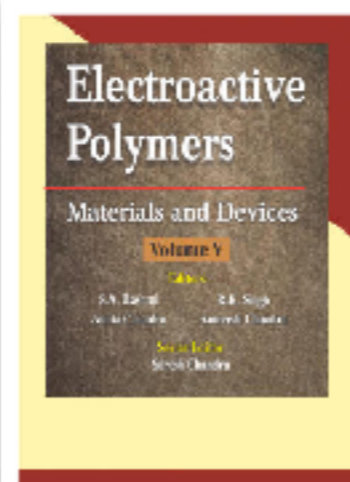
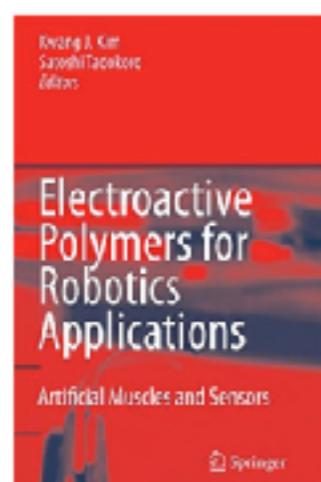
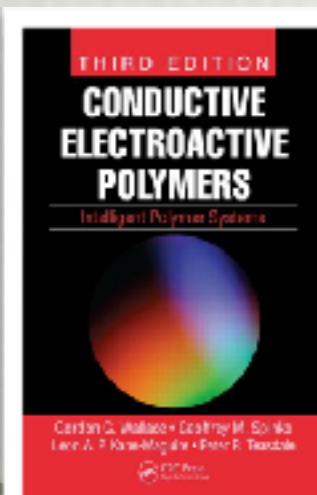
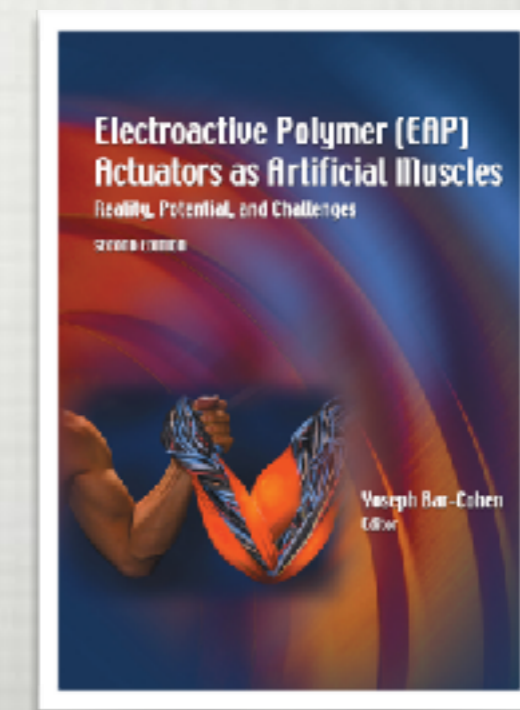
Multiple materials



Robotic applications

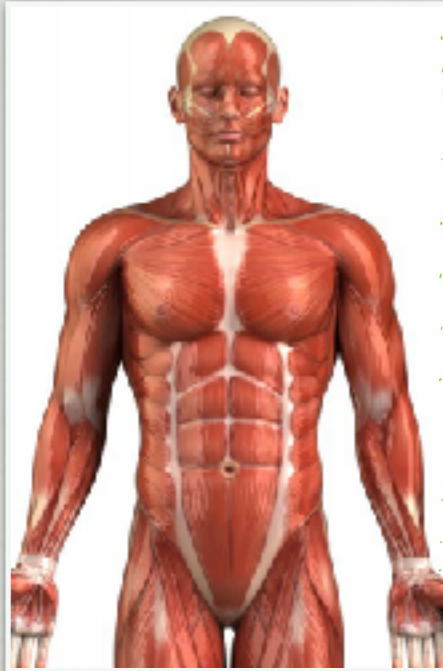


- ELECTRO-ACTIVE POLYMER (EAP) ARE BASED ON **ION-EXCHANGE PROPERTIES** OF RESINS
- EAP ARE USED IN SEVERAL **CHEMICAL & ROBOTIC APPLICATIONS - ARTIFICIAL MUSCLES**

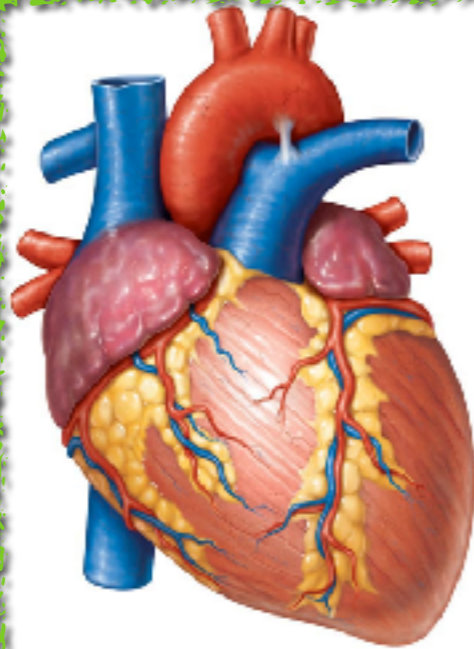


ACTIVE DEFORMABLE MEDIA

● BIOLOGICAL TISSUES



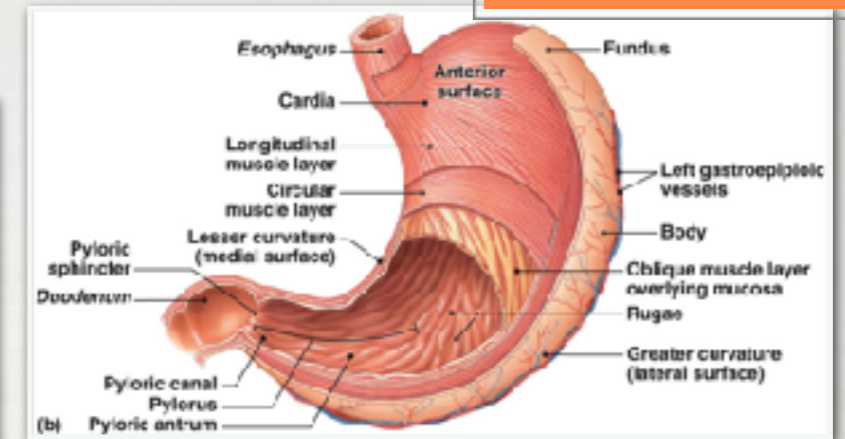
Skeletal Muscles



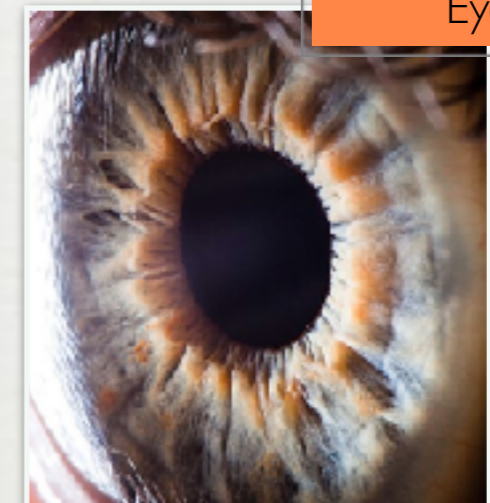
Heart



Intestines



Stomach



Eye's Iris

- **BIOLOGICAL ACTIVE TISSUES:** HEART, SKELETAL MUSCLE, GASTRO-INTESTINE, EYE'S IRIS...
- SHOW THE ABILITY TO DEVELOP **CONTRACTIONS**, PRODUCING THE MECHANICAL FORCES NECESSARY TO THE ORGAN'S FUNCTION.
- CONTRACTIONS ORIGINATED BY AN **ELECTRIC POTENTIAL** DUE TO TRANSMEMBRANE (K^- , Na^+) AND INTRACELLULAR **IONS** (Ca^{++})

EXCITATION-CONTRACTION

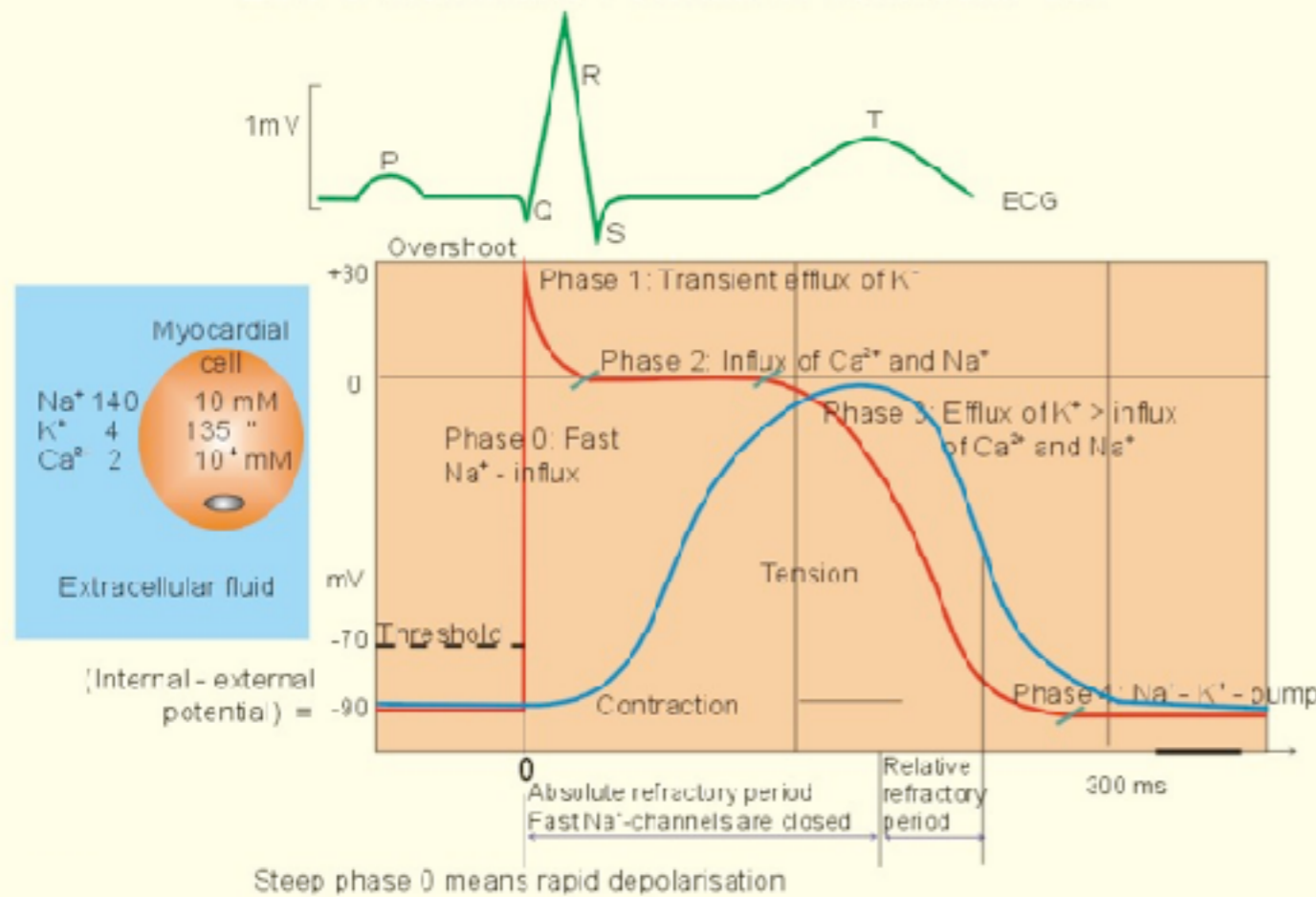
Biophysical Motivation

- **MACROSCOPIC** CORRELATION BETWEEN:

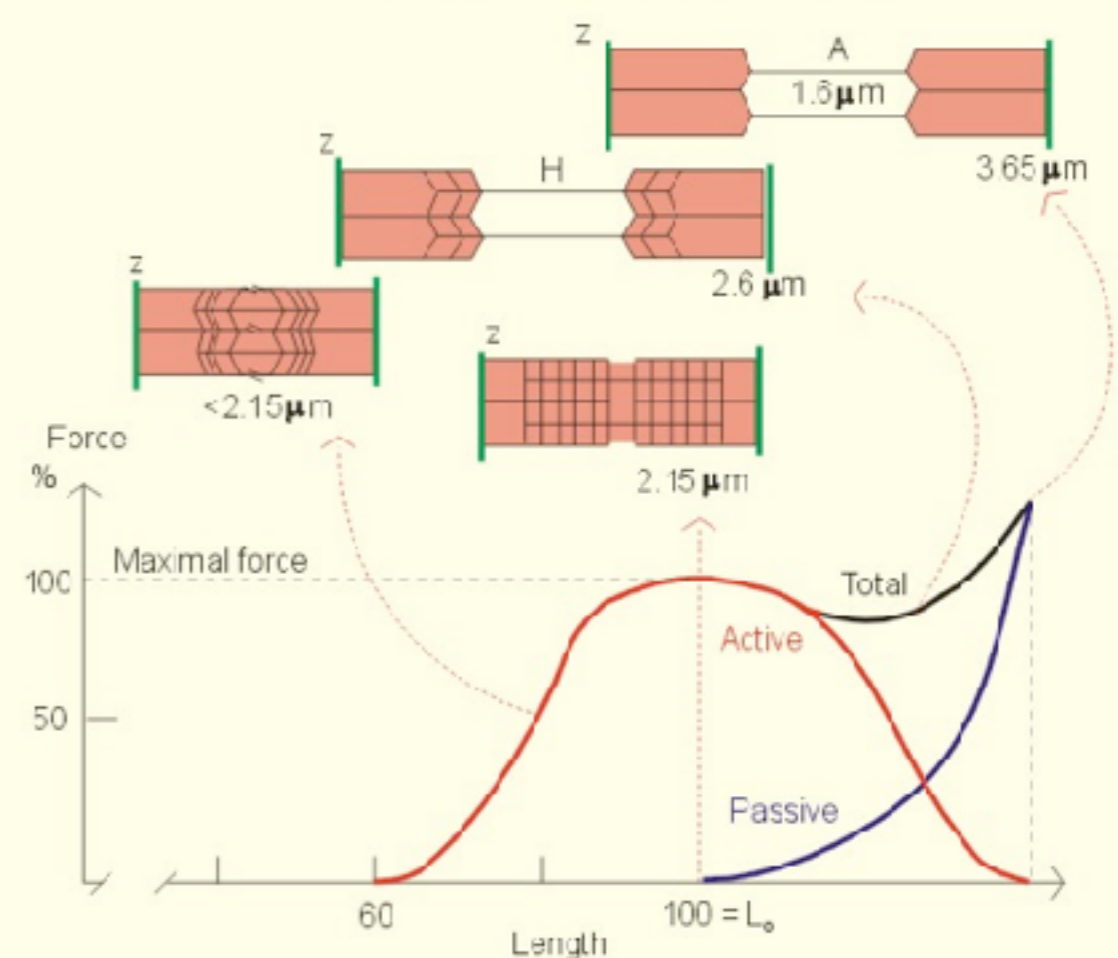
- * THE 5 ECG PHASES
- * THE TRANSMEMBRANE POTENTIAL
- * THE CONTRACTION WAVE FOR A MUSCULAR FIBER (CALCIUM)

- **MICROSCOPIC** RELATION BETWEEN: **ACTIVE - PASSIVE CONTRIBUTIONS**

ECG & Membrane Potential of Ventricular Cell



The Force-length Diagram



ATRIAL MEF

Biophysical Motivation

- ROLE OF **MECHANO-ELECTRIC FEEDBACK (MEF)** IN **ARRHYTHMOGENESIS**

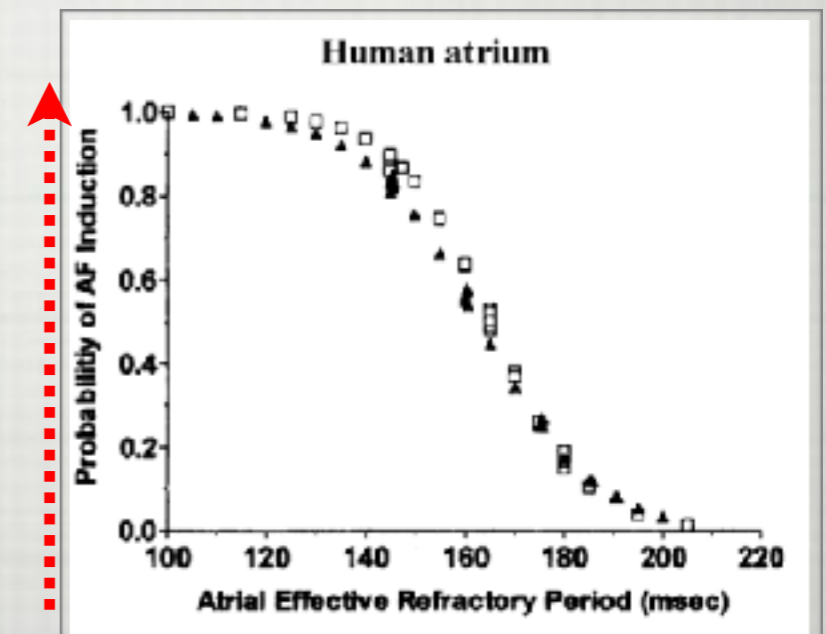
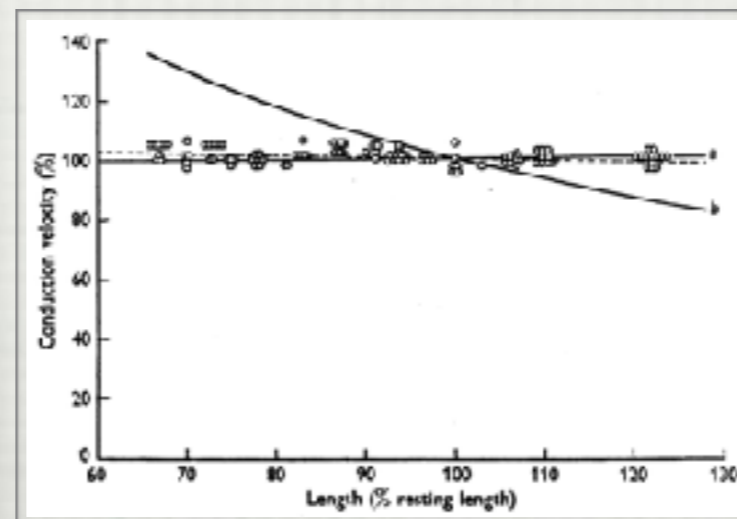
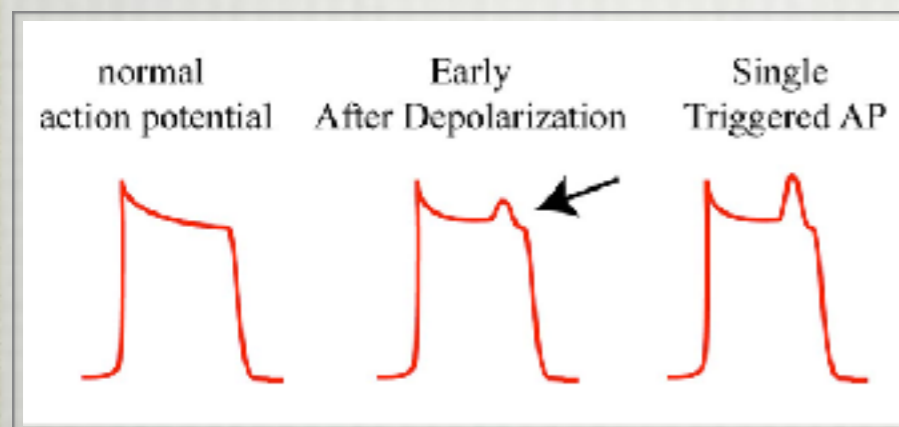
- **STRETCH** CONTRIBUTES TO:

- ✱ 1) FOCAL ARRHYTHMIAS BY INDUCING **EARLY AFTER-DEPOLARIZATIONS**

- ✱ 2) REENTRANT ARRHYTHMIAS BY

- ◆ SHORTENING THE **CONDUCTION VELOCITY**

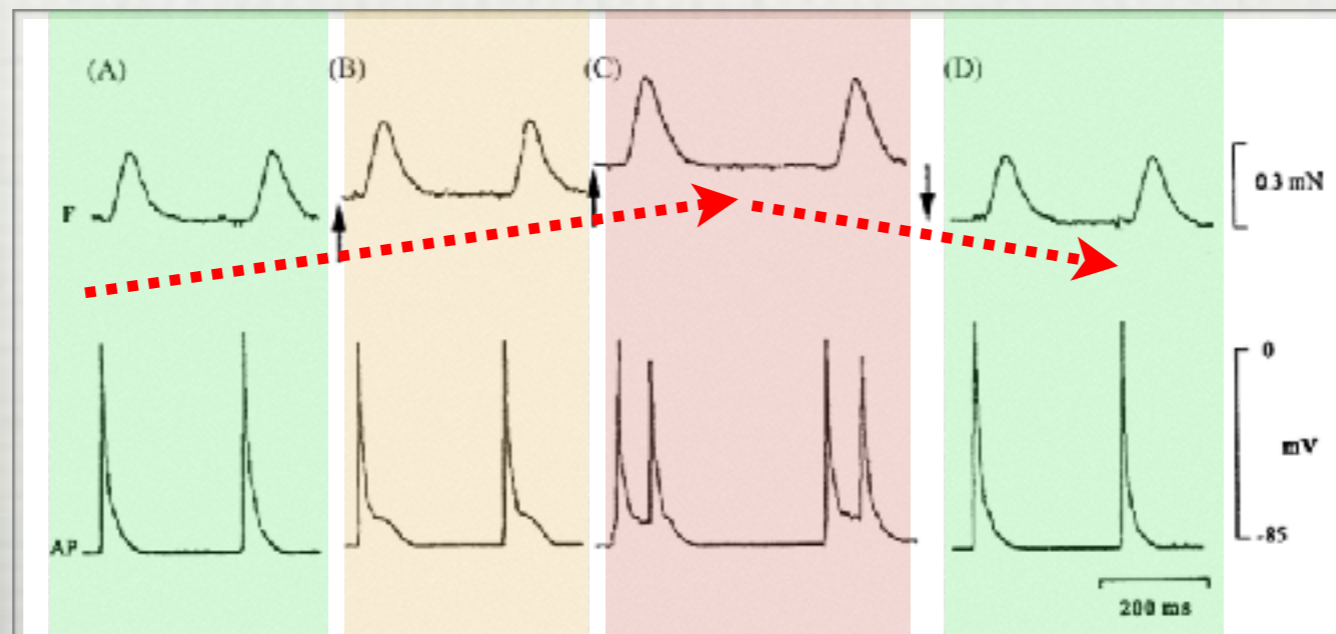
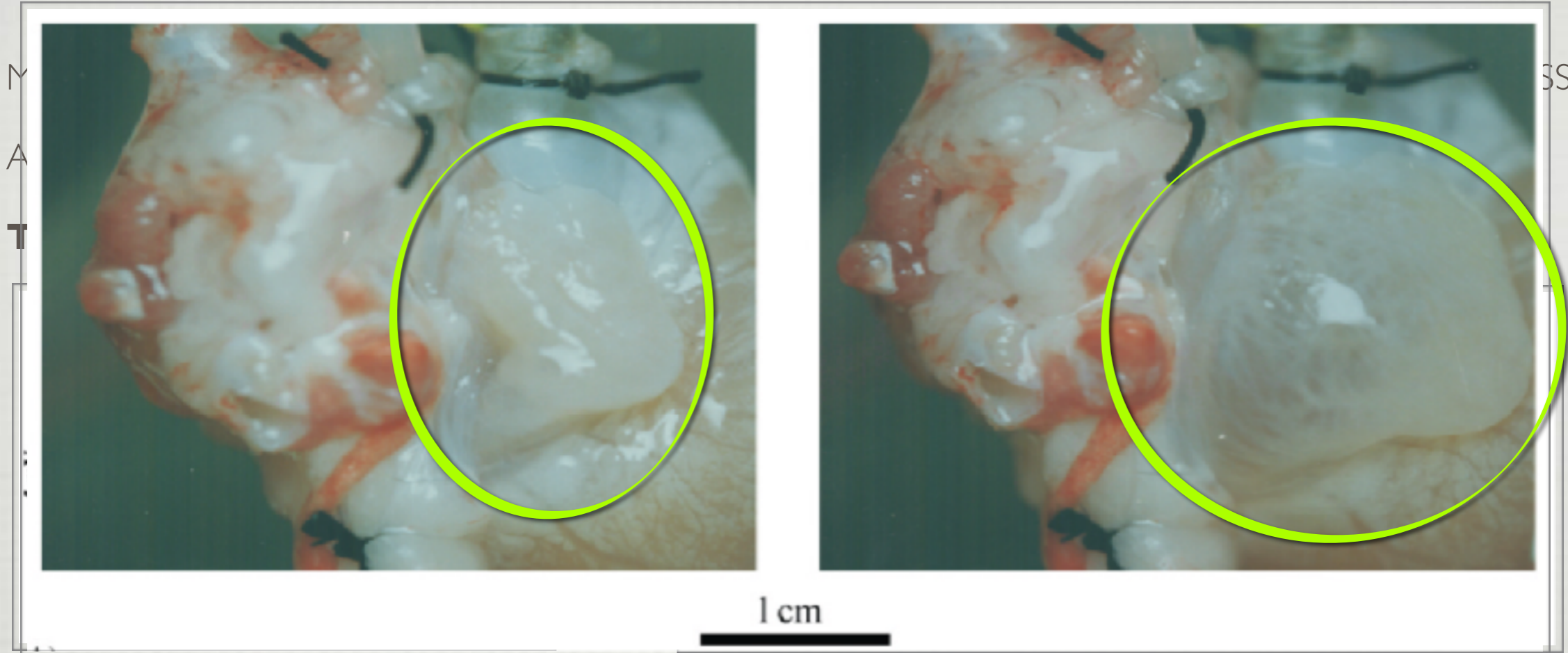
- ◆ SHORTENING **REFRACTORY PERIOD** INCREASING **SPATIAL DISPERSION**



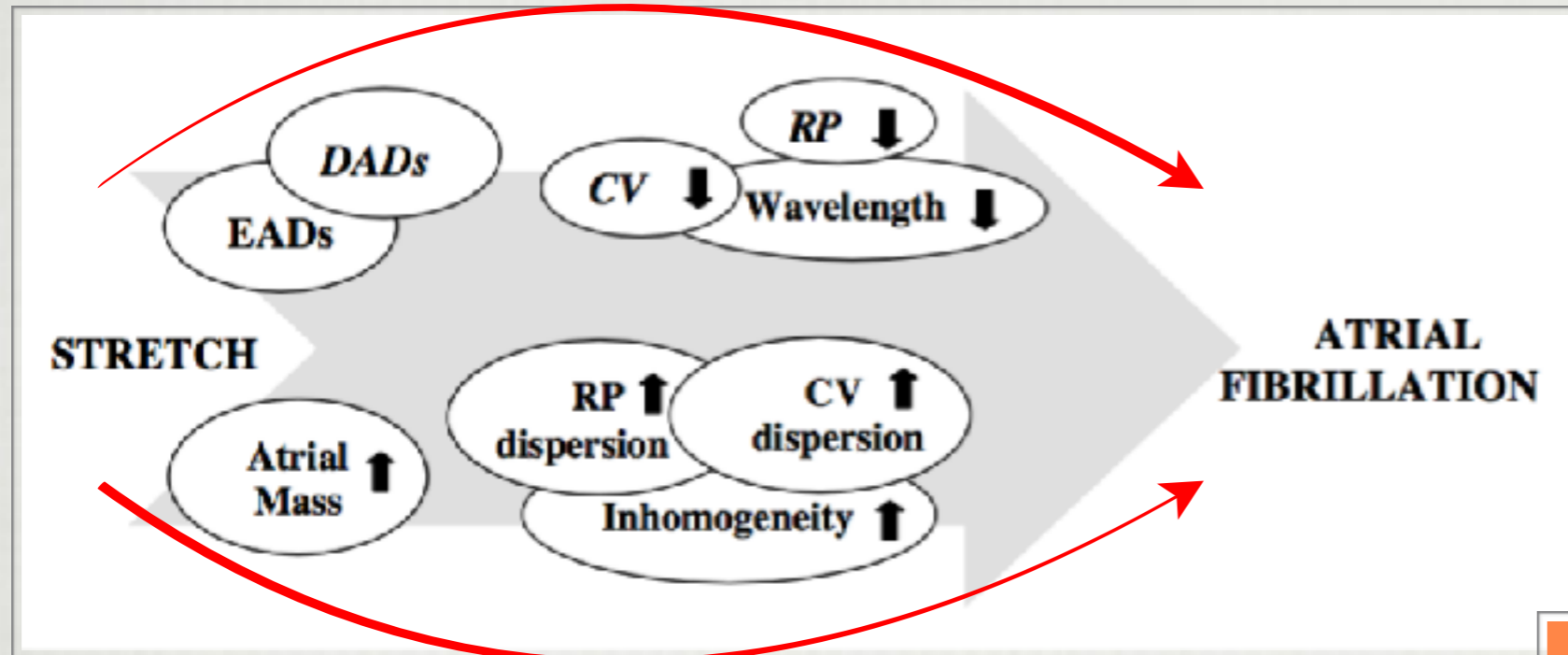
ATRIAL MEF

Biophysical Motivation

ISSUE



ATRIAL MEF



Complex Problem

Summary of the effects of acute stretch on atrial refractoriness in in vivo studies

Dilatation models	Species	ΔP (mmHg)	Atrial RP	Atrial RP dispersion	References
Simultaneous AV pacing	Dog	5	↑	—	Kaseda and Zipes (1988)
	Human	3.5	↑	—	Klein et al. (1990)
	Human	4	↑	↑	Chen et al. (1999)
	Human	4	↔	—	Calkins et al. (1992b)
	Human	8(peak)	↓	—	Calkins et al. (1992a)
	Human	7.4	↓	↑	Tse et al. (2001)
SVT	Human	2.7	↑	—	Klein et al. (1990)
	Human	4.5	↑	↑	Chen et al. (1998)
Volume overload	Dog	≥ 4	↑	—	Sideris et al. (1994)
	Dog	1.2	↑	↑	Satoh and Zipes (1996)
	Goat	4.5	↔	—	Wijffels et al. (1997)
Balloon inflation	Dog	8	↓	—	Solti et al. (1989)

SVT is supraventricular tachycardia, ΔP the average change in mean atrial pressure, RP refractory period. The parameter measured increased (↑), decreased (↓), or did not change (↔).

ACTIVE ELECTRO-MECHANICS (EM)

● **ELECTRO-ACTIVE SYSTEMS'** CHARACTERS:

- * DEFORMATIONS MAY INDUCE A **CHANGE** OF THE EVENTUAL **INITIAL ISOTROPY** OF A BODY
- * AN **INTERMEDIATE NON COMPATIBLE CONFIGURATION** IS USUALLY ACCOUNTED FOR (MULTIPLICATIVE DECOMPOSITION)
- * GENERALIZED CONSTITUTIVE EQUATIONS USUALLY IMPLY MULTIPLE PHYSICAL COUPLINGS (VISCOSITY, DAMAGE, GROWTH, TEMPERATURE)

● **ELECTRO-ACTIVE BIOLOGICAL SYSTEMS'** CHARACTERS:

- * THE **INTERNAL ACTIN-MYOSIN BINDING** (EXCITATION-CONTRACTION - MEF) CAN BE CONSIDERED AS A **MICRO-STRUCTURAL ALTERATION OF THE INTERNAL KINEMATIC STATE** OF THE MUSCLE FIBRE, WHICH LEADS TO CHANGES IN THE MACROSCOPIC BEHAVIOR

ACTIVE ELECTRO-MECHANICS (EM)

- **ACTIVE STRESS** (NASH & PANFILOV 2004)

Proof of Concept

- * MOST USED APPROACH IN CARDIAC ELECTRO-MECHANICS

- **ACTIVE STRAIN** (CHERUBINI ET AL. 2008, AMBROSI ET AL. 2011, RUIZ-BAIER ET AL. 2012)

- * REPRESENTS AN EIGENDEFORMATION APPROACH WELL ESTABLISHED FOR REALISTIC CARDIAC ELECTRO-MECHANICAL APPLICATIONS

- * **MULTIPLICATIVE DECOMPOSITION** OF THE DEFORMATION GRADIENT TENSOR

- **THERMODYNAMIC APPROACH** (GIZZI ET AL. 2015)

- * **RELATES** THE CONCEPTS OF **ACTIVE DEFORMATION** & **ACTIVE STRESS** THROUGH THERMODYNAMICAL ARGUMENTS

- * **ADDITIVE DECOMPOSITION OF THE STRAIN ENERGY**

- * **MAXWELL ELECTRO-STATICS**

LIMITATION

The active stress model does not derive from thermodynamical arguments

ADVANTAGES

Easy to implement in numerical codes for a proof of concept theoretical study

Active Stress Approach

$$\mathbf{P} = \mathbf{P}^p + \mathbf{P}^a$$

Passive Stress

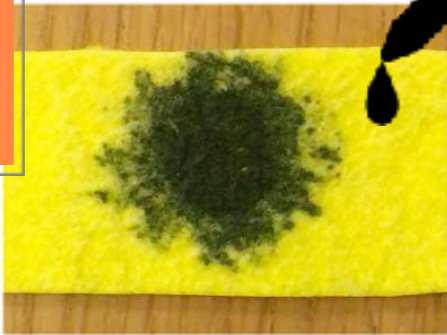
Active Stress

DIFFUSION IN SOLIDS

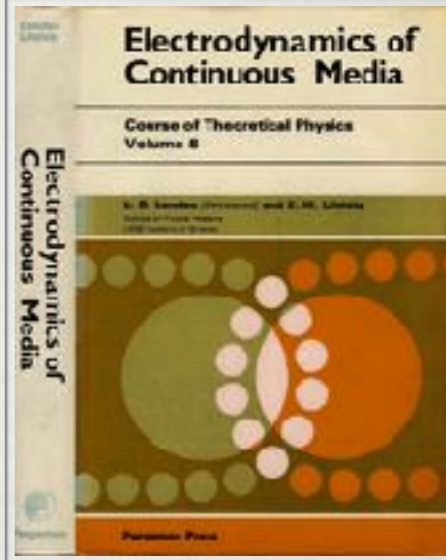
- **INFINITESIMAL ELECTRO-DYNAMICS**
- RATIONALIZE **DIFFUSION IN SOLIDS** BASED ON CONSERVATION OF MOMENTUM FOR THE DIFFUSING SPECIES
- THE BALANCE EQUATION CONTAINS:
 - * **STRESS SUPPORTED BY THE DIFFUSING SPECIES**
 - * **DIFFUSIVE FORCE** VECTOR FOR THE EXCHANGE OF MOMENTUM BETWEEN THE DIFFUSING SPECIES AND THE SOLID MATRIX
- **INTERDIFFUSING MATERIALS** ARE ACCOUNTED FOR BY THE CONSTITUTIVE PRESCRIPTIONS (APPLIED TO METALLURGY, POLYMER PHYSICS, GEOPHYSICS)

★ THESE TWO QUANTITIES ARE **NOT** IDENTIFIED IN THE **CLASSICAL DIFFUSION** INTERPRETATION

Homemade Proof of Concept



LANDAU, LIFSHITZ



COTTRELL, FLYNN

JOURNAL OF POLYMER SCIENCE: Polymer Physics Edition VOL. 11 (1973)
On the Stress-Dependent Diffusion Equation

Acta Mechanica 57, 265–283 (1980)
ACTA MECHANICA
 © by Springer-Verlag 1980
On the Problem of Diffusion in Solids
 By
 E. C. Aifantis, Urbana, Illinois

Acta Mechanica 45, 275–291 (1982)
ACTA MECHANICA
 © by Springer-Verlag 1982
On the Theory of Stress-Assisted Diffusion, I
 By
 R. K. Wilson and E. C. Aifantis, Urbana, Illinois

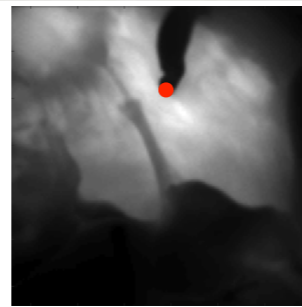
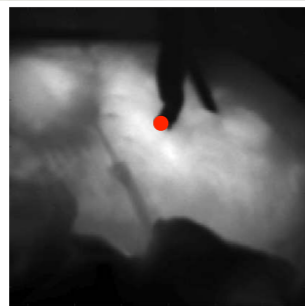
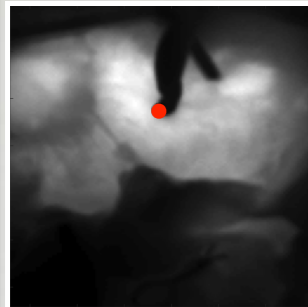
Acta Mechanica 47, 117–151 (1983)
ACTA MECHANICA
 © by Springer-Verlag 1983
On the Theory of Stress-Assisted Diffusion, II
 By
 D. J. Unger and E. C. Aifantis, Minneapolis, Minnesota

STRETCHED VENTRICLES

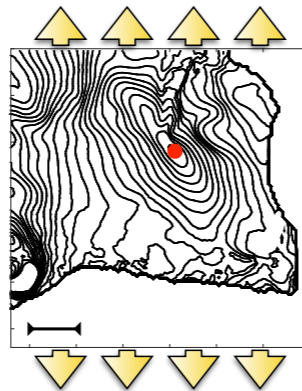
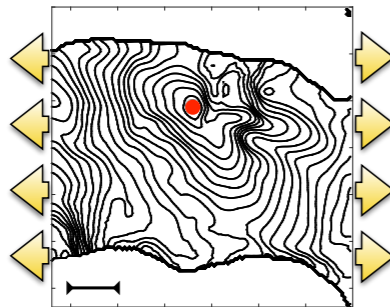
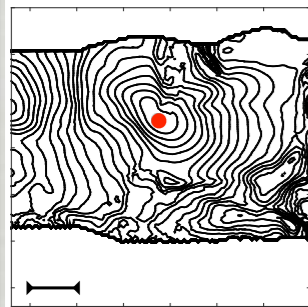
No stretch

H stretch

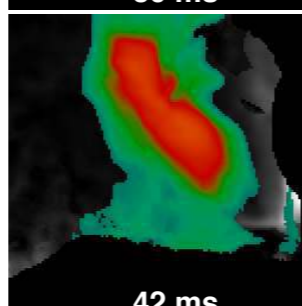
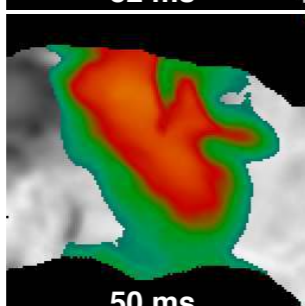
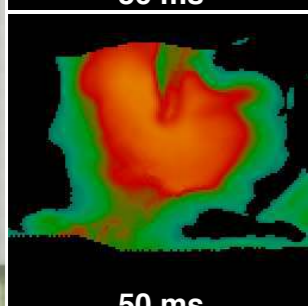
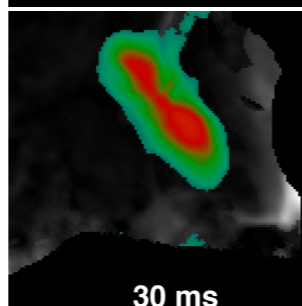
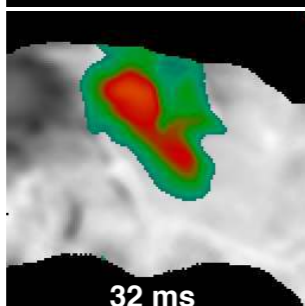
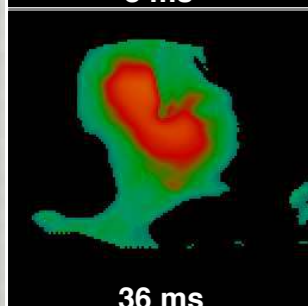
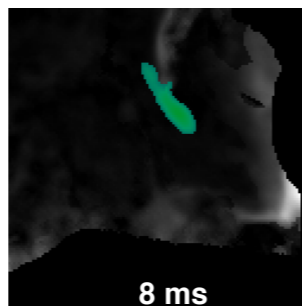
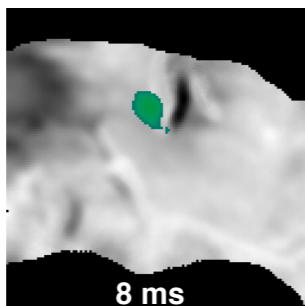
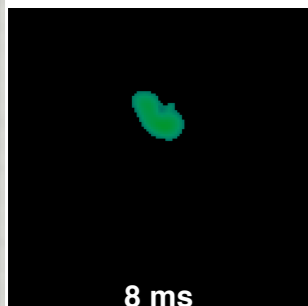
V stretch



**Pig Right Ventricle
Proof of Concept**



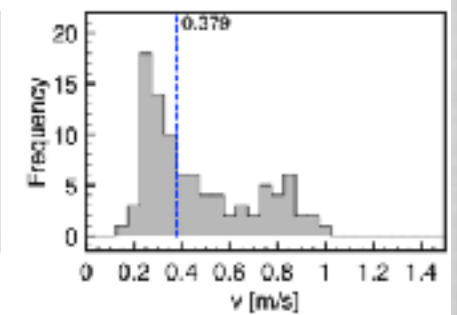
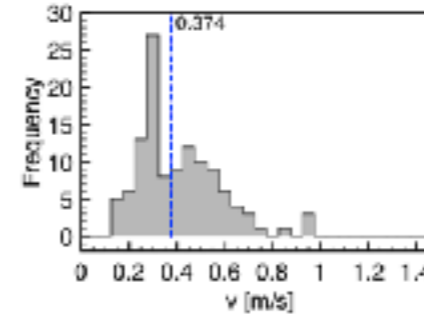
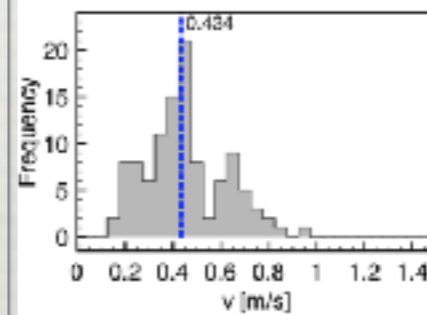
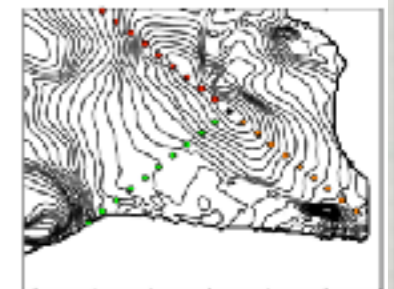
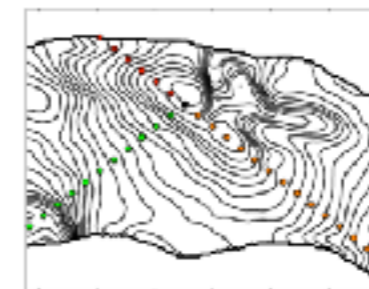
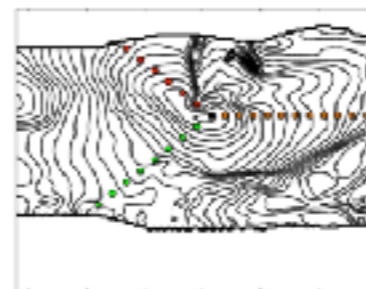
**Enhanced
Anisotropy**



No stretch

H stretch

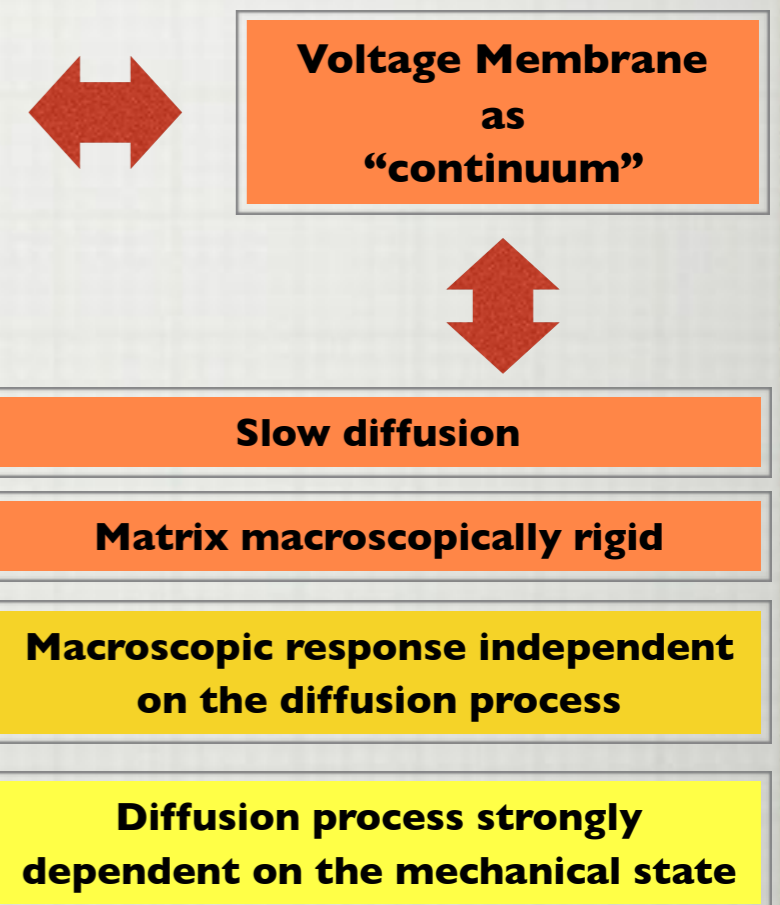
V stretch



Reduced CV

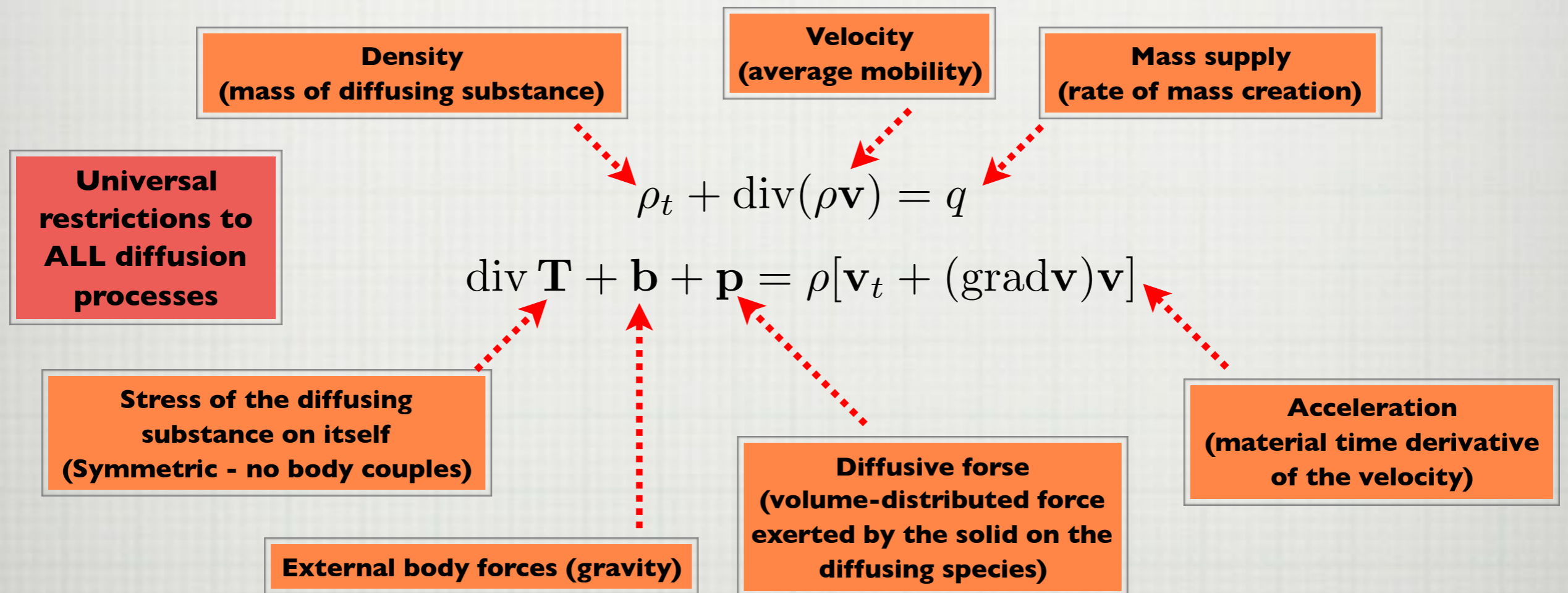
STRESS-ASSISTED DIFFUSION 'SAD'

- THE **BASIC PHENOMENOLOGY** IS THE SAME EITHER IF "MICROPOROSITY" OR "MACROPOROSITY" IS INVOLVED.
- DIFFUSION OF A **DILUTE SOLUTE IN A SOLID MATRIX**.
- GENERALIZE THE THEORETICAL WORK OF **FICK**
(A PHENOMENOLOGICAL DESCRIPTION OF DIFFUSION THAT PRECEDES ANY EXPERIMENTAL WORK) WHICH IS **CURRENTLY APPLIED** IN ANY ELECTROPHYSIOLOGICAL MODELING (CABLE EQUATION).
- THE THEORY OF **CONTINUOUSLY DISTRIBUTED MATTER** (EULER'S AXIOMS) LEADS TO THE **BALANCE OF MOMENTUM**.
- **CONSTITUTIVE EQUATIONS** WILL DIFFERENTIATE BETWEEN:
 - * **DIFFUSION IN STRESSED SOLIDS**
 - * **DIFFUSION OF VISCOUS SOLUTE**
 - * **DIFFUSION IN ELASTIC & INELASTIC MATERIALS**
- FRAMED WITHIN THE **THEORY OF MIXTURES**:
ATKIN & CRAINE, BOWEN, TRUSDELL, MAXWELL'S GASES THEORY



'SAD' THEORETICAL DERIVATION

Local form of Mass and Momentum balances



'SAD' THEORETICAL DERIVATION

Hp.1

Diffusive species do not support shear stress

Hp.2

Only the hydrostatic component of the stress affects the stress of the diffusing species

$$\rho_t + \text{div}(\rho \mathbf{v}) = \cancel{\rho_t} \quad \text{div } \mathbf{T} + \mathbf{b} + \cancel{\mathbf{r}} = \rho[\mathbf{v}_t + \cancel{(\text{grad } \mathbf{v})\mathbf{v}}] \quad \Rightarrow \quad \boxed{\rho_t + \text{div}(\mathbf{j}) = 0}$$

$$\text{div } \mathbf{T} + \mathbf{p} = 0$$

Representation theorems for non-linear isotropic or anisotropic vector and tensor functions (Spencer)

Nonlinear Stress (\mathbf{S}) in the solid

$$\Gamma_i = \gamma_{i1}(\rho, J_1, J_2, J_3)\mathbf{I} + \gamma_{i2}(\rho, J_1, J_2, J_3)\mathbf{S} + \gamma_{i3}(\rho, J_1, J_2, J_3)\mathbf{S}^2$$

J_1, J_2, J_3

The principal invariants of the stress tensor \mathbf{S} in the solid (frame indifferent)

$$\mathbf{j} = -(\alpha_0\mathbf{I} + \alpha_1\mathbf{S} + \alpha_2\mathbf{S}^2)\text{grad}\rho + (\alpha_0^*\mathbf{I} + \alpha_1^*\mathbf{S} + \alpha_2^*\mathbf{S}^2)\text{grad}(\cancel{t(\mathbf{S})})$$

Hp.3

Incompressibility

$$\mathbf{B} = \mathbf{F}\mathbf{F}^T$$

Strain measure

$$\mathbf{j} = -(\alpha_0\mathbf{I} + \alpha_1\mathbf{S} + \alpha_2\mathbf{S}^2)\text{grad}\rho$$



$$\mathbf{j} = -(\beta_0\mathbf{I} + \beta_1\mathbf{B} + \beta_{-1}\mathbf{B}^{-1})\text{grad}\rho$$

Named in the Literature:
"Stress-Assisted Diffusion Equation"

EM-SAD: A MINIMAL MODEL

Classical cubic
"Bistable" function

2v Reaction-Diffusion (RD) Model (Nash & Panfilov 2004)

RD Constitutive Equations

$$\frac{\partial V}{\partial t} = \frac{\partial}{\partial x_i} d_{ij}(\sigma_{ij}) \frac{\partial V}{\partial x_j} + I_{\text{ion}}(V, r) \star$$

$$\frac{dr}{dt} = f(V, r)$$

NO SAC

$$I_{\text{ion}} = -kV(V - a)(V - 1) - rV$$

$$f(V, r) = \left(\epsilon + \frac{\mu_1 r}{\mu_2 + V} \right) (-r - kV(V - b - 1))$$

Nonlinear
"Recovery" dynamics

Active Stress Formulation
(Nash & Panfilov 2004)

$$\frac{\partial T_a}{\partial t} = \epsilon(V)(k_{T_a} V - T_a)$$

Representation Formula for
2nd Order Isotropic Tensors

Two additional
Material Parameters

$$d_{ij}(\sigma_{ij}) = D_0 (\delta_{ij} + D_1 \sigma_{ij} + D_2 \sigma_{ik} \sigma_{kj})$$

Equilibrium

$$\frac{\partial \sigma_{ij}}{\partial x_i} = 0 \quad J = 1$$

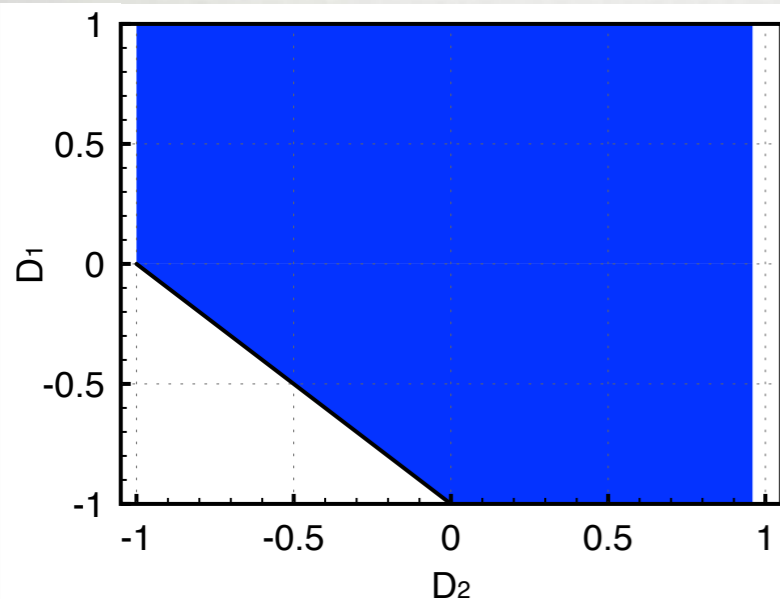
$$\sigma_{ij} = \sigma_{ij}^{\text{passive}} + \sigma_{ij}^{\text{active}} \quad \dots \rightarrow \quad \sigma_{ij} = 2c_1 b_{ij} - 2c_2 b_{ij}^{-1} - p \delta_{ij} + T_a \delta_{ij}$$

Isotropic Stress-Assisted Diffusion Model

Passive Neo-Hookean material with ISOTROPIC diffusion & ACTIVE stress

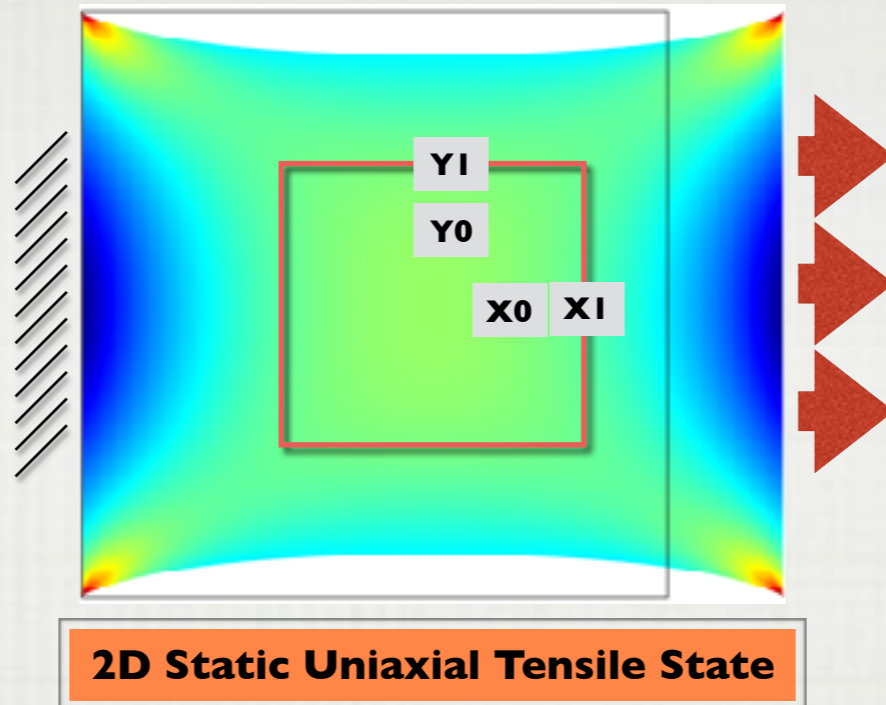
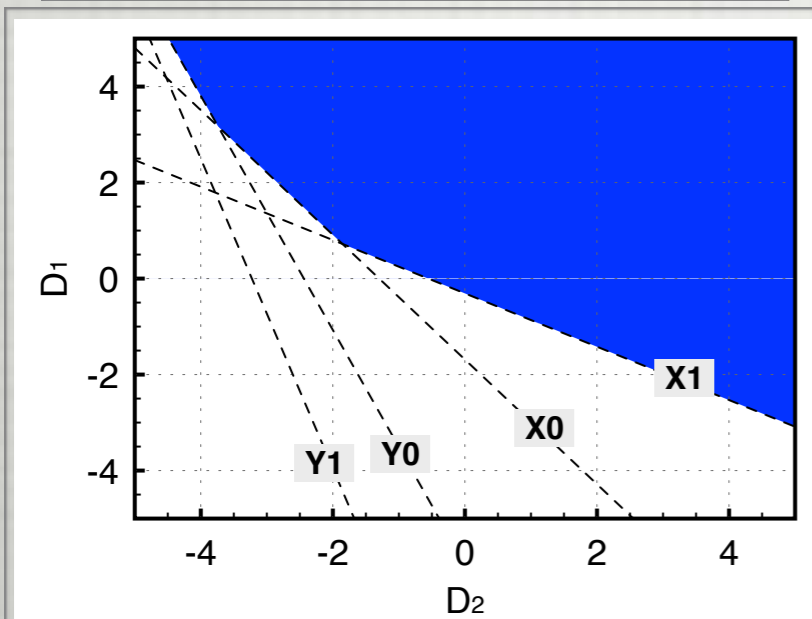
EM-SAD: PARAMETER SPACE

Theoretical Normalized Case



$$\sigma_1 = 1 \text{ or } \sigma_2 = 1$$

Computed Stress



Log von Mises stress

$$[\sigma_{ij}] = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$$

$$[\sigma_{ik}\sigma_{kj}] = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}$$

$$d_{ij}(\sigma_{ij}) = D_0 (\delta_{ij} + D_1 \sigma_{ij} + D_2 \sigma_{ik} \sigma_{kj})$$

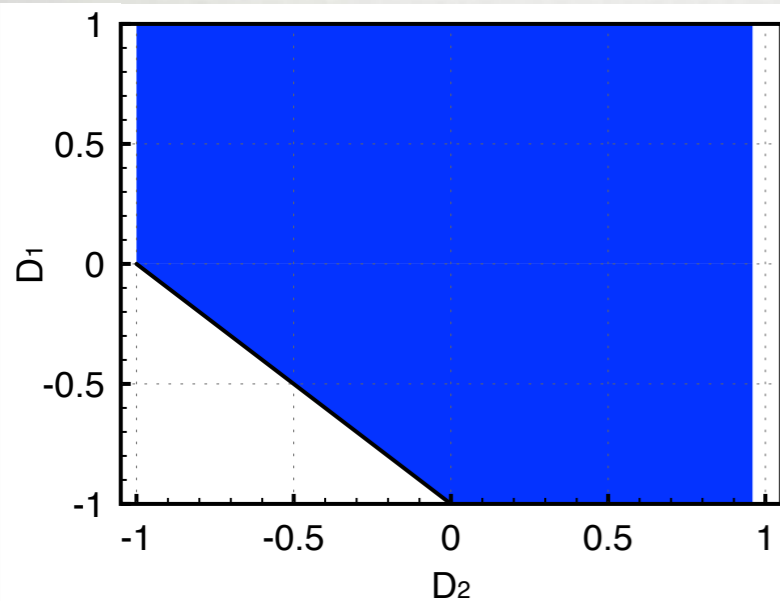
$$[d_{ij}] = \begin{bmatrix} 1 + D_1 \sigma_1 + D_2 \sigma_1^2 & 0 \\ 0 & 1 + D_1 \sigma_2 + D_2 \sigma_2^2 \end{bmatrix}$$

**Ellipticity regime
(Non-negative diffusion)**

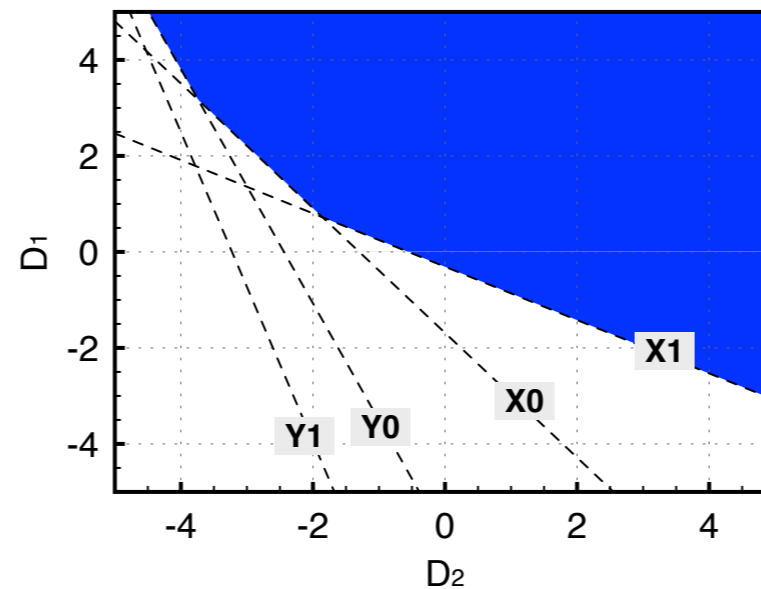
$$1 + D_1 \sigma_1 + D_2 \sigma_1^2 > 0, \quad \rightarrow \quad 4D_2 > D_1^2$$

EM-SAD: PARAMETER SPACE

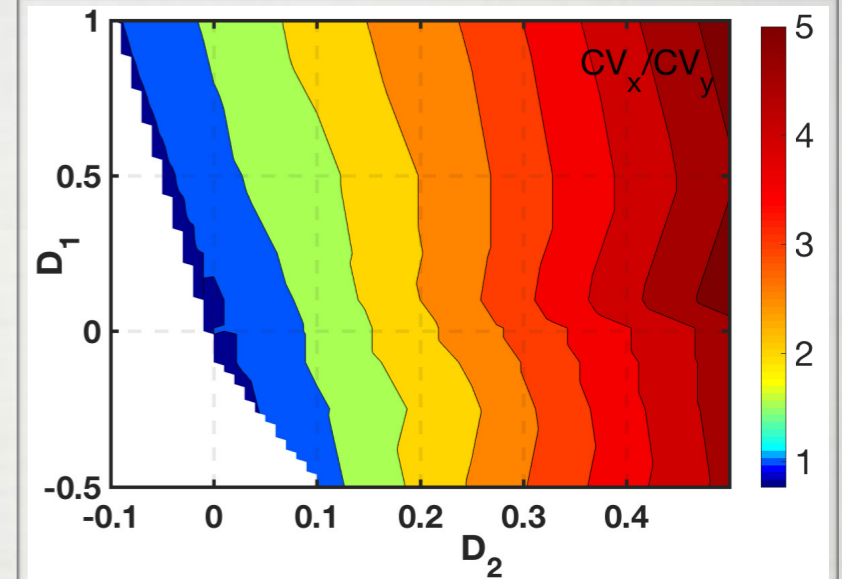
Theoretical Normalized Case



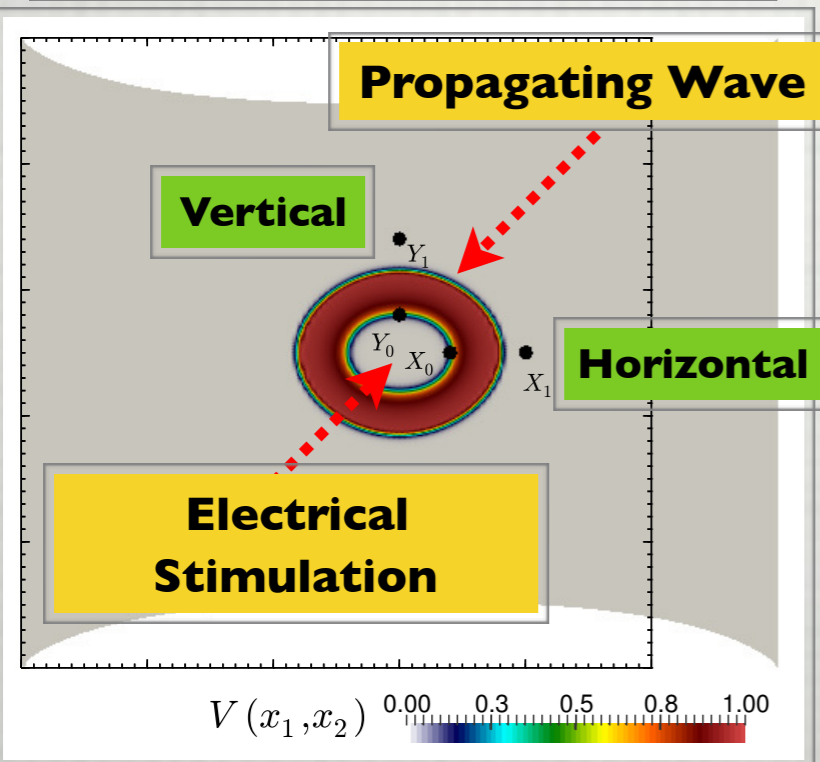
Computed Stress



Numerical-Physical CV



Conduction Velocity

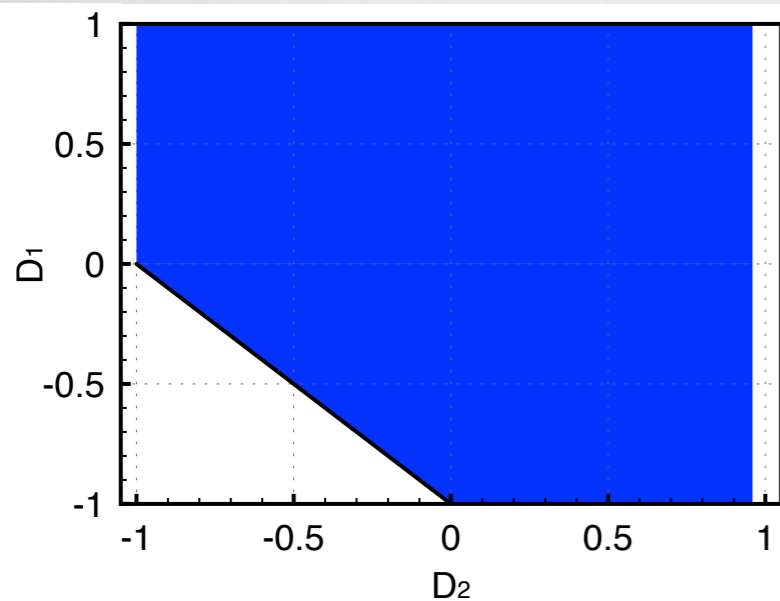


$$d_{ij}(\sigma_{ij}) = D_0 (\delta_{ij} + D_1 \sigma_{ij} + D_2 \sigma_{ik} \sigma_{kj})$$

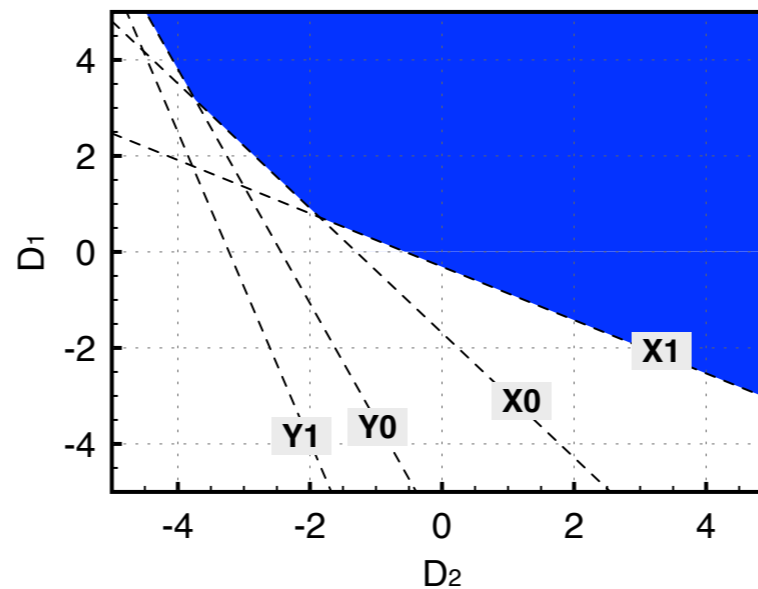
1. Calculate the Conduction Velocity of the Excitation Wave (Voltage) for different combinations of the Material Parameters
2. Compute the ratio CV_x/CV_y

EM-SAD: PARAMETER SPACE

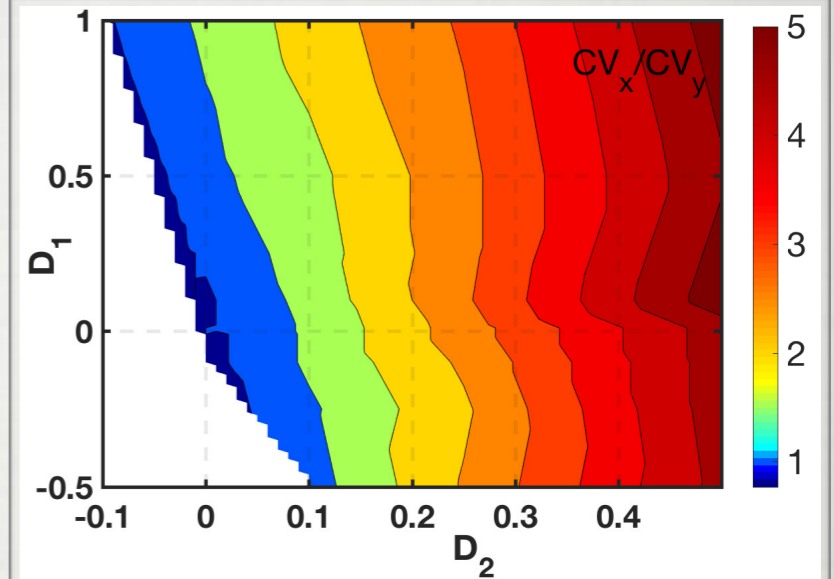
Theoretical Normalized Case



Computed Stress



Numerical-Physical CV

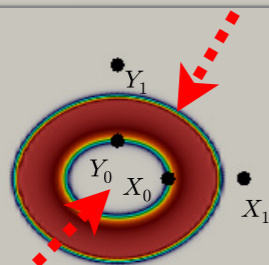


$$D_1 = 0, D_2 = 0$$

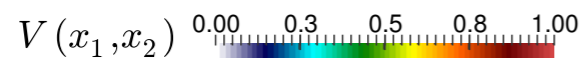
$$D_1 = -0.01, D_2 = 0.01$$

$$D_1 = -0.25, D_2 = 0.25$$

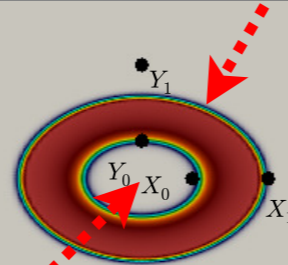
Propagating Wave



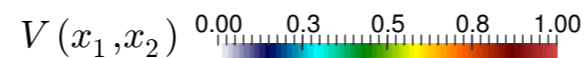
Electrical Stimulation



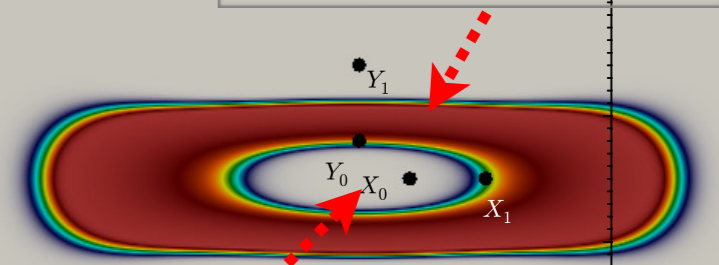
Propagating Wave



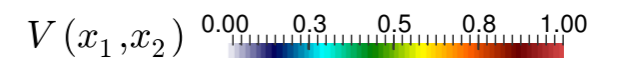
Electrical Stimulation



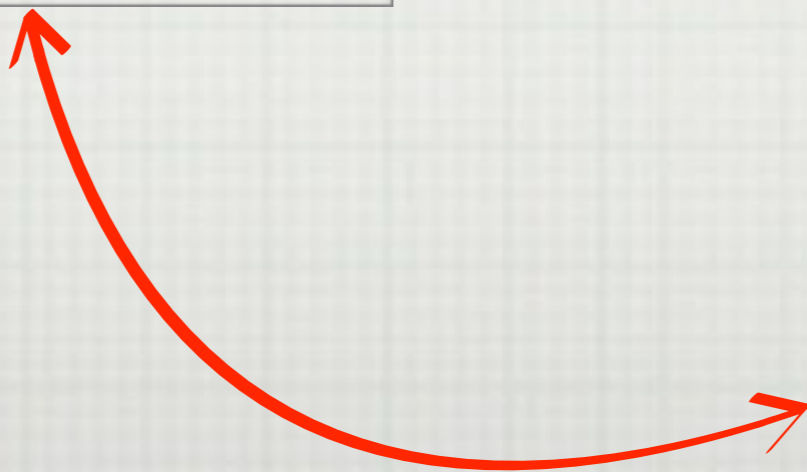
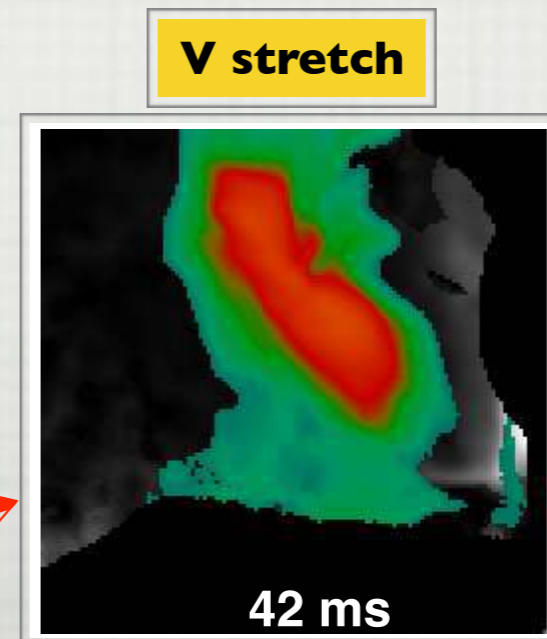
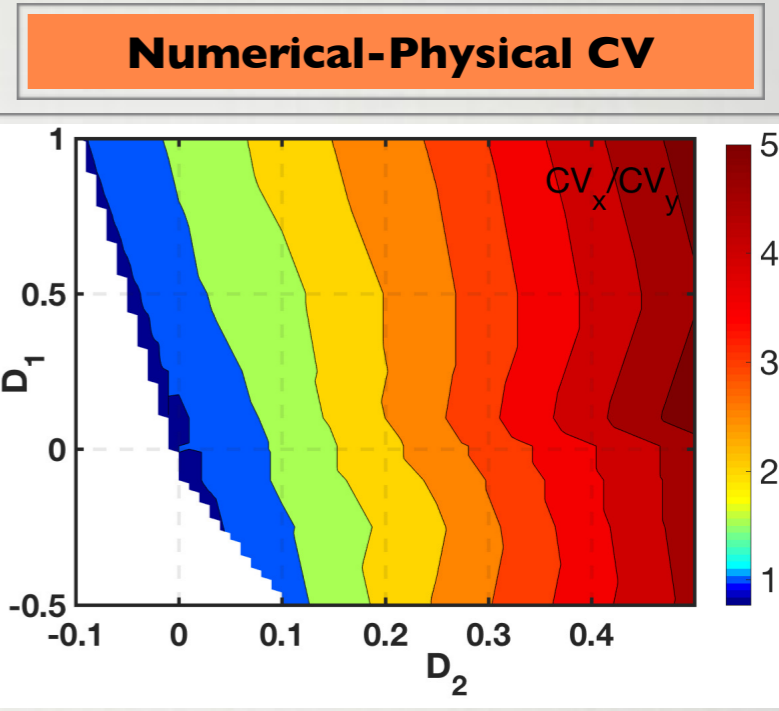
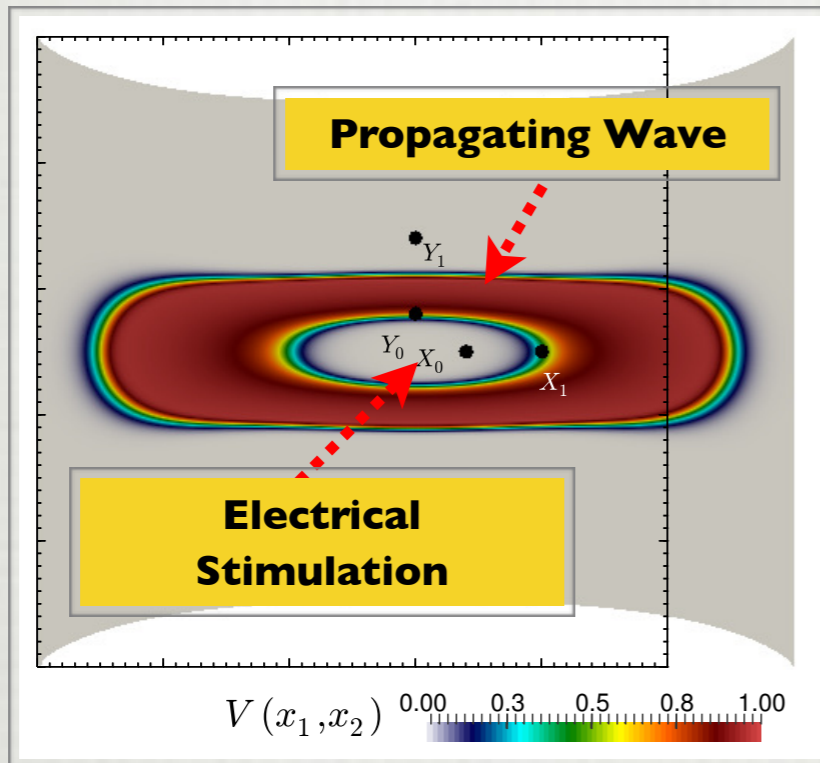
Propagating Wave



Electrical Stimulation

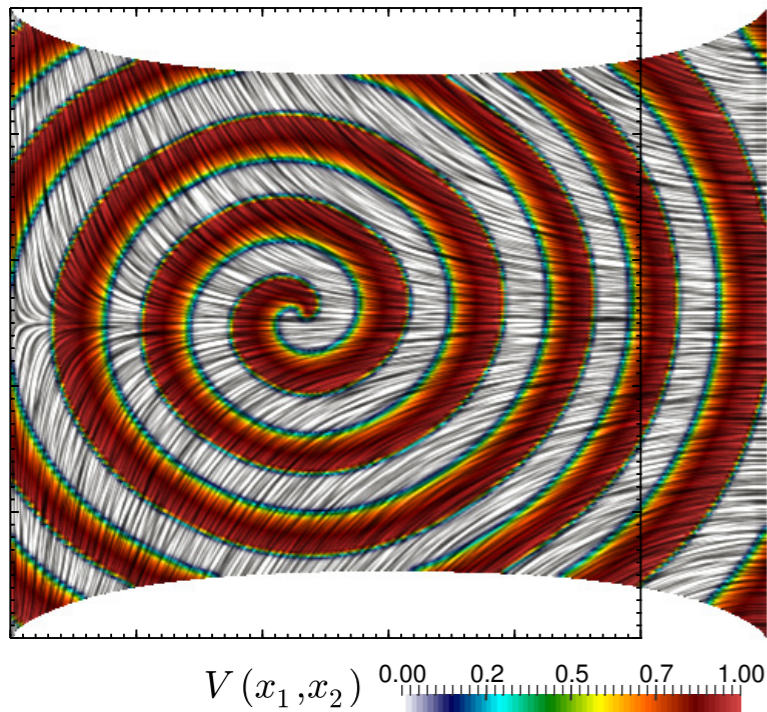


EM-SAD: PARAMETER SPACE



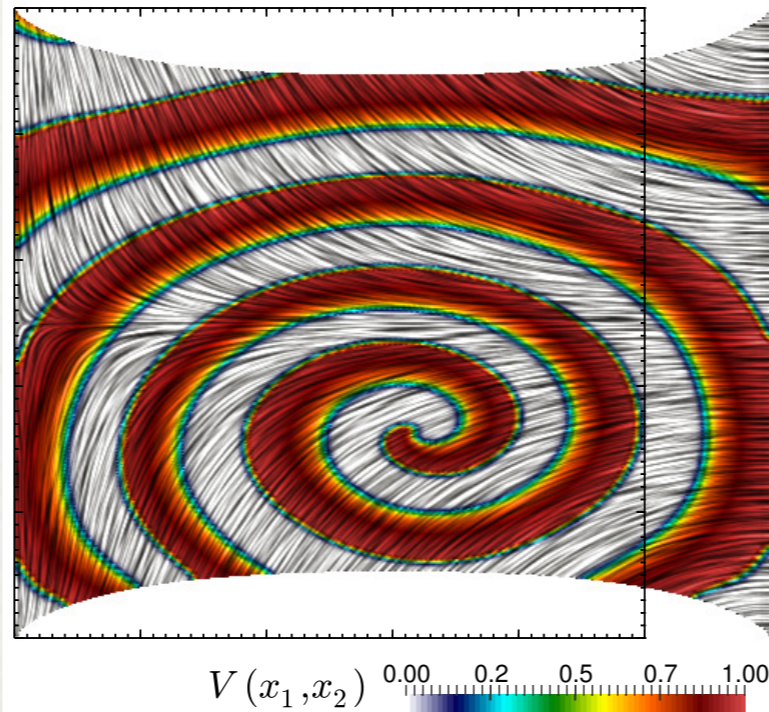
EM-SAD: INDUCED SPIRAL DRIFT

Nash & Panfilov 2004
Active Stress Model
NO DRIFT



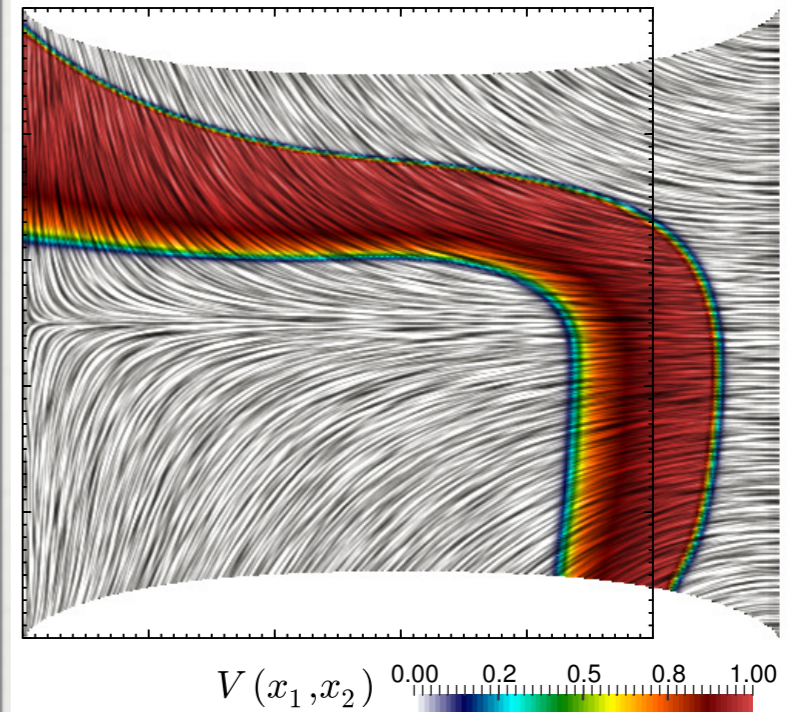
$$D_1 = 0, D_2 = 0$$

Novel
Stress-Assisted Diffusion
DIRECTIONAL SPIRAL DRIFT

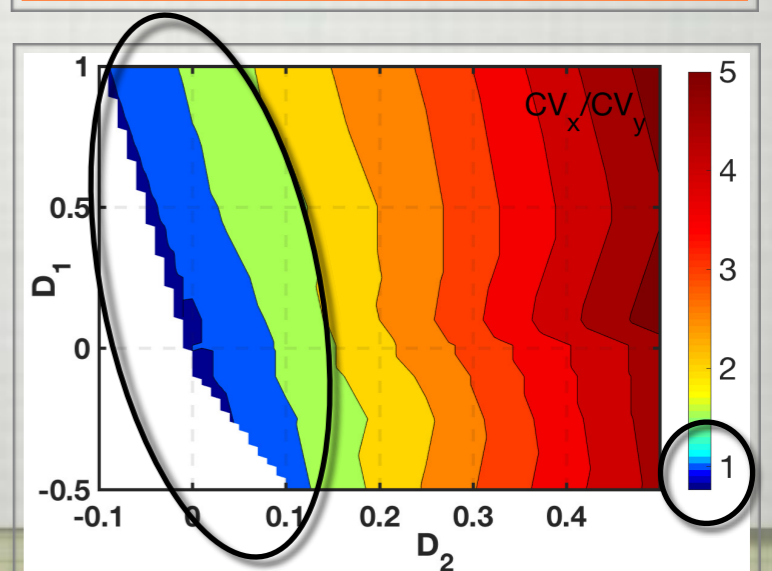


$$D_1 = -0.015, D_2 = 0.015$$

Novel
Stress-Assisted Diffusion
UNEXPECTED
DISTORTED EXCITATION



$$D_1 = 0.15, D_2 = 0.015$$

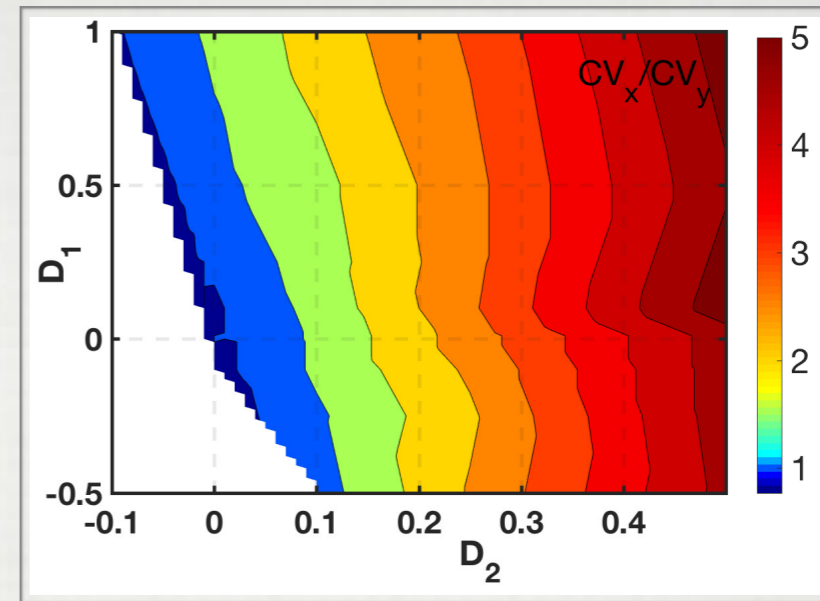


EM-SAD: ENHANCED MEANDERING

Uniaxial Sinusoidal Mechanical Loading



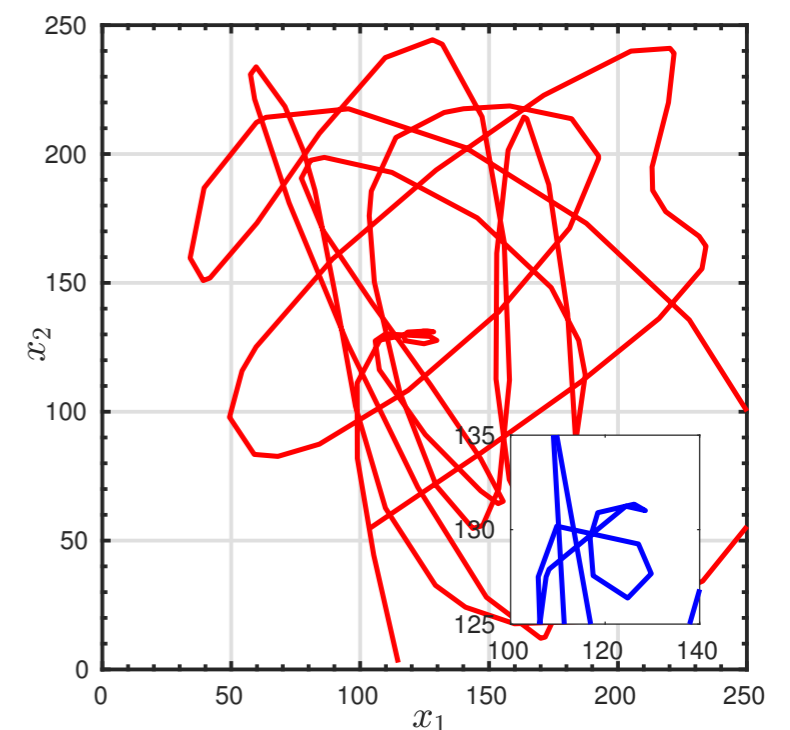
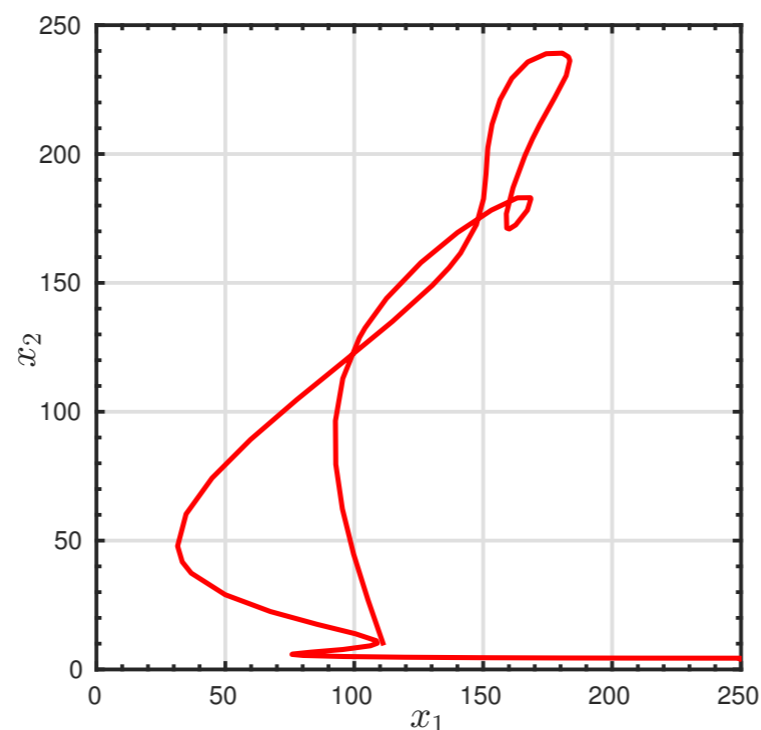
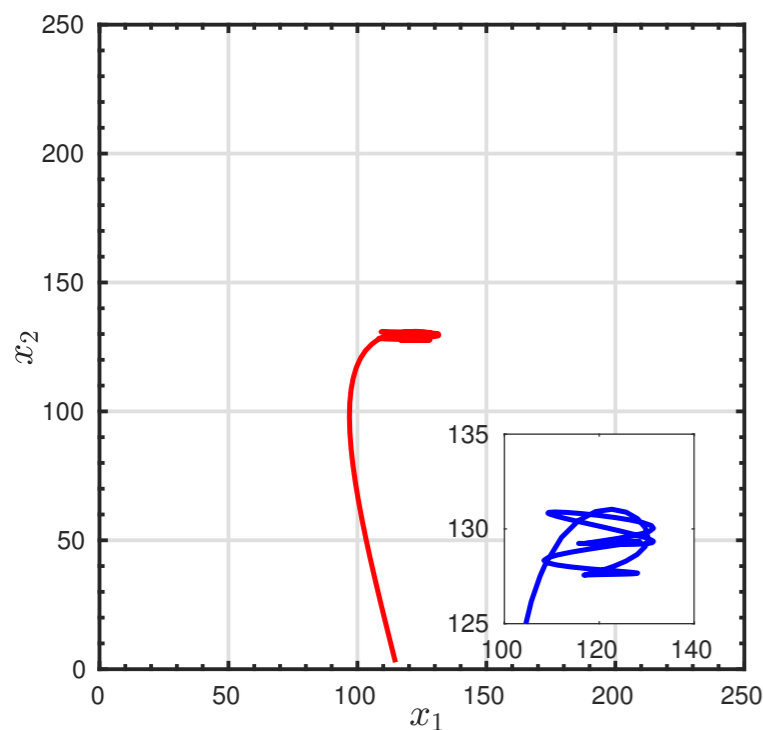
$$\mathbf{u} = \left[50 \sin^2 \left(\frac{\pi}{50} [t - t^*] \right), 0 \right]^T$$



$D_1 = -0.015, D_2 = 0.015$

$D_1 = 0.275, D_2 = -0.275$

$D_1 = -0.5, D_2 = 0.5$



EM - SAD & SAC (MINIMAL)

2v Reaction-Diffusion (RD) Model

$$\frac{\partial V}{\partial t} = \frac{\partial}{\partial x_i} d_{ij}(\sigma_{ij}) \frac{\partial V}{\partial x_j} + I_{\text{ion}}(V, r) + I_{\text{sac}}(\lambda, V)$$

$$\frac{dr}{dt} = f(V, r)$$

Active Stress Formulation

$$\frac{\partial T_a}{\partial t} = \epsilon(V)(k_{T_a} V - T_a)$$

RD Constitutive Equations

$$I_{\text{ion}} = -kV(V - a)(V - 1) - rV$$

$$f(V, r) = \left(\epsilon + \frac{\mu_1 r}{\mu_2 + V} \right) (-r - kV(V - b - 1))$$

Isotropic Stress-Assisted Diffusion Model

$$d_{ij}(\sigma_{ij}) = D_0 (\delta_{ij} + D_1 \sigma_{ij} + D_2 \sigma_{ik} \sigma_{kj})$$

Equilibrium

$$\frac{\partial \sigma_{ij}}{\partial x_i} = 0 \quad J = 1$$

$$\sigma_{ij} = 2c_1 b_{ij} - 2c_2 b_{ij}^{-1} - p \delta_{ij} + T_a \delta_{ij}$$

$$I_{\text{sac}}(\lambda, V) = G_s H_{\text{sac}} (\lambda - 1) (V_{\text{sac}} - V)$$

YES SAC

**Minimal model of
Stretch-Activated Currents**

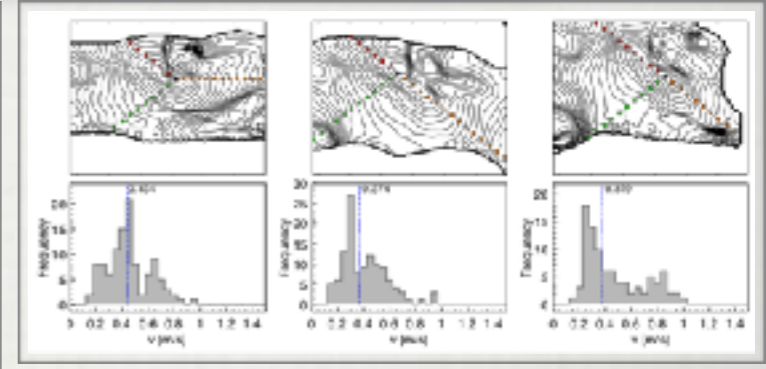
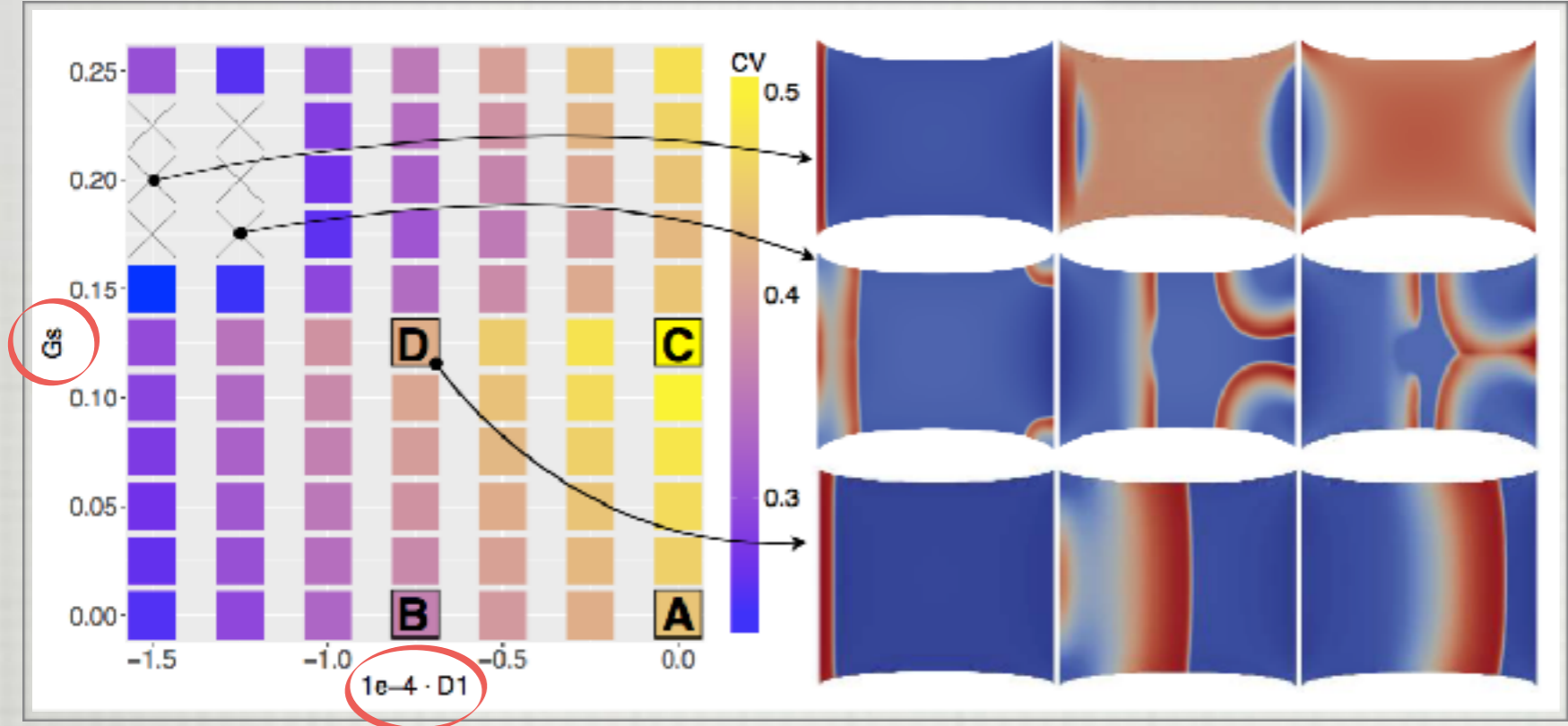
EM - SAD & SAC

fixed

$$d_{ij}(\sigma_{ij}) = D_0 (\delta_{ij} + D_1 \sigma_{ij} + D_2 \sigma_{ik} \sigma_{kj})$$

$$I_{\text{sac}}(\lambda, V) = G_s H_{\text{sac}}(\lambda - 1)(V_{\text{sac}} - V)$$

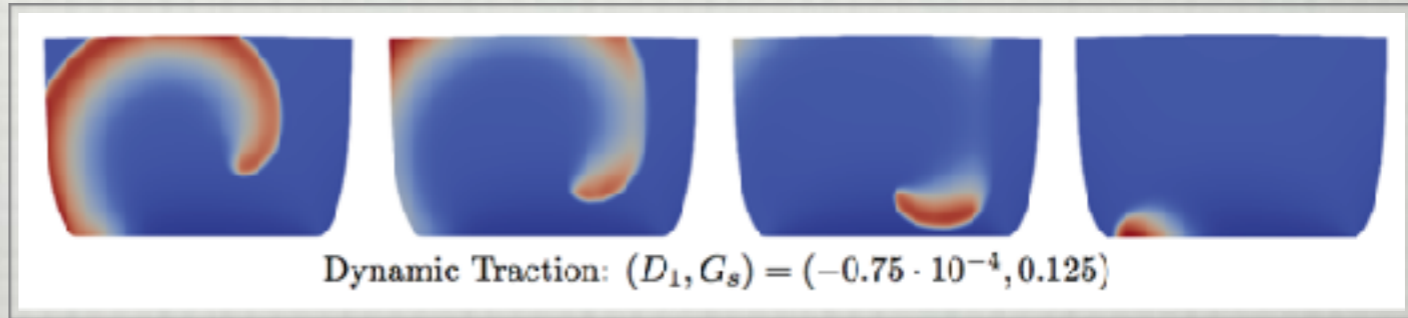
**Conduction Velocity Analysis
(planar wave)**



Two Competing Effects

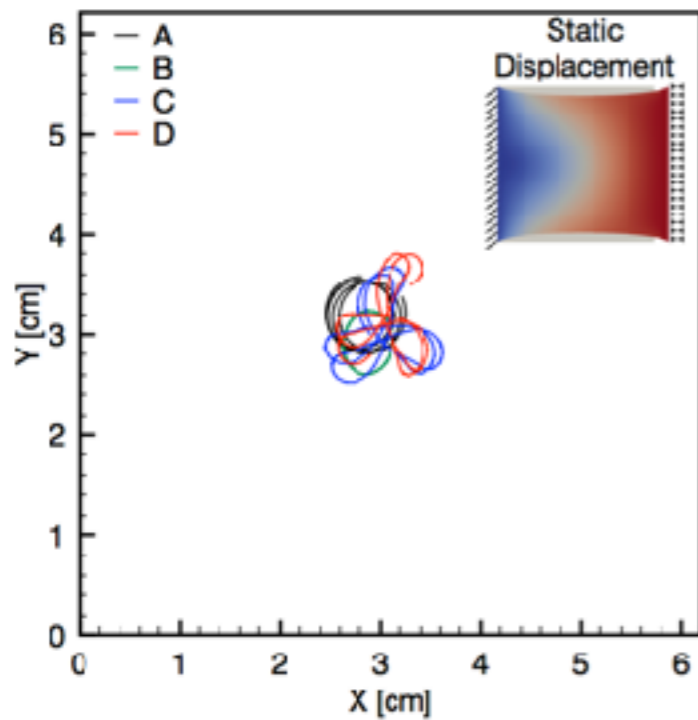
**Importance of “Mechanical”
Boundary Conditions**

**Static/Dynamic & Displacement/
Traction Boundary Conditions
greatly affects the emerging dynamics**

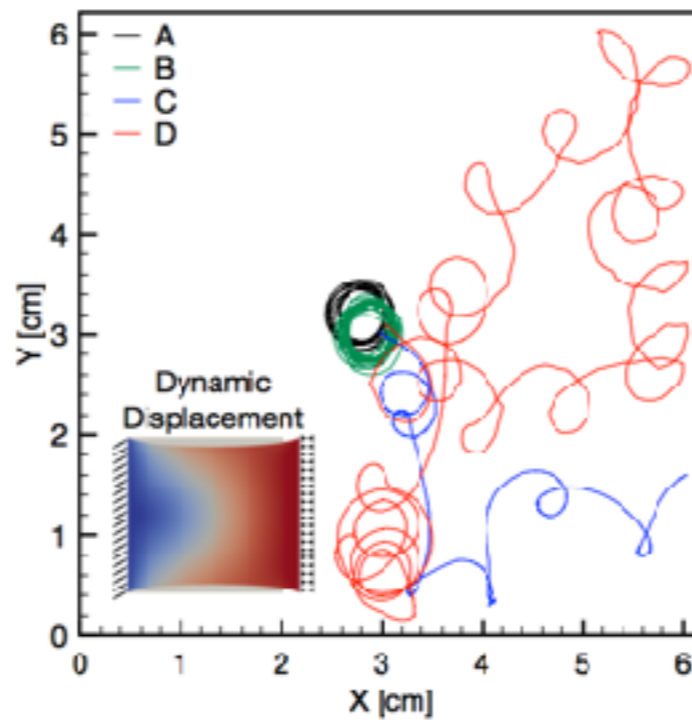


EM - SAD & SAC

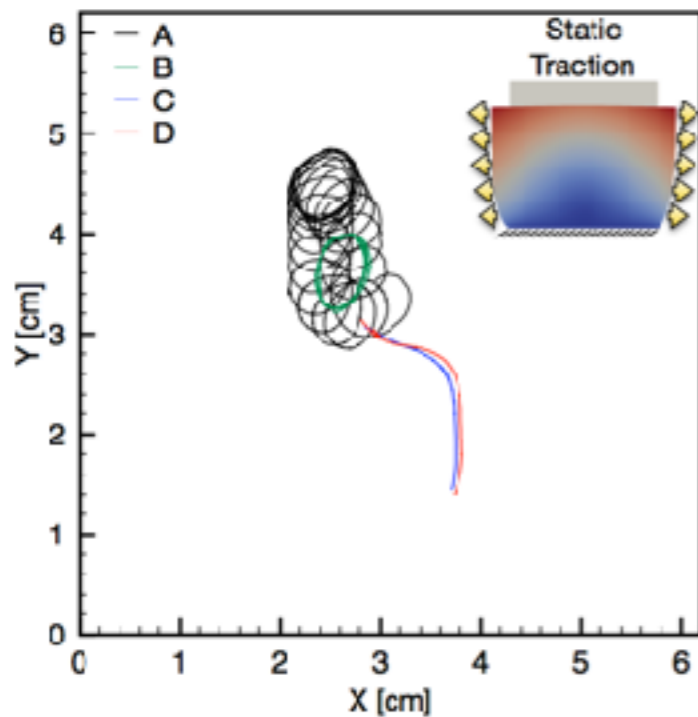
Spiral Tip Meandering



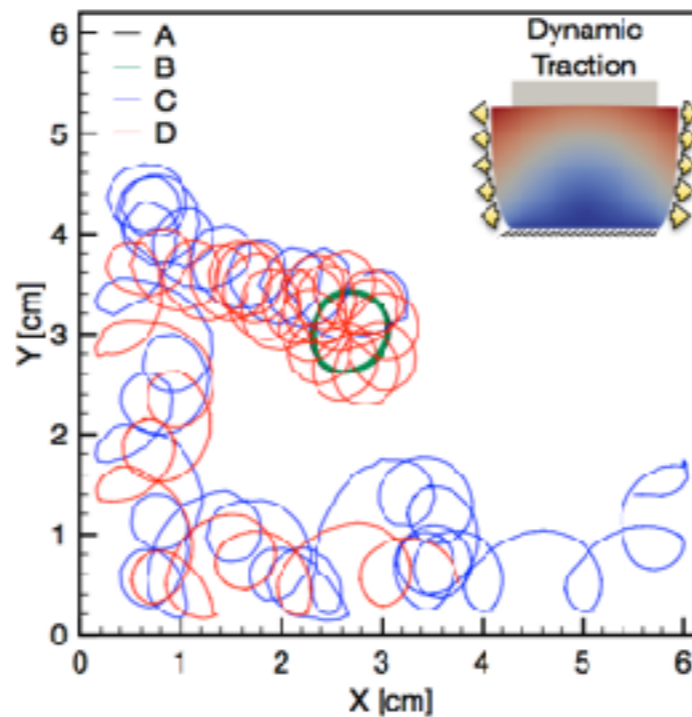
(a) Static Displacement



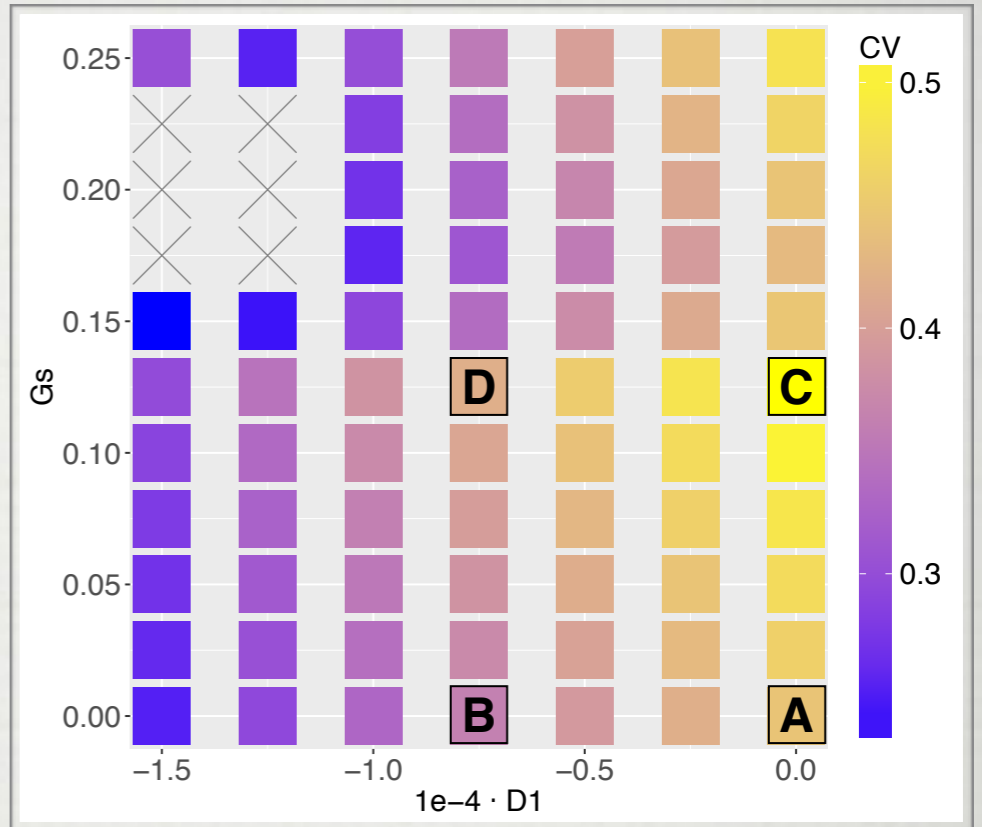
(b) Dynamic Displacement



(c) Static Traction



(d) Dynamic Traction



SAD stabilizes meandering

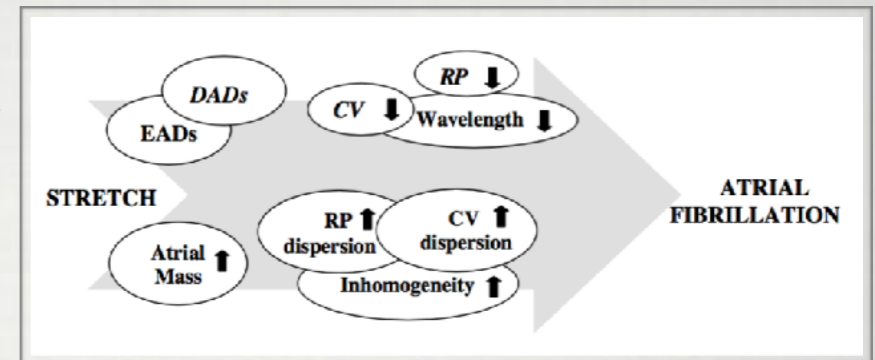
SAC generates irregular behavior

Static/Dynamic & Displacement/Traction BCs conduct to different scenarios

EM - SAD & SAC ... ?

✓ **GENERALIZED ELECTRO-MECHANICAL FRAMEWORK WITH MULTIPLE MEF EFFECTS**

Complex Problem



SAD

SAC

$$\frac{\partial V}{\partial t} = \frac{\partial}{\partial x_i} d_{ij}(\sigma_{ij}) \frac{\partial V}{\partial x_j} + I_{\text{ion}}(V, r) + I_{\text{SAC}}(\lambda)$$

Homogenized

Modify the "Diffusion"

Modify the "Reaction"

Local

Wave break & Arrhythmias Onset

Anisotropy & Conduction Block

Extra Activations & Hyperpolarization

Ectopic beats & Arrhythmias Onset

LIMITATIONS...

- ☑ GENERALIZED ELECTRO-MECHANICAL FRAMEWORK
- ☑ STRESS-ASSISTED DIFFUSION
- ☑ SAC VS. SAD
- ☑ ISOTROPIC HYPERELASTIC MATERIALS CAN UNDERGO ANISOTROPIC REACTION-DIFFUSION DYNAMICS
- ☑ MINIMAL PROOF OF CONCEPT

OPEN QUESTIONS...

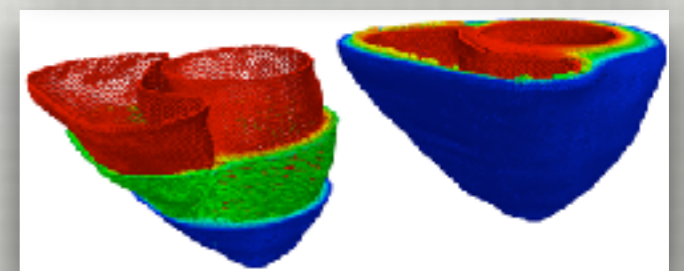
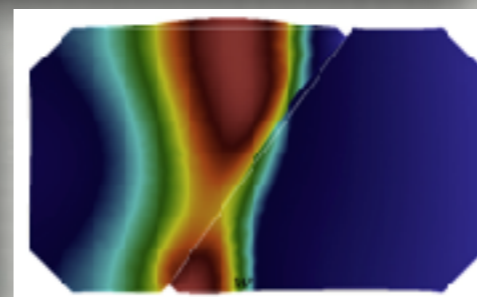
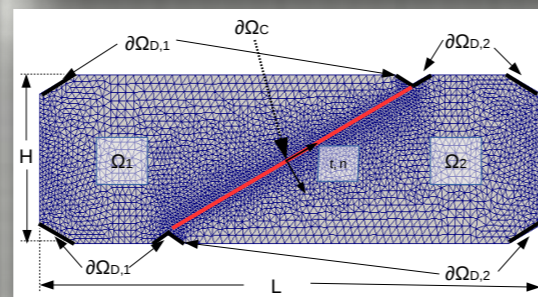
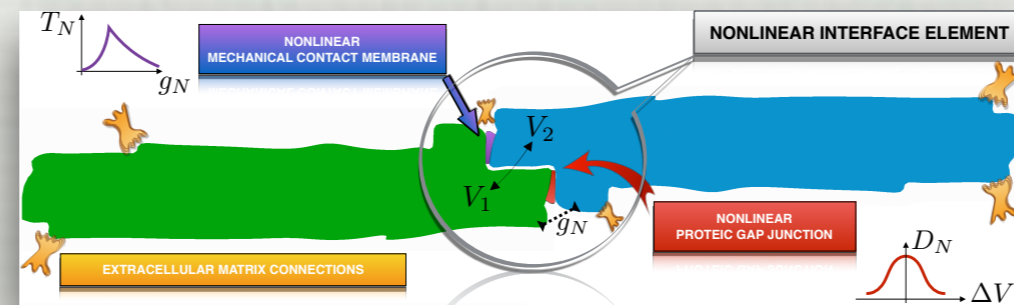
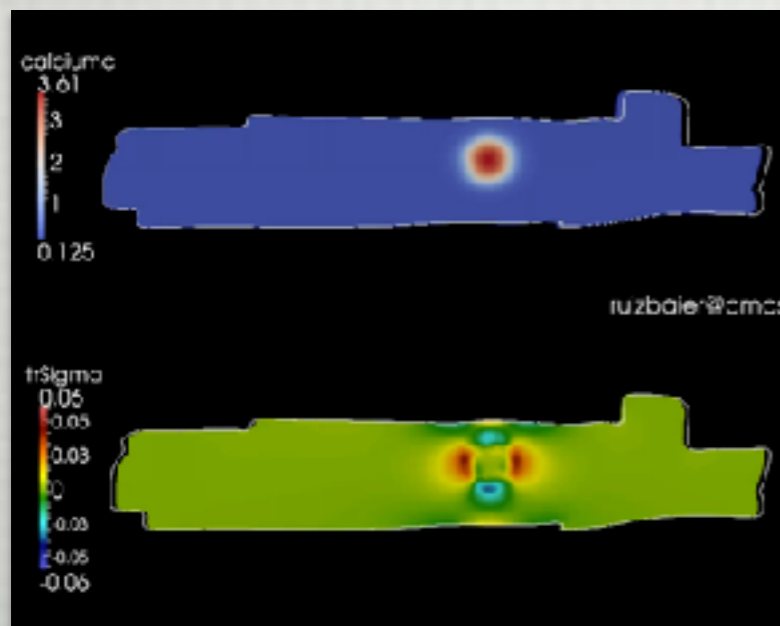
□ MULTISCALE MODELING

□ FROM CELL-CELL COUPLING TO TISSUE SCALE
(EMERGENT PHENOMENA)

□ PHYSIOLOGICAL MODELING (ROLE OF CALCIUM)

□ MODEL VALIDATION: SAC & SAD LIMITS

□ ...



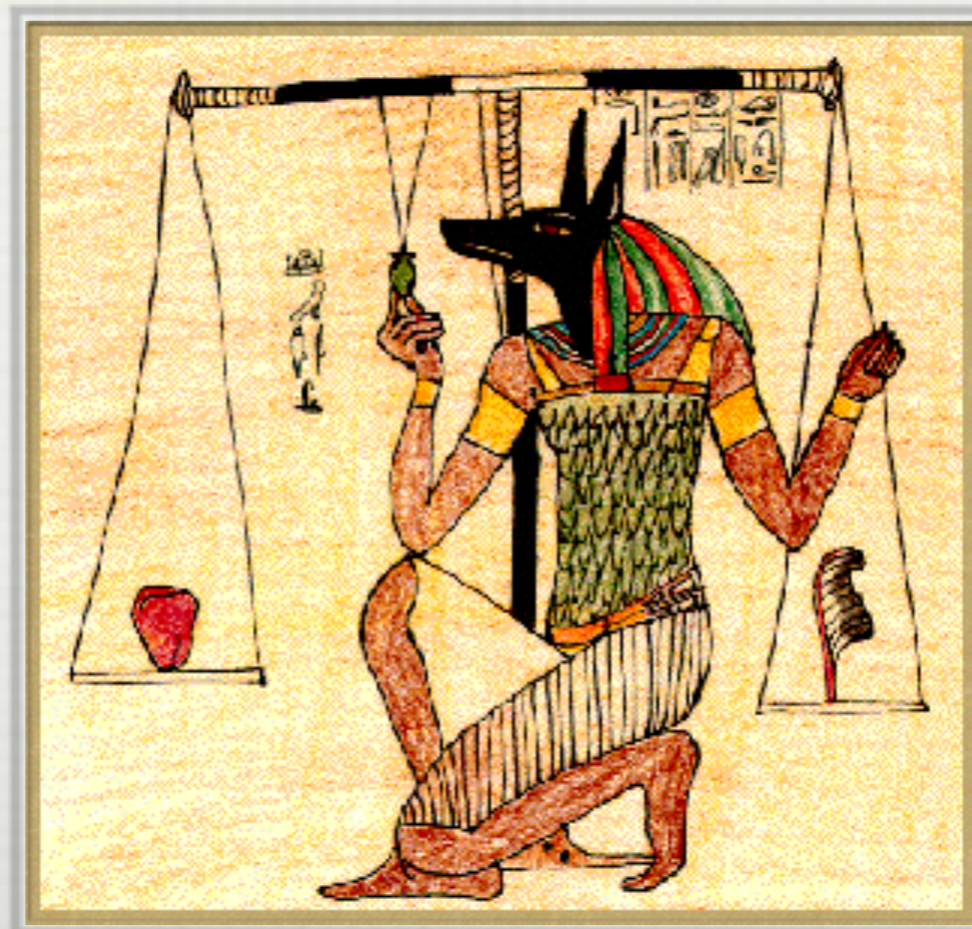
A. T. WINFREE, 2000

PRINCETON GRADUATE SCHOOL CENTENNIAL



ANUBIS IN THE DESERT EST OF EGYPT - 3RD MILLENNIUM BCE

“Weighing the heart against the feather of Truth”



we are still trying...

EM-SAD: INDUCED ANISOTROPY

Tensorial Glyph representation with Pressure levels

$$D_0 C_{IJ}^{-1}$$

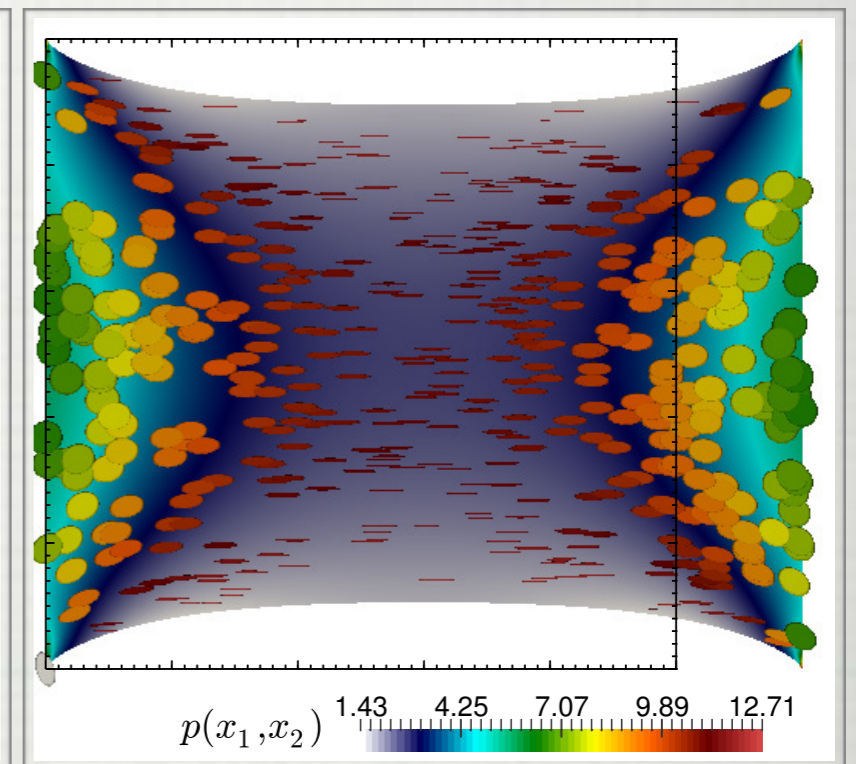
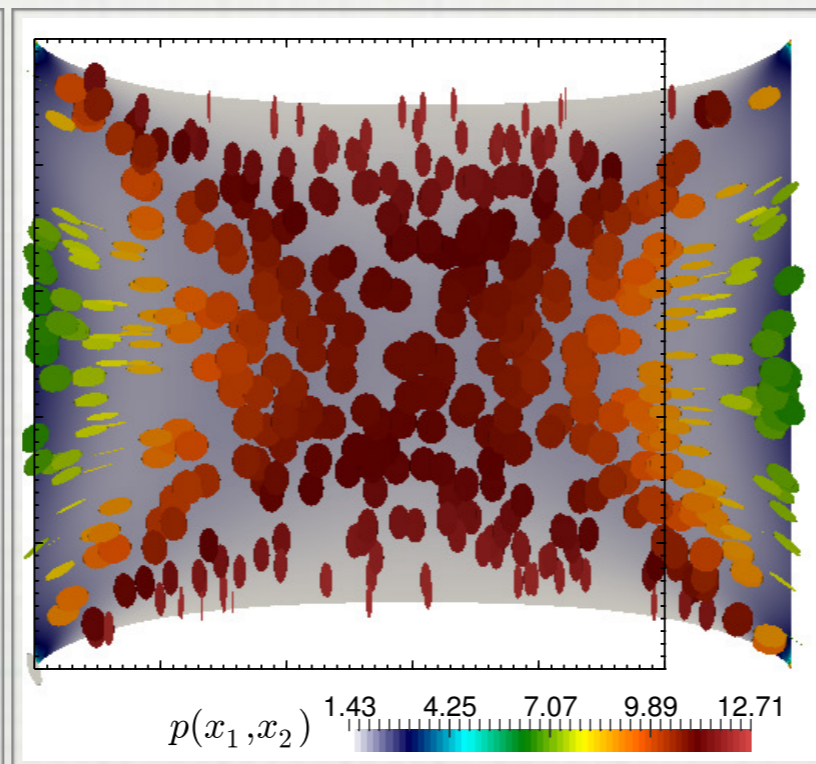
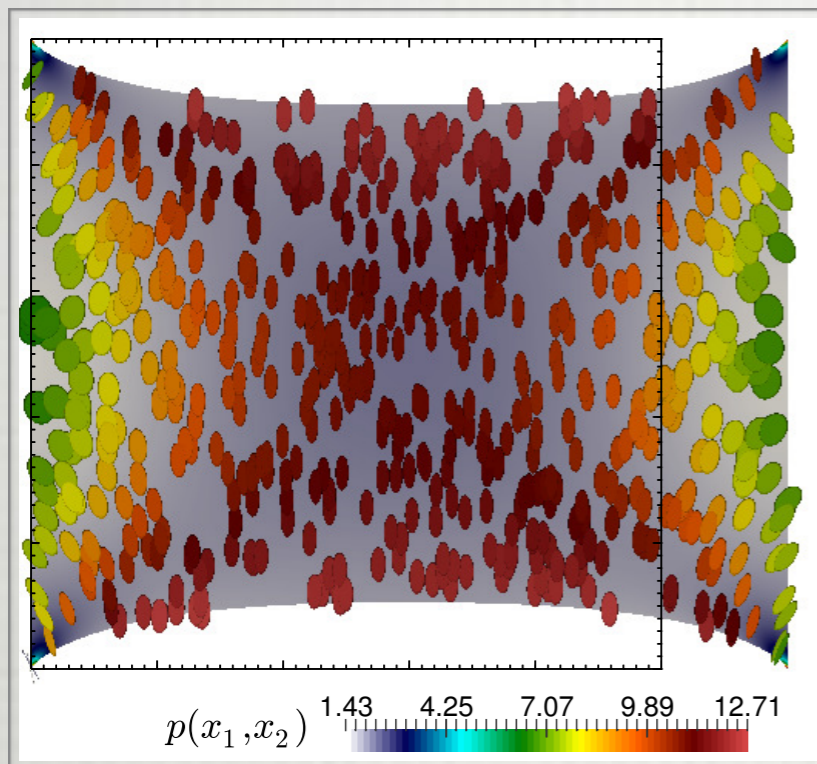
$$D_{IJ}$$

$$S_{IJ}$$

Isotropic Diffusion

Stress-Assisted Diffusion

Stress tensor



Eigenvalue/Eigenvector rendering of the a Second Order Tensor in a two-dimensional view

Solve local eigenvalue problems using the tensor at hand and display an ellipsoid whose shape and size depend on the local eigendirection

$$\sigma_{ij} = 2c_1 b_{ij} - 2c_2 b_{ij}^{-1} - p\delta_{ij} + T_a \delta_{ij}$$

$$d_{ij}(\sigma_{ij}) = D_0 (\delta_{ij} + D_1 \sigma_{ij} + D_2 \sigma_{ik} \sigma_{kj})$$